

Debt and government spending in ambiguous times*

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Abstract

Historically-high levels of public debt have recently raised numerous concerns regarding fiscal policy. In particular, should governments tax more, spend less, or do both to manage high public debt? Also, given the uncertainty about the strength of the economic recovery, what is the suitable timeframe over which to balance government budgets? To shed light on this debate, we take a dynamic optimal taxation approach and analyze the implications of our model for government expenditures, taxes, and debt. To capture the importance of uncertainty, we do so in a context of *ambiguity*, where government and households share doubts about the probability model of technology shocks.

Our contribution is threefold. First, we endogenize government expenditures and study their optimal provision and mix with distortionary taxes in an economy without ambiguity. Second, we show that uncertainty over the distribution of shocks generates procyclical or countercyclical allocation of distortions, whereas without ambiguity distortions would be acyclical. Third, we provide a quantitative evaluation of the fiscal plan with ambiguity and analyze its short- and long-run properties. Preliminary results suggest that optimal policy would, (1) delay distortions for the future, or, (2) accelerate distortions and converge to a balanced budget in the long-run, depending on the size of the intertemporal elasticity of substitution.

Keywords: Endogenous government expenditures, distortionary taxes, balanced budget, martingale, ambiguity aversion, multiplier preferences.

JEL classification: D80; E62; H21; H63.

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1 Introduction

The financial crisis of 2007-09, with the associated fiscal stimulus and the subsequent historically-high levels of public debt, has triggered heated debates about debt sustainability and the appropriateness of fiscal policy measures. In particular, should distortionary taxes increase, or government spending decrease, or both, to manage high levels of public debt?

The appropriate timing of fiscal policy has also appeared particularly controversial in the context of uncertainty about the strength of the recovery of the economy. This debate echoes the growing consensus that uncertainty, or else *ambiguity*, is of importance for fiscal policy. Technological uncertainty and fiscal uncertainty, with their effects on asset prices and macroeconomic quantities, have non-trivial implications on interest rates and surpluses, raising concerns for the long-term prospects of the economy.¹

To shed light on the effects of ambiguity on the design of fiscal policy, we take a dynamic optimal taxation approach and analyze the implications of ambiguity on the *joint* determination of government expenditures and taxes. There are several open questions that we want to answer. We are interested in investigating the size and the cyclical properties of government expenditures, taxes and surpluses, and the extent (if any) to which debt has to be reduced. Furthermore, we are interested in the optimal long-run behavior of these fiscal tools under ambiguity.

We build an economy without capital and complete markets as in Lucas and Stokey (1983). Government financing occurs through distortionary taxation of labor income and through state-contingent debt. We do not treat government expenditures as *exogenous* “waste” though, but allow them to provide utility to the representative household. This provides a simple and tractable way to generate an optimal policy problem where government expenditures are endogenous, and allows the study of the optimal mix of government expenditures and taxes.²

As far as ambiguity is concerned, we take the stance that there are doubts about the probability model of technology shocks, which are the only source of uncertainty in the economy. Any type of “fiscal” uncertainty will be an outcome of these doubts. We use the multiplier preferences of Hansen and Sargent (2001) to capture our household’s aversion towards this ambiguity.

As a first step, we provide an in-depth analysis of optimal policy in the case of an expected-utility household that has full confidence in the probability model. Somewhat surprisingly, there seems to be little work on this problem in the literature.

First-best optimality commands that the marginal rate of substitution of government for private consumption is equal to the marginal rate of transformation, which is unity. Thus, one additional unit of government consumption should provide the same marginal utility as an additional unit

¹See Epstein and Schneider (2010) for a survey of the implications of ambiguity aversion for asset prices. Ilut and Schneider (2011) focus on the business cycle implications, whereas Croce et al. (2012) study the effects of technological and fiscal uncertainty on long-run growth.

²In principle, we could allow government expenditures to affect the productive capacity of the economy as in the endogenous growth literature, but we abstract here from these type of channels that involve externalities.

of private consumption. We define a new wedge at the second-best, which we call the *public wedge*, that captures the deviation of the marginal rate of substitution from unity. A positive public wedge corresponds to the case where marginal utility of government consumption is larger than the marginal utility of private consumption. We will interpret this as a “tax” on government consumption relative to the first-best. Similarly, a negative public wedge corresponds to a “subsidy” of government consumption relative to the first-best.

We derive several lessons from the analysis of the second-best problem without ambiguity. Our first finding is that the optimal allocation, public wedge and tax rate are history-independent, reflecting how the smoothing motives in the Lucas and Stokey setup extend to environments where government expenditures provide utility. Using a typical homothetic specification for the utility of private and government consumption allows us to recast the analysis in terms of ratios. We find that the share of government consumption in output is *constant*. Thus, government expenditures become *procyclical* whereas their share becomes *acyclical*. Assuming in addition a constant Frisch elasticity of labor supply delivers a *constant* tax rate, extending therefore the procyclicality result to surpluses, and the acyclicity result to the share of surplus in output.

At a superficial level, one might think that optimal policy should prescribe a smaller ratio of government expenditures relative to the first-best, and consequently a positive public wedge, due to the cost that distortionary taxation imposes. We find though that the sign of the public wedge depends on the *substitutability* or *complementarity* of government and private consumption. When government and private consumption are substitutes, the public wedge is positive, leading to a ratio of government consumption in output that is smaller than in the first-best. In contrast, when government and private consumption are complements, the public wedge is negative and the government share in output is larger than in the first-best. Larger distortions at the government consumption margin translate to a smaller or larger government share relative to the first-best in the substitutes and complements case respectively.

There are stark differences when we turn to the analysis of the optimal provision of government expenditures in an environment of ambiguity. The household’s ambiguity aversion leads to the formation of *pessimistic* beliefs. A cautious household assigns high probability on low utility events, which are typically associated with adverse technological shocks.

The ambiguity premia that are generated substantially alter the optimal policy problem. A planner that maximizes the utility of the representative household is actively managing the household’s *endogenous* pessimistic expectations, and through them, asset prices. His objective is to properly affect the endogenous beliefs of the household in order to relax the government budget constraint by increasing the present value of surpluses. This allows him to minimize the welfare cost of taxation.

We find that the allocation of welfare distortions is *time-varying* and *persistent*, in contrast to the *constant* allocation of distortions when there is full confidence in the probability model of

technology shocks. Therefore, *both* the public wedge *and* the tax rate become time-varying and persistent. This translates to a share of surplus in output that is not constant anymore for our homothetic example with constant Frisch elasticity.

We have to understand the interplay of marginal utility of private consumption with the pessimistic household's beliefs, since they both determine state-contingent returns with ambiguity aversion. The key mechanism is that debt in marginal utility units becomes *less* costly when there is ambiguity aversion: histories where debt in marginal utility units is high decrease utility, making therefore the cautious household assign higher probability on them. As a result, prices of state-contingent claims increase, decreasing the return on debt and making it more attractive. This creates the incentive for the planner to shift welfare distortions towards histories with high debt in marginal utility units, and away from histories where debt in marginal utility units is low.

This price manipulation mechanism through the endogenous cautious beliefs is used only whenever the planner finds it optimal to differentiate his debt position in marginal utility units across shocks, or whenever he actually issues debt in the first place. Marginal utility is controlled for our parametric example by the elasticity of intertemporal substitution (IES). At the knife-edge case of a unitary IES, increases in surpluses are exactly offset by decreases in marginal utility, leading therefore to a constant surplus in marginal utility units and consequently a constant debt in marginal utility units across shocks. As a result, the planner does not find it optimal to differentiate the allocation of welfare distortions across states and dates, and chooses the same allocation, public wedge and tax rate as without doubts about the model. The same thing happens when the government runs a balanced budget, which happens for our utility function whenever there is zero initial debt.

Aside from these two knife-edge cases, the allocation of distortions varies across shocks. The size of IES controls the procyclicality or countercyclicality of the surplus in marginal utility units. When the IES is larger than unity, so when marginal utility is not very responsive to changes in consumption, surpluses in marginal utility units are procyclical. As a result, there is a procyclical allocation of distortions. The tax rate increases for favorable technology shocks, whereas the government share decreases (in the case of substitutes) or increases (in the case of complements). In contrast, when the IES is smaller than unity, there is high sensitivity of marginal utility to consumption, which leads to countercyclical distortions. The tax rate becomes countercyclical, whereas the government share is becoming procyclical for the substitutes case and countercyclical for the complements case. So whenever we have *both* intertemporal *and* intratemporal (between government and private consumption) substitutability or complementarity, the share of government expenditures in output becomes countercyclical.

We are in the process of evaluating the quantitative implications of ambiguity aversion numerically. Our optimal policy is driven by a variable that exhibits a martingale-like behavior with respect to the pessimistic beliefs of the household, which, besides inducing persistence, raises questions about its long-run behavior. Preliminary results suggest that for our example utility function and starting from a positive initial debt, we will have a *back-loading* of distortions when the IES is

larger than unity, so there will be a positive drift in the tax rate and a negative or positive drift in the government share in the substitutes and complements case respectively. In contrast, when the IES is smaller than unity, we expect distortions to be *front-loaded*. We conjecture that in the long-run optimal policy will converge to a balanced budget, which involves zero debt.

To summarize, our contribution is threefold: First, we endogenize government expenditures and provide a thorough analysis of their optimal provision and their optimal mix with distortionary taxes in an economy with full confidence in the probability model of technology shocks. Second, and most importantly, we analyze the optimal provision of government expenditures in an economy with doubts about the probability model of technology shocks.³ Third, we provide a quantitative evaluation of the optimal plan and analyze the short- and long-run properties.

2 Economy

Time is discrete and the horizon is infinite. We build an economy without capital and complete markets as in Lucas and Stokey (1983). Government expenditures are endogenous and provide utility to the representative household. Let s_t denote the technology shock at time t and let $s^t \equiv (s_0, s_1, \dots, s_t)$ denote the partial history of shocks up to period t with probability $\pi_t(s^t)$. There is no uncertainty at $t = 0$, so $\pi_0(s_0) \equiv 1$. The operator E denotes expectation with respect to π throughout the paper. We are not making any Markovian assumption for the technology shocks at this stage. The resource constraint of the economy reads

$$c_t(s^t) + g_t(s^t) = s_t h_t(s^t), \quad (1)$$

where $c_t(s^t)$ private consumption, $g_t(s^t)$ government consumption and $h_t(s^t)$ labor. The notation indicates the measurability of these functions with respect to the partial history s^t . Total endowment of time is normalized to unity, so leisure is $l_t(s^t) = 1 - h_t(s^t)$.

Household. The representative household derives utility from stochastic streams of private consumption, leisure and public consumption. Its preferences are

$$\sum_{t=0} \beta^t \sum_{s^t} \pi_t(s^t) U(c_t(s^t), 1 - h_t(s^t), g_t(s^t)) \quad (2)$$

³Fears of model misspecification feature also in Karantounias (2013a). In that study, government expenditures were treated as *exogenous*, not allowing therefore the analysis of the questions we are interested in. Furthermore, Karantounias analyzed a setup where the policymaker has full confidence in the model, whereas the household does not.

where U is monotonic and concave. We will explore specifications of U later. The household works at the pre-tax wage $w_t(s^t)$, pays proportional taxes on its labor income with rate $\tau_t(s^t)$ and trades in complete asset markets. Let $b_{t+1}(s^{t+1})$ denote the holdings of an Arrow security that promises one unit of consumption if the state of the world is s_{t+1} next period and zero otherwise. This security trades at the price of $p_t(s_{t+1}, s^t)$ in units of consumption at history s^t .

Given prices (p, w) and government policies (τ, g) , the household chooses $\{c_t(s^t), h_t(s^t), b_{t+1}(s^{t+1})\}_{t,s^t}$ to maximize (2) subject to

$$c_t(s^t) + \sum_{s_{t+1}} p_t(s_{t+1}, s^t) b_{t+1}(s^{t+1}) \leq (1 - \tau_t(s^t)) w_t(s^t) h_t(s^t) + b_t(s^t), \quad (3)$$

and the constraints $c_t(s^t) \geq 0, h_t(s^t) \in [0, 1]$, where b_0 is given. The household is also subject to the no-Ponzi-game condition

$$\lim_{t \rightarrow \infty} \sum_{s^{t+1}} q_{t+1}(s^{t+1}) b_{t+1}(s^{t+1}) \geq 0 \quad (4)$$

where $q_t(s^t) \equiv \prod_{j=0}^{t-1} p_j(s_{j+1}, s^j)$ denotes the price of an Arrow-Debreu contract at $t = 0$ with the normalization $q_0 \equiv 1$.

There is a representative competitive firm that operates the linear technology. The government is collecting tax revenues to finance government expenditures and trades with the household in Arrow securities. The government budget constraint reads

$$b_t(s^t) = \tau_t(s^t) w_t(s^t) h_t(s^t) - g_t(s^t) + \sum_{s_{t+1}} p_t(s_{t+1}, s^t) b_{t+1}(s^{t+1}). \quad (5)$$

Competitive equilibrium. A competitive equilibrium is a collection of prices (p, w) , a private consumption-labor allocation (c, h) , Arrow securities holdings b and government policies (g, τ) such that 1) given (p, w) and (g, τ) , (c, h, b) solves the households problem, 2) given w firms maximize profits, 3) prices (p, w) are such so that markets clear, i.e. the resource constraint (1) holds.⁴

2.1 Household's optimality conditions

Profit maximization of the competitive firm leads to a wage that is equal to the marginal product of labor, $w_t = s_t$. Turning to the household's problem, its labor supply decision is characterized by

⁴Note that we have not used a separate notation b_t^g for the government's asset holdings but have instead used the fact that in equilibrium $b_t^g = -b_t$. Using the resource constraint and the household's budget constraint delivers in equilibrium the government budget constraint, so it is redundant to include it in the definition.

$$\frac{U_l(c_t, 1 - h_t, g_t)}{U_c(c_t, 1 - h_t, g_t)} = (1 - \tau_t)w_t, \quad (6)$$

which equates the marginal rate of substitution of consumption and leisure with the after-tax wage. The optimal decision with respect to Arrow securities is characterized by

$$p_t(s_{t+1}, s^t) = \beta \pi_{t+1}(s_{t+1}, s^t) \frac{U_c(s^{t+1})}{U_c(s^t)}, \quad (7)$$

which equates the marginal rate of substitution of consumption at s^t and consumption at s^{t+1} with the price of an Arrow security. The respective price of an Arrow-Debreu contract at $t = 0$ is $q_t(s^t) = \beta^t \pi_t(s^t) \frac{U_c(s^t)}{U_c(s_0)}$. Note furthermore that the asymptotic condition (4) holds in equilibrium with equality, which leads to the exhaustion of the household's unique intertemporal budget constraint.

3 Ramsey problem with full confidence in the model

In the competitive equilibrium the household takes government expenditures as exogenously given. Consider now the problem of the Ramsey planner that chooses optimally the level of government expenditures, the level of distortionary taxes and the issuance of state-contingent debt in order to maximize the utility of the representative household at the competitive equilibrium. Before we proceed to this problem, it is instructive to understand the first-best allocation (c^*, h^*, g^*) , i.e. the allocation that could be sustained as a competitive equilibrium if lump-sum taxes were available.

3.1 First-best problem

The first-best problem is to choose $c_t, g_t \geq 0, h_t \in [0, 1]$ in order to maximize the utility of the representative household (2) subject to the resource constraint of the economy (1). The first-best allocation (c^*, h^*, g^*) is characterized by the resource constraint and two optimality conditions:

$$\frac{U_l(c_t, 1 - h_t, g_t)}{U_c(c_t, 1 - h_t, g_t)} = s_t, \quad (8)$$

which equates the marginal rate of substitution of leisure for consumption with the marginal rate of transformation, which is equal to the technology shock, and

$$\frac{U_g(c_t, 1 - h_t, g_t)}{U_c(c_t, 1 - h_t, g_t)} = 1, \quad (9)$$

which equates the marginal rate of substitution of government for private consumption with the respective marginal rate of transformation which is unity. Thus, at the first-best, the optimal provision of government expenditures requires that they provide the same marginal utility as private consumption.

3.2 Second-best problem

Consider now the Ramsey problem. We follow the primal approach of Lucas and Stokey (1983) by expressing prices and tax rates in terms of allocations and have the Ramsey planner choose (c, h, g) subject to the resource constraint and the associated implementability constraints that guarantee that the second-best allocation can be supported by a competitive equilibrium. Define first

$$\Omega(c, h, g) \equiv U_c(c, 1 - h, g)c - U_l(c, 1 - h, g)h. \quad (10)$$

The function Ω stands for consumption net of after-tax labor income in marginal utility of consumption units, after expressing the after-tax wage in terms of allocations through (6). Note that Ω is also equal to the primary government surplus in marginal utility units. Using the household's (or equivalently the government's) intertemporal budget constraint and substituting intertemporal marginal rates of substitution for equilibrium prices and intratemporal marginal rates of substitution for after-tax wages delivers the familiar implementability constraint

$$\sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi_t(s^t) \Omega(c_t(s^t), h_t(s^t), g_t(s^t)) = U_c(c_0, 1 - h_0, g_0) b_0. \quad (11)$$

Definition 1. *The Ramsey problem is to choose at $t = 0$ $c_t, g_t \geq 0, h_t \in [0, 1]$ in order to maximize (2) subject to the implementability constraint (11) and the resource constraint (1), where the initial shock s_0 and initial debt b_0 are given.*

3.3 Analysis

Let Φ denote the multiplier on the unique implementability constraint. We will call Φ the excess burden of taxation throughout the paper. At the first-best we have $\Phi = 0$. Let $\beta^t \pi_t(s^t) \lambda_t(s^t)$ denote the multipliers on the resource constraint at each t, s^t . First-order necessary conditions for $t \geq 1$

$$c_t(s^t) : \quad U_c(s^t) + \Phi \Omega_c(s^t) = \lambda_t(s^t) \quad (12)$$

$$h_t(s^t) : \quad -U_l(s^t) + \Phi \Omega_h(s^t) = -\lambda_t(s^t) s_t \quad (13)$$

$$g_t(s^t) : \quad U_g(s^t) + \Phi \Omega_g(s^t) = \lambda_t(s^t) \quad (14)$$

$\Omega_i, i = c, h, g$ stands for the respective partial derivative of the function Ω . The presence of initial debt modifies the first-order conditions for $t = 0$. In particular, we have

$$c_0 : \quad U_{c0} + \Phi(\Omega_{c0} - U_{cc0}b_0) = \lambda_0 \quad (15)$$

$$h_0 : \quad -U_{l0} + \Phi(\Omega_{h0} + U_{cl0}b_0) = -\lambda_0 s_0 \quad (16)$$

$$g_0 : \quad U_{g0} + \Phi(\Omega_{g0} - U_{cg0}b_0) = \lambda_0 \quad (17)$$

Eliminate now the multiplier λ_t from (12), (13) and (14) to get

$$\frac{U_l - \Phi\Omega_h}{U_c + \Phi\Omega_c} = s_t \quad (18)$$

$$\frac{U_g + \Phi\Omega_g}{U_c + \Phi\Omega_c} = 1 \quad (19)$$

These two expressions capture the *wedges* at the labor supply margin and government consumption margin and contrast to (8) and (9) of the first-best allocation (which correspond to $\Phi = 0$). Before we proceed to an analysis of the wedges, it is useful to note that by using (18) and (19) together with the resource constraint (1) allows us to solve for the optimal second-best allocation (c, h, g) in terms of the current technology shock s_t and the multiplier Φ , $c_t = c(s_t, \Phi)$, $h_t = h(s_t, \Phi)$, $g_t = g(s_t, \Phi)$, $t \geq 1$. Thus,

Proposition 1. *The optimal allocation (c, h, g) is history-independent.*

This result is similar to the result of Lucas and Stokey (1983), who treat the case of *exogenous* government expenditures. Using (15)-(17) we can find the respective wedges for $t = 0$:

$$\frac{U_{l0} - \Phi(\Omega_{h0} + U_{cl0}b_0)}{U_{c0} + \Phi(\Omega_{c0} - U_{cc0}b_0)} = s_0 \quad (20)$$

$$\frac{U_{g0} + \Phi(\Omega_{g0} - U_{cg0}b_0)}{U_{c0} + \Phi(\Omega_{c0} - U_{cc0}b_0)} = 1 \quad (21)$$

which lead to an initial allocation that depends on (s_0, b_0, Φ) . The value of the multiplier Φ is such so that the implementability constraint holds, i.e. the present value of government surpluses is equal to initial debt.

3.4 Public wedge

Define

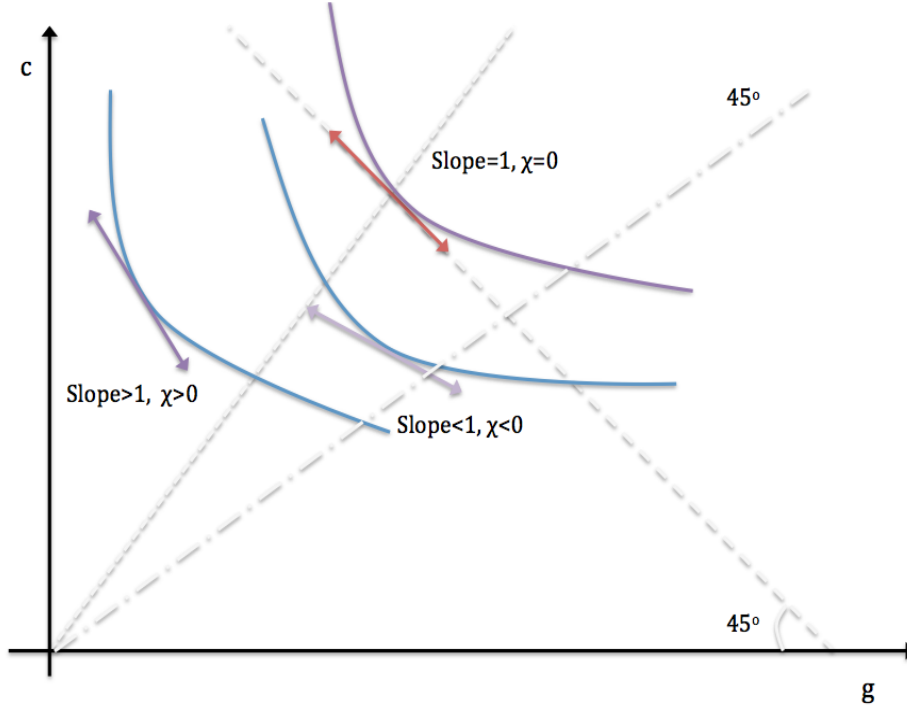


Figure 1: Illustration of a positive and negative public wedge. The zero public wedge corresponds to the first-best.

$$\chi \equiv \frac{U_g}{U_c} - 1. \quad (22)$$

We will call χ the *public wedge*, since it captures the deviation of the marginal rate of substitution of government consumption for private consumption from its first-best value that is unity. In particular, if $\chi > 0$, the marginal utility of government consumption is larger than the marginal utility of private consumption, $U_g/U_c > 1$, which is interpreted as a “tax” on government consumption versus private consumption relative to the first-best. For $\chi < 0$ we have $U_g/U_c < 1$, which is interpreted as a “subsidy” of government consumption versus private consumption.

Note that since the public wedge and the labor tax $\tau = 1 - U_l/(U_c s)$ are functions of the optimal allocation, they also inherit the history-independence property, $\chi_t = \chi(s_t, \Phi)$, $\tau_t = \tau(s_t, \Phi)$. We will express the optimal χ and τ as functions of *elasticities* that capture curvature properties of the period utility function U , and the excess burden of taxation Φ .⁵

Proposition 2. 1. *The optimal public wedge for $t \geq 1$ is*

⁵These formulas are in the spirit of the static analysis with exogenous government expenditures of Atkinson and Stiglitz (1972).

$$\chi = \frac{\Phi(1 - \epsilon_{cc} - \epsilon_{ch} - \epsilon_{gc} - \epsilon_{gh})}{1 + \Phi(\epsilon_{gc} + \epsilon_{gh})},$$

where $\epsilon_{cc} \equiv -U_{cc}c/U_c$, $\epsilon_{ch} \equiv U_{cl}h/U_c$, the own and cross elasticity (with respect to labor) of the marginal utility of private consumption, and $\epsilon_{gc} \equiv U_{gc}c/U_g$, $\epsilon_{gh} \equiv -U_{gl}h/U_g$ the cross elasticities of the marginal utility of government consumption with respect to private consumption and labor.

2. The optimal labor tax for $t \geq 1$ is

$$\tau = \frac{\Phi(\epsilon_{cc} + \epsilon_{ch} + \epsilon_{hh} + \epsilon_{hc})}{1 + \Phi(1 + \epsilon_{hh} + \epsilon_{hc})}$$

where $\epsilon_{hh} \equiv -U_{ll}h/U_l$, $\epsilon_{hc} \equiv U_{cl}c/U_l$, the own and cross elasticity (with respect to private consumption) of the marginal disutility of labor.

3. The formulas hold also for $t = 0$ when initial debt is zero, $b_0 = 0$. Otherwise, the corresponding wedges at $t = 0$ are

$$\begin{aligned}\chi_0 &= \frac{\Phi(1 - \epsilon_{cc} - \epsilon_{ch} - \epsilon_{gc} - \epsilon_{gh} + (\epsilon_{cc} + \epsilon_{gc})c_0^{-1}b_0)}{1 + \Phi(\epsilon_{gc} + \epsilon_{gh} - \epsilon_{gc}c_0^{-1}b_0)} \\ \tau_0 &= \frac{\Phi(\epsilon_{cc} + \epsilon_{ch} + \epsilon_{hh} + \epsilon_{hc} - (\epsilon_{cc} + \epsilon_{hc})c_0^{-1}b_0)}{1 + \Phi(1 + \epsilon_{hh} + \epsilon_{hc} - \epsilon_{hc}c_0^{-1}b_0)}\end{aligned}$$

4. The denominators in all expressions are positive, so the sign of the public wedge and the labor tax depends on the sign of the numerators.

Proof. Express (19) as $\frac{U_g}{U_c} \cdot \frac{1 + \Phi\Omega_g/U_g}{1 + \Phi\Omega_c/U_c} = 1$ and use the definition of the public wedge (22) to get $\chi = \frac{\Phi(\Omega_c/U_c - \Omega_g/U_g)}{1 + \Phi\Omega_g/U_g}$. Similarly, express the optimal wedge in labor supply (18) as $\frac{U_l}{U_c} \cdot \frac{1 - \Phi\Omega_g/U_l}{1 + \Phi\Omega_c/U_c} = s$, which can be written in terms of the labor tax as $\tau = \frac{-\Phi(\Omega_c/U_c + \Omega_h/U_l)}{1 - \Phi\Omega_h/U_l}$, since $\tau = 1 - U_l/(U_c s)$. The partial derivatives of Ω scaled by the respective marginal utilities take the form $\Omega_c/U_c = 1 - \epsilon_{cc} - \epsilon_{ch}$, $\Omega_h/U_l = -1 - \epsilon_{hh} - \epsilon_{hc}$ and $\Omega_g/U_g = \epsilon_{gc} + \epsilon_{gh}$. Use these expressions to finally get the optimal public wedge and labor tax stated in the proposition. Use (20) and (21) and follow the same steps for $t = 0$. For the signs of the denominators, use (13) and (14) to get $1 + \Phi(1 + \epsilon_{hh} + \epsilon_{hc}) = \lambda s/U_l > 0$ and $1 + \Phi(\epsilon_{gc} + \epsilon_{gh}) = \lambda/U_g > 0$ since $\lambda > 0$. Similarly, use (16) and (17) for $t = 0$. $\square$

The curvature properties of the utility function show up in the determination of the wedges because they capture how the surplus in marginal utility units Ω , which is the determinant of the

intertemporal budget constraint, is affected by the choices of c , h and g . The proposition shows that when elasticities are constant across states and dates, the public wedge and the labor tax become constant since they depend only on the constant excess burden of taxation. In the next section we will consider a utility function that delivers these results.

3.5 Example

Consider the period utility function

$$U = \frac{u^{1-\rho} - 1}{1-\rho} + v(l), \quad (23)$$

where u stands for a composite good of private and government consumption. Assume a constant elasticity of substitution (CES) aggregator u

$$u = [(1-\alpha)c^{1-\psi} + \alpha g^{1-\psi}]^{\frac{1}{1-\psi}}, \alpha \in (0, 1).$$

We will be silent for now about the subutility of leisure $v(l)$. The specification of the period utility function allows to separate intertemporal elasticity of substitution, which is controlled by $1/\rho$ and the intratemporal elasticity of substitution between private and government consumption, which is controlled by $1/\psi$. This will be important for our analysis later. We will call c and g *substitutes* when $\psi < 1$ and *complements* when $\psi > 1$. In the same vein, we are going to talk about *intertemporal* substitutability in terms of composite consumption when $\rho < 1$ and intertemporal complementarity when $\rho > 1$.

3.5.1 Public wedge

The homothetic specification in private and government consumption allows us to perform our analysis in terms of *ratios*. The marginal rate of substitution of government for private consumption is $U_g/U_c = A(g/c)^{-\psi}$, where $A \equiv \alpha/(1-\alpha)$. Let κ denote the ratio of government to private consumption, $\kappa \equiv g/c$. At the first best we have $\kappa^* = A^{1/\psi}$. At the second best, we have $\kappa < \kappa^*$ when there is a positive public wedge (“tax”) and $\kappa > \kappa^*$ when there is a negative public wedge (“subsidy”).

For the utility function in hand we have

$$\epsilon_{cc} = \lambda_c \rho + (1 - \lambda_c) \psi, \quad \epsilon_{gc} = (\psi - \rho) \lambda_c,$$

where $\lambda_c \equiv (1-\alpha)(\frac{c}{u})^{1-\psi} \in (0, 1)$.⁶ Thus, the elasticity of the marginal utility of private

⁶Use the CES aggregator u to get $1 = \lambda_c + \lambda_g$, with $\lambda_g \equiv \alpha(\frac{g}{u})^{1-\psi}$. The weight λ_c simplifies to $1 - \alpha$ for the

consumption is a weighted average of ρ and ψ . Most importantly, $\epsilon_{cc} + \epsilon_{gc} = \psi$, so the public wedge becomes

$$\chi = \frac{\Phi(1 - \psi)}{1 + \Phi(\psi - \rho)\lambda_c}.$$

The sign of χ is determined by the numerator according to proposition 2. Therefore, there is a positive public wedge in the case of *substitutes* ($\psi < 1$) and a negative public wedge in the case of *complements* ($\psi > 1$). For the Cobb-Douglas case of $\psi = 1$, the public wedge becomes zero and therefore the optimal ratio of government to private consumption becomes the same as in the first-best, which in this case is $\kappa^* = A$.

In order to determine the optimal value of χ we need to solve the equation $U_g/U_c = 1 + \chi$, which can be expressed in terms of κ as

$$A\kappa^{-\psi} = 1 + \frac{\Phi(1 - \psi)}{1 + \Phi(\psi - \rho)[1 + A\kappa^{1-\psi}]^{-1}}. \quad (24)$$

This equation is derived by expressing the weight λ_c as a function of κ , $\lambda_c(\kappa) = [1 + A\kappa^{1-\psi}]^{-1}$. Equation (24) does not depend on the shocks s and defines implicitly κ as a function of the excess burden of taxation Φ , $\kappa(\Phi)$, with $\kappa(0)$ denoting the first-best solution. Since Φ is constant, κ and the public wedge χ become *constant* at the second-best and do not vary across states and dates. Thus, the share of government consumption in output $\Lambda \equiv \kappa/(1 + \kappa)$ becomes a function of Φ , $\Lambda = \Lambda(\Phi)$, and does not vary across states and dates either. Government and private consumption are respectively

$$g_t = \Lambda y_t, \quad c_t = (1 - \Lambda)y_t, \text{ where } y_t \equiv s_t h_t.$$

Aside from the first-best, there is no closed-form solution of (24) unless specific parametric assumptions are made. For example, we showed before that for $\psi = 1$ we have $\kappa(\Phi) = \kappa(0) = A$. Furthermore, if we don't differentiate between intratemporal and intertemporal substitution and set $\psi = \rho$, we get $\chi = \Phi(1 - \psi)$ and $\kappa = (A/(1 + \Phi(1 - \psi)))^{1/\psi}$. More generally, we can use the implicit function theorem to show the existence of κ and its sensitivity with respect to the excess burden of taxation for small deviations from the first-best:

Lemma 1. $\kappa'(0) > 1$ if $\psi > 1$ and $\kappa'(0) < 1$ if $\psi < 1$, so $\text{sign } \Lambda'(0) = \text{sign}(\psi - 1)$.

Proof. Define $H(\kappa, \Phi) \equiv A\kappa^{-\psi} - 1 - \Phi(1 - \psi)[1 + \Phi(\psi - \rho)(1 + A\kappa^{1-\psi})^{-1}]^{-1}$ and write (24) as $H(\kappa, \Phi) = 0$. By the implicit function theorem, there exists a function $\kappa(\Phi)$ in a neighborhood of

Cobb-Douglas case of $\psi = 1$.

a solution of the equation with derivative $\kappa'(\Phi) = -H_\Phi/H_\kappa$ as long as $H_\kappa \neq 0$ at the solution. We have $H_\Phi = (\psi - 1)[1 + \Phi(\psi - \rho)\lambda_c]^{-2}$ and $H_\kappa = -A\kappa^{-\psi}[\psi\kappa^{-1} + (\psi - \rho)\Phi^2(1 - \psi)^2[\lambda_c^{-1} + \Phi(\psi - \rho)]^{-2}]$. The sign of H_Φ depends only on ψ being larger or smaller than unity, $\text{sign } H_\Phi = \text{sign}(\psi - 1)$. The partial H_κ is always negative for $\psi \geq \rho$. So for $\psi \geq \rho$ we have $\text{sign}(\kappa'(\Phi)) = \text{sign}(\psi - 1)$. For $\psi < \rho$ the sign of H_κ is ambiguous. But it is easy to see that around the first-best solution, we have $H_\kappa(\kappa^*, 0) = -\psi/\kappa^*$ and $\kappa'(0) = (\psi - 1)\kappa^*/\psi$. Since $\Lambda'(\Phi) = \kappa'(\Phi)/(1 + \kappa)^2$, we have $\text{sign } \Lambda'(0) = \text{sign } \kappa'(0) = \text{sign}(\psi - 1)$. $\square$

Lemma 1 provides an interpretation of increases in the excess burden of taxation Φ as increases in the distortions at the government consumption margin. In the complements case, where κ and therefore the share Λ are larger than the respective ones at the first-best, an increase in Φ leads to a larger share, so a larger “subsidy” to g . In the substitutes case, where Λ is smaller than its’ first-best value, a larger Φ leads to a smaller share, and therefore to a larger “tax” on g .

3.5.2 Labor tax, output and surplus

Turning to the labor tax, we need to specify the subutility of leisure. Consider the function $v(l) = -a_h(1 - l)^{1+\phi_h}/(1 + \phi_h) = -a_h h^{1+\phi_h}/(1 + \phi_h)$, which implies a constant Frisch elasticity of labor supply, $1/\phi_h$. The optimal tax rate in proposition 2 becomes

$$\tau = \frac{\Phi(\epsilon_{cc}(\kappa) + \phi_h)}{1 + \Phi(1 + \phi_h)} \quad (25)$$

The elasticity ϵ_{cc} depends on the ratio κ through the weight $\lambda_c(\kappa)$. A constant excess burden of taxation leads to a constant κ and therefore ϵ_{cc} does *not* vary across shocks. Therefore, the labor tax becomes *constant* across states and dates, $\tau_t = \tau(\Phi), t \geq 1$.

Differentiating the tax rate with respect to Φ delivers

$$\tau'(\Phi) = \frac{\epsilon_{cc} + \phi_h + \Phi\epsilon'_{cc}(\kappa)\kappa'(\Phi)(1 + \Phi(1 + \phi_h))}{(1 + \Phi(1 + \phi_h))^2}.$$

with $\epsilon'_{cc}(\kappa) = (\rho - \psi)(\psi - 1)A\kappa^{-\psi}\lambda_c^2$. For the case of $\psi = 1$ or the $\psi = \rho$, where we have $\epsilon_{cc} = \alpha + (1 - \alpha)\rho$ and $\epsilon_{cc} = \rho = \psi$ respectively, the tax rate becomes an *increasing* function of Φ . More generally, for a small deviation from the first-best we have $\tau'(0) = \epsilon_{cc}(\kappa^*) + \phi_h > 0$. As a result, an increase in Φ leads to larger distortions at both the government consumption margin and the labor supply margin.

Using the labor supply condition (6) and expressing the marginal utility of consumption as $U_c = (1 - \alpha)(\frac{c}{u})^{\rho-\psi} c^{-\rho}$ allows to get a quasi closed-form for the second-best labor and output,

$$\begin{aligned}
h(s, \Phi) &= s^{\frac{1-\rho}{\rho+\phi_h}} \left[\frac{1-\tau}{a_h} \frac{(1-\alpha) \left(\frac{c}{u}\right)^{\rho-\psi}}{(1-\Lambda)^\rho} \right]^{\frac{1}{\rho+\phi_h}} \\
y(s, \Phi) &= s^{\frac{1+\phi_h}{\rho+\phi_h}} \left[\frac{1-\tau}{a_h} \frac{(1-\alpha) \left(\frac{c}{u}\right)^{\rho-\psi}}{(1-\Lambda)^\rho} \right]^{\frac{1}{\rho+\phi_h}}.
\end{aligned}$$

Note that c/u is a function of κ , $c/u = [1 - \alpha + \alpha\kappa^{1-\psi}]^{\frac{1}{\psi-1}}$. Therefore the expression in the brackets depends only on Φ , which leads to a neat multiplicative separability. More importantly, despite the fact that there are taxes and government consumption in the economy and that the marginal utility of consumption is controlled by both ρ and ψ , the income and substitution effects in labor supply are controlled *only* by ρ . This is an outcome of both the homothetic specification that led to a constant share Λ , and the constant Frisch elasticity assumption that -together with the constant share- led to a constant tax rate.

Proposition 3. (*“Optimality of balanced budgets”*). *If initial debt is zero, then a balanced budget is optimal for every period. The size of the optimal tax rate does not depend on the stochastic properties of the shocks but only on preference parameters. If initial debt is positive, then surpluses are optimal for each $t \geq 1$, as long as the initial surplus does not cover the initial level of debt.*

Proof. Assume that $b_0 = 0$. Then the initial tax rate and government share are the same as in the subsequent periods, so $\tau_t = \tau(\Phi)$, $\Lambda_t = \Lambda(\Phi) \forall t \geq 0$. The intertemporal budget constraint reads $0 = (\tau(\Phi) - \Lambda(\Phi)) \sum_{t=0}^{\infty} \sum_{s^t} q_t(s^t) y_t(s_t)$ and therefore $\tau(\Phi) = \Lambda(\Phi)$. This equation, which is to be solved for Φ , does not depend on the shocks but only on the preference parameters $(\alpha, \rho, \psi, \phi_h)$. Thus, Φ and therefore the optimal tax rate and share Λ will not depend on stochastic properties of the shocks. If initial debt is positive, the intertemporal budget constraint can be rearranged to get $\tau(\Phi) - \Lambda(\Phi) = (b_0 - (\tau_0 - \Lambda_0)y_0) / \sum_{t=1}^{\infty} \sum_{s^t} q_t(s^t) y_t(s_t)$. If $b_0 > (\tau_0 - \Lambda_0)y_0$, then the government always runs surpluses $\tau(\Phi) > \Lambda(\Phi)$ for each $t \geq 1$. The value of the excess burden of taxation Φ that satisfies the budget constraint will depend on the properties of the shocks. $\square$

Turning to the surplus, we have $S_t \equiv \tau_t y_t - g_t = (\tau(\Phi) - \Lambda(\Phi))y(s, \Phi)$, $t \geq 1$, so we have a constant surplus-to-output ratio, $\tau - \Lambda$. When $\tau > \Lambda$, the surplus increases when technology shocks increase due to higher output. It will be useful for later purposes to consider also the optimal surplus in marginal utility units. Let $\Omega^*(s, \Phi) \equiv \Omega(c(s, \Phi), h(s, \Phi), g(s, \Phi))$ denote the optimal surplus as a function of (s, Φ) for $t \geq 1$. For our utility function it becomes

$$\Omega^*(s, \Phi) = U_c \cdot (\tau(\Phi) - \Lambda(\Phi))y(s, \Phi) = (1 - \alpha) \left(\frac{c}{u}\right)^{\rho-\psi} \frac{\tau - \Lambda}{(1 - \Lambda)^\rho} y^{1-\rho}. \quad (26)$$

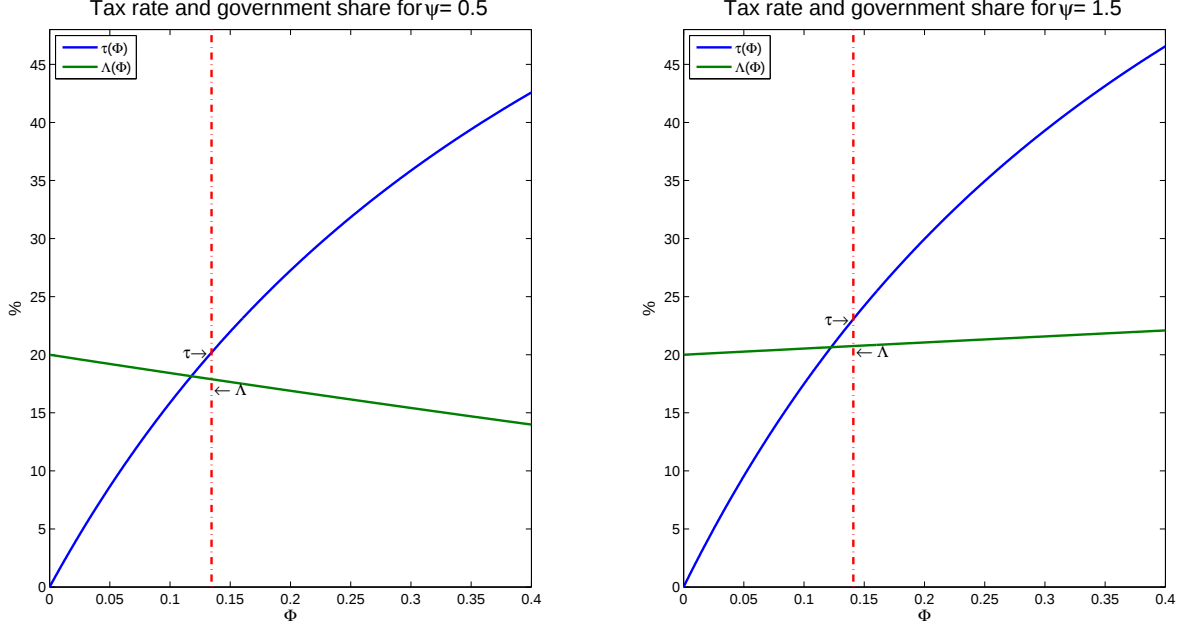


Figure 2: Plots of $\tau(\Phi)$ and $\Lambda(\Phi)$ for the substitutes (left graph) and the complements case (right graph). The vertical line indicates the Φ that satisfies the budget constraint with positive initial debt. The intersection of τ and Λ depicts the balanced budget policy.

As a result, for $\tau > \Lambda$, the surplus in marginal utility units increases with shocks when $\rho < 1$ and decreases with shocks when $\rho > 1$. The reason is that an increase in output and therefore in consumption is counteracted by a decrease in marginal utility which is controlled again only by ρ . When $\rho < 1$ the decrease in marginal utility is not large enough, and the surplus in marginal utility units increases, whereas for $\rho > 1$ the decrease in marginal utility dominates. For the knife-edge case of $\rho = 1$, even if the surplus S varies across shocks, it stays constant in marginal utility units.

Figure 2 plots the optimal tax rate and the government share Λ for the substitutes and the complements case as functions of Φ . We set $(\beta, \phi_h, \rho) = (0.96, 1, 1)$. Technology shocks take two values $s_L < s_H$, are *i.i.d* and equiprobable with average value unity. We set $(s_L, s_H) = (0.97, 1.03)$ which corresponds to a 3% standard deviation of output. For each ψ we set α and a_h so that so that at the first-best the government share is 20% and the household works 40% of its time. The shock at $t = 0$ is s_H and initial debt is set to 50% of first-best output. The figure shows that the tax rate is an increasing function of the excess burden of taxation whereas the government share is decreasing for the substitutes case ($\psi = 0.5$) and increasing for the complements case ($\psi = 1.5$). The balanced budget policy is at the intersection of τ and Λ . As we see from the figure, given that initial debt is positive, the government runs surpluses. For $\psi = 0.5$, the tax rate is 20.2% and the government share 17.9%. For $\psi = 1.5$, the corresponding tax rate is 23.07% and the government share 20.75%.

4 Doubts about the probability model

4.1 Preferences

Until now, we have analyzed an economy where agents have full confidence in the probability measure π that governs technology shocks. Consider now a situation where the household considers π (which we will call the reference measure from now on) as a good approximation of the true probability measure that describes uncertainty but entertains fears that π may be misspecified. In order to deal with the possibility of misspecification the household considers a *set* of alternative probability measures that are close to π . We are making the assumption that these measures are absolutely continuous with respect to π and express them as a change of measure. More specifically, the positive random variable m_{t+1} denotes a change of the conditional measure $\pi_{t+1}(s_{t+1}|s^t)$. In order to be a proper change of measure it has to integrate to unity, $E_t m_{t+1} = 1$. The unconditional change of measure is defined as $M_t \equiv \prod_{i=1}^t m_i$, $M_0 \equiv 1$, and is a martingale with respect to π .

We use the multiplier preferences of Hansen and Sargent (2001) in order to capture this *ambiguity* and the household's *aversion* towards it,⁷

$$V_t = U(c_t, 1 - h_t, g_t) + \beta \min_{m_{t+1} \geq 0, E_t m_{t+1} = 1} [E_t m_{t+1} V_{t+1} + \theta E_t m_{t+1} \ln m_{t+1}],$$

where $\theta > 0$. The parameter θ penalizes probability models that are far from the reference model in terms of relative entropy. Full confidence in the model, and therefore expected utility is captured by $\theta = \infty$.

4.2 Competitive equilibrium under ambiguity

The minimization problem of the household leads to an conditional change of measure that takes the form

$$m_{t+1}(s^{t+1}) = \frac{\exp(\sigma V_{t+1}(s^{t+1}))}{\sum_{s_{t+1}} \pi_{t+1}(s^{t+1}|s^t) \exp(\sigma V_{t+1}(s^{t+1}))} \quad (27)$$

where $\sigma \equiv -\theta^{-1} < 0$, with $\sigma = 0$ corresponding to the expected utility case. A cautious household assigns higher probability than the reference measure towards events that bear low continuation utility and smaller probability than the reference measure towards events with high continuation utility. So the household forms pessimistic beliefs that are endogenous, since they depend on continuation utility.

Turning to the household's maximization problem, the static labor supply condition (6) is unchanged. The intertemporal marginal rate of substitution is altered though with fears about model

⁷See Strzalecki (2011) for a decision-theoretic foundation of the multiplier preferences.

misspecification, leading to an optimality condition with respect to Arrow securities that takes the form

$$p_t(s_{t+1}, s^t) = \beta \pi_{t+1}(s_{t+1}|s^t) m_{t+1}(s^{t+1}) \frac{U_c(s^{t+1})}{U_c(s^t)}.$$

Therefore, the endogenous pessimistic beliefs of the household will determine equilibrium prices and alter the nature of the Ramsey problem.

4.3 Ramsey problem

The first-best allocation (c^*, h^*, g^*) with doubts about the probability model of technology shocks remains the same as without doubts, due to the essentially static nature of the problem.⁸

Proceeding with the Ramsey problem, we follow a recursive representation of the commitment problem from period one onward as in Karantounias (2013b). Let $z \equiv U_c b$ denote debt in marginal utility units and let $V(z, s)$ denote the value function of the planner. Assume that shocks are Markov with transition density $\pi(s'|s)$. Then V is described by the following Bellman equation:

$$V(z, s) = \max_{c, h, g, z'_{s'}} U(c, 1 - h, g) + \frac{\beta}{\sigma} \ln \sum_{s'} \pi(s'|s) \exp(\sigma V(z'_{s'}, s'))$$

subject to

$$z = \Omega(c, h, g) + \beta \sum_{s'} \pi(s'|s) \frac{\exp(\sigma V(z'_{s'}, s'))}{\sum_{s'} \pi(s'|s) \exp(\sigma V(z'_{s'}, s'))} z'_{s'} \quad (28)$$

$$c + g = sh \quad (29)$$

$$c, g \geq 0, h \in [0, 1], z'_{s'} \in Z(s') \quad (30)$$

Let Φ denote the multiplier on the dynamic implementability constraint (28) and let λ denote the multiplier on the resource constraint (29). The first-order necessary conditions are

⁸The first-best is characterized by $(-\partial V_0 / \partial h_t(s^t)) / (\partial V_0 / \partial c_t(s^t)) = s_t$ and $(\partial V_0 / \partial g_t(s^t)) / (\partial V_0 / \partial c_t(s^t)) = 1$. For the multiplier preferences we have $\partial V_0 / \partial h_t(s^t) = -\beta^t \pi_t M_t U_l(s^t)$, $\partial V_0 / \partial c_t(s^t) = \beta^t \pi_t M_t U_c(s^t)$ and $\partial V_0 / \partial g_t(s^t) = \beta^t \pi_t M_t U_g(s^t)$, which lead to (8) and (9).

$$c : \quad U_c + \Phi \Omega_c = \lambda \quad (31)$$

$$h : \quad -U_l + \Phi \Omega_h = -\lambda s \quad (32)$$

$$g : \quad U_g + \Phi \Omega_g = \lambda \quad (33)$$

$$z'_{s'} : \quad V_z(z'_{s'}, s')[1 + \sigma \eta'_{s'} \Phi] + \Phi = 0 \quad (34)$$

where $\eta'_{s'} \equiv z'_{s'} - \sum_{s'} \pi(s'|s) m'_{s'} z'_{s'}$. The variable $m'_{s'}$ stands for the conditional likelihood ratio, $m'_{s'} = \frac{\exp(\sigma V(z'_{s'}, s'))}{\sum_{s'} \pi(s'|s) \exp(\sigma V(z'_{s'}, s'))}$.

The variable $\eta'_{s'}$ can be thought of as the *net* debt ($\eta'_{s'} > 0$) or net asset ($\eta'_{s'} < 0$) position of the government in marginal utility units. Equivalently, it stands for the conditional *innovation* in debt in marginal utility units under the distorted measure, since $\sum_{s'} \pi(s'|s) m'_{s'} \eta_{s'} = \sum_{s'} \pi(s'|s) m'_{s'} z'_{s'} - \sum_{s'} \pi(s'|s) m'_{s'} (\sum_{s'} \pi(s'|s) m'_{s'} z'_{s'}) = 0$, since $\sum_{s'} \pi(s'|s) m'_{s'} = 1$.

Initial period problem. The problem from period one onward uses as an input the value of the state variable at $t = 1$, when the shock takes value is s , $z_{1,s}$. This value is chosen *optimally* at $t = 0$, together with the initial allocation (c_0, h_0, g_0) to solve the problem

$$\max_{c_0, g_0 \geq 0, h_0 \in [0, 1], z_{1,s} \in Z(s)} U(c_0, 1 - h_0, g_0) + \frac{\beta}{\sigma} \ln \sum_s \pi(s|s_0) \exp(\sigma V(z_{1,s}, s))$$

subject to

$$\begin{aligned} U_c(c_0, 1 - h_0, g_0) b_0 &= \Omega(c_0, h_0, g_0) + \beta \sum_s \pi(s|s_0) \frac{\exp(\sigma V(z_{1,s}, s))}{\sum_s \pi(s|s_0) \exp(\sigma V(z_{1,s}, s))} z_{1,s} \\ c_0 + g_0 &= s_0 h_0 \end{aligned}$$

The optimality conditions with respect to (c_0, h_0, g_0) are the same as in the problem without doubts (15-17), with the qualification that the multiplier on the initial implementability constraint indexed by $t = 0$, Φ_0 . Similarly, the optimality condition with respect to $z_{1,s}$ is given by (34) with the same qualification.

4.4 Analysis

The important element that doubts about the model contribute is an excess burden of taxation that is not *constant* anymore. Instead, it is *time-varying* and *persistent*. In particular, use the envelope condition $V_z(z, s) = -\Phi$ and rewrite the optimality condition with respect to debt in marginal

utility units next period (34) as

$$\Phi'_{s'} = \frac{\Phi}{1 + \sigma \eta'_{s'} \Phi}.$$

Taking inverses and using sequence notation leads to

$$\frac{1}{\Phi_{t+1}} = \frac{1}{\Phi_t} + \sigma \eta_{t+1}, t \geq 0 \quad (35)$$

where $\eta_{t+1} = z_{t+1} - E_t m_{t+1} z_{t+1}$. Note that the excess burden of taxation is increasing ($\Phi_{t+1} > \Phi_t$) if there is a net debt position in marginal utility units $\eta_{t+1} > 0$, i.e. when debt in marginal utility units z_{t+1} is larger than the average position, and it decreases ($\Phi_{t+1} < \Phi_t$) if there is a net asset position in marginal utility units, $\eta_{t+1} < 0$, so when z_{t+1} is smaller than the average position. As in the analysis with recursive preferences of Karantounias (2013b), debt in marginal utility units has an additional price effect that the policymaker is manipulating in order to make debt less costly. With doubts about the model, an increase in $z'_{s'}$ at the state of the world s' decreases utility and as a result it increases the probability that the household assigns to this state of the world. This leads to a higher price of an Arrow security. This increase in price is beneficial to the planner if he takes a net debt position (since the price at which he issues debt increases) and harmful if instead he takes a net asset position (since in this case the planner is a buyer on net). The fact that a relatively large z_{t+1} become less costly leads to a transfer of distortions (in the sense of the excess burden of taxation) towards this type of events, and therefore $\Phi_{t+1} > \Phi_t$. Similarly, the planner shifts away distortions from states of the world where z_{t+1} is relatively small, in order to increase the return on assets and $\Phi_{t+1} < \Phi_t$. Furthermore, Φ_t has a particular stochastic structure that induces persistence to the optimal plan:

Proposition 4. $1/\Phi_t$ is a martingale with respect to $\pi_t \cdot M_t$. Φ_t is a submartingale with respect to $\pi_t \cdot M_t$.

Proof. Take expectation according to the distorted measure in (35) and use the fact that $E_t m_{t+1} \eta_{t+1} = 0$ to get the martingale property. Furthermore, the function $1/x$ is convex for $x > 0$ which by applying the conditional Jensen's inequality leads to the submartingale result, $E_t m_{t+1} \Phi_{t+1} \geq \Phi_t$. $\square$

Using the optimality conditions of the recursive problem (31), (32) and (33) delivers the two equations that characterize the optimal wedges at the two margins, (18) and (19), with the crucial difference that the excess burden of taxation is indexed by time, Φ_t .⁹ This fact allows us to solve for (c, h, g) in terms of the shock s_t and a time-varying Φ_t , $c_t = c(s_t, \Phi_t)$, $h_t = h(s_t, \Phi_t)$, $g_t = g(s_t, \Phi_t)$.

⁹Using the initial period optimality conditions delivers also the respective initial period wedges (20) and (21) for the corresponding initial excess burden of taxation Φ_0 .

These functions are exactly the same functions of (s, Φ) as in the case without doubts about the model. Nevertheless, we need to solve the recursive problem stated above (and thus the actual policy functions in terms of (z, s)) in order to determine the law of motion of the time-varying Φ_t . Furthermore, we can express the optimal public wedge and the optimal tax rate as in proposition 2.

Proposition 5. *(Optimal wedges with doubts about the model) The optimal public wedge and the optimal tax rate are as in proposition 2, with an excess burden of taxation that follows now the law of motion (35).*

As a result, the formulas that we derived for the parametric example go through replacing the constant Φ with a time-varying Φ_t . An immediate implication is that the share of government consumption, the public wedge and the tax rate will *not* be constant anymore across states and dates.

5 Allocation of distortions over states and dates

5.1 Debt in marginal utility units and the excess burden of taxation

The endogenous pessimistic expectations provide to the planner the option to reduce the return on government debt and as a result make him allocate distortions towards events where he issues relatively more debt in marginal utility units. The reason why the relevant object of interest is debt in *marginal utility* units comes from the logic the intertemporal budget constraint of the government. The present discounted value of surpluses entails both an adjustment for model uncertainty through the pessimistic expectations and an adjustment for risk through marginal utility.

What is important to note is that this price manipulation mechanism is relevant only if the debt in marginal utility units *varies* across shocks or if it is actual necessary to issue debt. In the event that there is *no* variation of the optimal surpluses in marginal utility units and therefore no variation in debt in marginal utility units, or if debt is not necessary, the planner does not use anymore the additional channel of the worst-case beliefs to affect the allocation of welfare distortions and the excess burden of taxation becomes constant.

Proposition 6. *Assume that either 1) $\Omega^*(s, \Phi) = \Omega^*(s', \Phi), \forall \Phi, \forall s \neq s'$ or that 2) initial debt is zero and there exists a $\bar{\Phi}$ such that $\Omega^*(s, \bar{\Phi}) = \Omega^*(s', \bar{\Phi}) = 0, \forall s \neq s'$. Then $\Phi_t = \bar{\Phi}$, where $\bar{\Phi}$ is the excess burden of taxation of the economy with full confidence in the model. Therefore, the allocation (c, h, g) and the respective wedges are the same as in the economy without doubts. Only asset prices q are different.*

Proof. 1) We will show that, given the assumption, a constant Φ satisfies the optimality conditions of the Ramsey problem with doubts about the probability model. Debt in marginal utility units

is $z_t = E_t \sum_{i=0}^{\infty} \beta^i \frac{M_{t+i}}{M_t} \Omega^*(s_{t+i}, \Phi_{t+i})$. For a constant Φ we get $z_t = z = \Omega^*/(1 - \beta), t \geq 1$, since Ω^* does not vary across shocks for any given Φ . Thus η_{t+1} is identically zero $\forall t \geq 0$ and the law of motion for Φ_t (35) delivers $\Phi_t = \Phi, t \geq 0$, confirming that a constant Φ satisfies the optimality conditions. The constant Φ has to satisfy the implementability constraint at $t = 0$, which reduces to $U_{c0}b_0 = \Omega_0 + \beta\Omega^*/(1 - \beta)$. This is the same equation that Φ has to satisfy at the second-best with full confidence in the model. Let the solution to it be $\bar{\Phi}$ and the result follows.

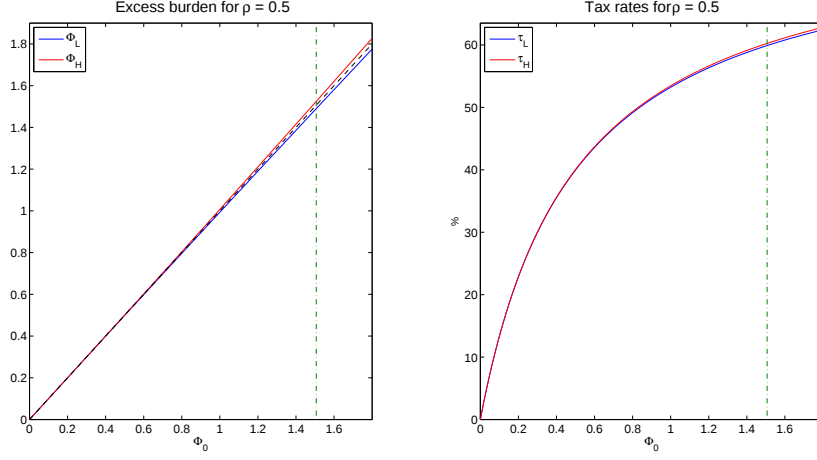
2) Given the assumption, there is a $\bar{\Phi}$ for which the government runs a balanced budget for every realization of the shock. For the given $\bar{\Phi}$ we have $z_t = 0 \forall t \geq 1$ and therefore $\eta_{t+1} \equiv 0, t \geq 0$. Thus, we have $\Phi_t = \bar{\Phi}, t \geq 0$ by (35). This $\bar{\Phi}$ satisfies the implementability constraint at $t = 0$ since initial debt is zero. This is the same condition as with full confidence in the model and the result follows. For $b_0 \neq 0$ the implementability constraint becomes $U_{c0}b_0 = \Omega_0$. Ω_0 depends on $(s_0, b_0, \bar{\Phi})$ through the initial allocation (c_0, h_0, g_0) and there is no guarantee that the constraint holds for the given $\bar{\Phi}$. □

Corollary. *When 1) the utility function is as in (23) with $\rho = 1$ or when 2) initial debt is zero and the utility function is as in (23) with constant Frisch elasticity, then doubts about the model leave the second-best allocation and wedges unaltered.*

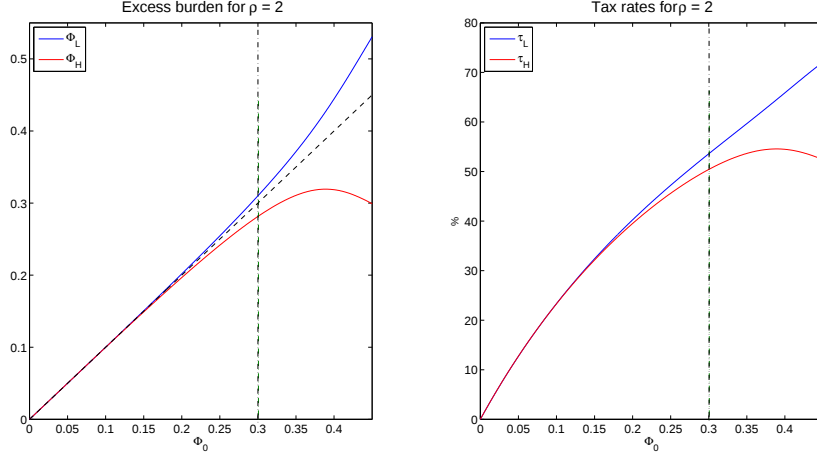
Proof. 1) We have already shown in (26) that the surplus in marginal utility units does not vary across shocks for $\rho = 1$ when we have a constant Frisch elasticity, so the first assumption of proposition 6 applies. We will show now that Ω^* doesn't vary across shocks for any subutility of leisure $v(l)$ if $\rho = 1$. For a generic $v(l)$ the elasticity of marginal disutility of leisure (which is the inverse of the Frisch elasticity) depends on h , $\epsilon_{hh}(h) = -v''(1 - h)h/v'(1 - h)$, which could in principle lead to a varying tax rate across shocks for a given Φ . We will show that for $\rho = 1$, optimal labor is only a function of Φ , which will ultimately deliver the result. For $\rho = 1$, $U_c = \lambda_c(\kappa)c^{-1}$ and $\epsilon_{cc}(\kappa) = \psi + (1 - \psi)\lambda_c(\kappa)$. Thus, the optimal wedge (18) becomes $\frac{v'(1-h)}{\lambda_c(\kappa)} \cdot \frac{1+\Phi(1+\epsilon_{hh}(h))}{1+\Phi(1-\epsilon_{cc}(\kappa))}c = s$. Setting $c = (1 - \Lambda)sh$, leads to the elimination of the shocks s from the optimal wedge equation, furnishing a labor that is only a function of Φ . As a result, the tax rate becomes only a function of Φ (albeit a different function than in (25)). The optimal surplus in marginal utility units becomes $\Omega^* = \lambda_c(\kappa)(\tau - \Lambda)c^{-1}y = \lambda_c(\kappa)(\tau - \Lambda)/(1 - \Lambda)$, which depends only on Φ . 2) Balanced budgets are optimal according to proposition 3 which delivers the second assumption of proposition 6. □

To summarize, we found sufficient conditions that *mute* the effect of the doubts about the model on the Ramsey plan. For our parametric example, this happens for the knife-edge cases of $\rho = 1$ or zero initial debt and a constant Frisch elasticity.¹⁰ Another way to state these results is that whenever the planner without doubts about the model was finding optimal to run either a constant

¹⁰Note that in the case of *exogenous* government expenditures it would be rare to obtain a surplus in marginal utility units that does not vary across shocks even in the knife-edge case of logarithmic utility. The reason is that the share of government expenditures wouldn't be constant.



(a) Cyclical allocation of distortions for $\rho < 1$.

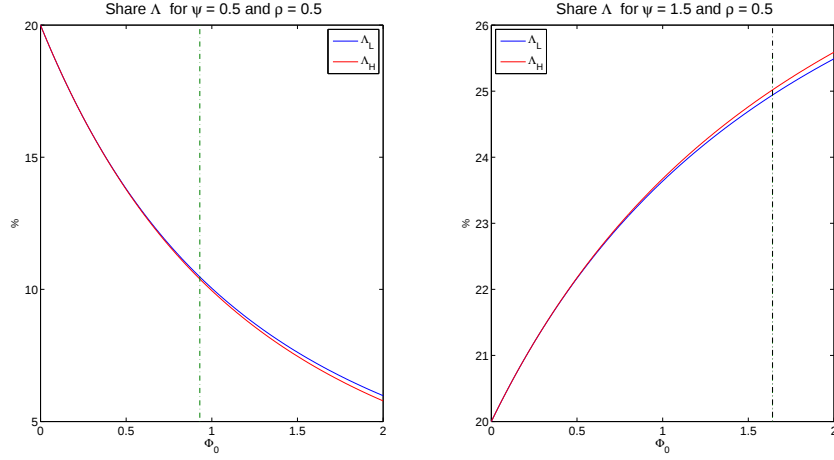


(b) Countercyclical allocation of distortions for $\rho > 1$.

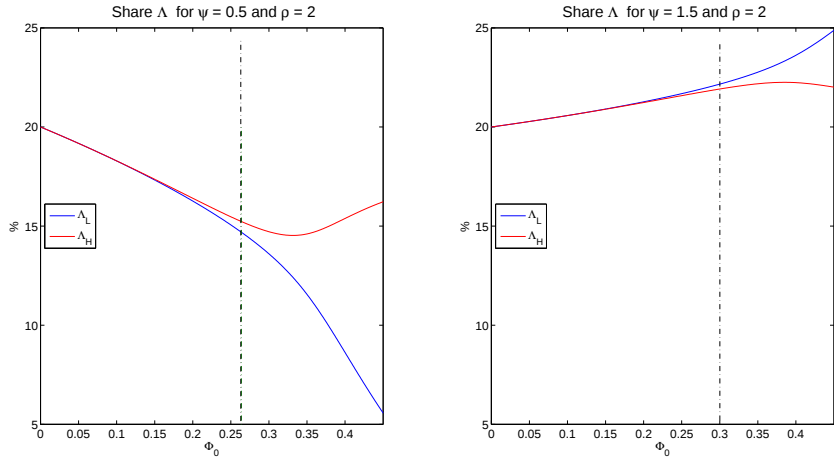
Figure 3: Φ_i and tax rates τ_i for the high and low shock for varying values of Φ_0 for $\rho = 0.5$ (top panel) and $\rho = 2$ (bottom panel). Without doubts about the model Φ_i would be on the dotted line which represents the 45° degree line. In each graph the vertical line depicts the actual Φ_0 that satisfies the budget constraint. For these graphs we have used $\psi = 1$. The ordering of Φ_i 's and τ_i 's is the same for $\psi < 1$ or $\psi > 1$ and the graphs are omitted.

surplus in marginal utility units or a balanced budget, he would still do so with doubts about the model and choose the same policies (g, τ) .

For the rest of our analysis, we are going to use our parametric example with constant Frisch elasticity, $\rho \neq 1$ and positive initial debt. These are necessary conditions for doubts about the model to kick in.¹¹



(a) Λ 's for $\rho < 1$.



(b) Λ 's for $\rho > 1$.

Figure 4: This figure depicts the shares of government consumption Λ_i for varying values of Φ_0 for $\rho = 0.5$ (top panel) and $\rho = 2$ (bottom panel). In each panel the left graph considers the case of substitutes ($\psi < 1$) and the right graph the case of complements ($\psi > 1$). The vertical line in each graph depicts the actual initial Φ_0 that satisfies the budget constraint. Without doubts about the model, we have $\Lambda_H = \Lambda_L$ for each pair (ρ, ψ) .

5.2 The cyclical allocation of distortions

To understand the allocation of distortions over the cycle it is useful to consider a two-period version of our model. In that case, debt in marginal utility units z simplifies to surplus in marginal utility units Ω . In particular, let the shocks take two values $s_L < s_H$. The excess burden of taxation at period one takes the form

$$\Phi_H = \frac{\Phi_0}{1 + \sigma\eta_H\Phi_0}, \quad \Phi_L = \frac{\Phi_0}{1 + \sigma\eta_L\Phi_0}$$

¹¹If we wanted to avoid the assumption of positive initial debt, we could use $v(l) = a_l(l^{1-\phi_l} - 1)/(1 - \phi_l)$, set initial debt to zero and $\rho \neq 1$. We will stick to the constant Frisch elasticity case since it implies the sharp result of perfect tax smoothing without doubts about the model.

where $\eta_i = \Omega_i - \sum_i \pi_i m_i \Omega_i$, $i = L, H$. Subscripts denote if we are at the high or low shock. If the government takes a higher position in marginal utility units when the shock is high, i.e. if $\Omega_H > \Omega_L$, then it allocates distortions towards the high shock, $\Phi_H > \Phi_L$. Similarly, if $\Omega_H < \Omega_L$ we have $\Phi_H < \Phi_L$.

The allocation of distortions depends on the covariance of Ω with the shocks. For our parametric example with constant Frisch elasticity, we saw from (26) that the cyclicalities or not of the surplus in marginal utility units depends on $\rho < 1$ or $\rho > 1$. With doubts about the model though, it is not clear anymore how the optimal Ω varies across shocks since Φ is also changing. However, for *small* doubts about the probability model we have the following lemma,

Lemma 2. *The planner is transferring distortions towards the shocks at which he runs a higher surplus in marginal utility units without doubts about the model.*

Proof. Express all variables in the law of motion of Φ as functions of σ to get $\Phi_i(\sigma)(1 + \sigma\eta_i(\sigma)\Phi_0(\sigma)) = \Phi_0(\sigma)$, $i = L, H$. Differentiate with respect to σ and set $\sigma = 0$ to get $\Phi'_i(0) = \Phi'_0(0) - \Phi(0)^2\eta_i(0)$, where $\Phi(0)$ and $\eta_i(0)$ correspond to the excess burden and the respective position without doubts about the model. To first-order we have $\Phi_i(\sigma) \simeq \Phi(0) + \sigma\Phi'_i(0)$ and $\Phi_0(\sigma) \simeq \Phi(0) + \sigma\Phi'_0(0)$. Therefore, $\Phi_i(\sigma) - \Phi_0(\sigma) = \sigma(\Phi'_i(0) - \Phi'_0(0)) = -\sigma\Phi(0)^2\eta_i(0)$. Thus $\Phi_H(\sigma) > (<) \Phi_L(\sigma)$ if $\Omega_H(0) > (<) \Omega_L(0)$. $\square$

Using (26) and the Lemma, we see that if the planner was running a surplus with full confidence in the model, then for small doubts about the model and $\rho < 1$ the planner would allocate distortions *procyclically* ($\Phi_H > \Phi_L$), whereas for $\rho > 1$ the planner would allocate distortions *countercyclically* ($\Phi_H < \Phi_L$).

Thus, the parameter ρ is determining the allocation of distortions over the cycle when we have doubts about the model. This is true also if we don't confine ourselves to an expansion as in lemma 2. Figure 3 displays the Φ_i 's and the respective tax rates $\tau_i = \tau(\Phi_i)$, which are increasing in the excess burden of taxation, for various values of the initial Φ_0 for $\rho < 1$ and $\rho > 1$.¹² The excess burden of taxation and the tax rate are procyclical when $\rho < 1$ and countercyclical when $\rho > 1$.

Turning to the share of government consumption $\Lambda_i = \Lambda(\Phi_i)$, $i = L, H$, we saw earlier that an increase in distortions translates either to a lower Λ (a larger tax on g) when we have substitutes ($\psi < 1$) or to a larger Λ (a larger subsidy on g) when we have complements ($\psi > 1$). As a result, when $\rho < 1$, which implies procyclical distortions, Λ will decrease for the high shock when we have substitutes, $\Lambda_H < \Lambda_L$, or increase when we have complements, $\Lambda_H > \Lambda_L$. In the opposite case of $\rho > 1$, where distortions become countercyclical, Λ will increase for the high shock when we have substitutes, $\Lambda_H > \Lambda_L$, whereas it will decrease when we have complements $\Lambda_H < \Lambda_L$. Figure 4

¹²For these figures we have set $(s_L, s_H) = (0.9, 1.1)$ and assumed that shocks are equiprobable. We have set $(\beta, \phi_h) = (0.96, 1)$. For each (ρ, ψ) that we consider, we set α and a_h so that so that the first-best share of government consumption in output is 0.2 and the household works 40% of its time when shocks take their average value of unity. The shock at $t = 0$ is s_H and initial debt is set to 50% of first-best output. For the illustration, we set the doubts about the model to $\sigma = -0.76$.

Table 1: Cyclicalities of τ and Λ .

	$\psi < 1$	$\psi > 1$
$\rho < 1$	τ : procyclical, Λ : countercyclical	τ : procyclical, Λ : procyclical
$\rho > 1$	τ : countercyclical, Λ : procyclical	τ : countercyclical, Λ : countercyclical

displays these patterns. Therefore, when $\rho < 1$ and $\psi < 1$ or when $\rho > 1$ and $\psi > 1$, i.e. when we have both intertemporal and intratemporal substitutability or complementarity, the share of government consumption in output becomes countercyclical. In the cases of either $\rho < 1$ and $\psi > 1$ or $\rho > 1$ and $\psi < 1$, the share of government expenditures becomes procyclical. Table 1 summarizes the results for the cyclicalities of the tax rate and government consumption.

Figures 3 and 4 show that for $\rho < 1$, the variation of the excess burden of taxation across shocks is small, which leads to a small variation of the tax rate and the government share. In contrast, for $\rho > 1$ this is *not* the case. So the variation of the surplus in marginal utility units seems to be small for $\rho < 1$ and large for $\rho > 1$. We will explore if this pattern emerges in infinite horizon for a proper calibration of the shocks and σ . Note that in infinite horizon, even if the variation in z_t turns out to be small for $\rho < 1$, the effects of doubts about the model can still be large over time, since the excess burden of taxation depends on the *cumulative* debt positions, as (35) shows.

5.3 Dynamics

In this last section, we conjecture the long-run dynamics of optimal fiscal policy, based on preliminary analytical and numerical results. Not surprisingly, the driver of the model long-run properties is the cyclical behavior of debt in marginal utility units z_t , as explained in Section 4.

First, assume $\rho < 1$. Based on the two-period model analysis, we conjecture that z_t will be procyclical. If this is so, the *change* in Φ_t will be procyclical, as the law of motion (35) shows. Therefore, we expect the change in tax rates to be procyclical and the change in the government share to be countercyclical for the substitutes and procyclical for the complements case respectively.

Remember that the household and the government assign lower probability than the reference measure on high utility events, which we expect them to be events where technology shocks are favorable. Thus, the government is issuing high debt in marginal utility units and therefore shifts distortions towards states of the world which happen with higher probability than expected, if we assume that the reference measure is also the true measure. This mechanism generates a positive drift in the excess burden of taxation.

This intuition can be shown formally. Proposition 4 stated that Φ_t is a submartingale with respect to the distorted measure which implies that $E_t \Phi_{t+1} \geq \Phi_t - Cov_t(m_{t+1}, \Phi_{t+1})$. But, if the

household assigns low probability on the good shocks, m_{t+1} is countercyclical, while, conditional on period t , Φ_{t+1} is procyclical. Therefore, the conditional covariance is negative, inducing a *positive* drift to the excess burden of taxation, $E_t \Phi_{t+1} \geq \Phi_t$. As a result, there is a *back-loading* of distortions over time. Tax rates increase on average over time, whereas the government share exhibits a negative drift in the substitutes case or a positive drift in the complements case.

In contrast, when $\rho > 1$, based again on the two-period model, we expect debt in marginal utility units to be countercyclical, making therefore the change in the excess burden of taxation and the change in the tax rate countercyclical. The change in the share Λ will be procyclical for the substitutes case and countercyclical for the complements case. When $\rho > 1$, the government is shifting distortions towards states of the world that do not happen so often according to the true measure (so the conditional covariance is now positive), which leads to the conjecture that there will be a negative drift in the excess burden of taxation, so a *front-loading* of distortions. If that is so, there will be a negative drift in taxes and a positive or negative drift in the government share in the substitutes and complements case respectively. Our conjecture is that in the long-run a balanced budget will be reached, which is an absorbing state given our earlier discussion in section 5.2.

6 Numerical results

[To be completed.]

7 Concluding remarks

[To be completed.]

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