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Debt Constraints and Unemployment*

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ABSTRACT

In the Great Contraction, regions of the United States that experienced the largest change in household debt to income ratios also experienced the largest drops in output and employment. Such output drops not only occurred for firms that sell primarily to a local region but also for regional establishments of nation-wide firms that sell highly traded goods to both the rest of the United States and abroad. These patterns are difficult to reconcile with standard models of financial frictions in which tightened financial constraints mainly affect firms' ability to borrow. We develop a Bewley-Huggett-Aiyagari incomplete market model with search and matching frictions of the Diamond-Mortenson-Pissarides type that generates such patterns. Critically, we allow for human capital acquisition by employed individuals, which generates realistic wage-tenure profiles. We show that with such upward sloping wage profiles, an unanticipated tightening of borrowing constraints leads consumers to value less the prospect of consumption when employed and, thus, to find employment relatively less attractive. In equilibrium, firms anticipate this behavior by consumers and consequently reduce the number of vacancies they post. The key result is that in equilibrium the tightening of borrowing constraints generates a path of increased unemployment that lingers, as consumers slowly adjust their asset positions given the tighter constraints, and seemingly 'sticky' wages, despite wages being continually renegotiated.
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As a number of authors, including Mian and Sufi (2009) and Midrigan and Phillipon (2011) have noted, in the most recent recession regions that experience the largest swing in household borrowing. This drop in output not only occured for firms that sell primarily to the local region but also for regional establishments of nation-wide firms that sell highly traded goods to both the rest of the United States and abroad. Such a pattern is difficult to reconcile with two classes of standard models. First it is hard to reconcile with models that stress firm side financial frictions. The reason is that since ownership of nation-wide firms are at the national level the financial constraints should impact the firm as a whole and when these frictions increase they reduce output for all establishments owned by such a firm. Such a pattern is not present in nation-wide firms in the United States. The smaller class are those from the sudden stop literature in which an increase in the difficulty in borrowing leads consumer to supply more labor. (See Chari, Kehoe, and McGrattan 2005 for a discussion.)

This evidence then suggest along the lines argued by Mian Sufi (2009) and Midrigan and Phillopon (2011) that a more promising class of models are those the have the financial frictions fall on the consumer side rather than the firm side. Motivated by such evidence we develop a Bewley-Huggett-Aiyagari incomplete market model with search and matching frictions of the Diamond-Mortenson-Pissarides structure that builds on Krusell, Mukoyama, and Sahin (2010), hereafter *KMS*. A key feature we add to that model is human capital so as to generate realistic wage-tenure profiles of the type documented by Buchinsky, Fougere, Kramarz, and Tchernis (2010). The key idea is that faced with such upward-sloping patterns of wages, an unemployed consumer that is close to the borrowing constraint realizes that accepting a job means accepting a path of consumption that tracks these wages. This path of consumption is relatively unattractive consumers that dislike curved paths for consumption. Either an unanticipated tightening of the borrowing constraint or an unanticipated fall in the value of a consumer's assets, say from an fall in house prices, makes such curved paths even less attractive to consumers.

The main reaction to this tightening comes from the firm side. After witnessing such unanticipated changes, firms think through what will be the outcome of their bargaining with consumers if they post a costly vacancy and get matched with a consumer. Such firms realized that after such changes any given wage they offer is relatively less attractive relative

to unemployment for the consumer they will be bargaining with. Hence, firms realize that the surplus from any match they make will be less, hence their anticipated profits will be less. In equilibrium then firms decrease the number of vacancies they offer and unemployment rises and output falls. This pattern continues during a lengthy transition as consumers slowly increase their precautionary savings so that better smooth consumption when they accept such jobs with curved profiles of wages. An interesting feature of this transition it that the tightening generates a path of increased unemployment that lingers and seemingly 'sticky' wages, despite wages being continuously renegotiated, as consumers slowly adjust their asset positions given the tighter borrowing constraints.

As emphasized by Hall (2013) the key unresolved aspect of the Mortenson and Pissarides model is that nature of the force that depresses the payoff of job creation in recessions. In most of the papers in this vein the key force is a drop in productivity. As Hall also emphasizes that explanation runs into two problems. First, productivity did not fall much in the Great Contraction. Second, given that small fall in productivity it is exceedingly difficult for the model to generate a large drop in output and employment. The driving force that we emphasize is an unexpected tightening of credit constraints on consumers.

As noted one key feature we added to the KMS framework is human capital. A second key feature is that we focus on Harsayni and Selten (1972) bargaining rather than the Nash bargaining that KMS focus on. In the Harsayni-Selten framework a consumer's assets is private information. Hence, when matched with a firm, the wage does not depend on the level of assets of the consumer but rather only on the consumer's human capital. In our context this bargaining setup both produces more realistic patterns for wages and savings than does standard Nash bargaining. Indeed, one drawback of standard Nash bargaining with observable assets is that paid wages for the same output differ widely across level of assets of consumers, a feature difficult to justify with micro data. Moreover, consumers adjust their savings to influence their future wages in a way that makes the savings behavior erratic.

Our work is related to a small literature that tries to link increases in financial frictions on the consumer side to regional economic downturns. In particular, Guerrieri and Lorenzoni (2010), Eggertson and Krugman (2011), and Midrigan and Phillipon (2011) study macroeconomic responses to a household-side credit crunch. All three of these papers find

that a credit crunch has only a minor effect on employment if the economy is away from a zero lower bound.

1. The Economy

We consider a version of the economy in Krusell et al. (2010) and we follow their notation closely. The key differences between our economy and theirs is that we allow human capital accumulation and posit an alternative bargaining protocol based on Harsayni and Selten (1972) in which a consumer's assets are private information. As we show, with human capital accumulation equilibrium interaction between the tightness of the borrowing constraints and consumer and firm behavior is critical for our results. The alternative bargaining prohe equilibrium interaction of the process for human capital accumulation and borrowing constraint is critical for our results.

The economy consists of a continuum of consumers of measure 1. In any period a consumer is either matched with a firm or is unemployed and searching for a match.

We begin by setting up and defining a stationary equilibrium for our economy using recursive notation in which primes denote next period variables. We assume a standard aggregate matching function $M(u, v)$ which represents the measure of matches in a period in which there are u unemployed workers and v vacancies in the current period. We assume M is homogenous of degree 1 and initially let $M(u, v) = Bu^{\eta}v^{1-\eta}$. The probability that a vacant job is filled in the current period is $\lambda_f(\theta) = M(1/\theta, 1) = M(u, v)/v$ where $\theta = v/u$ is the vacancy-unemployment ratio while the probability that an unemployed worker finds a match is $\lambda_w(\theta) = \theta\lambda_f(\theta) = M(u, v)/u$. In each match the consumer exogenously separates with probability σ so that the unemployment rate evolves according to

$$u' = (1 - \lambda_w(\theta))u + \sigma(1 - u)$$

A. Consumers

A consumer with human capital h that is matched with a firm produces

$$y_t = Ah_t.$$

If the match continues this capital evolves according to

$$h' = \lambda h + (1 - \lambda)\,\bar{h}$$

while if the match dissolves and the consumer is unemployed this capital is lost and the consumer produces b in home production in each period. A newly matched worker draws an initial human capital for the match $h = h_0$ from a distribution $F(h)$.

The consumer has access to uncontingent borrowing and lending as is subject to a borrowing limit. The budget constraint is

$$c + a' = (1 + r)a + nw + (1 - n)b + d$$

$$a' \geq \underline{a}$$

where $n = 1$ denotes the consumer is working and $n = 0$ denotes that the consumer is unemployed and d denotes dividends. A consumer is thus characterized by current assets and human capital,(a, h), together with the employment status. We will assume that the worker's assets are private information so that the firm sees only the worker's productivity. We will posit and then later characterize the equilibrium wages as the outcome of a certain bargaining problem described below that yields $w = \omega(h)$. For a given wage w, asset level a, and human capital h, an employed consumer chooses new assets subject to an a budget constraint and a borrowing constraint:

$$(1) \qquad \tilde{W}(w, a, h) = \max_{a' \geq \underline{a}} u(c) + \beta(1 - \sigma) W(a', h') + \beta\sigma U(a')$$

s.t.

$$c + a' = w + (1 + r) a + d$$

$$h' = \lambda h + (1 - \lambda) \bar{h}$$

where $u(c)$ is a increasing concave function.

The problem of an unemployed consumer is

$$(2) \qquad U(a) = \max_{a' \geq -\underline{a}} u(c) + \beta\lambda_w \int \max[W(a', h), U(a')] dF(h) + \beta(1 - \lambda_w) U(a')$$

s.t

$$(3) \qquad c + a' = b + (1 + r) a + d$$

(4) $a' \geq \underline{a}$.

In the bargaining problem we consider below, for any given distribution of assets there is a minimum initial human capital draw $\underline{h}$ below which the value of the surplus of a match between a firm and a worker is negative. If consumer matches with a firm and draws $h \geq \underline{h}$ the consumer proceeds to the bargaining stage in which the wage $\omega(h)$ is settled on, but if the consumer draws $h < \underline{h}$ the consumer immediately returns to unemployment. In (1) we define $W(a,h)$ as $\tilde{W}(\omega(h),a,h)$ for $h \geq \underline{h}$ and $U(a)$ otherwise. Letting the solution for asset holdings to (1) be denoted $a' = \tilde{a}'_e(w,a,h)$, we also define $a'_e(a,h) = \tilde{a}'_e(\omega(h),a,h)$. Let the solution for consumption bet denoted $\tilde{c}_e(w,a,h)$ and define $c_e(a,h) = \tilde{c}_e(\omega(h),a,h)$. Likewise let $a' = a'_u(a,h)$ and $c_u(a,h)$ denote a solution to (2).

Under this specification of $W(a,h)$ the value function of a unemployed consumer reduces to

$$(5) \qquad U(a) = \max_{a' \geq -\underline{a}} u(c) + \beta\lambda_w \int_{h \geq \underline{h}} W(a',h)\, dF(h) + \beta\left[\lambda_w F(\underline{h}) + (1-\lambda_w)\right] U(a')$$

subject to (3) and (4) with solution denoted $a' = a'_u(a)$ and $c_u(a)$.

Assuming $U(c) = c^{1-\alpha}/(1-\alpha)$ the Euler equations for (1) and (5) can be written as

$$c_e(a,h)^{-\alpha} \leq \beta(1+r)(1-\sigma)\, c_e(a',h')^{-\alpha} + \beta(1+r)\sigma c_u(a')^{-\alpha}$$

with equality if $a' > \underline{a}$ and

$$c_u(a)^{-\alpha} \leq \beta(1+r)\lambda_w \int_{h \geq \underline{h}} c_e(a',h)^{-\alpha}\, dF(h) + \beta(1+r)\left[\lambda_w F(\underline{h}) + (1-\lambda_w)\right] c_u(a')^{-\alpha}$$

with equality if $a' > \underline{a}$.

B. Firms

Firms form matches with consumer, produce output, and pay dividends to consumers. The objective of each firm is to maximize the discounted value of dividends. Each firm has one job that can either be vacant or filled. In any period a firm with an unfilled job can pay a fixed cost κ to create a vacancy and attract a worker at the beginning of the next period with probability $\lambda_f(\theta)$. If a matched consumer draws a human capital level $h \geq \underline{h}$ from the distribution $F(h)$ a match is formed and a wage w is paid, in which case the current value

of profits is $h - w$ and the match remains intact with probability $1 - \sigma$ and dissolves with probability σ. The value function of a firm with a vacancy is

$$V = -\kappa + \frac{1}{1+r}\lambda_f \int_{h \geq \underline{h}} J(h)\, dF(h) + \frac{1}{1+r}\left[\lambda_f F(\underline{h}) + (1 - \lambda_f)\right] V$$

where $J(h)$ denotes the value of a filled job at current wage w is given by

$$\tilde{J}(w,h) = h - w + \frac{1}{1+r}(1-\sigma) J(h') + \frac{1}{1+r}\sigma V \tag{6}$$

subject to

$$h' = \lambda h + (1 - \lambda)\bar{h}.$$

C. Bargaining

To set up the bargaining protocol let $G_e(a,h)$ be the measure of all employed consumers with assets a and human capital h, let $g_e(a,h)$ be the corresponding density, and let $\phi(h) = \int g_e(a,h)\, da$ denote the marginal density of consumers with human capital h. Let $G_u(a)$ be the measure of all unemployed consumers with assets a.

The Harsanyi Selten (1972) solution is as follows. For each level of human capital $h \geq \underline{h}$ a wage w is chosen each period to solve

$$S(h) = \max_w \gamma \int \log\left[\tilde{W}(w(h),a,h) - U(a)\right] \frac{g_e(a,h)}{\phi(h)} da + (1-\gamma)\log\left[\tilde{J}(w(h),h) - V\right] \tag{7}$$

where $S(h)$ is referred to as the surplus of the match between consumers with human capital h and the firm. The surplus is decreasing in h and $\underline{h}$ is defined as the value at which the surplus is zero. The solution to this problem is denoted $\omega(h)$. Clearly, the solution satisfies

$$\gamma \int \frac{\tilde{c}_e(w,a,h)^{-\alpha}}{\tilde{W}(w,a,h) - U(a)} \frac{g_e(a,h)}{\phi(a)} da = (1-\gamma)\frac{1}{\tilde{J}(w,h)}$$

To help develop intuition for how Harsanyi-Selten bargaining works suppose for simplicity that consumers at given level of human capital h have either asset levels a_1 or a_2. Then $p(a_i,h) = g_e(a_i,h)/\phi(h)$ denotes the fraction of consumers with human capital a_i that have asset level a_i. Taking the exponent of $S(h)$ for this case we get that the chosen wage maximizes

$$\left[(\tilde{W}(w,a_1,h) - U(a_1)^{p(a_1,h)} (\tilde{W}(w,a_2,h) - U(a_2)^{p(a_2,h)}\right]^{\gamma} \left[\tilde{J}(w,h) - V\right]^{1-\gamma}.$$

Thus, it is as if the wage $\omega(h)$ is set for the group of consumers with human capital h by a type of Nash bargaining in which the group of consumers with assets a_i have a Nash bargaining weight which is their fraction of the consumers of this type $p(a_i, h)$ times the Nash bargaining weight for consumers overall. Harsayni and Selten (1972) develop an axiomatic foundation for this type of bargaining. We prefer this type of bargaining of standard Nash bargaining in which the firm varies the wage for each consumer not only based on what the consumer produces but also based on how much the consumer has saved. We not only found this prediction intuitively and empirically unappealing but it also gave weird results: when the consumer decides new asset holdings the consumer takes the derivative of next period's wage with respect to this asset choice. The resulting functions were unstable.

For equilibrium we also need i) free entry: the value of a vacancy $V = 0$, ii) dividends: the value of dividends paid to consumers equals the value of dividends paid by firms

$$d = \int_{a,h} (h - \omega(h)) dG_e(a, h)$$

iii) stationarity: the distributions $G_e(a, h)$ and $G_u(a)$ reproduce themselves under the optimal decision rules. Finally, note that the conditions are sufficient since we are considering a small open economy that takes as given the interest rate r.

Note: the accompanying figures are for the Nash Bargaining protocol

2.

Calibration 1

	frequency	1 year
β	discount factor	0.96
r	i-rate	0.02
α	CRRA	2
σ	separation rate	0.10
A	growth rate of h	1.15
B	matching productivity	0.90
γ	worker bargaining share	0.50
θ	elasticity matching fn	0.50
μ	distribution initial h	0.15
$\bar{h}$	highest initial h	3
$h_{\max}$	upper bound h	10
b	home production	1.5
$\bar{a}$	asset limit	-3
θ	market tightness	1

fraction employed	0.76
Prob. worker matches	0.90
Prob. firm matches	0.90
Prob. match accepted	0.35
cutoff h when $a = 0$	1.14
vacancy posting cost, κ	0.18
fraction unempl. constrained	0.22
fraction workers constrained	0.09
income unempl	1.50
mean income worker	2.47
cons. unempl	1.85
cons. worker	2.36
assets unempl	-0.62
assets worker	0.00
average firm profits	2.12
average growth of w	1.028
average growth of h	1.052

Figure 1: Life Cycle wages and surplus

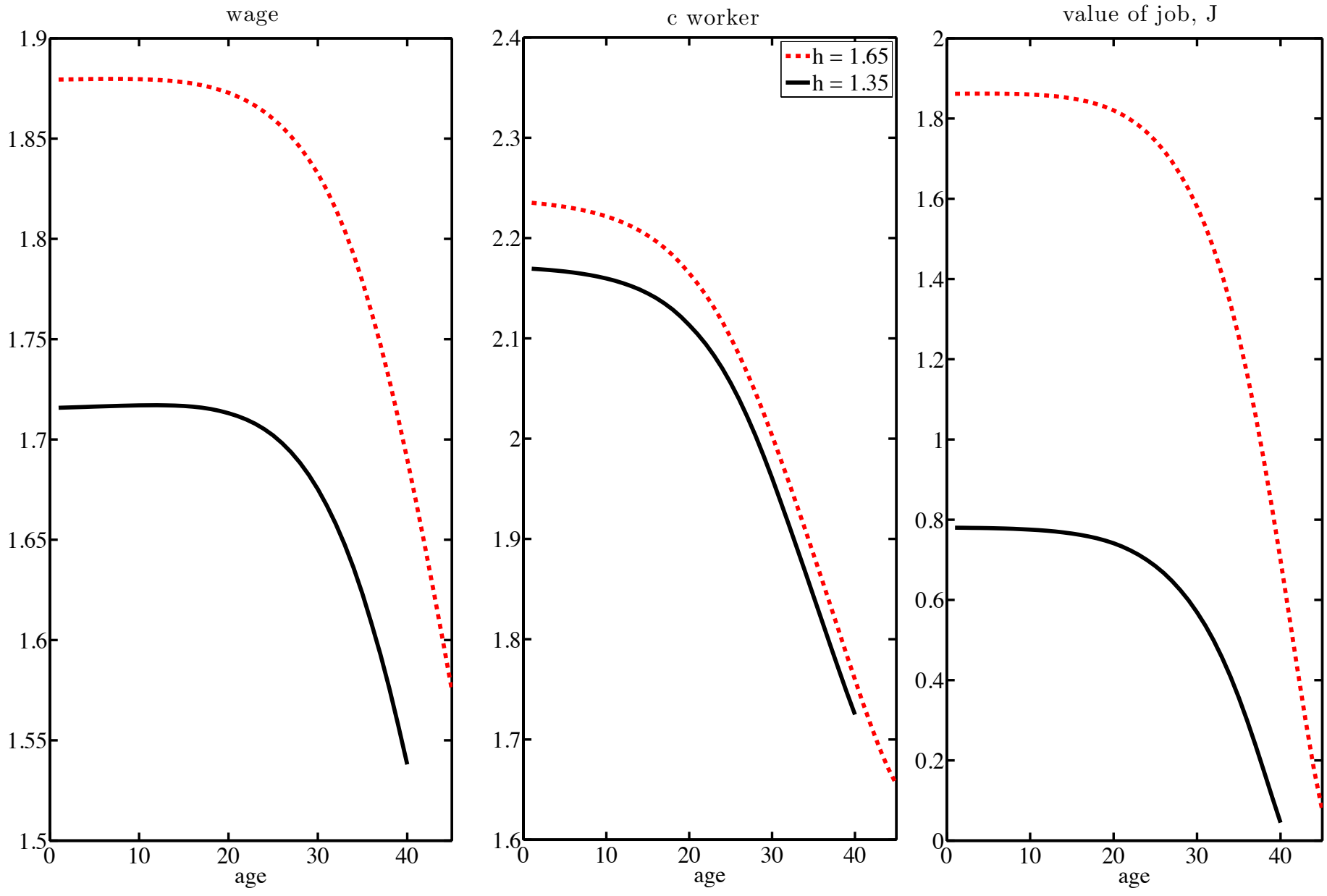


Figure 2: Wages vs. Human Capital

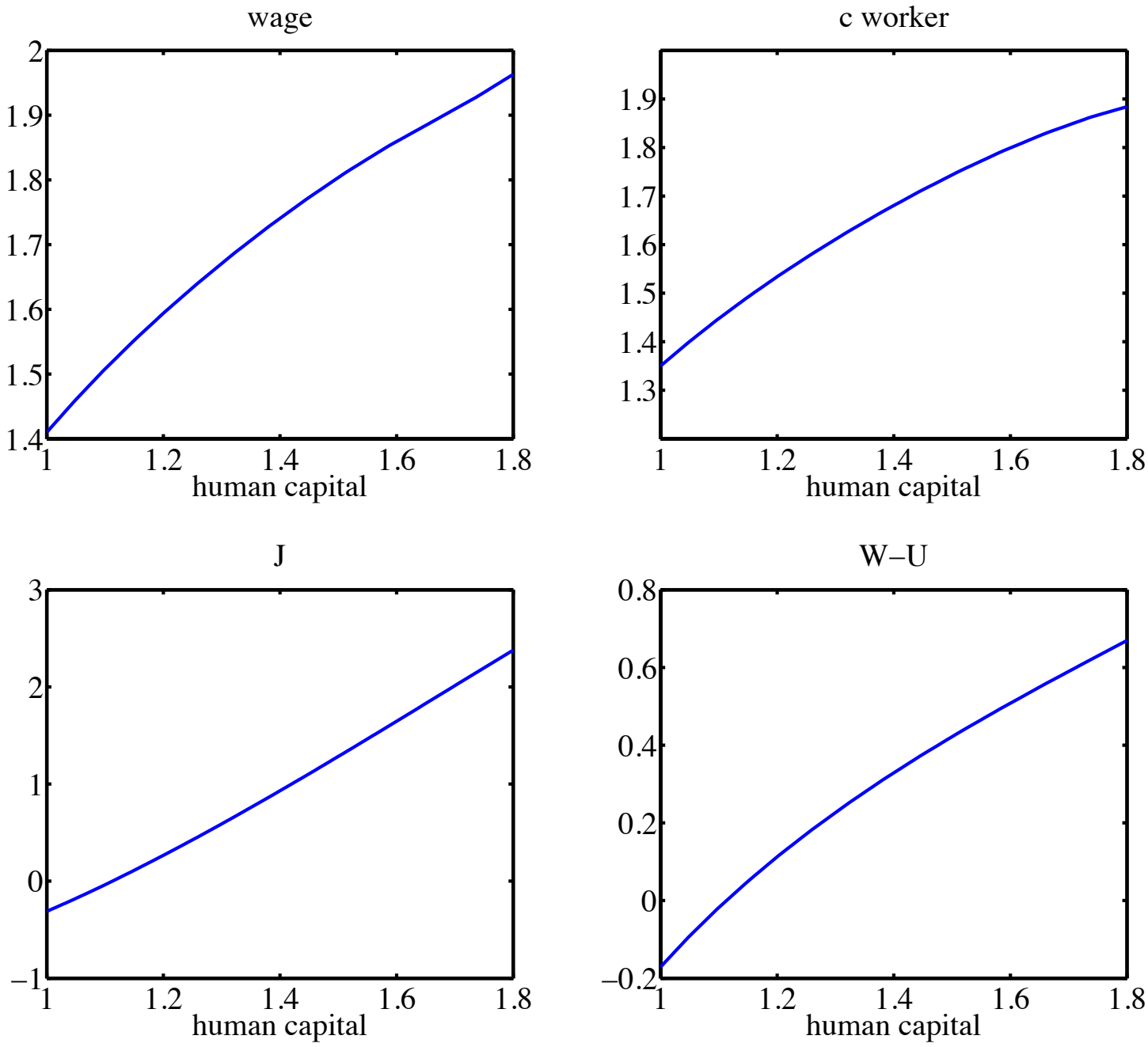


Figure 3: Wages vs. Assets, $h = 1.125$

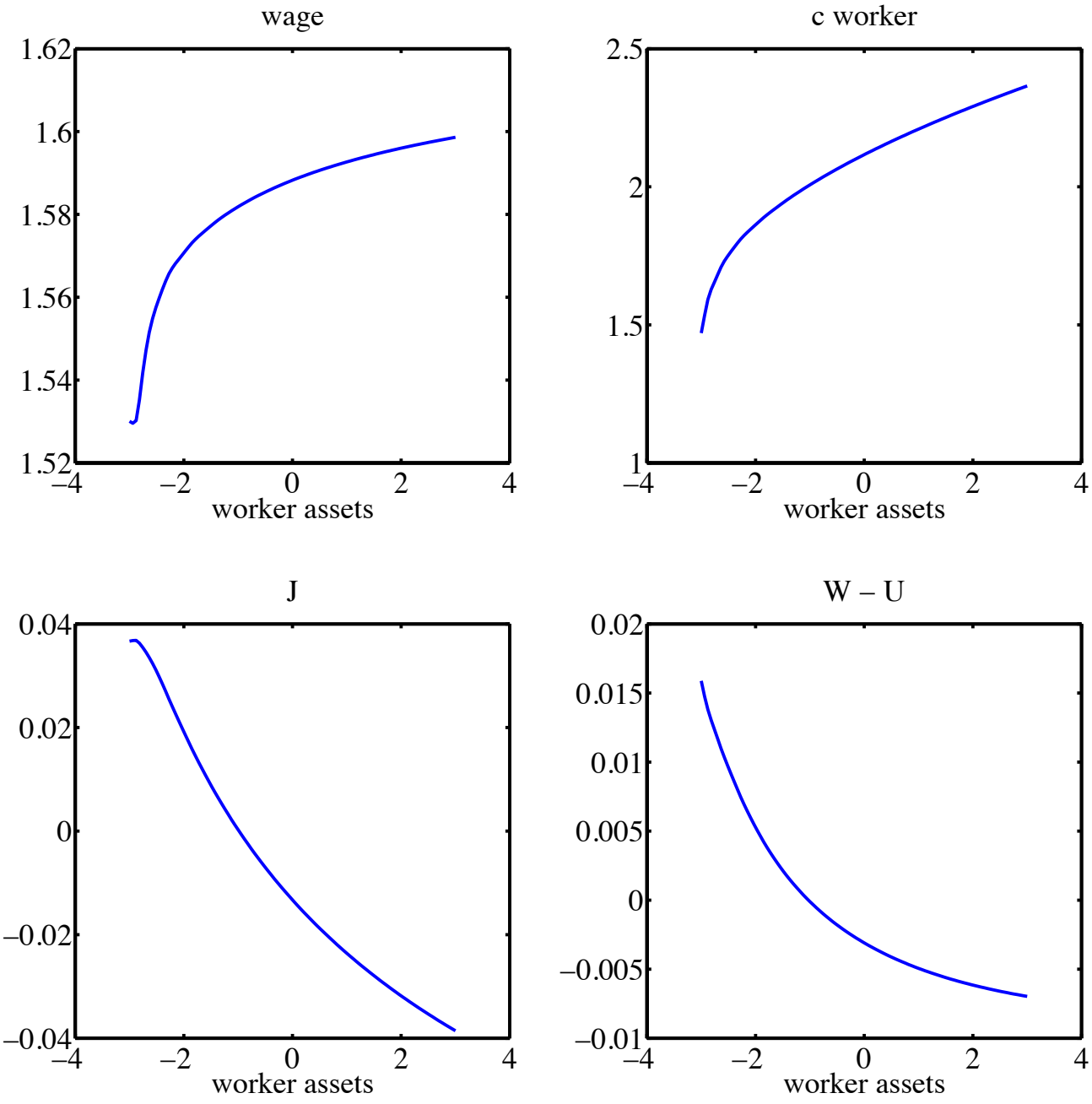
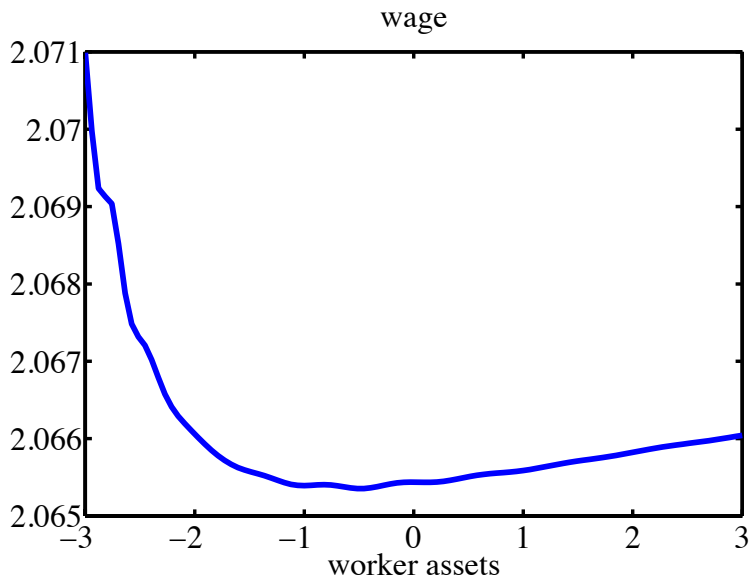
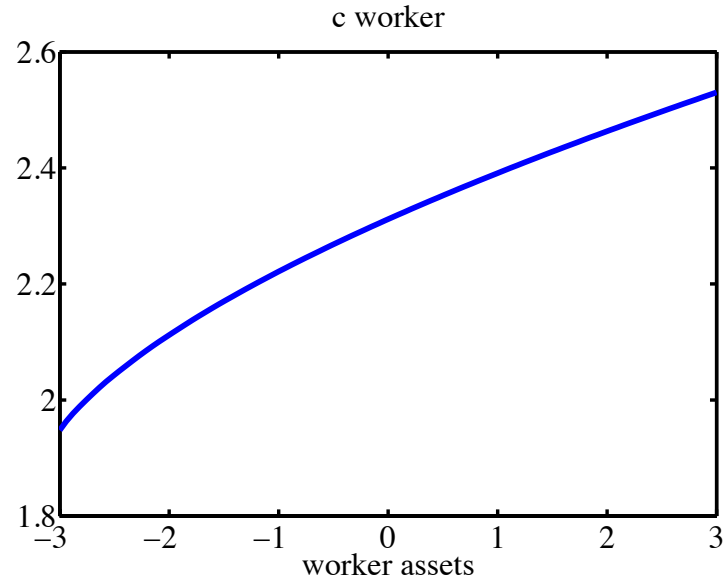
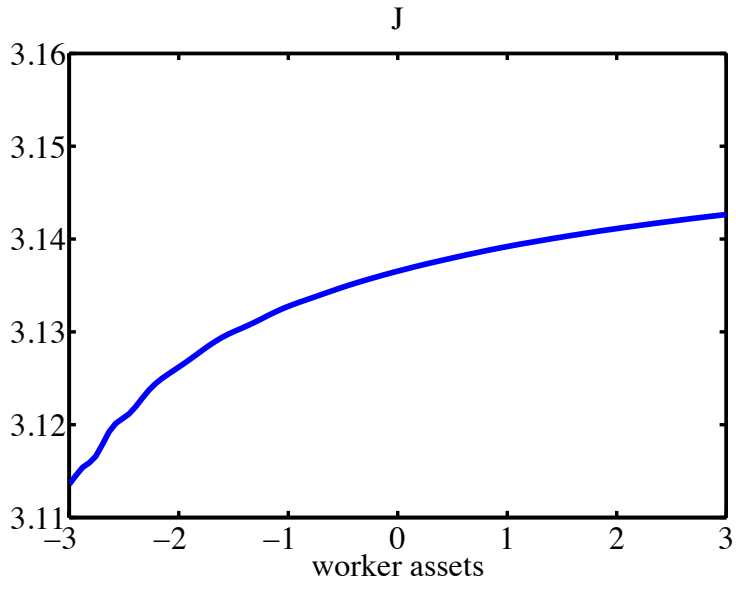


Figure 4: Wages vs. Assets, $h = 2$







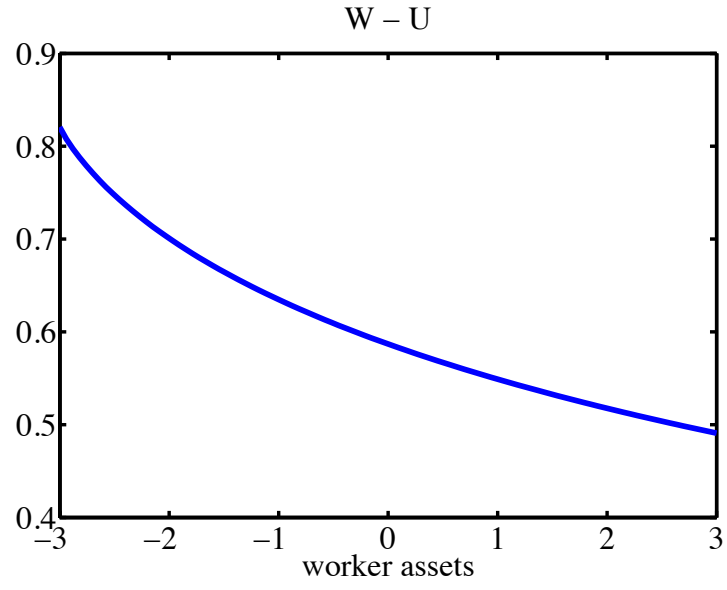
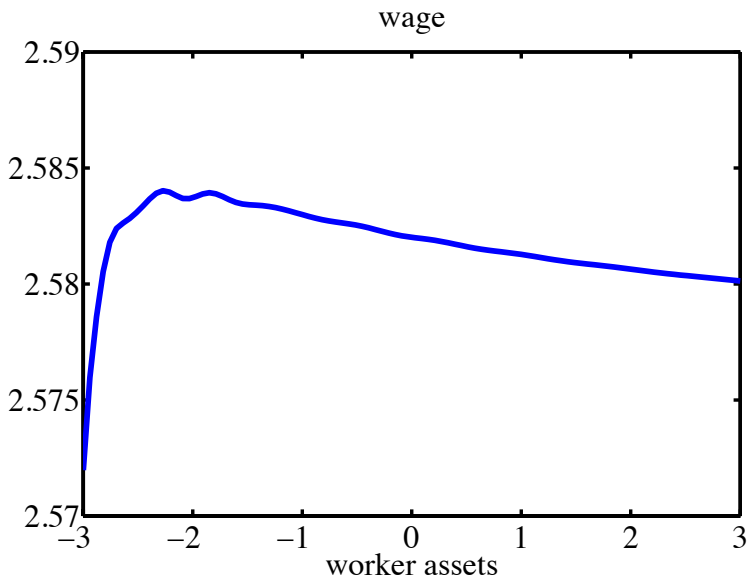
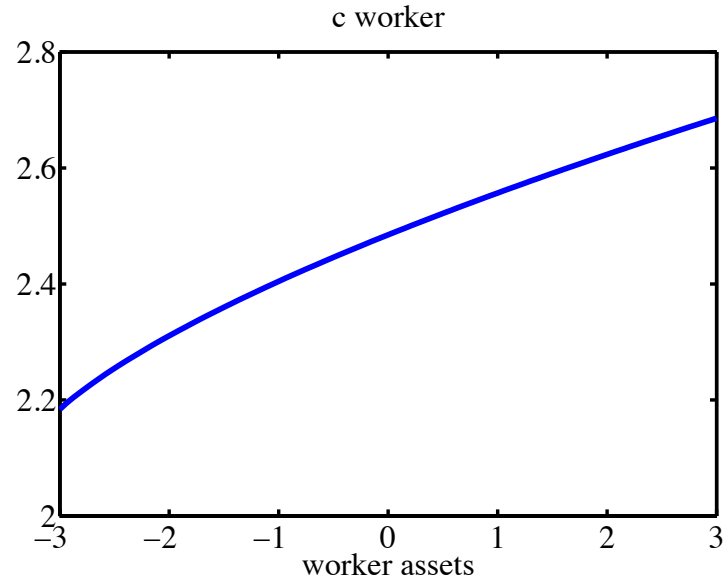
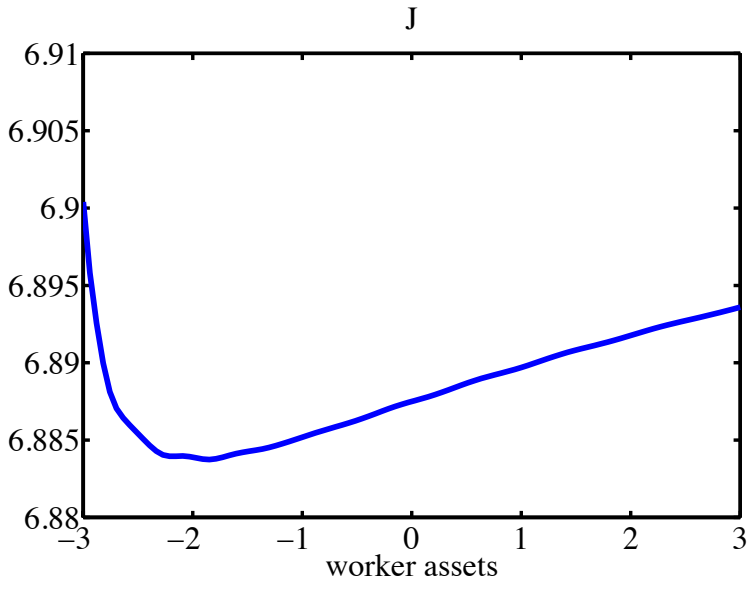
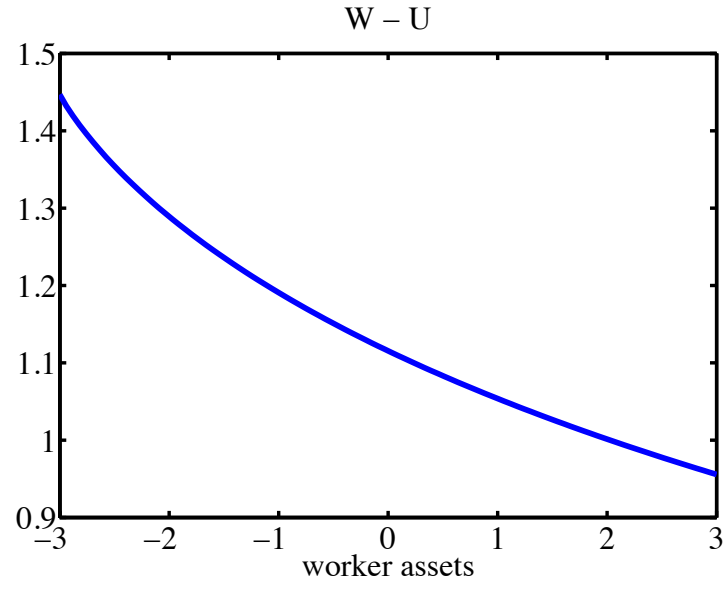


Figure 5: Wages vs. Assets, $h = 3$







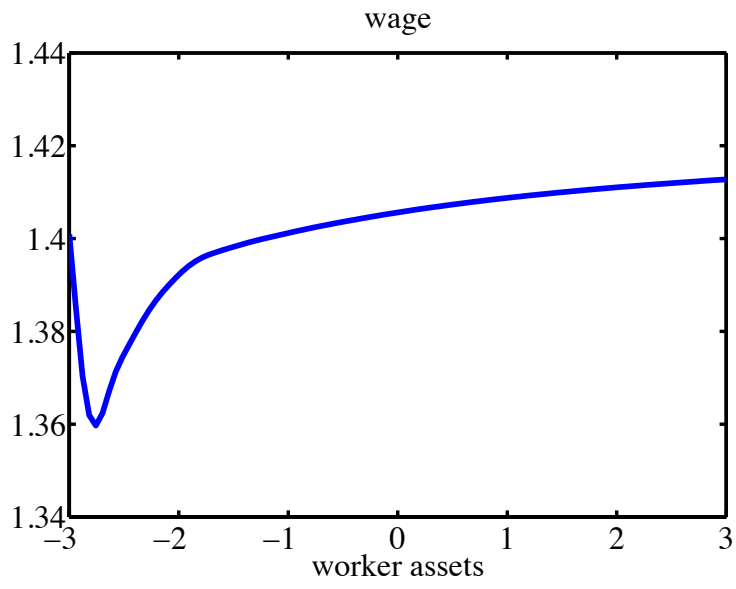


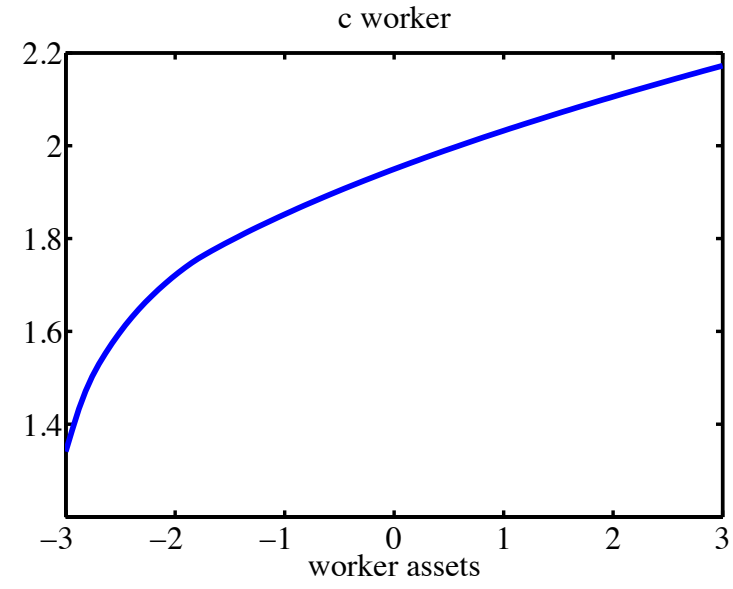
Calibration 2: lower matching probability

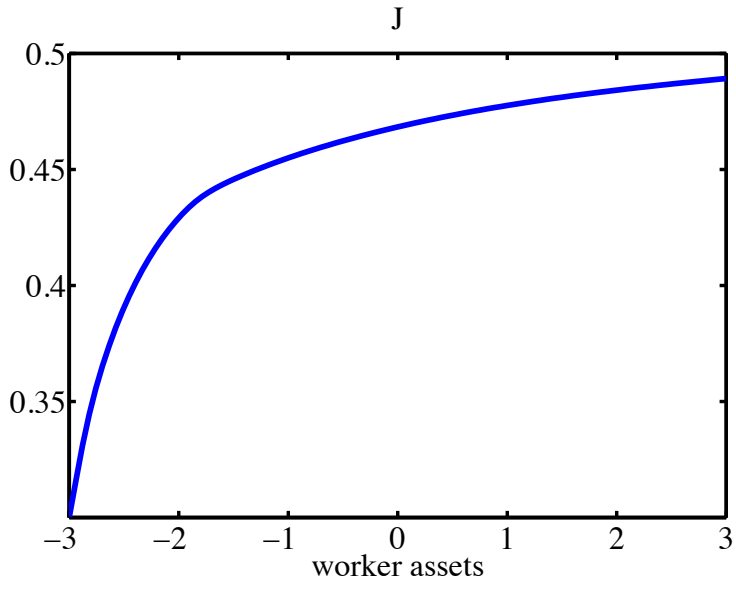
	frequency	1 year
β	discount factor	0.96
r	i-rate	0.02
α	CRRA	2
σ	separation rate	0.10
A	growth rate of h	1.15
B	matching productivity	0.25
γ	worker bargaining share	0.50
θ	elasticity matching fn	0.50
μ	distribution initial h	0.15
$\bar{h}$	highest initial h	3
$h_{\max}$	upper bound h	10
b	home production	1.5
$\bar{a}$	asset limit	-3
θ	market tightness	1

fraction employed	0.71
Prob. worker matches	0.25
Prob. firm matches	0.25
Prob. match accepted	1
cutoff h when $a = 0$	1
vacancy posting cost, κ	0.38
fraction unempl. constrained	0.30
fraction workers constrained	0.22
income unempl	1.50
mean income worker	2.16
cons. unempl	1.69
cons. worker	2.07
assets unempl	-1.04
assets worker	0.49
average firm profits	1.71
average growth of w	1.036
average growth of h	1.068

Figure 6: Low B and $h = 1$







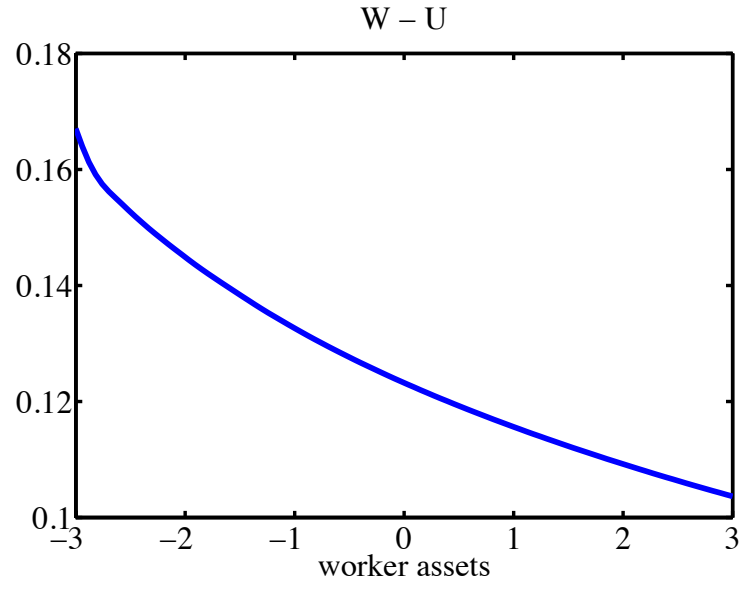


Figure 7: Low B and $h = 1.125$

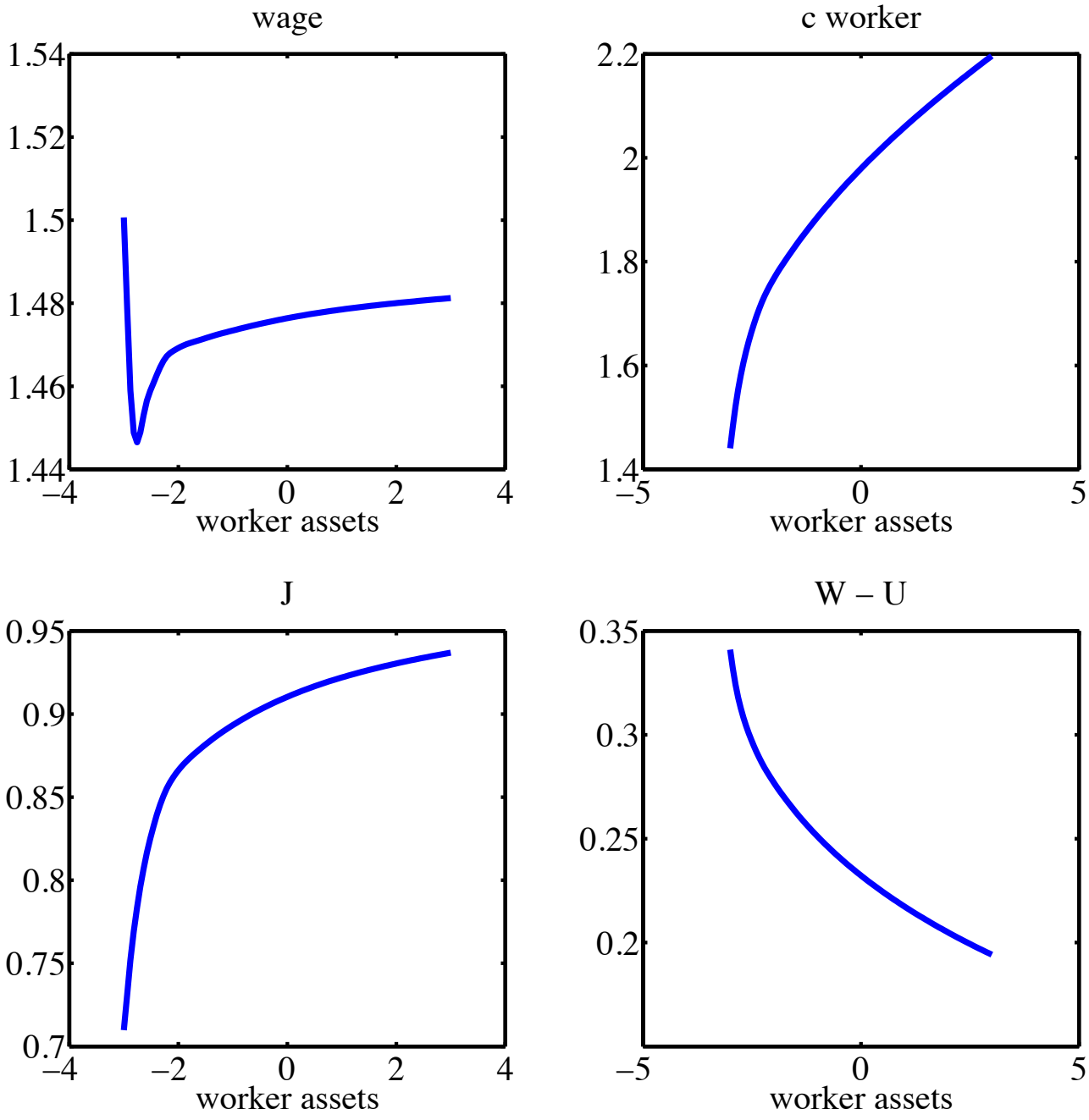


Figure 8: Low B and $h = 2$

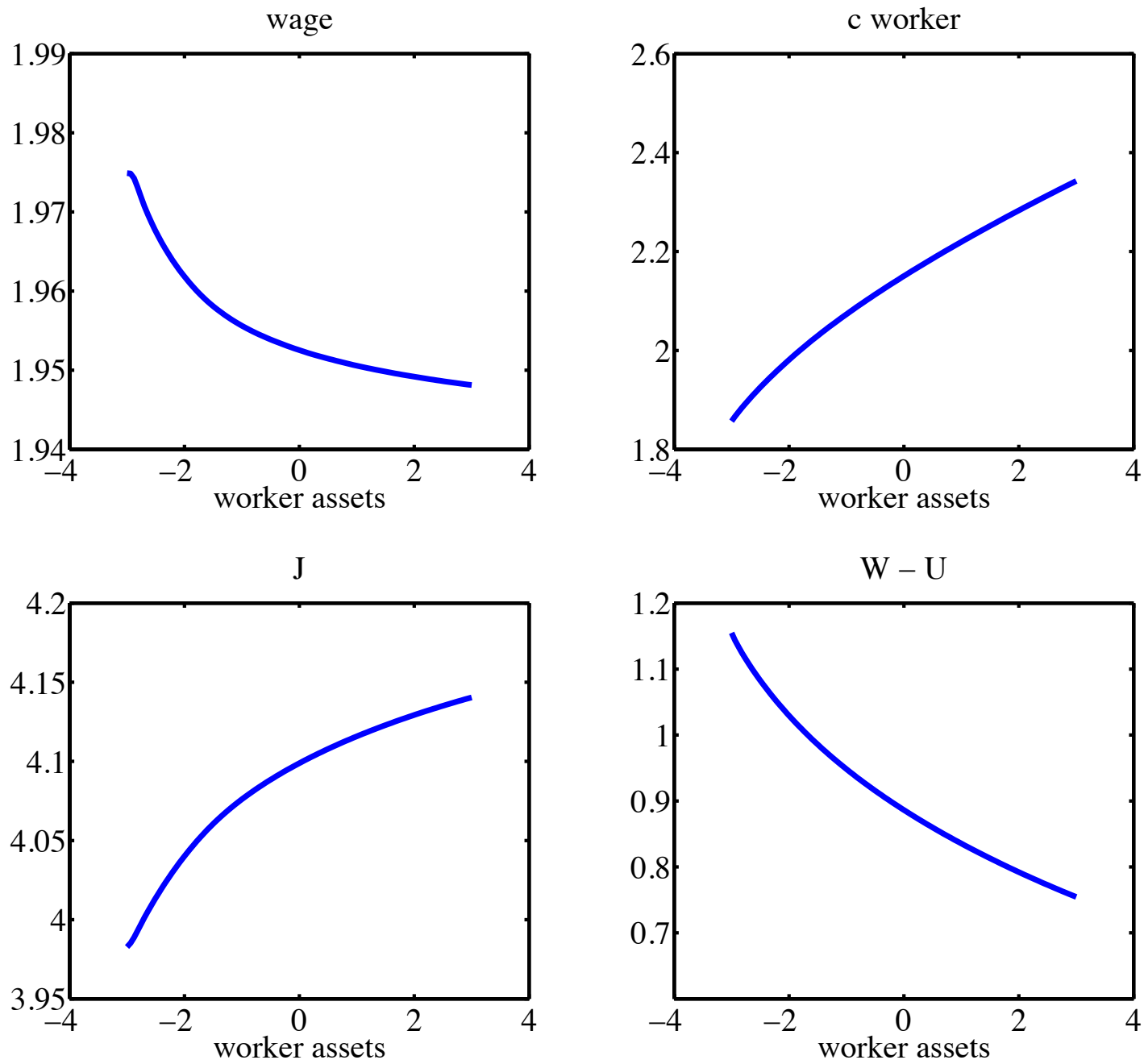


Figure 9: Low B and $h = 3$

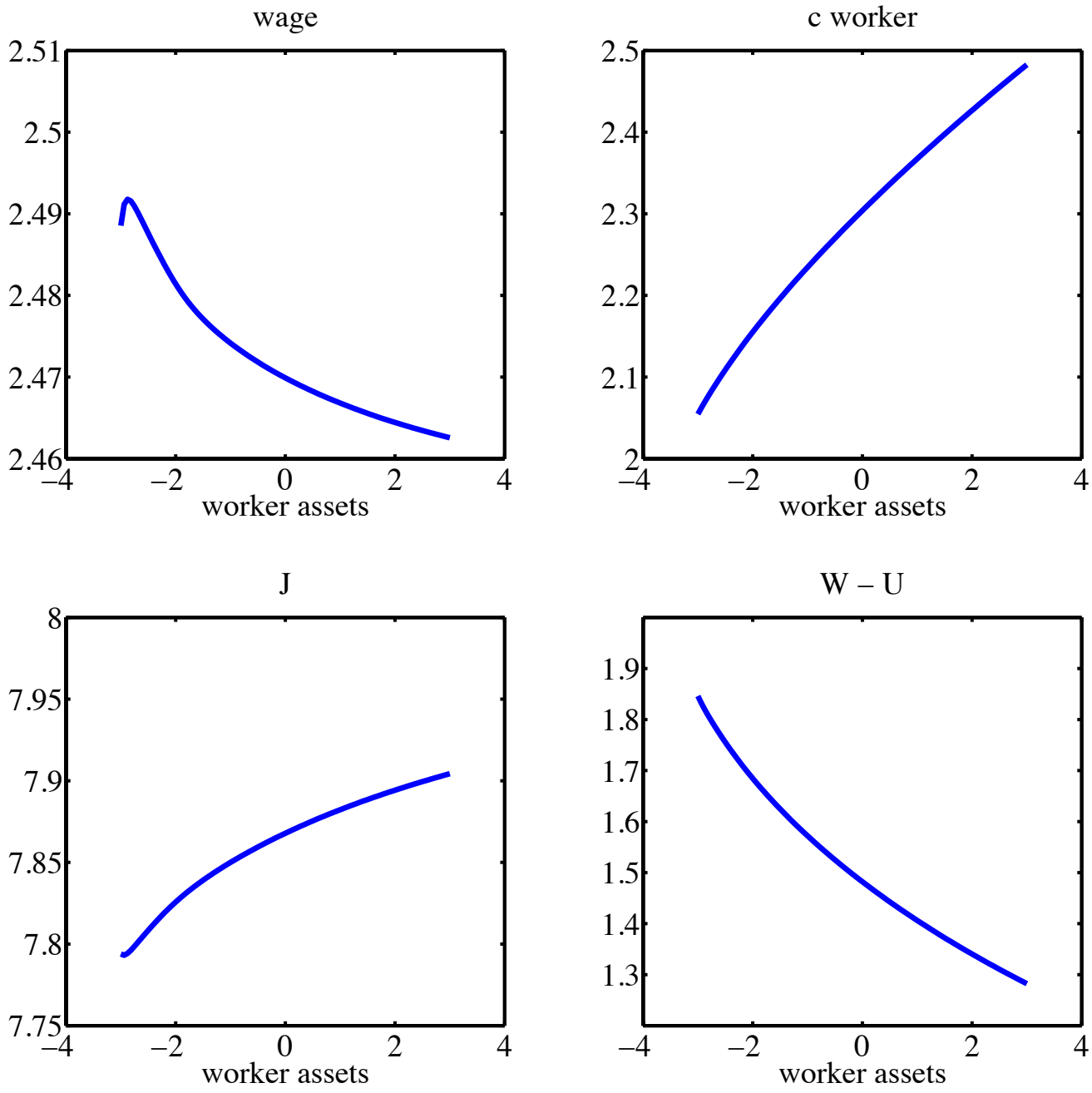
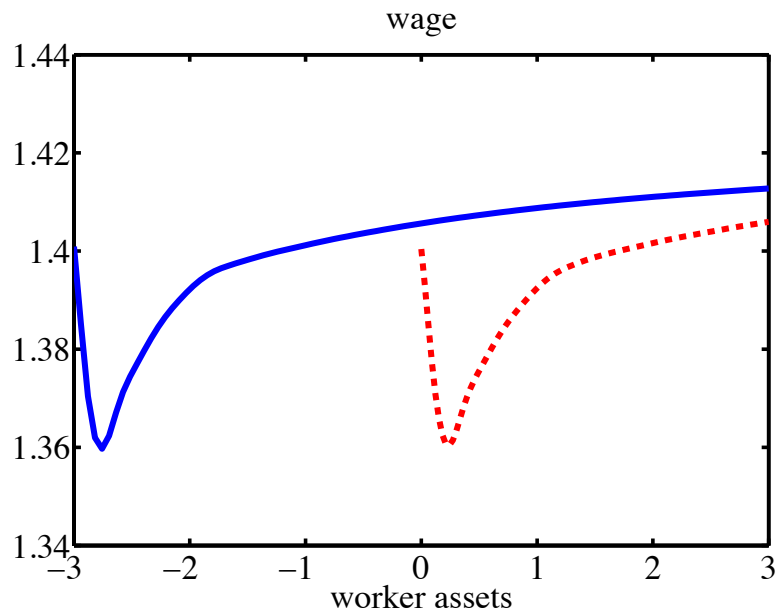
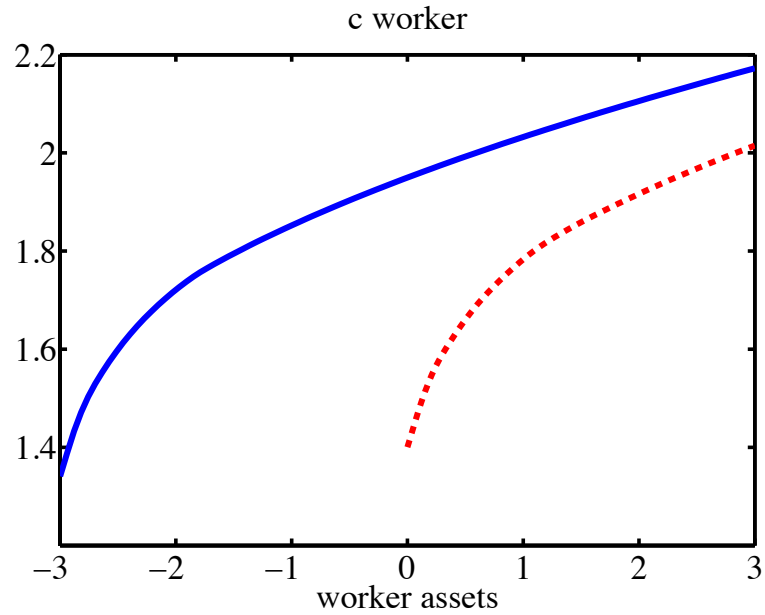
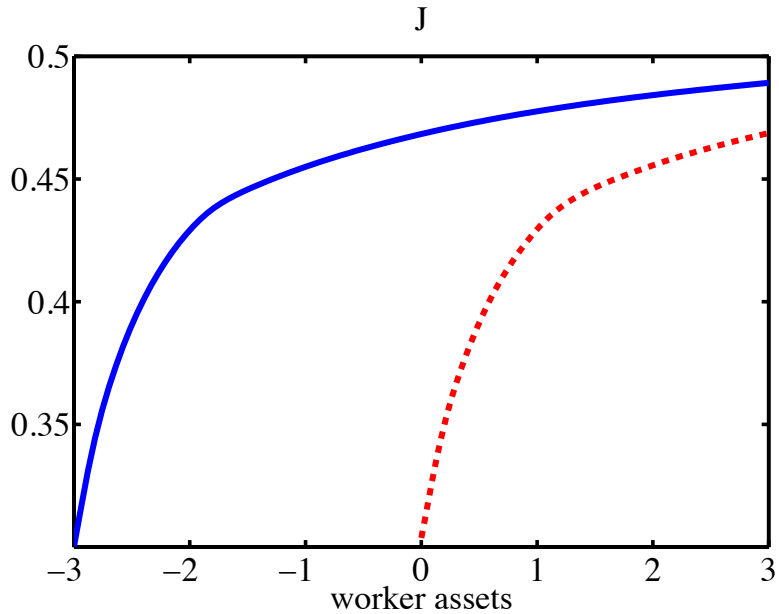


Figure 10: Changing SS debt limit from 3 to 0: $h = 1$







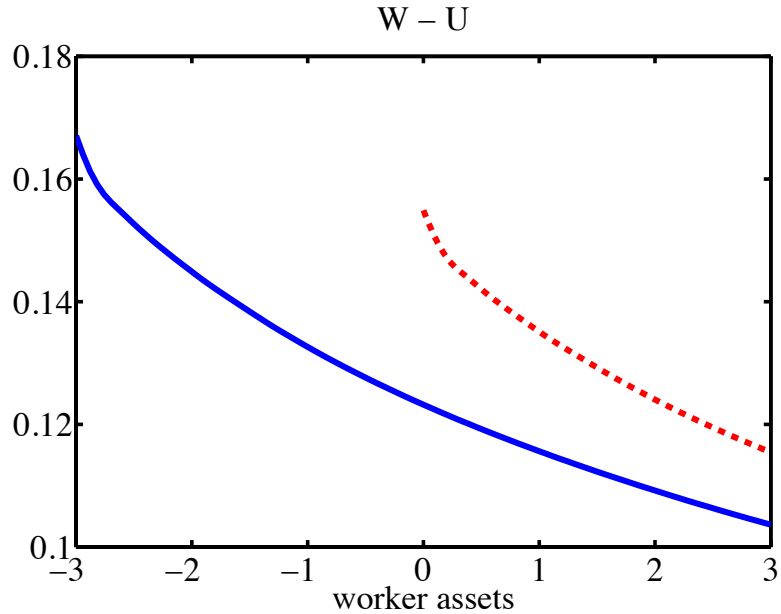
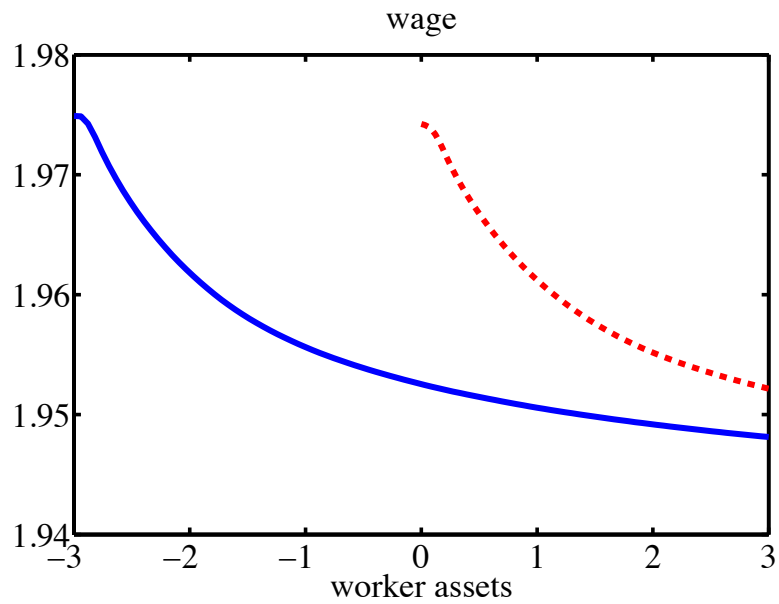
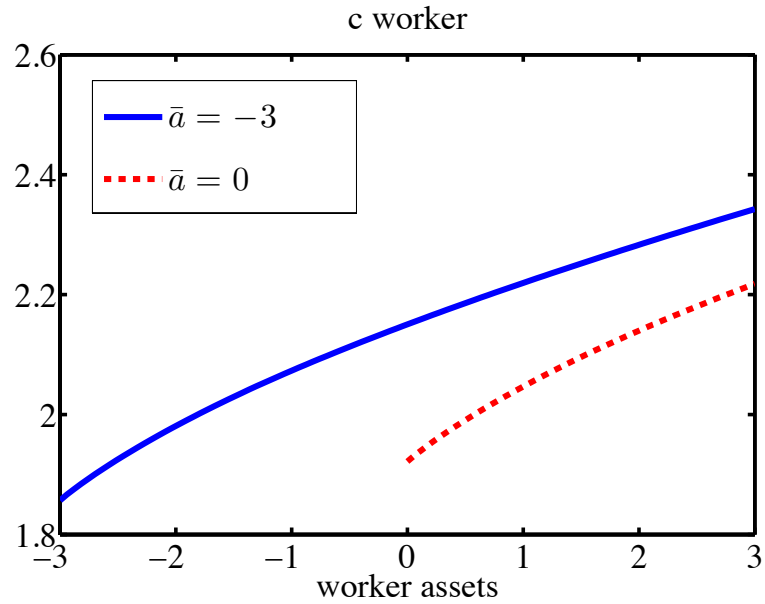
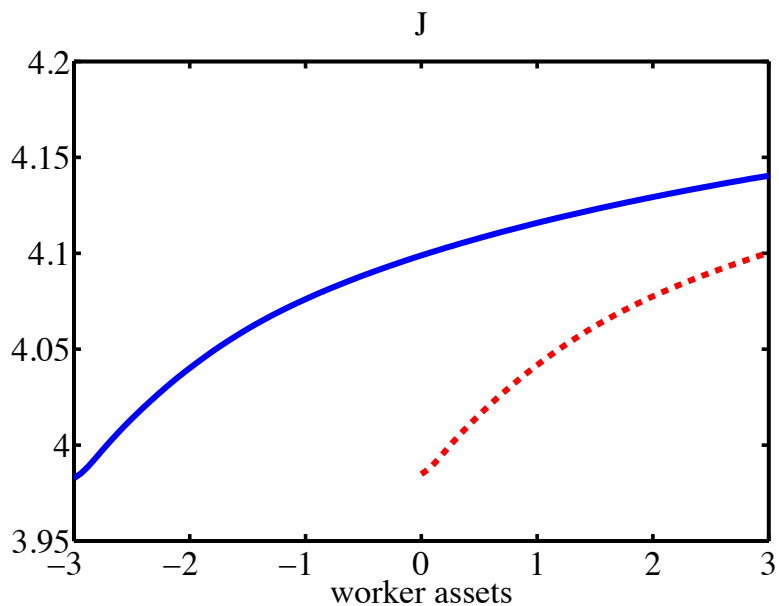
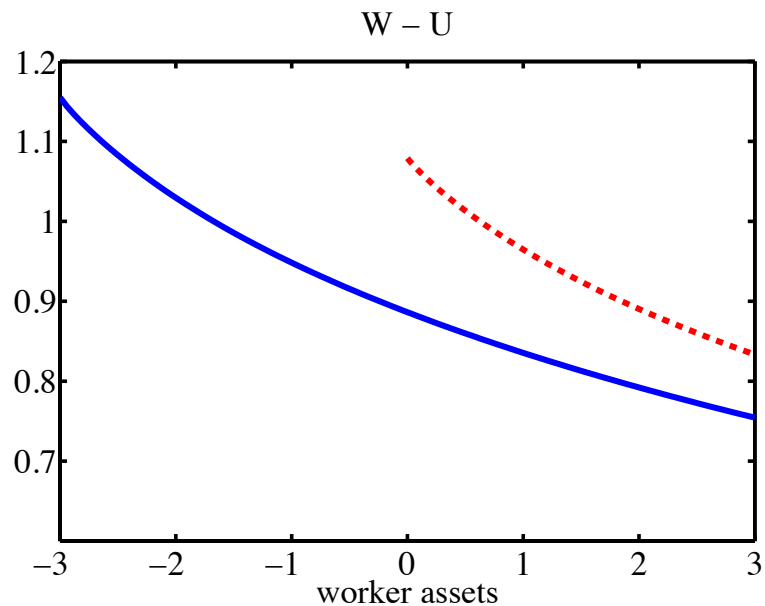


Figure 11: Changing SS debt limit from 3 to 0: $h = 2$









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