# Clearing, transparency, and collateral *

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**Abstract**

In an environment of Over-The-Counter trading with adverse selection we study traders' incentives to screen their counterparties under different clearing arrangements. When the clearing arrangement is also a choice, traders decide which types of transactions to clear under each arrangement, with significant consequences for transparency and collateral requirements.

The key trade-off is between insurance and the value of information: on one hand risk averse traders want to smooth consumption and on the other hand they want to extract the largest feasible surplus from their counterparties. Choosing an arrangement that provides insurance, however, may prevent them from taking advantage of the private information they learn about their counterparty. In fact, any arrangement involving risk pooling in equilibrium is inconsistent with any costly screening.

As a result, insurance comes at the cost of losing transparency: when clearing arrangements differ in the degree of risk sharing they implement, then they also differ in the degree of transparency arising in equilibrium. This has significant consequences on the role of collateral. In environments with limited commitment, collateral plays two roles for risk averse traders: it is a means to discipline incentives and to self-insure at the same time. When insurance is provided by a clearing arrangement that implements risk sharing, collateral is primarily used to discipline incentives, since risk sharing comes at the cost of less information about the limited commitment of the counterparty. The opposite is true in the equilibrium with a clearing arrangement that cannot provide risk sharing but gives traders sufficient incentives to screen their counterparties.

Keywords: Limited commitment, central counterparties, collateral

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# 1 Introduction

Relationships between traders are a key feature of financial markets: financial institutions can learn valuable information from multiple interactions with each other. In this paper we highlight a novel channel through which the information content of a financial transaction is affected: the clearing arrangement that is chosen for that transaction. Specifically, we study the consequences that different clearing arrangements have on the flow of information—which we refer to as transparency—to the clearing agent, and on the collateral required to insure against counterparty risk.

Modern financial institutions trade a variety of products bilaterally, such as over-the-counter (OTC) derivatives, repurchase agreements and even reserves at the central bank. Some of these markets are concentrated, with a few institutions holding a very large share of the total volume of trades on their balance sheets, and others are strongly connected, with most institutions having few counterparties. [1] Thus, relationships may be very important to counterparties in a financial transaction as unique information can be acquired, for example, by multiple interactions with a given institution. [2]

The recent financial crisis has highlighted the importance of these relationships and of the flow of information that they may carry through financial markets[3] and different clearing arrangements. [4]

Both academic research and policy makers[5]have argued that the lack of transparency in over-the-counter markets may have led to uncertainty in traders' positions during the crisis, which induced market participants to pull away from certain institutions and exacerbated their financial stress. In recent work Acharya and Bisin Acharya and Bisin (2014) argue that central counterparty clearing[6] has the potential to improve the degree of transparency in financial markets: the clearing entity interposes itself between the two parties to a transaction, becoming the buyer to every seller and the seller to every buyer. Therefore the clearing agent can observe all products traded by all the institutions for which it performs clearing, and can then offer individual institutions' pricing schedules for trades[7] that are contingent not just on publicly observable characteristics but also on its own knowledge of other trades. Also, because the central counterparty (CCP) makes the payments on behalf of the original trading counterparties, rather than trying to assess its exposure to all of its trading partners, a market participant needs to manage only its exposure to

[1]See ?, `http://www.newyorkfed.org/banking/tpr_infr_reform_data.html` (2014), `http://www.newyorkfed.org/markets/gsds/search.html` (2014); and for the Federal Funds market Afonso and Lagos (2012a), Afonso and Lagos (2012b), Bech and Atalay (2010).

[2]Either over time or across different types of financial transactions.

[3]Among many, see Caballero and Simsek (2009), Zawadowski (2011), and Zawadowski (2013).

[4]See Acharya and Bisin (2014), Biais, Heider, and Hoerova (2012a), Biais, Heider, and Hoerova (2012b), Koeppl, Monnet, and Temzelides (2012), Koeppl and Monnet (2013).

[5]See Acharya and Bisin (2014), and Powell (November, 21$^{st}$ 2013), Duffie, Li, and Lubke (2010), Jackson and Miller (2013)

[6]For a formal definition see for International Settlements (????).

[7]In practice, collateral arrangements and exposure limits.

the central counterparty (Biais, Heider, and Hoerova, 2012a). And, through its better knowledge of the market, a CCP can impose more effective risk controls on clearing members. Thus counterparty risk can be appropriately insured.

Other work (Pirrong (2009)) has suggested that costs arising from asymmetric information may instead be higher in centrally cleared markets than bilateral ones, especially for complex products traded by complex, opaque intermediaries. [8]

Our paper formalizes this idea and studies a mechanism, different from that in Acharya and Bisin (2014), through which central counterparty clearing can affect transparency in over-the-counter markets, resulting in the opposite effect. In our model, financial transactions are bilateral and subject to limited commitment and private information: trading partners can pay a cost to learn the severity of the commitment problem of their counterparty—their type—and can then condition the contract on that information. An agent chooses to learn his counterparty's type if the expected surplus he can extract from trading with such information exceeds its cost.

Also, because the product traded is subject to a shock, realized only after the contract is agreed upon, then a risk averse agent seeking to buy that product would also like to insure against the shock. When clearing is bilateral no such insurance is feasible. When clearing is done through a CCP, shocks that are idiosyncratic to a bilateral trade can be insured by mutualizing losses. [9]

Although it can provide risk pooling, the CCP may not have the same relationship with its members as bilateral counterparties have with each other. In particular, in our model the CCP cannot learn the severity of the commitment problem of the trading partners for which it does clearing. Thus, it has to elicit that information directly from the contract that the counterparties submit for clearing, and sets collateral requirements (i.e. margin) accordingly.

The key trade-off is then faced by trading partners with respect to their terms of trade: if they pay the cost to learn their counterparty's type and choose to clear the transaction centrally, then loss mutualization implies that they will not be able to extract the surplus that their trade generates by taking advantage of the private information they have learned. On the other hand, they will be insured against the idiosyncratic shock to the product they are about to purchase. As a result, when the value of insurance against the idiosyncratic shock is *large* then agents choose not to learn their counterparty's type and to clear the transaction centrally. In equilibrium collateral is then primarily used to discipline the limited commitment friction. When instead the value of idiosyncratic risk insurance is *small* relative to the expected gains from information (net of the cost of acquiring it) then agents choose to pay the cost to learn their counterparty's type. In equilibrium, if the counterparty turns out to be *good* then they choose to clear bilaterally: collateral is then used as insurance against the idiosyncratic shock. If the counterparty turns out to be

[8] Dealers in bilateral markets specialize in valuing complex derivatives contracts and have opaque and risky balance sheets. Thus they are more effective than a CCP at monitoring and pricing the counterparty risk of other dealers (Pirrong (2009), pg. 4).

[9] In practice this is done through a default or guarantee fund.

*bad* then they have nothing to lose from clearing centrally, so they choose to do so. The loss mutualization implemented by the CCP will in fact provide them with insurance against the idiosyncratic shock without any forgone gains from having to give up a *good* counterparty.

We also show that whenever central counterparty clearing is a feasible option for over-the-counter trades, transactions involving the riskiest assets are submitted to the central counterparty for clearing. On the other hand, the riskier the counterparty is ex-ante, the higher the value of information about its creditworthiness is. Thus, for a given distribution of asset returns, a riskier counterparty leads to stronger incentives for traders to acquire information about that counterparty and clear bilaterally to extract the value of such information.

Notice that our results on transparency and collateral are robust to different trading assumptions. Suppose that, rather than trading bilaterally, agents contract directly with a pool of counterparties, so to diversify the idiosyncratic shock. Then, as long as the screening can be done only by one agent and the information does not become publicly known, agents always have an incentive not to screen. Thus no equilibrium with risk sharing and screening exists. [10]

## 1.1 Related Literature

Our paper relates to a recent literature that studies how changes in financial market infrastructure influence the exposure to default risk, as well as market liquidity. Part of this literature has focused on the emergence of Central CounterParties as efficient institutions that help achieving Pareto superior allocations. Within this literature, Koeppl and Monnet (2010) focus on counterparty risk insurance as the key mechanism that affects welfare: central counterparty clearing improves on bilateral clearing by providing market participants with better insurance against counterparty default. Such insurance is provided by a mechanism that uses novation and mutualization of losses, therefore resembling a CCP. Acharya and Bisin (2014) stress the welfare enhancing effect of central clearing on transparency. The key friction in their paper is the non observability of the trading positions that a counterparty in a financial transaction has with third parties. When the exposure to third parties can cause a counterparty to default, any trader who cannot observe this exposure will fail in correctly accounting for the counterparty default risk. Thus the non observability of trading positions imposes an externality that cannot be captured by competitive prices. The introduction of a CCP can correct this externality: by observing all products traded by all the institutions for which it performs clearing, the CCP can offer prices that convey all the relevant information. Thus the equilibrium is efficient. Duffie and Zhu (2011) focus on the provision of efficient clearing services: by clearing through a CCP market participants benefit from the *multilateral netting* of financial contracts with the CCP members. However, by clearing

[10] The only exception being, in our model, the case where the value of idiosyncratic risk insurance is *small* and agents learn that they face a *bad* counterparty, so that they prefer to clear centrally and be insured. However, given an arrangement that provide risk sharing, rather than agents being able to choose it, screening never occurs.

through a CCP, market participants lose from not being able to bilaterally net different financial contracts with the same bilateral counterparty. If the former effect is stronger than the latter, then a CCP is welfare improving. Monnet and Nellen (2012) focus on two-sided limited commitment, and show that the introduction of a technology that allows to pledge collateral at a third party, a segregation technology, improves welfare with respect to bilateral clearing, and that a CCP is able to offer additional added-value through novation and mutualization. Finally, Carapella and Mills (2011) highlight a liquidity enhancing role for CCPs: by providing services that are valuable to market participants when they trade, CCPs reduce the (opportunity) costs related to trading. Thus they facilitate socially desirable transactions that would otherwise not occur (due to adverse selection arising endogenously in their model).

We focus on novation and mutualization of losses as key features of central clearing, and how they affect traders' incentives to learn information—that is socially valuable—about their trading partners.

In this respect, our paper is closer to Koeppl (2012) and Biais, Heider, and Hoerova (2012a), who study to what extent clearing can improve on the allocation of risk, and whether clearing should be bilateral, decentralized, or centralized. Koeppl (2012) considers an environment with moral hazard in which, depending on the severity of the moral hazard problem, market participants can use collateral either as an incentive device, or an insurance device. When a CCP cannot observe the degree of moral hazard, it can only use collateral either as an incentive or as an insurance device. If the CCP opts to use collateral as an incentive device, by preventing insurance contracts to take place, central clearing can have the unintended consequence of forcing collateral to increase for all contracts.

Biais, Heider, and Hoerova (2012a) study an environment with idiosyncratic risk, aggregate risk, and moral hazard. Specifically, in their paper, risk averse protection buyers own an asset that is subject to an idiosyncratic and to an aggregate shock. To cope with these shocks buyers can purchase insurance from risk neutral protection sellers; the problem is that protection sellers themselves may default. In this context, a clearing entity can insure protection buyers against the default of their counterparty, and when clearing is centralized a central clearing entity, a CCP, can offer insurance to all protection buyers. The benefit of central clearing is that it allows buyers to diversify their idiosyncratic risk: protection buyers can pool risk among themselves. Nevertheless, central clearing cannot provide insurance against aggregate risk, and sellers come into play. Because protection sellers differ in their probability of defaulting, and protection buyers can exert effort to find relatively safer protection sellers, when aggregate risk is significant, it is efficient to have buyers exerting effort to find safe counterparties. Nevertheless, when such effort is privately costly, incentive compatibility requires the CCP to not fully insure protection buyers against their idiosyncratic risk.

# 2 The Model

Time is discrete and consists of two periods, $t = 1, 2$. The economy is populated by two type of agents: a measure one of *lenders* and a measure one of *borrowers*, who have different preferences and have access to different technologies.

There are two goods, a consumption good and a capital good. The latter can be invested in time $t = 1$ and transformed, via a linear technology, into time $t = 2$ consumption good. Nevertheless, such a technology is available only to borrowers. This technology is indivisible and takes one unit of capital good, returning $\widetilde{\theta}$ units of consumption good in $t = 2$. The realization $\widetilde{\theta}$ is a random variable, with support $\{0, \theta\}$ and $p = Prob(\widetilde{\theta} = \theta)$, and it is unknown when the investment is undertaken.

On the other hand, the consumption good can be stored from $t = 1$ to $t = 2$ both by lenders and borrowers, but the latter enjoy more consumption in the first period. Specifically, a borrower preferences are defined over time $t = 1$ consumption $c_1$ and time $t = 2$ consumption $c_2$, and are represented by the utility function

$$U(c_1, c_2) = \alpha c_1 + c_2 \qquad \alpha > 1$$

In the first period, lenders receive an endowment of one unit of capital, while borrowers receive an endowment of $\omega$ units of consumption good.

In this economy resources are in the wrong hands: lenders have capital but they need borrowers to use their technology to transform it into consumption goods. This creates incentives from trade. Nevertheless, trade is subject to two frictions. First, each lender is randomly matched and can only contract with one borrower: trade is bilateral. Second, borrowers have limited commitment to repay: a borrower can repudiate a contract and, after default, get away with a fraction $1 - \lambda_i$ of the output realization. We assume that there are two types of borrowers: a measure $q$ of good borrowers, indexed by a parameter $\lambda_H$, and a measure $1-q$ of bad borrowers, indexed by the parameter $\lambda_L$, where $\lambda_H > \lambda_L$[11]. The type $\lambda_i$ is private information of the borrower, but can be learned by a lender by investing some resources into monitoring effort. Specifically, the preferences of a lender are defined only over second period consumption $x_2$, and time 1 monitoring effort $e_1$, according with the utility function

$$U(x_2) = u(x_2) - \gamma \cdot e_1$$

where $u$ is strictly increasing and strictly concave function, and $e_1 \in \{0, 1\}$. We further assume that $\lim_{x \to 0} u(x) = \infty$.

Contracts are negotiated bilaterally by a lender and a borrower in each match, but they can be settled bilaterally or centrally (via a Central counter-Party)in the second period. In the next sections, we take the market structure as given, and then we compare the properties of the solutions with central and bilateral clearing.

[11] Since $\lambda_H > \lambda_L$, borrowers with $\lambda_H$ can abscond a lower fraction of output. In this sense, $\lambda_H$ borrowers are good borrowers.

# 3 Bilateral clearing

The timing with bilateral clearing is the following: lenders and borrowers are randomly matched at the beginning of $t = 1$. In very match, a lender chooses whether to verify the counterparty type, $e_1 = 1$, nr not, $e_1 = 0$.

If the lender verifies the counterparty's type, he offers a contract $\left(x^i_{2,h}, x^i_{2,l}, c^i_1, c^i_{2,h}, c^i_{2,l}\right)$, where $x^i_{t,s}$ and $c^i_{t,s}$ are respectively the lender and the borrower consumption in time $t$ and state $s$, when the match is between a lender and a type $i$ borrower. A contract is then generally indexed by the borrower's type $i$, and second period consumption is indexed by idiosyncratic state $s \in \{h, l\}$. If the borrower accepts the contract, (i) the lender transfers the one unit of capital to the borrower, (ii) the borrower transfers the lender the consumption good $\omega$ and (iii) receives in exchange consumption $c^i_1$. We can think of the difference $\omega - c^i_1$ as collateral. The borrower then invests the one unit of capital in his technology, while the lender stores the consumption good $\omega - c^i_1$. In the second period, the borrower transfers $\widetilde{\theta}$ to the lender and the two consume $c^i_{2,s}$ (the borrower) and $x^i_{2,s}$ (the lender).

If the lender does not verify the counterparty type, he offers a contract $\left(x_{2,h},\ x_{2,l},\ c_1,\ c_{2,h},\ c_{2,l}\right)$, where consumption in the second period is indexed only by the idiosyncratic state $s \in \{h, l\}$. Then, the borrower transfers the consumption good $\omega$, and he receives the one unit of capital and consumption $\omega - c_1$. In the second period, after the shock realizes, the borrower transfers the realized investment $\widetilde{\theta}$, as long as $c_{2,s} \geq (1 - \lambda_i)\theta$. If the borrower transfers the consumption good, the lender consumes $x_{2,s}$; if not, he consumes 0.

## 3.1 The contract when the borrower is monitored

Lenders matched with a $\lambda_H$ borrower solve a similar problem to the one that lenders matched with $\lambda_L$ borrowers face, with the only difference of the limited commitment problem.

Let $V_i$ denote the value to a lender of a match with borrower of type $\lambda_i$, once the lender paid the cost $\gamma$ and knows the type of the borrower he's matched with. Then lenders choose contracts $\left(x^i_{2,h}, x^i_{2,l}, c^i_1, c^i_{2,h}, c^i_{2,l}\right)$ to maximize $V_i$ as follows:

$$V_i = \max_{\left(x^i_{2,h}, x^i_{2,l}, c^i_1, c^i_{2,h}, c^i_{2,l}\right)} \quad pu(x^i_{2,h}) + (1-p)u(x^i_{2,l}) - \gamma \tag{1}$$

$$s.t. \quad \alpha c^i_1 + p c^i_{2,h} + (1-p) c^i_{2,l} \geq \alpha\omega \tag{2}$$

$$c^i_1 \leq \phi + \omega \tag{3}$$

$$c^i_{2,h} + x^i_{2,h} \leq \omega - c_1 + \theta \tag{4}$$

$$c^i_{2,l} + x^i_{2,l} \leq w - c^i_1 \tag{5}$$

$$c^i_{2,h} \geq (1 - \lambda_i)\theta \tag{6}$$

$$c^i_{2,h}, c^i_{2,l}, c^i_1, x^i_{2,h}, x^i_{2,l} \geq 0 \tag{7}$$

Constraint (2) is the borrower's participation constraint: the borrower can always consume his endowment $\omega$ in the first period. Constraint (3) is time one feasibility, (5) and (4) are time two feasibility in states $h$ and $l$ respectively. Constraint (6) is the borrower individual rationality constraint: the borrower can default and run away with $1-\lambda_i$ of the consumption good (in the low state $\widetilde{\theta}=0$ the limited commitment to repay creates no problems). Finally, constraint (7) is a non-negativity constraint on consumption.

Notice that when $\alpha > 1$ the only reason for the lender to pay the borrower with less resources at $t = 1$ and more at $t = 2$ in the state with high output realization (i.e. limited commitment constraint is slack) is that storing resources from $t = 1$ to $t = 2$ is the only way for the lender to guarantee more consumption goods in the state with low output realization.

It is easy to see that at a solution it must be that (4) and (5) are binding:

$$\begin{aligned} c^i_{2,h} + x^i_{2,h} &= \phi + \omega - c_1 + \theta \\ c^i_{2,l} + x^i_{2,l} &= \phi + w - c^i_1 \end{aligned}$$

**Claim 1** *At a solution, constraint (3) is always slack.*

**Proof.** Suppose not. Then if $\left\{c^{i*}_1, c^{i*}_{2,h}, c^{i*}_{2,l}\right\}_{i=H,L}$ denotes a solution, it must be that $c^{i*}_{2,h} \geq (1-\lambda_i)\theta$ and by assumption $c^{i*}_1 = \phi + \omega$ . This implies that the participation constraint (2) is slack because $c^{i*}_1 > \omega$. Then we can construct an alternative allocation as follows: $\hat{c}^{i*}_1 = c^{i*}_1 - \varepsilon$, $\hat{c}^{i*}_{2,h} = c^{i*}_{2,h}$, $\hat{c}^{i*}_{2,l} = c^{i*}_{2,l}$.
Since the participation constraint was slack then it must still be satisfied under allocation $\left\{\hat{c}^{i*}_1, \hat{c}^{i*}_{2,h}, \hat{c}^{i*}_{2,l}\right\}_{i=H,L}$: the limited commitment constraint is still satisfied since the amount of consumption in the high state has not been altered. Therefore such allocation is in the constraint set to problem (1) but uses less resources than the original allocation $\left\{c^{i*}_1, c^{i*}_{2,h}, c^{i*}_{2,l}\right\}_{i=H,L}$. Such resources can be rebated to the lender so that $\left\{\hat{c}^{i*}_1, c^{i*}_{2,h}, c^{i*}_{2,l}\right\}_{i=H,L}$ attains a higher value of the objective function. Therefore $\left\{c^{i*}_1, c^{i*}_{2,h}, c^{i*}_{2,l}\right\}_{i=H,L}$ cannot be a solution. ■

So we can rewrite the problem as

$$\max_{\{c^i_1, c^i_{2,h}, c^i_{2,l}\}} \quad pu\left(\phi+\omega-c^i_1+\theta-c^i_{2,h}\right)+(1-p)u\left(\phi+\omega-c^i_1-c^i_{2,l}\right) \tag{8}$$

$$s.t.\,(\mu) \qquad \alpha c^i_1 + pc^i_{2,h} + (1-p)c^i_{2,l} \geq \alpha\omega \tag{9}$$

$$(\eta) \qquad c^i_{2,h} \geq (1-\lambda_i)\theta \tag{10}$$

$$c^i_{2,l}, c^i_1 \geq 0$$

Let $\mu$ and $\eta$ be the multipliers associated with (9) and (10) respectively. The first order conditions are

$$-pu'(\phi+\omega-c^i_1+\theta-c^i_{2,h}) + p\mu + \eta = 0 \tag{11}$$

$$- (1-p)u'(\phi+\omega-c_1^i-c_{2,l}^i)+(1-p)\mu \leq 0 \tag{12}$$

with equality if $c_{2,l}^i > 0$ and

$$- pu'(\phi+\omega-c_1^i+\theta-c_{2,h}^i)-(1-p)u'(\phi+\omega-c_1^i-c_{2,l}^i)+\alpha\mu \leq 0 \tag{13}$$

with equality if $c_1^i > 0$. Together with the complementary slackness conditions

$$\mu\{\alpha c_1^i + pc_{2,h}^i + (1-p)c_{2,l}^i - \alpha\omega\} = 0 \tag{14}$$

and

$$\eta\{c_{2,h}^i - (1-\lambda^i)\theta\} = 0 \tag{15}$$

these conditions characterize the solution to the problem.

**Claim 2** *At a solution* $c_{2,l}^{i*} = 0, \forall i$.

**Proof.** Suppose not. Then (12) must hold with equality and $\mu = u'(\phi^i+\omega-c_1^i-c_{2,l}^i) > 0$ that replaced in (11) gives

$$\eta = p[u'(\phi+\omega-c_1^i+\theta-c_{2,h}^i)-u'(\phi+\omega-c_1^i-c_{2,l}^i)]$$

Because $\eta \geq$, it must be $c_{2,h}^i \geq c_{2,l}^i+\theta > \theta > (1-\lambda^i)\theta$, hence $\eta = 0$ and $c_{2,h}^i = c_{2,l}^i+\theta$. Therefore, in (11) we obtain

$$\mu = u'(\phi+\omega-c_1^i-c_{2,l}^i) = u'(\phi+\omega+\theta-c_1^i-c_{2,h}^i)$$

that replaced in (13) gives $(\alpha-1)\mu \leq 0$ which contradicts $\mu > 0$ and $\alpha > 1$. ■

**Lemma 3** *If $u$ is strictly increasing then a solution to problem (1) $x_{2,h}^{i*} > x_{2,l}^{i*}$*
.

**Proof.** Suppose not. Then a solution to problem (1) is such that $x_{2,h}^{i*} = x_{2,l}^{i*}$ since it is never the case that $x_{2,h}^{i*} < x_{2,l}^{i*}$ because that would imply $c_{2,h}^{i*} > \theta$ and in turn that the limited commitment constraint is slack and that $c_1^{i*} < \omega$ (otherwise the participation constraint would also be slack, but then the lender could pay the borrower less so that it binds and attain higher utility). This can happen if and only if the limited commitment constraint is slack ($\eta = 0$) and $c_{2,h}^{i*} = \theta$. Then it must be that that $c_1^{i*} < \omega$ (otherwise the participation constraint would also be slack) and we can construct an alternative allocation such that $\hat{c}_1^{i*} = c_1^{i*}+\varepsilon$, $\hat{c}_{2,h}^{i*} = c_{2,h}^{i*} - \frac{\alpha\varepsilon}{p}$, $\hat{c}_{2,l}^{i*} = c_{2,l}^{i*} - \frac{\alpha\varepsilon}{1-p}$, $\hat{x}_{2,h}^{i*} = x_{2,h}^{i*} + \frac{\alpha\varepsilon}{p}$, $\hat{x}_{2,l}^{i*} = x_{2,l}^{i*} + \frac{\alpha\varepsilon}{1-p}$. This allocation is still feasible given that not all resources at $t = 1$ were used ($c_1^{i*} < \omega$) and it still satisfies the participation constraint and the limited commitment constraint for $\varepsilon > 0$ and arbitrarily small. Because $u$ is strictly increasing and because $\frac{\alpha}{p} > 1$, $\frac{\alpha}{1-p} > 1$ , then $\hat{x}_{2,h}^{i*}\hat{x}_{2,l}^{i*}$ yield a higher value of the objective function than $x_{2,h}^{i*}, x_{2,l}^{i*}$, which then cannot be a solution.
■

Since we know $c^i_{2,l} = 0$, the relevant first order conditions are (11) and (13), together with the complementary slackness conditions (14) and (15). It is easy to see that both the participation and the limited commitment constraint cannot be slack: if this was the case, the lender could increase its revenues just by decreasing $c^i_{2,h}$. Then there are three possible scenario.

1. limited commitment constraint is binding and the participation constraint is binding

2. limited commitment constraint is binding and the participation constraint is slack

3. limited commitment constraint is slack and the participation constraint is binding

1. Suppose that both constraints are binding: then $c^{i*}_{2,h} = \left(1-\lambda^i\right)\theta$ and $c^i_1 = \omega - \frac{p\left[1-\lambda^i\right]\theta}{\alpha}$. From (13) we have

$$\mu = \frac{pu'\left(\phi + \lambda^i\theta + \frac{(1-\lambda^i)p\theta}{\alpha}\right)}{\alpha} > 0$$

which replaced in (11) gives

$$\eta = \frac{p}{\alpha}\left[(\alpha - p)u'\left(\phi + \lambda^i\theta + \frac{(1-\lambda^i)p\theta}{\alpha}\right) - (1-p)u'\left(\phi + \frac{(1-\lambda^i)p\theta}{\alpha}\right)\right]$$

The consumption of the lender is

$$x^i_{2,h} = \phi + \lambda^i\theta + \frac{(1-\lambda^i)p\theta}{\alpha}$$
$$x^i_{2,l} = \phi + \frac{(1-\lambda^i)p\theta}{\alpha}$$

For this to be a solution, we need to check $c^i_1 \geq 0$ and $\eta \geq 0$. This is the a solution if and only if $\omega > \frac{\left[1-\lambda^i\right]p\theta}{\alpha}$ and

$$\frac{\alpha - p}{1-p} > \frac{u'\left(\phi + \frac{(1-\lambda^i)p\theta}{\alpha}\right)}{u'\left(\phi + \lambda^i\theta + \frac{(1-\lambda^i)p\theta}{\alpha}\right)}$$

2. Suppose that the limited commitment constraint is binding and the participation constraint is slack: then $\mu = 0$ and $c^{i*}_{2,h} = \left(1-\lambda^i\right)\theta$. Then, from (13) we have $c^i_1 = 0$ necessarily holds. From (11) we have $\eta = pu'\left(\phi + \omega + \theta - \left(1-\lambda^i\right)\theta\right) > 0$. Lender consumptions are

$$x^i_{2,h} = \phi + \omega + \lambda^i\theta$$
$$x^i_{2,l} = \phi + \omega$$

We only have left to check that the participation constraint is slack: $p(1-\lambda^i)\theta - \alpha\omega \geq 0$, hence this is a solution if and only if $\omega \leq (1-\lambda^i)p\theta$.

3. Suppose now that the limited commitment constraint is slack and the participation constraint is binding: then from (11)

$$\mu = u'(\phi + \omega - c_1^i + \theta - c_{2,h}^i) > 0$$

that replaced in (13) gives

$$p\mu + (1-p)u'(\phi + \omega - c_1^i) \geq \alpha u'(\phi + \omega - c_1^i + \theta - c_{2,h}^i) \tag{16}$$

with equality if $c_1^i > 0$.
If $c_1 = 0$ then from (14) we obtain $c_{2,h}^i = \frac{\alpha\omega}{p}$ and the consumption of the lender is

$$x_{2,h}^i = \phi + \omega + \theta - \frac{\alpha\omega}{p}$$
$$x_{2,l}^i = \phi + \omega$$

We have left to check that the limited commitment constraint is slack, and that (25) is satisfied:

$$\frac{\alpha\omega}{p} \geq \left(1 - \lambda^i\right)\theta$$
$$\frac{u'\left(\phi + \omega\right)}{u'\left(\phi + \omega + \theta - \frac{\alpha\omega}{p}\right)} > \frac{(\alpha - p)}{(1-p)}$$

If instead $c_1 > 0$, from (14) we have $c_{2,h}^i = \frac{\alpha - c_1}{p}$. Lender's consumption is

$$x_{2,h}^i = \phi + \omega - c_1 + \theta - \frac{\alpha(\omega - c_1)}{p}$$
$$x_{2,l}^i = \phi + \omega - c_1$$

where $c_1$ must solde (25) with equality:

$$\frac{\alpha - p}{1-p} = \frac{u'(\phi + \omega - c_1)}{u'\left(\phi + \omega - c_1 + \theta - \alpha\frac{\omega - c_1}{p}\right)}$$

We have left to check that the limited liability constraint is slack, which is the case if $c_{2,h}^i > (1-\lambda^i)\theta <==> \omega > \frac{(1-\lambda^i)p\theta}{\alpha}$.

### 3.2 Equilibrium when the borrower is not monitored

When the lender does not pay the cost $\gamma$ that is necessary to learn the borrower's $\lambda_i$, we assume that the game between the lender and the borrower changes in the following two aspects. First, and easily, the lender cannot condition the contract on the borrower's type $\lambda_i$. Second, and less easily, a type $\lambda_i$ borrower can behave as a type $\lambda_{-i}$ borrower.

The second aspect is not as straightforward as the first one, and the argument is the following: since the parameter $\lambda_i$ measures the (exogenous) level of pledgeable income, meaning the amount of income that the borrower can credibly pledge in the first period, without triggering default in the second period, we assume that, whenever this parameter is not common knowledge, a type $\lambda_i$ borrower can (ex-post) strategically behave as a type $\lambda_{-i}$ borrower, concerning his default decision. In other words, we are assuming that when the type $\lambda_i$ is not common knowledge, in the second period, after default, a $\lambda_i$ borrower can mimic a $\lambda_{-i}$ type and run away with a fraction $1-\lambda_{-i}$ of resources. [12]

Therefore, letting $W$ denote the value to a lender with endowment $\phi$ of a match with a borrower whose type is unknown, the lender chooses the contract to solve:

$$W = \max_{(x_{2,h},x_{2,l},c_1,c_{2,h},c_{2,l})} \quad pu(x_{2,h}) + (1-p)u(x_{2,l}) \tag{17}$$

$$s.t. \quad \alpha c_1 + pc_{2,h} + (1-p)c_{2,l} \geq \alpha\omega \tag{18}$$

$$c_1 \leq \phi + \omega \tag{19}$$

$$c_{2,h} + x_{2,h} \leq \phi + \omega - c_1 + \theta \tag{20}$$

$$c_{2,l} + x_{2,l} \leq \phi + \omega - c_1 \tag{21}$$

$$c_{2,h} \geq (1-\lambda_L)\theta \tag{22}$$

$$c_{2,h}, c_{2,l}, c_1, x_{2,h}, x_{2,l} \geq 0 \tag{23}$$

The solution to this problem is the same as the solution to problem (1) in section 3 as described in 1, apart from the lender not having to pay the cost $\gamma$. Then, $W = V_L + \gamma$, and it is easy to show the following:

**Claim 4** *If clearing is always bilateral, the lender pays the cost $\gamma$ and learns his borrower's type if and only if $q[V_H - V_L] \geq \gamma$.*

**Proof.** When clearing is always bilateral then the lender chooses whether to pay the cost of learning his borrower's type or not by comparing the expected payoff from trading under the terms of the contract that solves (1) once he knows $\lambda_i$ with $W = V_L + \gamma$. Not knowing what type the borrower is, the expected payoff from paying $\gamma$ is $qV_H + (1-q)V_H$. ■

[12] This argument can be rationalized if we derive $\lambda_i$, as in Holmstrom & Tirole, from a moral hazard problem between the lender and the borrower. In their model, after investment, the borrower can take an action that reduces the probability of success, but gives him a private benefit. If the lender doesn't monitor the borrower, the private benefit is $B > 0$,while if the lender does monitor the borrower, the private benefit reduces to $b < B$. The moral hazard, together with a limited liability constraint, reduces the amount of pledgeable income, as (exogenously) in our model. This level of pledgeable income is $\lambda_H$, when the lender monitors the borrower, and the private benefit is $b$, while it is $\lambda_L$, when there is no monitoring, and the private benefit is $B$. Hence, no monitoring leads automatically to the fraction $\lambda_L$ of pledgeable income.

### 3.3 Partitioning the state space

The partition of the state space depends on two key parameters: the collateral endowment $\omega$ and the value of the borrower $\lambda^i$ (or the borrower's temptation to default $1-\lambda^i$). The interaction of the two determines whether both the limited commitment and the participation constraint bind or only one of them binds.

In the bilateral problem $\omega$ plays a double role: it is the collateral needed for insurance purposes (against the high or low realization of $\tilde{\theta}$) and it is also a lower bound on the payoff the borrower needs to participate in the contract. The temptation to default $1-\lambda^i$ measures the severity of the commitment problem so that when $\lambda^i$ is relatively low the borrower is relatively bad, then the solution to (1) must be such that the limited commitment constraint binds. Then we can distinguish two scenarios: when $\omega$ is relatively low ($\omega \leq \frac{(1-\lambda^i)p\theta}{\alpha}$) then it is relatively easy to get the borrower to participate in the contract, that is to say the participation constraint is slack. Then the limited commitment constraint binds. On the other hand, because $\omega$ is relatively scarce, then it is valuable to provide insurance to the lender: thus $c_1 = 0$. This is solution 1 and area 4 in figure1.

When $\omega$ is relatively high the participation constraint starts binding: this is true for solutions $2a, 2b, 2c$ and areas $1, 2, 3$ in figure1. Whether the limited commitment constraint binds[13] or not[14] depends on the severity of the commitment problem with respect to a threshold $\lambda^*$: if $\lambda \leq \lambda^*$ then the borrower's temptation to default is high, so the limited commitment constraint binds. If $\lambda > \lambda^*$ then the borrower's temptation to default is low, so the limited commitment constraint is slack. In this last case, however, we can distinguish two subcases: the first where the borrower's endowment is more valuable as collateral rather than as an instrument to provide incentives ($\omega \leq \frac{(1-\lambda^*)p\theta}{\alpha}$) so that in equilibrium $c_1 = 0$. The second where the borrower's endowment is relatively abundant and is more valuable to provide incentives ($\omega > \frac{(1-\lambda^*)p\theta}{\alpha}$) so that in equilibrium $c_1 > 0$. Because the commitment problem is not severe (the limited commitment constraint is slack) in these two cases, then the relevant threshold for the borrower's endowment is a function of $\lambda^*$ rather than the actual temptation to default in the bilateral trade.

These results are summarized in the following lemmas.

**Lemma 5** *For a given $\lambda^i$, if $\omega \leq \frac{(1-\lambda^i)p\theta}{\alpha}$ then the solution to the bilateral problem (1) is such that the limited commitment constraint is binding, the participation constraint is slack and $c_1 = 0$. This is area 4 in figure 1.*

**Proof.** Suppose that the limited commitment constraint is binding and the participation constraint is slack: then $\mu = 0$ and $c_{2,h}^{i*} = \left(1-\lambda^i\right)\theta$. Then, from (13) we have $c_1^i = 0$ necessarily holds. From (11) we have $\eta = pu'\left(\phi + \omega + \theta - \left(1-\lambda^i\right)\theta\right) >$

[13] Solution $2a$ and area 3 in figure1.

[14] Solutions $2b, 2c$ and areas $1, 2$ in figure1.

0. Lender consumptions are

$$x_{2,h}^i = \phi + \omega + \lambda^i\theta$$
$$x_{2,l}^i = \phi + \omega$$

We only have left to check that the participation constraint is slack, which happens if and only if $\omega \leq \frac{(1-\lambda^i)p\theta}{\alpha}$. ■

Define $\lambda^*$ as the unique value of $\lambda$ satisfying:

$$\frac{\alpha - p}{1-p} = \frac{u'\left(\phi + \frac{(1-\lambda^*)p\theta}{\alpha}\right)}{u'\left(\phi + \theta - (1-\lambda^*)\frac{p\theta}{\alpha}(1-\frac{\alpha}{p})\right)} \tag{24}$$

Uniqueness is easy, because if we define the function

$$F(\lambda) = \frac{u'\left(\phi + \frac{(1-\lambda)p\theta}{\alpha}\right)}{u'\left(\phi + \theta - (1-\lambda)\frac{p\theta}{\alpha}(1-\frac{\alpha}{p})\right)}$$

easily this function is increasing, $F(1) > 0$ by assumption and $F(\lambda^*) = \frac{u'\left(\phi+\frac{(1-\lambda^*)p\theta}{\alpha}\right)}{u'\left(\phi+\theta-(1-\lambda^*)\frac{p\theta}{\alpha}(1-\frac{\alpha}{p})\right)}$.

**Lemma 6** *For a given $\lambda^i$, if $\omega > \frac{(1-\lambda^i)p\theta}{\alpha}$ and $\lambda^i < \lambda^*$ then both the limited commitment constraint and the participation constraint are binding. This is area 3 in figure 1. If $\omega > \frac{(1-\lambda^i)p\theta}{\alpha}$ and $\lambda^i \geq \lambda^*$ then the limited commitment constraint is slack, the participation constraint is binding and $c_1 > 0$ if also $\omega \geq \frac{(1-\lambda^*)p\theta}{\alpha}$, otherwise $c_1 = 0$. These are areas 1 and 2 respectively in figure 1.*

**Proof.** Suppose that both constraints are binding: then $c_{2,h}^{i*} = \left(1-\lambda^i\right)\theta$ and $c_1^i = \omega - \frac{p\left[1-\lambda^i\right]\theta}{\alpha}$. From (13) we have

$$\mu = \frac{pu'\left(\phi + \lambda^i\theta + \frac{(1-\lambda^i)p\theta}{\alpha}\right)}{\alpha} > 0$$

which replaced in (11) gives

$$\eta = \frac{p}{\alpha}\left[(\alpha-p)u'\left(\phi + \lambda^i\theta + \frac{(1-\lambda^i)p\theta}{\alpha}\right) - (1-p)u'\left(\phi + \frac{(1-\lambda^i)p\theta}{\alpha}\right)\right]$$

The consumption of the lender is

$$x_{2,h}^i = \phi + \lambda^i\theta + \frac{(1-\lambda^i)p\theta}{\alpha}$$
$$x_{2,l}^i = \phi + \frac{(1-\lambda^i)p\theta}{\alpha}$$

For this to be a solution, we need to check $c_1^i \geq 0$ and $\eta \geq 0$. This is the a solution if and only if $\omega > \frac{[1-\lambda^i]p\theta}{\alpha}$ and

$$\frac{\alpha - p}{1-p} > \frac{u'\left(\phi + \frac{(1-\lambda^i)p\theta}{\alpha}\right)}{u'\left(\phi + \lambda^i\theta + \frac{(1-\lambda^i)p\theta}{\alpha}\right)}$$

From the definition of $\lambda^*$ in (24) this is equivalent[15] to $\lambda^i < \lambda^*$.

Next suppose that the limited commitment constraint is slack and the participation constraint is binding: then from (11)

$$\mu = u'(\phi + \omega - c_1^i + \theta - c_{2,h}^i) > 0$$

that replaced in (13) gives

$$p\mu + (1-p)u'(\phi + \omega - c_1^i) \geq \alpha u'(\phi + \omega - c_1^i + \theta - c_{2,h}^i) \tag{25}$$

with equality if $c_1^i > 0$.
If $c_1 = 0$ then from (14) we obtain $c_{2,h}^i = \frac{\alpha\omega}{p}$ and the consumption of the lender is

$$\begin{aligned} x_{2,h}^i &= \phi + \omega + \theta - \frac{\alpha\omega}{p} \\ x_{2,l}^i &= \phi + \omega \end{aligned}$$

We have left to check that the limited commitment constraint is slack, and that (25) is satisfied:

$$\omega \geq \frac{\left(1-\lambda^i\right)p\theta}{\alpha} \tag{26}$$

$$\frac{u'\left(\phi + \omega\right)}{u'\left(\phi + \omega + \theta - \frac{\alpha\omega}{p}\right)} > \frac{(\alpha - p)}{(1-p)} \tag{27}$$

If instead $c_1 > 0$, from (14) we have $c_{2,h}^i = \frac{\alpha - c_1}{p}$. Lender's consumption is

$$\begin{aligned} x_{2,h}^i &= \phi + \omega - c_1 + \theta - \frac{\alpha(\omega - c_1)}{p} \\ x_{2,l}^i &= \phi + \omega - c_1 \end{aligned}$$

where (25) holds at equality:

$$\frac{\alpha - p}{1-p} = \frac{u'(\phi + \omega - c_1)}{u'\left(\phi + \omega + \theta - \frac{\alpha\omega}{p} + c_1(\frac{\alpha}{p} - 1)\right)} \tag{28}$$

[15] In fact

$$\begin{aligned} \phi + \lambda^i\theta + \frac{(1-\lambda^i)p\theta}{\alpha} &= \phi + \theta + \frac{(1-\lambda^i)p\theta}{\alpha} - (1-\lambda^i)\theta \\ &= \frac{(1-\lambda^i)p\theta}{\alpha}(1 - \frac{\alpha}{p}) \end{aligned}$$

We have left to check that the limited liability constraint is slack, which is the case if $c^i_{2,h} \geq (1-\lambda^i)\theta \iff \omega \geq \frac{(1-\lambda^i)p\theta}{\alpha}$.

Therefore necessary and sufficient conditions for $c_1 = 0$ are $\omega \geq \frac{(1-\lambda^i)p\theta}{\alpha}$ and (27), whereas for $c_1 > 0$ are $\omega \geq \frac{(1-\lambda^i)p\theta}{\alpha}$ and (28). Combining (28) with the definition of $\lambda^*$ (24) it follows that $\omega - c_1 = (1-\lambda^*)\frac{p\theta}{\alpha}$:

$$\begin{aligned}
\frac{\alpha-p}{1-p} &= \frac{u'\left(\phi + \frac{(1-\lambda^*)p\theta}{\alpha}\right)}{u'\left(\phi + \theta - (1-\lambda^*)\frac{p\theta}{\alpha}(1-\frac{\alpha}{p})\right)} && (29)\\
&= \frac{u'(\phi+\omega-c_1)}{u'\left(\phi+\omega+\theta-\frac{\alpha\omega}{p}+c_1(\frac{\alpha}{p}-1)\right)} && (30)\\
&\leq \frac{u'(\phi+\omega)}{u'\left(\phi+\theta+\omega(1-\frac{\alpha}{p})\right)} && (31)
\end{aligned}$$

Therefore, given strict concavity of $u$, if $\omega < (1-\lambda^*)\frac{p\theta}{\alpha}$ then $c_1 = 0$, and if $\omega \geq (1-\lambda^*)\frac{p\theta}{\alpha}$ then $c_1 > 0$. ■

These results are summarized graphically in figure 1, where we have that:

1. If $\omega < \frac{(1-\lambda^i)p\theta}{\alpha}$, then the limited commitment constraint is binding and the participation constraint is slack and the solution is:

$$\begin{aligned}
c^i_{2,h} &= (1-\lambda^i)\theta\\
c^i_1 &= 0\\
x^i_{2,h} &= \phi+\omega+\lambda^i\theta\\
x^i_{2,l} &= \phi+\omega
\end{aligned}$$

   which is area 4 in figure 1.

2. If $\omega \geq \frac{(1-\lambda^i)p\theta}{\alpha}$, then

   **2a** If $\frac{\alpha-p}{1-p} > \frac{u'\left(\phi+\frac{(1-\lambda^i)p\theta}{\alpha}\right)}{u'\left(\phi+\lambda^i\theta+\frac{(1-\lambda^i)p\theta}{\alpha}\right)}$, then the limited commitment constraint and the participation constraint are binding and the solution is:

$$\begin{aligned}
c^i_{2,h} &= (1-\lambda^i)\theta\\
c^i_1 &= \omega - \frac{(1-\lambda^i)p\theta}{\alpha}\\
x^i_{2,h} &= \phi+\lambda^i\theta+\frac{(1-\lambda^i)p\theta}{\alpha}\\
x^i_{2,l} &= \phi+\frac{(1-\lambda^i)p\theta}{\alpha}
\end{aligned}$$

   which is area 3 in figure 1.

**2.b** If $\frac{\alpha-p}{1-p} \leq \frac{u'(\phi+\omega)}{u'\left(\phi+\omega+\theta-\frac{\alpha\omega}{p}\right)}$, then the limited commitment constraint is slack, the participation constraint is binding and $c_1 = 0$ and the solution is:

$$\begin{aligned} c^i_{2,h} &= \alpha\omega/p \\ c^i_1 &= 0 \\ x^i_{2,h} &= \phi+\omega+\theta-\alpha\omega/p \\ x^i_{2,l} &= \phi+\omega \end{aligned}$$

which is area 2 in figure 1.

**2c** For $c_1^* > 0$ solving $\frac{\alpha-p}{1-p} = \frac{u'(\phi+\omega-c_1^*)}{u'\left(\phi+\omega-c_1^*+\theta-\frac{\alpha(\omega-c_1^*)}{p}\right)}$, then the limited commitment constraint is slack, the participation constraint is binding and $c_1 > 0$ and the solution is:

$$\begin{aligned} c^i_{2,h} &= \frac{\alpha(\omega-c_1^*)}{p} \\ c^i_1 &= c_1^* \\ x^i_{2,h} &= \phi+\omega-c_1^*+\theta-\alpha\frac{\omega-c_1^*}{p} \\ x^i_{2,l} &= \phi+\omega-c_1^* \end{aligned}$$

which is area 3 in figure 1.

# 4 CCP decision problem

Suppose that the CCP maximizes lenders' welfare [16] using the resources available in both time periods: suppose that both agents have to transfer their beginning of period endowments to the CCP and then they receive consumption payments, based on the contract they submit to the CCP for clearing [17].

With central clearing, each lender gets randomly matched with one borrower, that can be of type $\{\lambda_L, \lambda_H\}$. The lender decides whether to verify counterparty type, $e = 1$ or not $e = 0$. After verification, the lender reports the the type of the counterparty, $H, L, \phi$, where $\phi$ means he did not observe. The only two

[16] One could assume that this is a repeated game and the role of borrower and lender switches randomly (or not) so that even borrowers are happy to join the CCP since eventually they take advantage of it.

[17] We still need to prove that this is equivalent to having agents submitting a contract to the CCP and the CCP charging default fund contributions and margins accordingly. Agents would take into account such contributions and margin allocation rules when they agree on a contract, and then play their best response to such rules by choosing the terms of trade. Then the terms of trade are transfered to the CCP and the contributions and margining rules are applied. The conjecture is that such contributions and margining rules would implement the allocation that solves our CCP decision problem.

things that are contractible for the CCP are the message $m_L$ and the second period idiosyncratic state of a borrower $\{h, l\}$. After the message has been sent, the borrower can accept or he can reject the transaction. If the borrower rejects, both lender and borrower consume autarky payoff. If accepted, the CCP becomes counterparty of both lenders and borrowers. If the transaction is accepted, borrowers transfer the consumption good to the CCP, and lenders transfer the capital good to borrowers.

1. If $m_L \neq \phi$,
   - The CCP gives consumption $C_1^{m_L}$ to borrowers, where $m_L$ is the message that lenders sent.
   - In the second period, the idiosyncratic state $\{h, l\}$ realizes. The CCP asks borrowers to transfer 0 if the state is *low*, and to transfer $\theta$ if the state is *high*, where $m_L$ is the message sent by the lender in the firs period.
   - In *high* state, the borrowers can refus to repay the CCP, getting away with $(1 - \lambda_i)\theta$.
   - Out of its total revenues, the CCP then transfers consumption $X_2^{m_L}$ to lenders, and $C_{2,h}^{m_L}$ to borrowers if the state is *high*, and $C_{2,l}^{m_L}$ if the state is *low*.
2. If $m_L = \phi$
   - The CCP offers a contract $X_2^{\phi}$ to lenders
   - and a menu of contracts $\left(C_1^{hi}, C_{2,h}^{hi}, C_{2,l}^{hi}\right)$, $\left(C_1^{lo}, C_{2,h}^{lo}, C_{2,l}^{lo}\right)$

This is shown in Figure 2.

**Lemma 7** *The only incentive compatible contract, has the lender revealing that he did not learn the counterparty type:* $m_L = \phi$.
***Proof.*** *The proof is simple: suppose there is an incentive compatible contract with the lender exerting monitoring effort, and then revealing the information acquired. Then, after exerting effort,*

1. *When the lender learns* $\lambda_i = \lambda_L$, *he must truthfully report it:*
   $$x_2^L \geq x_2^H \qquad \text{if } \alpha C_1^H + p \max\{C_{2,h}^H, (1 - \lambda_L)\theta\} + (1 - p)C_{2,l}^H \geq \alpha\omega$$
   *Since* $\alpha C_1^H + p \max\{C_{2,h}^H, (1 - \lambda_L)\theta\} + (1 - p)C_{2,l}^H \geq \alpha C_1^H + pC_{2,h}^H + (1 - p)C_{2,l}^H \geq \alpha\omega$, *it must be* $X_2^L \geq X_2^H$.
2. *Similarly, when the lender learns* $\lambda_i = \lambda_H$, *he must truthfully report it:*
   $$x_2^H \geq x_2^L \qquad \text{if } \alpha C_1^L + p \max\{C_{2,h}^L, (1 - \lambda_H)\theta\} + (1 - p)C_{2,l}^L \geq \alpha\omega$$
   *Since* $\alpha C_1^L + p \max\{C_{2,h}^L, (1 - \lambda_H)\theta\} + (1 - p)C_{2,l}^L \geq \alpha C_1^L + pC_{2,h}^L + (1 - p)C_{2,l}^L \geq \alpha\omega$, *it must be* $X_2^H \geq X_2^L$.

*Combining the two,* $X_2^L = X_2^H$*. But then, it must be that the lender chooses to verify the counterparty type, instead of just reporting* $m_L(\phi) = \lambda_H$*, which means*

$$q \cdot u(x_2^H) + (1-q) \cdot u(X_2^L) - \gamma$$
$$= u(X_2^H) - \gamma \geq u(X_2^H) \qquad \textit{if } \alpha C_1^H + p \max\{C_{2,h}^H, (1-\lambda_L)\theta\} + (1-p)C_{2,l}^H \geq \alpha\omega$$

*But then, since* $\alpha C_1^H + p\max\{C_{2,h}^H, (1-\lambda_L)\theta\} + (1-p)C_{2,l}^H \geq \alpha C_1^H + pC_{2,h}^H + (1-p)C_{2,l}^H \geq \alpha\omega$*, this condition can never be satisfied.* ■

Hence, the only contract we need to characterize is the one in which the CCP offers a menu of contracts $\left(C_1^{hi}, C_{2,h}^{hi}, C_{2,l}^{hi}\right)$, $\left(C_1^{lo}, C_{2,h}^{lo}, C_{2,l}^{lo}\right)$ in which borrowers have incentives to self-select.

The CCP solves:

$$\begin{array}{lcl}
\max & qu\left(X_2^H\right) + (1-q)\,u\left(X_2^L\right) & \\
s.t. & & \\
(\mu^i) & \alpha C_1^i + pC_{2h}^i + (1-p)\,C_{2l}^i & \geq \alpha\omega \\
(\rho_1) & qC_1^H + (1-q)\,C_1^L & \leq \phi + \omega \\
(\rho_2) & q\left(X_2^H + pC_{2h}^H + (1-p)\,C_{2l}^H\right) + (1-q)\left(X_2^L + pC_{2h}^L + (1-p)\,C_{2l}^L\right) & \leq \\
 & \phi + \omega - qC_1^H - (1-q)\,C_1^L + p\theta & \\
(\eta^i) & C_{2h}^i & \geq \left(1-\lambda^i\right)\theta \\
(\beta^i) & \alpha C_1^i + pC_{2h}^i + (1-p)\,C_{2l}^i & \geq \\
 & \alpha C_1^{-i} + p\max\left(C_{2h}^{-i}, \left(1-\lambda^i\right)\theta\right) + (1-p)\,C_{2l}^{-i} &
\end{array}$$

Before jumping to characterizing the solution to this problem let us point out that:

1. because $\alpha > 1$ then it is always cheaper for lenders (and the CCP) to give incentives to the borrower by paying the via $C_1$rather than $C_2$. The only reason why $C_2$is paid is for limited commitment constraints
2. the resource constraint at $t = 2$ must bind
3. whether the resource constraint at $t = 1$ binds or not depends on whether the amount of $C_{2h}^i$ that needs to be paid to borrower $i$ because of the limited commitment problem is not sufficient for the participation constraint to be satisfied. In other words, if the limited commitment problems is very severe (i.e. $C_{2h}^i$ is very large), then lenders (and the CCP) may need less $t = 1$ consumption to pay to borrowers in order to induce them to agree to trade (to satisfy the participation constraint). Therefore the resource constraint at $t = 1$ is slack.

4. if the initial endowment of the borrower ($\omega$) is not very large then both the participation constraint and the resource constraint may be slack: in fact once $C^i_{2h}$ is paid to the borrower because of limited commitment problems, the remaining $C^i_1$ payments are solely driven by the participation constraint (and the truth telling constraint for the borrowers, in case the payments $C^i_{2h}$ are different for different $i$). When $\omega$ is low, then it may be the case that the $C^i_{2h}$ payment is large enough to induce the borrower to trade with no need of additional consumption in $t = 1$. This implies that the participation constraint is slack, and so is the resource constraint at $t = 1$. The resource constraint at $t = 1$ may be slack also in some cases when the participation constraint binds however: in fact lenders (and the CCP) may not need the entire amount of consumption goods available at $t = 1$ in order to induce the borrower to agree to trade.

Let $q^i = q$ if $i = H$ and $q^i = 1 - q$ if $i = L$. Before than characterizing the solution, we can simplify the problem as follows: first, the constraint $X^i_2 \geq X_2{-}i$ is redundant: if we solve the problem ignoring it we obtain $X^H_2 = X^L_2$, which satisfies the constraint above. Then, we can replace $X^H_2 = X^L_2 = X_2$. Second, the feasibility constraint ($\rho_2$) must bind: if not, just increase $X_2$. Then, we can replace

$$X_2 = \phi + \omega - q\Big(C^H_1 + pC^H_{2,h} + (1-p)C^H_{2,l}\Big) - (1-q)\Big(C^L_1 + pC^L_{2,h} + (1-p)C^L_{2,l}\Big)$$

Third, since maximizing $u(X_2)$ is equivalent to maximizing $X_2$, we can consider the CCP as de facto being risk-neutral. Finally, for $i = L, H$, constraints ($\beta^i$) and ($\eta^i$) can be rewritten in a more compact form.

**Lemma 8** *Constraints ($\beta^i$) and ($\eta^i$) hold if and only if*

$$\min\{C^H_{2,h}, C^L_{2,h}\} \geq (1-\lambda^L)\theta \tag{32a}$$

$$\alpha C^H_1 + pC^H_{2,h} + (1-p)C^H_{2,l} = \alpha C^L_1 + pC^L_{2,h} + (1-p)C^L_{2,l} \tag{32b}$$

**Proof.** Suppose first constraints ($\beta^i$) and ($\eta^i$) hold. Then we have to show that (32b) holds and for (32a) to hold it must be $C^H_{2,h} \geq (1-\lambda^L)\theta$. Because $\lambda^L < \lambda^H$, from ($\eta^L$) we have $C^L_{2,h} \geq (1-\lambda^L)\theta > (1-\lambda^H)\theta$. Hence constraint ($\beta^H$) becomes

$$\alpha C^H_1 + pC^H_{2,h} + (1-p)C^H_{2,l} \geq \alpha C^L_1 + pC^L_{2,h} + (1-p)C^L_{2,l} \tag{$\beta^{H'}$}$$

Combining this with ($\beta^L$) we obtain

$$\begin{aligned}
\alpha C^H_1 + pC^H_{2,h} + (1-p)C^H_{2,l} &\geq \alpha C^L_1 + pC^L_{2,h} + (1-p)C^L_{2,l} \\
&\geq \alpha C^H_1 + p\max\{C^H_{2,h}, (1-\lambda^L)\theta\} + (1-p)C^H_{c,l} \\
&<==> C^H_{2,h} \geq \max\{C^H_{2,h}, (1-\lambda^L)\theta\} \\
&<==> C^H_{2,h} \geq (1-\lambda^L)\theta
\end{aligned}$$

hence we proved (32a) holds. But then constraint $(\beta^L)$ becomes

$$\alpha C_1^L + pC_{2,h}^L + (1-p)C_{2,l}^L \geq \alpha C_1^H + pC_{2,h}^H + (1-p)C_{2,l}^H \qquad (\beta^{L'})$$

Combining $(\beta^{H'})$ with $(\beta^{L'})$ we obtain (32b).
Suppose now (32a) and (32b) hold. Then since $C_{2,h}^L \geq (1-\lambda^L)\theta > (1-\lambda^H)\theta$ we have that $C_{2,h}^L = \max\{C_{2,h}^L, (1-\lambda^H)\theta\}$ and

$$\begin{aligned} \alpha C_1^H + pC_{2,h}^H + (1-p)C_{2,l}^H &= \alpha C_1^L + pC_{2,h}^L + (1-p)C_{2,l}^L \\ &= \alpha C_1^L + p\max\{C_{2,h}^L, (1-\lambda^H)\theta\} + (1-p)C_{2,l}^L \end{aligned}$$

and constraint $(\beta^H)$ is satisfied. Similarly, since $C_{2,h}^H \geq (1-\lambda^L)\theta$, we have that $C_{2,h}^H = \max\{C_{2,h}^H, (1-\lambda^L)\theta\}$ and

$$\begin{aligned} \alpha C_1^L + pC_{2,h}^L + (1-p)C_{2,l}^L &= \alpha C_1^H + pC_{2,h}^H + (1-p)C_{2,l}^H \\ &= \alpha C_1^H + p\max\{C_{2,h}^H, (1-\lambda^L)\theta\} + (1-p)C_{2,l}^H \end{aligned}$$

and constraint $(\beta^L)$ is satisfied. Finally, $C_{2,h}^L \geq (1-\lambda^L)\theta$ by (32a), hence $(\eta^L$ is satisfied. Similarly $C_{2,h}^H \geq (1-\lambda^L)\theta > (1-\lambda^H)\theta$, hence $(\eta^H)$ is aslo satisfied.
■
From (32b) we can additionally ignore one participation constraint, say $(\mu^H)$, and rewrite the problem as

$$\begin{aligned} \max_{(C_1^i, C_{2,h}^i, C_{2,l}^i)} \quad & \phi + \omega + p\theta - \left[ q(C_1^H + pC_{2,h}^H + (1-p)C_{2,l}^H) + (1-q)(C_1^L + pC_{2,h}^L + (1-p)C_{2,l}^L) \right] \\ s.t. \quad & \alpha C_1^L + pC_{2,h}^L + (1-p)C_{2,l}^L \geq \alpha\omega & (\mu) \\ & qC_1^H + (1-q)C_1^L \leq \phi + \omega & (\rho_1) \\ & \min\{C_{2,h}^H, C_{2,h}^L\} \geq (1-\lambda^L)\theta & (\eta) \\ & \alpha C_1^H + pC_{2,h}^H + (1-p)C_{2,l}^H = \alpha C_1^L + pC_{2,h}^L + (1-p)C_{2,l}^L & (\beta) \\ & C_{2,l}^i, C_1^i \geq 0 \end{aligned}$$

**Lemma 9** *If* $\min\{C_{2,h}^H, C_{2,h}^L\} > (1-\lambda^L)\theta$, *the resource constraint must be slack:* $qC_1^H + (1-q)C_1^L < \phi + \omega$.

**Proof.** Suppose not: $\min\{C_{2,h}^H, C_{2,h}^L\} > (1-\lambda^L)\theta$ and $qC_1^H + (1-q)C_1^L = \phi + \omega$. Then either $C_1^H > \omega$, $C_1^L > \omega$ or both. In any case $\alpha C_1^L + pC_{2,h}^L + (1-p)C_{2,l}^L > \alpha\omega$ and the participation constraint $(\mu)$ must be slack. Decrease then $C_{2,h}^H$ and $C_{2,h}^L$ by the same amount $\epsilon$: all constraints are still satisfied and expected revenues of the CCP increase. ■

**Lemma 10** *If* $\min\{C_{2,h}^H, C_{2,h}^L\} > (1-\lambda^L)\theta$, *the resource constraint must bind:* $qC_1^H + (1-q)C_1^L = \phi + \omega$.

**Proof.** Suppose not: $\min\{C_{2,h}^H, C_{2,h}^L\} > (1-\lambda^L)\theta$ and $qC_1^H + (1-q)C_1^L < \phi + \omega$. Consider then the allocation we obtain defining $C_{2,h}^{L'} = C_{2,h}^L - \epsilon$ and $C_1^{L'} = C_1^L + \frac{p\epsilon}{\alpha}$. It is easy to check that all constraints are satisfied and expected revenues of the CCP strictly increase. ■

Combining the two results above we have

**Proposition 11** *Constraint* $(\eta)$ *always bind:* $\min\{C_{2,h}^H, C_{2,h}^L\} = (1-\lambda^L)\theta$.

**Lemma 12** *Consumption in the second period low state equals zero for both type of borrowers:* $C_{2,l}^H = C_{2,l}^L = 0$.

**Proof.** Suppose not. If both $C_{2,l}^H > 0$ and $C_{2,l}^L > 0$, then the participation constraint must bind, $\alpha C_1^L + pC_{2,h}^L + (1-p)C_{2,l}^L = \alpha\omega$: otherwise we could just decrease $C_{2,l}^H$ and $C_{2,l}^L$ by the same amount increasing expected revenues. But then, given that the participation constraint binds, it must be that $C_1^H < \omega$ and $C_1^L < \omega$, hence $qC_1^H + (1-q)C_1^L < \omega + \phi$ and the feasibility constraint is slack. Then consider the new allocation we obtain increasing $C_1^L$ by $\frac{\epsilon(1-p)}{\alpha}$ and decreasing $C_{2,l}^L$ by $\epsilon$. All constraints are satisfied and expected revenues increase. Then both $C_{2,l}^H, C_{2,l}^L > 0$ cannot hold.

Suppose then $C_{2,l}^i > C_{2,l}^{-i} = 0$. First, it should be that the participation constraint is slack. If not, hence the participation constraint binds, it follows $C_1^H < \omega$ and $C_1^L < \omega$ since both $C_{2,h}^H$ and $C_{2,h}^L$ are strictly positive by $(\eta)$. Then $qC_1^H + (1-q)C_1^L < \phi + \omega$. Consider then the allocation we obtain decreasing $C_{2,h}^i$ by $\epsilon$ and increasing $C_1^i$ by $\frac{\epsilon p}{\alpha}$. All constraints are satisfied and expected revenues strictly increase. Then the participation constraint must be slack.

Given the participation constraint is slack, either $C_1^H = 0$ or $C_1^L = 0$, or both: otherwise we can decrease both $C_1^H$ and $C_1^L$ by the same amount and increase expected revenues. I will show that it must be $C_1^H = C_1^L = 0$.

Suppose by contradiction that $C_1^i = 0$ and $C_1^{-i} > 0$. Then decrease $C_{2,l}^i$ and decrease $C_1^{-i}$ to satisfy $(\beta)$ with equality. Expected revenues increase and all constraints are satisfied.

Suppose instead $C_1^{-i} = 0$ and $C_1^i > 0$. Then, from $(\eta)$,$(\mu)$, which is slack, and $(\beta)$, we have $C_{2,h}^{-i} > C_{2,h}^i = (1-\lambda^L)\theta$. Decrease then $C_1^i$ and $C_{2,h}^{-i}$ to satisfy $(\beta)$ with equality. All constraints are satisfied and expected revenues increase.

Then it must necessarily be the case that $C_1^H = C_1^L = 0$. But then $C_{2,h}^{-i} > C_{2,h}^i = (1-\lambda^L)\theta$. Then decrease $C_{2,h}^{-i}$ by $\epsilon$ and increase $C_1^{-i}$ by $\frac{\epsilon p}{\alpha}$. All constraints are satisfied and expected revenues increase. Therefore, $C_1^i = 0$ for all $i = L, H$. ■

**Lemma 13** *The solution of the CCP problem satisfies* $C_{2,h}^H = C_{2,h}^L = (1-\lambda^L)\theta$.

**Proof.** We already know that the $\min\{C_{2,h}^H, C_{2,h}^L\} = (1-\lambda^L)\theta$. Suppose then by contradiction that $C_{2,h}^i > C_{2,h}^{-i} = (1-\lambda^L)\theta$. Then, by constraint

$(\beta)$ we have $C_1^{-i} > C_1^i \geq 0$. If the participation constraint $(\mu)$ is slack, then decrease $C_{2,h}^i$ and $C_1^{-i}$ to satisfy $(\beta)$ with equality: expected revenues increase. If instead the participation constraint $(\mu)$ is binding, then both $C_1^H < \omega$ and $C_1^L < \omega$, hence the feasibility constraint $(\rho)$ is slack. Then increase $C_1^i$ by $\frac{\epsilon p}{\alpha}$ and decrease $C_{2,h}^i$ by $\epsilon$: all constraints are satisfied and expected revenues increase. ■ Easily, once $C_{2,h}^H = C_{2,h}^L = (1-\lambda^L)\theta$ and $C_{2,l}^H = C_{2,l}^L = 0$, constraint $(\beta)$ gives $C_1^H = C_1^L$. This is pinned down by the participation constraint. In any case the feasibility constraint $(\rho)$ is slack. We summarize these findings in the next lemma.

**Lemma 14** *The feasibility constraint $(\rho)$ is always slack and first period consumption for borrowers is*

$$C_1^H = C_1^L = \max\left\{0, \omega - \frac{p(1-\lambda^L)\theta}{\alpha}\right\}$$

**Proof.** We already know that $C_1^H = C_1^L$ from constraint $(\beta)$, and $C_{2,h}^H = C_{2,h}^L = (1-\lambda^L)\theta$ from constraint $\eta$. Hence, the only two constraints left are the participation constraint $\alpha C_1^L + p(1-\lambda^L)\theta \geq \alpha\omega$ and the feasibility constraint $C_1^L \leq \phi + \omega$. The conclusion follows easily. ■
We summarize the result for the CCP problem in the following proposition.

**Proposition 15** *A solution to the CCP problem is such that the consumption of the lender is equal across states and is pinned down by the resource constraint at $t = 2$: $X_2^H = X_2^L = X_2$ which solves*

$$X_2 + pC_{2h} = \phi + \omega - C_1 + p\theta$$

So if:

1. $\omega \leq \frac{(1-\lambda_L)p\theta}{\alpha}$ , $C_1 = 0$,$C_{2h} = (1-\lambda_L)\theta$ and $X_2 = \phi + \omega + \lambda_L p\theta$. Denote by $X_{21}$ such value of $X_2$.

2. $\omega > \frac{(1-\lambda_L)p\theta}{\alpha}$ , $C_1 = \omega - \frac{(1-\lambda_L)p\theta}{\alpha}$,$C_{2h} = (1-\lambda_L)\theta$ and $X_2 = \phi + \left[\frac{(1-\lambda_L)}{\alpha} + \lambda_L\right]p\theta$. Denote by $X_{22}$ such value of $X_2$.

Notice that $X_{22} \geq X_{21}$.
Therefore the CCP

1. insures lenders against all risk (both counterparty default and exogenous output realization)

2. pays every borrower the same contract (we could interpret this as it sets uniform margin and default fund contributions regardless of whether borrowers have more or less incentives to deviate when it cannot observe such incentives).

3. the payments to the borrowers are state contingent (depending on the exogenous output realization)

Therefore there is no special role in the CCP for screening a $H, L$ borrower: there is no efficiency rationale here for screening because borrowers are not more or less productive depending to their type, they are just different in their temptation to deviate, but the stochastic process for output is the same for both of them. So the equilibrium with central clearing is equivalent to not screening at all but just making sure that nobody deviates assuming they are all the worst type $L$ (they all have the highest temptation to walk out of the contract ex post).

For the comparison with bilateral clearing:

1. I will map a parameters' space map of when the participation constraint is slack of binds for both borrowers and under both clearing arrangement

2. I will compare whether the mix of $C_1$ and $C_{2h}$ payments that borrowers get would be better for $H$ lenders under bilateral clearing: in fact $H$ lenders may be worse off under central clearing because of the schedule of payments forces them to pay more $C_2$ than they would need to pay with bilateral clearing in order to keep the borrower from walking out of the contract. Even in cases when the participation constraint binds under both clearing arrangements, the schedule of payments may make such $H$ - matched lenders worse off in a CCP, since it is cheaper for them to provide incentives via $C_1$. I find this very interesting because it is not only the amount that lenders end up paying to the borrowers that matters, but also the timing of payments.

3. Lenders are completely insured in a CCP, therefore they gain insurance from clearing centrally, even though they may lose from having to pay more for the limited commitment friction than the would if clearing bilaterally. This trade off may still be such that if the insurance gain is large enough then they want to clear centrally.

The goal of this paper would be to answer the question of whether lenders who are matched with $H$ type borrowers have incentive to clear through the CCP more risky assets: at least this should be true if they are sufficiently risk averse. The conjecture at this point would be that the more risk averse lenders are the more risky assets they will want to clear centrally.

# 5 Comparing CCP and bilateral clearing solutions

When clearing through the CCP both types of agents (borrower and lender) get the same allocation:

$$c_{2h} = \left(1 - \lambda^L\right) \theta$$

$$
\begin{aligned}
c_{2l} &= 0 \\
c_1 &= 0 \text{ if } \frac{\alpha\omega}{p} \leq \left(1-\lambda^L\right)\theta \\
c_1 &= \omega - \frac{\left(1-\lambda^L\right)p\theta}{\alpha} \text{ if } \frac{\alpha\omega}{p} > \left(1-\lambda^L\right)\theta
\end{aligned}
$$

And consequently the consumption to the lender differs according to whether the outside option has low or high value to the borrower. So if $\frac{\alpha\omega}{p} \leq \left(1-\lambda^L\right)\theta$ then $X_2 = \phi+\omega+\lambda_L p\theta$, whereas if $\frac{\alpha\omega}{p} > \left(1-\lambda^L\right)\theta$ then $X_2 = \phi + \left[\frac{(1-\lambda_L)}{\alpha} + \lambda_L\right]p\theta$.

With bilateral clearing the solution differs according to which partition of the parameters' space we are in.

## 5.1 Low outside option for both borrowers $\frac{\alpha\omega}{p} \leq \left(1-\lambda^H\right)\theta$

If $\frac{\alpha\omega}{p} \leq \left(1-\lambda^H\right)\theta$ then it is also the case that $\frac{\alpha\omega}{p} \leq \left(1-\lambda^L\right)\theta$ since $\lambda^H > \lambda^L$. The solution to the bilateral clearing problem is always such that the limited commitment constraint (LC) is binding and the participation constraint (PC) is slack: however there may be two different solutions for $X_2^i$:

$$
\begin{aligned}
X_{2h}^i &= \phi+\omega+\lambda^i\theta \\
X_{2l}^i &= \phi+\omega
\end{aligned}
$$

if and only if

$$
\begin{aligned}
-pu'\left(\phi+\omega+\lambda^i\theta\right)-(1-p)u'\left(\phi+\omega\right) &< 0 \\
\frac{\alpha\omega}{p} &\leq \left[1-\lambda^i\right]\theta
\end{aligned}
$$

With respect to the CCP solution ($X_2^{CCP} = \phi+\omega+\lambda_L p\theta$) we have that $X_{2l}^i < X_2^{CCP}$ and that $X_{2h}^i > X_2^{CCP}$ for both lenders, since $\lambda_L < \lambda_H$ and $p < 1$.

And

$$
\begin{aligned}
X_{2h}^i &= \phi+\omega-c_1^{i*}+\lambda^i\theta \\
X_{2l}^i &= \phi+\omega-c_1^{i*}
\end{aligned}
$$

if and only if

$$
\begin{aligned}
-pu'\left(\phi+\omega+\lambda^i\theta\right)-(1-p)u'\left(\phi+\omega\right) &> 0 \\
\frac{\alpha\omega}{p} &\leq \alpha c_1^* + \left[1-\lambda^i\right]\theta
\end{aligned}
$$

As when $c_1 = 0$, also when $c_1 > 0$ we have that with respect to the CCP solution $X_{2l}^i < X_2^{CCP}$ and that $X_{2h}^i > X_2^{CCP}$ for both lenders.

Therefore in this case both lenders prefer bilateral clearing ex post in the high state and prefer central clearing ex post in the low state. Ex ante lenders will clear centrally if the insurance they get through the CCP is large enough (or if they are risk averse enough).

For the case where $c_1 = 0$ it this is true if and only if:

$$u(\phi + \omega + \lambda_L p\theta) \quad > \quad pu\left(\phi + \omega + \lambda^i\theta\right) + (1-p)\,u(\phi + \omega)$$

Clearly, if this is satisfied for $\lambda^H$ lenders then it is also satisfied for $\lambda^L$ lenders. In general, lender of type $i$ ex ante prefers to clear centrally rather than bilaterally if and only if $\phi+\omega+\lambda_L p\theta > \overline{X}_2^i$ where $\overline{X}_2^i$ is defined as the certainty equivalent of the utility from bilateral clearing:

$$u\left(\overline{X}_2^i\right) \quad = \quad pu\left(\phi + \omega + \lambda^i\theta\right) + (1-p)\,u(\phi + \omega)$$

For the case where $c_1 > 0$ it this is true if and only if:

$$u(\phi + \omega + \lambda_L p\theta) \quad > \quad pu\left(\phi + \omega - c_1^{i*} + \lambda^i\theta\right) + (1-p)\,u\left(\phi + \omega - c_1^{i*}\right)$$

Similarly we can define the certainty equivalent of the utility from bilateral clearing, $\overline{X}_2^i$, as:

$$u\left(\overline{X}_2^i\right) \quad = \quad pu\left(\phi + \omega - c_1^{i*} + \lambda^i\theta\right) + (1-p)\,u\left(\phi + \omega - c_1^{i*}\right)$$

Notice that in this subset of the parameters' space the limited commitment constraint (LC) is binding and the participation constraint (PC) is slack, therefore the limited commitment friction is strong even though the borrowers' outside option to trading is not very high so that it is not very expensive for lenders to give them sufficient incentives to deal with the limited commitment friction.

## 5.2 High outside option for both borrowers $\frac{\alpha\omega}{p} > \left(1 - \lambda^L\right)\theta$

Central clearing through a CCP implies that both lenders get $X_2 = \phi + \frac{(1-\lambda_L)p\theta}{\alpha} + \lambda_L p\theta$.

**Claim 16** *Consider the case where both the limited commitment (LC) and the participation (PC) constraints are binding and $c_1^i = 0$. Both lenders are ex post better off with bilateral clearing if the realization of output is $h$ whereas are ex post better off with central clearing if the realization of output is $l$.*

This happens if and only if

$$\frac{(\alpha - p)}{(1-p)} \quad > \quad \frac{u'\left(\phi + \frac{p[1-\lambda^i]\theta}{\alpha}\right)}{u'\left(\phi + \frac{p[1-\lambda^i]\theta}{\alpha} + \lambda^i\theta\right)}$$

And for lenders we have that

$$\begin{aligned} X_{2h}^i &= \phi + \frac{\left[1-\lambda^i\right]p\theta}{\alpha} + \lambda^i\theta \\ X_{2l}^i &= \phi + \frac{\left[1-\lambda^i\right]p\theta}{\alpha} \end{aligned}$$

Lenders who are matched with $\lambda^L$ borrowers are ex post better off with bilateral clearing if the realization of output is $h$ whereas are ex post better off with central clearing if the realization of output is $l$.

Lenders who are matched with $\lambda^H$ borrowers are ex post better off with bilateral clearing if the realization of output is $h$ if and only if

$$\begin{aligned} \lambda^H - \lambda_L p &> \frac{\left(\lambda^H - \lambda_L\right) p}{\alpha} \\ \left(\lambda^H - \lambda_L\right) p + (1-p)\lambda^H &> \frac{\left(\lambda^H - \lambda_L\right) p}{\alpha} \\ \left(\lambda^H - \lambda_L\right) p \left(1 - \frac{1}{\alpha}\right) + (1-p)\lambda^H &> 0 \end{aligned}$$

which is always true. Whereas they are ex post better off with central clearing if the realization of output is $l$.

Therefore both lenders are ex post better off with bilateral clearing if the realization of output is $h$ whereas are ex post better off with central clearing if the realization of output is $l$.

As above we can define the certainty equivalent of the utility from bilateral clearing, $\overline{X}_2^i$, as:

$$u\left(\overline{X}_2^i\right) = pu\left(\phi + \frac{\left[1-\lambda^i\right] p\theta}{\alpha} + \lambda^i\theta\right) + (1-p)\, u\left(\phi + \frac{\left[1-\lambda^i\right] p\theta}{\alpha}\right)$$

so that central clearing is preferred by a lender matched with a borrower with type $\lambda^i$ if and only if

$$\phi + \frac{(1-\lambda_L)\, p\theta}{\alpha} + \lambda_L p\theta > \overline{X}_2^i$$

**Claim 17** *Consider the case where the limited commitment constraint (LC) is slack and the participation constraint (PC) is binding and* $c_1^i = 0$

This happens if and only if

$$\theta > \frac{\alpha\omega}{p} \geq \left(1-\lambda^i\right)\theta \tag{34}$$

$$\frac{u'(\phi+\omega)}{u'\left(\phi+\omega+\theta-\frac{\alpha\omega}{p}\right)} > \frac{(\alpha-p)}{(1-p)} \tag{35}$$

And for lenders we have that:

$$\begin{aligned} X_{2h}^i &= \phi + \omega + \theta - \frac{\alpha\omega}{p} \\ X_{2l}^i &= \phi + \omega \end{aligned}$$

Recall that in this subset of the parameters' space we have that $\frac{\alpha\omega}{p} > \left(1-\lambda^L\right)\theta$. Since $X_{2h}^i, X_{2l}^i$ are independent of $i$ then both lenders who are matched with

$\lambda^L$ borrowers and lenders who are matched with $\lambda^H$ borrowers have the same incentives to clear bilaterally or centrally.

Therefore lenders are ex post better off with bilateral clearing if the realization of output is $h$ if and only if

$$\begin{aligned} \frac{\theta\left(1-\left[\frac{(1-\lambda_L)}{\alpha}+\lambda_L\right]p\right)}{\left(\frac{\alpha}{p}-1\right)} &> \omega \\ \frac{p\theta}{\alpha}\frac{(\alpha-p)-(\alpha-1)\,p\lambda_L}{\alpha-p} &> \omega \end{aligned} \tag{36}$$

Whereas they are ex post better off with central clearing if the realization of output is $l$ if and only if

$$\begin{aligned} \omega-\frac{(1-\lambda_L)\,p\theta}{\alpha} &< \lambda_L p\theta \\ \frac{\alpha\omega}{p} &< [\alpha\lambda_L+(1-\lambda_L)]\,\theta \end{aligned} \tag{37}$$

Recall that this is the case where the LC is slack but still $c_1 = 0$ despite $\alpha > 1$ because the probability of the good outcome is relatively small: in bilateral clearing the only way to guarantee some high consumption to the lender in the low realization is to save, that is equivalent to paying the borrower less $c_1$. In this case obtaining insurance against the realization of output in the bilateral clearing setup is more costly because the tool of using $c_1$in order to give the borrower incentives needs to be employed. In the bilateral clearing case the problem is that the best instrument to provide incentives (collateral) is also the only instrument to provide insurance across states (storage). So this is a case where central clearing could be useful in any state (if $p$ is not too low, that is to say the probability of success of output, which is the total measure of additional output in the high state in the CCP clearing, because of full diversification of the output realization shock. So if there is not much to gain there, in terms of resources that can be used to provide insurance, there is no incentive to clear centrally). Or in other words, if we want to interpret the necessary and sufficient conditions for bilateral vs central clearing, (36) and (37), in terms of upper bounds on $\omega$, then central clearing is better unambiguously any time the borrower is not endowed with a lot of consumption good. This is so because in bilateral clearing if $\omega$ is large then there are relatively a lot of resources to store from one period to the next so that having to forgo the provision of incentives via $c_1$is not too costly. With central clearing instead, the participation constraint always binds, that is to say lenders always pay the borrowers their present discounted value of the outside option: when this is relatively large then the benefits from insurance through a CCP -coming from diversification (getting a lot of $\theta$regardless of the state each lender is in) rather than having to store more goods- are relatively small .

Also notice the upper bounds on $\omega$ needed in this case for the CCP to be always preferred even ex post, does not tighten the necessary and sufficient conditions (34) and (35) for the bilateral solution. In fact (34) is already taken into account, and if (35) is satisfied at a given $\overline{\omega}$ then it is always satisfied at any $\omega \leq \overline{\omega}$.

Clearly the opposite is also true: it doesn't matter how risk averse lenders are, they may always prefer to clear bilaterally if $\omega > \overline{\omega} = \max(\omega_1, \omega_2)$ where $\omega_1 = \frac{p\theta}{\alpha} \frac{(\alpha-p)-(\alpha-1)p\lambda_L}{\alpha-p}$, $\omega_2 = \frac{p\theta}{\alpha}\left[\alpha\lambda_L + (1-\lambda_L)\right]$ defined by (36) and (37).

**Claim 18** *Consider the case where the limited commitment constraint (LC) is slack and the participation constraint (PC) is binding and* $c_1^i > 0$

This happens if and only if

$$\theta > \frac{\alpha(\omega - c_{13})}{p} \geq \left(1-\lambda^i\right)\theta \tag{38}$$

$$\frac{u'(\phi+\omega)}{u'\left(\phi+\omega+\theta-\frac{\alpha\omega}{p}\right)} < \frac{(\alpha-p)}{(1-p)} \tag{39}$$

And for lenders we have that:

$$\begin{aligned} X_{2h}^i &= \phi+\omega-c_{13}+\theta-\frac{\alpha(\omega-c_{13})}{p} \\ X_{2l}^i &= \phi+\omega-c_{13} \end{aligned}$$

where $c_{13}$ solves

$$(1-p)u'(\phi+\omega-c_{13}) = (\alpha-p)\,u'(\phi+\omega-c_{13}+\theta-\frac{\alpha(\omega-c_{13})}{p})$$

Since $X_{2h}^i, X_{2l}^i$ are independent of $i$ then both lenders who are matched with $\lambda^L$ borrowers and lenders who are matched with $\lambda^H$ borrowers have the same incentives to clear bilaterally or centrally. In this case however things are different than with $c_1 = 0$ and are more interesting since it becomes apparent that the schedule of payments matters: in state $l$ the central clearing solution is preferred ex post, but in state $h$ the bilateral solution is ex post preferred by both lenders if and only if

$$\begin{aligned} -c_{13}-\frac{\alpha(\omega-c_{13})}{p} &> -\frac{\alpha\omega}{p} \\ c_{13}\left(1-\frac{\alpha}{p}\right) &< 0 \end{aligned}$$

which is always true.

Notice that with respect to the previous case the present value of consumption to the borrower is the same, because the participation constraint is binding, and also the insurance purpose of $c_1$ is still at play here because the limited commitment constraint is slack and $c_{13}$ is determined by the marginal rate of substitution between states. However lenders are optimally using some $c_1$ to provide incentives, which is a cheaper way to do so rather than using $c_{2h}$. This implies that in state $h$ lenders can consume more $c_{2h}$ which makes them prefer ex post the bilateral clearing solution.

Also notice that this solution arises when lenders are not very risk avers (i.e. (39) holds rather than (35)).

Ex ante lenders prefer the CCP clearing arrangement if and only if $\phi + \left[\frac{(1-\lambda_L)}{\alpha} + \lambda_L\right] p\theta > \overline{X}_2$ where $\overline{X}_2$ is defined as the certainty equivalent of the utility from bilateral clearing:

$$u\left(\overline{X}_2\right) = pu\left(\phi + \omega - c_{13} + \theta - \frac{\alpha\left(\omega - c_{13}\right)}{p}\right) + (1-p)\,u\left(\phi + \omega - c_{13}\right)$$

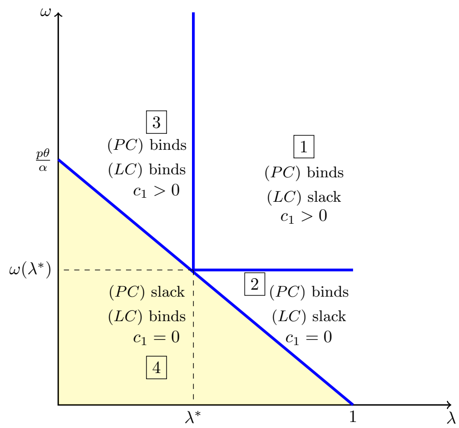


Figure 1: Solution to bilateral problem if $\frac{u'(\phi)}{u'(\phi+\theta)} > \frac{\alpha-p}{1-p}$.

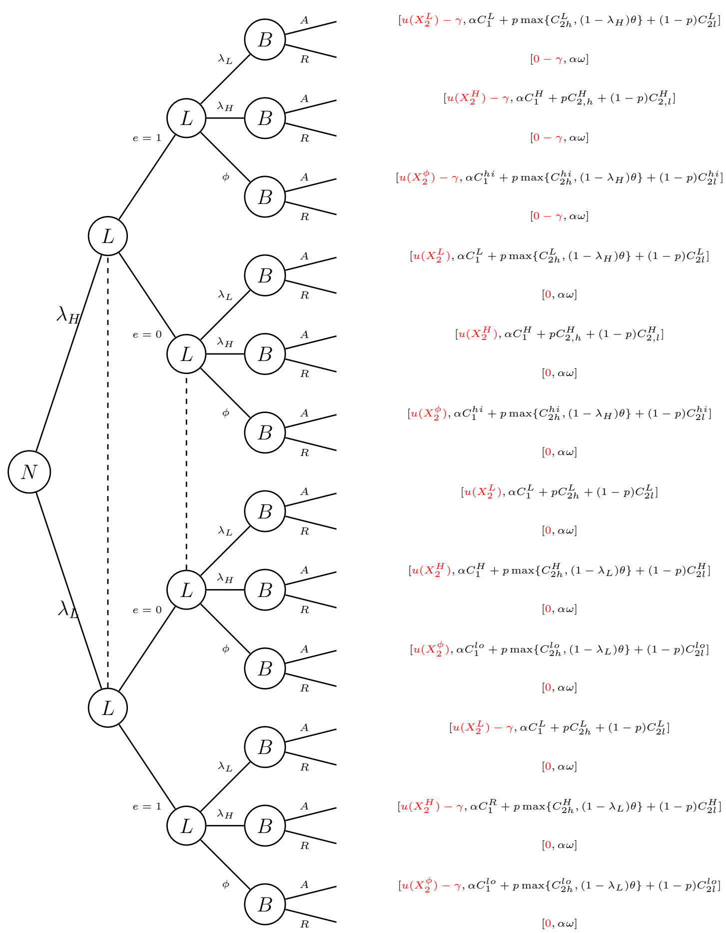


Figure 2: CCP problem.

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