

How Firms Affect Wages: a Structural Decomposition

Preliminary and Incomplete

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1 Introduction

There is a long literature that has studied the link between firm productivity, or TFP, and the wage they pay their workers. The typical finding in this literature is that more productive firms pay higher wages, and are larger in size. Two leading explanations for this observation are assortative matching and economic rents. The sorting explanation says that because of complementarities in production the more productive firms hire more skilled workers which explain the high wages these firms pay. The latter theory says that in markets with non-competitive features more productive firms generate more economic rents than less productive firms, and through bargaining workers capture a fraction of these rents.

A recent stream of papers including Lopes de Melo (13), Eeckhout and Kircher (10) and Lise, Meghir and Robin (12) has emphasized a different aspect of the productivity-wages relationship. These papers have stressed that in assignment models, after you condition on worker skill, wages are non-monotone in firm productivity. The explanation for this is that each worker has an ideal firm that he should be matched and the price system (wages) is what induces matching. For example, if the ideal firm for a mid-skilled worker is a mid-productivity firm his wage should be the highest at that firm. In particular, if this worker ends up mismatched at a more productive firm his wage should be lower, since this worker is more valuable at less productive firms. These papers have used this prediction of the theory to reconcile the predictions sorting models with evidence from reduced form worker-firm wage decompositions. However, they have not provided direct evidence of those non-monotonicities.

In this paper, we estimate a conditional wage function using a matched employer-employee dataset from Brazil to shed light on this question. In particular, we are interested in how much wages vary in firm productivity conditional on worker skill, and if that function is monotone or not. The idea is to use restrictions from the theory to construct an index of worker skill, an index of firm productivity and a third index which we label compensating differentials. This last index captures systematic differences in pay across firm, and are consistent with the compensation for a job amenity. Our indexes capture both observed and unobserved characteristics of firms and workers and they can be computed for subsets of workers within establishments. Once we compute those indexes we estimate the conditional wage function using non-parametric methods.

We introduce a search model with two-sided heterogeneity to motivate the assumptions that we use to create our indexes. The model is a version of Shimer and Smith (00) augmented with compensating differences relative to a firm characteristic. In this model, workers have a single dimension of heterogeneity that affects the productivity of matches, whereas firms have two dimensions of heterogeneity, one that affects match productivity and the other worker preferences. We focus on a few predictions of the theory. First, conditional on the firm characteristics wages are increasing in worker skill. Second, if complementarities in

production are strong, and either there are no compensating differentials, or the degree of frictions are low, then more productive firms hire better workers on average. Finally, conditional on worker skill and firm productivity wages are strictly decreasing in the compensating differential.

We first show how to compute our indexes in the model without compensating differentials. The method has two sequential steps. First, we use the insights of Hagedorn et al (12) to create the index of worker skill. The procedure involves first ranking workers within firms via their wages and then using the fact that workers switch firms in the panel to create a global rank of workers. Second, we use the monotone sorting assumption to create our ranking of firm productivity: we compute the mean level of worker skill for each firm, and then rank them according to this value. This procedure works both if the economy features positive assortative matching, or negative sorting.¹

Applying this method to a Brazilian matched employer employee dataset yields a number of results. First, we find that there is strong assortative matching in the Brazilian economy, as the worker and firm indexes are highly correlated. This is consistent with the results of Lopes de Melo (13), which look at patterns of sorting between workers and co-workers. Second, after conditioning on worker skill the share of wage dispersion due to differences in productivity is very small - less than 2% of overall wage dispersion. Third, for most levels of worker skill, the wage function is non-monotone in firm productivity. However, the peak of the wage function seems to be not increasing in worker skill, which goes against the theory. Fourth the estimated wage function only accounts for around 60% of overall wage dispersion, and it's residual is highly clustered across firms. This last observation is what motivates us to consider the extended model with compensating differentials, but the results with the augmented model are work in progress. These four results are robust to a number of ways we construct our sample, dividing it by age, education or by occupations.

2 Related Literature

To be written

3 The Model

The model shares the basic features of Shimer and Smith [?], plus a second dimension of firm heterogeneity which we label compensating differentials. It is a continuous time economy with heterogeneous agents, complementarities in production and search frictions. Workers have a single dimension of heterogeneity which represents their skill. Firms have a two-dimensions type, where the first component affects job productivity

¹In the case of negative sorting a high index firm will be a low-productivity firm.

whereas the second affects worker's preferences. Production in this economy happens in bilateral matches. Workers meet jobs according to a stochastic process, i.e. matching is not instantaneous because of search frictions. They observe each other's types, decide if they want to pair up or not, and, upon matching, produce output and split the proceeds according to a pre-designed rule. Similarly as in their model, matches are dissolved exogenously at a given rate.

3.1 The Environment

There is a continuum of workers and jobs, each indexed by a type. We exogenously assign a value $x \in [0, 1]$ for each worker, which can be interpreted as his/her human capital, and a vector $\{y, z\} \in [0, 1] \otimes [0, 1]$ for each job, which respectively affect match productivity and the preferences of workers. These exogenous random variables have atomless distributions $L : [0, 1] \rightarrow [0, 1]$ and $G : [0, 1] \otimes [0, 1] \rightarrow [0, 1]$, respectively, with densities $l(x)$ and $g(y, z)$. These types are assumed to be perfectly observable and the distributions are known to all agents. The mass of workers is normalized to 1, and the mass of jobs per worker is J .

Workers and firms discount time at rate r . Firms have linear preferences over flow profits, and workers have linear additive preference over flow wages and the compensating differential: $w + \tau(z)$, where $\tau(\cdot)$ is a known function. Unemployed workers receive a flow value $b(x)$ and vacant jobs receive zero flow value.

When a worker of type x and a firm of type (y, z) match they produce a flow output of $F(x, y)$ at every instant. The production function is an essential element to our model because it has strong implications for the sorting patterns of workers. Shimer and Smith [?] provide a characterization of sufficient conditions for positive assortative matching in the model without compensating differentials. In particular, they show that if the production function, the log of its first partial derivatives and the log of its cross derivative are all supermodular then the economy exhibits positive assortative matching. %%%#%\$

We assume a random search technology where worker/jobs meet each other at a finite Poisson rate. Upon meeting, they take a draw from the corresponding idle type distribution—e.g., if an unemployed worker meets an employer he/she takes a draw from the vacancies distribution. Unemployed workers meet vacancies at the rate λ^U . On the firm side, vacancies meet unemployed workers at a rate λ^F . Finally, all matches are exogenously destroyed at rate δ . Moreover, we assume that the economy is in steady state.

The presence of search frictions creates a temporary bilateral monopoly power in a match. We follow several others and adopt the generalized Nash bargaining solution to determine wages. We choose this particular mechanism because it simplifies the solution of the problem, while embedding the economic forces that we believe are important to explain the data.

Before describing how the equilibrium of the model is determined, we introduce the following notation.

An employed worker of type x working for a firm of type (y, z) has value $W(x, y, z)$, or, if this workers is unemployed, $U(x)$. A job of type (y, z) has value $J(x, y, z)$ if matched with a worker of type x , or $V(y, z)$ if vacant. The surplus of a match between worker x and job (y, z) is $S(x, y, z) \equiv [W(x, y, z) - U(x)] + [J(x, y, z) - V(y, z)]$. The wage in any match (x, y, z) is $w(x, y, z)$. The steady state measure of employed matches is $e(x, y, z)$. The measure of unemployed workers for each type is $u(x)$, and the measure of vacancies is $v(y, z)$.

3.2 Values and Decisions

An employed worker x at a firm (y, z) earns a flow wage of $w(x, y, z)$. At a rate δ his/her match is destroyed exogenously. This implies the following value equation:

$$rW(x, y, z) = w(x, y, z) + \tau(z) + \delta[U(x) - W(x, y, z)] \quad (3.1)$$

An unemployed worker of type x enjoys flow unemployment benefits $b(x)$ and receives job offers at rate λ^U :

$$rU(x) = b + \lambda^U \int_0^1 \int_0^1 \alpha(x, y', z') [W(x, y', z') - U(x)] \frac{v(y', z')}{v} dy' dz', \quad (3.2)$$

where $[X]^+$ denotes X if $X > 0$ or 0 otherwise, and

$$v = \int_0^1 \int_0^1 v(y', z') dy' dz'.$$

A productive job of type (y, z) , employing a worker of type x , enjoys flow profits $F(x, y) - w(x, y, z)$, but can be dissolved by the δ shock:

$$rJ(x, y, z) = F(x, y) - w(x, y, z) + \delta[V(y, z) - J(x, y, z)] \quad (3.3)$$

Finally, a vacancy does not earn any flow values, but can hire unemployed workers:

$$rV(y, z) = \lambda^F \int_0^1 [J(x', y, z) - V(y, z)]^+ \frac{u(x')}{u} dx', \quad (3.4)$$

where

$$u = \int_0^1 u(x') dx'.$$

As previously described, wages are determined by the Generalized Nash bargaining solution. Standard

results then imply that

$$S(x, y, z) = \frac{J(x, y, z) - V(y, z)}{(1 - \beta)} = \frac{W(x, y, z) - U(x)}{\beta}. \quad (3.5)$$

Plugging Equations (3.1) and (3.3) in this formula and rearranging we solve for the wage

$$w(x, y, z) = \beta [F(x, y) - rV(y, z)] + (1 - \beta) rU(x). \quad (3.6)$$

From this expression, we can see that wages are a function of output and the outside options of the worker and the firm.

When idle agents meet, they decide to pair up if doing so increases their values. It is easy to see from Equation (3.5) that this is equivalent to choosing the option that provides the largest surplus. Thus, when an unemployed worker meets a vacancy, the pair meet iff $S(x, y, z) > 0$.

3.3 Steady State Flows

We now describe the equilibrium equations that jointly determine the stationary measures $e(x, y, z)$, $u(x)$ and $v(y)$.

The first equilibrium equation that we describe is between the flows in and out of employed matches of type (x, y, z) . If $S(h, p) \leq 0$ then $e(x, y, z) = 0$. Otherwise, $e(x, y, z)$ is determined by equating the inflows to the outflows,

$$\lambda^U u(x) \frac{v(y, z)}{v} = \delta e(x, y, z), \quad (3.7)$$

Employed workers of type x have to equal the workers of that type minus the unemployed ones. This implies that

$$l(x) - u(x) = \int_0^1 \int_0^1 e(x, y', z') dy' dz'. \quad (3.8)$$

The same holds for jobs of type (y, z)

$$Ng(y, z) - v(y, z) = \int_0^1 e(x', y, z) dx'. \quad (3.9)$$

Equations 3.7 to 3.9 determine all endogenous objects in this model. However, for us to reach a steady state we still need to make sure that the number of unemployed workers finding jobs equal to the number of

vacancies meeting workers. This is true if λ^F satisfies

$$\lambda^F = \frac{u}{v} \lambda^U.$$

3.4 Equilibrium

Definition 1. *A steady state equilibrium in this economy consists of values for $e(x, y, z)$, $u(x)$, $v(y, z)$, $U(x)$, $V(y, z)$ such that Equations (3.2) to (3.9), are satisfied.*

Shimer and Smith [?] have a proof of equilibrium existence in the case where z is a singleton.

3.5 Properties of the Wage Function

It is useful at this point to establish some properties of the wage function.

Proposition 2. *In the Steady State equilibrium of this economy the wage function described by Equation 3.6 satisfies the following properties:*

1. *Wages are strictly increasing in worker skill: $\frac{dw(x, y, z)}{dx} > 0, \forall \{(x, y, z) | S(x, y, z) > 0\}$.*
2. *Wages decrease in jobs productivities for some matches: $\frac{dw(x, y, z)}{dy} < 0, \exists \{(x, y, z) | S(x, y, z) > 0\}$.*
3. *Wages are strictly decreasing with the job amenities: $\frac{dw(x, y, z)}{dz} < 0, \forall \{(x, y, z) | S(x, y, z) > 0\}$.*

Part 2 of his Proposition states that wages are non-monotone in firm productivity. These non-monotonicities are a natural reflection of the job scarcity and the fact that firms reject workers in equilibrium: the least skilled worker that a given firm hires generates zero surplus, and as a consequence earns his reservation wage.² Thus, for this worker it is actually preferable to find a job in a less productive firm. Moreover, our intuition suggests that this should extend beyond Nash bargaining and be true for any wage mechanism where matching is bilaterally efficient.

4 Identification Strategy With No Compensating Differences

In this Section we outline how to obtain empirical estimates for the heterogeneity indexes x, y using a matched employer-employee dataset. First, we study identification in the restricted model without compensating differentials, where z is singleton. In Section 7 we study how to obtain an empirical measure of z under the assumption that frictions are “small”. One conceptual and practical issue is that the model was formulated

²With positive sorting, this is true for any firm that only hires workers above the minimum skilled worker.

at the job level, whereas firms typically hire multiple workers. Thus, we assume that firms are clusters of jobs with the same level of y . In practice, these clusters can be defined as establishments, or groups of workers with the same characteristics within establishments (e.g. education or occupation). In what follows we outline the two-steps procedure which we use to retrieve those indexes.

First, we follow the insights of Hagedorn et al (13) to create a global rank of workers. The procedure involves ranking workers within firms and then using the fact that workers switch firms to create a global rank of workers. We can rank workers within firms because as seen in in Section 3 the wage function is increasing in x , conditional on y . Because wages are measured with error the wage rankings of workers may be inconsistent across firms. To reconcile that, Hagedorn et al (13) propose following a Bayesian approach and picking the rank of workers $\hat{x}$ that maximizes the likelihood of observed wages, conditional on a distribution of measurement error. For full details of the procedure see Hagedorn et al (13).

Second, we construct a rank of firm productivity using an assumption about assortative matching. The idea is that if complementarities in production are strong enough the equilibrium features positive/negative sorting, and more/less productive firms hire better workers. In practice, we make the following monotone sorting assumption:

- **Monotone sorting assumption:** $E[x|y]$ is strictly increasing or decreasing in y .

Given this assumption a natural way to rank firms is simply to compute the mean level of x at each firms, and then rank firms according to this value. In order to be consistent with our theory when constructing the firm rank we compute the share of jobs (not firms) with a mean x below a certain value. This yields our global rank of firms $\hat{y}$. Note that this method works both if we have positive assortative matching, or negative assortative matching. With positive sorting a high rank firm will be a high productivity firm, whereas with negative sorting a high rank firm is a low productivity firm.

5 Data

We use the labor market census RAIS (Relação Annual de Informações Sociais), an administrative dataset collected annually by the Brazilian labor ministry, which includes all firms in the Brazilian formal sector and provides information for all their workers. The ministry collects demographic information for workers, such as age, education and sex, some information about establishments, such as sector and location, and provides information about the job, such as the average wage earned during that year (the measure we use), the wage in December, the average number of hours worked, occupation, dates of admission and separation, type of contract, causes for separation.³ We include the observations of workers who work all year, and our measure

³The remaining variables are race, nationality, a measure of disability and the juridic nature of the firm.

Sample	# obs	# workers	# firms
Full Sample	3.39 <i>M</i>	1.02 <i>M</i>	38 <i>K</i>
High School	1.94 <i>M</i>	590 <i>K</i>	26 <i>K</i>
College	1.26 <i>M</i>	361 <i>K</i>	14 <i>K</i>
<i>Age</i> ≥ 35	1.59 <i>M</i>	500 <i>K</i>	19 <i>K</i>
Occupations	3.57 <i>M</i>	1.07 <i>M</i>	53 <i>K</i>

Table 1: Sample Statistics

of wages is the residual of a wage regression on time dummies and a polynomial for potential experience. Including or not the latter does not affect our results significantly.

We use 11 years of data, covering the years from 1995 to 2005. Although data from previous years were available, we choose this time-frame because until 1994 Brazil suffered from extremely high inflation, which caused serious measurement problems in variables such as wages, and also had structural implications for the macroeconomy. We restrict the sample to the state of São Paulo, which is the richest state of the country, covering over 13% of GDP and over 30% of industrial production. Sao Paulo has a much smaller level of informal employment than all the other regions of the country. Informality is an important feature in Brazil, just as it is in many developing countries, and is absent from our analysis. Furthermore, RAIS is an enormous dataset, and reducing the number of states makes the dataset more manageable.

We work with a baseline sample that includes workers with at least a high school degree who worked during the years of 1998 until 2002, and who worked in establishments with at least 5 workers and at most 1000 workers, all who satisfy the same criteria. In addition, we consider a number of alternative subsamples, divided by education, age and occupation categories. In the case of occupations the empirical counterpart of a firm is a combination of a establishment and 5 different levels of occupations, divided in a hierarchical way as in Menezes, Muendler and Ramey (08). Table 1 shows statistics of the sample.

6 Results

Next, we present the results when applying our methods to the Brazilian RAIS dataset, for each subsample described in Section 5. First, we compute the worker and firm ranks $\hat{x}$ and $\hat{y}$ using the method outlined in Section 4. Then we estimate the conditional wage function $w(x, y)$, first by dividing our indexes in a number of bins, and computing the mean level of wages within those bins. The results in this Section use 10 different values for the bins, but the results are not very sensitive to this choice.

The first column of Table 2 describes the degree of sorting in the economy. As we can see, for all subsamples we have a high degree of assortative matching as expressed by the high correlation between our

Sample	$Corr(x, y)$	$\frac{E[Var(w x)]}{Var(w)}$
Full Sample	0.563	0.017
High School	0.517	0.019
College	0.443	0.017
$Age \geq 35$	0.514	0.013
Occupations	0.715	0.01

Table 2: Assortative Matching and Wage Decomposition

indexes of skill and job productivity. The second column displays the degree of wage dispersion relative to firm-worker matching. We perform a simple wage decomposition between and within skill levels

$$Var(w(x, y)) = E[Var(w|x)] + Var[E(w|x)].$$

As we can see, only a very small share of wage dispersion- less than 2%- is attributed to differences in wages for a given level of worker skill.

Figures 6.1 and 6.2 illustrate the conditional wage function, respectively for the full sample and the subsamples. Each line in the graph corresponds to a decile of worker skill, and on the X axis we vary the level of firm productivity. The first thing to notice is that in all samples increasing the decile of worker skill increases the level of wages, confirming the prediction that wages are monotone in x conditional in y . Second, it is clear from the graphs that the wage function is very “flat” in y , which is in accord with the results from the wage decomposition illustrated in Table 2. Third, for most samples and levels of worker skill wages are non-monotone in firm productivity. The exception seems to be the sample of college graduates, where wages seem to be strictly increasing in productivity. Finally, the graphs seem to indicate that the peak of the wage function is not increasing in the level of worker skill. This is a prediction from the theory that doesn’t seem to hold in our sample, although this needs to be tested formally.

Our last set of results have to do with the explanatory power of the model and properties of the residual. The first column of Table 3 displays the fraction of explained variation by the model, which seems to be around 60% for most samples. This shows there is still a substantial amount of wage variation not captured by this model with two dimensions of heterogeneity. We investigate this further and estimate a regression of the residual of the model on firm dummies. The results are displayed in the second column of Table 3. As we can see, the firm dummies explain a sizable share of the wage dispersion not explained by our model. This suggests that firms affect wages in a systematic and monotone way, which is not captured by our baseline model. This motivates the introduction of compensating differentials into the model, as these

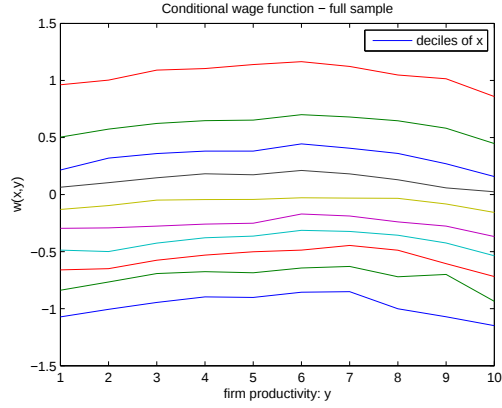


Figure 6.1: Conditional Wage Function - Full Sample

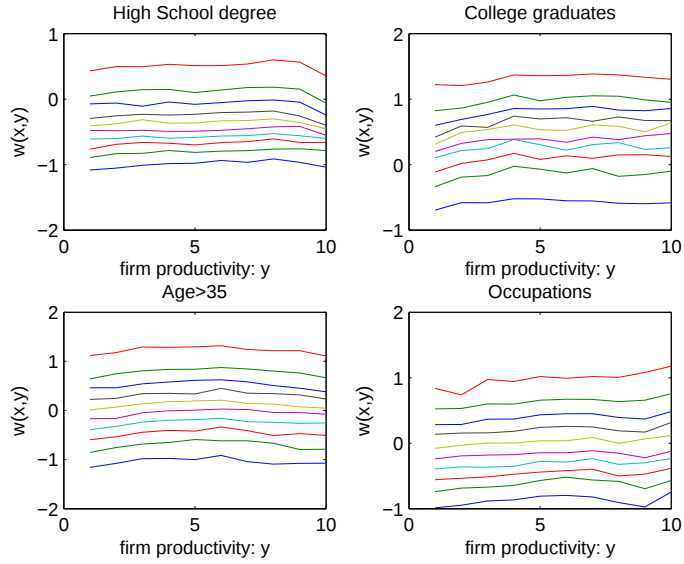


Figure 6.2: Conditional Wage Function - Subsamples

are firm characteristics that affect the wages of workers in a monotone way. This is studied in the next Section of the paper.

7 Compensating Differences

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Sample	R^2 - model	R^2 - resid on firm dummies
Full Sample	0.587	0.721
High School	0.562	0.683
College	0.439	0.741
$Age \geq 35$	0.617	0.701
Occupations	0.604	0.694

Table 3: Explanatory Power and Residuals

8 Conclusion

To be written