

Precautionary Saving and Aggregate Demand

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Abstract

This paper introduces incomplete insurance against idiosyncratic labour income risk into an otherwise standard New Keynesian business cycle model with involuntary unemployment. Following an adverse monetary policy shock that lowers aggregate demand, job creation is discouraged and unemployment risk persistently rises. Imperfectly insured households rationally respond to the rise in idiosyncratic income uncertainty by increasing precautionary saving, thereby cutting consumption and depleting aggregate demand even further; this in turn magnifies the initial labour market contraction and further raises unemployment risk. A Bayesian estimation of the model is used to assess the contribution of time-varying precautionary saving to movements in aggregate consumption.

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1 Introduction

Traditional business cycle analyses often emphasise the role of aggregate demand as a key driver of aggregate fluctuations (see, e.g., Woodford, 2003, Christiano et al. 2003, and Gali, 2010). In the basic New Keynesian model, monopolistic firms price goods above marginal cost; following an aggregate shock that lowers some components of aggregate demand, the firms that cannot fully adjust nominal prices find it worthwhile to passively adjust sales to changes in the demand. This lead to a fall in production and marginal costs, and thus to a increase in average markups as the economy is moving away from its (constrained-) efficient production level. While the basic model has a perfectly competitive labour market, the model has recently been extended to incorporate labour market frictions and involuntary unemployment (see Gali, 2011, for an overview of this strand of the literature). In the extended model, a fall in firms' sales does not mechanically manifest itself as a fall in firms' hours demand schedule and the equilibrium real wage; rather it lowers the expected benefit from hiring a worker which, given the presence of hiring costs, leads to a partial freeze in the labour market. The job-finding rate falls and the job-loss rate rises, i.e., the idiosyncratic unemployment risk faced by households rises.¹ However, even in the extended model, the maintained assumption that individuals' labour market transitions are fully insured within a "representative family" ensures that such transitions have little effect on households consumption choices; that is, aggregate demand affects idiosyncratic unemployment risk, but not the other way around.

In this paper, we analyse the feedback between precautionary saving and aggregate demand by removing the assumption of full risk sharing typically made in New Keynesian analyses. More specifically, we analyse the interactions between the three frictions that have been studied (separately) in earlier studies: imperfect competition and nominal rigidities in the goods market; search and matching frictions in the labour market; and incomplete asset markets (in the form of imperfect insurance and borrowing constraints) giving rise to a precautionary motive for holding wealth. Of course, interacting all three frictions would be of little interest if the model simply retained the known implications of each friction taken separately. We find that it is not the case, quite on the contrary: their interaction generates a powerful amplification mechanism for aggregate business cycle shocks, which is absent when only one or two frictions are considered. The reason why imperfect insurance has nontrivial effects on aggregate fluctuations when nominal and labour market frictions

¹In the New Keynesian model with involuntary unemployment, the increase in the period-to-period job-loss rate during a recession may have two sources; first, time aggregation, as a lower job finding rate makes it more difficult that an individual who falls into unemployment will rapidly exit that state; and second, endogenous destruction, as a larger fraction of low-productivity production units become inefficient. As in Gali (2011), we focus on the first source of fluctuations in the job-loss rate in the present paper.

are jointly introduced is straightforward and intuitive. Following an adverse aggregate (policy or productivity) shock that lowers aggregate demand, job creation is discouraged and unemployment risk (as summarised by the job-loss rate) persistently rises. Imperfectly insured households rationally respond to the rise in idiosyncratic income uncertainty by increasing precautionary saving, thereby cutting consumption and depleting aggregate demand even further; this in turn magnifies the initial labour market contraction, further raises unemployment risk, and so on. Therefore, the endogenous response of households' precautionary savings in an equilibrium that is partly driven by aggregate demand (due to nominal rigidities) and when idiosyncratic volatility is endogenous (due to labour market frictions) explains the potentially large response of the economy to the shocks.

The model is used to assess the contribution of time-varying precautionary saving to aggregate change in consumption. A Bayesian estimation is performed, and identifying the effect of the change in the idiosyncratic risk, it is found that precautionary saving contributes to an additional fall of 20% of consumption following a negative demand shock.

As is well known, the key difficulty with models assuming imperfect insurance is their lack of tractability. The reason is that, in the general case, the idiosyncratic labour market transitions faced by the households implies the existence of a full cross-sectional distribution of wealth, which enters the aggregate state vector (Aiyagari, 1994; Krusell and Smith, 1998). We circumvent this difficulty by means of two assumptions. First, we assume that (imperfectly insured) unemployed have limited ability to borrow, and face a borrowing limit that is tighter than the natural limit. Moreover, we focus our analysis on an equilibrium in which the borrowing constraint is sufficiently tight for the unemployed to face a binding borrowing constraint from the end of their first quarter of unemployment onwards. By way of consequence, all unemployed households face a binding borrowing constraint and have the same end-of-period asset wealth –rather than gradually depleting their asset wealth as the unemployment spell increases in length. Second, we assume that households enjoy periodic reinsurance, in the spirit of Lucas (1990). Namely, we assume that employed households face perfect insurance within their own “family”, while unemployed agents are excluded from the family (and hence from the cross-member insurance scheme) for the time that they remain unemployed. As a consequence, all employed households enjoy the same consumption levels and hold the same end-of-period asset wealth. This property –in fact the most minimal departure from the representative agent assumption–, jointly with the limited cross-sectional heterogeneity of wealth on the unemployed's side, implies that the overall cross-sectional wealth distribution has a small number of states. Consequently, the dynamics of the model can be summarised by a small-scale dynamic system. It is this reduction in the cross-sectional dimension of the problem that allows us to incorporate incomplete insurance and precautionary saving into a fully quantitative New Keynesian DSGE model.

Related literature. As stressed above, the amplification mechanism that we explore in this paper requires exactly three key frictions. Business cycle analyses have extensively studied the interactions between two of the three frictions discussed above. For example, models combining sticky nominal prices and frictional labour markets include Langot and Chéron (2000), Walsh (2005), Faia (2008), Trigari (2009), Gertler et al. (2008) and Blanchard and Gali (2010) –see Gali (2011) for an exhaustive survey of this strand of the literature. One important conclusion from this work is that labour market frictions per se do not significantly contribute to exacerbate aggregate fluctuations.² This is perhaps not surprising: as Shimer (2010) has pointed out, search frictions act as a particular type of labour adjustment cost; as such, they naturally tend to dampen, rather than amplify, the economy’s response to aggregate shocks. We use search frictions in the labour market as a way to generate involuntary unemployment and endogenously time-varying unemployment risk; as we show, once interacted with incomplete insurance, labour market frictions provide a powerful amplification mechanism for aggregate shocks, relative to an economy with nominal rigidities but no such frictions.

Krusell et al. (2011) and Nakajima (2012) analyse full-fledged incomplete-market, heterogeneous agents models with search frictions where the idiosyncratic income risk faced by the household is endogenised through firms’ job creation policy. These models assume flexible prices and perfect competition in the goods market; consequently, monetary policy shocks are neutral and there is no specific role for aggregate demand in shaping aggregate fluctuations.

As far as we are aware, the only paper that combines imperfect competition and nominal rigidities –and hence a role for aggregate demand– with incomplete insurance is Guerrieri and Lorenzoni (2011), who analyse the conditions for a liquidity trap within an heterogeneous-agent environment (with a competitive labour market). Their model is not tractable and is solved numerically under the assumption that nominal prices are constant. Our model considers a standard Phillips curve for the determination of inflation and transition rates in the labour market that respond endogenously to macroeconomic conditions. Other papers such as Iacoviello (2005) and more recently Bilbiie et al. (2012) study economies with nominal rigidities and a potentially binding borrowing constraint for impatient households (i.e., those with a subjective discount rate lower than the equilibrium interest rate), but do not have uninsured unemployment risk. Our model collapses into a version of theirs when we allow workers to enjoy full insurance against unemployment risk.

As far as we are, there are only two other papers that jointly introduce the frictions in goods, labour and asset markets discussed above: Gornemann et al. (2012) and Ravn and Sterk (2013).

²In Gali’s words, “Quantitatively realistic labor market frictions are likely to have, by themselves, a limited effect on the economy’s equilibrium dynamics [...] When combined with a realistic Taylor-type rule, the introduction of price rigidities in a model with labor market frictions has a limited impact on the economy’s equilibrium response to real shocks” (Gali, 2011, pp. 490-491).

There are important differences between these papers and ours, both in terms of focus and in terms of method. Gornemann et al. (2012) are essentially concerned with the redistributive impact of monetary policy shocks and show that an adverse such shock raises cross-sectional inequalities. Ravn and Sterk (2013) study the impact of job-separation shocks and argue that the feedback loop between precautionary saving and aggregate demand may explain the depth of the Great Recession.³ Unlike these papers, ours is tractable and hence can easily handle the large number of state variables and shocks that are required to make sense of the rich dynamics of actual business cycle fluctuations. We can thus perform a Bayesian estimation of the model with a rich structure of shocks.

The rest of the paper is organised as follows. Section 2 presents the models, with particular attention being paid to the risk sharing arrangement that makes our analysis tractable. Section 3 derives the model's steady state, analyse the existence conditions for the equilibrium that we focus on, and studies our economy's response to aggregate shocks. Section 4 concludes.

2 The Model

The model considers incomplete markets coupled with idiosyncratic employment shocks in an otherwise standard quantitative New Keynesian model. On the household side, workers face idiosyncratic and time-varying unemployment risk, which cannot be fully insured due to incomplete markets and borrowing constraints – as in, e.g., Krusell and Smith (1998); this will motivate precautionary saving behavior on the part of the workers, the intensity of which will depend on the extent of the idiosyncratic employment risk that they face. The production side is composed of three types of firms, as in, e.g., Trigari (2009) or Heer and Maussner (2010). More specifically, competitive intermediate goods firms hire labour in a market with matching frictions as in Mortensen and Pissarides (1994). This market produces the job transition rates that the workers take as given when choosing how much to consume and save. Intermediate goods are used as inputs by wholesale goods firms, each of which is the monopolistic supplier of the differentiated good it produces but faces Calvo-type nominal frictions when setting prices. Those firms sell goods to the competitive final good sector, which aggregates them into a single all-purpose good used for consumption and investment alike. The timing of events, presented in Figure 1 below, allows separated workers to find a job within the period. First matches are destroyed, then aggregated shocks are realized. Next vacancies are posted by intermediate goods firms and new matches are formed. Production takes places and income and unemployment benefit are paid to households. Finally, consumption and asset holding decisions are made.

We present the model recursively. We denote by X the vector containing the state variables.

³See also Carroll et al. (2012) for an empirical analysis of the role of precautionary savings in the Great Recession.

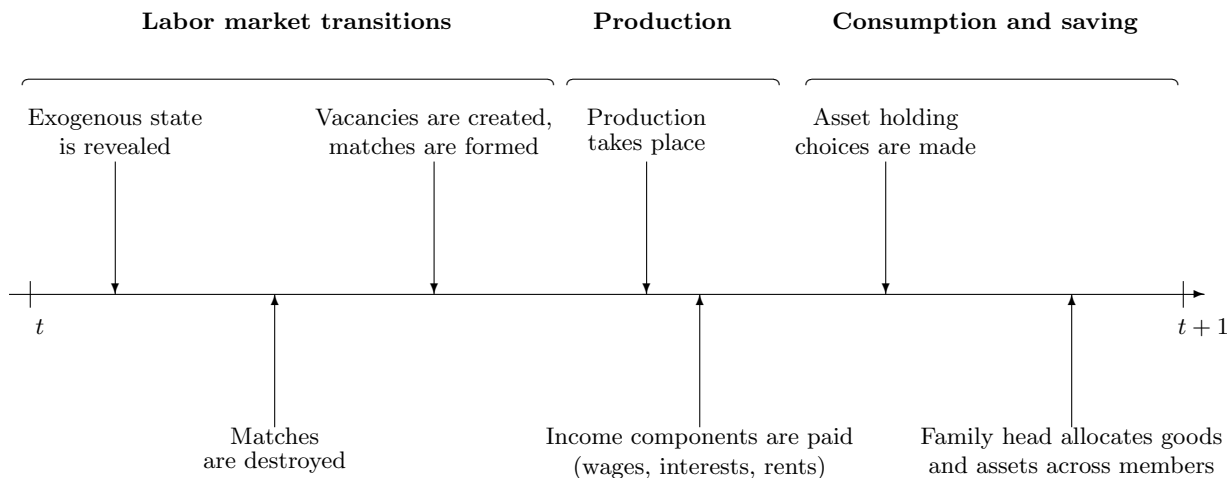


Figure 1: Model timeline

We assume that all agents know the law of motion for the aggregate state, which is necessary to form rational expectations. For expositional clarity we first present the problem of all agents conditional on X , and then summarize the content of X in Section 2.6.

2.1 Households

There are two types of households, employers, in mass ν , and workers in mass one.

2.1.1 Workers

We start by describing the behavior of the workers. Every worker has subjective discount factor $\beta^W \in (0, 1)$ and instant utility function $u(c, \mathfrak{c}) = u(c - h\mathfrak{c})$, defined over $c - h\mathfrak{c} > 0$ and satisfying $u'(c - h\mathfrak{c}) > 0$, $u''(c - h\mathfrak{c}) < 0$, where $c > 0$ is current consumption, $h \in [0, 1)$ a constant parameter and $\mathfrak{c} > 0$ is an external consumption habit. We will be more specific about the habit term below. At this stage we simply treat $\mathfrak{c}$ as an individual state variable that the worker takes as given.

Workers randomly transit between employment and unemployment. Every period a share $f(X)$ of the workers in the unemployment pool find a job, while a share $s(X)$ of employed workers loose their job. Households take these probabilities as given. Given these transitions and today's employment –after labour market transitions– n , employment in the next period will be:

$$n' = f(X') (1 - n) + (1 - s(X')) n. \quad (1)$$

Employed workers earn the net labour income $(1 - \tau(X))w(X)$, where $w(X)$ is the real wage rate and $\tau(X)$ the social contribution rate. Unemployed workers earn the unemployment benefit $b^u e^z$, which is indexed by productivity z to be defined below. The public unemployment insurance scheme is such that $b^u e^z < (1 - \tau)w(X)$ and it is balanced period by period

$$\tau(X)(w(X)n) = b^u e^z(1 - n).$$

It is assumed that workers do not enjoy any other unemployment insurance scheme; this is with no loss of generality, because any other unemployment insurance scheme, e.g., through family and social circles, can be taken into account by rescaling b^u appropriately. The only financial assets they have access to are one-period nominal bonds, which they may issue and trade subject to a borrowing constraint. More specifically, the real value of their nominal bond holdings at the end of the period cannot fall below an exogenous limit

$$\underline{a}(X) = \varsigma e^z, \tag{2}$$

where $\varsigma \leq 0$ is a constant indexing the tightness of the borrowing constraint. Crucially, we assume that

$$\varsigma > -\frac{b^u \beta^E}{1 - \beta^E},$$

where β^E is the subjective discount factor of employers. As will be discussed further below, this restriction on ς is a sufficient condition for the borrowing limit $\underline{a}(X)$ to be always tighter than the natural limit, provided that aggregate shocks have sufficiently small magnitude. This will imply that unemployed agents eventually hit the borrowing limit in finite time, a property that is a key in making our equilibrium tractable.

Workers faces idiosyncratic unemployment risk. We denote by $e \in \{0, 1\}$ the employment status of a worker, with $e = 1$ ($= 0$) if the worker is employed (unemployed), and a the worker's beginning-of-period wealth. We denote by $e \in \{0, 1\}$ the employment status of a worker, with $e = 1$ ($= 0$) if the worker is employed (unemployed), and a the worker's beginning-of-period wealth. The time-varying cross-sectional distribution of workers at the beginning of the period (i.e., before labour market transitions) is defined over $(e, a, \mathbf{c})$, i.e. $\tilde{\mu}^W = \tilde{\mu}^W(e, a, \mathbf{c})$, with $\sum_e \int_a \int_{\mathbf{c}} d\tilde{\mu}^W(e, a, \mathbf{c}) = 1$. For example, $\tilde{\mu}^W(1, a, \mathbf{c})$ is the measure of employed workers at the beginning-of-period with net wealth a and consumption habit $\mathbf{c}$. The time-varying cross-sectional distribution of workers after labour market transitions is defined over $(e, a, \mathbf{c})$, i.e. $\mu^W = \mu^W(e, a, \mathbf{c})$.

In general, incomplete markets coupled with idiosyncratic income shocks endogenously produces a cross-sectional distribution with a large number of wealth states, as accumulated wealth typically depends on the entire history of employment statuses (see, e.g., Huggett 1993, Aiyagari, 1994). Approximating this distribution when the economy is hit by aggregate shocks requires a drastic

reduction in the dimensionality of the exogenous state and some approximation techniques (Krusell and Smith, 1998). These techniques prohibit likelihood-based estimation.⁴ We circumvent this difficulty by positing a cross-household insurance scheme that retains incomplete insurance against idiosyncratic unemployment risk while maintaining tractability. More specifically, we assume that every employed workers is a member of a “family” and that there is a measure-1 continuum of such families. Every family behaves like a representative agent; as such, the family head pools assets and income from members and optimally allocates consumption and assets across members. When a worker who falls into unemployment, the worker leaves the family with its assets and is taken care of by the public insurance scheme. Once an unemployed worker finds a job, the worker randomly re-integrates one of the representative families. As there is a measure 1 of families, by the law of large numbers each family is of measure n .

This structure implies that there is periodic re-insurance amongst the workers, so that all employed workers’ share the same consumption and end-of-period asset levels, independently of their individual employment history (that is, workers who have just left unemployment do not consume less or hold less assets than workers having been employed long before). As we will show, this property and the fact that the borrowing limit is tighter than the natural limit will jointly imply that the support of $\tilde{\mu}^W$ and μ^W admits a finite number of wealth states, allowing the model to admit a simple state-space representation.

Unemployed workers. After labour market transitions have taken place, the Bellman equation for an unemployed worker with bond wealth a^u and consumption habit $\mathbf{c}_{-1}^u$ when the aggregate state is X is given by

$$V^u(a^u, \mathbf{c}_{-1}^u, X) = \max_{c^u, a^{u'}} e^{\xi_c u}(c^u, \mathbf{c}_{-1}^u) + \beta^W \mathbb{E} \left[(1 - f(X')) V^u(a^{u'}, \mathbf{c}^u, X') + f(X') \frac{V^e(n', A^{e'}, \mathbf{c}^{e'}, X')}{n'} \middle| X \right], \quad (3)$$

where c^u is the consumption of an unemployed worker, $V^e(n, A^e, \mathbf{c}^e, X)$ is the intertemporal utility of a typical family of employed workers, A^e the bond wealth of the family, and ξ_c a shock affecting the preferences of all households. Note that (i) because $V^e(n, A^e, \mathbf{c}^e, X)$ is the value function for the family, it must be divided by n to find its per-member analogue; (ii) because a worker is atomistic relative to the size of a family, the asset holding choice of an unemployed worker, $a^{u'}$, does not affect the assets of the family; and (iii) say something about $V^e(n, A^e, \mathbf{c}^e, X)$ along equilibrium.⁵ The unemployed worker faces the following budget and borrowing constraints:

$$a^{u'} + c^u = b^u e^z + (1 + r(X)) a^u \quad (4)$$

⁴See, however, McKay (2012).

⁵That is, we have $\partial V^e(\cdot) / \partial a^{u'} = V_A^e(\cdot) \partial A^{e'} / \partial a^{u'} = 0$.

and

$$a' \geq \underline{a}(X) \quad (5)$$

In equation (4),

$$r(X) = \frac{1 + \mathbf{R}_{-1}}{1 + \pi(X)}$$

is the ex post real return on nominal bond holdings, $\mathbf{R}_{-1}$ is the interest rate on nominal bonds from the previous to the current period (determined in the previous period), and

$$\pi(X) = \frac{P(X)}{\mathbf{P}_{-1}} - 1$$

is final good inflation –i.e., $P(X)$ is the price of final goods and $\mathbf{P}_{-1}$ the same price in the previous period. The unemployed worker head solves (3) subject to (4) and (5) and law of motion for n' , $X' = \Gamma(X, \epsilon')$ and $\mathbf{c}^u = \mathbf{c}^u(X)$, where ϵ' is the set of innovations to the structural shocks to be defined below. The solution to an unemployed worker's problem is given by the optimal policy functions $c^u(a^u, \mathbf{c}^u, X)$ and $a^u(a^u, \mathbf{c}^u, X)$.

Employed workers. As discussed above, we assume that all employed workers (for whom $e = 1$) are members of a family. In every family, the family head pools resources and allocate consumption goods so as to maximize the intertemporal utility of all the family members. We assume that all employed workers share the same level of consumption habits $\mathbf{c}^e$, so the latter does not produces any heterogeneity in marginal consumption utility across such workers.

Let A^e denote the wealth of the representative family at the very beginning of the period. The Bellman equation for the family after labour market transitions is given by

$$\begin{aligned} V^e(n, A^e, \mathbf{c}_{-1}^e, X) = & \max_{c^e, A^{e'}} e^{\xi_e} n u(c^e, \mathbf{c}_{-1}^e) \\ & + \beta^W \mathbb{E}[(1 - s(X')) V^e(n', A^{e'}, \mathbf{c}^e, X') + ns(X') V^u(a^{u'}, \mathbf{c}^{u'}, X') | X], \end{aligned} \quad (6)$$

where c^e is the consumption of each member of the family and a' here denotes the assets that an workers who falls into unemployment takes away when leaving the family. Note that the family head values current members' potential utility loss associated with them becoming unemployed in the next period; there will be $ns(X')$ such members, hence the corresponding weight before $V^u(a^{u'}, \mathbf{c}^{u'}, X')$ in the above Bellman equation.

The family head allocates goods and assets across members in the ignorance of whom will fall into unemployment in the next period, since idiosyncratic employment shocks will only take place in the next period – see Figure 1. Because currently employed workers are perfectly symmetric, the family head optimally gives identical end-of-period wealth to all members, i.e., $a^{u'}$ in $V^u(a^{u'}, \mathbf{c}^{u'}, X')$ must be such that

$$a^{u'} = A^{e'}/n. \quad (7)$$

Let us now turn to the budget constraint of the family. After the very beginning of the current period the process of job destruction and creation takes place, causing $s(X) \mathbf{n}_{-1}$ workers to fall into unemployment and leave the family and $f(X)(1 - \mathbf{n}_{-1})$ workers to find a job and enter the family, where $\mathbf{n}_{-1}$ is employment before labour market transitions (i.e., $\mathbf{n}'_{-1} = n$). From (7), every worker leaving the family takes away $a^u = A^e/n$, so the total amount of wealth taken away is $s(X)na^u$. We denote by $B(X)$ the total amount of wealth that is brought into the family by those who join it, i.e., the newly employed. Integrating over all unemployed workers, we find it to be such that

$$B(X) = f(X) \int_a \int_{\mathbf{c}} a d\tilde{\mu}^W(0, a, \mathbf{c}),$$

where $\mu^W(e, a, \mathbf{c})$ is the cross-sectional distribution of workers *after* labour market transitions, i.e.,

$$\begin{aligned} \mu^W(1, a, \mathbf{c}) &= f(X) \tilde{\mu}^W(0, a, \mathbf{c}) + (1 - s(X)) \tilde{\mu}^W(1, a, \mathbf{c}), \\ \mu^W(0, a, \mathbf{c}) &= s(X) \tilde{\mu}^W(1, a, \mathbf{c}) + (1 - f(X)) \tilde{\mu}^W(0, a, \mathbf{c}). \end{aligned}$$

It follows from these flows that the bond wealth of the family after the process of job destruction and creation is $A^e - s(X)na + B(X)$, so the budget constraint of the family is

$$A^{e'} + nc^e = (1 - \tau(X))w(X)n + (1 + r(X))(A^e - s(X)\mathbf{n}_{-1}a + B(X)), \quad (8)$$

where $(1 - \tau(X))w(X)n$ is the total (net) labour income of the family and c is the consumption level of every employed worker. Finally, the family is also subject to the borrowing constraint, so that we must have

$$A^{e'} \geq n\underline{a}(X). \quad (9)$$

The family head solves (6) subject to (8)–(9). The optimal solution is given by the optimal policy functions $c^e(A^e, \mathbf{c}^e, X)$ and $A^{e'}(A^e, \mathbf{c}^e, X)$.

2.1.2 Employers

Employers share the same instant utility function as the workers but are more patient: their subjective discount factor is $\beta^E \in (\beta^W, 1)$. They do not earn any wage income or unemployment benefit, but own and rent out the capital stock and also own and run all the firms. We also assume that their (private) equity holdings in the firms is fully diversified. These two features jointly imply that employers' total income is fully insulated from any source of idiosyncratic risk. Finally, we assume that all employers share the same external stock of habit $\mathbf{c}^E$, which is equal to the average consumption of employers in the previous period; hence all employers share the same marginal utility of consumption. Taken together, these assumptions imply that employers are perfectly

symmetric. Employers participate in the market for nominal bonds (just like the workers), and we denote by k and a^E an employer's capital and real bond wealth at the beginning of the period.⁶

An employers' individual capital builds up according to the accumulation equation

$$k' = (1 - \delta)k + e^{\xi_i} (1 - S(i/\mathbf{i}_{-1})) i, \quad (10)$$

where $\mathbf{i}_{-1}$ is the employer's investment level in the previous period and i that chosen in the current period, respectively, k is beginning-of-period capital (which depreciates at rate $\delta \in [0, 1]$) and ξ_i is an investment-specific shock. $S(\cdot)$ is an investment adjustment cost function satisfying $\partial^2 S(\cdot) / \partial (i/\mathbf{i}_{-1})^2 > 0$ and $S(g_i) = \partial S(\cdot) / \partial (i/\mathbf{i}_{-1})|_{i/\mathbf{i}_{-1}=g_i} = 0$, where g_i is the steady-state value of $i/\mathbf{i}_{-1}$. Note that $\mathbf{i}_{-1}$ is an individual state variable since it affects current investment i (via its impact on investment adjustment costs).

The employer maximizes its intertemporal utility:

$$W^E(k, a^E, \mathbf{i}_{-1}, \mathbf{c}^E, X) = \max_{a^{E'}, i', u} e^{\xi_c} u(c^E, \mathbf{c}^E) + \beta^E \mathbb{E}[W^E(k', a^{E'}, \mathbf{i}'_{-1}, \mathbf{c}^{E'}, X') | X], \quad (11)$$

where ξ_c is the preference shock of households, and subject to the capital accumulation equation (10) and the budget constraint

$$c^E + i + e^{-\xi_i} \eta(u) k + a^{E'} = r_k(X) u k + (1 + r(X)) a^E + \Upsilon.$$

In the latter equation c^E is the consumption of employers, Υ denotes the firms' profits that are rebated to the employer as dividends, $r_k(X)$ the real rental rate of capital, $u \in [0, 1]$ the capital utilization rate (a choice variable for the employer), and $\eta(u)k$ the real cost that this utilization rate entails. The function η is such that $\eta(1) = 0$ and $\partial \eta / \partial u, \partial^2 \eta / \partial^2 u > 0$. The policy functions that solve (11) are $c^E(k, a^E, \mathbf{i}_{-1}, \mathbf{c}^E, X)$, $i(k, a^E, \mathbf{i}_{-1}, \mathbf{c}^E, X)$, $a^{E'}(k, a^E, \mathbf{i}_{-1}, \mathbf{c}^E, X)$ and $u(k, a^E, \mathbf{i}_{-1}, \mathbf{c}^E, X)$. Note that since employers are perfectly symmetric, their individual state vector $(k, a^E, \mathbf{i}_{-1}, \mathbf{c}^E)$ is in fact included in the aggregate state X , so we may simply write the policy functions as $c^E(X)$, $i(X)$, $a^{E'}(X)$ and $u(X)$. This in turn allows us to write the stochastic discount factor of the employers, which serves to price any future payoff coming from the firms, as follows:

$$M^E(X, X') = \beta^E e^{\Delta \xi_c'} \frac{u'(c^{E'}(X') - h c^E(X))}{u'(c^E(X) - h \mathbf{c}^E)}. \quad (12)$$

2.2 Firms

The economy has three production layers: a final goods sector buys differentiated products from the wholesale sector and combines them into a single final good that is sold to the households (for

⁶We could impose, for the sake of symmetry, that employers real wealth be such that $k' + a^{E'} \leq \underline{a}(X)$. However, as will become clear below such a constraint is never binding in the equilibrium that we are considering, so we simply ignore from the onset.

consumption and capital utilisation) and intermediate goods firms (for investment and vacancy creation). The wholesale sector buys undifferentiated goods from intermediate goods firms. Finally, the intermediate goods sector uses labour to produce the undifferentiated intermediate goods. Each types of firms forms a continuum of measure one.

2.2.1 Final goods firms

The final good is produced by a continuous of identical and competitive producers that combine wholesale goods, uniformly distributed on the unit interval $i \in [0, 1]$ according to the production function

$$y = \left(\int_0^1 y_i^{(\theta-1)/\theta} di \right)^{\theta(\theta-1)}, \quad (13)$$

where $\theta > 1$ is the cross partial elasticity of substitution between any two wholesale goods. Let $P_i(X)$ and $P(X)$ respectively denote the nominal price of wholesale good i and that of the final good. Both price are functions of the aggregate state and are taken as given by final goods producers. The program of the representative final good producer is

$$\max_y P(X) y - \int_0^1 P_i(X) y_i di. \quad (14)$$

subject to (13). From the optimal choices of final good firms, one can deduce the demand function for the wholesale good firms, $i \in [0, 1]$:

$$y_i(X, P_i) = \left(\frac{P_i}{P(X)} \right)^{-\theta} y(X), \quad (15)$$

where $y(X)$ is the total demand for final goods, and where

$$P(X) = \left(\int_0^1 P_i(X)^{1-\theta} di \right)^{1/(1-\theta)}$$

is the nominal price of final goods obtained from imposing zero profits on the final good producers.

2.2.2 Wholesale goods firms

The wholesale sector is imperfectly competitive. Wholesale firm $i \in [0, 1]$ is the monopolistic supplier of the good it produces, which it does by means of a linear production function with a fixed cost:

$$y_i = x_i - \kappa_y e^z, \quad (16)$$

where x_i is the quantity of intermediate goods used in production, and where the fixed cost $\kappa_y e^z$, measured in units of the intermediate goods. Firm i ' current real profit is given by

$$\Xi(P_i, X) = \left(\frac{P_i}{P(X)} - p_m(X) \right) y_i(X, P_i) - p_m(X) \kappa_y e^z,$$

where $p_m(X)$ is the real price of intermediate goods, which is taken as given by wholesale goods firms.

Firm i chooses P_i (or equivalently y_i) to maximise the present discounted value of future profits taking as given the demand curve (15). Following Calvo (1985), we also assume that in every period every wholesale goods firm can be in one of the following two idiosyncratic states: either the firm can freely reoptimize its price, or it cannot and simply rescales the existing price according to the indexation rule $P'_i = (1 + \bar{\pi})^{1-\gamma}(1 + \pi)^\gamma P_i$, where $\gamma \in (0, 1)$ measures the degree of indexation to the most recently available final good inflation measure and $\bar{\pi}$ is steady state inflation. The ex ante probability of being able to reoptimize the price in the next period is constant and equal to $1 - \alpha \in [0, 1]$, irrespective of the time elapsed since the period in which the price was last revised.

It follows from this price adjustment mechanism that the behaviour of the firm can be described by two Bellman equations, corresponding to the two idiosyncratic state in which the firm can be. The value of a firm that is allowed to reset its price is given by $\mathbb{W}^I(X)$ and only depends on the vector of states variables. The value of a firm for which last period's selling price is P_i , and which is not allowed to reset its price, is denoted as $\mathbb{V}^I(P_i, X)$. These value functions are

$$\begin{aligned} \mathbb{W}^I(X) &= \max_{P_i} \Xi(P_i, X) \\ &+ \alpha \mathbb{E} [M^E(X, X') \mathbb{V}^I(P_i, X')] + (1 - \alpha) \mathbb{E} [M^E(X, X') \mathbb{W}^I(X') | X], \end{aligned} \quad (17)$$

and

$$\mathbb{V}^I(P_i, X) = \Xi(P'_i, X) + \alpha \mathbb{E} [M^E(X, X') \mathbb{V}^I(P'_i, X')] + (1 - \alpha) \mathbb{E} [M^E(X, X') \mathbb{W}^I(X')].$$

where $P'_i = (1 + \bar{\pi})^{1-\gamma}(1 + \pi)^\gamma P_i$.⁷ The solution to wholesale goods firms' problem is the optimal nominal reset price common to all price resetting firms, $P^*(X)$.

In every period the price of final goods weights the selling prices of wholesale goods firms that reoptimise their prices and those which do not. This implies law of motion for final goods prices is given by

$$P(X') = (1 - \alpha) (P^*(X'))^{1-\theta} + \alpha ((1 + \bar{\pi})^{1-\gamma}(1 + \pi(X))^\gamma P(X))^{1-\theta}.$$

The Calvo price-setting mechanism implies the existence of a whole distribution of nominal prices, since the selling price of a firm not reoptimising its price depends on the time that has elapsed since the last time it set its price optimally. However, the price dispersion index

$$\Delta(X) = \int_0^1 \left(\frac{P_i}{P(X)} \right)^{-\theta} di \quad (18)$$

⁷This recursive formulation of wholesale goods firms' problem makes use of the property, specific to the Calvo price setting mechanism, that firm's selling price in the last period summarises its relevant idiosyncratic history.

is a sufficient statistics to capture the relevant properties of the price distribution. $\Delta(X)$ evolves according to the law of motion:

$$\Delta(X') P(X') = (1 - \alpha) P^*(X')^{-\theta} + \alpha ((1 + \bar{\pi})^{1-\gamma} (1 + \pi(X))^\gamma P(X))^{-\theta} \Delta(X).$$

2.2.3 Intermediate goods firms

The market for intermediate goods is perfectly competitive, and firms in that sector are perfectly symmetric. A typical firm $j \in [0, 1]$ in that sector produces with the Cobb-Douglas technology

$$y_{m,j} = (\bar{k}_j)^\phi (e^z n_j)^{1-\phi}, \quad \phi \in (0, 1), \quad (19)$$

where n_j and $\bar{k}_j$ are employment and capital services at firm j , respectively.

The labour market is plagued with search frictions, and our timing convention about job destruction and creation is as in Walsh (2005), Gali (2011) and many others –see Figure 1 again. More specifically, at the beginning of every period, a fraction $\rho(\xi_\rho)$ of existing matches is destroyed, where ξ_ρ is a shock to the job destruction rate. After this destruction phase employment is $(1 - \rho(\xi_\rho)) \mathbf{n}_{-1}$, so the size of the unemployment pool is $1 - (1 - \rho(\xi_\rho)) \mathbf{n}_{-1}$. Given its knowledge of X , firm j chooses how many vacancies v_j to post and then how much capital services $\bar{k}_j$ to rent, at unit costs (in terms of the final good) $\kappa_v e^z$ and $r_k(X)$, respectively. Let $\lambda(X)$ denote the economywide vacancy-filling rate, i.e., the fraction of posted vacancies that are filled within the period, which the firm takes as given when choosing v_j . Employment at firm j evolves as:

$$n_j = (1 - \rho(\xi_\rho)) \mathbf{n}_{-1,j} + \lambda(X) v_j, \quad (20)$$

where $\mathbf{n}_{-1,j}$ is employment at the firm at the very beginning of the period, i.e., before the job destruction and creation process –so that total employment is $\mathbf{n}_{-1} = \int_0^1 \mathbf{n}_{-1,j} dj$. Let us denote by $\mathbb{V}(\mathbf{n}_{-1,j}, X)$ the value of a generic intermediate goods firms at that moment. The firms maximises value and thus solves:

$$\begin{aligned} \mathbb{V}(\mathbf{n}_{-1,j}, X) = \max_{v_j, \bar{k}_j} & p_{m,j}(X) y_m - w(X) n_j - r_k(X) \bar{k}_j - \kappa_v e^z v_j \\ & + \mathbb{E}(M^{Et}(X, X') \mathbb{V}(n_j, X') | X), \end{aligned} \quad (21)$$

subject to (19) and (20), and where $p_m(X)$ is the price of intermediate goods in terms of the final good. The solution to this problem takes the form of optimal policy rules for capital services and vacancies, $\bar{k}(X)$ and $v(X)$, where the j index can now be dropped thanks to the symmetry of those functions with respect to j .

While every intermediate goods firm takes $\lambda(X)$ as given, in equilibrium the latter is determined by the vacancy opening policy of all the firms. This aggregate relationship is determined by the

matching technology, which produces job matches out of unemployment and the total number of vacancies according to the function

$$m(X) = \bar{m}e^{\xi_m}(1 - (1 - \rho(\xi_\rho))\mathbf{n}_{-1})^\chi v(X)^{1-\chi},$$

where $(1 - \rho(\xi_\rho))\mathbf{n}_{-1}$ is the size of the unemployment pool at the time the matching market opens, $\bar{m} > 0$ and $\chi \in (0, 1)$ are a scale and elasticity parameter, respectively, and ξ_m determines the productivity of the matching process. It follows that the job-finding and vacancy-filling rates are given by, respectively:

$$f(X) = \frac{m(X)}{1 - (1 - \rho(\xi_\rho))\mathbf{n}_{-1}}, \quad \lambda(X) = \frac{m(X)}{v(X)}. \quad (22)$$

Finally, note that under our timing workers that are separated can find a job within the period. Hence the period-to-period separation rate $s(X)$ is given by:

$$s(X) = \rho(\xi_\rho)(1 - f(X)). \quad (23)$$

2.3 Wage

As is now well understood, the presence of search frictions in the labour market implies that there exist a full bargaining set over which an employer and an employee find it mutually profitable to trade. However, the theory does not pin down the specific way in which the match surplus is shared among the parties. For the sake of tractability, we assume here the type of wage rule suggested by Hall (2005):

$$w(X) = w(z, \xi_w) = \bar{w}e^{z+\xi_w}, \quad (24)$$

where ξ_w is a shock to the real wage. Importantly, we assume that i) all agents take $w(z, \xi_w)$ as given, and ii) we make sure that this wage always remains within the relevant bargaining set (so that it is never the case that the two parties would mutually agree to change the given wage).

2.4 Market clearing

There are two assets (nominal bonds and capital), three goods (intermediate goods, wholesale goods and final goods) in the economy and a unique labor type in the economy.

Asset markets. Employers are symmetric, in measure ν and each of them supplies uk units of capital services. The demand for capital services by intermediate goods firms is $\bar{k}$. Clearing of the market for capital thus requires

$$\nu uk = \int_0^1 \bar{k}_j dj. \quad (25)$$

All the households participate in the market for nominal bonds, subject to the borrowing constraint. Clearing of this market thus requires:

$$A^{W'} + \nu a^{E'} + \int_a \int_{\mathbf{c}} a^{u'}(a, \mathbf{c}, X) d\mu(0, a, \mathbf{c}) = 0, \quad (26)$$

where $\mu(0, a, \mathbf{c})$ is the measure of unemployed agents with wealth a and a consumption habits $\mathbf{c}$.

Goods markets. The aggregate demand for final goods is made of total investment (by employers), the consumption of all types of households (employers as well as employed and unemployed workers), as well as capital utilisation and vacancy costs. Clearing of this market requires that demand be equal to supply, i.e.,

$$\nu \left(c^E + i + e^{-\xi_i} \eta(u) k \right) + n c^e + \int_a \int_{\mathbf{c}} c^u(a, \mathbf{c}, X) d\mu(0, a, \mathbf{c}) + \kappa_v e^z v = y, \quad (27)$$

where $c^u(a, \mathbf{c}, X) \equiv b^u e^z + (1 + r(X)) a - a^{u'}(a, \mathbf{c}, X)$ is the consumption policy function for the unemployed.

The total demand for wholesale goods by the final good sector is $\int_0^1 y_i(X, P_i(X)) di = y(X) \Delta(X)$ (see (15) and (18)). The total supply of intermediate goods is equal to $\int_0^1 y_i di = \int_0^1 (x_i) di - \kappa_y e^z$ (see (16)). Hence, clearing of the market for wholesale goods requires:

$$y(X) \Delta(X) = \int_0^1 x_i di - \kappa_y e^z. \quad (28)$$

Finally, the supply of intermediate goods is given by (19), while the wholesale sector demands one unit of intermediate goods for any unit of wholesale goods. Hence the market-clearing condition for this market is:

$$\int_0^1 x_i di = \int_0^1 (\bar{k}_j)^\phi (e^z n)^{1-\phi} dj. \quad (29)$$

Labor market. Finally, total labour demand by intermediate goods firm must equal worker's total labour supply, i.e.

$$\int_0^1 n_j dj = n.$$

2.5 Central Bank

We assume that the Central Banks sets the nominal interest rate according to a “Taylor rule” of the form

$$\log \left(\frac{1 + R}{1 + \bar{R}} \right) = \rho_R \log \left(\frac{1 + \mathbf{R}_{-1}}{1 + \bar{R}} \right) + (1 - \rho_R) \left[\frac{1}{4} a_\pi \log \left(\frac{1 + \pi^a(X)}{(1 + \bar{\pi}^a) e^{\xi_\pi}} \right) + \frac{1}{4} a_y \log \left(\frac{g^a(X)}{\bar{g}^a} \right) \right] + \xi_R, \quad (30)$$

where $\rho_R \in (0, 1)$ indexes the degree of nominal interest rate smoothing by the Central Bank, $\pi^a(X)$ and $g^a(X)$ are the inflation and output growth indicators to which the Central Bank responds (with

$(\bar{\pi}^a, \bar{g}^a)$ their steady state counterparts), (a_π, a_y) are the reaction coefficients to those variables and ξ_R is an exogenous monetary policy term. In addition, we introduce a shock to the inflation target ξ_π . Following Justiniano et al. (2013), in our empirical implementation of the model we will assume that the Central Bank sets R in every period, but in so doing responds to realized inflation and output growth accumulated over several periods. For example, if R is set at the quarterly frequency but reacts to annualized inflation we would have $\pi^a(X) = (1 + \pi(X))(1 + \pi_{-1})(1 + \pi_{-2})(1 + \pi_{-3}) - 1$, denote accumulated inflation over four quarters. Similarly $1 + g^a$ is the annual growth rate of output over the last four quarters, and the quarterly growth rate of output is denoted $g^y =$.

2.6 State variables and equilibrium definition

We may now summarise the content of the aggregate state X , the components of which were gradually introduced above. Let us first write $X = \{S, \Omega\}$, where S and Ω are the endogenous and exogenous states, respectively. The exogenous aggregate state is composed of the following 9 forcing variables:

$$\Omega = \{z, \xi_i, \xi_c, \xi_R, \xi_w, \xi_m, \xi_\rho, \xi_\pi\},$$

which represent, respectively, the stochastic trend (z), an investment-specific productivity term (ξ_i), impatience terms for households (ξ_c), a nominal interest rate shock (ξ_R), the matching productivity term (ξ_m), and the exogenous component of the job separation rate (ξ_ρ), a shock to the inflation target (ξ_π).

The endogenous aggregate state is:

$$S = \{\Delta, k, a^E, \mathbf{c}^E, \tilde{\mu}^W, w, \mathbf{P}_{-1}, \mathbf{R}_{-1}, \mathbf{n}_{-1}, \mathbf{i}_{-1}, \pi_{-1}, \pi_{-2}, \pi_{-3}, g_{-1}^y, g_{-2}^y, g_{-3}^y\}.$$

Note that at this stage S contains the (time-varying) cross sectional distribution of workers, as in Krusell and Smith (1998).

The definition of the recursive equilibrium is the following.

1. The policy functions $a^w(a, \mathbf{c}, X)$, $A^e(A^e, \mathbf{c}^e, X)$, $i(X)$, $a^{E'}(X)$, and $u(X)$ solve the households' programs (equations (3), (6) and (11))
2. The policy functions $y_i(X, P_i)$, $P^*(X)$, $\bar{k}(X)$ and $v(X)$ solve the firms' programs (equations (14), (17) and (21)).
3. The job finding rate, $f(X)$, the vacancy filling rate $\lambda(X)$ and the job separation rate $s(X)$ satisfy the equilibrium conditions (22) and (23).
5. The real wage w follow the exogeneous law of motion (24) and the nominal interest rate R follows the Taylor rule (30).
6. The market clearing conditions (25)-(29) are satisfied.

7. The state vector X follows a law of motion consistent with optimal policy functions and equilibrium flows.

3 Model Solution

3.1 Distribution without consumption habits

We now derive the cross-sectional distribution of workers over the individual state vector (e, a, c) . We first consider the case where $h = 0$ (i.e., households do not form consumption habits), and then discuss the class of consumption reference points that preserves the equilibrium without habit.

We first note first that from our risk-pooling assumption about *employed* workers, all of them not only share the same individual end-of-period asset holding level $a^e = A^e/n'$ (see (7)) but also the same individual consumption level. From (8)), the latter is given by

$$c^e = [(1 - \tau) wn' + (1 + r)((1 - s)A^e + B) - A^e]/n'.$$

Hence, whatever the degree of cross-sectional heterogeneity there might be, it will be concentrated among *unemployed* workers. But since we have assumed that the borrowing limit $\underline{a}(X)$ is tighter than the natural limit, it will end up binding after a finite number of period in the idiosyncratic state associated with a low income (unemployment here), implying that cross-sectional heterogeneity among the unemployed will also be limited.

In what follow, we focus on the simplest equilibrium with limited cross-sectional heterogeneity: one in which the borrowing limit becomes binding after one period spent in the state of unemployment.⁸ We use a guess-and-verify strategy. First, we characterise the equilibrium under consideration and then we derive a sufficient condition for its existence.

If the constraint is binding for all unemployed workers, then the policy rule for assets is simply

$$a^u(X) = \underline{a}(X),$$

where $\underline{a}(X)$ is defined by (2). This in turn implies that there can be only two possible types (i.e., consumption levels) for the unemployed. From (4) and (7), those who are falling into unemployment (denoted as *eu* agents) in the current period have assets A^e/n and hence consume

$$c^{eu} = b^u c^z + (1 + r) \frac{A^e}{n} - a^u,$$

where $a^u = \underline{a}(X)$. The number of those agents is $n^{eu} = n\rho(\xi_\rho)$.

⁸We differ until the quantitative Section the discussion as to why this is the most plausible length, but one could easily extend it to a finite number of periods without losing tractability.

On the other hand, those who were already unemployed in the previous period (denoted as uu agents) have assets a^u and hence consume

$$c^{uu} + a^{u'} = b^u e^z + (1 + r) a^u - a^{u'}.$$

The number of those agents evolves according to $n^{uu'} = n^{eu} (1 - f(X'))$. We can now explicit the total wealth of workers joining the family, defined in Section 2.1.1). As all unemployed workers choose the same end of period wealth $a^{u'} = \underline{a}(X)$, it implies that the total wealth of workers who will join the family next period is simply

$$B(X') = f(X') (1 - n) \underline{a}(X)$$

When an worker is employed (is of the “e” type), she may remain so in the next period and hence consume $c^{e'}$, or become of the “eu” type and consume $c^{eu'}$. Assuming that the demand for nominal bonds by employed workers is interior (we will examine later on the conditions under which this is indeed the case), the following bond Euler equation must hold:

$$\mathbb{E} [M^e(X, X') (1 + r(X')) | X] = 1,$$

where

$$M^e(X, X') = \beta^W e^{\Delta \xi'_c} \frac{(1 - s(X')) u'(c^{e'}) + s(X') u'(c^{eu'})}{u'(c^e)} \quad (31)$$

is the stochastic discount factor of employed workers.

This stochastic discount factor reflects the sensitivity of workers’s asset holding decisions to both future aggregate conditions and their potential changes in idiosyncratic employment status. More specifically, while a currently employed worker enjoys marginal utility $u'(c^e)$, it may either stay employed in the next period –with probability $1 - s(X')$ – and hence enjoy marginal utility $u'(c^{e'})$, or fall into unemployment –with probability $s(X')$ – and enjoy marginal utility $u'(c^{eu'})$. Under incomplete consumption insurance, the latter is greater than the former, which motivates precautionary asset holdings by the family in excess of the borrowing limit (despite the fact that workers are impatient relative to employers).

To get some further intuition about how uninsured idiosyncratic risk affects the asset demand of workers, log-linearise the SDF of employed workers in (31) to get:

$$\hat{M}^e(X, X') = \underbrace{\left[\frac{u'(\bar{c}^{eu'}) - u'(\bar{c}^{e'})}{u'(\bar{c}^{e'})} \right]}_{>0} (s' - \bar{s}) - \sigma (1 - \bar{s}) [\hat{c}^{e'} - \hat{c}^e] - \bar{s} \sigma [\hat{c}^{eu'} - \hat{c}^e],$$

where variables with bars and hats denote steady state values and proportional deviations from the steady state, respectively. The key transmission mechanism in our model shows up in the ratio inside the square brackets. The latter represent the proportional increase in the steady-state

marginal utility associated with a change of idiosyncratic state from employment to unemployment. Incomplete consumption insurance implies that this term is strictly positive, i.e., falling into unemployment is associated with a consumption loss, or equivalently to an upward shift in marginal utility. In this situation, changes in the likelihood that such an event will occur (the job separation rate in the next period, s') has a first-order effect on the SDF. By way of consequences, it has a first-order effect on the quantity of assets held by employed workers for precautionary purpose.

3.2 Distribution with consumption habits

In the equilibrium just described, there are exactly three different consumption levels for the workers (c^e , c^{eu} , c^{uu}), depending on the unemployment status of the worker in the current and the last period. Clearly, a necessary condition for the existence of an equilibrium with the same property in the presence of consumption habits is that two households experiencing the same consumption level in the economy without habit share the stock of habit in the economy with habits; otherwise they could choose different consumption levels due to different habit terms.⁹ We preserve the equilibrium with limited cross-sectional heterogeneity described above if we assume, when $h > 0$, that the stocks of habit correspond to the consumption of the same group of workers in the last period. That, is, by assuming:

$$\mathbf{c}^{e'} = c^e, \mathbf{c}^{eu'} = c^{eu}, \mathbf{c}^{uu'} = c^{uu}$$

In this scenario, the stochastic discount factor derived above for the case where $h = 1$ must be generalised as follows:

$$M^e(X, X') = \beta^W e^{\Delta \xi_c'} \frac{(1 - s(X'))u'(c^{e'} - hc^e) + s(X')u'(c^{eu'}(X') - hc^{eu}(X))}{u'(c^e - hc^e)},$$

where $\mathbf{c}^e$ is an individual state variable such that $\mathbf{c}^{e'} = c^e$.

3.3 Reduced state vector

We can now exhibit the form of the state vector in our equilibrium. Due to the equilibrium with reduced heterogeneity, it can be written as

$$X = \{\Delta, k, a^E, \mathbf{c}^E, \tilde{\mu}^W, w, \mathbf{P}_{-1}, \mathbf{R}_{-1}, \mathbf{n}_{-1}, \mathbf{i}_{-1}, \pi_{-1}, \pi_{-2}, \pi_{-3}, g_{-1}^y, g_{-2}^y, g_{-3}^y, \Omega\}$$

where Ω is as before the vector of exogenous forcing variables. To reproduce the distribution of wealth we need to follow the asset choice of the family. We also need the stock of habit for each type of agents, and $\mathbf{n}^{eu}$, the number of eu the previous period ($\mathbf{n}^{eu'} = \mathbf{n}^{eu}$), to keep track of the population structure. The vector X now includes a finite number of state variables (and thus no continuous distributions).

⁹This issue only matters for those workers whose consumption-saving choice is interior. The equilibrium described above would break down

3.4 Existence conditions

The construction of the cross-sectional distribution of workers above started from the notion that all unemployed workers faced a binding borrowing constraint. There are two types of unemployed workers, i.e., eu and uu workers, and both are borrowing-constrained if and only if

$$\mathbb{E} [M^{ju} (X, X') (1 + r(X)) | X] < 1, \quad j = e, u, \quad (32)$$

where M^{ju} is the SDF of ju workers, $j = e, u$, which is given by

$$M^{ju} (X, X') = \frac{((1 - f(X)) u'(c^{uu} - h\mathbf{c}^{uu}) + f(X) (u'(c^{eu} - h\mathbf{c}^{eu})))}{u'(c^{ju} - h\mathbf{c}^{ju})}, \quad j = e, u.$$

In the empirical implementation of the model below we will make sure that the inequalities in (32) hold in the steady state; then, this will imply that they also hold –and consequently the finite state space equilibrium exists– provided that aggregate shocks are sufficiently small. Note that in steady state it is necessarily the case that $u'(c^{uu} - h\mathbf{c}^{uu}) > u'(c^{eu} - h\mathbf{c}^{eu})$ whenever $a^{W'} > \underline{a}(X)$ (i.e., employed workers hold some buffer stock saving), in which case $M^{eu} > M^{uu}$ and the condition $\mathbb{E} [M^{eu} (1 + r) | X] < 1$ become sufficient for both inequality in (32) to hold.

The second existence condition requires that employed workers do not face a binding borrowing constraint. In other words, it must indeed be the case that in equilibrium¹⁰

$$\mathbb{E} [M^e (X, X') (1 + r') | X] = 1. \quad (33)$$

In the quantitative implementation of the model we will make sure that conditions (32)–(33) hold in the steady state. Then, this will also imply that they hold along the equilibrium with aggregate shocks provided that their magnitude is sufficiently small.

4 Empirical Analysis

4.1 Solution Method and Econometric Strategy

Before proceeding, we must specify the functional forms adopted for the utilization cost function $\eta(\cdot)$ and for the investment adjustment cost function $S(\cdot)$. We assume

$$\eta(u) = \frac{\bar{r}_k}{\nu_\eta} [\exp(\nu_\eta(u - 1)) - 1]$$

and

¹⁰When the borrowing constraint is also binding for employed workers, which occurs, e.g., when workers are too impatient relative to employers (i.e., β^W/β^E is too low), then the equilibrium still exists but features purely “hand-to-mouth” workers (as in, e.g., Iacoviello, 2005), rather than “buffer-stock saving” workers.

$$S\left(\frac{i}{\mathbf{i}_{-1}}\right) = \frac{\nu_i}{2} \left(\frac{i}{\mathbf{i}_{-1}} - g_i\right)^2.$$

Also, we impose the functional form

$$\rho(\xi_\rho) = \frac{1}{1 + \exp(\bar{\rho} - \xi_\rho)},$$

where $\bar{\rho}$ is a constant that pins down the steady-state value of ρ . With this functional form, we ensure that ρ varies only in the compact set $[0, 1]$.

Here $\bar{r}_k$ is the steady-state value of the rental rate of capital r_k , $\nu_\eta > 0$ is the curvature of the utilization cost, and ν_i is the curvature of the investment adjustment cost function. These function forms ensure that in a steady state with $i/\mathbf{i}_{-1} = g_i$ and $u = 1$, both the adjustment costs and the utilization costs vanish.

In order to simplify the presentation of the empirical analysis we are going to index variables by a time subscript, where t will denote current variables and $t - 1$ lagged ones. Also, before, taking the model to the data, we first induce stationarity by normalizing the model by z_t and we linearize the resulting system in the neighborhood of the resulting steady state.¹¹ Then, let $\hat{X}_t$ denote the vector collecting the deviation from steady state of the normalized state variables and let ϵ_t denote the vector collecting the innovations of the structural shocks. The law of motion of $\hat{X}_t$ is of the form

$$\hat{X}_t = \mathbf{F}(\boldsymbol{\psi})\hat{X}_{t-1} + \mathbf{G}(\boldsymbol{\psi})\epsilon_t, \quad (34)$$

where $\boldsymbol{\psi}$ is the vector of model's parameters. The matrices $\mathbf{F}(\boldsymbol{\psi})$ and $\mathbf{G}(\boldsymbol{\psi})$ are functions of the model's parameters.

We use as observable variables in estimation the growth rate of output $\Delta \log(y_t)$, the growth rate of investment $\Delta \log(i_t)$, inflation π_t , the nominal interest rate R_t , wage inflation $\Delta \log(W_t)$, the separation rate s_t , and the job-finding rate f_t . The measurement equation is

$$\begin{pmatrix} \Delta \log(y_t) \\ \Delta \log(i_t) \\ \pi_t \\ R_t \\ \Delta \log(W_t) \\ s_t \\ f_t \end{pmatrix} = \mathbf{M}(\boldsymbol{\psi}) + \mathbf{H}(\boldsymbol{\psi})\hat{X}_{t-1} + \mathbf{J}(\boldsymbol{\psi})\epsilon_t. \quad (35)$$

We use $\mathbf{M}(\boldsymbol{\psi})$ to denote the vector of mean of observed variables. We follow the Bayesian approach to estimate the model's parameters. Based the state-space representation for the dynamic

¹¹See Appendix for further details on the procedure used to induce stationarity.

system, the Kalman filter is then used (i) to evaluate the likelihood of the observed variables at any value of ψ and (ii) to form the its posterior distribution by combining the likelihood function with a joint density characterizing some prior beliefs.

Given the specification of the model, the posterior distribution cannot be recovered analytically but we may numerically draw from it, using a Monte-Carlo Markov Chain (MCMC) sampling approach. More specifically, we rely on the Metropolis-Hastings algorithm to obtain a random draw of size 400,000 from the posterior distribution of the parameters.

4.2 Data and Stochastic Structure

The data used for estimation come from the Federal Reserve Bank of St. Louis' FRED II database and from the Bureau of Labor Statistics website. They consist of private investment, deflated by the implicit GDP deflator (GDPDEF). Private investment is defined as the sum of gross private domestic investment (GPDI) and personal consumption expenditures on durable goods (PCDG). Output is defined as real GDP. For the labor-market transition probabilities, we proceed as follows. First, we compute monthly job- finding probabilities using CPS data on unemployment and short-run unemployment, using the two-state approach of Shimer (2005, 2012). (As suggested by Shimer, 2012, the short-run unemployment series is made homogenous over the entire sample by multiplying the raw series by 1.1 from 1994M1 onwards). We then compute monthly separation probabilities as residuals from a monthly flow equation similar to equation (1). Using these two series, we construct transition matrices across employment statuses for every month in the sample, and then multiply those matrices over the three consecutive months of each quarter to obtain quarterly transition probabilities (this naturally implies that cyclical fluctuations in the quarter-to-quarter separation rate partly reflect changes in the underlying monthly job- finding probability). All the series are converted to per-capita terms by dividing them by the civilian population, age 16 and over (CNP16OV), when relevant. All the series are seasonally adjusted except for population. Our sample runs from 1964:1 to 2008:3.

The stochastic shocks considered at the estimation stage are the following. First, given the persistence properties of s_t and f_t over the sample considered in this paper, we impose the following structure on ξ_m and ξ_ρ . For $h \in \{m, \rho\}$, we have

$$\xi_{h,t} = \tilde{\rho}_h \xi_{h,t-1} + \tilde{\sigma}_h \tilde{\epsilon}_{h,t} + \sigma_h \epsilon_{h,t}.$$

We calibrate $\tilde{\rho}_h = 0.99$ and $\tilde{\sigma}_h = 0.01$, so that these process help capture the low-frequency behavior of s_t and f_t , without compromising the estimation. Finally, we assume that ϵ_m and ϵ_ρ are simply *Niid*, with standard errors σ_ρ and σ_m , respectively. The approach taken for ξ_π is closely related,

except that the process is further restricted to

$$\xi_{\pi,t} = \tilde{\rho}_{\pi}\xi_{\pi,t-1} + \tilde{\sigma}_{\pi}\tilde{\epsilon}_{\pi,t},$$

where we impose $\tilde{\rho}_{\pi} = 0.99$ and $\tilde{\sigma}_{\pi} = 0.01$. We do not add an extra innovation ϵ_{π} to this process, because the latter would not be separately identifiable as long as ϵ_R is taken into account in the Taylor-like rule.

The shocks ξ_c , ξ_i and ξ_w are assumed to follow AR(1) processes of the form

$$\xi_{h,t} = \rho_h \xi_{h,t-1} + \sigma_h \epsilon_{h,t}, \quad \epsilon_{h,t} \sim \text{Niid}(0, 1), \quad h \in \{c, i, w\}.$$

The technology shock z_t follows a random walk

$$z_t = \mu_z + z_{t-1} + \sigma_z \epsilon_{z,t}, \quad \epsilon_{z,t} \sim \text{Niid}(0, 1).$$

The monetary policy shock $\xi_{R,t}$ is *Niid*

$$\xi_{R,t} = \sigma_R \epsilon_{R,t}, \quad \epsilon_{R,t} \sim \text{Niid}(0, 1).$$

4.3 Calibrated Parameters

The vector of parameters ψ is split in two subvectors ψ_1 and ψ_2 . The first one, $\psi_1 = (\sigma, \beta^E, \beta^W, \delta, \nu_{\eta}, \phi, \bar{w}, \kappa_v, b^u, \chi, \bar{m}, \bar{\rho}, \theta, \varsigma, \gamma)$, contains parameters calibrated prior to estimation. Typically, these are parameters difficult to estimate in our framework. We impose $\sigma = 1$, so that the utility function is logarithmic. The subjective discount factor of employers $\beta^E = 0.9985$ is set so that the real interest rate is close to 2 percent. The subjective discount factor of workers $\beta^W = 0.9835$ is set so that the ratio of individual consumptions c^{eu}/c^e is 80 percent, meaning that upon reaching unemployment, workers loose 20 percent of consumption compared with workers who stay employed. The depreciation rate $\delta = 0.025$ implies a 10 percent annual depreciation of physical capital. The curvature of the utilization cost function, ν_{η} , is set to 1. We set $\phi = 0.36$. We pin down $\bar{w}$ and κ_v such that (i) the labor income share in output is 63.35 percent and (ii) the share of vacancy costs in output $\kappa_v v/y$ is 0.65 percent. The parameter b^u is set so as to impose a replacement rate of 50 percent. The elasticity of the matching function with respect to vacancies is set 0.5, so that $\chi = 0.5$. We choose θ such that the markup is 20 percent. We restrict borrowing by setting $\varsigma = 0$. Finally, we set the price-indexation parameter γ to 1. This parameter was initially estimated but was converging towards its upper theoretical value of 1, which we impose henceforth. The parameters $\bar{m}$ and $\bar{\rho}$ are pinned down by imposing that the steady-state value of s and f coincides with their empirical means. Since these means depend on the value of the estimated parameters $\bar{m}$ and $\bar{\rho}$ will be functions of the posterior draw such that s and f coincides with their empirical means for every draw from the posterior.

Table 1: Prior and Posterior Distributions

Parameter	Prior shape	Prior Mean	$100 \times$ Prior s.d.	Posterior Mean	$100 \times$ Posterior std
$100\mu_z$	Normal	0.41	10.00	0.3999	3.3437
$100\bar{\pi}$	Normal	0.97	10.00	0.9789	6.8057
ν_i	Normal	2.50	50.00	1.2183	28.7220
h	Beta	0.75	10.00	0.8757	4.0295
α	Beta	0.75	10.00	0.7434	1.6116
ρ_R	Beta	0.75	15.00	0.8089	2.4931
a_π	Gamma	1.50	15.00	1.4449	12.6284
a_y	Gamma	0.50	15.00	0.7307	10.0817
ρ_c	Beta	0.75	10.00	0.9086	3.4335
ρ_w	Beta	0.75	10.00	0.9843	0.5721
ρ_i	Beta	0.75	10.00	0.6680	5.2679
σ_w	Inverse Gamma	0.01	50.00	0.0045	0.0337
σ_i	Inverse Gamma	0.01	50.00	0.0270	0.3268
σ_ρ	Inverse Gamma	0.01	50.00	0.0412	0.4877
σ_z	Inverse Gamma	0.01	50.00	0.0051	0.0369
σ_c	Inverse Gamma	0.01	50.00	0.7479	23.1497
σ_R	Inverse Gamma	0.01	50.00	0.0025	0.0164
σ_m	Inverse Gamma	0.01	50.00	0.2100	1.7235

4.4 Estimated Parameters

The remaining parameters, contained in $\psi_2 = (\mu_z, \bar{\pi}, \nu_i, h, \alpha, \rho_R, a_\pi, a_y, \rho_c, \rho_w, \rho_i, \sigma_w, \sigma_i, \sigma_\rho, \sigma_z, \sigma_c, \sigma_R, \sigma_m)$, are estimated. Estimation results are reported in Table 1.¹² Classically, we impose Inverse-Gamma priors for the standard errors, with prior mean set to 1 percent and prior standard deviation set to 0.5 percentage points. The serial correlation parameters have Beta priors centered on 0.75, with standard error 0.10. The prior are somewhat tilted toward a strong persistence. We use the same prior shape and parameters for h , α , and ρ_R . Hence, we guarantee that serial correlation parameters, the degree of interest rate smoothing, the habit parameter, and the Calvo probability are in the interval $(0, 1)$. Finally, we use normal priors for the growth rate of technology and the inflation target, each centered on the empirical mean of output growth and inflation, respectively. The standard deviation of the prior is set to 0.1, so as to penalize large deviations from these empirical means. We also use a normal prior for ν_i , with mean 2.5 and standard error 0.5.

Table 1 also reports the posterior moments. As the reader can observe the posterior standard

¹²At this stage, the table only reports preliminary results based on the posterior maximization.

Table 2: Serial Correlation of Aggregate Variables

Order	1	2	3	4	5
$\Delta \log(y_t)$	0.2587	0.2167	0.1606	0.1609	0.1460
$\Delta \log(i_t)$	0.6579	0.3733	0.1604	0.0190	-0.0645
π_t	0.8822	0.7944	0.7196	0.6552	0.5988
R_t	0.9224	0.7867	0.6369	0.5096	0.4316
$\Delta \log(W_t)$	0.5553	0.4971	0.4476	0.4051	0.3680
s_t	0.9999	0.9998	0.9997	0.9995	0.9993
f_t	0.9999	0.9998	0.9997	0.9995	0.9993
$\Delta \log(W_t)$	0.6450	0.3925	0.2286	0.1243	0.0535
n_t	0.9999	0.9998	0.9997	0.9995	0.9993

deviation is always smaller than the prior standard deviation, indicating that the estimated parameters are somewhat identified. As expected the growth rate of technology and inflation target posteriors are centered around their empirical means and the posterior standard deviation is quite small compared with the prior standard deviation. The reported posterior mean of the degree of curvature of the adjustment cost of investment function is quite small. Also the habit formation is estimated to be large and also very accurately. Price rigidity and nominal interest rate smoothness are estimated to values standard in the literature. Taylor rule posterior means are also estimated to values accepted to be normal. The posterior mean of the persistence and standard deviation of all the structural shocks are not surprising except for the standard deviation of the demand shocks, σ_c , that it is quite large. This is due to the fact that we have two types of consumers and both face the same demand shock.

Table 2 reports the serial correlation of observed variables generated by the model evaluated at the posterior mean. Table 3 reports the same correlations as found in the data. As it can be observed the model does a good job matching most of the the observed serial correlation. The most important problem is that the model generates too much persistence of s_t , f_t , and n_t . This is related to the calibrated parameters $\tilde{\rho}_h$ for $h \in \{m, \rho\}$.

4.5 Impulse Response Function to a Monetary Policy Shock

To investigate the behavior of the model, we analyze the response of aggregate variables when the economy is hit by a positive monetary policy shock. We parametrize the model to the mean of the posterior distribution. The impulse response function of the main variables are reported in Figure 2. As it can be seeing, the model behaves as expected. A increase in interest rates increase, as expected, output, consumption, investment and inflation fall. The behavior of the variables

Table 3: Serial Correlation of Aggregate Variables - Empirical Data

Order	1	2	3	4	5
$\Delta \log(y_t)$	0.3078	0.2534	0.1205	0.0936	-0.0475
$\Delta \log(i_t)$	0.3148	0.1847	0.0918	0.0210	-0.1285
π_t	0.8836	0.8337	0.7995	0.7734	0.7116
R_t	0.9623	0.9062	0.8585	0.8048	0.7436
$\Delta \log(W_t)$	0.5713	0.4648	0.4780	0.4284	0.4496
s_t	0.9683	0.9298	0.8824	0.8296	0.7835
f_t	0.9799	0.9540	0.9101	0.8627	0.8068
$\Delta \log(W_t)$	0.4301	0.2128	0.1364	0.0703	0.0472

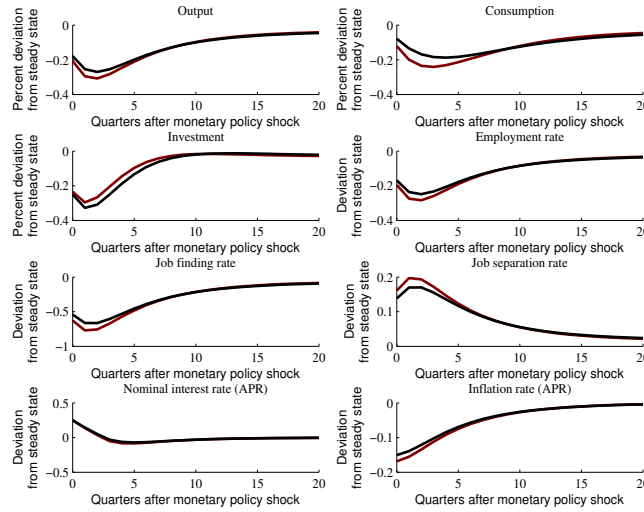


Figure 2: Impulse Response to a Monetary Policy Shock (in percent)

related to the labor market also follows the expected pattern: Employment fall because job finding rate decreases while job separation rate increases. A positive monetary policy shock is a standard demand shock that will allow us to highlight the propagation mechanism build into the model. In other words, this demand shock will allow us to highlight the relationship between precautionary saving and aggregate demand.

4.6 Precautionary Saving after a Monetary policy shock

We now investigate the contribution of time-varying idiosyncratic risk to the drop of aggregated variables after a positive monetary policy shock. We compare the impulse response functions of consumption reported in Figure 2 with the evolution of the consumption, if households did not face time-variations of the idiosyncratic risk. We call the impulse response function reported in Figure

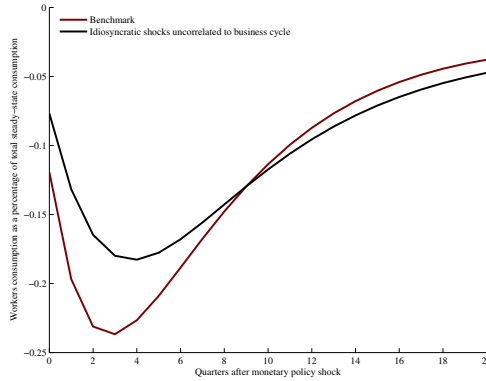


Figure 3: Fall in consumption of workers for the benchmark economy and for the economy without time-varying risk, as a fraction of steady-state total consumption.

2 the benchmark case. The comparison is done in Figure 3. To do such comparison we perform the following experiment in an alternative economy where idiosyncratic risk is constant.

We construct an economy where the family of workers receive all labor income and where workers out of the family receive a transfer plus the remuneration of saving (as before), but where the probability to leave the family (s) and the probability to re-enter the family (f) are equal to their estimated steady-state value. As a consequence

$$f = \bar{f}, s = \bar{s}$$

In this economy the size of the family is constant (equal to the steady-state value) and is thus different from the number of employed workers in the economy, n . In other words, the family head is able to pool all the labor income, but it is unable to insure family members from the probability to exit the family, which is constant. In this alternative the idiosyncratic risk (measured by the probability to exit the family) is constant.

The impulse response functions to a positive monetary policy shock associated to this alternative economy are computed using the same value of the parameters as Section 4.5. The size of the monetary policy shock is also the same. Figure 3 plots the comparison.

As it can be observed, the fall in consumption of workers is 20% higher due to the time-variation of the idiosyncratic risk.

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