Bidding for Teams*

Benoit Julien†, John Kennes‡, and Moritz Ritter§

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Abstract

The production possibility frontier of an organization can exhibit either increasing, constant, or decreasing returns to scale, depending on its current workforce. This paper considers the hiring problem of an organization in a competing job auction environment where each organization can set a 'reserve price' at each date in its life cycle. We find (i) increasing returns to scale organizations advertise negative reserve prices for new slots, which is equivalent to advertising an amenity (for example, free health club membership), (ii) constant returns to scale organizations advertise zero reserve prices (Wages are determined by simple Bertrand competition), and (iii) decreasing returns to scale organizations advertise positive reserve prices, which is equivalent to an insider-outsider wage schedule.

1 Introduction

A team is any group of people linked together in a common purpose. The team might be required to be of sufficient size as when a group of hunters attempt to corner a stag, or be of sufficient specialization as when a group of musicians play in a symphony orchestra. More prosaic examples of a team are a group of employees working together inside a firm, or the members of a social networking site sharing stories and making joint plans of action.

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†UNSW; Email: benoit.julien@unsw.edu.au

‡Department of Economics and Business, Aarhus University, Universitetsparken, Building 1322, DK-8000 Aarhus C, Denmark; Email: jkennes@econ.au.dk

§Temple University; Email: moritz@temple.edu

The standard theory of the firm taught in microeconomics places no emphasis on the process by which teams are created. Therefore, intital conditions such as how many workers are currently employed by a firm cannot influence hiring policies, because the firm can instanteously hire and fire people whenever a change in technology and preferences makes this new assignment optimal. One objection of this standard theory is that it makes no allowance for the amount of coordination that is embodied in the current distribution of workforce. Therefore, this theory does not treat coodination as either an investment that reflects the costs incurred in past hiring and relocation decisions, or a hurdle that must be overcome when a change in this distribution of employees across firms is required to achieve greater ends.

The present paper studies the creation of teams using a theory of competing mechanisms (McAfee 1993, Wolinsky 1988, Peters and Severinov 1997). One of the most basic results is the equivalence between random urn-ball assignment and the mixed strategy equilibrium of reserve price competition in auctions (Julien, Kennes and King 2000). Therefore, a competing auction environment offers a foundation for the matching technology of an economy.[1] The concept of island competition is also used to describe how isolated platforms obtain, and transform, various local means into useful ends (Lucas and Prescott 1974). Therefore, islands may also have an important multi-sided function to attract and enable the creation of teams (Rochet and Tirole 2006).

This paper aims to develop a richer theory of island matching, which incorporates insights from the two sided matching literature, and extends the analysis of competing auctions to team formation. We draw a distinction between two fundamental competitive environments - anti-coordination versus coordination. Existing models of directed search have focused attention on anti-coordination games. In such cases, in the mixed strategy equilibrium, there is a trade-off between advertising a higher price (or advertising lower quality) and obtaining a lower probability of trade (Refer to Shimer 2005). However, many important two sided matching problems are not anti-coordination games. For example, if a man and woman wish to meet, and must independently choose between two islands, then restricting attention to a mixed strategy equilibrium means that the better island gets a lower probability of trade than the worse island. Therefore, the directed search equilibrium concept offers a much less appealing solution to these matching problems.

[1] These competitive environments are also closely related to models of price posting models and directed search (Montgommery 1991, Burdett, Shi and Wright 2001, Shimer 2005).

We categorize a large island matching problem in competing auctions as either a game of coordination, or anti-coordination, depending on the difference between 1) the probability of an island-team match, and 2) the probability of more bidders than slots relative to the elasticity of the matching technology. This distinction is useful for questions related to the selection of auction mechanisms. In a competing auction environment, the anti-coordination matching games always feature positive reserve prices while the coordination matching games have negative reserve prices. Therefore, the nature of island competition for teams can be profoundly different depending on whether it is a directed search game of coordination or anti-coordination. On the one hand, in the coordination game, the competing auctioneers will try to sweeten the deal by posting amenities (a negative reserve price). On the other hand, in an anti-coordination game, the competing auctioneers will generally seek methods of rent extraction, such as a positive reserve price, to recoup their setup costs.

The details of a team production technology are essential for understanding how we disentangle the rates by which islands are matched, and the rates by which island have more bidders than slots. For example, one basic type of team is a male-female pair. If males and females seek to be matched on islands, it becomes readily apparent that additional islands may actually reduce the overall rate matching, and thus an island tax is needed to preserve efficiency under the assumption that island rents are determined by auctions (Refer to Mortensen 2009). Other types of team formation have a similar effect. For example, if a firm must hire a critical mass of workers, then simple job auctions may also lead to an equilibrium with too many firms relative to job searchers. However, alternative teams structures can produce results that go the other way - giving too little island entry in the simple auction equilibrium. This occurs, for example, if a firm can hire and produce with any number of workers up to some capacity (Refer to Burdett, Shi, and Wright 2001). In such cases, the social value of additional firms is greater than their private value in the simple auction equilibrium, because on each island there is generally more slots than workers, and thus little competitive upward pressure on wages.

An auction is a dynamic pricing mechanism that responds to local market conditions, and which requires no commitment to a particular price (but perhaps commitment to a level of service, which can be advertised). An alternative solution to price determination in directed search equilibrium is to assume that islands can make and advertise price commitments. However, unlike auctions, quality competition does not emerge in the price posting equilibrium, because

the island sellers can commit to a vector of rent divisions corresponding to all possible ex post outcomes. We can then conclude that the possibility of price commitment, or lack thereof, can have important real effects on how resources are allocated in alternative models of directed search equilibrium.

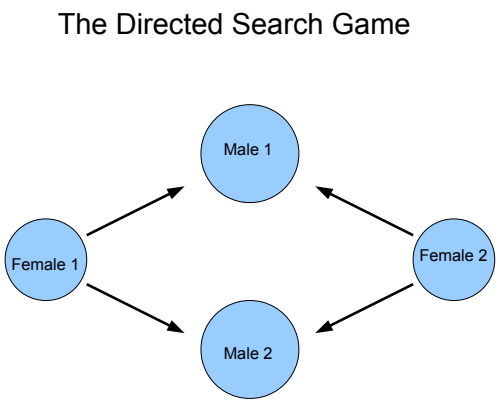
The paper is organized as follows. In section 1, we present the problem of island assignment and discuss how this environment can be characterized as either a game of coordination or anti-coordination. We also consider finite replications to a larger market and explain how we refine the directed search equilibrium. In section 2, we consider a large economy and derive the Mortensen tax, which ensures equivalence between the social planner and decentralized equilibrium with simple auctions. We then consider how the Mortensen tax is implemented by competing auctions and compare the price posting outcome. In section 3, we derive results for alternative team structures when agents are homogenous. In section 4 we derive limiting results for larger islands. Section 5 consider the problem of matching teams with asymmetric agents. The final section offers concluding remarks.

2 Island coordination

This section discusses two canonical matching problems that are relevant to the analysis of team formation in directed search equilibrium.

2.1 The anti-coordination game

The standard model of directed search can be cast as a simple problem of coordinating the matching of two agents to two locations. An obvious example is when there are two spatially separated males and two spatially separate females, where the females must simultaneously choose one of these male location in order to create a match. This can be characterized as an anti-coordination game where the female gets a payoff of zero if the other female chooses the same location (Competion with the other female bids away the male rents), and a payoff of y_i if she chooses male $i \in \{1, 2\}$ and the other female chooses the other male. This game is illustrated as follows.



The normal form of the game is given by

F_1/F_2	1	2
1	$0, 0$	y_1, y_2
2	y_2, y_1	$0, 0$

In the mixed strategy equilibrium of this assignment game, we find that probability π_1 that a female visits male 1 is given by

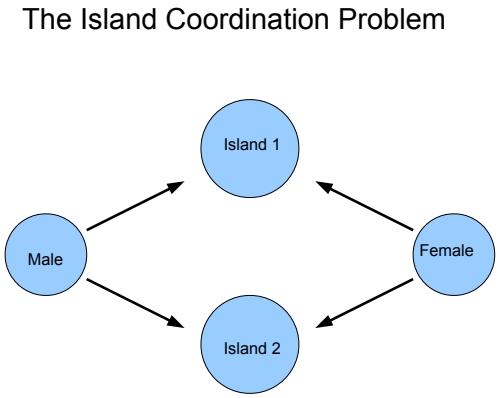
$$\pi_1 = \frac{y_1}{y_1 + y_2}$$

This assignment game has the 'nice' property that a more attractive male has a higher probability of trade than the less attractive male. In this sense, the directed assignment equilibrium is better than purely random matching. This assignment is also considered to be the reasonable outcome of this game because it is the only equilibrium of the game where the identical females are assumed to adopt identical mixed strategies. The outcome of this refinement is known as the directed search equilibrium (Refer to Shimer 2005).

2.2 The coordination game

Another variation of the location game is to assume that the problem of coordination involves two islands, a man, and a woman. In this game, if both the man and the woman choose the same island, they can both enjoy $y/2$ units of output

and if they choose different islands then they receive nothing. The question then is how do they choose islands if they are uncoordinated over which island to visit.



An obvious point is that this is a simple battle of the sexes game with two pure strategy equilibria and mixed strategy equilibrium. The normal form game is simply

M/F	1	2
1	$\frac{y}{2}, \frac{y}{2}$	$0, 0$
2	$0, 0$	$\frac{y}{2}, \frac{y}{2}$

In this case, the mixed strategy is the one where the men and women randomize over the two islands. To see how this game differs from the standard directed search game, consider the case where one island is better than another island so that the return on island 1 is greater than on island 2 - thus $y_1 > y_2$. This changes the normal form of the coordination game as follows.

M/F	1	2
1	$\frac{y_1}{2}, \frac{y_1}{2}$	$0, 0$
2	$0, 0$	$\frac{y_2}{2}, \frac{y_2}{2}$

The logic of the directed search equilibrium is to assume that agents choose to play the mixed strategy equilibrium. Therefore, the expected payoffs of visiting the two islands are equated. This means that probability that a male or female

visits an island is given by

$$\pi^1_{Naive} = \frac{y_2}{y_1 + y_2}$$

In this version of a directed search equilibrium, a reduction in the quality of an island has the perverse effect of increasing the probability that men and women locate on this island. If agents follow this strategy, then we have the perverse effect that an increase in the quality of one island will lower the overall value of trade (Note that we can easily interpret high y_1 as a high wage offer to a couple.)

Of course we can argue that the mixed strategy equilibrium is pushing the idea of anonymity towards unrealistic equilibrium. There is another directed search equilibrium that is perhaps more sensible. We might assume that agents play a strategy of choosing the best island. This means that

$$\pi^2_{Sophisticated} = \begin{cases} 0 & if \quad y_1 > y_2 \\ 1/2 & if \quad y_1 = y_2 \\ 1 & if \quad y_2 > y_1 \end{cases}$$

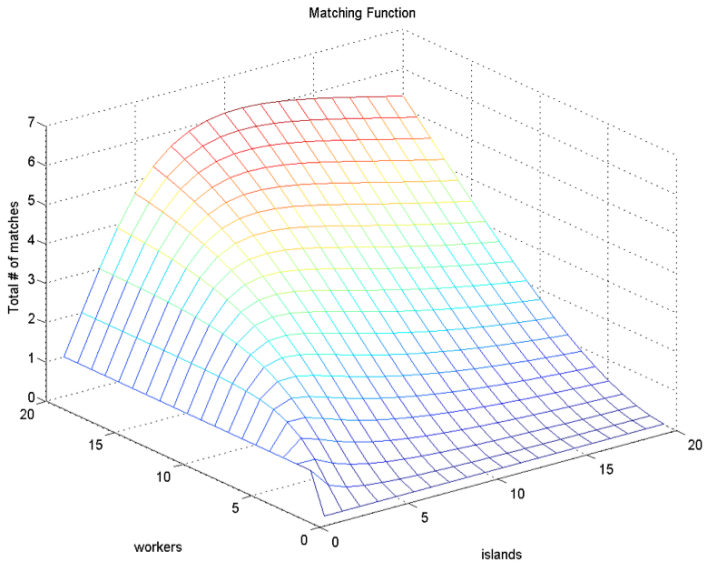
In this case, the equilibrium is to have both agents visit the high quality island. The idea suggested by this equilibrium is that the islands can pursue a higher quality (or lower prices) as a means to facilitate coordination. This includes any credible commitments/announcements by any two agents.

2.3 When is the matching technology well-behaved?

More generally we can see the problem of coordination and team formation by mapping out the matching technology for an economy where a firm must hire at least two agents to produce and the agents are randomizing over island locations. The matching technology for this environment is

$$X(M, N) = N\left(1 - (1 - 1/N)^M - M(1 - 1/N)^{M-1}\right)$$

where N is the number of firms, $(1 - 1/N)^M$ is the probability no worker visits the firm, and $M(1 - 1/N)^{M-1}$ is the probability exactly one worker visits. The matching technology for this problem of critical mass teams formation is illustrated below.



The figure illustrates that number of matches first increases then decreases as the number of firms decreases.

- The matching technology $X(N, M)$ is well behaved in the region that $\partial X/\partial N > 0$ and $\partial X/\partial M > 0$.

This example serves to illustrate that the problem of team coordination is different than partnership coordination. The fundamental issue concerns the value of critical mass in team formation, which may differ fundamentally from that in partnership formation economies. Nevertheless, the analysis in the subsequent sections will demonstrate that directed search equilibrium are generally found in the region where the matching function is well-behaved. Therefore, such models are anti-coordination games even when teams require critical mass or complementary inputs.

3 Compementary inputs

Assume that there is a fixed and equal number of men and women, which we can normalize to one, and that islands can enter this environment if they pay a cost c. Production requires exactly one man, one woman and one slot (i.e. room) on an island. If an island enters the market, it has a single room that it can sell. If n islands enter this economy, the average queue length of people of each sex at the islands is $q = 1/n$. If the men and women treat all islands

equally and pursue identical mixed strategies, the aggregate matching function is given by

$$X(q) = n\left(1 - e^{-q}\right)^2 \tag{1}$$

where $q = 1/n$ is the expected queue length of men and women at each island, and the rate at which an island is filled is then given by $x(q) = \left(1 - e^{-q}\right)^2$.

3.1 The social planning problem

The social planner's problem is to maximize the number of expected matches times their output subject to the costs of entry for islands. Therefore, the social planner chooses the number of islands $n = 1/q$ to maximize:

$$S_P^* = \max_{\{q\}} \frac{\left(1 - e^{-q}\right)^2}{q} y - \frac{c}{q}. \tag{2}$$

where the first term to the right of the max operator is the number of matches times the output of a match, and the second term is the total cost of the $n = 1/q$ islands. This is a well defined maximization problem with a solution

$$yx\left(q\right)\eta_{X,q} = c \tag{3}$$

where

$$\eta_{X,q} = \frac{\partial X\left(q\right) q}{\partial q X\left(q\right)}$$

which states that the social planner equates the social cost of each island with the output of a match weighted by the hiring probability and the elasticity of the matching function. The solution to the maximization problem is illustrated below

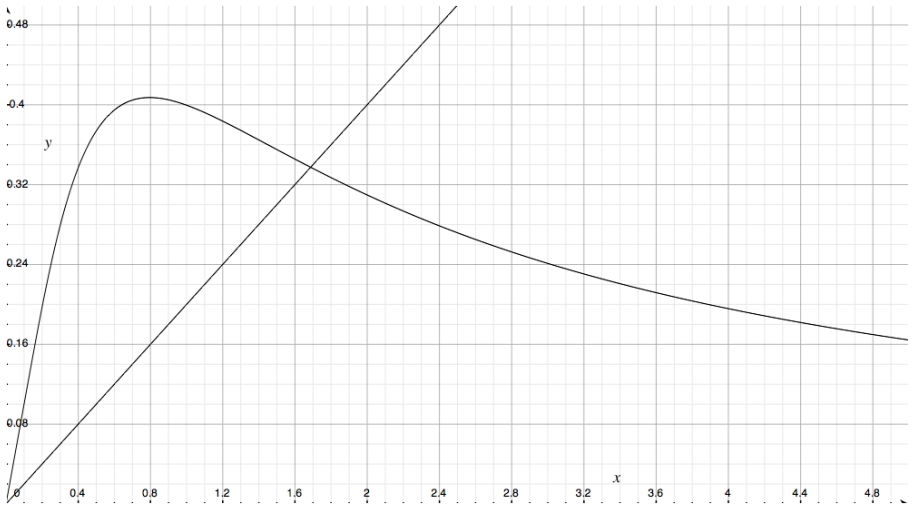


Figure 3: Value of matches versus cost of additional islands

3.2 Simple auctions

Suppose that islands earn the surplus of the island whenever there are more couples than slots. In this case, the couples extract all the surplus whenever there is one couple on the island. The free entry condition for islands in this equilibrium is given by

$$\pi_1^1 y = c \tag{4}$$

where $\pi_1^1 = (1 - e^{-q} - qe^{-q})^2$ is the probability of more couples than slots (i.e. multiple couples) - here the subscript and superscript refers to the valuation of the slot by the first and second highest valuation couple, respectively. The seller earns zero profits when there is no bidding couple, which occurs with probability $\pi_0^0 = (e^{-q})^2$ and when there is only one couple, which occurs with probability $\pi_1^0 = (1 - e^{-q} - qe^{-q})^2 - (e^{-q})^2$. If we use this condition to determine the number of matches, then there might be more islands than would maximize the number of matches.

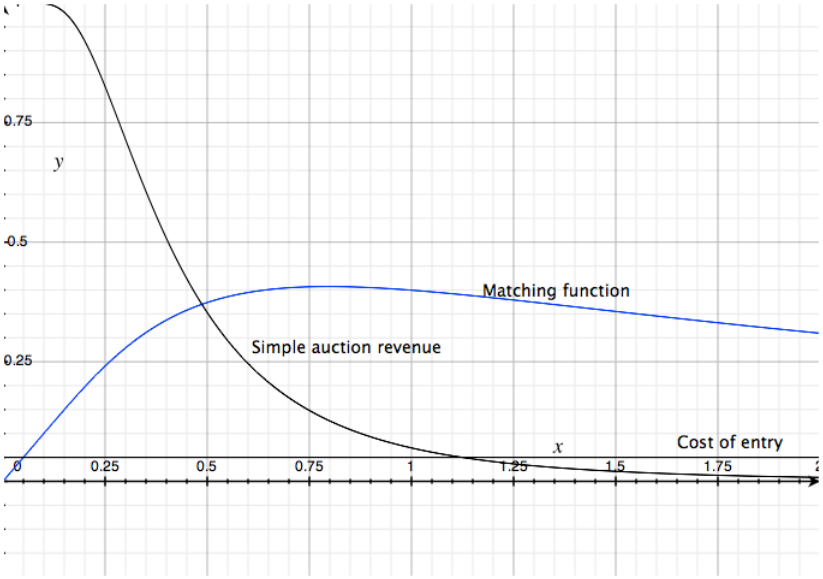


Figure 4: Simple Auction equilibrium and excess islands

So the simple auction equilibrium can easily contradict the refinement, which we developed in the last section. For example, if the cost of island entry is close to zero, then the equilibrium expressed by the equation (1) will be to have more islands than would maximize the total matches number of matches. So we might assume that the auction equilibrium in this case is equal to the maximum number of matches.

3.3 The Mortensen tax

The ordered seller refinement considered in the subsection on replications goes some distance to resolving the problem of excess islands. For example, if the cost of island entry is close to zero, then the equilibrium expressed by the equation (1) will be to have more islands than would maximize the total matches number of matches. However, if there are costs of island entry, then the equilibrium will not be efficient. Consider the case where island entry costs are sufficiently high to keep the equilibrium to the left of the match maximizing number of islands. We can take equation (8) and rewrite it as follows.

$$\left[\eta_{X,q} x\left(q\right) + \left(qe^{-q}\right)^2\right] y = c \tag{5}$$

Therefore, comparing this to the social planning problem, we observe too many islands in the simple auction equilibrium. One solution to this problem is to tax islands by an amount

$$tax_{Mortensen} = \left(qe^{-q}\right)^2 y,$$

which is the same rule proposed by Mortensen (2009). The Mortensen tax in the unit capacity island environment is equivalent to the probability that there is one couple at the island times the output of the island.

3.4 Advertised ammenities and Bertrand pricing

In this section we assume that the islands may offer amenities to the agents who locate on their islands. The idea is that the island could offer an additional 'free service' that is of value to a couple if they decide to locate on the island. An example might be a firm that offers access to a recreational centre in addition to the tasks needed to complete the job. Under an auction, a team will collect this benefit only if there are no other competing teams on the island. The timing of this posted subsidy game is as follows.

1. Simultaneous entry of islands

2 Simultaneous posting of the extra benefits d at each island.

3. Men and women the posted benefits at each island and make simultaneous choices over which island to visit.

4. Couples bid for the slots on the islands

Suppose that all island offer amenities d. If there are $n = 1/q$ islands in the economy, the expected queue length at each is q. The payoff to an island offering amenities d and enjoying expected queue length q is given by

$$\pi(q,d) = \left(1 - e^{-q} - qe^{-q}\right)^2 y - d\left(2qe^{-q}\left(1 - e^{-q}\right) - \left(qe^{-q}\right)^2\right) \tag{6}$$

and the expected return to a female (or male) locating on this island is given by

$$F(q,d) = \left(1 - e^{-q} - qe^{-q}\right)e^{-q}(y+d) + qe^{-q}e^{-q}\left(\frac{y+d}{2}\right) \tag{7}$$

Now consider the return to an island that deviates and offers amenities d'. The expected returns on both islands must be the same.

$$F(q,d) = F(q',d') \tag{8}$$

Therefore, the dividend of the deviating firm and the associated queue are then related as follows

$$d' = d(q',q,d) = \frac{F(q,d)}{\left(1 - e^{-q'} - \frac{q'e^{-q'}}{2}\right)e^{-q'}} - y \tag{9}$$

We can use the implicit function theorem to verify that $\partial q'/\partial d' > 0$. We can also cast the deviating island's problem as a choice of q' to maximize

$$q^* = \arg\max_{q'} \pi\left(q', d'(q',q,d)\right) \tag{10}$$

The first order condition of this seller's problem is given by

$$\left(1 - e^{-q*}\right)e^{-q*}y - \left(1 - e^{-q} - \frac{qe^{-q}}{2}\right)e^{-q}\left(y + d(q^*,q,d)\right) = 0 \tag{11}$$

In equilibrium, the expected queue length of the deviating island is equal to the expected queue length of the non-deviating islands. Thus

$$q^* = q$$

Therefore, we can use the first order condition of the seller's maximization problem to solve for the equilibrium amenity, d^*. We find that

$$d^* = \frac{\left(\frac{q^*e^{-q^*}}{2}\right)}{\left(1 - e^{-q^*} - \frac{q^*e^{-q^*}}{2}\right)}y$$

This gives revenue to the competing auction of

$$\pi\left(q^*, d^*\right) = yx\left(q^*\right)\left(1 - \eta_{x,q}\right)$$

which is equivalent to the returns required by the social planner. So the decentralized advertisement of amenities and the selling of island slots by an auction can function in the same way as the Mortensen tax.

3.5 Posted prices

An alternative pricing mechanism to an auction is to assume that the island commits to a price for each slot. In this case, the male and female of a couple each gets a payoff of (1-p)/2 if they are assigned to a room and zero otherwise. The owner of the island gets a payoff of p if the room is sold and zero otherwise. The timing of the price posting game is as follows

1. Simultaneous entry of islands

2 Simultaneous posting of hotel prices p at each island.

3. Simultaneous choice of islands by men and women

4. One couple is selected at random.

We solve the pricing game of islands after they have paid the sunk cost of entry. If all islands advertise a price p and buyers randomize over these islands, the payoff to an island advertising price p and expecting a queue length of q is given by

$$\pi^P(p,q) = p\left[\left(1-e^{-q}\right)\left(1-e^{-q}\right)\right]$$

The expected payoff of a woman (or man) of choosing an island with posted price p and expected queue length q is given by

$$F^P(q,p) = \left(\frac{1-e^{-q}}{q}\right)\left(1-e^{-q}\right)\left(\frac{1-p}{2}\right)$$

Now consider the possibility that a seller deviates and advertises a price p' other than p. In the mixed strategy equilibrium, the men and women are indifferent to visiting either type of seller. Thus

$$F^P(q,p) = F^P(q',p')$$

Therefore, we can solve for the following 'reaction function'

$$p'(q',p,q) = 1 - \frac{\left(1-e^{-q}\right)^2}{\left(1-e^{-q'}\right)^2}\frac{q'}{q}(1-p)$$

This reaction function of buyers can be substituted into the island's pricing problem. The seller can then be seen to choosing an optimal q^* to maximize

$$q^* = \arg\max_{q'} \pi^P \left(p \left(q', p, q \right), q' \right)$$

The first order condition of the pricing problem is

$$2\left(1 - e^{-q*}\right) e^{-q*} - \left(1 - e^{-q}\right)^2 \frac{(1-p)}{q} = 0$$

In equilibrium, all sellers are identical and have the same q, thus

$$q* = q$$

Therefore, using the seller's first order condition

$$2\left(1 - e^{-q}\right) e^{-q} - \left(1 - e^{-q}\right)^2 \frac{(1-p)}{q} = 0$$

or

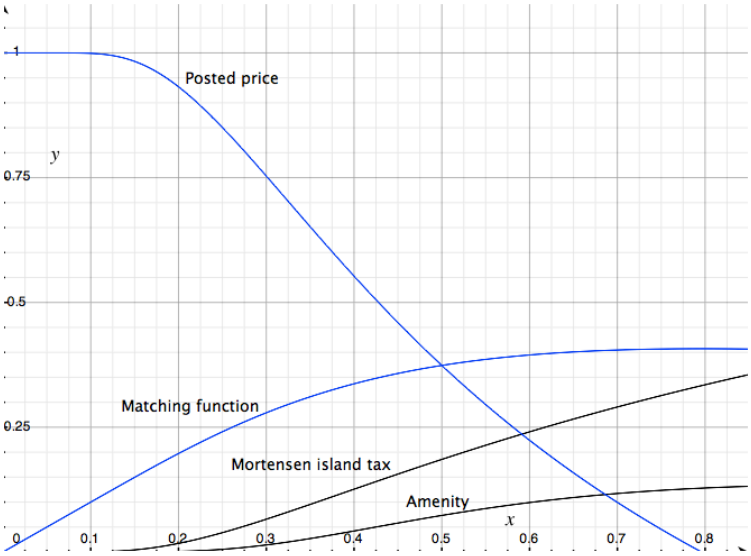
$$p^* = 1 - 2\frac{qe^{-q}}{1 - e^{-q}}$$

We can then substitute this price and the associated queue length into the island's payoff function to get

$$\pi^P(p^*, q^*) = p\left[\left(1 - e^{-q}\right)\left(1 - e^{-q}\right)\right]$$

Thus the equilibrium posted price gives each island seller identical revenues as the receive when they advertise auctions with benefits, and as the social planner would give them to encourage efficient entry.

The following figure summarizes the results from the analysis of price posting and the analysis of competing auctions with benefits, which was developed in the last section.



4 Returns to Scale

This section derives general results for organizations of arbitrary size. For simplicity, we focus on the problem of hiring one type of worker.

4.1 An Environment with Island Entry

1. Islands produce $y(1)$ if there is 1 worker, $y(2)$ if there are 2 workers,...
2. Let $\Delta_i = y(i) - y(i-1)$ denote the marginal productivity of the ith worker.
3. $y(0) = 0$ and cost $c \geq 0$ to enter.
4. Islands have capacity k, i.e. output of $k, k+1, k+2, ...$ workers is $y(k)$, i.e. $\Delta_j = 0 \ \forall j > k$.

We say that the organization is constant to scale if Δ_i is constant, increasing returns to scale if Δ_i is increasing, and decreasing returns to scale if Δ_i is decreasing.

4.1.1 Planner's Problem

$$\begin{aligned} S &= \frac{1}{q}\left[y(1)qe^{-q} + y(2)\frac{q^2}{2!}e^{-q} + y(3)\frac{q^3}{3!}e^{-q} + ... \right. \\ &\qquad \left. + y(k)\left(1 - e^{-q} - \frac{q}{2}e^{-q} - ... - \frac{q^{k-1}}{(k-1)!}e^{-q}\right)\right] - \frac{c}{q} \end{aligned}$$

FOC:

$$\begin{aligned} c &= \left[y(1)qe^{-q} + y(2)\frac{q^2}{2!}e^{-q} + y(3)\frac{q^3}{3!}e^{-q} + ... \right. \\ &\qquad \left. + y(k)\left(1 - e^{-q} - \frac{q}{2}e^{-q} - ... - \frac{q^{k-1}}{(k-1)!}e^{-q}\right)\right] \\ &\quad + q\left(-y(1)\left(e^{-q} - qe^{-q}\right) - y(2)\left(qe^{-q} - \frac{q^2}{2!}e^{-q}\right) - y(3)\left(\frac{q^2}{2!}e^{-q} - \frac{q^3}{3!}e^{-q}\right) - ... \right. \\ &\qquad \left. - y(k)\frac{q^{k-1}}{(k-1)!}e^{-q}\right) \\ &= y(1)q^2e^{-q} + y(2)\left(\frac{q^3}{2!}e^{-q} - q^2e^{-q}\left(1 - \frac{1}{2!}\right)\right) + y(3)\left(\frac{q^4}{3!}e^{-q} - q^3e^{-q}\left(\frac{1}{2!} - \frac{1}{3!}\right)\right) + ... \\ &\qquad + y(k)\left(1 - e^{-q} - \frac{q}{2}e^{-q} - ... - \frac{q^{k-1}}{(k-1)!}e^{-q} - \frac{q^k}{(k-1)!}e^{-q}\right) \end{aligned}$$

4.1.2 Decentralization via Simple Auction

An island profits receives a surplus only if there are more workers than slots, in which case total output is $y(k)$. Hence, the free entry condition is given by

$$c = y(k)\left(1 - e^{-q} - \frac{q}{2}e^{-q} - ... - \frac{q^k}{k!}e^{-q}\right)$$

4.1.3 Efficiency

The *negative* of Mortensen Tax on island entry to ensure efficiency is given by

$$\begin{aligned}
-tax &= y(1)q^2e^{-q} + y(2)\left(\frac{q^3}{2!}e^{-q} - q^2e^{-q}\left(1-\frac{1}{2!}\right)\right) + y(3)\left(\frac{q^4}{3!}e^{-q} - q^3e^{-q}\left(\frac{1}{2!}-\frac{1}{3!}\right)\right) + ... \\
&\qquad -y(k)\left(q^ke^{-q}\left(\frac{1}{(k-1)!}-\frac{1}{k!}\right)\right) \\
&= \sum_{i=1}^{k-1} y(i)\frac{q^{i+1}}{i!}e^{-q} - \sum_{i=2}^{k} y(i)q^ie^{-q}\frac{k-1}{k!} \\
&= q^2e^{-q}\left(y(1)-y(2)\right) + \frac{q^3}{2!}e^{-q}\left(y(2)-y(3)\right) + ... + \frac{q^k}{(k-1)!}e^{-q}\left(y(k-1)-y(k)\right) \\
&\qquad + \sum_{i=2}^{k} y(i)\frac{q^i}{i!}e^{-q} \\
&= \sum_{i=2}^{k} y(i)\frac{q^i}{i!}e^{-q} - \sum_{i=2}^{k}\Delta_i\frac{q^i}{(i-1)!}e^{-q} \\
&= \sum_{i=2}^{k}\frac{q^i}{(i-1)!}e^{-q}\left(\frac{y(i)}{i}-\Delta_i\right)
\end{aligned}$$

So the tax is given by

$$tax = \sum_{i=2}^{k}\frac{q^i}{(i-1)!}e^{-q}\left(\Delta_i - \frac{y(i)}{i}\right)$$

Definition. A prodtion technology has increasing (decreasing) returns to scale if

$$\frac{y(i)}{i} < (>)\Delta_i \quad \forall\, i \in \{1,2,...k\}$$

That is, the average product is smaller (exceeds) than the marginal product. In that case, the Mortensen Tax is positive (negative).

Discussion. Entry of additional islands lowers the matching probability for all existing islands and increases the matching probability for workers (congestion externality). In addition, if the production technology exhibits non constant returns to scale, entry also changes the average product of all existing islands, conditional on matching. The simple auction prices the congestion externality (i.e. the impact of the matching rate correctly) but does not take the effect

on the average product into account. If the technology exhibits increasing (decreasing) returns, additional entry represents a negative (positive) productivity externality.

Conjecture. Pricing mechanism with ex-ante competition such as competing auctions or price posting internalize the productivity externality.

4.1.4 Special Cases

If islands have capacity $k = 2$, the tax is given by

$$tax = q^2 e^{-q} \left(y(2) - y(1) \right) - \frac{q^2}{2} e^{-q} y(2) = q^2 e^{-q} \left(\frac{y(2)}{2} - y(1) \right)$$

Thus the tax is $\gtreqless 0$ if $y(1) \lesseqgtr 0.5y(2)$.

- BSW teams have $y(1) = 0.5y(2)$ and hence the tax is zero. The simple auction leads to the efficient allocation.
- Critical mass teams have $y(1) = 0$ and hence require a positive tax on island entry to restore efficiency.
- Couples (heterogeneous teams) are special cases of critical mass teams and hence also require a positive tax.

5 Dynamics

In order to develop a dynamic analysis it is important to consider the case of heterogeneous Islands.

1. There is a fixed number of islands.
2. Islands vary by type (capacity); islands can of type $1, 2, ..., k$.
3. Islands of type k produce $y(k, 1)$ if there is 1 worker, $y(k, 2)$ if there are 2 workers,...
4. Let $\Delta_i^k = y(k, i) - y(k, i-1)$ denote the marginal productivity of the ith worker.
5. $y(i, 0) \ \ \forall i. \ \Delta_j^k = 0 \ \forall j > k$
6. Workers can choose which type of island to apply to and randomize within each type.

Note: free entry of islands is not possible as the free entry condition and the workers' indifference condition independently determine the relative queue lengths between the types.

5.0.5 Planner's Problem

The planner chooses the distribution of workers over island types $M_1, M_2, ..., M_k$ to maximize total surplus:

$$\begin{aligned} S &= N_1\left(y(1,1)(1-e^{-q_1})\right) + N_2\left(y(2,1)q_2e^{-q_2} + \frac{y(2,2)}{2}\left(1-e^{-q_2}-q_2e^{-q_2}\right)\right) + ... \\ &\quad + N_k\left(\sum_{i=1}^{k-1} y(k,i)\frac{q_k^i}{i!}e^{-q_k} + y(k,k)\left(1-e^{-q_k}-\sum_{i=1}^{k-1}\frac{q_k^{i-1}}{(i-1)!}e^{-q_k}\right)\right) \end{aligned}$$

where $q_k = M_k/N_k$.

Let $\Lambda_k \equiv \frac{\partial S}{\partial M_k}$ denote the marginal value of a worker on island of type k.

$$\begin{aligned} \Lambda_1 &= y(1,1)e^{-q_1} \\ \Lambda_2 &= y(2,1)\left(e^{-q_2}-q_2e^{-q_2}\right) + y(2,2)q_2e^{-q_2} \\ &= y(2,1)e^{-q_2} + q_2e^{-q_2}\left(y(2,2)-y(2,1)\right) \\ \Lambda_k &= \sum_{i=1}^{k-1} y(k,i)\left(\frac{q_k^{i-1}}{(i-1)!}e^{-q_k} - \frac{q_k^i}{i!}e^{-q_k}\right) + y(k,k)\frac{q_k^{k-1}}{(k-1)!}e^{-q_k} \\ &= y(1,1)e^{-q_k} + \sum_{i=2}^{k}\frac{q_k^{i-1}}{(i-1)!}e^{-q_k}\Delta_i^k \end{aligned}$$

5.0.6 Workers' Problem with Simple Auction

Workers receive the following expected payoff from applying to type j islands:

$$\begin{aligned} W_1 &= y(1,1)e^{-q_1} \\ W_2 &= y(2,1)e^{-q_2} + \frac{y(2,2)}{2}q_2e^{-q_2} \\ W_k &= \sum_{i=1}^{k}\frac{y(k,i)}{i}\frac{q_k^{i-1}}{(i-1)!}e^{-q_k} \end{aligned}$$

5.0.7 Efficiency

The planner's solution and the simple auction equilibrium coincide if

$$\sum_{i=2}^{k} \frac{y(k,i)}{i} \frac{q_k^{i-1}}{(i-1)!} e^{-q_k} = \sum_{i=2}^{k} \frac{q_k^{i-1}}{(i-1)!} e^{-q_k} \Delta_i^k$$

As before with island entry, efficiency hinges on the returns to scale of the production technology. The worker decides based on the average product she will receive, whereas the planner considers the marginal product. With CRS, the simple auction equilibrium is efficient, whereas with CRS and DRS it is not. With DRS, there will be too many workers searching for jobs on the larger islands. In other words, with DRS, there are too few islands relative to workers (the queue length is too long), just as with island entry. With DRS the worker has a negative externality on the other workers that the auction does not price.

Efficiency can be restored by taxing entry of workers into type k islands. The tax at type k islands that restores efficiency is given by:

$$tax^{Worker}(k) = \sum_{i=2}^{k} \frac{q^i}{(i-1)!} e^{-q} \left(\frac{y(i)}{i} - \Delta_i^k \right)$$

Which is the negative of the tax on island entry.

5.1 Block recursivity

Suppose that an organization lives for T periods. Let $\Lambda(k)$ denote the continuation value of a firm of size k. In this case, we can solve the ..

Using the result from the previous section, we see that this is a block recursive problem.

6 Conclusions

This paper has shed some light on the following issues. The provision of perks and rewards for seniority are two important employment practices inside a firm. This paper has provided a simple explanation of these phenononmen based on the ....

- The Mortensen tax and team structure
  - No Mortensen taxes if constant returns to scale organization

  - Positive Mortensen tax if increasing returns to scale
  - Negative Mortensen tax if decreasing returns to scale

- Implementation by auctions and price posting
  - Zero reserve price for if constant returns to scale organization
  - Negative reserve prices if increasing returns to scale organization
  - Positive reserve prices if decreasing returns to scale organization

- Dynamics
  - Mortensen tax is increasing in the island separation rate, decreasing if the cost of advertising 'rooms' increases.
  - Price dispersion is greater in the competing ammenities equilibrium.

- Policy
  - Richer theory of the resource cost need for creating teams
  - Team formation may require large up front expenditures as a means of coordination. Are firms able to access funding for such activities? What is the role of credit constraints in problems of coordination?

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