

Precautionary Saving and Aggregate Demand (Preliminary)

Edouard Challe* Julien Matheron[†] Xavier Ragot[‡]
Juan F. Rubio-Ramirez[§]

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Abstract

This paper introduces incomplete insurance against idiosyncratic labour income risk into an otherwise standard New Keynesian business cycle model with involuntary unemployment. Following an adverse monetary policy shock that lowers aggregate demand, job creation is discouraged and unemployment risk (as summarised by the job-loss rate) persistently rises. Imperfectly insured households rationally respond to the rise in idiosyncratic income uncertainty by increasing precautionary saving, thereby cutting consumption and depleting aggregate demand even further; this in turn magnifies the initial labour market contraction, further raises unemployment risk, and so on. A calibrated version of the model suggests that the aggregate demand–precautionary saving feedback loop may significantly amplify the impact of aggregate shocks on unemployment, relative to the full-insurance case.

1 Introduction

Traditional business cycle analyses often emphasise the role of aggregate demand as a key driver of aggregate fluctuations (see, e.g., Woodford, 2003, Christiano et al. 2003, and Gali, 2010). In the basic New Keynesian model, monopolistic firms price goods above marginal

*CNRS, Ecole Polytechnique, CREST and Banque de France; Email: edouard.challe@gmail.com.

[†]Banque de France. Email: julien.matheron@banque-france.fr.

[‡]CNRS and Paris School of Economics. Email: xavier.ragot@gmail.com.

[§]Duke University and Federal Reserve Bank of Atlanta. Email: juan.rubio-ramirez@duke.edu.

cost; following an aggregate shock that lowers some components of aggregate demand, the firms that cannot fully adjust nominal prices find it worthwhile to passively adjust sales to changes in the demand. This leads to a fall in production and marginal costs, and thus to an increase in average markups as the economy is moving away from its (constrained-) efficient production level. While the basic model has a perfectly competitive labour market, the model has recently been extended to incorporate labour market frictions and involuntary unemployment (see Gali, 2011, for an overview of this strand of the literature). In the extended model, a fall in firms' sales does not mechanically manifest itself as a fall in firms' hours demand schedule and the equilibrium real wage; rather it lowers the expected benefit from hiring a worker which, given the presence of hiring costs, leads to a partial freeze in the labour market. The job-finding rate falls and the job-loss rate rises, i.e., the idiosyncratic unemployment risk faced by households rises.¹ However, even in the extended model, the maintained assumption that individuals' labour market transitions are fully insured within a "representative family" ensures that such transitions have little effect on households consumption choices; that is, aggregate demand affects idiosyncratic unemployment risk, but not the other way around.

In this paper, we analyse the feedback between precautionary saving and aggregate demand by removing the assumption of full risk sharing typically made in New Keynesian analyses. More specifically, we analyse the interactions between the three frictions that have been studied (separately) in earlier studies: imperfect competition and nominal rigidities in the goods market; search and matching frictions in the labour market; and incomplete asset markets (in the form of imperfect insurance and borrowing constraints), which give rise to a precautionary motive for holding wealth. Of course, interacting all three frictions would be of little interest if the model simply retained the known implications of each friction taken separately. We find that it is not the case, quite on the contrary: their interaction generates a powerful amplification mechanism for aggregate business cycle shocks, which is absent when only one or two frictions are considered. The reason why imperfect insurance has nontrivial effects on aggregate fluctuations when nominal and labour market frictions are jointly introduced is straightforward and intuitive. Following an adverse aggregate (policy or produc-

¹In the New Keynesian model with involuntary unemployment, the increase in the period-to-period job-loss rate during a recession may have two sources; first, time aggregation, as a lower job finding rate makes it more difficult that an individual who falls into unemployment will rapidly exit that state; and second, endogenous destruction, as a larger fraction of low-productivity production units become inefficient. As in Gali (2011), we focus on the first source of fluctuations in the job-loss rate in the present paper.

tivity) shock that lowers aggregate demand, job creation is discouraged and unemployment risk (as summarised by the job-loss rate) persistently rises. Imperfectly insured households rationally respond to the rise in idiosyncratic income uncertainty by increasing precautionary saving, thereby cutting consumption and depleting aggregate demand even further; this in turn magnifies the initial labour market contraction, further raises unemployment risk, and so on. Therefore, the endogenous response of households' precautionary savings in an equilibrium that is partly driven by aggregate demand (due to nominal rigidities) and when idiosyncratic volatility is endogenous (due to labour market frictions) explains the potentially large response of the economy to the shocks. A calibrated version of the model suggests that this aggregate demand/unemployment risk/precautionary saving feedback loop may significantly amplify the short-run impact of monetary policy shocks, relative to a comparable full-insurance (i.e., representative-agent) economy.

As is well known, the key difficulty with models assuming imperfect insurance is their lack of tractability. The reason is that, in the general case, the idiosyncratic labour market transitions faced by the households implies the existence of a full cross-sectional distribution of wealth, which enters the aggregate state vector (Aiyagari, 1994; Krusell and Smith, 1998). We circumvent this difficulty by means of two assumptions. First, we assume that (imperfectly insured) unemployed have limited ability to borrow, and face a borrowing limit that is tighter than the natural limit. Moreover, we focus our analysis on an equilibrium in which the borrowing constraint is sufficiently tight for the unemployed to face a binding borrowing constraint from the end of their first quarter of unemployment onwards. By way of consequence, all unemployed households face a binding borrowing constraint and have the same end-of-period asset wealth –rather than gradually depleting their asset wealth as the unemployment spell increases in length. Second, we assume that households enjoy periodic reinsurance, in the spirit of Lucas (1980). Namely, we assume that employed households face perfect insurance within their own “family”, while unemployed agents are excluded from the family (and hence from the cross-member insurance scheme) for the time that they remain unemployed. As a consequence, all employed households enjoy the same consumption levels and hold the same end-of-period asset wealth. This property, jointly with the limited cross-sectional heterogeneity on the unemployed's side, implies that the overall cross-sectional wealth distribution has a small number of states, so that the dynamics of the model can be summarised by a small-scale dynamic system.

Related literature. As stressed above, the amplification mechanism that we explore in this

paper requires exactly three key frictions (one per market), the interactions of which have not yet been analysed, to the best of our knowledge. Models combining sticky nominal prices (and, at times, wages) and frictional labour markets include Langot and Chéron (2000), Walsh (2005), Faia (2008), Trigari (2009), Gertler et al. (2008) and Blanchard and Gali (2010) –see Gali (2011) for an exhaustive survey of this strand of the literature. One important conclusion from this work is that labour market frictions per se do not significantly contribute to exacerbate aggregate fluctuations.² This is perhaps not surprising: as Shimer (2010) has pointed out, search frictions act as a particular type of labour adjustment cost; as such, they naturally tend to dampen, rather than amplify, the economy’s response to aggregate shocks. We use search frictions in the labour market as a way to generate involuntary unemployment and endogenously time-varying unemployment risk; as we show, once interacted with incomplete insurance, labour market frictions provide a powerful amplification mechanism for aggregate shocks, relative to an economy with nominal rigidities but no such frictions.

Krusell et al. (2011) and Nakajima (2012) analyse full-fledged incomplete-market, heterogeneous agents models with search frictions where the idiosyncratic income risk faced by the household is endogenised through firms’ job creation policy. These models assume flexible prices and perfect competition in the goods market; consequently, monetary policy shocks are neutral and there is no specific role for aggregate demand in shaping aggregate fluctuations.

As far as we are aware, the only paper that combines imperfect competition and nominal rigidities –and hence a role for aggregate demand– with incomplete insurance is Guerrieri and Lorenzoni (2011), who analyse the conditions for a liquidity trap within an heterogeneous-agent environment (with a competitive labour market). Their model is not tractable and is solved numerically under the assumption that nominal prices are constant. Our model considers a standard Phillips curve for the determination of inflation and transition rates in the labour market that respond endogenously to macroeconomic conditions. Other papers such as Iacoviello (2005) and more recently Bilbiie et al. (2012) study economies with nominal rigidities and a potentially binding borrowing constraint for impatient households (i.e., those with a subjective discount rate lower than the equilibrium interest rate), but do not have uninsured unemployment risk. Our model collapses into a version of theirs when

²In Gali’s words, “Quantitatively realistic labor market frictions are likely to have, by themselves, a limited effect on the economy’s equilibrium dynamics [...] When combined with a realistic Taylor-type rule, the introduction of price rigidities in a model with labor market frictions has a limited impact on the economy’s equilibrium response to real shocks” (Gali, 2011, pp. 490-491).

we allow workers to enjoy full insurance against unemployment risk.

The rest of the paper is organised as follows: Section 2 presents the models, with particular attention being paid to the risk sharing arrangement that makes our analysis tractable. Section 3 derives the model's steady state, analyse the existence conditions for the equilibrium that we focus on, and studies our economy's response to aggregate shocks. Section 4 concludes.

2 The Model

2.1 Firms

The economy has three production layers:³ a retail (or final goods) sector buys differentiated products from the wholesale sector and combines them into a single final good that is sold to the household; the wholesale sector buys undifferentiated goods from intermediate goods firms and turn them into the differentiated products bought by the retail sector; and the intermediate goods sector uses labour to produce the undifferentiated intermediate goods.

2.1.1 Intermediate goods firms

The market for intermediates goods is perfectly competitive. At every date t , the representative firm in that sector produces according to the CRS technology

$$y_{m,t} = (\bar{k}_t)^\phi (e^{\psi_t} n_t)^{1-\phi},$$

where $\phi \in (0, 1)$ is the elasticity of production with respect to capital services and ψ_t a deterministic productivity growth trend such that $\Delta\psi_t = \mu_\psi > 0$.

The firm hires labour in a market with search frictions. The timing convention is as in Walsh (2005), Blanchard and Gali (2010) and Gali (2011): at the very beginning of date t , a fraction ρ_t of existing matches are exogenously destroyed, so beginning-of-period employment and unemployment are $\tilde{n}_t = (1 - \rho_t) n_{t-1}$ and $1 - \tilde{n}_t = 1 - (1 - \rho) n_{t-1}$, respectively. The unemployed have a probability f_t to find a job within the same period, and stay unemployed until the end of the period with probability $1 - f_t$. It follows that the period-to-period job-loss rate is $s_t = \rho(1 - f_t)$, and is primarily affected by the ease at which households

³Since the demand for final goods will emanate from heterogeneous households, it is convenient to aggregate differentiated products into a single final good that is bought by the households (rather than having households directly buy differentiated products).

avoid unemployment spells by being immediately re-employed after a job destruction. Given its knowledge of $\tilde{n}_t$ and the (Markovian) exogenous state vector $\mathbf{\Omega}_t$, the representative firm posts v_t vacancies at cost $\kappa_v \exp^{\psi_t} > 0$ each, and a fraction λ_t of which are filled in the current period –see figure 1.

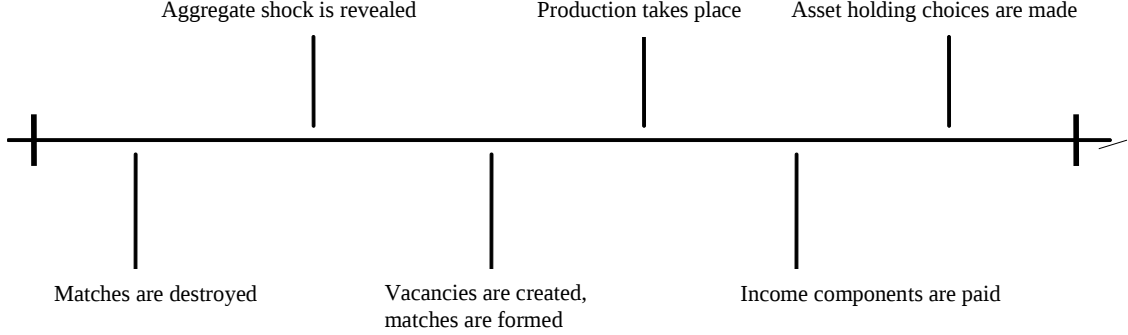


Figure 1: Sequence of events at date t .

Total employment at the end of date t is thus:

$$n_t = (1 - \rho) n_{t-1} + \lambda_t v_t. \quad (1)$$

The vacancy filling rate λ_t is related to the vacancy opening policy of the firm via the matching technology. The number of matches during time t is $m_t = \bar{m}(1 - \tilde{n}_t)^\chi v_t^{1-\chi}$, where $\bar{m} > 0$ is a scaling parameter and $\chi \in (0, 1)$ the elasticity of m_t with respect to the size of the unemployment pool, $1 - \tilde{n}_t$. Hence, the job-finding and vacancy-filling are given by:

$$f_t = \bar{m} \left(\frac{v_t}{1 - \tilde{n}_t} \right)^{1-\chi}, \quad \lambda_t = \bar{m}^{\frac{1}{1-\chi}} f_t^{-\frac{\chi}{1-\chi}}. \quad (2)$$

The representative intermediate-good firm maximises value and thus solves:

$$V(\tilde{n}_t, \mathbf{\Omega}_t) = \max_{v_t, \bar{k}_t} D_t + E_t(M_{t+1}^E V(\tilde{n}_{t+1}, \mathbf{\Omega}_{t+1})) \quad (3)$$

subject to $\tilde{n}_{t+1} = (1 - \rho)(\tilde{n}_t + \lambda_t v_t)$ and

$$D_t = p_{m,t} y_{m,t} - w_t n_t - r_{k,t} \bar{k}_t - c v_t, \quad (4)$$

where M_{t+1}^E is the stochastic discount factor of intermediate goods firms' owner (determined below), D_t the dividend paid to these owners. Labor is paid the real wage rate w_t and the real rental rate of capital services is $r_{k,t}$, both measured in units of the final good. From (4), the optimal demand for capital $\bar{k}_t$ must satisfy

$$r_{k,t} = p_{m,t} \phi \left(\frac{\bar{k}_t}{\exp^{\psi_t} n_t} \right)^{\phi-1} \quad (5)$$

The second choice variable of the firm is the number of vacant jobs to create, v_t , which by implication determines the equilibrium job-finding and vacancy-filling rates f_t and λ_t . From the first-order and envelop conditions associated with problem (3) and the constant-return to scale assumption, we obtain the following recursion on the job-finding rate:

$$f_t^{\frac{\chi}{1-\chi}} = \Upsilon \left((1 - \phi) p_{m,t} \left(\frac{\bar{k}_t}{\exp^{\psi_t} n_t} \right)^\phi - w_t \exp^{z_t} \right) + (1 - \rho) \mathbb{E}_t \left(M_{t,t+1}^E f_{t+1}^{\frac{\chi}{1-\chi}} \right), \quad (6)$$

where $\Upsilon = \bar{m}^{1/(1-\chi)}/\kappa_v$. Given n_{t-1} (inherited from the past) and f_t (endogenously determined in the current period), equation (6) gives the equilibrium number of vacancies, $v_t = (1 - (1 - \rho) n_{t-1}) (f_t/\bar{m})^{1/(1-\chi)}$.

2.1.2 Final good firms

The final good is produced by competitive firms which combine retail goods $i \in [0, 1]$ according to

$$y_t = \left(\int_0^1 y_t(i)^{(\theta-1)/\theta} di \right)^{\theta(\theta-1)}, \quad (7)$$

where $\theta > 1$ is the elasticity of substitution between any two retail goods. Let $P_t(i)$ denote the nominal price of retail good i , which final good producers take as given. Let P_t denote the price of the final good. The nominal profit of the representative final good producer firm is

$$P_t y_t - \int_0^1 P_t(i) y_t(i) di$$

The first order condition of the final good producer generates the following demand curve for retail good i

$$y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} y_t. \quad (8)$$

The following relationship between final good price and retail prices

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta} di \right)^{1/(1-\theta)} \quad (9)$$

is a direct consequence of (8) and the zero profit condition for final good producers.

2.1.3 Wholesale firms

There is a continuum of wholesale firms $i \in [0, 1]$. Firm i uses the intermediate good as an input, $x_t(i)$, in the production of good i , $y_t(i)$, according to

$$y_t(i) = x_t(i) - \kappa_y \exp^{z_t},$$

where $\kappa_y \exp^{z_t}$ is a fixed production cost measured in units of intermediate good.

The nominal period t profits of the retail firm i is

$$P_t(i)y_t(i) - P_{m,t}(y_t(i) + \kappa_y \exp^{z_t}).$$

Retailer i chooses $P_t(i)$ to maximize the present discounted value of future profits. However, we assume that in each period of time, a retail firm can reoptimize its price only with probability $1 - \alpha$, irrespective of the elapsed time since it last revised its price. If the firm cannot reoptimize its price, the latter is rescaled according to the simple revision rule

$$P_t(i) = (1 + \bar{\pi})^{1-\gamma}(1 + \pi_{t-1})^\gamma P_{t-1}(i),$$

where $\gamma \in (0, 1)$ measures the degree of indexation to the most recently available final good inflation measure $\pi_t = P_t/P_{t-1} - 1$ and $\bar{\pi}$ is steady state inflation.

2.2 Households

The economy has two types of agents: employers (in small mass $\nu > 0$), who have the ability to run firms and make a living out of the profits they collect, and workers (in mass 1), who supply inelastically one labour unit to employers if they are employed and none otherwise. There are frictions in the financial markets: first, idiosyncratic income risk is imperfectly insured; second, unemployed workers have limited ability to borrow against future income.

2.2.1 Employers

We assume that employers hold perfectly symmetric (private) equity portfolios of both intermediate goods and wholesale goods firms, as well as the capital of intermediate goods firms k_t .⁴ The representative patient household has utility defined by

$$\mathbb{E}_0 \sum_{t \geq 0} (\beta^E)^t \nu u \left(\frac{c_t^E - h \underline{c}_{t-1}^E}{\nu} \right) \quad (10)$$

where $\beta^E \in (0, 1)$ is the employers' subjective discount factor and c_t^E/ν is the individual consumption of an employer, $\underline{c}_{t-1}^E$ is the aggregate consumption of employers, and $h \in [0, 1)$ is the degree of external habit formation.

⁴We could of course allow employers to trade a stock market index aggregating the profits of these firms. Then, the price of this index would adjust endogenously and no trade would occur in equilibrium, leaving the dynamics of the model unaffected.

The representative patient household faces the nominal budget constraint

$$P_t(c_t^E + i_t + \eta(u_t)k_{t-1}) + A_t^E = P_t r_{k,t} u_t k_{t-1} + (1 + R_{t-1})A_{t-1}^E + \text{Profits}_t \quad (11)$$

where i_t denotes investment, u_t is the utilization rate of capital, k_{t-1} is the available stock of capital, Profits_t denotes total profits rebatted to the patient household, and A_t^E is quantity of bonds acquired at t , maturing at $t + 1$, and paying the nominal interest rate R_t at $t + 1$. Utilization entails a cost (measured in units of final good) $\eta(u_t)k_{t-1}$, which is assumed to be proportional to k_{t-1} and scaled down by the investment-specific shock. The function η is such that $\eta(1) = 0$ and $\eta'(\cdot), \eta''(\cdot) > 0$.

Physical capital accumulates according to

$$k_t = (1 - \delta)k_{t-1} + \left(1 - S\left(\frac{i_t}{i_{t-1}}\right)\right) i_t, \quad (12)$$

where the function S is an adjustment cost function such that $S(g_i) = S'(g_i) = 0$ where g_i is the strady-tate gross rate of growth of i_t , and $S''(\cdot) > 0$.

Employers' choice of physical investment k_t and capital utilisation u_t are standard and described in the technical appendix). Employers' holdings of nominal bonds obeys the Euler equation

$$E_t (M_{t+1}^E (1 + r_{t+1})) = 1, \quad (13)$$

where

$$M_{t+1}^E = \beta^E \frac{u' \left(\frac{c_{t+1}^E - h c_t^E}{\nu} \right)}{u' \left(\frac{c_t^E - h c_{t-1}^E}{\nu} \right)} \quad (14)$$

denotes employers' pricing kernel and $r_t \equiv (i_{t-1} - \pi_t) / (1 + \pi_t)$ is the (ex post) real return on nominal bonds.

2.2.2 Workers

There is a continuum of mass one of families of workers. Each family is itself composed of a continuum of members in mass 1, with every worker in the family randomly transiting between employment or unemployment. Employed workers are in number n_t (both in every family and in the aggregate) and earn the net wage income $w_t (1 - \tau_t)$, where τ_t is the social contribution rate. Unemployed workers are in number $1 - n_t$ and earn the unemployment benefit $b^u e^{\psi_t}$, with $0 < b^u e^{\psi_t} < w_t (1 - \tau_t)$. We assume that the unemployment insurance scheme is balanced, i.e.,

$$n_t w_t \tau_t = (1 - n_t) b^u e^{\psi_t}. \quad (15)$$

Workers' ability to smooth individual consumption in the face of idiosyncratic labour market shocks is limited, for two reasons. First, the unemployed have limited ability to borrow against future income. Second, idiosyncratic transitions across labour market statuses cannot be insured *ex ante* (except for the unemployment benefit b^u). In order to keep the analysis tractable, we adopt a minimal departure from the complete-market assumption, in the spirit of Lucas (1990). Namely, we think of each family as providing full consumption insurance (via the pooling of assets and goods) to *employed* workers. In contrast, workers who fall into unemployment are temporarily excluded from the family (which they leave with their fair share of the families assets) and stay so as long as they remain in that state, but they reintegrate the family once they leave unemployment. There is thus periodic re-insurance amongst workers. This device insures that employed workers' individual consumption and assets are independent of their individual history (that is, workers who have just left unemployment do not consume less or hold less assets than workers having been employed in the past). This property, coupled with the fact that unemployed workers face a borrowing limit that is tighter than the natural limit –so that there is a finite number of possible wealth levels for these workers–, will imply that the cross-sectional distribution of wealth amongst workers has a finite number of states. We start by describing unemployed workers' asset holding choices, and then turn to those of a typical family.

The unemployed. Denote by $\tilde{c}_t$ and $\tilde{a}_t$ the consumption and end-of-period assets of an unemployed worker, respectively (here, we use tildas to refer to individual variables). An unemployed worker solves

$$V^u(\tilde{a}_{t-1}, \Omega_t) = \max_{c_t} \left\{ u(\tilde{c}_t - h\tilde{c}_{t-1}) + \beta^W \mathbf{E}_t \left[(1 - f_{t+1}) V^u(\tilde{a}_t, \Omega_{t+1}) + f_{t+1} V^e(a_t^W, \Omega_{t+1}) \right] \right\}$$

subject to the budget constraint $\tilde{a}_t = b^u + (1 + r_t)\tilde{a}_{t-1} - \tilde{c}_t$, the borrowing constraint $\tilde{a}_t \geq \varsigma e^{\psi_t}$, $\varsigma \leq 0$, and the nonnegativity constraints $\tilde{c}_t > 0$. $\beta^W \in (0, 1)$, $\beta^W < \beta^E$, is the workers' subjective discount factor and $\tilde{c}_t$ is the aggregate per capita consumption of unemployed workers in the same reference group as that of the unemployed worker under consideration (details below). $V^u(\tilde{a}_{t-1}, \Omega_t)$ is the value function for an unemployed worker, while $V^e(a_t^W, \Omega_{t+1})$ is the value functions for the head of a family of employed workers. The latter depends on the family's next period assets a_t^W , which is described below.

Importantly, an unemployed worker is infinitesimally small relative to the overall size of the family. As a consequence, an unemployed worker's consumption choices $\tilde{c}_t$ will not affect

the family's wealth a_t^W , i.e.

$$\frac{\partial V^e(a_t^W, \Omega_{t+1})}{\partial \tilde{c}_t} = 0.$$

We will conjecture (and then verify the condition for) an equilibrium where unemployed workers face a binding borrowing constraint by the end of their first period of unemployment. Denote by $\tilde{a}_{t-1}^{eu}$ the asset wealth after having left the family. Under the conjectured equilibrium, the unemployed can be of two types only, with consumption levels

$$\tilde{c}_t^{eu} = b^u e^{\psi_t} - \varsigma e^{\psi_t} + (1 + r_t) \tilde{a}_{t-1}^{eu}$$

or

$$\tilde{c}_t^{uu} = b^u e^{\psi_t} - \varsigma e^{\psi_t} + (1 + r_t) \varsigma,$$

$\tilde{c}_t^{uu}$ is the consumption of an agent who is currently unemployed and was so in the previous period. This worker had bond holdings ςe^{ψ_t} in the previous period and thus earns $(1 + r_t) \varsigma e^{\psi_t}$ in period t . He also decides to hold ςe^{ψ_t} bonds at the end of period t . Similarly, $\tilde{c}_t^{eu}$ is the consumption of a worker transiting from employment to unemployment, taking $\tilde{a}_{t-1}^{eu}$ from the family and holding asset wealth ςe^{ψ_t} at the end of the period. Since this worker consumes any additional unit of beginning-of-period wealth, we have

$$\frac{\partial V^u(\tilde{a}_{t-1}^{eu}, \Omega_t)}{\partial \tilde{a}_{t-1}^{eu}} = (1 + r_t) u'(\tilde{c}_t^{eu} - h \tilde{c}_{t-1}^{eu}). \quad (16)$$

The employed. Employed are part of the family, which is run by a benevolent family head. The latter solves

$$V^e(a_{t-1}^W, \Omega_t) = \max_{c_t^e} \left\{ n_t u(\tilde{c}_t^e - h \tilde{c}_{t-1}^e) + \beta^W \mathbb{E}_t \left[(1 - s_{t+1}) V^e(a_t^W, \Omega_{t+1}) + s_{t+1} n_t V^u(\tilde{a}_t^{eu}, \Omega_{t+1}) \right] \right\}$$

where $\tilde{c}_t^e = c_t^e / n_t$ and c_t^e are the per-member and overall consumption levels of a typical family of employed workers, respectively, and a_t^W is end-of-period wealth of this family. Note that the family head does not value unemployed workers' current consumption (since these are excluded from the family, by assumption), but it does value the members' potential utility loss associated with them becoming unemployed in the next period. There will be $s_{t+1} n_t$ such members at date $t + 1$, hence the weight before V^u in the above Bellman equation.

The family's bond holdings a_t^W evolve according to

$$a_t^W = (1 - \tau_t) w_t n_t + (1 + r_t) \left[a_{t-1}^W - s_t n_{t-1} \tilde{a}_{t-1}^{eu} + \int \tilde{a}_{t-1}^{ue} dF(\tilde{a}_{t-1}^{ue}) \right] - c_t^e, \quad (17)$$

where $s_t n_{t-1} \tilde{a}_{t-1}^{eu}$ is the wealth taken away by members who became unemployed during date t and $\int \tilde{a}_{t-1}^{ue} dF(\tilde{a}_{t-1}^{ue})$ is the wealth brought back by workers who exited unemployment during date t , with cdf F . Since all such members have bond holdings ς and are in overall mass $f_t(1 - n_{t-1})$, it must be the case that

$$\int \tilde{a}_{t-1}^{ue} dF(\tilde{a}_{t-1}^{ue}) = f_t(1 - n_{t-1})\varsigma.$$

Importantly, the family head is supposed to choose a_t^W at the end of t , after the unemployed have left the family and $s_t n_{t-1} \tilde{a}_{t-1}^{eu}$ bond units have been taken away. This implies that

$$\frac{\partial a_t^W}{\partial a_{t-1}^W} = 1 + r_t.$$

Also, notice that in period $t+1$, a newly unemployed workers will take away $\tilde{a}_t^{eu} = a_t^W / n_t$. Substituting this value of $\tilde{a}_t^{eu}$ into (??) and rearranging, we find that the optimal holdings of nominal bonds by a family of employed workers is given by

$$\mathbb{E}_t (M_{t+1}^W (1 + r_{t+1})) = 1, \quad (18)$$

where

$$M_{t+1}^W = \beta^W \frac{(1 - s_{t+1})u'(\tilde{c}_{t+1}^e - h\tilde{c}_t^e) + s_{t+1}u'(\tilde{c}_{t+1}^{eu} - h\tilde{c}_t^{eu})}{u'(\tilde{c}_t^e - h\tilde{c}_{t-1}^e)} \quad (19)$$

This pricing kernel reflects the sensitivity of workers's asset holding decisions to both future aggregate conditions and their potential changes in idiosyncratic employment status. More specifically, while a currently employed worker enjoys marginal utility $u'(\tilde{c}_t^e - h\tilde{c}_{t-1}^e)$, it may either stay employed in the next period –with probability $1 - s_{t+1}$ – and hence enjoy marginal utility $u'(\tilde{c}_{t+1}^e - h\tilde{c}_t^e)$, or fall into unemployment –with probability s_{t+1} – and enjoy marginal utility $u'(\tilde{c}_{t+1}^{eu} - h\tilde{c}_t^{eu})$. Under incomplete consumption insurance, the latter is greater than the former, which motivates precautionary asset holdings by the family in excess of the borrowing limit (despite the fact that workers are impatient relative to employers).

2.2.3 Wage

As is now well understood, the presence of search frictions in the labour market implies that there exist a full bargaining set over which an employer and an employee find it mutually profitable to trade. However, the theory does not pin down the specific way in which the match surplus is shared among the parties. For the sake of tractability, we assume here the type of wage rule suggested by Hall (2005) and subsequently adopted by Blanchard and Gali

(2010); namely, i) we assume that all agents take the real wage as given, and ii) we make sure that this wage always remains within the relevant bargaining set (so that it is never the case that the two parties would mutually agree to change the given wage). Following Blanchard and Gali (2010), we assume the real wage to be unresponsive to demand shocks, i.e.,

$$w_t = we^{\psi_t}, \quad (20)$$

This specification implies that the real wage does not respond to monetary policy shocks –or, more generally, to any demand shock that is uncorrelated with productivity. Using a structural VAR approach and interpreting as demand shocks those who do not change output in the long run, Gali (2011) argues that the real wage response to supply shocks is positive (as is consistent with (20)), while the response to demand shocks is negative. This makes our specification in (20) conservative in that a positive real wage response to an contractionary monetary policy shock (as is implied by Gali’s estimates) would further contract firms’ surplus, thereby amplifying the impact of the shock relative to the constant wage specification.

2.3 Monetary authority

The Central Bank sets R_t according to the Taylor-like rule

$$\log \left(\frac{1 + R_t}{1 + \bar{R}} \right) = \rho_R \log \left(\frac{1 + R_{t-1}}{1 + \bar{R}} \right) + (1 - \rho_R) \left[a_\pi \log \left(\frac{1 + \pi_t^a}{1 + \bar{\pi}^a} \right) + a_y \log \left(\frac{y_t}{y_{t-4}} \right) \right] + \sigma_R \epsilon_{R,t}, \quad (21)$$

where $\epsilon_{R,t} \sim \text{Niid}(0, 1)$ is a nominal interest rate shock and

$$1 + \pi_t^a \equiv (1 + \pi_t)(1 + \pi_{t-1})(1 + \pi_{t-2})(1 + \pi_{t-3}). \quad (22)$$

2.4 Market clearing

The market for final goods must clear, taking into account the fact that some of the final goods being produced are used to pay vacancy costs, and that a more intensive use of the capital stock also uses final goods. Let c_t^W denote total workers’ consumption. Aggregating the heterogenous consumption levels above weighted by the corresponding population shares are rearranging, we find

$$\begin{aligned} c_t^W &= (1 - f_t) (1 - n_{t-1}) \tilde{c}_t^{uu} + s_t n_{t-1} \tilde{c}_t^{eu} + c_t^e \\ &= w_t n_t + (1 + r_t) [a_{t-1}^W + (1 - n_{t-1}) \varsigma] - [a_t^W + (1 - n_t) \varsigma] \end{aligned}$$

Market clear then implies

$$y_t = c_t^E + c_t^W + i_t + \eta(u_t)k_{t-1} + \kappa_v v_t. \quad (23)$$

where

$$\Delta_t y_t = (u_t k_{t-1})^\phi (e^{\psi_t} n_t)^{1-\phi} - \kappa_y$$

and $\Delta_t = \int_0^1 (P_t(i)/P_t)^{-\theta} di$ is a measure of price dispersion with dynamics

$$\Delta_t = \alpha \left(\frac{1 + \pi_t}{(1 + \bar{\pi})^{1-\gamma} (1 + \pi_{t-1})^\gamma} \right)^\theta \Delta_{t-1} + (1 - \alpha) \left(\frac{P_t^*}{P_t} \right)^{-\theta}.$$

Equivalently (by Walras law), the bond market must clear, i.e.,

$$a_t^E + a_t^W + (1 - n_t) \varsigma = 0. \quad (24)$$

2.5 Existence conditions

The equilibrium under consideration was constructed under the conjecture that unemployed workers faced a binding borrowing constraint from their first period of unemployment onwards. We must thus check that this condition holds ex post. This will indeed be the case whenever

$$u'(\tilde{c}_t^{eu}) > \beta^W \mathbb{E}_t \left\{ (f_{t+1} u'(c_{t+1}^e) + (1 - f_{t+1}) u'(\tilde{c}_{t+1}^{uu})) (1 + r_{t+1}) \right\}, \quad (25)$$

where the left hand side of this equation is the marginal utility of a worker who falls into unemployment in the current period and the right hand side his expected, appropriately discounted future marginal utility, given aggregate uncertainty and the idiosyncratic transitions (back to employment or still unemployed) that this worker may face in the next period.⁵ This existence condition is checked numerically below. Intuitively, the borrowing constraint is all the more likely to be binding that the left hand side of this equation is low. As long as $c^e > \delta$ (that is, the employed consume more than the long run unemployed), a high job finding rate or a high level of employed workers' consumption both raise expected future expected consumption, thereby causing unemployed workers to be willing to borrow against future income and making it more likely that the constraint will bind. The same is true of a higher value of δ . In contrast, a low value of δ implies that workers who fall into unemployment fear staying in that state in the next period, and are thus more likely to be willing hoard assets in excess of the borrowing limit.

⁵Note from (??)–(??) that $c_t^{uu} < c_t^{eu}$, and hence $u'(c_t^{uu}) > u'(c_t^{eu})$. This implies that if the borrowing constraint is binding for *eu* workers (those currently falling into unemployment), then it is binding for *all* unemployed workers.

3 Aggregate dynamics

3.1 Calibration

The period is a quarter. All variables are made stationary by multiplying the trending variables by $e^{-\mu_\psi}$, where μ_ψ is set to $\log(1.005)$. Trend inflation $\bar{\pi}$ is set to $1.02^{1/4} - 1$.

Preferences. In our baseline specification we assume a unit elasticity of intertemporal substitution and a habit formation parameter $h = 0.8$. Hence the instant utility function is

$$u(c_t^i - \underline{c}_t^i) = \log(c_t^i - 0.8\underline{c}_{t-1}^i),$$

where $\underline{c}_{t-1}^i$ is the individual consumption of a household who was in the same reference group as household i in the last period (e.g., a eu worker has utility $u(c_t^{eu}) = \log(c_t^{eu} - \underline{c}_{t-1}^{eu})$ at date t , where $\underline{c}_{t-1}^{eu}$ is the individual consumption of a typical eu worker at date $t - 1$). Employers' subjective discount factor determines the steady state gross real interest rate, $1 + r$. We target $r = 1.04^{1/4} - 1$, and consequently we set $\beta^E = e^{\mu_\psi} / (1 + r) = 0.9952$.

From (18)–(19) and the fact that $\beta^E = e^{\mu_\psi} / (1 + r)$, we can rewrite the steady state counterpart of the Euler equation for impatient households as follows:

$$\beta^W = \frac{\beta^E}{e^{\mu_\psi} [1 - s + s\tilde{c}_t^e / \tilde{c}_{t+1}^{eu}]}$$

We set β^W so that the proportional consumption fall associated with falling into unemployment be 20%, once trend growth has been removed (see, e.g., Challe and Ragot (2013) and the discussion therein). This gives $\beta^W = 0.9836$.

Labour market frictions. We first calibrate the steady state values of f , s (or, equivalently, ρ), and n . To compute f , we first compute monthly transition rates using the two-state model of Shimer (2005) over the 1948–2011 period. We then multiply the three consecutive monthly transition matrices belonging to the same quarter to obtain a quarter-to-quarter transition matrices over the whole period. The average quarter-to-quarter job-finding rate is 79%, to which the value of f is set. The average quarter-to-quarter job loss rate is $s = 0.047$, so we set $\rho = 0.047 / (1 - 0.79)$. This produces a mean unemployment rate of $1 - n = s / (f + s) = 5.65\%$. Following Gali (2011), the matching elasticity ξ is set to 0.5, while the unit hiring cost is set to 4.5% of the quartely real wage, i.e., $\kappa_v / \lambda = 0.045w$. The matching parameter $\bar{m}$ is normalised to 1. This implies that we must set the (detrended) real wage rate to 1.7732.

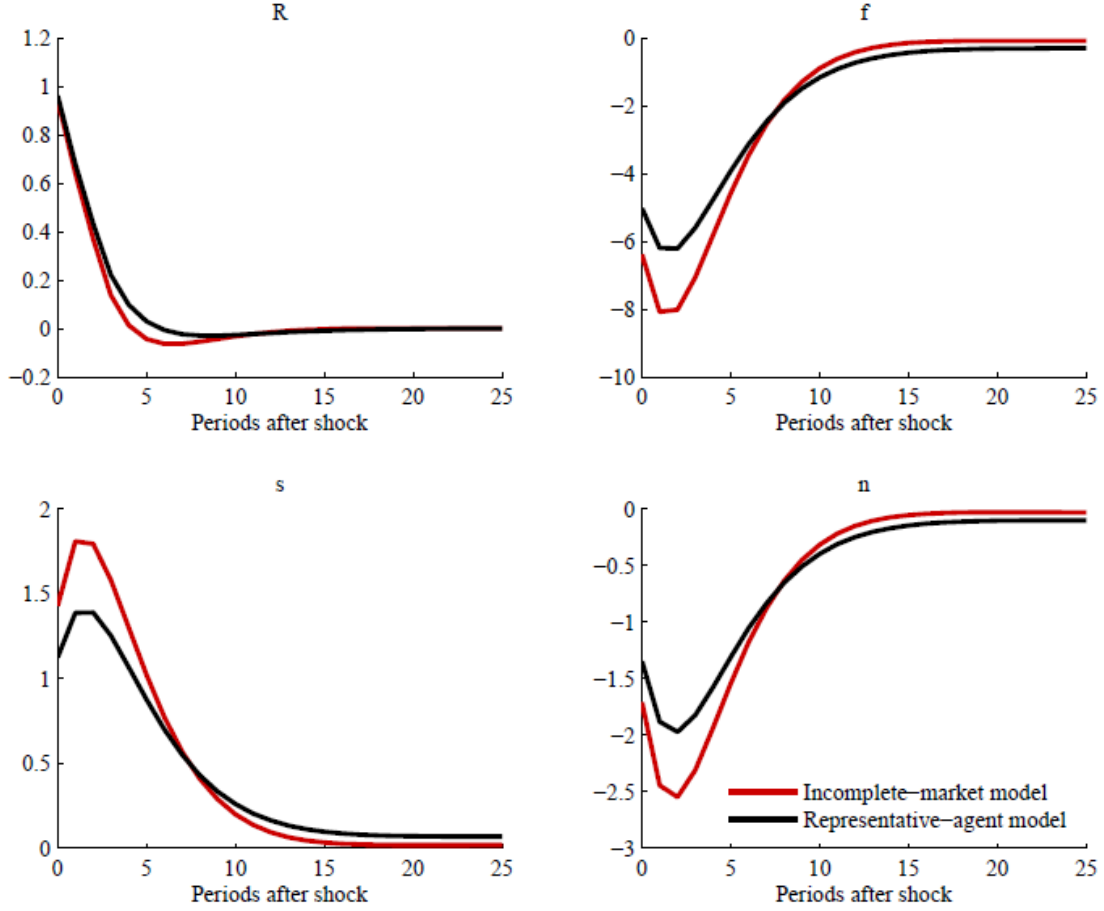
Insurance. We set the replacement ratio to 0.5, so that $b^u = 0.5w$. We assume that $\varsigma = -1$, which implies a borrowing capacity for the unemployed (ςe^{ψ_t}) slightly above the unemployment benefits ($b^u e^{\psi_t} = 0.89e^{\psi_t}$).

Production, pricing. Following Blanchard and Gali (2010), the cross-partial elasticity of substitution between good varieties is set to $\theta = 6$, while the share of wholesale goods firms not resetting prices optimally in a particular quarter is set to $\alpha = 0.75$. For those firms, we set γ (the parameter measuring the relative weight of steady state versus past inflation in the indexation rule) to 0.5.

Monetary policy rule. Finally, the elasticities of the policy reaction to inflation and the output gap are set to $a_\pi = 1.5$ and $a_y = 0.125$, respectively. The persistence of the nominal interest rate is set to $\rho_R = 0.8$.

3.2 Impulse-response analysis

Figures 2 and 3 display the economy's response to a monetary policy shock in the incomplete-market model versus the representative agent model. The latter economy is constructed by pooling workers and employers resources and by assuming that all agents share the same subjective discount factor (that is $\beta^W = \beta^E$, so that $r \simeq 1\%$ at the quarterly frequency). It is thus formally similar to the model of Walsh (2005) and Gali (2011), except for the simple wage-setting rule (20) and the absence of endogenous job destruction (which is present in Walsh, 2005). The incomplete-market specification illustrates the amplification mechanism emphasised in this paper. Following a monetary policy (i.e., demand) shock, labour market variables and workers' total demand for goods all respond much more to the initial monetary policy impulse (about twice as much), as the rise in idiosyncratic volatility and the fall in aggregate demand mutually reinforce each other. The fall in aggregate demand is driven by the fall in aggregate consumption, and more specifically by the fall in the consumption of precautionary savers (who raise their asset hoarding in the face of worsening labour market conditions). In contrast, investment fluctuations are buffered by the precautionary motive, which urges agents to save more during recessions.



Responses of the nominal interest rate (R), the job-finding rate (f), the job-loss rate (s) and the employment rate (n) to a monetary policy shock (ϵ_R)

4 Concluding remarks

In this paper, we have explored the joint impact of goods market (i.e., imperfect competition and price rigidities), labour market (i.e., search and matching) and asset market (i.e., incomplete insurance and borrowing constraint) frictions for the propagation of business cycle shocks. Nominal frictions generate a role for fluctuations in aggregate demand, while labour market frictions implies that such fluctuations are reflected in the job creation policy of the firms and hence in the idiosyncratic income risk faced by individuals. When this risk is imperfectly insured, countercyclical variations in idiosyncratic labour income risk endogenously trigger countercyclical changes in households precautionary assets, which in turn feed back to aggregate demand and labour market conditions. Under our baseline calibration, credit market frictions and the feedback loop that they generate amplify the impact of a typical

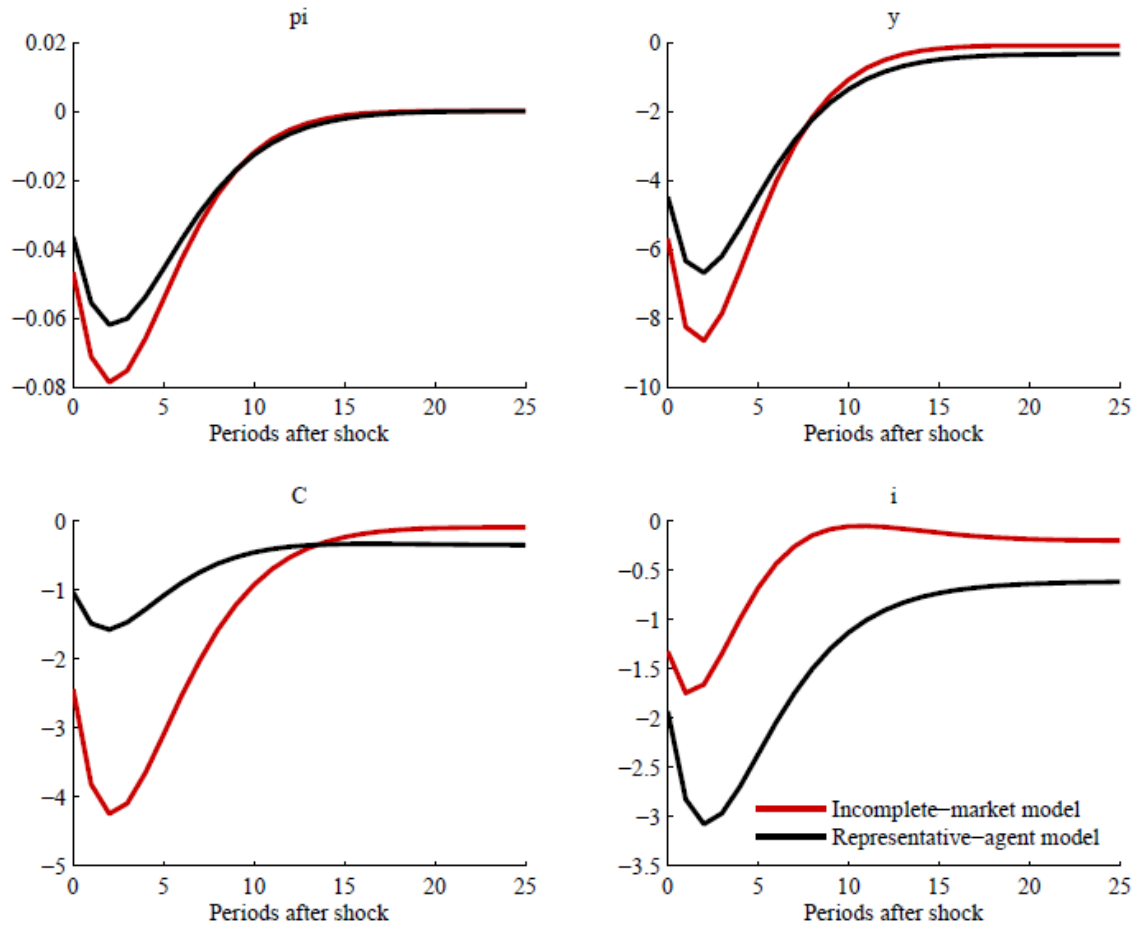


Figure 2: Responses of the inflation rate (π), output (y), aggregate consumption (C) and aggregate investment (i) to a monetary policy shock (ϵ_R)

monetary policy shock on labour market variables (the job-finding rate, the job loss rate and the unemployment rate) and the consumption demand of imperfectly insured agents. The stronger short-run response of aggregates to the shock under incomplete markets and borrowing constraints suggests that the interactions of the three frictions under consideration may have important implication for the optimal design of macroeconomic policies, since those may have to adequately stabilise precautionary savings to maximise welfare –rather than directly targeting output, employment and inflation.

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