A Unified Model of Structural Adjustments and International Trade: Theory and Evidence from China

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Abstract

In this paper, we first document the patterns of structural adjustments in Chinese manufacturing production and exports using a firm level data from China. We find that manufacturing production became more capital intensive while exports became more labor intensive from 1999 to 2007. To explain these patterns, we unify major international trade theories by embedding the Melitz (2003) model of heterogeneous firm into the Dornbusch-Fischer-Samuelson model with both Ricardian and Heckscher-Ohlin (1977, 1980) comparative advantage. We discuss the equilibrium patterns of production and international trade. Then we solve the model numerically and analyze other equilibrium properties. Finally we study the comparative statics to examine the effects of trade liberalization, capital deepening and technology changes.

Key Words: Structural Adjustments, Heterogeneous Firm, Comparative Advantage

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1 Introduction

In this paper we study the interactions between the changes in firm's distribution within a sector and resource reallocations across sectors. Using the firm level data in China from 1999 to 2007, we document seemingly puzzling data patterns: comparing the data in 2007 with that in 1999, Chinese manufacturing productions became more capital intensive. On the other hand, Chinese exports became more labor intensive.

Following Schott (2003), we define industries as "HO aggregates" and regroup firms into 100 industries according to their capital share. Comparing the data in 2007 with that in 1999, the share of firm numbers (the number of firms in an industry/total number of firms), the share of employment (industrial employment/total employment) and the share of output (industrial output/total output, output measured by value added or sales) all increase in capital intensive industries but decline in labor intensive industries. However, across industries, the share of exporting firm number (the number of exporting firms in an industry/total number of exporting firms) increases in labor intensive industries but decreases in capital intensive industries. Furthermore, firms in the labor intensive industries export a larger fraction of their total output while firms in the capital intensive industries export a smaller fraction of their total output.

China was clearly more capital abundant in 2007 than in 1999. According to the classical Heckscher-Ohlin theory, China should produce and export more capital intensive goods. Thus the changes in China's production structure are consistent with the classical HO theory, but the changes in the export structure seem to contradict the theory. To understand the seemingly puzzling data pattern, we introduce the firm heterogeneity into the HO framework, and provide an explanation: When China becomes more capital abundant, firms exit from the labor intensive sectors and enter the capital intensive sectors. Since exiting and entering firms are marginal firms and therefore less productive, they are less likely to be exporters. Thus, while *firm switching* makes the capital intensive industries expand and the labor intensive industries shrink, it increases the fraction of exporters in the labor intensive industries and reduces those in capital intensive industries. Moreover, trade liberalizations including joining WTO in 2001 strengthen China's existing comparative advantage in labor intensive industries and drive up their exports. At the same time, the new exporters in capital intensive sectors are not as competitive as foreign competitors and only export a little.

The discussion above highlights the interesting interactions between changes in firms' distribution within a sector and resource reallocations across sectors. To study such an interaction, we introduce Melitz-type of firm's heterogeneity into the DFS continuum goods framework of Ricardian and Heckscher-Olin model (Dornbusch, Fischer and Samuelson 1977, 1980). We show that there are two industrial factor intensity cut-offs: the most capital intensive industries and labor intensive industries are specialized by the capital abundant country and labor abundant country respectively; for industries with intermediate factor intensities, both country produce. In industries that a country specialize, we show that the export participation (measured by export probability or export intensity) remain *constant* and does not vary with industrial factor intensity. In industries that both countries produce, the export participation *decreases* with capital intensity in the labor abundant country while it *increases* with capital intensity in the capital abundant country. The Chinese data supports the theoretical predictions on labor abundant country.

Using the framework, we find the numerical solution and perform comparative statics on capital deepening, trade liberalization and technology changes. The goal is to disentangle the effects of these channels and shed light on the Chinese data patterns. We find that capital deepening and technology changes make productions and exports become more capital intensive in a labor abundant country: it produces and exports more in capital intensive industries and *vice versa* in labor intensive industries. However, trade liberalization makes productions and exports more labor intensive goods since its comparative advantage is strengthened. Given that we observe Chinese production became more capital intensive while export became more labor intensive, none of these comparative statics alone could explain what we observe in the Chinese data.

Our paper is closely related to four papers. Romalis (2004) introduces monopolistic competition into the DFS framework in a HO model. We extend his model by allowing firms to be heterogeneous within a sector. Bernard, Redding and Schott (2007) incorporate Melitz (2003) model into the two-sector HO framework. We extend their work to the continuum sectors of DFS framework. Comparing to these two papers, our model is one step closer to the data, and therefore allows us to quantitatively analyze Chinese firm level data. Lu (2010) also incorporates heterogeneous firm into a Heckscher-Ohlin model with multiple industries, and Fan et al (2011) combine DFS' Ricardian model with the Melitz type heterogeneous firm. Our model is more general than their papers in the sense that we endogenize both trade patterns and firm distribution across sectors. More importantly, we focus on the interactions between the changes in firm's distribution within a sector and resource reallocations across sectors overt time, which is new in the literature.

The rest of the paper are organized as follows. Section 2 presents the data patterns in the Chinese firm level data. Section 3 develops the model, and Section 4 contains the equilibrium analysis. We preform comparative statics and study the effect of trade liberalization, capital deepening and technology changes in Section 5. Section 6 concludes.

2 Motivating Evidences

In this section we present several stylized facts about the adjustments in the production and trade structure over time in China. The data we use is the Chinese Annual Industrial Survey. It covers all the State Own Enterprise (SOE) and non-SOEs with sales higher than 5 million RMB Yuan. The dataset provides information on balance sheet, profit and loss, cash flow statements of firms, and firm's identification, ownership, export, employment, capital stock, etc. Our focus is on the manufacturing firms (thus excludes utility and mining firms) which contribute more than 90% of the total Chinese manufacturing exports in the aggregate trade data. To clean the data, we follow Brandt *et al* (2011) and drop firms with missing, zero, or negative capital stock, export and value added, and only include firms with employment larger than 8. We also drop firms with the capital share larger than one or less than zero where the capital share is defined as: $1 - \frac{wage}{value\_added}$.[1] Since the focus of this paper is the changes overtime, we compare the data for 1999 and 2007.[2] The Statistics Summary of the data after cleaning is shown in Table 1.

Table 1: Statistical Summary of Main Variables

Variables	mean in 1999	mean in 2007
number of firms	117702	291473
revenue(¥*1,000*)	50808	117744
value_added(¥*1,000*)	14098	31942
newly_sales(¥*1,000*)	49187	115296
export(¥*1,000*)	8880	23896
number of employee	328	218
total profit(¥*1,000*)	1854	6804
wage(¥*1,000*)	3363	5417
profit/revenue	0.011	0.043
proportion of exporters	0.252	0.248
proportion of SOE	0.258	0.041
capital share	0.669	0.707

Notes: This is for the sample after data cleaning.

As we mentioned earlier, industries are defined as "HO aggregate" following Schott (2003). That is, we put all firms in the same year together and then regroup them according to their capital share. For example, firms with the capital share between 0 and 0.01 are

[1] Wage is defined as the sum of wage_payable, labor and employment insurance fee, and total employee benefits payable. In the 2007 data, there are also information about housing fund and housing subsidy, endowment insurance and medical insurance, and employee educational expenses. Adding these 3 variables would increase the average labor share but only slightly (from 0.293 to 0.308). To be consistent, we don't include them.

[2] We don't use year 2008 and years after because we don't have the complete data and the aftermath of the financial crisis is of great concern.

lumped together and defined as industry 1. In total, we have 100 industries.

Schott (2003) looks at the product level variations, while we investigate the variations at the firm level. From Table 2, we find there are indeed large variations of capital share within the 2 digit Chinese Industry Classification (CIC) of industry and significant difference in capital intensity between exporters and non-exporters. Interestingly, we find that except for Tobacco (industry 16) and Recycling (industry 43), capital share is significantly lower for exporters.[3] This is different from Alvarez and López (2005)'s finding that Chilean exporters are more capital intensive than non-exporters. It is in line with Bernard *et al's* (2007b) speculation that exporters in developing countries should be more labor intensive than non-exporters given their comparative advantage in the labor intensive goods.[4]

Fact 1: *Under the industry definition according to final end (CIC), there are large variations of capital shares within each industry. Exporters are less capital intensive than non-exporters (exceptions are tobacco and recycling industry in 2007).*

2.1 Production Structures

This subsection describes how the overall production structures change between 1999 and 2007. In the figures below, industries are defined according to firms' capital share and we regroup firms into 100 industries. The horizontal axis is the industry index and higher numbers correspond to higher capital share.

Fact 2: *Compared with 1999, the overall Chinese production became more capital intensive in 2007.*

[3] On average, exporters are less capital intensive than non-exporters for all firms. The gap is larger in 2007 than 1999.

[4] For the same data, Ma *et al* (2011) use capital labor ratio (or capital wage payment ratio) as indicator of factor intensity. They also find Chinese exporters are less capital intensive than non-exporters. Based on transaction data, they find exporters choose to produce more labor intensive products which is consistent with the comparative advantage of China. Thus our finding is consistent with their findings.

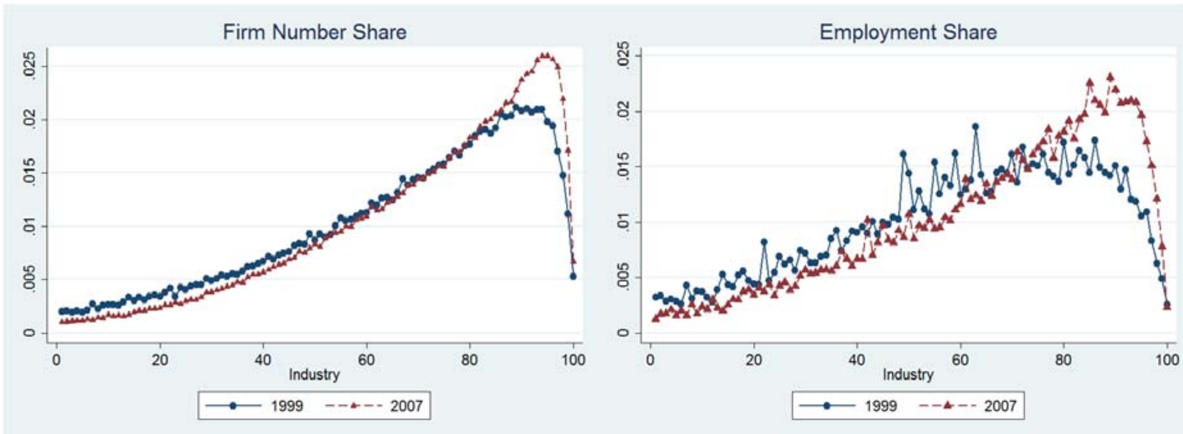


Figure 1: *Firm Number Share and Employment Share*

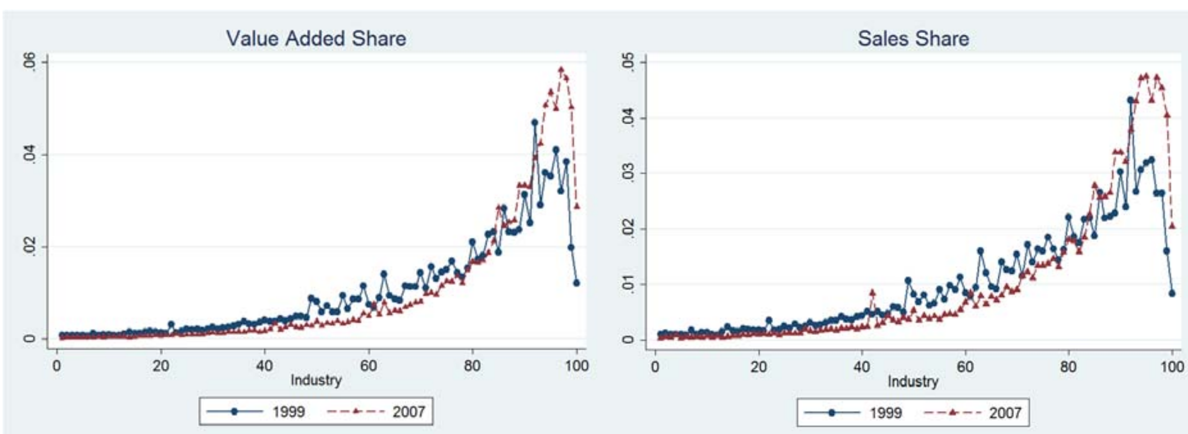


Figure 2: *Value Added Share and Sales Share*

Table 2: Captial Share of Firms in 2007

2 digit industry code	description	Capital Share of non-exporters	Std. Dev. of capital share for non-exporters	Capital Share for exporters	Std. Dev. of capital share for exporters
13	Processing of Food from Agriculture	0.83	0.18	0.76	0.21
14	Foods	0.76	0.20	0.71	0.22
15	Beverages	0.80	0.18	0.78	0.18
16	Tobacco	0.74	0.19	0.90	0.11
17	Textile	0.72	0.20	0.63	0.22
18	Wearing Apparel	0.60	0.24	0.51	0.24
19	Leather, Fur, Feather	0.64	0.25	0.53	0.23
20	Timber, Wood,etc	0.74	0.20	0.69	0.21
21	Furniture	0.69	0.23	0.56	0.23
22	Paper and Paper Products	0.73	0.19	0.65	0.22
23	Printing & Recording Media	0.67	0.21	0.59	0.22
24	Culture, Education related products	0.64	0.24	0.54	0.24
25	Processing of Petroleum, Coking & Nuclear Fuel	0.85	0.16	0.78	0.20
26	Raw Chemical Materials & Chemical Products	0.79	0.19	0.75	0.19
27	Medicines	0.78	0.19	0.74	0.19
28	Chemical Fibers	0.80	0.17	0.77	0.19
29	Rubber	0.73	0.21	0.61	0.23
30	Plastics	0.72	0.21	0.60	0.23
31	Non-metallic Mineral Products	0.74	0.20	0.63	0.22
32	Smelting and Pressing of Ferrous Metals	0.82	0.17	0.82	0.15
33	Smelting and Pressing of Non-ferrous Metals	0.82	0.18	0.78	0.19
34	Metal Products	0.71	0.21	0.61	0.22
35	General Purpose Machinery	0.72	0.20	0.65	0.20
36	Special Purpose Machinery	0.72	0.21	0.63	0.21
37	Transport Equipment	0.70	0.21	0.65	0.21
39	Electrical Machinery and Equipment	0.73	0.21	0.61	0.23
40	Communication Equipment, Computers	0.65	0.23	0.58	0.25
41	Measuring Instruments and other Machines	0.69	0.22	0.56	0.23
42	Artwork and Other Manufacturing	0.66	0.23	0.57	0.24
43	Recycling and Disposal of Waste	0.79	0.21	0.81	0.18

Notes: This is the 2-digit industry definition from Chinese National Bureau of Statics.

A direct evidence shown in Table 1 is that the average of capital share is 0.669 in 1999 and 0.707 in 2007. Thus the aggregate production became more capital intensive.[5] We also find

[5] Thus the overall production is very concentrated on capital intensive industries. Hsieh and Klenow (2009) point out that labor share is significantly less than aggregate labor share in manufacturing reported

other pieces of evidence for this change in the production structure using different measures. We first look at the number of firms and labor employment for each industry. As shown in Figure 1, during 1999-2007, more firms are producing in the capital intensive industries while fewer firms are producing in the labor intensive industries. At the same time, workers are moving out of labor intensive industries into more capital intensive industries. There is a significant reallocation of labor into the capital intensive industries. In terms of the output (Figure 2), we find that firms in the capital intensive industries account for a larger fraction of value added and sales in 2007 than in 1999. In Table 3, we compute the share of firms with a capital share higher than the average capital share in 1999. Clearly, the production structures become more capital intensive in 2007.

Table 3: Structural Adjustment of Production

Variable	firm number share	employment share	value added share	sales share
2007	0.648	0.585	0.860	0.860
1999	0.588	0.459	0.744	0.706
Difference	0.061	0.126	0.116	0.154

Notes: The numbers in the 1st and 2nd row are the corresponding share for firms with capital share higher than the average capital share in 1999 (0.669). The 3rd row is the difference between 2007 and 1999 (2007 minus 1999).

We also compare the labor productivity between the two years in Figure 3. Labor productivity is measured in terms of real value added per worker to make it comparable over years. Real value added is calculated using the input and output pricing index constructed by Brandt *et al* (2011). From the left panel, the labor productivity is higher for the capital

in the Chinese input-output tables and the national accounts (roughly 50%). They argue that it could be explained by non-wage compensation and assume it a constant fraction of a plant's wage compensation and adjust it to be the same as aggregate reports. Since we only care about the distribution, a constant adjustment would not help thus we simple use the original value.

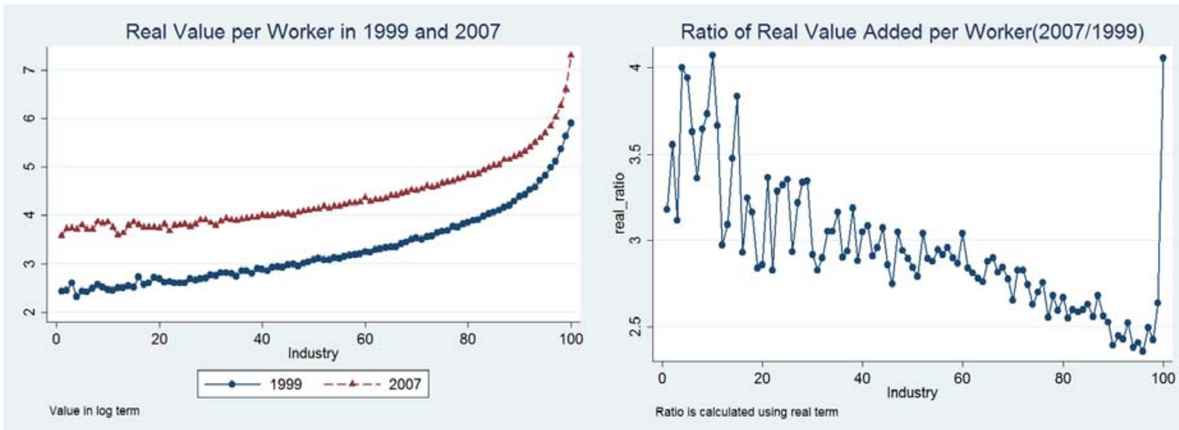


Figure 3: *Labor Productivity*

intensive industries and it increases from 1999 to 2007 for all industries. From the right panel, it is clear that the labor productivity increases more in the labor intensive industries.

Fact 3: *The magnitude of labor productivity growth from 1999 to 2007 decreases with capital intensity; that is, labor productivity grows faster in labor intensive industries.*

2.2 Trade Structure

In this subsection, we focuses on how the trade structure changes over time. The most important findings are:

Fact 4: *From 1999 to 2007, the exporting firm number share became more labor intensive.*

Fact 5: *Export participation (measured by fraction of exporters and sales exported) increases in labor intensive industries while the opposite is true in capital intensive industries.*

In Figure 4, we plot the export share in terms of firm number and value of export. From the left panel, we find the number share of exporters decreases slightly in the capital intensive industries. From the right panel, we find the value share of exports across industries is more

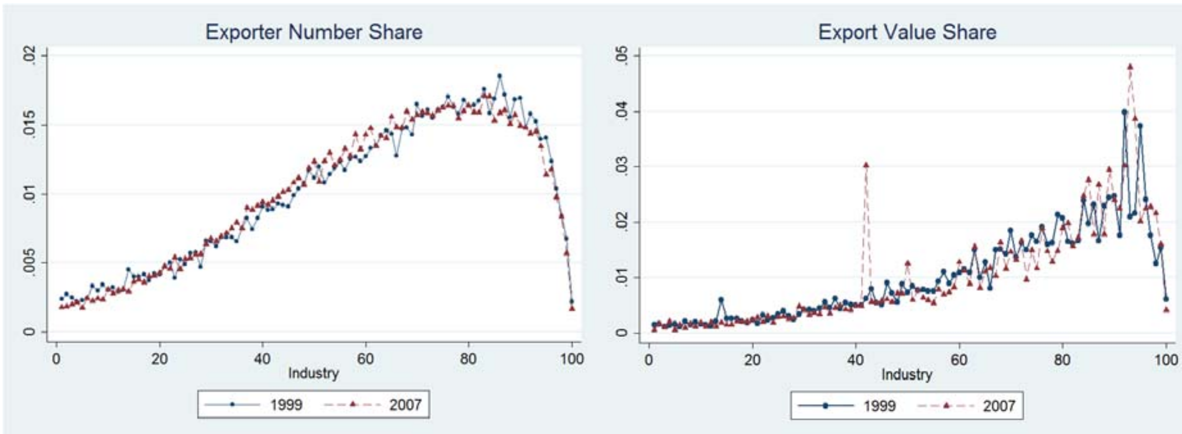


Figure 4: *Export Share in Terms of Firm Number and Value*

or less the same for both years. In Figure 5, we focus on how exports change *within* each industries. From the left panel, we find the proportion of exporters in 2007 is higher than 1999 in the labor intensive industries while the opposite is true for the capital intensive industries. In terms of the fraction of sales from exporting, we find it increases in general over time but more so for the labor intensive industries. In fact, for the most capital intensive ones, it decreases.

The messages from Figure 4 and Figure 5 can also be summarized by Table 4. By comparing with it with Table 3, we find a **puzzling pattern.** The production clearly became more capital intensive in 2007 than 1999 (if measured by export value share, the difference is +0.013 between 2007 and 1999 while the difference of total sales share +0.154 in Table 3). However, exports did not become as capital intensive as production does. In fact there is evidence that exports become more labor intensive (if measured by export number share difference, it is -0.018 versus firm number share difference +0.061 in Table 3). As we have said in the introduction, it is puzzling in the sense that from the standard trade theory, we expect the exports also become more capital intensive when the production becomes more capital intensive.

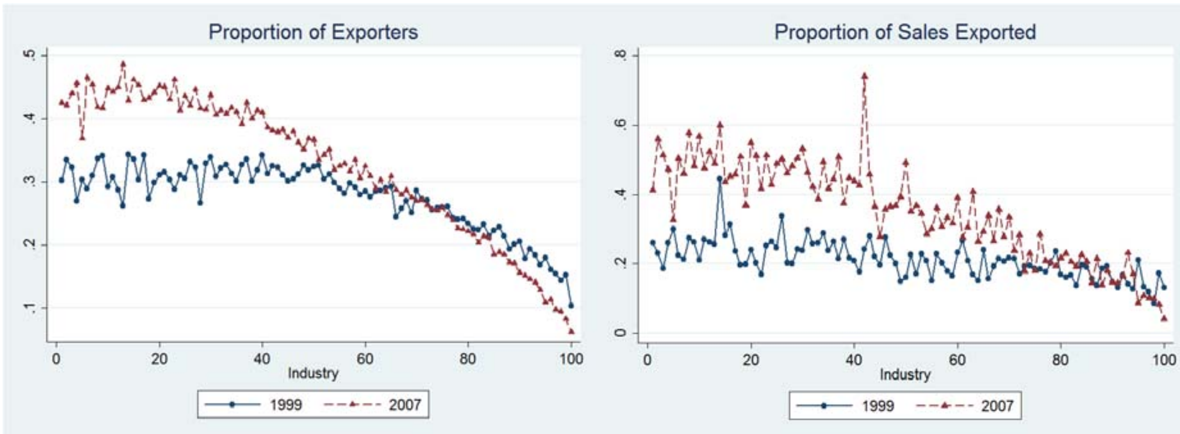


Figure 5: Export Participation

Our finding that Chinese exports didn't become more capital intensive seems to contradict existing works on the rising sophistication of Chinese export (Schott 2008, Wang and Wei 2010). Though China might expand its exports by increasing the extensive margin on more capital intensive industries, there is no guarantee that the overall export value share or exporter number share in the capital intensive industries also increase. If more firms became exporters in labor intensive and their export value increased more, the overall Chinese exports could indeed become more labor intensive. In fact, Schott (2008) finds that though Chinese exports overlap more and more with OECD countries, they also become cheaper and cheaper in terms of the unit value. A possible explanation he suggests is that firms produce according to their comparative advantage and focus on their core competencies. This view is elaborated by Ma *et al* (2011) where they focus on the *product switching* within firms while we focus on the *firm switching* within and across industries.

In Figure 6, we also compare the ages of firms over years. From the left panel, we find that exporters are on average younger than others for all industries. However, from the right panel, we find that exporters are as old as others in the labor intensive industries but significantly older than nonexporters in the capital intensive industries. On average, the capital intensive firms are younger, and firms in 2007 are younger than firms in 1999, indicating that lots of new firms have come in. We will look at the firm entry and exit in the next subsection.

Fact 6: *Firm age decreases with the capital intensity for both years. In 1999, exporters were younger than non-exporters. However, exporters became significantly older than non-exporters in the capital intensive industries in 2007.*

Table 4: Structural Adjustment of Export

Variable	exporter number share	export value share	average fraction of exporter
2007	0.487	0.667	0.194
1999	0.505	0.654	0.217
Difference	-0.018	0.013	-0.023

Notes: The numbers in the 1st and 2nd row are the corresponding share for firms with capital share higher than the average capital share in 1999 (0.669). The 3rd row is the difference between 2007 and 1999 (2007 minus 1999).

2.3 Entry and Exit

In this subsection we look at the firm entry and exit. [6] Entry firms are defined as those appear in 2007 but not in 1999 and *vice versa* for exit firms. An entrant exporter is defined as an entry firm that exports or a continuing firm that started to export in 2007. An export exit is an exit firm that exported in 1999 or a continuing firm that stopped exporting in 2007. Figure 7 plots the ratio of the number for the two types of firms. From the left panel, we find that there are relatively more firms entering the capital intensive industries than the labor intensive industries. From the right panel, we find that the ratio of entry and

[6] The concept of entry and exit is different from the literature since we are comparing two distant years. Another concern is that this might not be real entry or exit since the survey is censored at sales above 5 million Yuan.

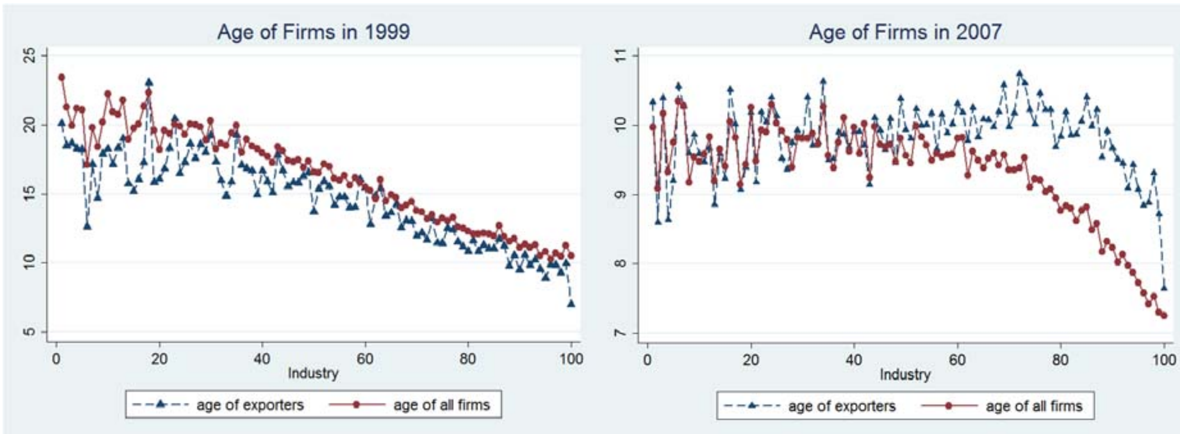


Figure 6: Firm Ages

exit exporters has an inverse "U" shape: it first increases and declines afterwards with the capital intensity. This finding is summarized as follows:

Fact 7: *The ratio of firm entry and exit increases with capital intensity while exporting firm entry exit ratio firstly increases then decreases with capital intensity.*

In principle, firm entry and exit could explain much of the facts above. For example, a significant entry in capital intensive industries naturally leads to an increase in the firm number share and employment share of the capital intensive industries, as demonstrated in Figure 1 and Figure 2. And if the entry firms are less likely to export, then the proportion of exporters should decrease in the capital intensive industries given more entries in those industries. And since for the labor intensive industries the entry of exporter is greater than the overall entry, we expect the fraction of exporters would increase in the labor intensive industries. Finally, more firms entering the capital intensive industries will drive down the average age. Relatively older exporters in the most capital intensive industries means that the age of exporters in the capital intensive industries will remain higher.

However, why firm entry and exit demonstrate such a pattern is still a question. Is it caused by capital deepening or trade liberalization? Moreover, the figures above indicate a

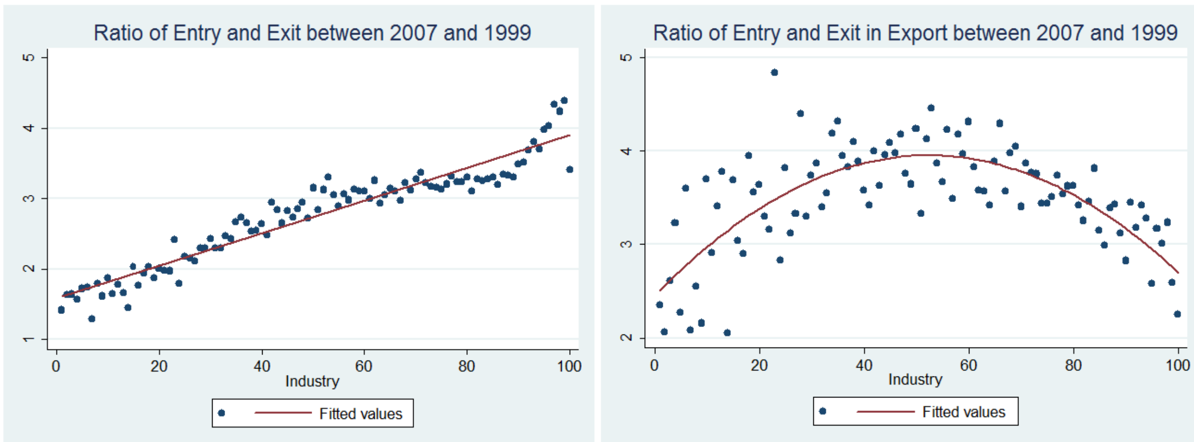


Figure 7: *Firm Entry and Exit*

key role played by factor intensity. In the section below, we will present a model with firm heterogeneity and multi-sectors to examine the empirical data patterns.

3 Model Set Up

Our model incorporates heterogenous firms (Melitz 2003) into a Ricardian and Heckscher-Ohlin theory with a continuum of industries (Dornbusch, Fisher and Samuelson 1977, 1980). There are two countries: North and South. We assume the home country to be South. The two countries only differ in their technology and factor endowment. Without lost of generality, we assume that home country is labor abundant, that is: $L/K > L^*/K^*$, and has Ricardian comparative advantage in more labor intensive industries. [7] There is a continuum of industries z on the interval of $[0, 1]$. The index z is also the industry's capital intensity. A higher z stands for a higher capital intensity. Each industry is inhabited by heterogeneous firms which produce different varieties of goods and sell in a market with monopolistic competition.

[7] Variables with "*" are foreign country (North country) variables. To simplify the notation, we omit it except where important.

3.1 Demand Side

The economy is inhabited by a continuum of identical and infinitely lived households that can be aggregated into a representative household. The representative household's preference over different goods is summarized by the following Cobb-Douglas utility function:

$$U = \int_0^1 b(z) \ln Q(z) dz, \quad \int_0^1 b(z) dz = 1$$

where $b(z)$ is the expenditure share on each industry and $Q(z)$ is the lower-tier utility function over the consumption of individual varieties $q_z(\omega)$ given by the following CES aggregation. $P(z)$ is the dual price index of $Q(z)$ defined over the price of different varieties $p_z(\omega)$

$$Q(z) = (\int_{\omega \in \Omega_z} q_z(\omega)^\rho d\omega)^{1/\rho}, P(z) = (\int_{\omega \in \Omega_z} p_z(\omega)^{1-\sigma} d\omega)^{1/(1-\sigma)}$$

Here Ω_z is the set of varieties available for industry z and $0 < \rho \leq 1$ so that the elasticity of substitution $\sigma = \frac{1}{1-\rho} > 1$. The aggregates can be used to derive the demand function for individual varieties

$$q_z(\omega) = Q(z)(\frac{p_z(\omega)}{P(z)})^{-\sigma} \tag{3.1}$$

3.2 Production

Following the standard assumptions of Melitz(2003), we assume that production incurs a fixed cost each period which is the same for all firms in the same industry, and the variable cost varies with the firm productivity. Firm productivity is denoted as $A(z)\varphi$ where $A(z)$ is a common component for all firms in an industry z, while φ, the heterogeneous productivity, is drawn randomly by firms from a distribution $G(\varphi)$. Following Romalis (2004) and Bernard *et al* (2007a), we assume that the fixed cost is paid using capital and labor with the same factor intensity as in the production in that industry. Specifically, we assume that the total cost function is:

$$\Gamma(z,\varphi) = (f_z + \frac{q(z,\varphi)}{A(z)\varphi}) r^z w^{1-z} \tag{3.2}$$

And we assume that the relative industry specific productivity for home and foreign $\varepsilon(z)$ is:

$$\varepsilon(z) \equiv \frac{A(z)}{A^*(z)} = \lambda A^z, \lambda > 0,\ A > 0 \tag{3.3}$$

Here λ is a parameter capturing the absolute advantage of home country: a higher λ means that home country has a higher relative industry specific productivity for all industries. A is the parameter capturing the comparative advantage. If $A > 1$, home country is relatively more productive in the more capital intensive industries and thus has Ricardian comparative advantages in these industries. If $A = 1$, then $\varepsilon(z)$ doesn't vary with z and there is no role for Ricardian comparative advantage. Given our assumption that Home has Ricardian comparative advantage in the more labor intensive industries, we have $0 < A < 1$.

The presence of a fixed cost implies that each firm produces only one variety. The profit maximization implies that the equilibrium price is a constant mark-up over the marginal cost. Trade is costly and firms need to ship τ units of goods for 1 unit of goods to arrive in the foreign market. This is the standard "iceberg cost" assumption. Then we have,

$$p_{zx}(\varphi) = \tau p_{zd}(\varphi) = \tau \frac{r^z w^{1-z}}{\rho A(z)\varphi} \tag{3.4}$$

where $p_{zx}(\varphi)$ and $\tau p_{zd}(\varphi)$ are the exporting and domestic price respectively. Given the pricing rule, a firm's revenues from the domestic and foreign market are:

$$\begin{aligned} r_{zd}(\varphi) &= b(z)R(\frac{\rho A(z)\varphi P(z)}{r^z w^{1-z}})^{\sigma-1} && (3.5)\\ r_{zx}(\varphi) &= \tau^{1-\sigma}(\frac{P(z)^*}{P(z)})^{\sigma-1}(\frac{R^*}{R}) r_{zd}(\varphi) && (3.6)\end{aligned}$$

Where R and R^* are the aggregate revenue for home and foreign respectively. Then the revenue of firms are:

$$r_z(\varphi) = \begin{cases} r_{zd} & \text{if it sells only domestically} \\ r_{zx} + r_{zd} & \text{if it exports} \end{cases}$$

For firms that export, they need to pay a per-period fixed cost $f_{zx} r^z w^{1-z}$ which requires both labor and capital. Therefore, the firms' profits could be divided into portions earned from domestic and foreign market:

$$\begin{aligned} \pi_{zd}(\varphi) &= \frac{r_{zd}}{\sigma} - f_z r^z w^{1-z} \\ \pi_{zx}(\varphi) &= \frac{r_{zx}}{\sigma} - f_{zx} r^z w^{1-z} \end{aligned} \tag{3.7}$$

So the total profit is given by:

$$\pi_z(\varphi) = \pi_{zd}(\varphi) + \max\{0, \pi_{zx}(\varphi)\} \tag{3.8}$$

Then a firm with the productivity φ produces if its revenue at least covers the fixed cost, that is $\pi_{zd}(\varphi) \geq 0$, and it exports if $\pi_{zx}(\varphi) \geq 0$. We thus defines the **zero-profit productivity cut-off** $\bar{\varphi}_z$ and **costly trade zero profit productivity cut-off** $\bar{\varphi}_{zx}$ which satisfy:

$$r_{zd}(\bar{\varphi}_z) = \sigma f_z r^z w^{1-z} \tag{3.9}$$

$$r_{zx}(\bar{\varphi}_{zx}) = \sigma f_{zx} r^z w^{1-z} \tag{3.10}$$

Using the two equations above and equation (3.5) (3.6), we could derive the relationship between the two productivity cut-offs:

$$\bar{\varphi}_{zx} = \Lambda_z \bar{\varphi}_z, \text{ where } \Lambda_z = \frac{\tau P(z)}{P(z)^*} \left(\frac{f_{zx} R}{f_z R^*}\right)^{\frac{1}{\sigma-1}} \tag{3.11}$$

$\Lambda_z > 1$ implies a selection into the export market: only the most productive firms export. The empirical literature strongly supports selection into market and we focus on

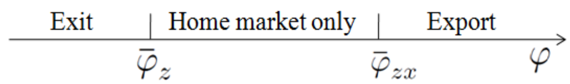


Figure 8: Production and Trade Pattern

parameters where exporters are always more productive following Melitz (2003) and Bernard *et al* (2007a).[8] Then the firms' decision on production and exporting are shown in Figure 8. For firms that enter in a period, a fraction $G(\bar{\varphi}_z)$ of them exit upon entry since they do not earn positive profit at all. And $1 - G(\bar{\varphi}_{zx})$ fraction of firms export since they draw sufficiently high productivity and earn positive profit from both domestic and foreign sales. As for firms whose productivity is between $\bar{\varphi}_{zx}$and $\bar{\varphi}_z$, they only sell in domestic market. So the *ex ante* probability of exporting conditional on successful entry is

$$\chi_z = \frac{1 - G(\bar{\varphi}_{zx})}{1 - G(\bar{\varphi}_z)} \tag{3.12}$$

3.3 Free entry

If a firm does produce, it faces a constant probability δ of bad shock in every period that would force it to exit. The steady-state equilibrium is characterized by a constant mass of firms entering an industry M_{ez} and constant mass firms producing M_z.Then in the steady state equilibrium, the mass of firms that enter must equal to the firms that die:

$$(1 - G(\bar{\varphi}_z))M_{ez} = \delta M_z \tag{3.13}$$

In an equilibrium with a positive production, we require that the value of entry V_z equals to the cost of entry: $f_{ez}r^z w^{1-z}$. We assume that the entry cost $f_{ez}r^z w^{1-z}$ also uses

[8] Lu(2010) explore the possibility that $\Lambda_z < 1$ and documents that in the labor intensive sectors of China, exporters are less productive. But our own empirical findings in the following section provides little support that. In fact, according to Dai *et al* (2011), Lu's result is solely driven by processing exporters. And using TFP as productivity measure instead of value added per worker, even including processing exporters still support that exporters are more productive.

capital and labor. The expected profit of entry V_z comes from two parts: (1) the *ex ante* probability of successful entry times the expected profit from domestic market until death; (2) *ex ante* probability of exporting times the expected profit from the export market until death. Then we have the following free entry condition

$$V_z = \frac{1 - G(\bar{\varphi}_z)}{\delta}(\pi_{zd}(\widehat{\varphi}_z) + \chi_z \pi_{zx}(\widehat{\varphi}_{zx})) = f_{ez} r^z w^{1-z} \tag{3.14}$$

where $\pi_{zd}(\widehat{\varphi}_z)$ and $\chi_z \pi_{zd}(\widehat{\varphi}_{zx})$ are the expected profitability of a successful entry. And $\widehat{\varphi}_z$ is the average productivity of all producing firms while $\widehat{\varphi}_{zx}$ is the average productivity of all exporting firms in industry z. They are defined as follows:

$$\begin{aligned}
\widehat{\varphi}_z &= [\frac{1}{1 - G(\bar{\varphi}_z)} \int_{\bar{\varphi}_z}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi]^{\frac{1}{\sigma-1}} \\
\widehat{\varphi}_{zx} &= [\frac{1}{1 - G(\bar{\varphi}_{zx})} \int_{\bar{\varphi}_{zx}}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi]^{\frac{1}{\sigma-1}}
\end{aligned} \tag{3.15}$$

Combining with the zero profit condition (3.9), (3.10), we have (3.16) below which determines the two productivity cut-offs with the equation (3.11).

$$\frac{f_z}{\delta} \int_{\bar{\varphi}_z}^{\infty} [(\frac{\varphi}{\bar{\varphi}_z})^{\sigma-1} - 1] g(\varphi) d\varphi + \frac{f_{zx}}{\delta} \int_{\bar{\varphi}_{zx}}^{\infty} [(\frac{\varphi}{\bar{\varphi}_{zx}})^{\sigma-1} - 1] g(\varphi) d\varphi = f_{ez} \tag{3.16}$$

3.4 Market Clearing

In equilibrium, we require that the sum of domestic and foreign spending on domestic varieties equals to the value of domestic production (total industry revenue, R_z) for every industry in both countries:

$$R_z = b(z) R M_z (\frac{p_{zd}(\widehat{\varphi}_z)}{P(z)})^{1-\sigma} + \chi_z b(z) R^* M_z (\frac{\tau p_{zd}(\widehat{\varphi}_{zx})}{P(z)^*})^{1-\sigma} \tag{3.17}$$

where the price index $P(z)$ is given by the equation below. R_z^* and $P(z)^*$ follow symmetric definitions.

$$P(z) = [M_z(p_{zd}(\widehat{\varphi}_z))^{1-\sigma} + \chi_z^* M_z^*(\tau p_{zd}(\widehat{\varphi}_{zx}^*)^*)^{1-\sigma}]^{\frac{1}{1-\sigma}} \tag{3.18}$$

The factor market clearing condition is:

$$\begin{aligned} L &= \int_0^1 l(z)dz,\ L^* = \int_0^1 l^*(z)dz \\ K &= \int_0^1 k(z)dz,\ K^* = \int_0^1 k^*(z)dz \end{aligned} \tag{3.19}$$

Before we proceed, we make a few assumptions so as to simplify the algebra. First, we assume that the productivity distribution is Pareto and the density function is given by

$$g(\varphi) = a\theta^a \varphi^{-(a+1)}, a > \sigma + 1$$

where θ is a lower bar of productivity: $\varphi \geq \theta$.Second, we assume that the coefficients of fixed costs are the same for all industries:[9]

$$f_z = f_{z'},\ f_{zx} = f_{z'x},\ f_{ez} = f_{ez'}, \forall z \neq z'.$$

Finally, we assume that the expenditure share $b(z)$ is the same for all industries at home and abroad, that is:

$$b(z) \equiv b(z^{'}), \forall z \neq z'.$$

[9] f_z , f_{ez}, f_{zx} could still differ from each other.

3.5 Equilibrium

The equilibrium consists of the vector of $\{\bar{\varphi}_z,\ \bar{\varphi}_{zx},\ P(z),\ p_z(\varphi),\ p_{zx}(\varphi),\ r,\ w,\ R,\ \bar{\varphi}_z^*,\ \bar{\varphi}_{zx}^*,$ $P(z)^*,\ p_z(\varphi)^*,\ p_{zx}(\varphi)^*,\ r^*,\ w^*,\ R^*\}$ for $z \in [0, 1]$. The other endogenous variables are given by these variables. The equilibrium vector is determined by the following conditions for each country:

(a) Firms' pricing rule (3.4) for each industry and each country;

(b) Free entry condition (3.14) and relationship between zero profit productivity cut-off and costly trade zero profit productivity cut-off (3.11) for each industry and both countries;

(c) Factor market clearing condition (3.19);

(d) The pricing index (3.18) implied by consumer and producer optimization;

(e) The goods market clearing condition of world market (3.17).

Proposition 1 *There exists a unique equilibrium given by* $\{\bar{\varphi}_z,\ \bar{\varphi}_{zx},\ P(z),\ p_z(\varphi),\ p_{zx}(\varphi),$ $r,\ w,\ R,\ \bar{\varphi}_z^*,\ \bar{\varphi}_{zx}^*,\ P(z)^*,\ p_z(\varphi)^*,\ p_{zx}(\varphi)^*,\ r^*,\ w^*,\ R^*\}$.

Proof. *See Appendix.*■

4 Equilibrium Analysis

The presence of trade cost, multiple factors, heterogeneous firm, asymmetric countries and continuum of industries make it very difficult to find a close-form solution to the model. In this section, we first derive several analytical properties. Then we find the numerical solution and solve the equilibrium factor prices and other endogenous variables.

4.1 Analytical Properties

Proposition 2 *(a) As long as home and foreign country are sufficiently different in their endowment or technology, there exist two factor intensity cut-offs* $0 \leq \underline{z} < \overline{z} \leq 1$ *such that the labor abundant home country specializes in the production within* $[0, \underline{z}]$*, the capital* *abundant foreign specializes in the production within $[\overline{z}, 1]$, and both countries produce within $(\underline{z}, \overline{z})$.*

(b) If there is no variable trade cost ($\tau = 1$) and the fixed cost of export equals to the fixed cost of production for each industry ($f_{zx} = f_z, \forall z$), then $\underline{z} = \overline{z}$. This is the classic case of complete specialization.

Proof. *See Appendix.*■

This proposition is on the production and export pattern for each country. The basic result is illustrated in the Figure 9. Countries engage in inter-industry trade for industries within $[0, \underline{z}]$ and $[\overline{z}, 1]$ due to specialization. This is where the comparative advantage in factor abundance or technology (classical trade power) dominates trade costs and the power of increasing return and imperfect competition (new trade theory). Countries engage in intra-industry trade for industries within $(\underline{z}, \overline{z})$. This is where the power of increasing return to scale and imperfect competition dominates the power of comparative advantage (Romalis, 2004). Thus if the two countries are very similar in their technology and endowments, we would expect the power of comparative advantage is very weak. Then there will be no specialization and only intra-industry trade between the two countries. That is to say, $\underline{z} = 0$ and $\overline{z} = 1$.

In the classical DFS model with zero transportation costs, factor price equalization (FPE) prevails and the geographic patterns of production and trade are not determined when the two countries are not too different. With costly trade and departure from FPE, we are able to determine the pattern of production. This is the property of Romalis model (2004) which we are able to inherit. However, his assumption of homogeneous firm leads to the stark feature that all firms export. With the introduction of firm heterogeneity, we have the following proposition 3 and 4 on the variation of export participation across industries.

Proposition 3 *(a)Within $(\underline{z}, \overline{z})$, the zero profit productivity cut-off φ_z decreases with the capital intensity while the costly trade zero profit cut-off φ_{zx} increases with the capital intensity in home country and vice versa in foreign country.*

(b) Both cut-offs remain constant in the industries that either country specializes.

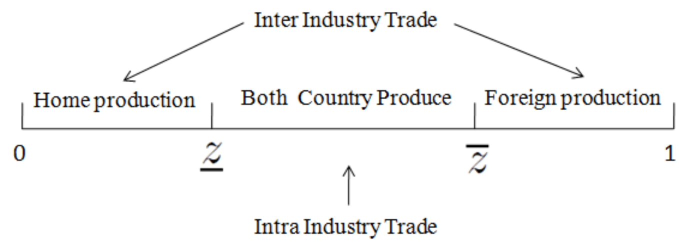


Figure 9: *Productivity Cutoffs and Firm Decision*

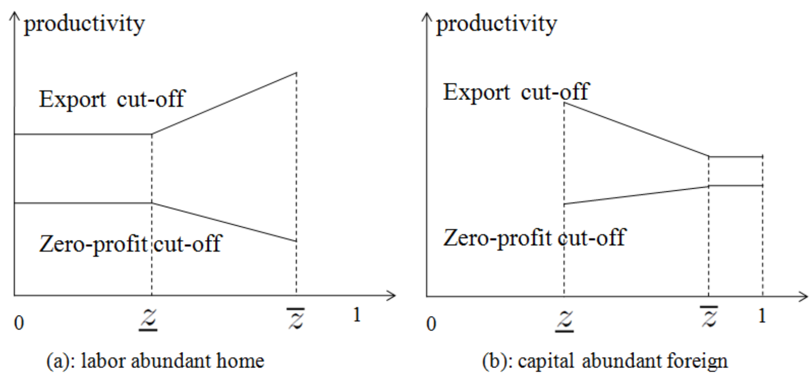


Figure 10: *Productivity Cut-offs across Industries in Home and Foreign Countries*

Proof. *See Appendix.*■

Figure 10 illustrates the result of this proposition. The property (a) of Proposition 3 does not depend on the assumption of Pareto distribution for firm specific productivity. It is a direct extension of Bernard *et al* (2007a). They prove that under the two-industry case, the productivity cut-offs are closer in the comparative advantage industry. We generalize their result in the model with a continuum of industries. An important finding is that the cut-offs do not vary with the factor intensity in industries that countries specialize.

Proposition 4 *(a)Within the specialization zone* $[0, \underline{z}]$ *and* $[z, 1]$*,the export probability* χ_z *is a constant. For the industries that both country produce* $(\underline{z},\overline{z})$*,the export probability* χ_z

decreases with the capital intensity in the labor abundant country and vice versa in the capital abundant country. To be specific, we have

$$\chi_z = \begin{cases} \frac{R^*}{fR} & z \in [0, \underline{z}] \\ \frac{\widetilde{\tau}^{-a} f - \varepsilon^a g(z)}{\varepsilon^a f g(z) - \widetilde{\tau}^a} & z \in (\underline{z}, \overline{z}) \end{cases}$$

where $g(z) = (\frac{w}{w^*}(\frac{r/w}{r^*/w^*})^z)^{\frac{a\sigma}{1-\sigma}}$ *and*

$$\frac{\partial \chi_z}{\partial z} = \frac{(1 - \widetilde{\tau}^{-2a} f^2)\varepsilon^a g a}{(\varepsilon^a f g(z) - \widetilde{\tau}^a)^2}(\ln(A) - \frac{\sigma}{\sigma - 1}\ln(\frac{r/w}{r^*/w^*})), if\ z \in (\underline{z}, \overline{z})$$

(b) The export intensity is: $\gamma_z = \frac{f\chi_z}{1+f\chi_z}$ *which follows the same pattern as* χ_z*.*

Proof. *See Appendix.* ■

Proposition 4 is a straightforward result following Proposition 3. In general, it tells us that the stronger the power of comparative advantage is, the more that firms participate in international trade. However, for industries that countries specialize, export participation is a constant. Figure 11 depicts this idea. And we also find that the sign of $\frac{\partial \chi_z}{\partial z}$ depends on two terms within $(\underline{z}, \overline{z})$: the *Ricardian Comparative Advantage* $\ln(A)$ and the *Heckscher-Ohlin Comparative Advantage* $\ln(\frac{r/w}{r^*/w^*})$. The magnitude of the HO Comparative Advantage depends on σ, the elasticity of substitution between varieties, or the degree of imperfect competition: the smaller σ is, the larger that different industries differ in their export participation. Since $A < 1$ and $\frac{K}{L} < \frac{K^*}{L^*}$(or $\frac{r/w}{r^*/w^*} > 1$),home country has both Ricardian Comparative Advantage and Heckscher-Ohlin Comparative Advantage in the more labor intensive industries. Thus we expect $\frac{\partial \chi_z}{\partial z} < 0$ and export probability decreases with the capital intensities in home country. However, if $A > 1$, home country has Ricaridan Comparative Advantage in the more capital intensive industries. Then the sign of $\frac{\partial \chi_z}{\partial z}$ depends on which comparative advantage is more powerful. If Ricardian Comparative Advantage is so strong that it overturns the Heckscher-Ohlin Advantage, then home country will export more in the more capital intensive industries.

Fan *et al* (2011) incorporate Melitz (2003) into the DFS model (1977) with Ricardian Comparative Advantage and get very similar prediction on the export participation. The

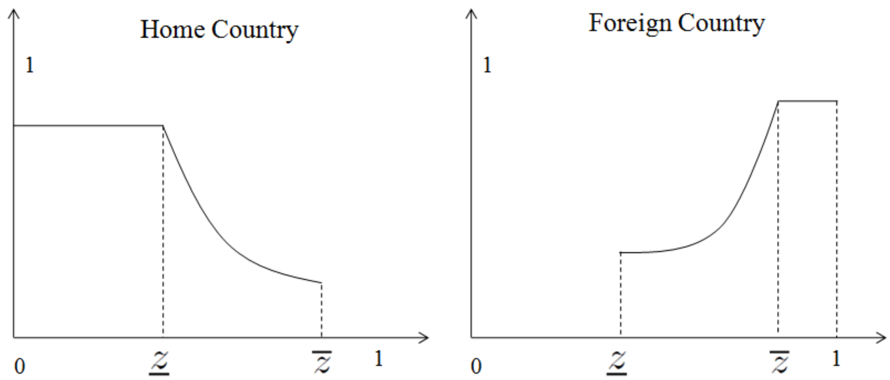


Figure 11: *Export Probability or Export Intensity in Home Country and Foreign Country*

key insight from Melitz model is that within sector resource reallocation generates productivity gain. Bernard et al (2007) find that the strength of reallocation is stronger in the industry uses more of the country's abundant factor. Such heterogeneous reallocation will generate endogenous Ricardian Comparative Advantage. From the last paragraph, we find that such endogenous comparative advantage could even overturn the exogenous Ricardian Comparative Advantage. This is elaborated in next proposition.

Proposition 5 *(a)The average of firm specific productivity in each industry is*

$$\widehat{\varphi}_z = (\frac{a}{a+1-\sigma})^{\frac{1}{\sigma-1}}[\frac{(\sigma-1)\theta^a}{\delta(a+1-\sigma)\widetilde{f}}(1+f\chi_z)]^{1/a}$$

It is a constant within the specialization zone $[0, \underline{z}]$ *and* $[\underline{z}, 1]$. *Within* $(\underline{z},\overline{z})$, *it decreases with the capital intensity for the labor abundant country and vice versa for the capital abundant country.*

(b)The magnitude of Recardian Comparative Advantage could be amplified by the endogenous technology difference generated by reallocation if the Heckscher-Ohlin Comparative Advantage is in line with it. Otherwise, it is dampened.

Proof. *See Appendix.*■

Conclusion (a) of Proposition 5 enables us to decompose industrial average productivity. $A(z)$ is the industry-specific productivity while $\widehat{\varphi}_z$ is the average of firm specific productivity. From the expression of $\widehat{\varphi}_z$, it is quite obvious that opening to trade leads to a productivity gain since χ_z increases from zero to a positive number. Also, the reallocation effect is stronger when there are more firms exporting in that industry. And the resulting average productivity would also be higher holding the industry specific productivity $A(z)$ constant. Then with Proposition 4, conclusion (b) is straightforward: if $\ln(A) > 0$ while $\frac{\partial \chi_z}{\partial z} < 0$, $\frac{A(z)}{A^*(z)}$ will increase with z and $\frac{\widehat{\varphi}_z}{\widehat{\varphi}_z^*}$ decreases z, then the overall average industry productivity ratio $\frac{A(z)\widehat{\varphi}_z}{A^*(z)\widehat{\varphi}_z^*}$ could become a decreasing function of z if the reallocation effect is very strong. If this is the case then the Ricardian comparative advantage is dampened. Otherwise, it is amplified.

4.2 Numerical Solution

In this subsection, we find the numerical solution to the model and discuss other equilibrium properties of the model. The algorithm is in Appendix 6 and the parameters chosen are in Table 5. The equilibrium factor prices and cut-off industries are in table 6. It is easy to see that $\frac{r/w}{r^*/w^*} = 1.165 > 1$. Also $A < 1$, thus we would expect $\frac{\partial \chi_z}{\partial z} < 0$ between $(\underline{z},\overline{z})$. From Figure 12, we find this is exactly the case: export probability and intensity first stays constant and then decreases with the capital intensity, the opposite is true for foreign country. We also find that industrial output and industrial export follow the same pattern: countries tend to produce and export more in its comparative advantage industries. Firm mass follows similar patterns but it doesn't stay constant in industries that countries specialize. We should point out that firm mass, industrial output and export also depend on household's expenditure share $b(z)$: for industries with a higher demand, the firm mass, industrial output, and exports will also be higher. Since we normalize $b(z)$ to be 1 for all industries, this channel is shut down.

Table 5: Parameters used in simulation

Variables	meaning	value
K	home capital stock	100
L	home labor stock	300
K^*	foreign capital stock	300
L^*	foreign labor stock	100
f	relative fixed cost of export $\frac{f_{zx}}{f_z}$	6.5
$\widetilde{f}$	relative fixed cost of entry $\frac{f_{zx}}{f_z}$	$20 * f$
τ	iceberg cost	1.05
a	shape parameter of Pareto Distribution	3.8
θ	lower bound of Pareto Distribution	0.2
δ	exogenously death probability of firms	0.025
σ	elasticity of substitution	3.4
A	strength of comparative advantage	0.5
λ	strength of absolute advantage	1

Notes: most of parameters follow Bernard et al (2007a) and Romalis (2004). A is chosen to be less than 1 so that home country has Ricardian comparative advantage in more labor intensive industries.

Table 6: Equilibrium Factor Prices and Cut-off Industry

Variables	meaning	value
r	domestic interest rate	0.06807
w	domestic wage rate	0.07045
r^*	foreign interest rate	0.07989
w^*	foreign wage rate	0.0964
$\underline{z}$	lower cut-off industry	0.2756
$\overline{z}$	higher cut-off industry	0.6547

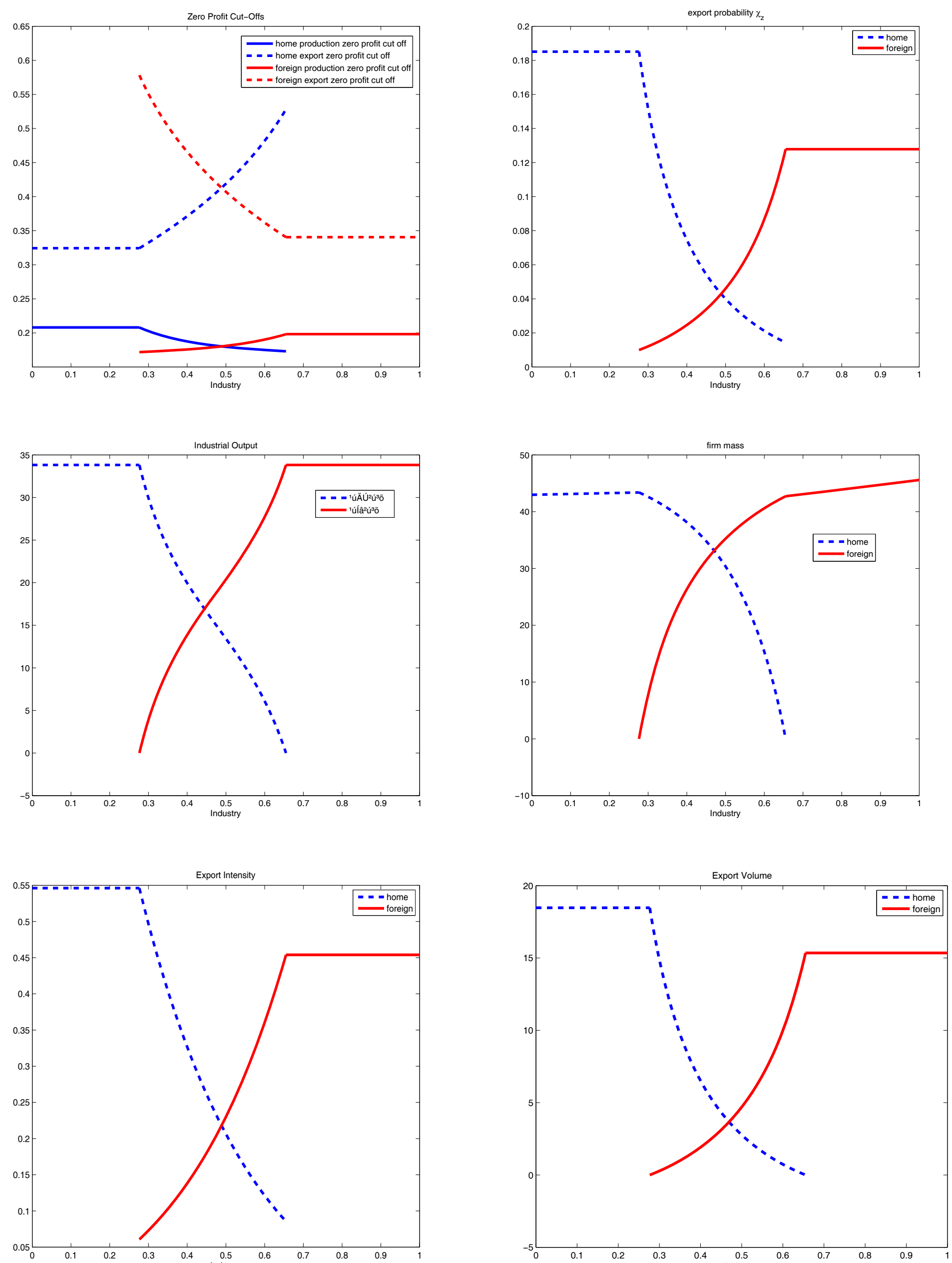


Figure 12: Baseline Simulation

5 Comparative Statics

In this section, we conduct comparative statics by changing the value of some exogenous variables and study how the economy responds to such changes. The aim of these exercises is to help us to disentangle different channels of structure adjustments and shed light on what we observe in the Chinese data. The three main channels we are interested in are capital deepening, trade liberalization and technology change.

5.1 Capital Deepening

By keeping all other variables unchanged and increasing home country capital stock K, we increases the capital labor ratio of home country. The effects are shown in Figure 13 and Figure 14. From the left panel of Figure 13, we find that as home country becomes more capital abundant, $\overline{z}$ increases till it stops at 1. Thus home country begins to produce more capital intensive goods. Also $\underline{z}$ decreases till it stops at 0, which means that foreign country also becomes to produce labor intensive goods. And $\overline{z}-\underline{z}$ increases steadily. Thus home and foreign production and export structure become more and more similar as their endowment structures converge. From the middle panel of Figure 13, we find that wage increases in both home and foreign country while capital rental rate decreases in both countries. And from right panel, we find that home country has higher and higher welfare as its capital stock increases but foreign country's welfare deteriorates.

The intuition is that as home country becomes more capital abundant, capital becomes cheaper in home country and its comparative advantages in capital intensive industries increases. Thus we expect it expands its production and export to more capital intensive industries. But since labor supply is fixed and home country has to reallocate labor from the labor intensive industries to more capital intensive industries, home exports of labor intensive goods decrease, forcing foreign to pick up these industries in order to satisfies the demand. In general, the relative demand for foreign goods decreases thus foreign country incurs losses.

Figure 14 illustrates in greater details the effect of capital deepening on production and trade patterns. As we see, firm mass and industrial output both increases for home country and the magnitude is larger in the more capital intensive industries. And foreign production moves into more capital intensive industries. As for export, we find home country begins to export more and more in the capital intensive industries: export probability, export intensity and export volume all increase. The opposite is true for the foreign country.

Then the question is: could capital deepening explain what we observe in the Chinese data. The answer is NO. If we believe that China is becoming more capital abundant comparing with rest of world, then Chinese production becomes more capital intensive. Chinese exports should also become more capital intensive which is not consistent with what we observe!

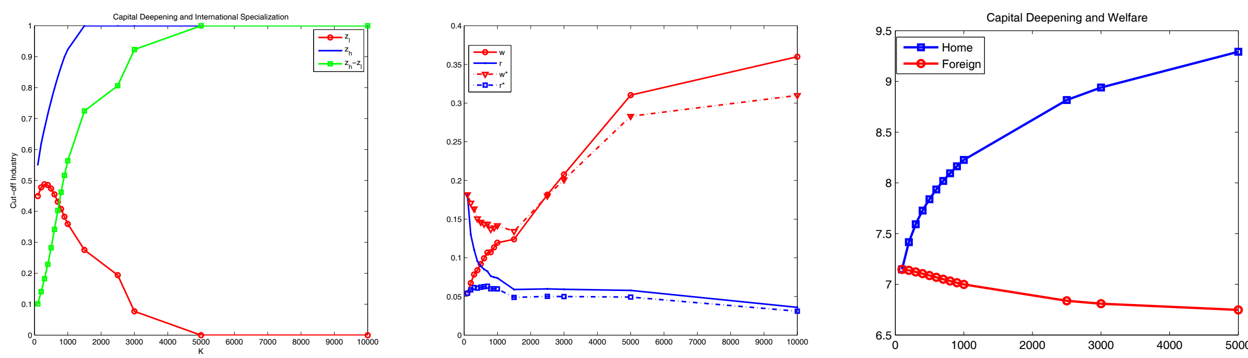
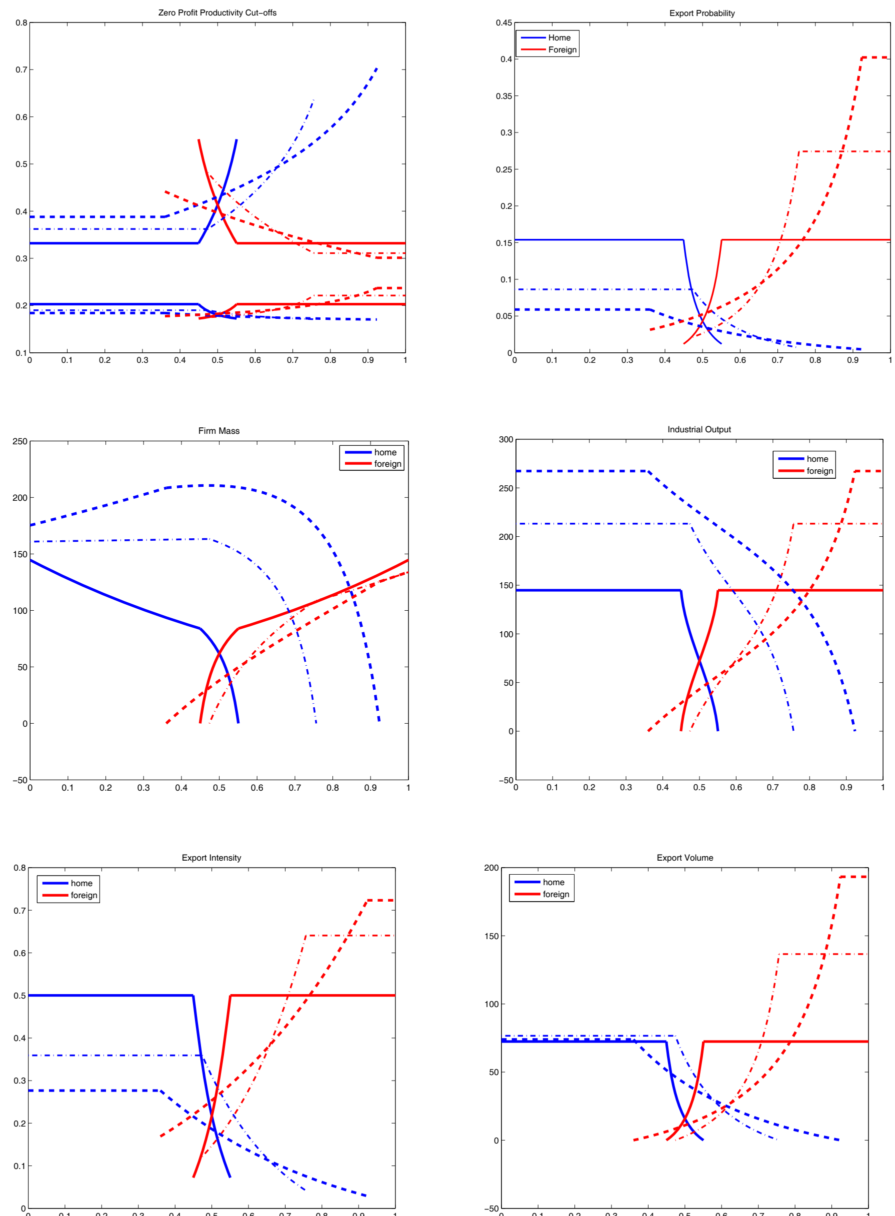


Figure 13: Capital Deepening



Notes: from solid lines to thin dash lines & thick dash lines is the direction that K rises.

Figure 14: Capital Deepening and Equilibrium Outcomes

5.2 Trade Liberalization

In this subsection, we focus on trade liberalization by studying the effect of trade cost reduction. There are two types of trade costs: fixed costs f_{zx} and variable costs τ which we investigate respectively.

5.2.1 Reduction in Fixed Costs $\mathbf{f_{zx}}$

Before we go on to look at the result, one notion worthy of mention is that the overall fixed cost of export is $f_{zx}r^z w^{1-z}$, thus a decrease of f_{zx} only means that the fixed costs of export become relatively less comparing with fixed cost of production, or say f decreases. From Figure 15, we find that as f decreases, $\overline{z} - \underline{z}$ also decreases: production and exports become more and more dissimilar and both countries become more and more concentrated on their comparative advantage industries. Also the return to their abundant factors increase for both countries *vice versa* for the scarce factor. Finally, both countries gain from trade. From Figure 16, we have the following lessons. First, the reduction in the fixed trade costs boosts international trade, more in one's comparative advantage industries: export volume, export probability and export intensity all increase and the changes increase with comparative advantage. Second, as the fixed costs of trade reduces, productions and exports both move to ones' more comparative advantage industries.

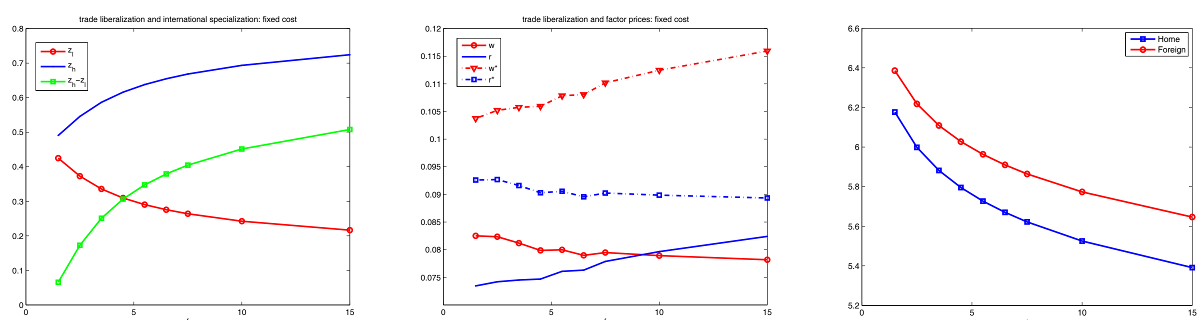
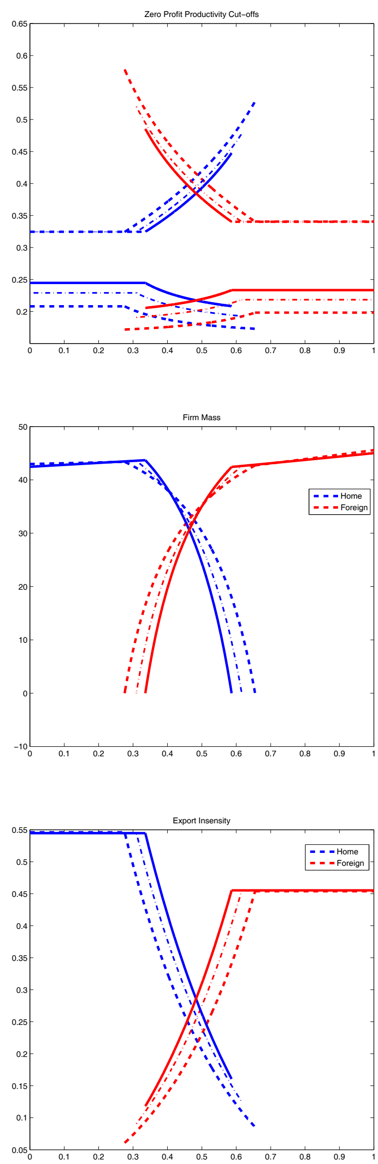


Figure 15: Reduction of Fixed Trade Costs



Notes: from solid lines to thin dash lines & thick dash lines is the direction that f_{zx} rises.

Figure 16: Trade Liberalization: Reduction in Fixed Cost

5.2.2 Reduction in Variable Trade Costs τ

Another form of trade liberalization is reduction in variable costs, or transportation cost τ. From Figure 17, we find what we learned above from the reduction fixed costs is still true: trade liberalization is welfare improving for both countries and increases return to ones' abundant factor and drives more specializations. Looking at Figure 18, it is also quite similar to Figure 16. For brevity, we don't repeat the findings here.

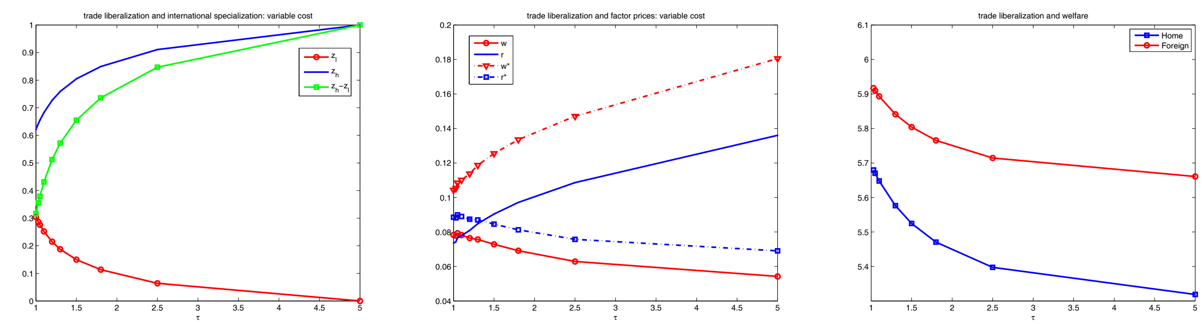
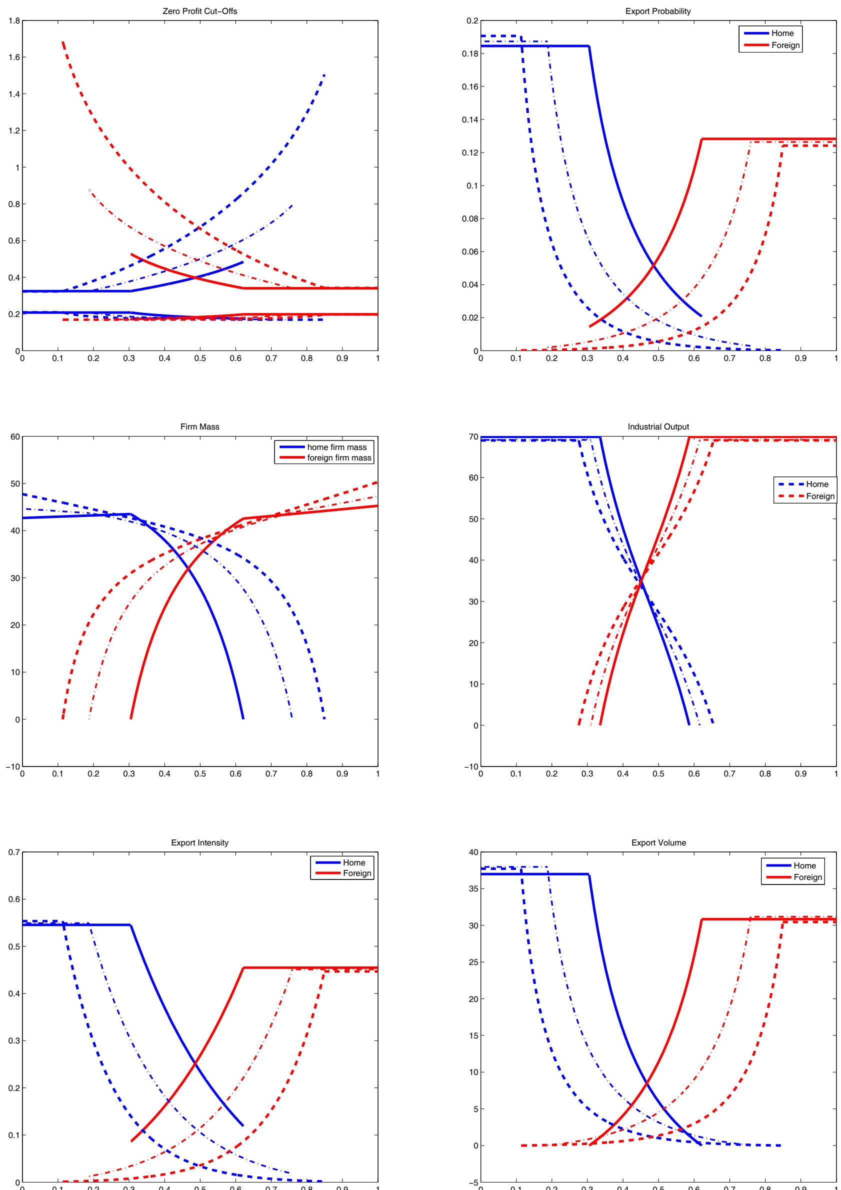


Figure 17: Reduction of Variable Trade Costs: τ



Notes: from solid lines to thin dash lines & thick dash lines is the direction that τ rises.

Figure 18: Trade Liberalization: Reduction in variable Cost

5.3 Technology Changes

In this subsection, we study the effect of technological changes. There are two types of technology changes: absolute advantage λ and comparative advantage A.

5.3.1 Increases in Absolute Advantages

Technology changes in absolute advantages are captured by changing the value of λ. The results are illustrated in Figure 19 and Figure 20. From the left panel of Figure 19, it is obvious that home country begins to produce in more and more industries while foreign is cutting the labor intensive industries as home country becomes relatively more productive in every industry. However, the overlap industries stays stable as λ increases. From the middle panel, the factor return to both factors in foreign country dramatically decrease, while the capital return increases and wage stays almost unchanged at home. Then it is natural to understand why home gains while foreign loses from such changes.

From Figure 20, we find that export volume, export intensity, and export probability all decrease in the labor intensive industries but increase in the capital intensive industries as λ increases. Thus export become more capital intensive in home country and so does production.

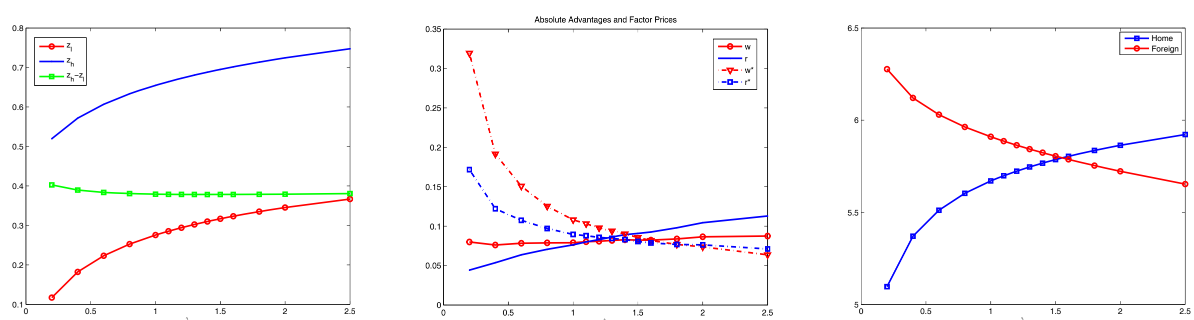
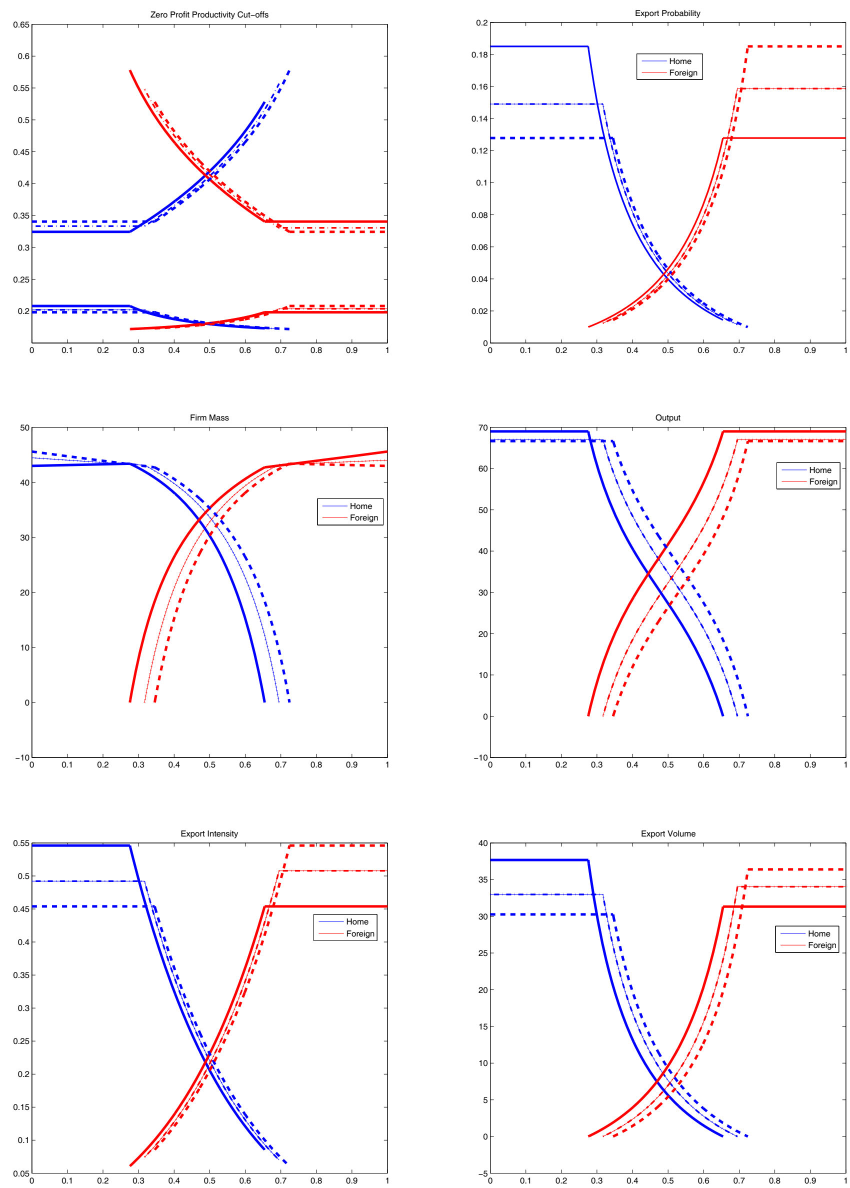


Figure 19: Increases in Absolute Advantage: λ

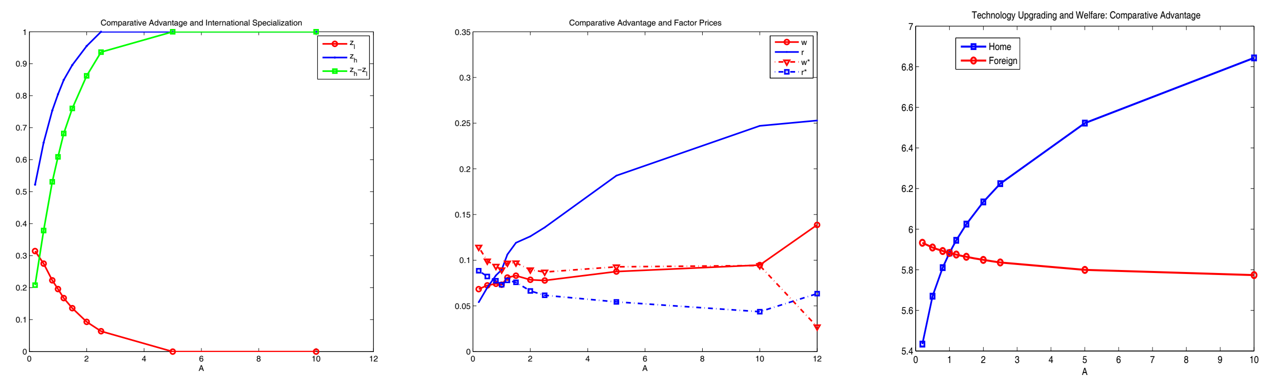


Notes: from solid lines to thin dash lines & thick dash lines is the direction that λ rises.

Figure 20: Technology Innovation: Absolute Advantage: λ

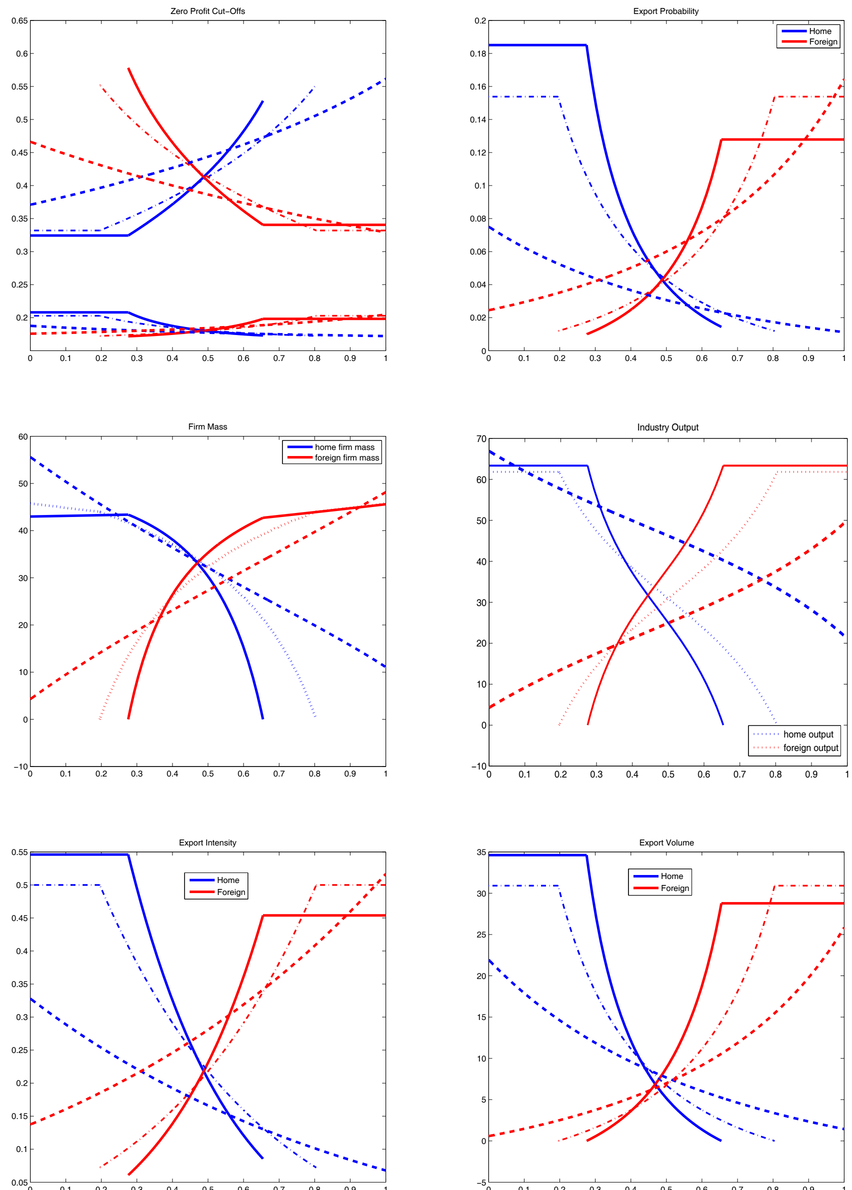
5.3.2 Increases in Comparative Advantages

The effect of increases in the Ricardian comparative advantages is capture by changing the value of A. As A increases, home country becomes relatively more productive in the capital intensive industries than foreign country. First, from the left panel of Figure 21, home country will produce more and more capital intensive goods and foreign country will also pick up labor intensive industries. Thus their productions overlap more and more. Secondly, the factor return to both factors in home country increases but wage remains stagnant and capital decreases in foreign. And we find home country's welfare increases dramatically and foreign country's welfare decreases modestly. Finally, we depict the structural adjustments of production and export as A increases. It is obvious that home production and exports become more and more capital intensive as home country gains technological comparative advantage in capital intensive industries.



Notes: in middle panel when A=12, there is a structural break: home country's technology comparative advantages in capital intensive industries begins to dominate comparative disadvantage in endowment and specializes in capital intensive goods.

Figure 21: Increases in Comparative Advantage: A



Notes: from solid lines to thin dash lines & thick dash lines is the direction that A rises.

Figure 22: Increases in Comparative Advantage: A

6 Bring the Model to Data

6.1 Export Probability and Export Intensity

Proposition 4 predicts that export probability and export intensity do not change with capital intensity for the industries that a country specializes. For industries that both country produce, countries participate relative more in their comparative advantage industries. Looking at Figure 5, we do see that the both remain relatively flat in the most labor intensive industries and then decrease with capital intensity. In this subsection, we test this prediction using econometric method instead of using "regression by eyes".

Proposition 2 predicts that there is industrial breaks $\underline{z}$ and $\overline{z}$ which determine the production pattern[10]. We detect this break using the method developed by Zivot and Andrews (1992). In the test below, we use export probability which corresponds to proportion of firms that export in the data to find the break. The result is shown in Table 6.

Table 6: Zivot-Andrew Test and Cut off industries $\underline{z}$

Year	1999	2007
Cut off Industry $\underline{z}$	48	11

Notes: The test is done using the STATA toolbox.

We find that the specialization zone shrinks from 48 in 1999 to 11 in 2007. It implies that the relative distance between $\underline{z}$ and $\overline{z}$ became larger, or say, there is an increase of overlap between the production (trade) of China and capital abundant countries. This is consistent with Schott (2008), Wang and Wei (2010) that Chinese exports are more and more sophisticated in the sense that it overlaps more and more with exports of developed countries.

Then by regression, we could test whether the proportion of firms export remains constant for the industries that it specializes and decreases with capital intensity. Basically, we run the regression below

[10] We presume that China is the labor abundant country and specializes in the labor intensive industries.

$$\chi_i = \alpha + \beta * Industry_i + \epsilon_i$$

χ_i is proportion of firms export for industry i and Industry$_i$ is the industry index. We expect that $\beta = 0$ between $[0, \underline{z}]$ and $\beta < 0$ between $[\underline{z}, 1]$ from our theory. In Table 7 below, we see this is exactly the case. From the regression, we find that the proportion of export firms increases from 0.306 to 0.426 and the slope decreases from -0.0033 to -0.0042 which is what we see in Figure 5.

Table 7: Regression on Proportion of Firms Export

Year	1999		2007	
Coefficients	industry<=48	industry>48	industry<=11	industry>11
β	.00027	-.0033***	.0012	-.0042***
	(.0002)	(.0001)	(.0027)	(.0001)
α	.306***	.489***	.426***	.547***
	(.0059)	(.011)	(.018)	(.0059)
observations	48	52	11	89
R square	.03	.91	.02	.96

*Notes: (a)***,**,*stand for significance level at 1%, 5% and 10% respectively;.(b) numbers in parentheses are standard errors.*

6.2 Calibration

To be Added.

7 Concluding Remarks

In this paper, we first document the seemingly puzzling patterns of structural adjustments in Chinese production and exports based on a comprehensive Chinese firm level data: the overall manufacturing production became more capital intensive while exports became more labor intensive between 1999-2007. It counters our understanding from Rybczynski Theorem of HO theory. To explain these findings, we embed Melitz-type heterogeneous firm into the Ricardian and Heckscher-Ohlin trade theory with *continuous* industries. The theory predicts that export probability and export intensity decrease with comparative advantages. And they remain *constant* for inter-industry trade industries where countries specialize. Such predictions are supported by data.

But the question remains how we could explain the structure adjustments we observe in the Chinese data. From our comparative statics, we find that production and export keep moving in the same direction if only one parameter changes: both become more capital intensive or become more labor intensive. This is at odds with our empirical findings. Thus we expect that to explain the data patterns, it is necessary to find out the actual changes in the endowments, technologies and trade costs and feed in these changes into our model to see how much they explain the structural adjustments.

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8 Appendix

8.1 Proof of Proposition 1

The proof goes in this way: suppose that factor prices $\{w, w^*, r, r^*\}$ are known, and we find the factor demands as functions of them. Then market clearing condition will pin down the unique equilibrium. Firstly, we have the national revenue for home country and foreign country: $R = wL + rK$ and $R^* = w^*L^* + r^*K^*$. Potentially, there could be industries that either country specializes.[11] The factor demands in home country for these industries are $l(z) = (1-z)b(z)(R+R^*)/w$, $k(z) = zb(z)(R+R^*)/r$. Factor demands in foreign country have symmetric expressions. For industries that both country produce, the industry revenue function is given by equation(3.17) , thus we need to know the firm mass M_z, M_z^*,the pricing index $P(z)$ and $P(z)^*$ and industry average productivity $\widehat{\varphi}_z$ and $\widehat{\varphi}_z^*$ (average price p($\widehat{\varphi}_z$) and p($\widehat{\varphi}_z^*$)) in order to find its factor demand. Firstly, from equation (3.17), we find that:

$$\frac{r(\widehat{\varphi}_z)}{r(\widehat{\varphi}_z^*)} = \widetilde{p}_z^{1-\sigma} \frac{(\frac{P(z)}{P(z)^*})^{\sigma-1} + \frac{R^*}{R}\tau^{1-\sigma}\chi_z^{\frac{a+1-\sigma}{a}}}{\frac{R^*}{R} + \chi_z^{*\frac{a+1-\sigma}{a}}\tau^{1-\sigma}(\frac{P(z)}{P(z)^*})^{\sigma-1}} \tag{8.1}$$

Here $r(\widehat{\varphi}_z) = \frac{R_z}{M_z}$ is the average firm revenue and $\widetilde{p}_z \equiv \frac{p_{zd}(\widehat{\varphi}_z)}{p_{zd}(\widehat{\varphi}_z^*)} = \frac{\widehat{\varphi}_z^* w}{\varepsilon(z)\widehat{\varphi}_z w^*}(\frac{r/w}{r^*/w^*})^z$ is the relative average domestic price between the two countries. Using the zero profit condition(3.9),(3.10) and $\frac{r(\widehat{\varphi}_z)}{r(\bar{\varphi}_z)} = (\frac{\widehat{\varphi}_z}{\bar{\varphi}_z})^{\sigma-1}$,[12] it is obvious that $r(\widehat{\varphi}_z) = (f_z(\frac{\widehat{\varphi}_z}{\bar{\varphi}_z})^{\sigma-1} + \chi_z f_{zx}(\frac{\widehat{\varphi}_{zx}}{\bar{\varphi}_{zx}})^{\sigma-1})\sigma r^z w^{1-z}$. Combing with the free entry condition, we could find that the average productivity between home and foreign country is $\frac{\widehat{\varphi}_z^*}{\widehat{\varphi}_z} = (\frac{1+f\chi_z^*}{1+f\chi_z})^{\frac{1}{a}}$while $f \equiv \frac{f_{zx}}{f_z}$.Using the Pareto distribution assumption, we can easily solve that $\frac{\widehat{\varphi}_z}{\bar{\varphi}_z} = \frac{\widehat{\varphi}_{zx}}{\bar{\varphi}_{zx}} = (\frac{a}{a+1-\sigma})^{\frac{1}{\sigma-1}}$ and $\chi_z = \frac{1-G(\bar{\varphi}_{zx})}{1-G(\bar{\varphi}_z)} = \Lambda_z^{-a}$ while Λ_z is the productivity cut-off ratio given by (3.11). Then we have:

$$\frac{r(\widehat{\varphi}_z)}{r(\widehat{\varphi}_z^*)} = \widetilde{p}_z(\frac{1+f\chi_z}{1+f\chi_z^*})^{\frac{a+1}{a}} \tag{8.2}$$

Using the definition of $\widetilde{p}_z$ and combining (8.1) and (8.2), we have:

$$\chi_z = \frac{\widetilde{\tau}^{-a}f - \varepsilon^a g(z)}{\varepsilon^a f g(z) - \widetilde{\tau}^a} \tag{8.3}$$

[11] We are going to show how to determine the specialization pattern in proposition 2. And greater detailed could be found in the algorithm of numerical solution.

[12] This is a typical property of Melitz(2003) type model.

Here $g(z) \equiv (\frac{w}{w^*}(\frac{r/w}{r^*/w^*})^z)^{\frac{a\sigma}{1-\sigma}}$ while $\widetilde{\tau} \equiv \tau f^{\frac{1}{\sigma-1}}$.[13] From (8.3), we see that χ_z is a function of the factor price. From equation (3.11) we have $\Lambda_z = \chi_z^{-1/a} = \frac{\tau P(z)}{P(z)^*}(\frac{fR}{R^*})^{1/(\sigma-1)}$,then $\frac{P(z)}{P(z)^*} = \frac{\chi_z^{-1/a}}{\tau}(\frac{R^*}{fR})^{1/(\sigma-1)}$.So we can find that for those industries that both country produce:

$$
\begin{aligned}
R_z &= b(z)[\frac{R}{1-\widetilde{\tau}^{-a}\varepsilon^a f g(z)} - \frac{fR^*}{\widetilde{\tau}^a\varepsilon^a g(z) - f}] && (8.4) \\
R_z^* &= b(z)\varepsilon^a g(z)[\frac{R^*}{\varepsilon^a g(z) - f\widetilde{\tau}^{-a}} - \frac{fR}{\widetilde{\tau}^a - \varepsilon^a f g(z)}] && (8.5)
\end{aligned}
$$

So both could be written as a function of the factor price. Again using $l(z) = (1-z)b(z)R_z/w$ and $k(z) = zb(z)R_z/r$. Then the factor demand for industries that both country produce as:

$$
\begin{aligned}
\int_{I(s)} (1-z)\frac{b(z)(R+R^*)}{w}dz + \int_{I(b)} (1-z)\frac{R_z}{w} &= L \\
\int_{I(s)} z\frac{b(z)(R+R^*)}{r}dz + \int_{I(b)} z\frac{R_z}{r} &= K
\end{aligned}
$$

Another 2 symmetric equations could be written for the case of foreign country. I(s) is set of the industries that home country specializes and while I(b) is the set of industries that both countries produce. It is determined where either domestic or foreign firm mass is zero. From the definition of price index (3.18), we have $\frac{M_z}{M_z^*} = \widetilde{p}_z^{\sigma-1}\frac{(\frac{P(z)}{P(z)^*})^{1-\sigma} - \chi_z^{-\frac{a+1-\sigma}{a}}\widetilde{\tau}^{-2(a+1-\sigma)}\tau^{1-\sigma}}{1-\chi_z^{\frac{a+1-\sigma}{a}}\tau^{1-\sigma}(\frac{P(z)}{P(z)^*})^{1-\sigma}}$. Thus it is also determined by factor prices.[14] So there are 4 equations for 4 unknowns, given reasonable parameters the equilibrium factor prices could be uniquely pinned down.■

8.2 Proof of Proposition 2

In the proof of Proposition 1, we mention that the relative firm mass at home and abroad is:

[13] Here it can be proved that $\frac{\partial \chi_z}{\partial z} < 0$ which is one of the conclusions in proposition 3. However, here we rely on the Pareto distribution while proposition 3 doesn't need that.

[14] We provide more details in next proof.

$$\frac{M_z}{M_z^*} = \widetilde{p}_z^{\sigma-1} \frac{(\frac{P(z)}{P(z)^*})^{1-\sigma} - \chi_z^{-\frac{a+1-\sigma}{a}} \widetilde{\tau}^{-2(a+1-\sigma)} \tau^{1-\sigma}}{1 - \chi_z^{\frac{a+1-\sigma}{a}} \tau^{1-\sigma} (\frac{P(z)}{P(z)^*})^{1-\sigma}}$$

Since $\frac{P(z)}{P(z)^*} = \frac{\chi_z^{-1/a}}{\tau} (\frac{R^*}{fR})^{1/(\sigma-1)}$ and $\widetilde{p}_z = \frac{\widehat{\varphi}_z^* w}{\varepsilon(z) \widehat{\varphi}_z w^*} (\frac{r/w}{r^*/w^*})^z$, we find it could be further simplified as:

$$\frac{M_z}{M_z^*} = \varepsilon^{1-\sigma} (\frac{1 + f\chi_z^*}{1 + f\chi_z})^{\frac{\sigma-1}{a}} [\frac{w}{w^*} (\frac{r/w}{r^*/w^*})^z]^{\sigma-1} \frac{\frac{fR}{R^*} - \chi_z^{-1} \widetilde{\tau}^{-2a} f^2}{1 - \chi_z \frac{fR}{R^*}} \tau^{\sigma-1} \chi^{\frac{\sigma-1}{a}}$$

Then $\exists\, \chi_z = \frac{R^*}{fR} (\frac{f}{\widetilde{\tau}^a})^2$ such that $\frac{M_z}{M_z^*} = 0$.Since $M_z^* > 0$ $(M_z^* \neq 0)$, it must be that $M_z = 0$. And as χ_z decreases such that $\chi_z < \frac{R^*}{fR} (\frac{f}{\widetilde{\tau}^a})^2$,it must be that$\frac{M_z}{M_z^*} < 0$. If χ_z increases such that χ_z approaches $\frac{R^*}{fR}$ we have $\frac{M_z}{M_z^*} \rightarrow +\infty$,or say $\frac{M_z^*}{M_z} \rightarrow 0$, so again we have $M_z^* = 0$.If χ_z further increases such that $\chi_z > \frac{R^*}{fR}$, we again have $\frac{M_z^*}{M_z} < 0$.Thus to maintain positive firm mass for both home and foreign in certain industry z, we must have:

$$\frac{R^*}{fR} (\frac{f}{\widetilde{\tau}^a})^2 < \chi_z < \frac{R^*}{fR}$$

where $\frac{f}{\widetilde{\tau}^a} = \frac{f}{\tau^a f^{\frac{a}{\sigma-1}}} < \frac{f}{f^{\frac{a}{\sigma-1}}} < 1, (a > \sigma + 1 > \sigma - 1 > 0)$,if $\tau > 1$ and $f > 1$. And if χ_z falls out of this range. One of the countries' firm mass is zero (it cannot be negative which is meaningless) and the other is positive. This is where specialization happens! For industries that both country produces, we have

$$\chi_z = \frac{\widetilde{\tau}^{-a} f - \varepsilon^a g(z)}{\varepsilon^a f g(z) - \widetilde{\tau}^a} \tag{8.6}$$

which is a continuous and monotonic function between $[\underline{z}, \overline{z}]$,[15] Then we have

$$\chi_{\underline{z}} = \frac{R^*}{fR} \text{ and } \chi_{\overline{z}} = \frac{R^*}{fR} (\frac{f}{\widetilde{\tau}^a})^2$$

[15] This is true given our assumption of home country is labor abundant and has Ricardian comparative advantage in more labor intensive industries.

and $(\underline{z}, \overline{z})$ are given by:

$$\underline{z} = \frac{\ln(\frac{\chi_{\underline{z}}\widetilde{\tau}^a + f\widetilde{\tau}^{-a}}{1+f\chi_{\underline{z}}}) - \frac{a\sigma}{1-\sigma}\ln(\frac{w}{w^*}) - a\ln(\lambda)}{\frac{a\sigma}{1-\sigma}\ln(\frac{r/w}{r^*/w^*}) + a\ln(A)}$$

$$\overline{z} = \frac{\ln(\frac{\chi_{\overline{z}}\widetilde{\tau}^a + f\widetilde{\tau}^{-a}}{1+f\chi_{\overline{z}}}) - \frac{a\sigma}{1-\sigma}\ln(\frac{w}{w^*}) - a\ln(\lambda)}{\frac{a\sigma}{1-\sigma}\ln(\frac{r/w}{r^*/w^*}) + a\ln(A)}$$

And if $\tau = f = 1$, we have $\chi_{\underline{z}} = \chi_{\overline{z}} = \frac{R^*}{R}$. So $\underline{z} = \overline{z}$ and there are intra-industry trade.■

8.3 Proof of Proposition 3

Let's focus on the labor abundant home country: for any 2 industries z and z', suppose z<z'. From the definition of Λ_z (3.11) and using the assumption that trade costs and fixed costs are the same all industries, we have:

$$\frac{\Lambda_z}{\Lambda_{z'}} = \frac{P(z)/P(z')}{P(z)^*/P(z')^*}.$$

Thus if $\frac{P(z)}{P(z')} < \frac{P(z)^*}{P(z')^*}$, or say labor intensive products are relatively cheaper in home country, then we have $\Lambda_z < \Lambda_{z'}$. This is exactly what we are going to do. If $\frac{P(z)}{P(z')} < \frac{P(z)^*}{P(z')^*}$ under autarky and $\frac{P(z)}{P(z')} = \frac{P(z)^*}{P(z')^*}$ under free trade, then the costly trade case will fall between and establishes our proof. When there is free trade (no variable costs or fixed costs of trade), all firms will export, the price of each variety and number of varieties will be the same for both countries. Thus the pricing index P(z)=P(z)* for all industries and $\frac{P(z)}{P(z')} = \frac{P(z)^*}{P(z')^*}$. On the other extreme of close economy, no firms export and from (3.18) we have $P(z) = M_z^{\frac{1}{1-\sigma}} p_{zd}(\widehat{\varphi}_z)$.Firm mass for each industry is $M_z = \frac{b(z)R}{r(\widehat{\varphi}_z)} = \frac{b(z)R}{r(\bar{\varphi}_z)}(\frac{\bar{\varphi}_z}{\widehat{\varphi}_z})^{\sigma-1}$.So $\frac{P(z)}{P(z')} = (\frac{w}{r})^{(z'-z)/\rho}(\frac{b(z)}{b(z')})^{\frac{1}{1-\sigma}}\frac{A(z')\bar{\varphi}_{z'}}{A(z)\bar{\varphi}_z}$. Using (3.16) we have homogeneous cut-offs for all industries under autarky:$\bar{\varphi}_{z'} = \bar{\varphi}_z$.Then it can be verified that

$$\frac{P(z)/P(z')}{P(z)^*/P(z')^*} = (\frac{w/r}{w^*/r^*})^{\frac{z'-z}{\rho}} A^{z'-z}$$

Since z'>z and $A < 1$, then$\frac{w}{r} < \frac{w^*}{r^*} \Longleftrightarrow \frac{P(z)}{P(z')} < \frac{P(z)^*}{P(z')^*}$. So our next task is to prove $\frac{w}{r} < \frac{w^*}{r^*}$ under autarky. Because of the factor market clearing condition and the Cobb-Douglas production function for production, entry and payments of fixed costs, we find that:

$$\frac{K}{L} = \frac{w}{r} \frac{\int_0^1 zb(z)dz}{\int_0^1 (1-z)b(z)dz}, \quad \frac{K^*}{L^*} = \frac{w^*}{r^*} \frac{\int_0^1 zb(z)dz}{\int_0^1 (1-z)b(z)dz}$$

Thus $\frac{K}{L} < \frac{K^*}{L^*} \Longleftrightarrow \frac{w}{r} < \frac{w^*}{r^*}$ and we establish that $\Lambda_z < \Lambda_{z'}$, or say Λ_z increases with z in home country. For industries that home country specializes: $\Lambda_z = \chi_z^{-1/a} = (\frac{fR}{R^*})^{1/a}$ and doesn't vary with z. This is also true for foreign country.

As for intra-industry trade zone, by referring back to (3.16) which determines the two cut-offs, we see that the first term of left hand side is a decreasing function of $\bar{\varphi}_z$. Since Λ_z increases with z, it can be easily shown that $\bar{\varphi}_z' > 0$ or $\bar{\varphi}_z' = 0$ cannot maintain the equation, so it must be the case that $\bar{\varphi}_z' < 0$. Then the first term will increase as z increases. To maintain the equation the second term must decrease with z. So $\bar{\varphi}_{zx} = \Lambda_z \bar{\varphi}_z$ should be an increasing function of z. Applying the same logic, we can get the opposite results for foreign country: $\bar{\varphi}_z^{*\prime} > 0$ and $\bar{\varphi}_{zx}^{*\prime} < 0$. And this result rely on any assumption of the distribution here.$\blacksquare$

8.4 Proof of Proposition 4

From the proof of proposition 4, we know that $\Lambda_z < \Lambda_{z'}$ if $z < z\prime$ within the intra-industry trade region. Within the specialization zone, it can be easily found that $\Lambda_z = (\frac{fR}{R^*})^{1/a}$ which doesn't do with z. Since exporting probability $\chi_z = \frac{1-G(\bar{\varphi}_{zx})}{1-G(\bar{\varphi}_z)} = \Lambda_z^{-a}$ $(a > 1)$, conclusion (a) is obvious. For industries that both country produce, we know that $\chi_z = \frac{\tilde{\tau}^{-a} f - \varepsilon^a g(z)}{\varepsilon^a f g(z) - \tilde{\tau}^a}$from the proof of proposition 1. Using chain rule, we have

$$\frac{\partial\chi_z}{\partial z} = \frac{(1-\widetilde{\tau}^{-2a}f^2)\varepsilon^a ga}{(\varepsilon^a fg(z)-\widetilde{\tau}^a)^2}(\ln(A) - \frac{\sigma}{\sigma-1}\ln(\frac{r/w}{r^*/w^*}))$$

For average export intensity $\gamma_z \equiv \frac{\chi_z r(\widehat{\varphi}_{zx})}{r(\widehat{\varphi}_z)+\chi_z r(\widehat{\varphi}_{zx})} = \frac{\chi_z f_{zx}(\frac{\widehat{\varphi}_{zx}}{\bar{\varphi}_{zx}})^{\sigma-1}\sigma r^z w^{1-z}}{(f_z(\frac{\widehat{\varphi}_z}{\bar{\varphi}_z})^{\sigma-1}+\chi_z f_{zx}(\frac{\widehat{\varphi}_{zx}}{\bar{\varphi}_{zx}})^{\sigma-1})\sigma r^z w^{1-z}} = \frac{f_{zx}\chi_z}{f_z+f_{zx}\chi_z} = \frac{f\chi_z}{1+f\chi_z}$, thus $\frac{\partial\gamma_z}{\partial\chi_z} = \frac{f}{(1+f\chi_z)^2} > 0$.So γ_z is a monotonic increasing function of χ_z and should follow the same pattern.■

8.5 Proof of Proposition 5

Again from equation (3.16), we could calculate that:

$$\widehat{\varphi}_z = (\frac{a}{a+1-\sigma})^{\frac{1}{\sigma-1}}\bar{\varphi}_z = (\frac{a}{a+1-\sigma})^{\frac{1}{\sigma-1}}[\frac{(\sigma-1)\theta^a}{(a+1-\sigma)\delta\widetilde{f}}(1+f\chi_z)]^{\frac{1}{a}}$$

where $\widetilde{f} = \frac{f_{ez}}{f_z}$.Again it is monotonic function of χ_z and should follow the same pattern of it. Since we assume A(z) is the same for all industries, conclusion (a) is established. For conclusion (b), the average productivity for exporters and non-exporters are given by:

$$\begin{aligned}
\widehat{\varphi}_{zx} &= [\frac{1}{1-G(\bar{\varphi}_{zx})}\int_{\bar{\varphi}_{zx}}^{\infty}\varphi^{\sigma-1}g(\varphi)d\varphi]^{\frac{1}{\sigma-1}} = (\frac{a}{a+1-\sigma})^{\frac{1}{\sigma-1}}\bar{\varphi}_{zx} \\
\widehat{\varphi}_{znx} &= [\frac{1}{G(\bar{\varphi}_{zx})-G(\bar{\varphi}_z)}\int_{\bar{\varphi}_z}^{\bar{\varphi}_{zx}}\varphi^{\sigma-1}g(\varphi)d\varphi]^{\frac{1}{\sigma-1}} = (\frac{a}{a+1-\sigma})^{\frac{1}{\sigma-1}}(\frac{1-\Lambda_z^{\sigma-1-a}}{1-\Lambda_z^{-a}})^{\frac{1}{\sigma-1}}\bar{\varphi}_z
\end{aligned}$$

Thus the ratio of average productivity for exporters and non-exporters are:

$$\begin{aligned}
\frac{\widehat{\varphi}_{zx}}{\widehat{\varphi}_{znx}} &= \Lambda_z(\frac{1-\Lambda_z^{\sigma-1-a}}{1-\Lambda_z^{-a}})^{-\frac{1}{\sigma-1}} \\
&= \chi_z^{-\frac{1}{a}}(\frac{1-\chi_z}{1-\chi_z^{\frac{1+a-\sigma}{a}}})^{\frac{1}{\sigma-1}}
\end{aligned}$$

It is a decreasing function of χ_z and follows the opposite pattern of it within the intra-industry zone and remain constant within the specialization zone.■

8.6 Numerical Solution

Given the exogenous parameters, the algorithm below will enable us to solve the equilibrium variables. The idea is very much the proof of Proposition 1: suppose that the wage factor {w, w*, r, r*} is known, we could find the factor demand as a function of it. Then market clearing condition will pin down the unique solution. We set b(z)=1 for all z so as to satisfied $\int_0^1 b(z) = 1$ and in principle we specify other kind of utility functions. But this is the simpliest one to use.

The aggregate revenue for home and foreign are:

$$\begin{aligned} R &= wL + rK \\ R^* &= w^*L^* + r^*K* \end{aligned}$$

Factor intensity cut offs are:

$$\begin{aligned} \underline{z} &= \frac{\ln(\frac{\chi_{\underline{z}}\widetilde{\tau}^a + f\widetilde{\tau}^{-a}}{1+f\chi_{\underline{z}}}) - \frac{a\sigma}{1-\sigma}\ln(\frac{w}{w^*}) - a\ln(\lambda)}{\frac{a\sigma}{1-\sigma}\ln(\frac{r/w}{r^*/w^*}) + a\ln(A)} \\ \overline{z} &= \frac{\ln(\frac{\chi_{\overline{z}}\widetilde{\tau}^a + f\widetilde{\tau}^{-a}}{1+f\chi_{\overline{z}}}) - \frac{a\sigma}{1-\sigma}\ln(\frac{w}{w^*}) - a\ln(\lambda)}{\frac{a\sigma}{1-\sigma}\ln(\frac{r/w}{r^*/w^*}) + a\ln(A)} \end{aligned}$$

where $\chi_{\underline{z}} = \frac{R^*}{fR}$ and $\chi_{\overline{z}} = \frac{R^*}{fR}(\frac{f}{\widetilde{\tau}^a})^2$ are what we find in the proof of proposition 2. We also know that the equation solving home exporting probability within the intra-industry trade region is:

$$\chi_z = \frac{\widetilde{\tau}^{-a}f - g(z)}{fg(z) - \widetilde{\tau}^a} \tag{8.7}$$

where $g(z) \equiv (\frac{w}{w^*}(\frac{r/w}{r^*/w^*})^z)^{\frac{a\sigma}{1-\sigma}}$.

Then the factor demand within the specialization region are:

$$L_s = \int_0^{\underline{z}} l(z)dz = (\underline{z} - \frac{1}{2}\underline{z}^2)\frac{R+R^*}{w}$$

$$K_s = \int_0^{\underline{z}} k(z)dz = \frac{1}{2}\underline{z}^2\frac{R+R^*}{r}$$

$$L_s^* = \int_{\overline{z}}^{1} l^*(z)dz = (\frac{1}{2} - \overline{z} + \frac{1}{2}\overline{z}^2)\frac{R+R^*}{w^*}$$

$$K_s^* = \int_{\overline{z}}^{1} k^*(z)dz = (1-\overline{z}^2)\frac{R+R^*}{2r^*}$$

Using (8.4) we find that the factor demand within the intra-industry trade region are:

$$L_{int} = \int_{\underline{z}}^{\overline{z}} \frac{(1-z)R_z}{w}dz = \frac{1}{w}\int_{\underline{z}}^{\overline{z}}(1-z)[\frac{R}{1-\widetilde{\tau}^{-a}\varepsilon^a fg(z)} - \frac{fR^*}{\widetilde{\tau}^a\varepsilon^a g(z) - f}]dz$$

$$K_{int} = \int_{\underline{z}}^{\overline{z}} \frac{zR_z}{r}dz = \frac{1}{r}\int_{\underline{z}}^{\overline{z}} z[\frac{R}{1-\widetilde{\tau}^{-a}\varepsilon^a fg(z)} - \frac{fR^*}{\widetilde{\tau}^a\varepsilon^a g(z) - f}]dz$$

$$L_{int}^* = \int_{\underline{z}}^{\overline{z}} \frac{(1-z)R_z^*}{w^*}dz = \frac{1}{w^*}\int_{\underline{z}}^{\overline{z}}(1-z)\varepsilon^a g(z)[\frac{R^*}{\varepsilon^a g(z) - f\widetilde{\tau}^{-a}} - \frac{fR}{\widetilde{\tau}^a - \varepsilon^a fg(z)}]dz$$

$$K_{int}^* = \int_{\overline{z}}^{1} \frac{zR_z^*}{r^*}dz = \frac{1}{r^*}\int_{\underline{z}}^{\overline{z}} z\varepsilon^a g(z)[\frac{R^*}{\varepsilon^a g(z) - f\widetilde{\tau}^{-a}} - \frac{fR}{\widetilde{\tau}^a - \varepsilon^a fg(z)}]dz$$

In the equations above we use the goods market clearing condition and the definition of $P(z)$ and $P^*(z)$ to find out R_z^* and R_z.The factor Market Clearing condition is:

$$
\begin{aligned}
L_s + L_{int} &= L \qquad (8.8) \\
K_s + K_{int} &= K \qquad (8.9) \\
L_s^* + L_{int}^* &= L^* \qquad (8.10) \\
K_s^* + K_{int}^* &= K^* \qquad (8.11)
\end{aligned}
$$

From the market clearing condition we then pin down the equilibrium factor prices and other variables are simply function of factor prices.