Mixed search

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Abstract

In search theory, an important distinction can be drawn between models with competitive search and with random search; empirical work on job search models overwhelmingly assume random search. This paper uses linked register data on workers and firms to explore patterns of wages at job to job transitions. We argue that models with random search provide insufficient structure to match the tight correspondence in the data between the firm wage in the old and the new job. In an archetypical model with directed on-the-job search, we find that the pure job ladder equilibrium of this model is also unable to account for observed wage patterns, this time by predicting too little variation in the conditional distribution of new wages after a job change.

Next, we present a model where workers and firms meet through both directed and undirected search; we call this mixed search. This model is qualitatively consistent with patterns of wages across job-to-job transitions with wage gains. To account for wage cuts at job-to-job transitions, an extended model version makes search behavior contingent on realized match specific productivity, revealed to workers and firms after the match is formed.

Keywords: competitive search, random search, directed search, register data

JEL Classification Numbers: J30, J60

1 Introduction

In search theory, an important distinction can be made between models with random search on the one hand, and models with directed/competitive search on the other hand. The two models have a different role of wages. In competitive search equilibrium, firms can post higher wages in order to influence the workers' ex ante search decisions. When workers are randomly matched with firms however, wages can only affect the worker's ex post decision of whether or not to accept the offer. The distinction is important, as the two modes of wage determination have different predications for the welfare properties of labor market equilibrium.

Empirical work using microdata for structural estimation of job search models overwhelmingly assume random search (see, e.g. Postel-Vinay & Robin 2002, Cahuc et al. 2006). We argue that the random search assumption has implications for predicted wage dynamics at job-to-job transitions. In random search models with wage posting, workers typically accept all job offers where the wage offer is higher than their current job. Then, the expected distribution of firm wages after a job-to-job transition corresponds to the job offer distribution, truncated at the worker's current job. This also implies that the wage distribution after a job-to-job transition conditional on the new wage being above some threshold $\bar{w}$ should be independent of firm wage in the old job so long as the old firm wage is below $\bar{w}$.

To illustrate this point, consider two workers i, j earning w_i, w_j before a job-to-job transition, and assume $w_i < w_j < \bar{w}$. With random search and wage posting, i and j have the same contingent distribution of new wages, conditional on the wage in the new job being at least $\bar{w}$. Looking at firm average wages, a similar argument can be made for a wider range of random search models with different assumptions on wage setting. In the first part of this paper, we use linked administrative register data on workers and firms to explore patterns of wages at job-to-job transitions. First, most job-to-job transitions are characterized by a small upward wage change. Second, some workers move many steps up the job ladder. Third, a substantial number of transitions are associated with a wage cut. These three patterns hold whether we look at individual wages or firm average wages.

We argue that these patterns indicate to that the relationship between the new and the old wage has a close structure relative to the predictions of random search models. Specifically, the observed correspondence between wages in the old job and wages in the new job is too tight compared to what random search models would

Motivated by this, we set up an archetypical directed search model with on-the-job search and endogenous productivity distribution. Equilibrium is characterized by a strict job ladder where the wage in the new job is uniquely determined by the wage in the old job. Compared to what we find in the data, this model with directed search only provides too much structure, and is thus unable to account for the significant share of job-to-job transitions with large jumps in wages.

Next, we introduce a model with on-the-job search where workers search both random and directed. We call this mixed search. This model is able to qualitatively match patterns of job-to-job transitions with wage gains: the mixed search model predicts that while most job-to-job transitions will have small upward movements in wages, others will experience large jumps in wages. However, this model is still unable to explain the incidence of job-to-job transitions with wage cuts.

To account for this, we present an extension where the worker's search behavior is made contingent on realized match-specific productivity, which is revealed to both parties after the match is formed. In this version of the model, workers whose realized match specific productivity is sufficiently low will direct their

search towards less productive firms posting lower wages and, as their reservation wage will be lower, accept randomly drawn job offers from less productive firms. We argue that with match specific productivity, the mixed search model seems qualitatively consistent with observed patterns of wages at job-to-job transitions.

Related work includes the model in Garibaldi & Moen (2010) of competitive on-the-job search, though the present paper considers only the case where the productivity distribution is endogenous. There is a significant literature on the effects of job specific productivity shocks on job creation and destruction in random matching models (Mortensen & Pissarides 1994), while Guerrieri (2007, 2008) also feature models of competitive search with shocks to worker effort/training costs.

The mixture of random and competitive search is in some ways a labor market analogy to search theroetic models of product markets with informed and uninformed consumers, e.g. Lester (2011).

The paper is organized as follows: Section 2 discusses predicted wage dynamics of random search models. Linked employer employee microdata is then used to characterize the empirical patterns of wage dynamics at job to job transitions. Section 3 presents an archetypical model of directed on-the-job search. Section 4 presents our mixed search model and characterizes equilibrium. An extended version of the model with match specific productivity is presented in section 5 Section 6 presents some concluding remarks. A detailed presentation of the dataset as well as all proofs can be found in the appendix.

2 Wage dynamics

As discussed in the introduction, most empirical work on job search models assume random search. In this section, we first give a brief review of what this class of models predicts for the pattern of wages at job to job transitions. Next, microdata on workers and firms is used to explore empirical regularities in wages before and after job search. Specifically, this data allows us to compute empirical distributions of wages in the new job conditional on wages in the old job. These patterns can then be compared to the predictions of models with random on-the-job search.

2.1 Wage dynamics in random search

To characterize wage dynamics in random search models, consider first models with posted wages, i.e.Burdett & Mortensen (1998). In this model, workers and firms match at random, with the worker accepting all wages above his reservation wage, given by his current wage. It follows that the distribution of wages in the new job conditional on a worker's wage in the old job is equal to the distribution of wages across vacancies truncated at his old wage. Ranking the vacancies from lowest wage to highest wage, the worker has an equal probability of moving to each job ranked higher than his current job.

More generally, while various models random search with wage setting through bargaining may yield a similar dependence of new wages on old wages, if we were to rank jobs by productivity instead of wage, the argument above extends to a wider class of models. In models where workers and firms meet randomly, workers will accept any job with a higher productivity than his current job. Again, ranking vacancies from least to most productive, workers should have an equal probability of moving to each job ranked higher than his current job.

These predictions can be compared to patterns observed in data using linked samples of wages and job durations. As the distribution of vacancies is unobservable, we use the distributions of individual wages and firm wages in the population. In a stationary environment there is a unique correspondence between rank in the vacancy distribution and in the distribution across filled jobs, so the implications sketched out above should still hold. The following section will present our data and results.

2.2 Data

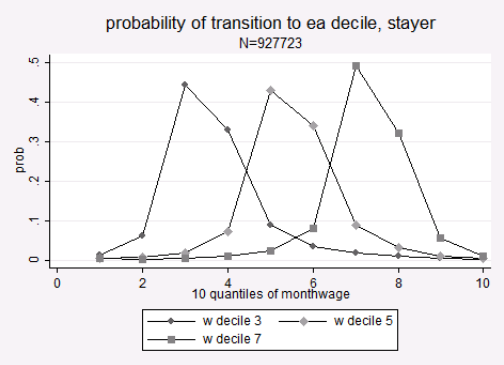
This paper uses administrative register data on workers and firms in an explicit search theoretic model. In this section, the sampling procedure linking labor market histories to wage data is described in some detail. Then, we consider some cross sectional properties of the data. Finally, we exploit the panel dimension of the data to look at individual wage dynamics. Data points towards a job ladder, with most job-to-job transitions leading to small upwards adjustment in wages. On the other hand, wage cuts are also relatively common.

Our principal source of data is Norwegian administrative registers on workers and firms. We use data from 2003 to 2007; while longer time series is available, we have chosen to limit the sampling period, as we will use moments of these data to calibrate a steady-state model and the steady-state assumption is unlikely to hold for longer time series. Unique person and firm identifiers allow use to construct two samples: one sample of individual labor market histories, recorded continuously, and one panel of individual wages and average firm productivity, recorded annually.

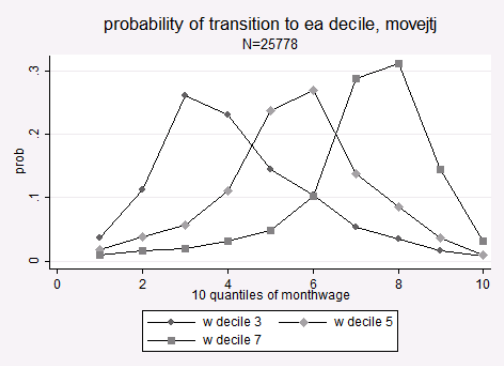
Each job in the wage sample is classified into one of ten bins by decile of the distribution of monthly wages in the population. Then, we consider observations where we have wage data available for two consecutive years. For each decile of origin, $i = 1, ..., 10$, we caluculate the probability that a person with a wage at time t in a certain decile i of the population wage distribution is observed at time $t+1$ with a new wage in decile $j = 1, ..., 10$ of the same distribution by the ratio of transitions from i to j to the total number of transitions originating in decile i. Figure 1 plots results for deciles 3,5 and 7; i.e. for jobs in the lower, medium and higher ends of the wage distribution. Looking at the second panel in figure 1, most job to job transitions appear to be associated with small wage changes, with most jobs ending up in the next or the same decile of the wage distribution. Compared to stayers, job to job transitions are more spread out.

Unobserved individual heterogeneity would cause the same pattern; but then we would expect this to be the case also for cases where the person has a

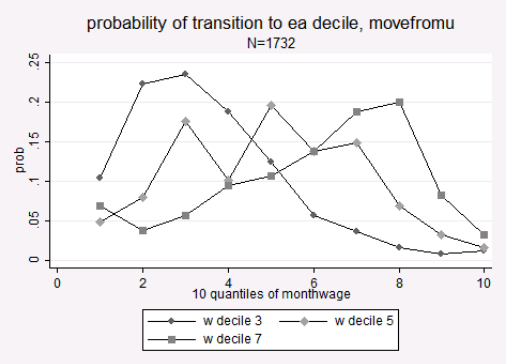
Figure 1: New wage rank, by rank of old job



(a) Stayers



(b) Movers jtj



(c) Movers utj

Note: Figure illustrates computed distribution of wages in year t conditional on the wage year $t-1$. Distributions calculated by classifying each job into one of ten binds by decile of the distribution of monthly wages in the population. For each decile of origin i , the fraction of job-to-job transitions where the new wage is in decile j is computed. Figures plot these fractions for transitions where the old job was in decile 3, 5 and 7. The upper panel represents persons who stayed at the same firm, middle panel represents persons who made a job-to-job transition and the lower panel represents persons who changed employers with an intervening period of unemployment.

registered unemployment spell between the two jobs. Comparing panels 2 and 3 of figure 1, we see that for transitions where there is an intervening period of unemployment between two jobs, there is less of a correspondence between the old wage and the new wages, and new jobs tend to be concentrated in the lower quantiles of the wage distribution.

Finally, models with state dependent bargaining power would predict similar patterns for individual wages. However, if we were to consider firm average wages instead of individual wages, these differences should average out. Figure 2 shows transition probabilities between deciles of the distribution of average firm wages. Looking at the second panel, workers whose old firm pays an average wage in decile j appear more likely to make a job-to-job transition to a firm in decile j or $j+1$ compared to higher deciles, which is difficult to explain using most models of random search with identical workers.

To summarize, while some workers experience larger wage jumps and others experience wage cuts, most job-to-job transitions are characterized by small upwards wage adjustments,. This also holds looking at firm average wages, which can be seen as a proxy of firm productivity. Recalling the discussion in section 2.1, this tight correspondence between the rank of the old job and the rank of the new job is difficult to explain using models of random search. The next section presents a simple model of directed on-the-job search with endogenous productivity distribution.

3 Directed on-the-job search

This section presents a simple model of directed on-the-job search. Before entry, firms choose from a menu of entry costs and associated production flow in filled jobs. This generates an endogenous productivity distribution and a pure job ladder in equilibrium. Workers will always move up the job ladder one step at a time, with zero probability of moving more than one step up the job ladder. We argue this is at odds with observed wage dynamics, where larger movements up the job ladder are fairly common.

The model is one of submarkets, where a market is composed of all firms paying a certain wage w and all workers who are searching in that market. The matching function in each submarket is

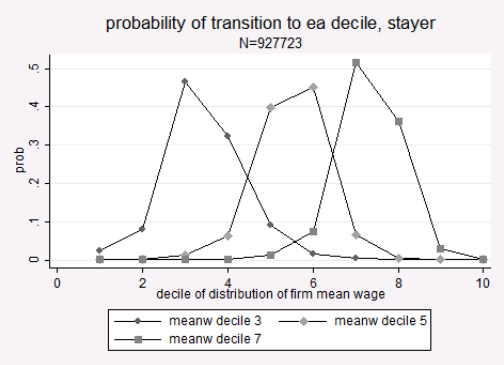
$$m = Au^{\beta}v^{1-\beta} \tag{1}$$

where u and v are the numbers of workers and firms searching in that submarket.

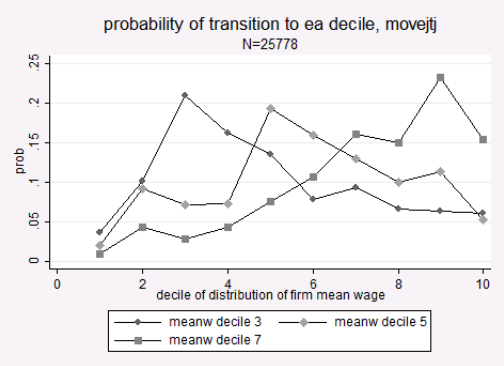
Firms pay investment costs K_i before they enter the market; we assume firms constantly post one vacancy. We denote by π_i the share of vacancies posted by type i firms. The total number of workers in the economy is normalized to 1: we denote by u_j the measure of workers employed in firms of type j, and let u_0 denote the unemployment rate.

Wage contracts specify a wage w, and on-the-job search possibilities. In the following, W_i^p denotes the expected NPV of contracts posted by type i firms.

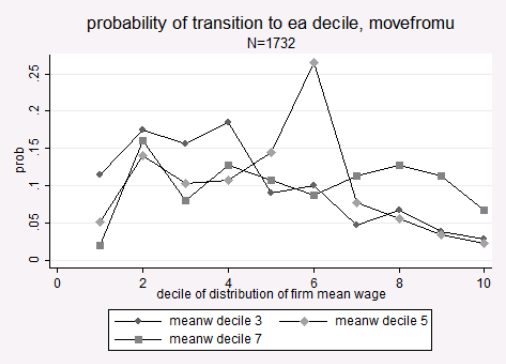
Figure 2: New firm average wage rank, by rank of old job



(a) Stayers



(b) Movers jtj



(c) Movers utj

Note: Figure illustrates computed distribution of firm average wages in jobs in year t conditional on the firm average wage year $t-1$. Distributions calculated by classifying each person-year into one of ten binds by decile of the distribution of firm average monthly wages in the population. For each decile of origin i , the fraction of observation transitions where the new firm wage is in decile j is computed. Figures plot these fractions for transitions where the old job was in decile 3, 5 and 7. Upper panel represents persons who stayed at the same firm, middle panel represents persons who made a job-to-job transition and the lower panel represents persons who changed employers with an intervening period of unemployment.

3.1 Workers

We assume workers search efficiently on the job, that is, there are no internal coordination problems in the firm (Moen & Rosén 2004), meaning workers search on the job so as to maximize joint income. A further implication of this assumption is that the worker's reservation wage is given by the value of his current job. Unemployed workers receive (exogenous) unemployment benefits y_0.

A worker employed in a firm with flow value of production y_i searches for jobs so as to maximize gains from search:

$$rM_i = y_i + s(M_0 - M_i) + \max_k p(\theta_k)(W_k^p - M_i) \tag{2}$$

Here, the first term y_i represents flow value of production on the job [1], the second term is expected loss from separation. The third term $p(\theta_k)(W_k - M_i)$ is the expected gain from directed job search; $p(\theta_k)$ is the arrival rate of job offers with posted contracts W_k^p to workers.

From equation (2) it is clear that different combinations of job offer arrival rates and expected values of posted contract give the same value M_i. Rearranging, we get the indifference curve for each worker type - combinations of θ, W yielding the same expected gain from job search $p(\theta_k^c)(W - M_i)$

$$\begin{aligned} \theta_i &= g_i(W; M_i, M_0) \\ &= p^{-1}\left(\frac{rM_i - y_i - s(M_0 - M_i)}{W_k^p - M_i}\right) \end{aligned} \tag{3}$$

In equilibrium, the relationship between advertised wage and θ is given by the lower envelope of the indifference curves

$$\theta(W) = \min_i(g_i(W; M_i, M_0)) \tag{4}$$

We assume workers employed in the most productive firms do not search at all, so the joint value of a type J job is given by

$$rM_J = y_J + s(M_0 - M_J) \tag{5}$$

3.2 Firms

We assume firms continuously post a single vacancy (the "fishing line" assumption). Firms of each type (investment level) posts wage contracts to maximize the value of the vacancy taking into account that the expected arrival rate of workers to vacancies depends on posted wages.

The value of a vacancy can then be written

$$rV_i = q(\theta_i^c)(M_i - W_i^p) \tag{6}$$

[1] unemployment benefits for $i = 0$

At entry, firms choose investment level and corresponding flow value of production in productive matches $\{y, K\}$ to maximize net profits; we assume free entry in each market.

Claim 1. *Any equilibrium where more than one investment level is chosen in equilibrium will be characterized by a job ladder where workers always wish to search for jobs in the submarket immediately above the one they currently work in. That is, only type $i-1$ workers direct their search toward type i firms. (See proof in appendix)*

Type i firms posting wage contracts maximize the value of vacancy given by (15) subject to the indifference curve (3) of type $i-1$ workers

The first order condition is:

$$\theta_i'(W^p) = \frac{q(\theta_i)}{q'(\theta_i)(M_i - W^p)}$$

3.3 Solving for equilibrium

First, we characterize optimal posted contracts. Firms of type i set wage W_i^p to maximize the value of a vacancy given by (15) taking into account that type $i-1$ workers search on-the-job so as to maximize the joint value of their current job, M_{i-1}.

Solving for optimal posted contract of a type i firm, W_i^{p*}:

$$W_i^{p*} = \beta M_i + (1-\beta) M_{i-1} \tag{7}$$

In equilibrium, (2) must hold for all i. Inserting for equilibrium posted contracts $W_i^c, i = 1, ..., J$ gives

$$rM_i = y_i + s(M_0 - M_i) + p(\theta_{i+1})\beta(M_{i+1} - M_i)$$

these make up a system of equations implicitly defining job offer arrival rates as functions of the vector M of job asset values, the distribution of vacancies π and the parameters of the model.

$$p_i = p_i(M, \pi)$$

which also gives market tightness and arrival rates of workers to firms: $\theta_i = \theta_i(M, \pi)$, $q_i = q(\theta_i(M, \pi)) = q_i(M, \pi)$

Also, inserting for W_i^p, W_i^r, the equilibrium expected value of a type i vacancy

$$rV_i^* = q(\theta_i) \times (1-\beta)(M_i - M_{i-1})$$

is a function of M and the distribution of workers across jobs (u_js).

Free entry implies the zero profit condition must hold in each submarket, so in equilibrium, the value of a vacancy must equal investments:

$$V_i = K_i \tag{8}$$

Moreover, in steady state, the stocks of workers in each firm type as well as the measure of unemployed workers have to be constant:

$$\dot{u}_i = 0 \tag{9}$$

In addition, aggregate consistency requires the following definition of market tightness in the competitive market of type i firms to hold

$$\theta_i = \frac{v_i}{u_{i-1}} \tag{10}$$

as well as aggregate market tightness

$$\Theta = \frac{\sum_i v_i}{1 - u_J} \tag{11}$$

Definition 1. *Equilibrium is vectors of contracts W, asset values M and market tightness θ that satisfies*

- *The first order conditions of workers and firms, i.e. satisfying equation* (7)
- *The free entry condition* (8)
- *The aggregate consistency conditions* (9), (10) *and* (11)

Claim 2. *Equilibrium exists*

3.4 Wages and mobility

Let Λ_{kl} denote the transition rate from jobs paying wage w_k to jobs paying wage w_l. In this stylized model, it holds that

$$\Lambda_{kl} = \begin{cases} 0 & \text{if } l \leq k \\ p(\theta_l) & \text{if } l = k+1 \\ 0 & \text{if } l > k+1 \end{cases}$$

That is, the model predicts a pure job ladder where workers always move up the job ladder one step at a time.

As discussed in the previous section, observed job-to-job transitions tend to be associated with small upward movements in wages, but large jumps in wages are also common, as are wage cuts. This model with pure directed search predicts zero probability of moving more than one step up the job ladder. In other words, while models with only random search are unable to generate enough correspondence between the wage in the old and new job, this model with purely directed search appears to generate too little variation.

To illustrate, figure 3 plots predicted shares of job-to-job transitions to each decile of the job rank distribution, for workers whose old job rank is in deciles 3, 5 and 7. The upper panel shows an example of predictions of directed search models: all job-to-job transitions are associated with small movements up the

job ladder, with zero probability of moving more than one step up the ladder. The lower panel shows the predictions of random search models, where transition probabilities are uniform above the decile of the worker's current job. Comparing these predictions to the plots in figures 1 and 2 illustrates how neither models of random matching nor our archetypical model of directed on-the-job search is able to match both the tendency of small upward movements in wages and the occurence of large wage jumps. In order to better account for such empirical patterns, the next section will show a model where workers and firms can meet both through random and directed search.

4 Mixed search model

This model is an extended version of the model presented above, with similar endogenous productivity distribution and associated tendency toward a pure job ladder. However in this workers and firms have two channels of matching. As before, workers search actively, choosing which submarket to direct their search to to maximize the value of search, matching according to the matching function in (1). At the same time, at rate p^r, workers randomly meet firms, making a random draw from the distribution of vacancies, accepting all job offers with a contract greater than the current joint NPV of the job. This model version is able to qualitatively match upward movements in wages at job to job transitions.

The number of "random" matches is given by the matching function m^r:

$$m^r = A^r u^\beta v^{1-\beta} \tag{12}$$

where u and v are the total number of searching workers and vacancies in the economy. That is, all searching workers in the economy face the same arrival rate of job offers through random matches, given by

$$p^r = \frac{m^r}{u} = A^r \Theta^{1-\beta}$$

As before, firms choose fixed cost K_i paid before entry and get the corresponding flow value of production y_i in filled matches. W_i^p still denotes the expected NPV of contracts posted by type i firms, W_i^r refers to expected NPV of contracts for workers and firms who meet through random matching.

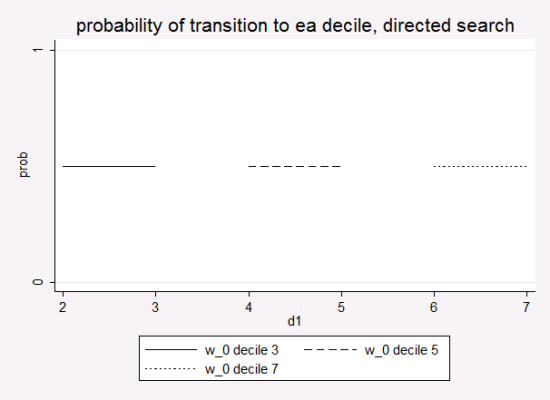
4.1 Workers

Workers employed in a firm with flow value of production y_i search for jobs so as to maximize gains from search:

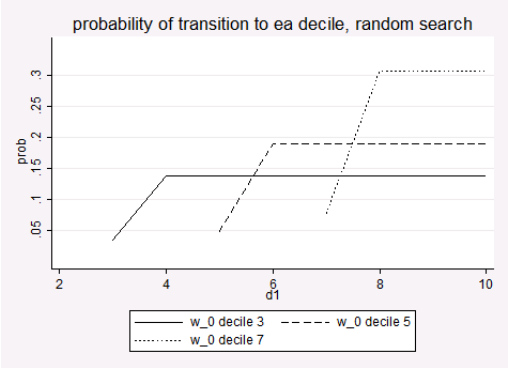
$$rM_i = y_i + s(M_0 - M_i) + \max_k p(\theta_k)(W_k^p - M_i) + p^r \sum_{j>i} \pi_j (W_j^r - M_i) \tag{13}$$

The final term here represents the expected gain from job search through random matching. At rate p^r, a worker and employer meet at random, with the

Figure 3: New job rank, by rank of old job, directed and random search



(a) Pure directed search



(b) Pure random search

Note: Figure illustrates examples of predicted distribution of new wages conditional on old wages for models with random search and directed search. The distributions have been computed by simulating a very simple job search process where jobs pay wages $w = 1, ..., 20$ and workers either accept all jobs paying strictly higher wages (corresponding to random search) or always search one submarket up (corrresponding to directed search).

working accepting all wage offers which are more valuable than their current job. With this additional term, the worker's indifference curve reflecting the tradeoff between queue length and wages can be written

$$\begin{aligned}\theta_i &= g_i(W; M_i, M_0) \\ &= p^{-1}\left(\frac{rM_i - y_i - s(M_0 - M_i) - p^r \sum_{j>i} \pi_j (W_j^r - M_i)}{W_k^p - M_i}\right)\end{aligned} \tag{14}$$

4.2 Firms

Like workers, firms have two ways of meeting workers, through directed search (wage posting) and through random matching. In addition to posting wages, firms are also randomly matched with workers at rate $\tilde{a}$; we assume efficient contracts so workers will accept the job if the value of the contract W_i^r is greater than the joint value of their current job.

The value of a vacancy can then be written

$$rV_i = q(\theta_i^c)(M_i - W_i^p)] + \tilde{q}_i(M_i - W_i^r) \tag{15}$$

where $\tilde{q}_i$ is the rate that type i firms hire workers through random matching:

$$\tilde{q}_i = q^r h_i$$

where Θ is the ratio of vacancies to searching workers in the economy as a whole and h_i is the probability that a worker is hired once a match is made. Once a match happens, the probability that the worker is hired is equal to the probability that their current job has a joint value less than W_i^r. Wages in random matches are assumed to be set through bilateral bargaining, so workers will accept any job where there is a surplus to be shared, that is, where the joint value of their current job has a joint value less than M_i, which will be the case as long as current match productivity is lower than y_i:

$$h_i = \frac{\sum_{j<i} u_j}{1 - u_J}$$

At entry, firms choose investment level and corresponding flow value of production in productive matches $\{y, K\}$ to maximize net profits; we assume free entry in each market. As long as there is some directed search, i.e. $A > 0$, the job ladder property still holds, and only type $i-1$ workers will direct their search toward type i firms. Therefore, the first order condition of the firms remains the same as in the pure directed search model.

4.3 Solving for equilibrium

First, we characterize optimal posted contracts and wages in random matches. The optimality conditions of workers and firms are unchanged, so the optimal posted contracts of type i firms are given by equation (7).

$$W_i^{p*} = \beta M_i + (1 - \beta) M_{i-1}$$

Optimal posted wage contracts can be solved for without considering random matching, as the processes are separate. This simplifies the analysis considerably, however it is worth noting that this need not be the case. If the two processes are linked, e.g. if firms have to offer the posted wage to random matches, and random search is sufficiently important, i.e. when p^r is large, some higher type firms may find it optimal to post a contract worth less than W_i^{p*}, and attract only random matches. The argument is essentially the same as in Lester (2011). The existence of such "rip-off" firms would complicate solving for equilibrium, however the qualitative conclusions of our model would not be affected, as it would always be optimal for some firms to post wages W_i^{p*} as outlined above as long as $A > 0$.

To avoid these issues, we make some assumptions on wage setting in random matches:

Assumption 1. *In random matches, wages are set through Nash bargaining under Hosios, with the worker's outside option is given by the joint value of the highest type job of which a worker would want to join the poaching firm.*

That is, unemployed workers meeting a type 2 firm will bargain over the distribution of M_2 using M_1 as their outside option. To rationalize this, one story can be that while a worker can observe type of the firm before bargaining takes place, the firm cannot observe the type of the worker (i.e. his current employer). This assumption simplifies the model as it ensure wages in jobs resulting from random matches pay the same wage as wages in jobs where the worker and firm meet through competitive search, that is, we will have a one-to-one relationship between firm type and wage.

$$W_i^{r*} = \beta M_i + (1-\beta)M_{i-1} = W_i^{p*} \tag{16}$$

In equilibrium, (13) must hold for all i. Inserting for $W_i^r, W_i^c, i = 1, ..., J$ gives

$$\begin{aligned} rM_i =& y_i + s(M_0 - M_i) + p(\theta_{i+1})\beta(M_{i+1} - M_i) \\ &+ p^r \sum_{j>i} \pi_j(\beta M_j + (1-\beta)M_{j-1} - M_i) \end{aligned}$$

forming a system of equations implicitly defining job offer arrival rates, market tightness and arrival rates of workers to firms, as functions of the vector M of job asset values, the distribution of vacancies π and the parameters of the model:

$$p_i = p_i(M, \pi), \theta_i = \theta_i(M, \pi), q_i = q(\theta_i(M, \pi)) = q_i(M, \pi)$$

Also, inserting for W_i^p, W_i^r, the equilibrium expected value of a type i vacancy

$$\begin{aligned} rV_i^* = &\left[q(\theta_i) + q^r \frac{\sum_{j=0}^{i-1} u_i}{1 - u_J} \right] \\ &\times (1-\beta)(M_i - M_{i-1}) \end{aligned}$$

is a function of M and the distribution of workers across jobs (u_js).

To close the model, the zero profit condition (8) must hold in each submarket and the aggregate consistency conditions (9)-(11) must be satisfied.

4.4 Wages and mobility

The model presented above have empirically relevant implications for the distribution of durations of unemployment spells and job spells, and for transition rates between different kinds of jobs.

Let Λ_{kl} denote the transition rate from jobs paying wage w_k to jobs paying wage w_l.

$$\Lambda_{kl} = \begin{cases} 0 & \text{if } l \leq k \\ p(\theta_l) + p^r \pi_l & \text{if } l = k+1 \\ p^r \pi_l & \text{if } l > k+1 \end{cases}$$

To see how this compares to related models, consider first the special case when there is no random search, that is, when $A^r \to 0$, the probability of transitions more than one step up the wage ladder approaches zero. This corresponds to Garibaldi and Moen (2010) with endogenous productivity distribution, with a pure job ladder where workers always move up the job ladder one step at a time. When $A \to 0$, the model gives similar predictions to random search with wage posting, with $\Lambda_{jl} = \Lambda_{kl}$ for any two $j, k < l$. That is, with no directed search, the transistion probabilities to higher paying jobs depend only on the distribution of vacancies π.

Next, consider the more general case when $A^r, A > 0$, that is, when both random and competitive search are important. Then $\Lambda_{kl} > \Lambda_{jl}$ when $k = l-1, j < l-1$. Put differently, directed search means that the relative transition probabilities to the submarket one step up the job ladder is higher than what would be predicted by the distribution of vacancies/jobs across submarkets.

With this model, we have managed to account for two of the three patterns of job to job transitions, however we are still unable to explain job to job transitions with wage cuts. For this reason, we extend the model to include stochastic match specific productivity.

5 Stochastic match productivity

To account for the substantial incidence of wage cuts around job to job transitions, we extend the model to allow for stochastic match productivity. In less productive matches, workers are pushed to search for lower paying jobs to maximize the current value of the job. Through this mechanism, some workers in high paying firms will be observed making job-to-job transitions from high to low wage firms.

Match productivity is uncertain at the time of hiring. A firm investing $K = K_i$ at the time of entry generates a production flow $y = y_i$ in productive matches. With probability δ_{ij}, the hire is a mismatch, yielding a lower productivity flow $y_j < y_i$, for $j = 0, ..., i-1$. With probability $1 - \sum_j \delta_{ij}$ the match is productive, yielding the higher production flow y_i corresponding to the firm's investment

level, for the duration of the match. We denote by u_j^i the measure of workers employed in firms of type j with realized match productivity flow y_i, and let u_0^0 denote the unemployment rate.

While the flow wage w is independent of realized productivity, on-the-job search possibilities specified in posted and bargained contracts are conditional on realized match productivity.

Once a match is formed (and the contract is signed), true match quality is revealed to both the worker and the firm. There is no recall employment, i.e. workers cannot return to their old employer, but have to keep searching while employed in the unproductive match with the new employer.

The realized joint value of a match to a worker and a firm will only depend on realized match productivity. However the realized division of surplus between the worker and the firm will depend on both type of firm/investment level as well as realized productivity, as the wage w is determined before true productivity is revealed.

5.1 Workers

Workers' optimal job search strategy is given by (2) as before, however now workers in the same firm type K_i will have different optimal search behaviors depending on realized match productivity. In random matches, workers will accept all jobs where the flow productivity in productive matches is higher than the flow productivity in their current match. To be clear, workers will always move from less productive matches to matches with higher expected productivity, however they might move from high to low productivity firms (as measured by average productivity.)

5.2 Firms

The expected value of a vacancy of a firm with investment level i will depend on the distribution of match specific productivities as measured by the parameters δ_{ij}:

$$\begin{aligned} rV_i =& q(\theta_i^c)(E[M|K=K_i] - W_i^p)] + \tilde{q}_i(E[M|K_i] - W_i^r) \\ =& q(\theta_i)\left[M_i + \sum_j \delta_{ij}(M_j - M_i) - W_i^p\right] \\ &+ q^r \frac{\sum_k \sum_{j<i} u_j^k}{1-u_J}\left[M_i + \sum_j \delta_{ij}(M_j - M_i) - W_i^r\right] \end{aligned} \tag{17}$$

Firms post wages W_i^p to maximize (17), taking into account that the arrival rate of workers $q(\theta(W))$ is a function of the posted wage.

5.3 Equilibrium

Equilibrium posted wages W_i^{p*} satisfy the first order conditions of the maximization problem of workers and firms, satisfying

$$\begin{aligned} W_i^{p*} &= \beta E[M|K = K_i] + (1-\beta)M_{i-1} \\ &= \beta(M_i + \sum_j \delta_{ij}(M_j - M_i)) + (1-\beta)M_{i-1} \end{aligned} \tag{18}$$

When solving for equilibrium, we impose the more restrictive wage setting rule in random matches, that is we impose assumption 1. Inserting for equilibrium contracts, W_i^p, W_i^r, in the asset value equation for vacancy V_i, the free entry condition when match productivity is uncertain is

$$\begin{aligned} rV_i^* = &\left[q(\theta_i) + q^r \frac{\sum_{j=0}^{i-1} u_i}{1-u_J}\right] \\ &\times (1-\beta)(M_i + \sum_j \delta_{ij}(M_j - M_i) - M_{i-1}) = rK_i \end{aligned} \tag{19}$$

Finally, the aggregate consistency conditions now require constant stocks of workers in each i,j-state to be constant:

$$\dot{u}_i^j = 0 \tag{20}$$

and the definition of market tightness in each submarket as well as aggregate market tightness have to be consistent:

$$\theta_i = \frac{v_i}{\sum_j^J u_{i-1}^j} \tag{21}$$

as well as aggregate market tightness

$$\Theta = \frac{\sum_i v_i}{1-u_J^J} \tag{22}$$

Definition 2. *With stochastic match productivity, equilibrium is a vectors of contracts W, asset values M and market tightness θ that satisfies*

- *The first order conditions of workers and firms, i.e. satisfying equation* (18)
- *The free entry condition* (19)
- *The aggregate consistency conditions* (20), (21) *and* (22)

5.4 Wage dynamics

Let Λ_{kl} denote the transition rate from jobs paying wage w_k to jobs paying wage w_l.

$$\Lambda_{kl} = \begin{cases} \delta_{kl-1} p(\theta_l) + p^r \pi_l \left(\sum_{j=0}^{l-1} \delta_{kj} \right) & \text{if } l \leq k \\ \left(1 - \sum_j \delta_{kj} \right) p(\theta_l) + p^r \pi_l & \text{if } l = k+1 \\ p^r \pi_l & \text{if } l > k+1 \end{cases}$$

When $\delta_{kj} = 0, \forall k, j$, the model's predicted wage dynamics are the same as in the basic model in section 4.4. When $\delta_{kj} > 0$, with some positive probability the match will have a low productivity and some workers will direct their search towards lower paying jobs as well as accept randomly drawn job offers from lower type firms. Through this mechanism, this extended model will generate job-to-job transitions with wage cuts.

6 Conclusions

Predicted wage dynamics in models with on-the-job search are different depending on whether one assumes random or directed search. Linked administrative register data on workers and firms allows us to empirically investigate the dynamics of firm average wages at job-to-job transitions. There is a tendency of wages in new jobs to be close to the wage in the old job; however large jumps in wages are not uncommon. These patterns remain even when we look at firm average wages, suggesting that the interior mode in the distribution of new jobs conditional on the wage in the old job is not only driven by unobserved heterogeneity. At the same time, a substantial share of job transitions are associated with a wage cut.

We argued these three facts are hard to account for using models with pure random or pure directed search. A model was presented where workers and firms meet both through directed search and random matches. To account for wage cuts, the model was extended to include stochastic match specific productivity. Characterizing equilibrium, we show that this allows us to qualitatively account for the pattern of wages at job-to-job transitions. The two channels of matching alone are sufficient to explain the shape of upward wage mobility, with an interior mode and long right tail of the conditional distribution of new jobs. While wage cuts can be explained by workers drawing low values of match specific productivity searching for less productive firms.

Appendix

Data appendix

Job spells are constructed using data from the employment register. A job is defined as a unique person-firm pair, using recorded start and stop dates to determine duration of the job spell. Unemployment spells come from data on

registered unemployment. In the Norwegian unemployment insurance system, registered unemployment can be full time or part time, permanent or temporary. To be counted as unemployed in our data, a person has to be registered as full time permanently unemployed for at least part of the unemployment spell.

The data source for hourly wages is Statistics Norway's "Wage Statistics". For private sector workers, the data source is questionnaires on individual workers filled out by all firms included in the annual sampling. Large firms are oversampled in the survey; in total, hourly wage data is available for 50-65% of workers in each industry each year. The dataset contains monthly earnings and contracted hours for each worker; full time equivalent monthly wages are computed using a standard working hours corresponding to 162.5 hours per month. All nominal figures, i.e. wages and accounting data, are deflated using the CPI.

As we are interested in how wages and productivity evolve as workers move out of unemployment and between jobs, it is important that we correctly identify job-to-job, job-to-unemployment and unemployment-to-job transitions. In particular, we have to be careful not to incorrectly classify an involuntary separation as a job to job transition. Two criteria are used to determine type of transition. First, we require that the recorded gap between stop of last spell and start of the next spell be less than 30 days. Second, we restrict the degree of overlap between spells, that the "new spell" should be ongoing for some time after the "old spell" has ended. Moreover, we exclude firms with observed mass layoffs, defined as a decrease in employment of 10% or greater, from the sample. The reason for doing this is that we believe that many of the observed job to job transitions from these firms may be involuntary, i.e. workers who have been fired and found new employment so fast, during the time of notice, that they are never recorded as unemployed. In order to avoid problems with changing firm identifiers leading to continuing jobs incorrectly being classified as job-to-job transitions, we require firms to appear in the employment register the calendar year before. Finally, we notice that the firm size distribution in the data is extremely skewed, with a few very large firms. To avoid these firms having an excess influence on population, the ten largest firms each year are excluded from the sample.

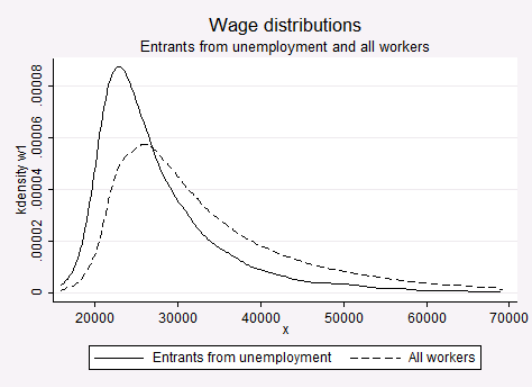
We restrict the sample further by excluding all individuals who changed education level during the sample period. In order to avoid the impact of life cycle phenomena on wage dynamics, we exclude workers younger than 25 and older than 60. Observations with missing wage data are excluded from the sample, as are observations which could not be linked to spell data (e.g. in wage statistics, but not in demographics file).

In our empirical analysis, we implicitly assume that the observed wage correctly reflects the flow value of the job to the worker. This assumption is potentially problematic if workers receive some income from ownership of the firms, which is especially relevant for small firms and family firms. To deal with this, persons who at some point have registered income from self employment are excluded from the sample; and we require firms to have at least 20 full time equivalent workers in order to be included in the wage sample. For the same reason we exclude observations on wages for workers employed in the public

Table 1: Descriptives, persons in sample

	Mean	Std. dev
Age (2003)	40.69	9.856
Higher education	27.4%	
High school education	61.8%	
Basic education only	10.8%	
Number of spells	1.941	1.385
Any unemployment	16.1%	
Any job to job transition	29.1%	
Any job to unemployment transition	5.71%	
Observations	534,821	

Figure 4: Density plot, wages



sector (central or local government, public schools or health care); to avoid complications comparing wages in the public and private sector (compensating differentials, pension arrangements etc.)

The final sample contains data on 534,821 individuals. Table 1 contains some summary statistics on these persons. The average person is registered with 1.94 spells in the durations sample. Using the criteria outlined above, job-to-job transitions are substantially more common than job to unemployment-transitions: while 29.1% of persons are registered with one or more job-to-job transitions, only 5.71% of persons have one or more job-to-unemployment transition during the 5 years individuals are followed in our sample.

Considering first the cross sectional properties of the data, figure 4 shows density plots of individual wages for entrants from unemployment and for all workers in the sample. The distribution of wages for entrants from unemployment lies to the left of the population distribution; it is also more highly skewed and has a lesser variance.

Table 2 summarizes the evolution of individual and firm average wages for persons observed in "wage statistics" for two (or more) consecutive years, clas-

Table 2: Descriptives, by group

	Stayers		Unemployment-to-job		Job-to-job	
	mean	sd	mean	sd	mean	sd
Monthly wage w	33884.6	10261.3	27346.6	7703.2	35203.8	11024.0
$\Delta \ln(w)$	0.0388	0.111	0.00502	0.214	0.0494	0.177
Wage cut	25.1%		45.6%		33.5%	
Monthly firm wage $\bar{w}$	33986.8	7851.1	29858.1	7284.5	35723.0	9897.0
$\Delta \ln(\bar{w})$	0.0311	0.0574	0.00385	0.242	0.0503	0.238
Firm wage cut	21.5%		46.0%		39.4%	
Observations	927723		1732		25778	

sified into three groups by whether they are stayers or movers, and if they are movers, whether the transition was job-to-job or with an intervening period of registered unemployment. The number of observations in the second column in particular is quite small: this is in part due to the requirement that the worker should be observed in the wage sample for two consecutive years and have an intervening unemployment spell ending with a transition to employment.

Compared to stayers, job-to-job movers have higher wage growth on average, however the variance of wage growth is also higher, so the share of transitions with a wage cut is higher, at 33.5%. Transitions with an intervening period of unemployment have, on average, a wage growth close to zero. The last three rows of table 2 summarize levels and growth of a proxy for firm average wages. The pattern here is fairly similar; entrants from unemployment work in firms with lower average wages, and experience lower growth in firm average wages compared to stayers and workers moving directly between jobs.

Proof of pure job ladder in equilibrium

Proof of existence

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