

# Risk Aversion Heterogeneity, Risky Jobs and Wealth Inequality\*

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## Abstract

This paper considers the macroeconomic implications of a set of empirical studies finding a high degree of dispersion in preference heterogeneity. It develops a model with risk aversion heterogeneity, uninsurable idiosyncratic income risk, and self-selection into risky jobs to quantify their effects on wealth inequality. The results show that, when estimating the risk aversion distribution with the appropriate PSID data on income lotteries, the model can match the observed degree of wealth inequality in the U.S., accounting for the wealth Gini index in several cases. The model replicates well many features of the wealth distribution, such as its quintiles. However, the share of wealth held by the top 1% is still substantially lower than in the data. Quantitatively, with fairly persistent income processes, the variance of the income shocks greatly matters in generating enough wealth inequality. It is also shown that models without risk aversion heterogeneity underestimate the size of precautionary savings by up to 14 percentage points, and that they account for up to a 55% increase of the complete markets capital stock.

**JEL Classification Codes:** E21, D52, D58, C68.

**Keywords:** Wealth Inequality, Heterogeneous Agents, Incomplete Markets, Computable General Equilibrium.

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# 1 Introduction

How much do observable preference heterogeneity in risk aversion and labor income risk account of the U.S. wealth inequality? This paper addresses this question within a macroeconomic model with incomplete markets, heterogeneous agents and (with or without) self-selection into risky jobs.

It is a well known fact that wealth is highly concentrated in the U.S., its Gini index being estimated in the 0.78–0.82 range for the period 1992–2007. At the same time, income and labor earnings are less unequally distributed, as discussed by Wolff (1998), Cagetti and De Nardi (2008), and Diaz-Gimenez, Glover, and Rios-Rull (2011).

The wealth distribution and its determinants have important implications for capital accumulation and growth, the design of optimal taxation schemes and their welfare consequences. These issues have been studied, for example, by Imrohoroglu (1998), Ventura (1999), Heathcote (2005), and Conesa, Kitao and Krueger (2009).

The role of entrepreneurship, Quadrini (2000) and Cagetti and De Nardi (2006), uninsurable income risk, Castaneda, Diaz-Gimenez, and Rios-Rull (2003), and intergenerational links, De Nardi (2004), have been found to be key in explaining the high concentration of wealth.<sup>1</sup> One aspect that has not been fully explored in accounting for wealth inequality is the role of preference heterogeneity. This is interesting, particularly in light of the findings of a few recent studies trying to elicit individuals' preferences and determine some measures of their dispersion.

This paper takes these empirical results seriously and quantifies how much wealth inequality is accounted for by agents' heterogeneity in their risk aversion. An otherwise standard heterogeneous agents macroeconomic model with incomplete markets and uninsurable idiosyncratic income risk is extended to allow for risk aversion heterogeneity. The framework is used to compute the effects of the latter on measures of wealth inequality and, more generally, on the overall wealth distribution, with or without endogenous sorting into jobs that differ in their implied labor income risk.

A recent body of empirical research has aimed at estimating individuals' risk aversion. These contributions have relied on either special modules included in large and well established surveys, or on experimental set-ups. Surveys such as the Health and Retirement Study (HRS) and the Panel Study of Income Dynamics (PSID) for the U.S. (Barsky, Juster, Kimball, and Shapiro (1997), Kimball, Sahm, and Shapiro (2008), and Kimball, Sahm, and Shapiro (2009)), and the Survey of Household Income and Wealth (SHIW) for Italy (Chiappori and Paiella (2011)), have been used to provide estimates of the risk aversion distributions for the underlying populations. In the HRS and, to a less

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<sup>1</sup>There is a vast literature developing models that try to account for the dispersion of wealth, reviewed in Quadrini and Rios-Rull (1997), and more recently in Cagetti and De Nardi (2008), and Heathcote, Storesletten and Violante (2009).

extent, in the PSID the respondents have been asked to answer a sequence of questions describing hypothetical lotteries on their *lifetime* income. Under appropriate assumptions, these answers provide a direct measure of the respondent's risk aversion. Differently, in the SHIW it is possible to exploit the households' portfolio composition and, crucially, its change over time (which is a rare feature for large surveys) to identify the distribution of risk aversion. In this case the empirical methodology does not measure preferences directly, rather it backs them out from actual intrinsically risky economic choices. The alternative experimental set-ups have been conducted either in a lab environment with a limited number of Danish individuals, Harrison, Lau, and Rutstrom (2007), or with an on-line interface targeting a large set of Dutch households, von Gaudecker, van Soest, and Wengstrom (2011). These experiments were designed for the participants to win or lose some *small* monetary stakes, by selecting the lottery they preferred from a menu of options that differed in their degree of risk.

The possible studies' limitations notwithstanding, a common and seemingly robust finding is the high dispersion of people's attitude towards risk. Both the survey and the experimental approaches to measuring preference dispersion have found considerable heterogeneity in the estimated risk aversion parameter. Figure (1) displays the risk aversion distributions resulting from the estimation procedures implemented by Kimball, Sahm, and Shapiro (2009) and Chiappori and Paiella (2011). I think it is fair to say that this plot provides compelling evidence on the dispersion of risk preferences.

**[Figure (1) about here]**

Identifying and estimating the risk aversion distribution is a challenging procedure. Pervasive measurement error and limited stakes in the experimental lotteries are unavoidable obstacles that make it a very hard empirical problem. However, in order to avoid some potential drawbacks of the analysis that will be discussed below, it is preferable to rely on sources of risk aversion heterogeneity that are empirically grounded, rather than treating it as unobserved heterogeneity.

First, I am going to consider a model where all individuals face the same stochastic income process. In this framework, two different specifications for preference heterogeneity are going to be proposed. The first case (KSS) will rely on the methodology proposed by Kimball, Sahm, and Shapiro (2009) to estimate the distribution of risk tolerance from PSID data. A second case (CP) will make use of the estimates on some moments of the risk aversion distribution provided by Chiappori and Paiella (2011), who rely on the SHIW, a panel dataset that tracks a representative sample of Italian households. The two estimated distributions of risk preferences differ in some essential features, and it is worthwhile exploring the implications of both cases.

Not only the individuals' attitude towards risk is an important ingredient in shaping the consumption and saving decisions, but also the amount of risk faced in the labor market will affect their

behavior. Labor income risk can be either considered as purely exogenous, or it can be partially driven by self-selection into jobs that differ in their earnings risk.

In the first case, for both preference distributions, several assumptions on the parameters representing the stochastic process for labor earnings are going to be considered.

Interestingly, with the available estimates of the risk aversion distribution, the model can match some salient features of the U.S. wealth distribution, while missing others. Compared to a model without preference heterogeneity, the degree of wealth concentration increases substantially. For a standard homogeneous preferences model the wealth Gini index lies in the 0.41–0.49 range (depending on the assumed stochastic process for income risk), while for the heterogeneous preferences model the range changes to 0.59–0.74. With an income process which is fairly persistent and with a relatively large variance, the PSID risk preference specification leads to a Gini index ranging from 0.72 to 0.74, which is quite close to what is observed in the data. However, substantially less persistent income processes imply a lower wealth concentration.

Overall, several features of the wealth distribution are replicated well, such as the bottom and top quintiles, unlike the share of wealth held by the top 1%, which is substantially underestimated in all specifications. Given that the richest households in the U.S. hold approximately one third of the total wealth, this is an important finding. Preference heterogeneity alone does not fully account for the determination of the top of the wealth distribution.

Although convenient, the assumption that individuals do not sort themselves into jobs that differ in their income volatility according to their preference for risk seems to be strong, and at odds with some recent evidence discussed in Guiso, Jappelli and Pistaferri (2002), Fuchs-Schundeln and Schundeln (2005), Bonin, Dohmen, Falk, Huffman, and Sunde (2007), and Schulhofer-Wohl (2011). The PSID data can be used to analyze how people’s actual earnings instability changes with their risk aversion. This paper also provides a first attempt to tackle this complex empirical issue. It will be assumed that only two different careers are available for every worker to embark on, whose entailed risks are perfectly known to the workers before entering the labor market. The workers are going to sort themselves into the two different jobs, by choosing one of the two different stochastic processes.

A calibrated version of the model with endogenous sorting generally confirms the findings obtained in the simpler version of the model. However, the results related to both wealth and consumption inequality are found to be closer to the data. The wealth Gini index is now in the 0.73–0.82 range, while the consumption Gini index in the 0.20–0.21 one.

Some additional results show that precautionary savings are underestimated in models without preference heterogeneity, with the bias being up to 14 percentage points. Furthermore, considering the homogeneous preferences counterpart of the heterogeneous preferences economy does lead to equilibria

and allocations that are quite different. The interest rate in the heterogeneous preferences economies is between 0.22 and 0.74 percentage points lower, always leading to larger capital stocks, output and consumption.

## 1.1 Related literature

In a seminal paper, Becker (1980) showed that in a deterministic growth model with infinitely lived agents that differ in their discount factors, the whole capital stock is held by the most patient dynasty. However, this result is mainly driven by the absence of uncertainty and by the assumption of complete markets.

There is limited quantitative work on the aggregate effects of preference heterogeneity in stochastic growth models. Notable exceptions are Krusell and Smith (1998), Cagetti (2003), Coen-Pirani (2004), Guvenen (2006), and Hendricks (2007). However, there are a few differences between my framework and theirs.

Krusell and Smith (1998) consider an RBC model with heterogeneous agents. They propose two versions of their model, one with preference heterogeneity and one without. Limited ad-hoc heterogeneity in the discount factors across generations is shown to have a major impact on wealth inequality, with the model being able to account for the observed Gini index only in this case.

Cagetti (2003) and Hendricks (2007) propose life-cycle models with incomplete markets, stochastic incomes, but without aggregate uncertainty. They focus on preference heterogeneity arising from differences in discount factors, and Cagetti (2003) allows for (some) heterogeneity in the risk aversion parameter as well. A structural estimation/calibration procedure is implemented to match a set of moments computed from the PSID and SCF samples. Cagetti (2003) estimates the preference parameters by matching the median age/wealth profile for three educational groups. However, he considers a partial equilibrium model. Hendricks (2007) picks the discount factors and their distribution to match the wealth Gini index profiles over the life-cycle. Their findings show that this type of preference heterogeneity accounts for the dispersion in wealth holdings of observationally equivalent households, and matches the high concentration of wealth.

Coen-Pirani (2004) and Guvenen (2006) study economies with aggregate fluctuations, multiple assets (a risk free asset and a risky one) and with a recursive utility framework. Their agents can potentially differ in both their risk aversions and in their Elasticities of Intertemporal Substitution (EIS). Although interesting, I don't follow this approach, because there are no reliable estimates for the EIS distribution for the overall U.S. population.<sup>2</sup> Coen-Pirani (2004) considers an endowment

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<sup>2</sup>The results discussed in Barsky, Juster, Kimball, and Shapiro (1997) are based on a very small sample (less than 200 observations), and apply to a selected sample, namely people in the later stages of their lives.

economy with preference heterogeneity only in the risk aversion parameter: his findings show that, contrary to the results obtained using standard expected utility, for some parameter values the long run distribution of wealth is dominated by the more risk averse agents. Guvenen (2006) proposes an RBC model with heterogenous EIS reconciling the findings of empirical studies using aggregate consumption data that estimate low EIS, and those of calibrated models designed to match growth and fluctuations facts that require a higher EIS. His results arise because of limited participation in stock markets together with an EIS increasing in wealth. The EIS estimated on simulated aggregate consumption data is small, because it reflects the EIS of the majority of the households, that are asset poor.

Compared to the literature, I allow for a form of preference heterogeneity which is empirically grounded, for GE effects, and for endogenously chosen risky careers. However, for tractability, I do not consider the effects of aggregate shocks.

In the empirical literature, Lawrance (1991) and Vissing-Jorgensen (2002) provide estimates of the consumption Euler equations that find heterogeneity in time preferences and in the EIS. Furthermore, the results in Chiappori and Paiella (2011) support the notion that individuals' relative risk aversion is constant: they find no significant response of the portfolio structure to changes in financial wealth.

The rest of the paper is organized as follows. Section 2 presents the theoretical model. Section 3 is devoted to the definition of the equilibrium concept used in the model. Section 4 presents the calibration procedure. Section 5 provides the main results and predictions of the baseline model, while Section 6 is devoted to the extension with endogenous sorting into risky jobs. Section 7 concludes. Three Appendices discuss the details of the numerical methods and provide some additional results.

## 2 The Economy

### 2.1 Risk Aversion Heterogeneity and Inequality: the Mechanics

The intuition behind this paper is easily understood with a simple graph.

[Figure 2 about here]

Figure (2) compares the endogenous wealth distributions that would arise in two different economies. Both economies share the same features, namely incomplete markets, an occasionally binding exogenous borrowing constraint and uninsurable labor income risk, as in Aiyagari (1994). However, they

differ in one aspect: in the first economy all agents share the same risk aversion parameter  $\gamma$ , while in the second economy there are both high risk aversion types ( $\gamma_h$ ), and low risk aversion ones ( $\gamma_l$ ).<sup>3</sup> Figure 3 displays utility functions of the CRRA class for different degrees of constant relative risk aversion  $\gamma$ : the higher  $\gamma$ , the more concave the utility function and the stronger the precautionary savings motive (Huggett and Ospina (2001)).

**[Figure 3 about here]**

For the sake of the argument, let's assume that both types have the same mass  $\mu_{\gamma_h} = \mu_{\gamma_l} = \frac{1}{2}$ , and that the types are permanent, meaning that there is no evolution in their risk aversion.

In the first economy the wealth distribution is non-degenerate because agents want to self-insure against the future risk represented by income fluctuations. This leads to a wealth distribution which could be represented as the one in the middle of Figure (2). Asset rich agents are the ones who experienced a long sequence of good income shocks, unlike the asset poor ones, allowing them to pile up a large stock of wealth. In the second economy, agents face the same uncertainty, i.e. the same stochastic process for labor income. However, the endogenous wealth distribution is going to change from the first economy: low risk averse types have a lower desire to smooth consumption across states of the world, hence they are going to accumulate less assets for self-insurance purposes. Differently, the more risk averse agents are going to save more, in order to achieve a better consumption smoothing. If the two agent types were to live in isolation, their wealth distributions would look like the ones in Figure (2). The wealth distributions of the two types are still non-degenerate, because labor income risk is still present and with concave utility functions some self-insurance is going to take place. In the absence of General Equilibrium (GE) effects, the economy would display a wealth distribution represented by the mixture of the two underlying ones. An outcome is clear: moving from the first economy to the second one, wealth inequality is going to increase, because the supports of the heterogeneous types wealth distributions will start diverging. The higher the difference in their risk aversion, the stronger the tendency for the two groups to accumulate different wealth levels.

However, the economy with heterogeneous types doesn't necessarily aggregate into its homogeneous types counterpart, with the average (or median) risk aversion: risk aversion heterogeneity is likely to

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<sup>3</sup>The analysis will rely on utility functions of the CRRA class. This choice is dictated by the set of results discussed in Kimball, Sahm, and Shapiro (2008) and Kimball, Sahm, and Shapiro (2009), obtained assuming this type of preferences, and in Chiappori and Paiella (2011), who directly test this assumption. Notice that the first two papers provide estimates for the risk tolerance parameter  $\tau$ , not the risk aversion  $\gamma$ . However, there is a simple relationship between the two:  $\gamma = \frac{1}{\tau}$ . Consequently, with CRRA utility, risk tolerance and elasticity of intertemporal substitution coincide. Moreover, recall that when a random variable is lognormally distributed, its reciprocal is lognormally distributed as well, with the same parameter  $\sigma_\gamma^2$  but opposite  $\mu_\gamma$ :  $\gamma \sim LN(\mu_\gamma, \sigma_\gamma^2) \rightarrow \tau = \frac{1}{\gamma} \sim LN(-\mu_\gamma, \sigma_\gamma^2)$ .

lead to different aggregate capital supplies, triggering GE effects. A strong enough precautionary saving motive of the high risk aversion types leads to an increase in the aggregate capital supply, driving down the equilibrium interest rate. As a consequence, the saving motive for intertemporal reasons is reduced, because of the decreased rate of return: this GE effect leads to a further change in wealth inequality.

This simple example explains the mechanics of how wealth inequality responds when preference heterogeneity in risk aversion is included in the framework. At the same time, this also raises a potential problem. With enough flexibility, virtually any degree of wealth inequality (far enough from perfect equality) can be achieved. By picking appropriately the degree of uncertainty, the risk aversion parameters, and the distribution of types any degree of wealth concentration can be obtained, a point made by Quadrini and Rios-Rull (1997).

By way of an example, let's consider the case of maximum degree of inequality, that is a wealth Gini coefficient that is arbitrarily close to 1. In order to achieve this outcome, it is sufficient to impute a high degree of risk aversion for the high types, say  $\gamma_h = 10$ , and assign them just an almost negligible positive mass  $\mu_{\gamma_h} = \epsilon_\mu > 0$ . Likewise, the low types are assigned a small degree of risk aversion, say  $\gamma_l = \epsilon_\gamma > 0$ , and a mass  $\mu_{\gamma_l} = 1 - \epsilon_\mu \approx 1$ . This economy would display a wealth Gini coefficient close to 1, as the high degree of risk aversion for the high types would lead them to accumulate a lot of assets and at the same time with little dispersion, while the low types with preferences close to linear would not engage in extensive precautionary savings.

Given these considerations, the computational experiment carried out in this paper could be flawed. Treating preferences as unobserved heterogeneity can potentially lead to identification issues. Different preference distributions, with different implied equilibria, could match the very same moments. Moreover, considering preference heterogeneity as parametric unobserved heterogeneity, whose parameters are pinned down by a set of moments in the wealth distribution, can lead to some unpleasant implications. According to such theories, trends and fluctuations in the wealth distribution could be explained by changes in preferences. A by-product of this approach is that it could lead to virtually impossible relative assessments of different policy interventions, and their welfare implications. Unless the researcher knows how preferences evolve over time, and what triggers their change, welfare analysis is not a viable option.

In order to avoid the potential drawbacks of this framework, a lot of discipline is imposed in the model's parameterization, in order to avoid issues of the type mentioned above.

As for the amount of uncertainty that the agents are going to face, several estimates for wage processes that are routinely used in the quantitative literature on macroeconomics with heterogeneous agents are going to be considered. If the wage processes are truly exogenous with respect to the agents'

degree of risk aversion, this step is a valid one. Since this assumption could be violated, an extension considers a model with endogenous sorting into risky jobs.

As explained above, the analysis will rely on sources of risk aversion heterogeneity that are based on a set of empirical findings trying to measure it. Although feasible, a calibration of the preference distribution by matching some features of the saving behavior (hence some moments of the wealth distribution) will not be attempted, to impose a lot of discipline in the quantitative implementation of the model.

## 2.2 The Baseline Model

First, a model with only one exogenous stochastic income process is going to be proposed. A simple extension with endogenous sorting into two jobs that differ in their degree of risk will follow.

## 2.3 Demographics

Time is discrete. The economy is populated by a measure one of infinitely lived agents (workers) denoted by  $i$ .<sup>4</sup>

## 2.4 Preferences

Agents' utility function is defined over stochastic consumption sequences  $c_t$

$$\begin{aligned} E_0 U(c_0, c_1, \dots; \gamma_i) &= E_0 \sum_{t=0}^{\infty} \beta^t u(c_t; \gamma_i) \\ u(c_t; \gamma_i) &= \frac{c_t^{1-\gamma_i} - 1}{1 - \gamma_i}, i \in [0, 1] \end{aligned} \tag{1}$$

the future is discounted at rate  $\beta \in (0, 1)$ , which is the common discount factor. The per period utility  $u(\cdot)$  is strictly increasing, and strictly concave. Differently from models without preference heterogeneity, it depends explicitly on the risk aversion parameter  $\gamma_i$ . Every agent  $i$  is born with an innate attitude towards risk, as captured by their CRRA parameter, which is a permanent feature.<sup>5</sup>

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<sup>4</sup>Data limitation suggests to consider infinitely lived agents. In the PSID the risk aversion question was asked in 1996 only, making it hard to know if the higher risk aversion observed for parents vs. their offspring is due to a cohort effect, or if it is an intrinsic demographic trait, possibly due to risk aversion increasing with age instead. Furthermore, Kimball, Sahm, and Shapiro (2009) find a positive correlation between the risk aversion of parents and that of their children.

<sup>5</sup>In the HRS, it is possible to keep track of how an individual answers the same risk aversion question over time. As shown by Kimball, Sahm, and Shapiro (2008), many people change their answers across waves, and they propose an econometric methodology addressing the survey response error. Treating the switches as measurement error is a way of rationalizing this outcome.

## 2.5 Endowments

There is a stochastic process for the effective units of labor  $\varepsilon$  a worker is going to supply in the labor market. This process is assumed to be an exogenous continuous first order Markov process.<sup>6</sup>

## 2.6 Technology

The production side of the model is represented by a constant returns to scale technology of the Cobb-Douglas form, which relies on aggregate capital  $K$  and labor  $L$  to produce the final output  $Y$ :

$$Y = F(K, L) = K^\alpha L^{1-\alpha}.$$

Capital depreciates at the exogenous rate  $\delta$  and firms hire capital and labor every period from competitive markets. The first order conditions of the firm provide the expressions for the net real return to capital  $r$  and the wage rate per efficiency unit  $w$ :

$$r = \alpha \left( \frac{L}{K} \right)^{1-\alpha} - \delta, \quad (2)$$

$$w = (1 - \alpha) \left( \frac{K}{L} \right)^\alpha. \quad (3)$$

Notice that the marginal productivity of labor is always positive, hence firms will rely on the total sum of the efficiency units of labor. It follows that in the steady-state:

$$L = \int \varepsilon d\mu_L(\varepsilon)$$

where  $\mu_L(\varepsilon)$  is the stationary distribution over the labor endowments implied by the Markov process.

## 2.7 Other market arrangements

The final good market is competitive. Firms hire capital every period from a competitive market. Capital is supplied by rental firms that borrow from workers at the risk-free rate  $r$  and invest in physical capital.

There are no state-contingent markets to insure against income risk, but workers can self-insure by saving into the risk-free asset. The agents also face a borrowing limit, denoted as  $b \geq 0$ .

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<sup>6</sup>The analysis will focus on steady states, hence from now on time subscripts will be suppressed.

### 3 Stationary Equilibrium

First, the problem of a worker and the problem of the firm are defined. The individual state variables are the labor endowment  $\varepsilon \in \mathcal{E} = [0, \bar{\varepsilon}]$ , and asset holdings  $a \in \mathcal{A} = [-b, \bar{a}]$ . Moreover, risk aversion  $\gamma \in \Gamma = [\underline{\gamma}, \bar{\gamma}]$  represents a permanent state for the agents. The stationary distribution is denoted by  $\mu(\varepsilon, a; \gamma)$ .

#### 3.1 Problem of the agents

This Section first defines the problem of the agents in their recursive representation, then it provides a formal definition of the recursive competitive equilibrium.

##### 3.1.1 Problem of the workers

The value function of an agent whose current asset holdings are equal to  $a$ , whose current labor endowment is  $\varepsilon$ , and with innate risk aversion  $\gamma$  is denoted with  $V(\varepsilon, a; \gamma)$ . The problem of these agents can be represented as follows:

$$V(\varepsilon, a; \gamma) = \max_{c, a'} \{u(c; \gamma) + \beta E_{\varepsilon'|\varepsilon} V(\varepsilon', a'; \gamma)\} \quad (4)$$

*s.t.*

$$\begin{aligned} c + a' &= (1 + r)a + w\varepsilon \\ \log \varepsilon' &= \rho_y \log \varepsilon + \eta', \eta \sim iid N(0, \sigma_y^2) \\ a_0 \text{ given, } c &\geq 0, \quad a' > -b \end{aligned}$$

Agents have to set optimally their consumption/savings plans. They enjoy utility from consumption, and face some uncertain events in the future. In the next period they will still have the same risk aversion parameter, but their labor income can go up or down, depending on the future realizations of the earnings shock  $\eta$ .

#### 3.2 Recursive Stationary Equilibrium

**Definition 1** *A recursive stationary equilibrium is a set of decision rules  $\{c(\varepsilon, a; \gamma), a'(\varepsilon, a; \gamma), k = \frac{K}{L}\}$ , value functions  $V(\varepsilon, a; \gamma)$ , prices  $\{r, w\}$  and a set of stationary distributions  $\mu(\varepsilon, a; \gamma)$  such that:*

- Given relative prices  $\{r, w\}$ , the individual policy functions  $\{c(\varepsilon, a; \gamma), a'(\varepsilon, a; \gamma)\}$  solve the household problem (4) and  $V(\varepsilon, a; \gamma)$  are the associated value functions.
- Given relative prices  $\{r, w\}$ ,  $k$  solves the firm's problem (2)-(3).
- The asset market clears

$$K = \int_{\mathcal{E} \times \mathcal{A} \times \Gamma} a d\mu(\varepsilon, a; \gamma)$$

- The goods market clears

$$F(K, L) = \int_{\mathcal{E} \times \mathcal{A} \times \Gamma} c(\varepsilon, a; \gamma) d\mu(\varepsilon, a; \gamma) + \delta K$$

- The stationary distributions  $\mu(\varepsilon, a; \gamma)$  satisfy

$$\mu(\varepsilon', a'; \gamma) = \sum_{\varepsilon} \int_{a: a'(\varepsilon, a; \gamma) = a'} \pi(\varepsilon', \varepsilon) d\mu(\varepsilon, a; \gamma), \forall \gamma \in \Gamma \quad (5)$$

In equilibrium the measure of agents in each state is time invariant and consistent with individual decisions, as given by the equation (5) above.<sup>7</sup>

### 3.3 Discussion

As with many other dynamic problems, the Euler equations help getting an intuition of the main intertemporal trade-offs that the agents in this economy are facing. They offer a more formal argument underlying the intuition provided in Figure (2). The optimal consumption functions  $c(\varepsilon, a; \gamma)$  satisfy the following optimality condition, where the equality holds whenever the agents are not borrowing constrained:

$$c(\varepsilon, a; \gamma)^{-\gamma} \geq \beta(1+r) E_{\varepsilon'|\varepsilon} [c(\varepsilon', a'; \gamma)^{-\gamma}]$$

$c(\cdot)^{-\gamma}$  is a convex function, hence  $E_{\varepsilon'|\varepsilon} [c(\varepsilon', a'; \gamma)^{-\gamma}] \geq \{E_{\varepsilon'|\varepsilon} [c(\varepsilon', a'; \gamma)]\}^{-\gamma}$  because of Jensen's inequality. In order to take care of this property, it is always possible to define a positive number  $\eta(\gamma)$  such that  $E_{\varepsilon'|\varepsilon} [c(\varepsilon', a'; \gamma)^{-\gamma}] = \{E_{\varepsilon'|\varepsilon} [c(\varepsilon', a'; \gamma)\eta(\gamma)]\}^{-\gamma}$ . More precisely, the CRRA parameter affects the strength of Jensen's inequality, and for any  $\gamma$  there is an associated  $\eta(\gamma)$  satisfying  $0 < \eta(\gamma) < 1$  and  $\frac{d\eta(\gamma)}{d\gamma} < 0$ . Substituting this expression in the optimality condition and rearranging gets:

$$\frac{E_{\varepsilon'|\varepsilon} c(\varepsilon', a'; \gamma)}{c(\varepsilon, a; \gamma)} \geq \frac{[\beta(1+r)]^{\frac{1}{\gamma}}}{\eta(\gamma)}.$$

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<sup>7</sup>Notice that the equation already exploits the Markov Chain representation of the continuous process for  $\varepsilon$ .

The LHS represents the expected consumption growth, which depends on  $\beta, \gamma$  and  $r$ . It is easy to show that, for a given interest rate, the expected consumption growth is increasing in both the discount factor and the risk aversion.<sup>8</sup> The higher  $\gamma$ , the more convex the function  $c(\cdot)^{-\gamma}$ , and the lower the number  $\eta(\gamma)$ , which increases the expected consumption growth because of the increased precautionary savings. At the same time, the RHS is highly non-linear in  $\gamma$ : this property can rationalize a result that is previewed now, and discussed in more detail in Section 6. Krusell and Smith (1998) found that approximate aggregation holds in their economy even in the case with heterogeneous  $\beta$ 's. In the economy considered here, often this will not be the case. The aggregate allocations of the heterogeneous preferences economy are going to be quantitatively quite different from the ones of its homogeneous preferences counterpart. This is going to be true both for the model with only one risky job, when the variance of the shocks is large enough, and for the model with multiple risky jobs.

There are two possible explanations. The first one relates to their calibration, i.e.  $\gamma = 1$ . With this value, expected consumption growth is linear in  $\beta$  for unconstrained individuals, who are almost all the agents in their economy. As a consequence, the aggregate allocations of the heterogeneous preferences economy are going to be extremely close to the ones of the homogeneous preferences one. This result does not necessarily hold for an economy where the heterogeneity pertains the  $\gamma$ 's, in which (because of the non-linearity) the average of all  $\gamma$ -types consumption growths can be pretty far from the consumption growth of the average  $\gamma$ . Overall, this explanation seems to be of second order.

A more relevant explanation is related to the amount of labor market risk, which in this economy is considerably higher than in theirs for many stochastic income processes. When solving the economy with a stochastic process similar to the one used in Krusell and Smith (1998), precautionary savings shrink to only a fraction (1.5%) of the capital stock in the complete markets economy (vs. up to 17.1% for the benchmark model, and 54.6% for the two risky jobs economy), leading to allocations that are extremely close to each other both in the heterogeneous and in the homogeneous preferences cases.<sup>9</sup>

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<sup>8</sup>Notice that  $\frac{dLHS}{d\beta} = \frac{[\beta(1+r)]^{\frac{1}{\gamma}}}{\beta\gamma\eta(\gamma)} > 0$  and  $\frac{dLHS}{d\gamma} = -\frac{[\beta(1+r)]^{\frac{1}{\gamma}} \ln[\beta(1+r)]}{\gamma^2\eta(\gamma)} - \frac{[\beta(1+r)]^{\frac{1}{\gamma}} \eta'(\gamma)}{[\eta(\gamma)]^2} > 0$ . The sign of the last expression is obtained by recognizing that in incomplete markets economies  $\beta(1+r) < 1$ , and  $\ln[\beta(1+r)] < 0$ . Moreover  $\frac{d\eta(\gamma)}{d\gamma} < 0$ , given that  $\eta(\gamma) = \frac{\{E_{\varepsilon'|\varepsilon}[c(\varepsilon', a'; \gamma)^{-\gamma}]\}^{-\frac{1}{\gamma}}}{E_{\varepsilon'|\varepsilon}[c(\varepsilon', a'; \gamma)]}$  and thanks to the properties of generalized means  $M_{(\cdot)}$  for non-negative random variables. The numerator of  $\eta(\gamma)$  is the generalized mean of order  $-\gamma$  while the denominator is the mean of order 1, and generalized means are such that  $M_{(p)} < M_{(q)}$ , for  $p < q$ . Finally,  $\eta(\gamma)$  is bounded above by 1 because  $\gamma > 0$ , and it is bounded below by 0 because  $M_{-\infty} = \min c(\cdot) \geq 0$ .

<sup>9</sup>This case considers a quarterly model, with only two labor market states, and a calibration broadly consistent with theirs. Unemployment ( $u$ ) income is equal to zero, and employment ( $e$ ) income is equal to  $0.3271 * w$ . The transition probabilities are  $\pi(e', e) = 0.9565$  and  $\pi(u', u) = 0.5$ ,  $\beta = 0.99$  and  $\delta = 0.025$ . In the working paper version of Krusell and Smith (1998), they study a case with ad-hoc heterogeneity in risk aversion, and find that approximate aggregation holds also in that case. Their finding, together with mine, suggests that the amount of uninsurable income risk seems

## 4 Parameterization

This section describes the calibration strategy. The length of the model period is set to one year. The calibration starts by taking as given the two elements characterizing the uncertainty in this economy: how volatile and persistent agents' labor income is and how agents relate to the uncertainty arising from the postulated stochastic income process. Several cases are going to be considered and discussed in the following.

The remaining parameters are set such that in the steady state equilibrium the incomplete markets economy matches some characteristics of aggregate level data. It is worth stressing that no parameters are chosen to match selected features of the wealth distribution. The complete parameterization of the model is reported in Table 1.

[Table 1 about here]

### 4.1 Parameterizing Uncertainty

Uncertainty plays a double role in the model. On the one hand the agents are going to face stochastic income sequences with a given persistence and variance, on the other hand they are going to react to these possible income histories differently, according to their innate CRRA parameter.

#### 4.1.1 Income Uncertainty

For each risk aversion distribution, four different specifications of the exogenous process for the labor efficiency endowments are going to be considered. These are reported in Table 2.

[Table 2 about here]

The stochastic process for labor earnings is a key element in the analysis. Different degrees of uncertainty, both in terms of the shocks size and how persistent these shocks are, matter for the incentives to save and for the degree of wealth inequality. In order to perform some robustness checks and to gauge how much earnings uncertainty matters for the results, four different processes are considered. These are all similar in their econometric specification: they all share the same AR(1)

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to be essential for the approximate aggregation results.

process, which has two parameters. The persistence parameter is denoted by  $\rho_y$ , while the variance of the innovations by  $\sigma_y^2$ . Table 2 outlines the complete parameterization of the earnings processes.<sup>10</sup>

The label *FLIN* in Table 2 refers to the specification estimated on PSID data by Floden and Linde (2001), while the label *FREN* refers to the specification estimated by French (2005). Finally, *GHIP* and *GRIP* are the two estimates provided by Guvenen (2009). The former refers to the Heterogenous Income Profile case, while the latter to the Restricted Income Profiles, in his terminology.

These four processes were selected because they imply quite a different persistence and variance of the innovations:  $\rho_y$  ranges from 0.82 in the *GHIP* case, to 0.988 in the *GRIP* case, while  $\sigma_y$  ranges from 0.12 in the *FREN* case, to 0.21 in the *FLIN* case.

#### 4.1.2 Risk Aversion Distribution

As for the risk aversion distributions, two different specifications are going to be considered. These are reported in Table 3.

[Table 3 about here]

First, the distribution for the risk aversion parameter is estimated following the procedure outlined by Kimball, Sahm, and Shapiro (2009), and this case will be denoted as KSS. Under the assumption that the risk preference parameter is log-normally distributed in the population,  $\gamma \sim LN(\mu_\gamma, \sigma_\gamma^2)$ , it is possible to exploit the answers to the survey gambles in the PSID, and estimate with maximum likelihood an ordered probit model with known thresholds. Given the answers to the sequence of gambles, each individual in the PSID who was administered the labor income gambles falls into one out of six possible risk categories. The MLE procedure finds the parameters of the log-normal distribution that maximize the probability of having observed the actual choice categories. The two parameters' point estimates are  $\mu_\gamma = 1.07$  and  $\sigma_\gamma^2 = 0.76$ .<sup>11</sup>

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<sup>10</sup>Table 11 in Appendix C provides some statistics and outcomes implied by the four processes. Tauchen's discretization of an AR(1) process implies a stationary distribution of the markov chain that is symmetric. This is why the values for  $\mu_L(\varepsilon_7) - \mu_L(\varepsilon_{11})$  are not reported.

<sup>11</sup>Kimball, Sahm, and Shapiro (2009) find  $\mu_\gamma = 1.05$ , because they do not restrict the sample to the household heads. The results are almost identical when using their estimate. Moreover, Tables 3 and 4 in Kimball, Sahm, and Shapiro (2008) show the estimates for the two parameters with HRS data:  $\sigma_\gamma$  is drastically reduced (from 1.76 to 0.73) when correcting the estimator for response error. In the PSID it is possible to apply the same correction procedure only by using some information from the HRS data, because the risk aversion question was asked only once. As for the other parameter, the value of  $\mu_\gamma = 1.98$  is substantially larger in the HRS. These alternative estimates, in turn, lead to first order effects on the wealth Gini index. However, the HRS respondents are not a representative sample of the overall economically active U.S. population. Exploiting these estimates for the whole U.S. economy could lead to large biases.

Although Kimball, Sahm, and Shapiro (2008) and Kimball, Sahm, and Shapiro (2009) proposed and implemented a sophisticated econometric technique, inferring a continuous preference distribution from a limited sequence of lotteries and implied outcomes is a challenging task. Many problems can potentially bias the results. For example, the log-normal parametric assumption could give too little flexibility, possibly forcing the tails to be too fat or too thin.<sup>12</sup> Or, more fundamentally, the survey’s respondents might not be able to assess accurately the amount of risk involved by the lifetime lotteries they are facing in the questionnaires. This could lead to a systematic overestimate of the risk parameter.

With these caveats in mind, a finding based on the PSID (and HRS) data is that people show very high risk aversions. According to the parameters’ estimates, there are few individuals whose  $\gamma$  is around 1, with the average CRRA being 4.28. More in detail, 10.77% of the population has  $\gamma \leq 1$ , 22.0% has  $\gamma \leq 1.5$ , 33.0% has  $\gamma \leq 2$ , and 72.9% has  $\gamma \leq 5$ . With the class of preferences assumed in this paper, this result could lead to a counterfactual implication. The implied elasticity of intertemporal substitution could be at odds with the values typically estimated in the literature, and surveyed by Attanasio and Weber (2010). Although the average CRRA is extremely high, due to a manifestation of Jensen inequality, the average EIS is equal to  $0.498 \gg \frac{1}{4.28}$ , a value consistent with the empirical findings.<sup>13</sup>

The second specification, denoted as CP, will make use of the estimates on some moments of the risk aversion distribution provided by Chiappori and Paiella (2011). This alternative case is interesting, because it differs in some crucial aspects from the KSS one. Their study does not rely on U.S. data, but on the SHIW, a panel dataset that tracks a representative sample of Italian households. The authors back out the distribution of the risk aversion parameter from actual choices on the portfolio composition. They report two statistics: the median risk aversion ( $\gamma^{Med} = 1.7$ ), and the third quartile ( $\gamma^{0.75} = 3.0$ ). I use these two pieces of information to identify a parametric distribution: a *Beta*( $\mu_\gamma, \sigma_\gamma$ ) specification proved to be an excellent solution. A minimum distance estimator matching these two moments was implemented, with the support of the distribution being  $[0.5, 14]$ . The two estimated parameters are  $\mu_\gamma = 0.80$  and  $\sigma_\gamma = 5.58$ .<sup>14</sup>

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<sup>12</sup>When focusing on the household heads, in the PSID the six risk tolerance categories have the following number of observations: 1765, 1024, 869, 835, 761, 368.

<sup>13</sup>In a previous version of the paper, I considered a third log-normal preference specification matching a median EIS of 0.5 (higher than the value 0.34 implied by the KSS case) and the variance found in the PSID data. The results were quantitatively between the ones of the other two distributions and are not worth reporting.

<sup>14</sup>A log-normal specification was attempted at first, but the results were poor. Comparing the fitted distribution with Figure 2 in Chiappori and Paiella (2011) showed that in order to match the center of the distribution, the log-normal specification was clearly missing the behavior at the lower and upper ends of the support. This is consistent with the formal tests reported in Chiappori and Paiella (2011), which rejected both normality and log-normality. Eyeballing the

Figure (1) provides the plots of the two preference heterogeneity densities used in the solution of the model. Comparing these distributions it is clear that the KSS case has the highest mean, because it has a lot of mass for high values of  $\gamma$ . The CP case shows a density whose mode is at the lower bound of its support, with a lot of mass concentrated for relatively low values of  $\gamma$ , representing a population that is more risk tolerant than in the other specification. In this case, 27.5% of the population has  $\gamma \leq 1$ , 44.7% has  $\gamma \leq 1.5$ , 57.6% has  $\gamma \leq 2$ , and 92.5% has  $\gamma \leq 5$ , and the average CRRA is 2.20.

## 4.2 Calibration

The remaining parameters are calibrated as follows. The capital depreciation rate is set to replicate an investment/output ratio of approximately 24%. This is achieved with  $\delta = 0.08$ . I assume a Cobb-Douglas production function, hence the capital share is captured by the parameter  $\alpha = 0.36$ , a common value for the U.S. economy.

For the rate of time preference  $\beta$ , I target an equilibrium interest rate that is always in the 3 – 4% range irrespective of the stochastic income process.<sup>15</sup> This is obtained with  $\beta = 0.9605$  for the estimated risk aversion distributions based on the PSID data, and with  $\beta = 0.9612$  for the CP case.

The borrowing limit  $b$  is set at its most stringent value, namely  $b = 0$ . This choice was mainly dictated to facilitate the comparison with previous contributions in the literature that used such a value, e.g. Aiyagari (1994) and Castaneda, Diaz-Gimenez, and Rios-Rull (2003). However, it is not clear if this value is fully satisfactory. In the data, between 6% and 15% of U.S. households have a negative net worth, as reported by Wolff (1998) and Cagetti and De Nardi (2008). This is potentially important for two reasons: 1) a less tight borrowing constraint allows people to better smooth consumption, relying on debt rather than on precautionary savings as a way to equalize the marginal utility of consumption across all possible states of the world, 2) on the one hand a less stringent borrowing constraint decreases the incentive to save, possibly reducing the upper value of the support of wealth, on the other hand it shifts mechanically the lower value of the support of wealth. By affecting the range of wealth, the borrowing constraint can have a non-trivial effect on some measures of wealth dispersion.<sup>16</sup>

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fitted *Beta* distribution, it is clear that it now captures the features of the ones estimated by Chiappori and Paiella (2011). The results are robust to different lower bounds of the support. Notice that I did not attempt to use the results provided by Harrison, Lau, and Rutstrom (2007) and von Gaudecker, van Soest, and Wengstrom (2011). As for the former paper, the number of people participating in the experiments was very limited, making the estimation for the whole preference distribution prone to large errors. Differently, the latter did not rely on expected utility theory, making their preference distributions inconsistent with the framework used here.

<sup>15</sup>From (2), for a given labor share and capital depreciation, this is in fact equivalent to matching the capital/output ratio. This is the only wealth related target that is used in the parameterization of the model.

<sup>16</sup>This is one of the results found in a global sensitivity analysis for the Aiyagari (1994) economy reported in Cozzi

## 5 Results

This Section presents the main results for the baseline model. It is shown how introducing preference heterogeneity, in the form of non degenerate distributions of innate risk aversion, does alter considerably typical measures of wealth inequality.

[Table 4 about here]

Table 4 compares the Gini index and the Coefficient of Variation obtained in the two models with preference heterogeneity, KSS and CP, to the inequality measures computed in the corresponding economies without preference heterogeneity.

Overall, the model economy moves from a wealth Gini index in the 0.41 – 0.49 range (for the homogeneous preferences case) to a wealth Gini index in the 0.59 – 0.74 range, depending on the actual stochastic income process considered. The KSS model almost matches the observed degree of wealth inequality in the U.S. with the *FLIN* income process, while the results for the CP model show that wealth is always less concentrated in this case.<sup>17</sup>

As expected, when moving from the homogeneous risk aversion set-up to the heterogeneous one, the inequality measures do increase for all endogenous variables. Consumption, income and wealth are more concentrated in the heterogeneous preferences economies. Although the effect on wealth inequality tends to be pronounced, and a non-negligible increase in the income and consumption Gini indices is obtained, the concentration of the last two variables is still far from the data. In particular, the consumption Gini index is now between 0.10 and 0.14.

As for the other measure of wealth inequality, the Coefficient of Variation, the model does not match the actual figure of 6.02 found by Diaz-Gimenez, Glover, and Rios-Rull (2011) in the SCF. The values implied by the model are in the 1.18 – 1.86 range. The reason behind this result is that the model misses the share of wealth held by the richest households. Given that the wealthiest households in the U.S. hold approximately one third of the total wealth, this is an important finding. Preference

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(2011). Incidentally, results related to different calibrations of the borrowing limit are not reported. A relatively low  $\beta$  makes borrowing a more attractive option, which in turn leads to wild changes in the percentage of people in debt when considering slight changes in  $b$  and  $\beta$ , making the comparisons among the different stochastic processes less transparent. Finally, in this model the effect of  $b$  on the wealth Gini index was found to be small.

<sup>17</sup>Notice, however, that the *GHIP* cases are somewhat misleading. Unlike Guvenen (2009), I am just considering the AR(1) component of the econometric specification for labor income, that is just the restricted income profile. In order to gauge the predictions of the model for this case, I should focus only on the residual inequality, namely the one that is not accounted for by the heterogenous income profiles, which are not considered in this model.

heterogeneity alone does not fully account for the determination of the top of the wealth distribution. In the KSS and CP cases, the share of wealth held by the top 1% is never higher than 10.8%.

To see this in more detail, Table 5 reports a set of statistics related to the wealth distribution. The five quintiles ( $Q1$ - $Q5$ ) and a breakdown of the top quintile are shown for both the U.S. data, and the various models. The first two rows report the quintiles obtained from the SCF in 1998 and the PSID in 1999.<sup>18</sup> When focusing on the same income process, e.g. *FLIN*, the two risk aversion distributions (presented in the second panel) show results in terms of wealth concentration and shares of wealth held by a set of quantiles that are relatively similar. Both the KSS and the CP distributions have almost identical bottom quintiles,  $Q1$  and  $Q2$ , while a larger difference is observed for the share of wealth held by the top two quintiles,  $Q4$  and  $Q5$ . The KSS and CP wealth distributions differ markedly in  $Q4$ , the share of wealth held by the fourth quintile (16.3% Vs. 20.4%), which is well above the data (12.2% – 12.6%). A similar comment applies to  $Q5$ , with both cases underestimating the actual value, but with the KSS specification being substantially closer to the data (77.4% Vs. 81.7% – 83.2%).

[Table 5 about here]

Unlike the share of wealth held by the top 1%, several other features of the wealth distribution are replicated fairly well, such as the bottom quintiles. The U.S. data show that the bottom three quintiles hold very little wealth. The first quintile has negative asset holdings, the second quintile has between 0.8% and 1.3% of the total wealth, while the third quintile has between 4.2% and 5.0%. In the model, the left tail of the distributions display some discrepancies with the data. However, these discrepancies are small. The bottom quintile is prevented to be in debt, which explains the 0% figure Vs. the negative one in the data. A similar pattern is observed for the second and third quintiles. With the KSS preference distribution, the bottom 60% holds as little as 6.3% of the total wealth, while the same figure is 10% with the CP distribution. The model predicts that wealth-poor agents hold very little wealth, although slightly more than what is found in the data, because the bottom quintiles are dominated by risk prone agents, that save little for two different reasons. On the one hand, their willingness to face consumption profiles that are not very smooth drives their precautionary savings down. At the same time, the more risk averse agents are accumulating a lot of assets, driving up the capital supply in the economy, and reducing the rate of return of savings, making future consumption

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<sup>18</sup>The same statistics for the SCF 2002, SCF 2007, PSID 1994, and PSID 2007 data are quite similar, and are reported in Appendix C. For a systematic study on how the PSID and SCF wealth data differ because of the different survey designs, see Bosworth and Anders (2008). Their analysis shows that, apart from the top 5%, the two wealth distributions are very close.

relatively more expensive than current consumption. This GE effect triggers a reduction in the interest rate that drives further down the incentives to save for agents that have a low risk aversion. These two effects combined explain why in this economy there are several agents that have little or no assets at all, as the Table shows.

## 5.1 Lorenz Curves

It is informative to compare the Lorenz curve related to the model without risk aversion heterogeneity to the one obtained with heterogeneous preferences.

[Figure (4) about here]

Figure (4) plots three Lorenz curves. One curve, drawn with a solid line, represents the PSID data on net worth in 1999.<sup>19</sup> The second curve, drawn with dots, refers to the KSS economy for the *FLIN* income process. Finally, the third Lorenz curve displays another economy, which has the same amount of uncertainty (*FLIN*), the same trading opportunities, the same baseline parameters, but it is without risk aversion heterogeneity (i.e.  $\gamma = 4.28$ ).

By comparing the Lorenz curves it is clear that the heterogeneous preferences economy displays a wealth distribution that is substantially more unequal, and that until the 60<sup>th</sup> percentile it tracks well the empirical one. This is an alternative way of looking at the change in the Gini coefficient, which confirms this finding: it increases from 0.45 in the homogeneous CRRA case to 0.74 in the heterogeneous one. A similar set of comments and plots apply when considering the other preference distribution, and the other income processes, although the fit with the PSID data is somewhat poorer.

## 5.2 Discussion

At least three aspects of the analysis call for a further discussion. The first one is related to the assumption of risk aversion being a permanent feature. The second is why I consider preference heterogeneity in only one parameter, the risk aversion. The final aspect refers to risky jobs, preference for risk and sorting into careers, which is dealt with in the next section.

An infinitely lived agents model refers to dynasties. In the PSID, Kimball, Sahm, and Shapiro (2009) find that risk aversion is positively correlated among different generations of the same family. However, this covariation is not perfect. The second remark is related to a life-cycle element. It is

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<sup>19</sup>Comparing the model to the PSID wealth data seems appropriate, as the stochastic income processes are all estimated with this dataset.

possible that individuals change their attitude towards risk after having gone through some specific stages in their life. Unfortunately, data limitations do not allow to capture if such dynamics are indeed taking place. Neither the HRS nor the PSID allow to single out the life-cycle component vs. a cohort one. As Kimball, Sahm, and Shapiro (2008) point out, some individuals in the HRS are asked to assess their attitudes towards risk more than once. In quite a few cases these individuals change their answers. With the available information it is hard to say whether this is a form of measurement error, or whether this reflects the fact that individuals change their attitudes towards risk in response to some economically relevant events (e.g., new information on the future income streams, or aggregate and idiosyncratic shocks). However, all the people in the HRS sample are in the later stages of their lives, that is they are in a situation where there is little uncertainty about lifetime income, at least in the form of labor earnings. This is why the measurement error approach seems to be a valid one.

As for the second aspect, the modeling choice was mainly driven by data limitations. Barsky, Juster, Kimball, and Shapiro (1997) find heterogeneity not only as far as risk aversion is concerned, but also in the elasticity of intertemporal substitution, and in the rate of time preference. However, their sample size is really limited, making the estimation of the distributions for the last two parameters too unreliable. It goes without saying that, with CRRA preferences, once heterogeneity in risk aversion is allowed for, heterogeneity in the elasticity of intertemporal substitution mechanically follows. The distributions of the two parameters are just a monotone transformation.

In the benchmark model, the amount of income uncertainty from the point of view of a single agent is exogenously determined by the stochastic process for labor income. However, Guiso, Jappelli and Pistaferri (2002), Fuchs-Schundeln and Schundeln (2005), Bonin, Dohmen, Falk, Huffman, and Sunde (2007), and Schulhofer-Wohl (2011), among others, pointed out that different attitudes towards risk can also imply different career choices. High risk averse individuals can self-select into safer jobs, while low risk averse ones can be willing to take jobs with a higher variance of earnings. This endogenous sorting could have profound implications for the degree of inequality. However, such a research avenue poses some challenges on how to model the career specific stochastic processes for earnings. An attempt along these lines is proposed in the next section.

## 6 Endogenous Sorting into Risky Jobs

Part of the results obtained above could be driven by the absence of self-selection into jobs differing in their income risk. Schulhofer-Wohl (2011) exploits the question about risk tolerance in the HRS to document that less risk averse individuals have the largest earnings fluctuations during their working life, suggesting that preference heterogeneity may be an important factor in occupational choices

and the allocation of risk. This section proposes a simple extension to deal with this issue. It is now assumed that there are two mutually exclusive careers, and that all the workers have perfect knowledge on the degree of income risk associated with these options.<sup>20</sup>

The workers can now self-select into different jobs, according to their characteristics. For simplicity, it is assumed that the workers make a once in a lifetime decision on their career. Upon becoming economically active, workers choose which career they want to undertake, by solving the following problem:

$$\max \{V^h(\varepsilon, a; \gamma), V^l(\varepsilon, a; \gamma)\}$$

where the  $V^j(\varepsilon, a; \gamma)$  represent the expected lifetime utility derived when choosing career  $j = h, l$ . Associated with this choice there is an optimal policy function  $\Phi(\varepsilon, a; \gamma)$ , which is an indicator function equal to one whenever  $V^h(\varepsilon, a; \gamma) \geq V^l(\varepsilon, a; \gamma)$ , namely when an agent decides to work in the riskier job.<sup>21</sup> Hence, in equilibrium the endogenous share of workers choosing to work in the riskier job ( $\phi$ ) is represented by:

$$\phi = \int_{\varepsilon \times \mathcal{A} \times \Gamma} \Phi(\varepsilon, a; \gamma) d\mu(\varepsilon, a; \gamma)$$

Since there are two Markov processes for the effective units of labor, each one is going to have its own stationary distribution, and the equilibrium conditions related to the labor input have to be changed accordingly. In the steady-state the expression for  $L$  becomes:

$$L = \phi \int \varepsilon d\mu_L^h(\varepsilon) + (1 - \phi) \int \varepsilon d\mu_L^l(\varepsilon)$$

where  $\mu_L^j(\varepsilon)$  stands for the stationary distribution over the labor endowments implied by either markov process  $j = h, l$ .

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<sup>20</sup>It goes without saying that this simple specification neglects considerations related to shopping for a career in the early stages of a worker's labor market experience, and learning about the relative riskiness of a job. Moreover, data limitation suggests to restrict the number of jobs to two. An alternative approach could be to use the career definitions available in the PSID. Unfortunately, for most of the respondents, it is not possible to observe their labor market entry, making the comparison between the theory and the data somewhat problematic.

<sup>21</sup>This formulation assumes that the worker observes the realization of the first income shock before making the career choice. If this decision is made before any interaction with the labor market, the appropriate formulation is  $\max \{E_\varepsilon V^h(\varepsilon, a; \gamma), E_\varepsilon V^l(\varepsilon, a; \gamma)\}$ . Given the same parameters, solving this alternative case leads to slightly different Gini coefficients. Notice also that the income shocks are career dependent, and they would evolve independently for the two careers  $j = h, l$ , with only one of the two histories of shocks being actually experienced by the worker.

## 6.1 Two Labor Endowment Processes

The two postulated careers are represented by two different stochastic processes, which govern the dynamics of the effective units of labor  $\varepsilon$  a worker is going to supply in the labor market. It follows that

$$\log \varepsilon' = \rho_{y,j} \log \varepsilon + \eta', \eta \sim iid N(0, \sigma_{y,j}^2), j = h, l$$

which highlights how the parameters  $\rho_{y,j}$  and  $\sigma_{y,j}^2$  can now differ in the two stochastic processes. It goes without saying that the riskier job ( $j = h$ ) is such that  $\sigma_{y,h}^2 > \sigma_{y,l}^2$ . As for the persistence parameter, postulating an inequality is less straightforward. However, the intuition to justify the restriction  $\rho_{y,h} \geq \rho_{y,l}$  goes as follows. The lower the persistence parameter, the more transitory the income shocks will be, because their effect dies out faster. Differently, more persistent processes imply that a shock of a given size will have a more long lasting effect. More risk averse individuals prefer smoother consumption profiles, hence they consider riskier a stochastic income process whose shocks have a higher variance and that are more persistent, namely shocks that cannot be easily offset by their saving behavior.<sup>22</sup>

Whether these assumptions have an empirical foundation is an open issue. However, some evidence from PSID data can be used in order to support them, together with the results reported by Schulhofer-Wohl (2011), who relies mainly on HRS data, showing that less risk averse individuals have larger income shock variances.

## 6.2 Empirical Evidence

In the PSID, it is possible to compute the autocorrelation of labor income together with the variance of the innovations for individuals belonging to different risk aversion categories. Starting from the SRC sample I follow a set of standard steps, outlined for example in Heathcote, Perri and Violante (2010). First, I select the PSID respondents according to the following criteria: I consider only the household head, between the age of 24 and 62. For each year  $t = 1968, \dots, 1996$ , I run a first stage regression of labor earnings on a set of controls,  $\log y_{it} = \theta_t X_{it} + \varepsilon_{it}$ , with the controls  $X_{it}$  being age, education, state of residence, race, gender and number of children. With the estimated parameters  $\hat{\theta}_t$  I compute the earnings residuals  $\hat{\varepsilon}_{it}$  and estimate an AR(1) panel regression on them. The econometric model  $\log \hat{\varepsilon}_{it}^{RA} = \rho_{y,RA} \log \hat{\varepsilon}_{it-1}^{RA} + \eta_{it}^{RA}$  is estimated separately for two risk aversion categories, represented

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<sup>22</sup>The labor processes imply that the two career-dependent average incomes have to be the same: perhaps surprisingly, this assumption is supported by the PSID data. The median (average) annual labor earnings are \$38,521 (\$45,255) for the low risk aversion group and \$38,550 (\$44,776) for the high risk aversion one.

by  $RA = \{\gamma_h, \gamma_l\}$ . The breakdown in the data is chosen such that each group has approximately 50% of the respondents.<sup>23</sup>

Before reporting the results, it is worth stressing that this estimation procedure does not recover the structural parameters of the model. From the PSID data it is possible to track the yearly labor earnings of individuals according to their risk category. However, it is not possible to estimate exogenously two (or more) potential stochastic processes from the data and impose them in the model. The endogeneity of a person's career prevents this estimation procedure from recovering the true processes, and suggests not to use them as an input in the model. Some form of indirect inference method is needed to estimate the structural parameters in equilibrium, because only the actual volatility of earnings is observed, with agents of the same risk aversion but with different labor income shocks and asset holdings selecting different jobs upon labor market entry.

[Table 6 about here]

The first remark is that different estimators give relatively different answers, as shown in Table 6. As for the standard deviation of the shocks, the results are relatively stable (also with respect to the sample selection rules) and always such that  $\hat{\sigma}_{y,\gamma_l} > \hat{\sigma}_{y,\gamma_h}$ . For this econometric model, (Pooled) OLS is inconsistent because it does not control for unobserved heterogeneity, but it is reported as a benchmark comparison:  $\hat{\sigma}_{y,\gamma_l}^{POLS} = .420$  and  $\hat{\sigma}_{y,\gamma_h}^{POLS} = .396$ . In a dynamic panel also the LSDV estimator is inconsistent, but some Monte-Carlo studies show that in short panels it can outperform GMM based estimators in terms of efficiency. The estimates in this case are  $\hat{\sigma}_{y,\gamma_l}^{LSDV} = .388$  and  $\hat{\sigma}_{y,\gamma_h}^{LSDV} = .363$ . The Arellano-Bond and the Blundell-Bond estimators give somewhat lower standard deviations of the idiosyncratic component of the shock, with  $\hat{\sigma}_{y,\gamma_l}^{A-B} = .344$  and  $\hat{\sigma}_{y,\gamma_h}^{A-B} = .317$  for the first estimator, and with  $\hat{\sigma}_{y,\gamma_l}^{B-B} = .347$  and  $\hat{\sigma}_{y,\gamma_h}^{B-B} = .317$  for the second one. Overall, these results are reassuring, because they confirm the assumptions used in the model: less risk averse individuals do have larger income shock variances. However, compared to the income shocks used in the previous calibrations, these shocks appear to be larger. The difference is due to two factors. First, these estimates concern earnings rather than wages, and earnings are more dispersed than wages. Second, even when focusing on wages, for this sample and with these estimators, the wage shocks are somewhat bigger than the estimates available in the literature, and used in the calibration of the baseline model.

The results related to the persistence parameters are as follows. The estimates are  $\hat{\rho}_{y,\gamma_l}^{POLS} = .694$  and  $\hat{\rho}_{y,\gamma_h}^{POLS} = .711$ ,  $\hat{\rho}_{y,\gamma_l}^{LSDV} = .393$  and  $\hat{\rho}_{y,\gamma_h}^{LSDV} = .386$ ,  $\hat{\rho}_{y,\gamma_l}^{A-B} = .341$  and  $\hat{\rho}_{y,\gamma_h}^{A-B} = .292$ ,  $\hat{\rho}_{y,\gamma_l}^{B-B} = .360$

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<sup>23</sup>E.g., 1,131 individuals and 13,175 observations for the most risk averse group ( $\gamma_h$ ), and 1,232 individuals and 12,440 observations for the other group ( $\gamma_l$ ) for the Arellano-Bond estimator. See Table 6.

and  $\hat{\rho}_{y,\gamma_h}^{B-B} = .291$ . Since these results do not seem to be very robust, they won't be used as targets in the calibration of the model with self-selection into risky jobs.

### 6.3 Calibration of the Endogenous Sorting Model

Although appealing, structurally estimating the four parameters characterizing the two jobs (and possibly the parameters of the risk aversion distribution) is computationally too burdensome. A more pragmatic approach is taken instead. For each risk aversion distribution, the calibration of the labor income processes specifies two given persistence parameters, while the two shock variances are recovered to match the variances of the income shocks computed in the PSID, for the two risk aversion categories defined above. More in detail, three moments are matched in equilibrium: an interest rate of 4%, and the two income shock variances of the agents above and below the 52<sup>nd</sup> percentile in the risk aversion distribution.<sup>24</sup> The three parameters calibrated in equilibrium are  $\beta, \sigma_{y,h}^2$ , and  $\sigma_{y,l}^2$ . With a Nelder-Mead optimization algorithm, a highly parallelized code, and searching only over these three parameters, the calibration takes up to 100 hours to complete. As for  $\rho_{y,h}$  and  $\rho_{y,l}$ , an extensive robustness analysis is performed, by recalibrating the model under different assumptions on their values.

[Table 7 about here]

Table 7 reports the parameters and the calibration targets of the four different cases which are going to be discussed. These are labeled as *ES1-ES4*. Cases *ES1* and *ES2* refer to the KSS preference distribution, while cases *ES3* and *ES4* to the CP one.

The calibrated rate of time preference  $\beta$  deserves some comments. As expected, in this economy high risk aversion agents accumulate a lot of assets, when facing good earnings shocks. This leads to a supply of savings which is higher than in its homogeneous preference parameters counterpart. With a yearly time period, if a typical value for  $\beta \approx 0.96$  were used, the equilibrium interest rate would be close to 0%, to dissuade saving. In order to avoid this counterfactual outcome, I target an equilibrium interest rate equal to 4%. The calibrated  $\beta$  now ranges between 0.927 and 0.952. A lower value of  $\beta$  makes the agents less patient: by giving less weight to the future, agents consume more out of their current income and decrease their savings, preventing the interest rate to become implausibly small.

As for the two income shock variances, notice that the heterogeneity in wealth at labor market entry, together with different initial productivity shocks, lead agents with the same risk aversion to possibly make different career decisions. Consequently, the actual earnings instability for agents with

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<sup>24</sup>The discreteness of the risk aversion categories makes it impossible to rely on the median as the threshold value.

a given risk aversion parameter results from a weighted average of the calibrated values of  $\sigma_{y,h}^2$  and  $\sigma_{y,l}^2$ .

In the *ES1* case, the two parameters of the autoregressive component are set to  $\rho_{y,h} = \rho_{y,l} = 0.95$ , while the two calibrated standard deviations are  $\sigma_{y,h} = 0.478$  and  $\sigma_{y,l} = 0.228$ . Case *ES2* sets  $\rho_{y,h} = \rho_{y,l} = 0.9$ , and the two calibrated standard deviations are now  $\sigma_{y,h} = 0.459$  and  $\sigma_{y,l} = 0.245$ . The logic behind the calibration of the two CP cases is similar. In *ES3* the autocorrelations are set to 0.95, while in *ES4* to 0.9. The values of the calibrated standard deviations of the earnings shocks are very similar in both the *ES3* and *ES4* specifications, and happen to be a mix of the two KSS cases.

## 6.4 Wealth and Consumption Inequality

Table 8 shows the values for the wealth Gini index in the model with endogenous sorting into risky jobs.

In this case, the model accounts for the observed concentration of wealth with both the KSS and CP risk aversion distributions. The calibrations setting highly persistent income shocks, *ES1* and *ES3*, display a wealth Gini index of 0.823 and 0.766, respectively. The other two cases, *ES2* and *ES4*, have lower Gini indices of 0.783 and 0.732, because of the lower persistence of the shocks.

[Table 8 about here]

As for  $\phi$ , the endogenous share of workers selecting the risky career, it is found to be relatively similar across specifications, ranging from 41% for the case *ES1* to 47% for the *ES4* one. As expected, overall there is a strong negative relationship between the share of agents in the riskier job and their risk aversion. However, it is interesting to notice that for some calibrations (e.g. *ES1*) several high risk aversion agents select the riskier job. This happens because in the steady-state they start their working life with a lot of accumulated assets, which already allow them to smooth consumption well. Hence, they are more willing to take the bet of a risky career, once a good initial earnings shock is realized.

Selected quantiles of the wealth distributions are shown at the bottom of Table 5. Comparing the outcomes of the models without endogenous sorting to the models with two risky jobs, a major difference is worth stressing. Although the first two quintiles are relatively similar, there is a substantial change in the share of wealth held by the top two quintiles. In the *ES1-ES4* models, the top quintile holds between 75.7% and 87.0% of the total wealth, which represents an increase of no less than 10 percentage points, compared to the model without endogenous sorting.

It is interesting to notice that, although they now better capture the top of the wealth distribution, the *ES1-ES2* specifications do so by sacrificing somewhat the fit of the bottom three quintiles, the third one in particular. This does not happen in the *ES3-ES4* cases, because the economy with the CP preference distribution still falls short of matching the top quintile of the wealth distribution, although by a small amount.

With such a high wealth concentration, the model does show a higher share of wealth held by the top 1%. This statistic now reaches at most 16.3%, which is still very far from what is observed in the data.

**[Figure (5) about here]**

Figure (5) plots three Lorenz curves. As before, the curve drawn with a solid line represents the PSID data. The curve drawn with a dashed line now refers to the KSS economy with the more persistent income process, namely case *ES1*. Finally, the third Lorenz curve refers to case *ES2*. Compared to the baseline model, the improved fit of the two risky jobs extension is very noticeable. The Lorenz curve for the data and the corresponding ones implied by the model are hard to distinguish for a large portion of the graph.

An important shortcoming of the model without self-selection is represented by the concentration of consumption, which is too low, being in the 0.10 – 0.14 range. Differently, the model with endogenous sorting achieves a consistently higher consumption inequality, with the Gini index now being in the 0.20 – 0.21 range, which represents a remarkable 50% – 100% increase. However, these findings suggest that the model is possibly limited by the absence of the life-cycle dimension. As for the data, Heathcote, Perri and Violante (2010) report that in the CEX, since 1990, the Gini index of consumption of non-durable goods has been above 0.3, with a maximum value of 0.32, reached in 2004 and 2005.

The model implies that average consumption is increasing in the degree of risk aversion: the higher  $\gamma$ , the higher the stock of accumulated savings, and the higher the capital income used for consumption smoothing purposes. In terms of the data, it is hard to test whether this feature of the model is supported by the PSID, as this dataset provides limited information on consumption, which is confined only to the food category. Taking this empirical evidence with caution, however, the data show that the average food consumption is slightly higher for the individuals in the higher risk aversion category. After eliminating some clear outliers (with annual food consumption/expenditure less than \$365 and higher than \$100,000) the average food consumption for the high risk aversion group is \$7,661, while the corresponding figure for the high risk aversion group is \$7,457. The former

group enjoys a higher consumption, which is 2.74% more than the one of more risk prone individuals. The *ES1* model counterparts of these statistics are 1.685 and 1.273, a consumption level which is 32.36% higher for the more risk averse agents. The model is qualitatively consistent with the data. However, it is hard to assess it on a quantitative basis, given that the definition of consumption in the model is much broader than the one in the PSID. Although not conclusive, these results can be used as *prima facie* evidence supporting the model's predictions.

## 6.5 Robustness checks

An extensive robustness analysis with respect to the persistence parameters is addressed in Figure (6), and in other calibrations reported in Appendix C. All these calibrations show that the results are not overly sensitive to alternative assumptions on the configuration of  $\rho_{y,h}$  and  $\rho_{y,l}$ . In particular, Figure (6) displays the Gini indices for wealth, earnings and consumption for several different combinations of the persistence parameters. This robustness check refers to case ES1, because apart from  $\rho_{y,h}$  and  $\rho_{y,l}$  all the other parameters are identical to the ones considered in that case.

[Figure (6) about here]

From the Figure it is apparent that the Gini indices grow almost linearly in the  $\rho_{y,h}/\rho_{y,l}$  ratio, but quantitatively they do not show very large changes. The largest response is for the wealth Gini index, which ranges from 0.81 to 0.83.

## 6.6 Comparison of Equilibria: Homogeneous Vs. Heterogeneous Preferences

An interesting feature of this model is that the allocations in the economies with heterogeneous preferences do not coincide with the allocations of the corresponding economy with homogeneous preferences.<sup>25</sup> A comparison of the equilibria reported in Table 9 shows that the two sets of allocations are quantitatively rather different.<sup>26</sup>

[Table 9 about here]

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<sup>25</sup>In the latter economy, I set  $\beta$  to the calibrated value in the corresponding heterogeneous preferences case, and  $\gamma$  to the average value of the relevant preference distribution. The parameters in the income process are set to the value implied by the underlying calibrated variances weighted by the endogenous share of risky jobs.

<sup>26</sup>In the interest of space, the results for the baseline model are included in Appendix C.

For any income shock persistence/preferences distribution pair, the equilibrium interest rate in the heterogeneous preferences case is lower by no less than 0.22 percentage points, and by up to 0.74 percentage points. It follows that the capital stock, total output, and aggregate consumption are always substantially higher in the economies with heterogeneous preferences.<sup>27</sup>

For the KSS distribution, output (consumption) moves from 1.863 to 1.940 (from 1.442 to 1.474) in the *ES1* specification, and from 1.889 to 1.962 (from 1.461 to 1.490) with the *ES2* one. Overall, the increases in output are 3.9% and 4.1%, while the increases in consumption are 2.0% and 2.2%. For the CP distribution, the results are similar, but are quantitatively less strong.

## 6.7 Precautionary Savings

The measurement of the size of precautionary savings represents a natural application in which heterogeneous-agent models are extensively used. It is informative to study how the heterogeneous preferences cases compare to their homogeneous preferences counterparts, which is the content of Table 10.

[Table 10 about here]

In the Table, the value of aggregate capital is reported under the assumption of both incomplete ( $K^{IM}$ ) and complete markets ( $K^{CM}$ ). The capital allocations of each risk preference distribution is compared to its homogeneous case counterpart. A clear result is that models with homogeneous risk aversion underestimate the size of precautionary savings. The extent of this bias ranges between 3.8 percentage points for the CP distribution, and 13.9 percentage points for the KSS distribution. A second aspect concerning precautionary savings is related to their size. Typically, in standard heterogeneous agent models, it is found that precautionary savings account for a small increase in the aggregate savings, usually below 2%, as discussed in Aiyagari (1994). This is not true for this model, in which the percentage increase of the aggregate capital stock, when moving from the complete markets economy to the incomplete markets one, is above 15.0% in all four cases, with a maximum of 54.6%. According to the endogenous sorting model, precautionary savings are quantitatively sizable, not only because of the larger variances of the income shocks, but also because of the stronger consumption smoothing motive of the agents with a relatively high risk aversion parameter.

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<sup>27</sup>It goes without saying that it is possible to find a representative risk preferences economy that has the same equilibrium prices and allocations. However, it is interesting to notice that the outcome of this aggregation exercise does not coincide with the economy with the average risk aversion.

## 7 Conclusions

This paper contributed to the literature on wealth inequality. A model of incomplete markets and precautionary savings was extended to allow for the type of preference heterogeneity found in the PSID/SHIW data, and for self-selection into risky jobs.

Does preference heterogeneity account for a substantial part of the U.S. wealth inequality? The findings show that the answer is not clear-cut. A result of this paper is that some forms of empirically grounded preference heterogeneity go a long way in accounting for several features of the U.S. wealth distribution. Compared to a model without preference heterogeneity, the left tail of the distribution gets closer to the little share of wealth held by the first three quintiles. At the same time, also the top quintiles are replicated relatively well, especially in the model with endogenous sorting into risky careers. A consequence of this result is that the Gini index in the model and in the data are very close to each other.

Other features of the wealth distribution are less satisfactory. In particular, the share of wealth held by the top 1% is at most 16.3%, which is well below the observed share of more than 30%. Entrepreneurship still seems to be the candidate channel leading to such large estates, although preference heterogeneity can provide a microfoundation of why some workers self-select into entrepreneurial activities while others decide not to work as self-employed. Such a model can be considered as an alternative way of microfounding the models of entrepreneurship and wealth accumulation developed in Quadrini (2000) and Cagetti and De Nardi (2006).

The results of this paper also show that neglecting the risk preference heterogeneity channel can have a first order effect also on the aggregate allocations and on the macroeconomic outcomes. This points to the need for further applied research aimed at eliciting more accurately individuals' risk preferences in large longitudinal and cross-sectional data, also because preference heterogeneity introduces another layer for heterogeneous welfare effects stemming from a policy change.

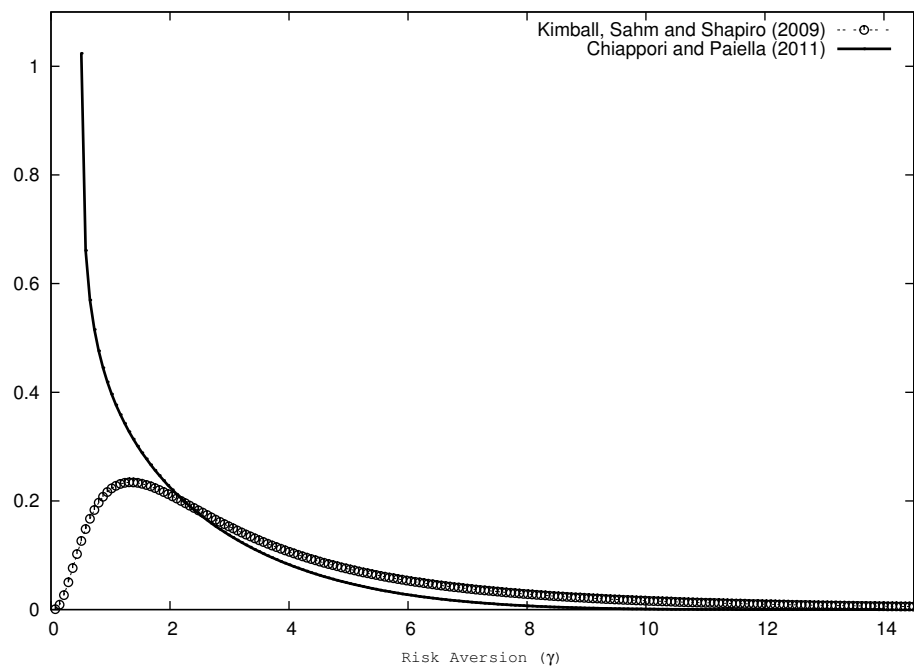


Figure 1: Risk Aversion Heterogeneity, Estimated Densities from PSID and SHIW data.

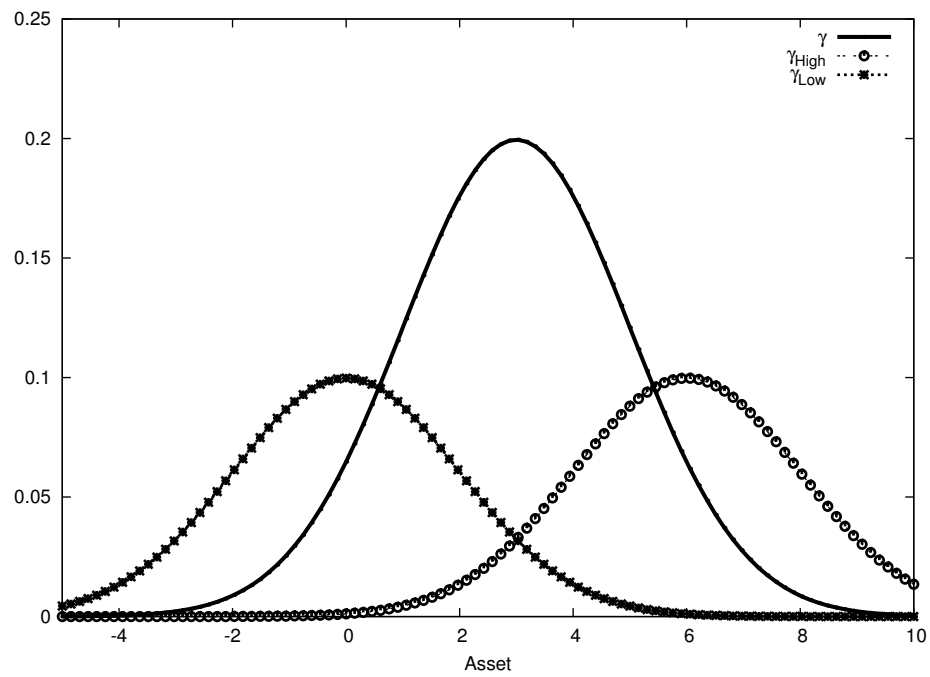


Figure 2: Heterogeneous Risk Aversion and Wealth Inequality

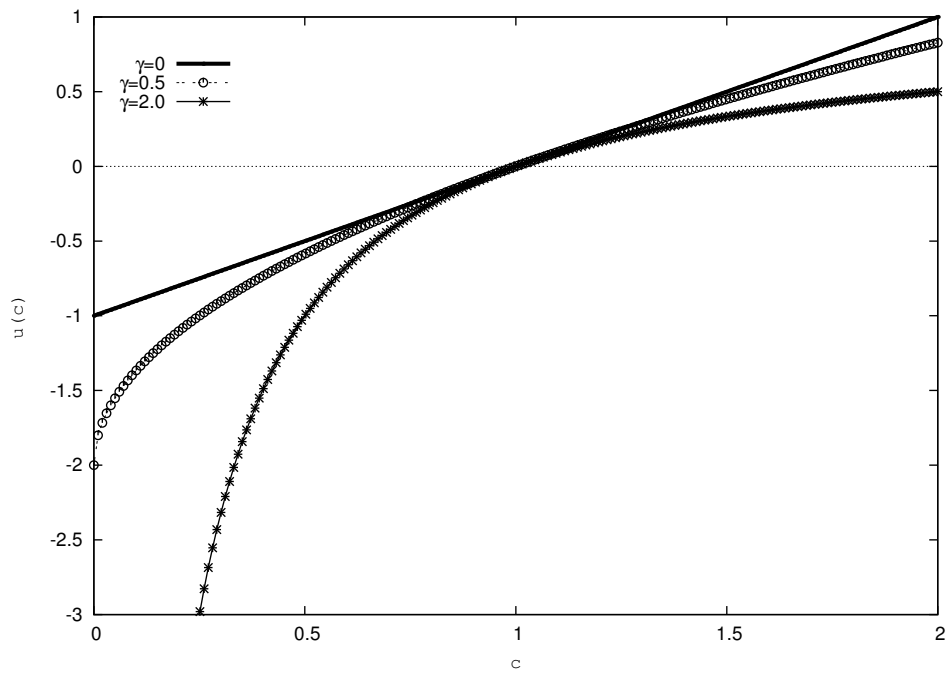


Figure 3: CRRA Utility Functions,  $\gamma = \{0, 0.5, 1\}$

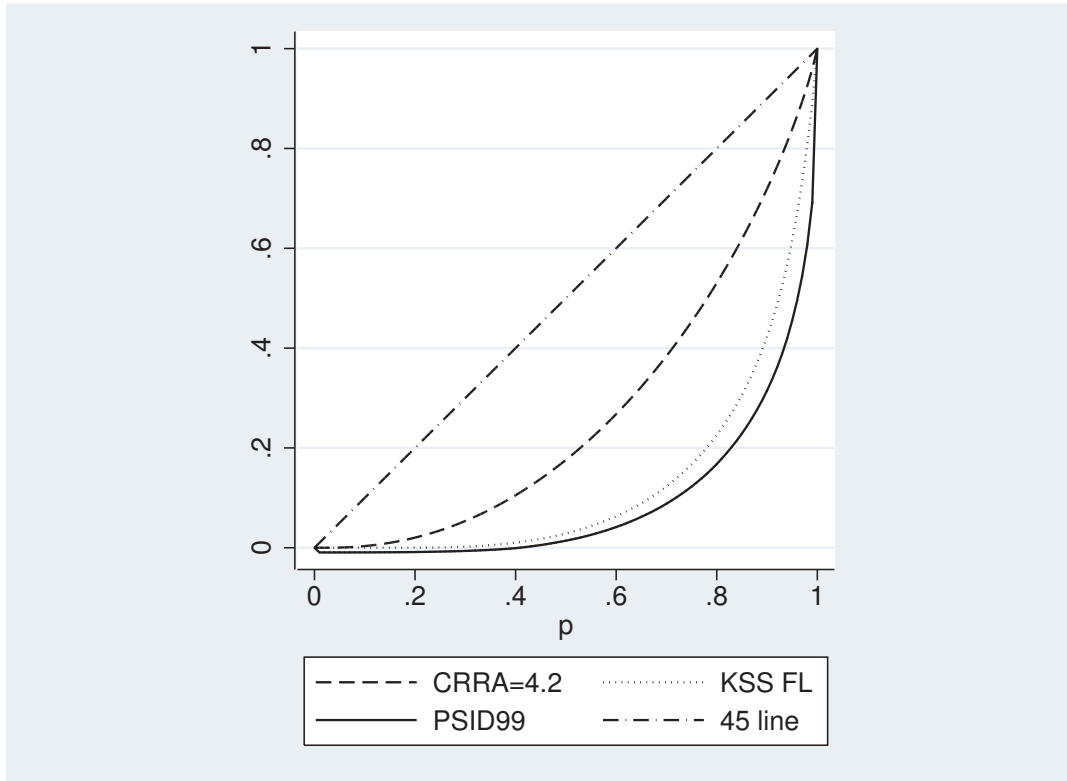


Figure 4: Lorenz Curves for Assets: Homogeneous Preferences, RA Heterogeneity (KSS - FLIN case), and PSID data

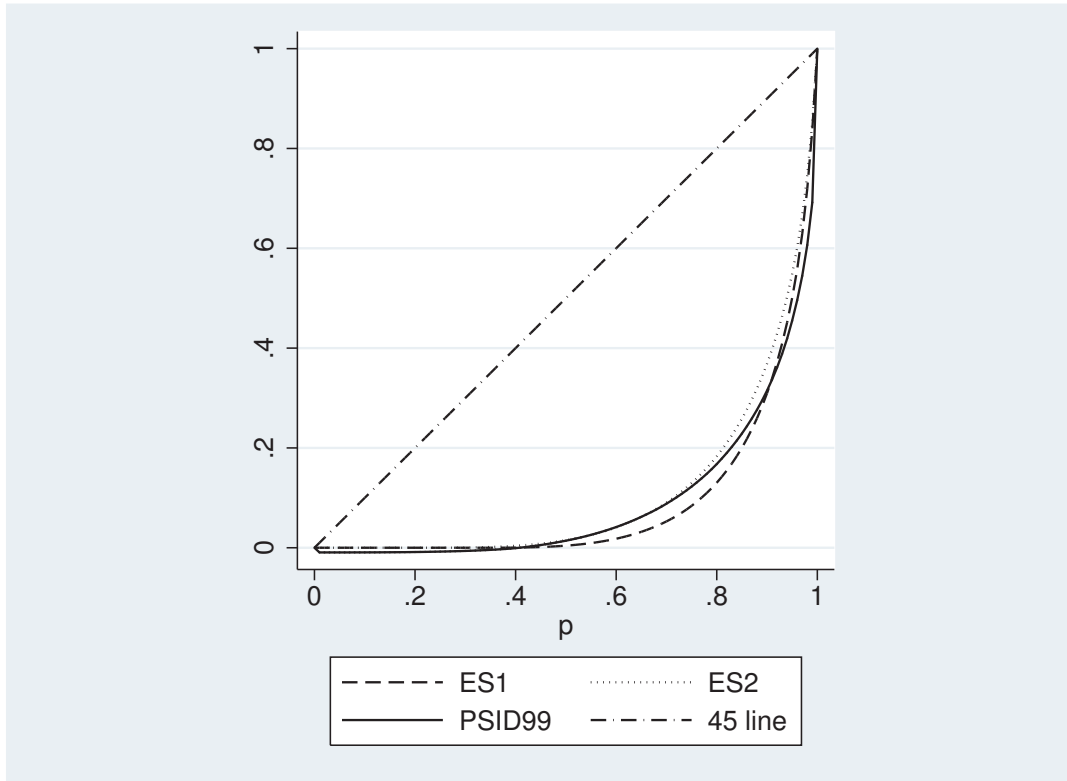


Figure 5: Lorenz Curves for Assets: RA Heterogeneity (ES1 and ES2 cases), and PSID data

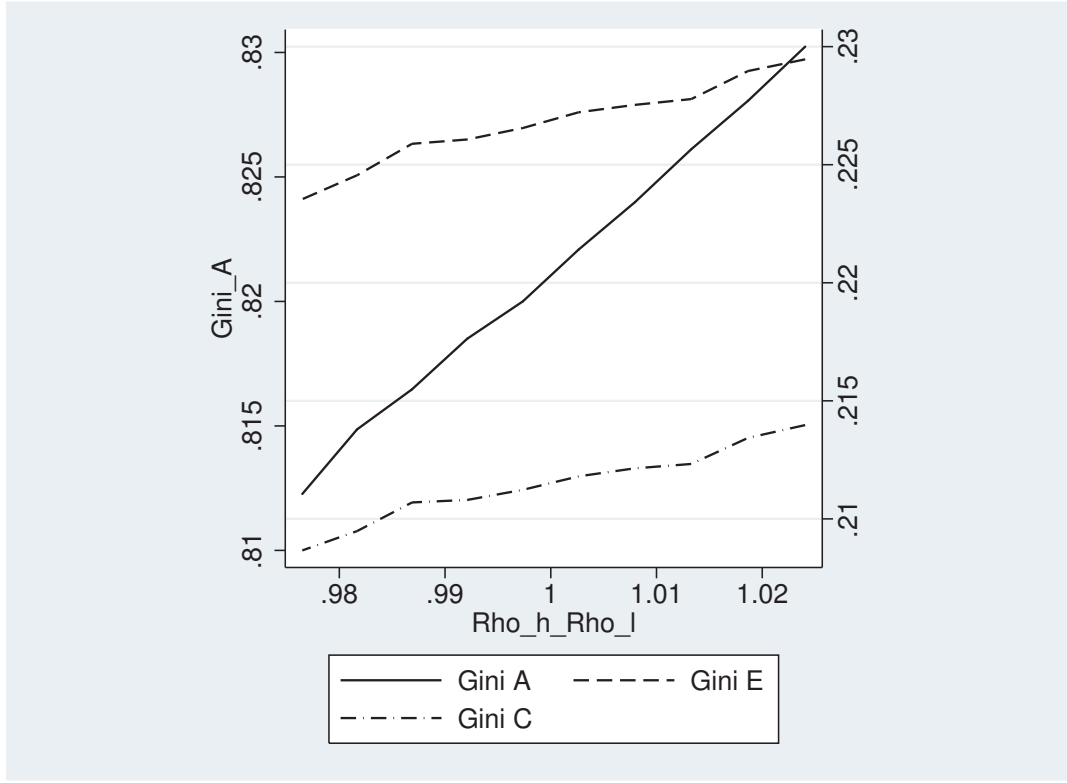


Figure 6: Robustness Analysis with respect to the income shocks persistence  $\rho_{y,h}/\rho_{y,l}$  ( $\rho_y^{Avg} = .945, \sigma_{y,l} = .228, \sigma_{y,h} = .478$ ),

| <i>Parameter</i>                             | <i>Value</i>         | <i>Target</i>                                                           |
|----------------------------------------------|----------------------|-------------------------------------------------------------------------|
| <i>Model Period</i>                          | <i>Yearly</i>        |                                                                         |
| $\varepsilon$ - <i>Worker's productivity</i> | <i>See Table 2</i>   | <i>AR(1) Stochastic Process for earnings</i>                            |
| $\gamma$ - <i>CRRA</i>                       | <i>See Table 3</i>   | <i>Distributions from PSID and SHIW data</i>                            |
| $\beta$ - <i>Rate of time preference</i>     | $\{0.9605, 0.9612\}$ | <i>Annual interest rate <math>\approx 3\text{-}4\%</math> {KSS, CP}</i> |
| $\delta$ - <i>Capital depreciation rate</i>  | 0.08                 | <i>Investment/Output ratio <math>\approx 24\%</math></i>                |
| $\alpha$ - <i>Capital share</i>              | 0.36                 | <i>Capital Share of Output</i>                                          |
| $b$ - <i>Borrowing limit</i>                 | 0                    |                                                                         |

Table 1: Benchmark Model, Calibration - U.S.

| <i>Income Process</i> | (1) <i>FLIN</i>            | (2) <i>FREN</i>      | (3) <i>GHIP</i>       | (4) <i>GRIP</i>       |
|-----------------------|----------------------------|----------------------|-----------------------|-----------------------|
| <i>Source</i>         | <i>Floden-Linde</i> (2001) | <i>French</i> (2005) | <i>Güvenen</i> (2009) | <i>Güvenen</i> (2009) |
| $\rho_y$              | 0.92                       | 0.977                | 0.821                 | 0.988                 |
| $\sigma_y$            | 0.21                       | 0.12                 | 0.17                  | 0.122                 |

Table 2: Labor Income Stochastic Processes

| <i>Case</i>                                                     | <i>Distribution</i>                         | $\mu_\gamma$ | $\sigma_\gamma$ |
|-----------------------------------------------------------------|---------------------------------------------|--------------|-----------------|
| <i>KSS - PSID Data</i>                                          | <i>LN</i> ( $\mu_\gamma, \sigma_\gamma^2$ ) | 1.07         | 0.87            |
| <i>CP - SHIW Data</i> ( $\gamma^{Med}=1.7, \gamma^{0.75}=3.0$ ) | <i>Beta</i> ( $\mu_\gamma, \sigma_\gamma$ ) | 0.80         | 5.58            |

Table 3: Risk Aversion Distributions, author's calculations

| <i>RRA Heterogeneity:</i>                | <i>Case</i>     | <i>Asset</i>    | <i>Income</i>   | <i>Consumption</i> |
|------------------------------------------|-----------------|-----------------|-----------------|--------------------|
|                                          |                 | <i>Gini, CV</i> | <i>Gini, CV</i> | <i>Gini, CV</i>    |
| <i>Yes - KSS:</i>                        | (1) <i>FLIN</i> | 0.743, 1.865    | 0.160, 0.310    | 0.141, 0.284       |
|                                          | (2) <i>FREN</i> | 0.717, 1.644    | 0.130, 0.274    | 0.126, 0.269       |
|                                          | (3) <i>GHIP</i> | 0.686, 1.676    | 0.146, 0.291    | 0.123, 0.262       |
|                                          | (4) <i>GRIP</i> | 0.666, 1.422    | 0.118, 0.240    | 0.115, 0.236       |
| <i>No - <math>\gamma = 4.28</math>:*</i> | (1) <i>FLIN</i> | 0.455, 0.842    | 0.131, 0.232    | 0.099, 0.176       |
|                                          | (2) <i>FREN</i> | 0.472, 0.874    | 0.091, 0.164    | 0.085, 0.154       |
|                                          | (3) <i>GHIP</i> | 0.412, 0.763    | 0.112, 0.200    | 0.079, 0.143       |
|                                          | (4) <i>GRIP</i> | 0.462, 0.850    | 0.087, 0.156    | 0.083, 0.151       |
| <i>Yes - CP:</i>                         | (1) <i>FLIN</i> | 0.678, 1.565    | 0.159, 0.302    | 0.139, 0.272       |
|                                          | (2) <i>FREN</i> | 0.635, 1.339    | 0.116, 0.229    | 0.112, 0.223       |
|                                          | (3) <i>GHIP</i> | 0.612, 1.370    | 0.139, 0.265    | 0.114, 0.231       |
|                                          | (4) <i>GRIP</i> | 0.588, 1.179    | 0.105, 0.201    | 0.101, 0.197       |
| <i>No - <math>\gamma = 2.20</math>:*</i> | (1) <i>FLIN</i> | 0.488, 0.914    | 0.110, 0.243    | 0.108, 0.193       |
|                                          | (2) <i>FREN</i> | 0.488, 0.913    | 0.094, 0.170    | 0.088, 0.161       |
|                                          | (3) <i>GHIP</i> | 0.441, 0.823    | 0.116, 0.208    | 0.085, 0.154       |
|                                          | (4) <i>GRIP</i> | 0.475, 0.880    | 0.089, 0.162    | 0.085, 0.156       |

Table 4: Equilibrium - Inequality Measures in the one job model: Gini Index and Coefficient of Variation, an asterisk denotes that all the parameters are the same as in the corresponding heterogeneous preferences economy

|                                             | <i>Q1</i> | <i>Q2</i> | <i>Q3</i> | <i>Q4</i> | <i>Q5</i> | <i>90 – 95th</i> | <i>95 – 99th</i> | <i>100th</i> |
|---------------------------------------------|-----------|-----------|-----------|-----------|-----------|------------------|------------------|--------------|
| <i>Data:</i>                                |           |           |           |           |           |                  |                  |              |
| <i>U.S. - SCF98</i>                         | −.3       | 1.3       | 5.0       | 12.2      | 81.7      | 11.3             | 23.1             | 34.7         |
| <i>U.S. - PSID99</i>                        | −.8       | 0.8       | 4.2       | 12.6      | 83.2      | 14.0             | 23.7             | 30.8         |
| <i>Exogenous Earnings:</i>                  |           |           |           |           |           |                  |                  |              |
| <i>KSS (FLIN)</i>                           | .00       | 1.0       | 5.3       | 16.3      | 77.4      | 19.3             | 27.6             | 10.8         |
| <i>CRRA=4.28* (FLIN)</i>                    | 2.0       | 8.5       | 16.3      | 26.3      | 46.9      | 11.9             | 12.0             | 4.1          |
| <i>CP (FLIN)</i>                            | .01       | 2.1       | 7.9       | 20.4      | 69.5      | 17.2             | 21.9             | 9.4          |
| <i>CRRA=2.20* (FLIN)</i>                    | 1.3       | 7.3       | 15.5      | 26.4      | 49.5      | 12.6             | 12.9             | 4.5          |
| <i>Endogenous Sorting:</i>                  |           |           |           |           |           |                  |                  |              |
| <i>ES1 (KSS, <math>\rho_y = .95</math>)</i> | .00       | .04       | 1.8       | 11.2      | 87.0      | 19.1             | 33.6             | 16.3         |
| <i>ES2 (KSS, <math>\rho_y = .90</math>)</i> | .00       | .39       | 3.7       | 14.2      | 81.7      | 18.0             | 30.4             | 14.7         |
| <i>ES3 (CP, <math>\rho_y = .95</math>)</i>  | .00       | .30       | 3.8       | 16.0      | 79.8      | 19.4             | 27.0             | 12.5         |
| <i>ES4 (CP, <math>\rho_y = .90</math>)</i>  | .01       | .97       | 5.6       | 17.8      | 75.7      | 18.2             | 25.1             | 11.8         |

Table 5: Data Vs. Equilibria - Statistics of the Wealth Distribution, author’s calculations, the SCF figures are from Diaz-Gimenez, Glover and Rios-Rull (2011), an asterisk denotes that all the parameters are the same as in the corresponding heterogeneous preferences economy

| <i>Parameter</i>            | <i>POLS</i> | <i>LSDV</i> | <i>A-B</i> | <i>B-B</i> |
|-----------------------------|-------------|-------------|------------|------------|
| $\hat{\sigma}_{y,\gamma_l}$ | .420        | .388        | .344       | .347       |
| $\hat{\sigma}_{y,\gamma_h}$ | .396        | .363        | .317       | .317       |
| $\hat{\rho}_{y,\gamma_l}$   | .694        | .393        | .341       | .360       |
| $\hat{\rho}_{y,\gamma_h}$   | .711        | .386        | .292       | .291       |
| $Obs_{\gamma_l}$            | 14,667      | 14,667      | 12,440     | 14,167     |
| $Individuals_{\gamma_l}$    | —           | 1,330       | 1,232      | 1,416      |
| $Obs_{\gamma_h}$            | 15,295      | 15,295      | 13,175     | 14,769     |
| $Individuals_{\gamma_h}$    | —           | 1,212       | 1,131      | 1,208      |

Table 6: Estimates of the Labor Earnings Shocks Variance and Persistence by two Risk Aversion Categories in the PSID (1968-1996), author's calculations

| <i>Parameter</i> | <i>ES1</i> | <i>ES2</i> | <i>ES3</i> | <i>ES4</i> | <i>Targets</i>                                    |
|------------------|------------|------------|------------|------------|---------------------------------------------------|
| $\gamma$         | <i>KSS</i> | <i>KSS</i> | <i>CP</i>  | <i>CP</i>  | -                                                 |
| $\sigma_{y,h}$   | 0.4783     | 0.4591     | 0.4604     | 0.4600     | $\sigma_y   (\gamma \geq \gamma^{52nd}) = 0.3442$ |
| $\sigma_{y,l}$   | 0.2277     | 0.2448     | 0.2245     | 0.2239     | $\sigma_y   (\gamma < \gamma^{52nd}) = 0.3167$    |
| $\beta$          | 0.9340     | 0.9275     | 0.9516     | 0.9492     | $r = 0.04$                                        |

Table 7: Two Risky Jobs Model, Calibration - U.S.

| <i>RRA Heterogeneity:</i>                | <i>Case</i>                   | <i>Asset</i>    | <i>Income</i>   | <i>Consumption</i> |
|------------------------------------------|-------------------------------|-----------------|-----------------|--------------------|
|                                          |                               | <i>Gini, CV</i> | <i>Gini, CV</i> | <i>Gini, CV</i>    |
| <i>Yes - KSS:</i>                        | <i>ES1</i> ( $\rho_y = .95$ ) | 0.823, 2.437    | 0.227, 0.487    | 0.212, 0.459       |
|                                          | <i>ES2</i> ( $\rho_y = .90$ ) | 0.783, 2.203    | 0.234, 0.483    | 0.210, 0.436       |
| <i>No - <math>\gamma = 4.28</math>:*</i> | ( $\rho_y = .95$ )            | 0.479, 0.880    | 0.184, 0.325    | 0.152, 0.268       |
|                                          | ( $\rho_y = .90$ )            | 0.453, 0.842    | 0.195, 0.351    | 0.150, 0.267       |
| <i>Yes - CP:</i>                         | <i>ES3</i> ( $\rho_y = .95$ ) | 0.766, 1.976    | 0.218, 0.436    | 0.199, 0.398       |
|                                          | <i>ES4</i> ( $\rho_y = .90$ ) | 0.732, 1.841    | 0.229, 0.455    | 0.198, 0.391       |
| <i>No - <math>\gamma = 2.20</math>:*</i> | ( $\rho_y = .95$ )            | 0.519, 0.973    | 0.187, 0.331    | 0.153, 0.271       |
|                                          | ( $\rho_y = .90$ )            | 0.499, 0.942    | 0.200, 0.360    | 0.152, 0.270       |

Table 8: Equilibrium - Inequality Measures in the two risky jobs model: Gini Index and C. of V., an asterisk denotes that all the parameters are the same as in the corresponding heterogeneous preferences economy

| <i>Case</i> | <i>RRA Heterogeneity</i> | <i>r (%)</i> | <i>Y</i> | <i>I</i> | <i>C</i> | <i>K/Y</i> |
|-------------|--------------------------|--------------|----------|----------|----------|------------|
| (1)         | <i>No - CRRA=4.28*</i>   | 4.748        | 1.863    | 0.421    | 1.442    | 2.824      |
|             | <i>Yes - KSS ES1</i>     | 4.001        | 1.940    | 0.465    | 1.474    | 3.000      |
| (2)         | <i>No - CRRA=4.28*</i>   | 4.709        | 1.889    | 0.428    | 1.461    | 2.833      |
|             | <i>Yes - KSS ES2</i>     | 4.011        | 1.962    | 0.470    | 1.490    | 2.997      |
| (3)         | <i>No - CRRA=2.20*</i>   | 4.229        | 1.909    | 0.449    | 1.459    | 2.944      |
|             | <i>Yes - CP ES3</i>      | 4.007        | 1.939    | 0.465    | 1.472    | 2.998      |
| (4)         | <i>No - CRRA=2.20*</i>   | 4.222        | 1.934    | 0.456    | 1.478    | 2.945      |
|             | <i>Yes - CP ES4</i>      | 4.001        | 1.969    | 0.473    | 1.494    | 3.000      |

Table 9: Homogeneous Vs. Heterogeneous Preferences: Equilibrium Allocations, an asterisk denotes that all the parameters are the same as in the corresponding heterogeneous preferences economy

|                             | <i>Case</i>    | $K^{IM}$     | $K^{CM}$ | <i>Prec. Saving (%)</i> | <i>Prec. Saving (<math>\Delta</math>)</i> |
|-----------------------------|----------------|--------------|----------|-------------------------|-------------------------------------------|
| <i>KSS vs. CRRA=4.28*</i> : | (1) <i>ES1</i> | 5.821, 5.260 | 4.081    | 42.64, 28.89            | 13.75                                     |
|                             | (2) <i>ES2</i> | 5.880, 5.351 | 3.802    | 54.65, 40.74            | 13.91                                     |
| <i>CP vs. CRRA=2.20*</i> :  | (3) <i>ES3</i> | 5.812, 5.620 | 5.055    | 14.97, 11.17            | 3.80                                      |
|                             | (4) <i>ES4</i> | 5.907, 5.697 | 4.966    | 18.95, 14.72            | 4.23                                      |

Table 10: Precautionary Savings: Heterogeneous Vs. Homogeneous Preferences, an asterisk denotes that all the parameters are the same as in the corresponding heterogeneous preferences economy

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## Appendix A - Computation

- All codes solving the economies were written in the FORTRAN 2003 language, relying on the Intel Fortran Compiler, build 11.1.048 (with the IMSL library). They were compiled selecting the O3 option (maximize speed), and without automatic parallelization. Both the simulations of a sample of agents and the computation of the career dependent value functions were parallelized with the OpenMP directives. They were executed on a 64-bit PC platform, running Windows 7 Professional Edition, with an Intel *i7 - 2600k* Quad Core processor clocked at 4.6 Ghz.
- Depending on the parameters (essentially on the discount factor  $\beta$ , and the precision of the initial guess) without endogenous sorting the model solution took between 30 minutes and 3 hours to complete (between 4 and 15 iterations on the interest rate are needed to find each equilibrium). With endogenous sorting, thanks to the parallelization of the decision problem, the model solution took on average one hour. Hence, the moment matching procedure required up to 100 hours to complete.
- In the actual solution of the model, I need to discretize the three continuous state variables  $\varepsilon$ ,  $a$ , and  $\gamma$ . As for  $\varepsilon$ , I rely on Tauchen's method, which approximates the  $AR(1)$  process for the efficiency units with a Markov chain. I use an eleven-state chain, which guarantees that the errors in Tauchen's approximation are not systematically worse than with other discretization procedures. As for  $a$ , I rely on an unevenly spaced grid, with the distance between two consecutive points increasing geometrically. In order to keep the computational burden manageable, I use 151 grid points on the asset space, the lowest value being the borrowing constraint and the highest one being a value  $a_{\max} > \bar{a}$  high enough for the saving functions to cut the 45 degree line ( $a_{\max} = 150$ ). This is done to allow for a high precision of the policy rules at low values of  $a$ , where the change in curvature is more pronounced. As for  $\gamma$ , I discretize its support using 100 unevenly spaced points, the lowest value being close to zero ( $\gamma_{\min} = 0.001$ ) and the highest one being 11 ( $\gamma_{\max} = 11$ ). Overall, I need to solve a functional equation at  $11 \times 151 \times 100 = 166,100$  points in the state space.
- The model is solved with a 'successive approximation' procedure on the set of value functions. I start from a set of guesses  $V(\varepsilon, a; \gamma)_0$ . I compute the vector of parameters  $\Omega$  representing the Schumaker spline approximations of the value functions. I solve the constrained maximization problems and retrieve the policy functions,  $a'(\varepsilon, a; \gamma)$ . Notice that I do not restrict the agents' asset holdings to belong to a discrete set. As for the approximation method, I rely on the quadratic spline approximations for the future value functions, when evaluated at the chosen

saving level. I keep on iterating until a fixed point is reached, i.e. until two successive iterations satisfy:

$$\sup_a |V(\varepsilon, a; \gamma)_{n+1} - V(\varepsilon, a; \gamma)_n| < \epsilon, \forall \varepsilon \text{ and } \forall \gamma$$

where  $\epsilon$  is a small convergence criterion. In a previous version of the paper I solved the model with a time iteration scheme on the policy functions. For several parameters, this method showed numerical instability for high values of the risk aversion parameter. With value function iteration, the model can be solved more reliably with larger upper bounds for  $\gamma$ .

- In the model without endogenous sorting, the stationary distributions  $\mu(\varepsilon, a; \gamma)$  are computed by simulating a large sample of 100,000 individuals for 3,000 periods, which ensure that the statistics of interest are stationary processes. With endogenous sorting, the model needs longer simulations of 10,000 periods, with the 100,000 agents re-optimizing their career choices every 1,500 periods.
- The first step of the simulation is a random draw from the risk aversion distribution for each agent, whose risk aversion type is permanent, hence it will not evolve over time. Once the risk aversion is assigned, I compute the fixed weights for the interpolation in the  $\gamma$  dimension. I then start simulating the sample of agents by drawing sequences of efficiency units from the Markov chain, and compute the capital supply together with the inequality measures.
- In the model with endogenous sorting, I apply Tauchen's method twice, once for each exogenous earnings process. The value functions become job dependent,  $V^j(\varepsilon, a; \gamma)$   $j = h, l$ , but the solution method is similar to the one outlined above. In this case I need to solve a functional equation at  $2 \times 11 \times 151 \times 100 = 332,200$  points in the state space. I also need to simulate two independent sequences of shocks for each individual.
- As for the approximation method, I rely on a tetra-linear approximation scheme for the saving and consumption functions, for values of  $a$  and  $\gamma$  falling outside the grid. Notice that I interpolate also in the  $\gamma$  dimension. Some experimenting showed that a linear interpolation implied relatively large errors for lower values of  $\gamma$ , suggesting to have several points for  $\gamma < 1$ . Numerically, it still seems more appropriate to have a large number of gridpoints, with a slowly growing distance between gridpoints, rather than a coarser grid with an aggressive interpolation scheme. It goes without saying that this impacts quite substantially the computational burden. While I make sure that the asset grid is not binding, I extrapolate the saving choices in the risk aversion dimension up to  $\bar{\gamma} = 14.5$ , so that there are no constrained individuals in the CP case and a small number in the KSS one.

## Appendix B - Solution Algorithm

The computational procedure used to solve the baseline model can be represented by the following algorithm:

- Generate a discrete grid over the CRRA space  $[\gamma_{\min}, \dots, \gamma_{\max}]$ .
- Generate a discrete grid over the asset space  $[-b, \dots, a_{\max}]$ .
- Generate a discrete grid over the efficiency units space  $[\varepsilon_{\min}, \dots, \varepsilon_{\max}]$ .
- Get the aggregate labor supply  $L$ .
- Guess the interest rate  $r_0$ .
- Get the capital demand  $k$ .
- Get the wage rate per efficiency units  $w$ .
- Get the saving functions  $a'(\varepsilon, a; \gamma)$ .
- Get the stationary distributions  $\mu(\varepsilon, a; \gamma)$ .
- Get the aggregate capital supply.
- Check the asset market clearing; Get  $r_1$ .
- Update  $r'_0 = \varpi r_0 + (1 - \varpi) r_1$  (with  $\varpi$  an arbitrary weight).
- Iterate until market clearing.
- Get the consumption functions  $c(\varepsilon, a; \gamma)$ .
- Check the final good market clearing.

The computational procedure used to solve the two risky jobs model needs three additional steps: the computation of two job-dependent value functions  $V^j(\varepsilon, a; \gamma)$ , the computation of the optimal job policy functions  $\Phi(\varepsilon, a; \gamma)$ , and the related share of agents  $\phi$  choosing the less risky career.

- Generate a discrete grid over the CRRA space  $[\gamma_{\min}, \dots, \gamma_{\max}]$ .
- Generate a discrete grid over the asset space  $[-b, \dots, a_{\max}]$ .
- Generate two discrete grids over the efficiency units space  $[\varepsilon_{\min}^j, \dots, \varepsilon_{\max}^j]$ ,  $j = h, l$ .
- Guess the share of risky jobs  $\phi_0$ .
- Get the aggregate labor supply  $L$ .
- Guess the interest rate  $r_0$ .
- Get the capital demand  $k$ .
- Get the wage rate per efficiency units  $w$ .
- Get the saving functions  $a'_j(\varepsilon, a; \gamma)$ ,  $j = h, l$ .
- Get the job-dependent value functions  $V^j(\varepsilon, a; \gamma)$ .
- Get the optimal job decisions  $\Phi(\varepsilon, a; \gamma)$ .
- Get the stationary distributions  $\mu(\varepsilon, a; \gamma)$ .
- Get the aggregate capital supply.
- Check the asset market clearing; Get  $r_1$  and  $\phi_1$ .
- Update  $r'_0 = \varpi_r r_0 + (1 - \varpi_r) r_1$  (with  $\varpi_r$  an arbitrary weight).
- Update  $\phi'_0 = \varpi_\phi \phi_0 + (1 - \varpi_\phi) \phi_1$  (with  $\varpi_\phi$  an arbitrary weight).
- Iterate until both market clearing and  $\phi'_0 \approx \phi_0$ .
- Get the consumption functions  $c(\varepsilon, a; \gamma)$ .
- Check the final good market clearing.

## Appendix C - Additional Results

| <i>Wage Process</i> | $\rho_y$ | $\sigma_y$ | $\varepsilon_1$        | $\varepsilon_2$        | $\varepsilon_3$        | $\varepsilon_4$        | $\varepsilon_5$        | $\varepsilon_6$        | $\varepsilon_7$ | $\varepsilon_8$ | $\varepsilon_9$ | $\varepsilon_{10}$ | $\varepsilon_{11}$ |
|---------------------|----------|------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|-----------------|-----------------|-----------------|--------------------|--------------------|
|                     |          |            | $\mu_L(\varepsilon_1)$ | $\mu_L(\varepsilon_2)$ | $\mu_L(\varepsilon_3)$ | $\mu_L(\varepsilon_4)$ | $\mu_L(\varepsilon_5)$ | $\mu_L(\varepsilon_6)$ | .               | .               | .               | .                  | .                  |
| (1) <i>FLIN</i>     | .92      | .21        | .65                    | .72                    | .79                    | .86                    | .93                    | 1.00                   | 1.08            | 1.17            | 1.27            | 1.38               | 1.53               |
| <i>Floden-Linde</i> |          |            | 2.65%                  | 5.53%                  | 8.71%                  | 11.75%                 | 13.97%                 | 14.78%                 | .               | .               | .               | .                  | .                  |
| (2) <i>FREN</i>     | .977     | .12        | .88                    | .90                    | .92                    | .95                    | .98                    | 1.00                   | 1.02            | 1.05            | 1.08            | 1.11               | 1.14               |
| <i>French</i>       |          |            | 7.16%                  | 8.74%                  | 9.30%                  | 9.73%                  | 10.01%                 | 10.11%                 | .               | .               | .               | .                  | .                  |
| (3) <i>GHIP</i>     | .821     | .17        | .60                    | .68                    | .76                    | .83                    | .91                    | 1.00                   | 1.09            | 1.20            | 1.32            | 1.47               | 1.65               |
| <i>Güvenen</i>      |          |            | 0.40%                  | 2.08%                  | 6.11%                  | 12.37%                 | 18.50%                 | 21.09%                 | .               | .               | .               | .                  | .                  |
| (4) <i>GRIP</i>     | .988     | .122       | .91                    | .93                    | .95                    | .96                    | .98                    | 1.00                   | 1.02            | 1.04            | 1.05            | 1.08               | 1.10               |
| <i>Güvenen</i>      |          |            | 8.51%                  | 9.36%                  | 9.21%                  | 9.17%                  | 9.16%                  | 9.17%                  | .               | .               | .               | .                  | .                  |

Table 11: Labor Efficiency Units - Ergodic Distributions from the Tauchen Markov Chains

| <i>E. Process</i> | $\rho_y$ | $\sigma_y$ | $\varepsilon_1$<br>$\mu_L(\varepsilon_1)$ | $\varepsilon_2$<br>$\mu_L(\varepsilon_2)$ | $\varepsilon_3$<br>$\mu_L(\varepsilon_3)$ | $\varepsilon_4$<br>$\mu_L(\varepsilon_4)$ | $\varepsilon_5$<br>$\mu_L(\varepsilon_5)$ | $\varepsilon_6$<br>$\mu_L(\varepsilon_6)$ | $\varepsilon_7$<br>. | $\varepsilon_8$<br>. | $\varepsilon_9$<br>. | $\varepsilon_{10}$<br>. | $\varepsilon_{11}$<br>. |
|-------------------|----------|------------|-------------------------------------------|-------------------------------------------|-------------------------------------------|-------------------------------------------|-------------------------------------------|-------------------------------------------|----------------------|----------------------|----------------------|-------------------------|-------------------------|
| (1) <i>ES2</i>    |          |            |                                           |                                           |                                           |                                           |                                           |                                           |                      |                      |                      |                         |                         |
| <i>High Risk</i>  | .90      | .459       | .35<br>1.82%                              | .45<br>4.59%                              | .56<br>8.24%                              | .69<br>12.12%                             | .83<br>15.11%                             | 1.00<br>16.25%                            | 1.20<br>.            | 1.46<br>.            | 1.77<br>.            | 2.20<br>.               | 2.82<br>.               |
| <i>Low Risk</i>   | .90      | .245       | .57<br>1.82%                              | .66<br>4.59%                              | .74<br>8.24%                              | .82<br>12.12%                             | .91<br>15.11%                             | 1.00<br>16.25%                            | 1.10<br>.            | 1.22<br>.            | 1.36<br>.            | 1.52<br>.               | 1.74<br>.               |
| (2) <i>ES1</i>    |          |            |                                           |                                           |                                           |                                           |                                           |                                           |                      |                      |                      |                         |                         |
| <i>High Risk</i>  | .95      | .478       | .46<br>4.55%                              | .55<br>7.16%                              | .65<br>9.20%                              | .76<br>10.88%                             | .87<br>12.00%                             | 1.00<br>12.39%                            | 1.15<br>.            | 1.32<br>.            | 1.53<br>.            | 1.80<br>.               | 2.17<br>.               |
| <i>Low Risk</i>   | .95      | .228       | .69<br>4.55%                              | .76<br>7.16%                              | .82<br>9.20%                              | .87<br>10.88%                             | .94<br>12.00%                             | 1.00<br>12.39%                            | 1.07<br>.            | 1.14<br>.            | 1.23<br>.            | 1.32<br>.               | 1.45<br>.               |
| (3) <i>ES4</i>    |          |            |                                           |                                           |                                           |                                           |                                           |                                           |                      |                      |                      |                         |                         |
| <i>High Risk</i>  | .90      | .460       | .35<br>1.82%                              | .45<br>4.59%                              | .56<br>8.24%                              | .69<br>12.12%                             | .83<br>15.11%                             | 1.00<br>16.25%                            | 1.20<br>.            | 1.46<br>.            | 1.78<br>.            | 2.20<br>.               | 2.83<br>.               |
| <i>Low Risk</i>   | .90      | .224       | .60<br>1.82%                              | .68<br>4.59%                              | .76<br>8.24%                              | .83<br>12.12%                             | .91<br>15.11%                             | 1.00<br>16.25%                            | 1.09<br>.            | 1.20<br>.            | 1.32<br>.            | 1.47<br>.               | 1.66<br>.               |
| (4) <i>ES3</i>    |          |            |                                           |                                           |                                           |                                           |                                           |                                           |                      |                      |                      |                         |                         |
| <i>High Risk</i>  | .95      | .460       | .47<br>4.55%                              | .57<br>7.16%                              | .66<br>9.20%                              | .76<br>10.88%                             | .87<br>12.00%                             | 1.00<br>12.39%                            | 1.14<br>.            | 1.31<br>.            | 1.51<br>.            | 1.76<br>.               | 2.11<br>.               |
| <i>Low Risk</i>   | .95      | .225       | .69<br>4.55%                              | .76<br>7.16%                              | .82<br>9.20%                              | .88<br>10.88%                             | .94<br>12.00%                             | 1.00<br>12.39%                            | 1.07<br>.            | 1.14<br>.            | 1.22<br>.            | 1.32<br>.               | 1.44<br>.               |

Table 12: Labor Efficiency Units with Endogenous Sorting - Ergodic Distributions from the Two Tauchen Markov Chains

| <i>RRA Heterogeneity:</i>                           | <i>Case</i>                   | <i>Asset</i>    | <i>Income</i>   | <i>Consumption</i> |
|-----------------------------------------------------|-------------------------------|-----------------|-----------------|--------------------|
|                                                     |                               | <i>Gini, CV</i> | <i>Gini, CV</i> | <i>Gini, CV</i>    |
| <i>Yes - KSS:</i>                                   | <i>ES5</i> ( $\rho_y = .85$ ) | 0.747, 2.035    | 0.230, 0.468    | 0.197, 0.405       |
| <i>Yes - CP:</i>                                    | <i>ES6</i> ( $\rho_y = .85$ ) | 0.685, 1.643    | 0.223, 0.435    | 0.181, 0.352       |
| <i>Yes - KSS:</i>                                   | <i>ES7</i> ( $\rho_y = .60$ ) | 0.624, 1.504    | 0.205, 0.394    | 0.140, 0.283       |
| <i>No - <math>\gamma = 2.0, \beta = .96</math>:</i> | (1) <i>FLIN</i>               | 0.492, 0.924    | 0.138, 0.246    | 0.110, 0.197       |
|                                                     | (2) <i>FREN</i>               | 0.489, 0.915    | 0.095, 0.173    | 0.090, 0.164       |
|                                                     | (3) <i>GHIP</i>               | 0.445, 0.831    | 0.118, 0.210    | 0.087, 0.158       |
|                                                     | (4) <i>GRIP</i>               | 0.475, 0.880    | 0.091, 0.165    | 0.087, 0.159       |

Table 13: Robustness Analysis, Equilibrium - Inequality Measures: Gini Index and C. of V.

|                                             | $Q1$ | $Q2$ | $Q3$ | $Q4$ | $Q5$ | $90 - 95th$ | $95 - 99th$ | $100th$ |
|---------------------------------------------|------|------|------|------|------|-------------|-------------|---------|
| <i>U.S. - SCF07</i>                         | −.2  | 1.1  | 4.5  | 11.2 | 83.4 | 11.1        | 26.7        | 33.6    |
| <i>U.S. - SCF92</i>                         | −.4  | 1.7  | 5.7  | 13.4 | 79.5 | 12.6        | 23.9        | 29.5    |
| <i>U.S. - PSID01</i>                        | −0.9 | 0.7  | 4.2  | 13.2 | 82.8 | 14.3        | 24.1        | 28.6    |
| <i>U.S. - PSID94</i>                        | −1.2 | 0.8  | 5.0  | 14.7 | 80.7 | 15.2        | 24.8        | 23.9    |
| <i>ES5 (KSS, <math>\rho_y = .85</math>)</i> | .01  | 1.1  | 5.4  | 16.0 | 77.4 | 17.1        | 28.1        | 13.7    |
| <i>ES6 (CP, <math>\rho_y = .85</math>)</i>  | .1   | 2.1  | 7.7  | 19.8 | 70.3 | 16.9        | 22.4        | 10.4    |
| <i>ES7 (KSS, <math>\rho_y = .60</math>)</i> | .9   | 4.5  | 9.8  | 19.8 | 64.9 | 15.3        | 21.0        | 9.9     |

Table 14: Data Vs. Equilibria - Statistics of the Wealth Distribution

| <i>Case</i>     | <i>RRA Heterogeneity</i> | <i>r (%)</i> | <i>Y</i> | <i>I</i> | <i>C</i> | <i>K/Y</i> |
|-----------------|--------------------------|--------------|----------|----------|----------|------------|
| (1) <i>FLIN</i> | <i>No - CRRA=4.28</i>    | 3.223        | 1.963    | 0.504    | 1.459    | 3.208      |
|                 | <i>Yes - KSS</i>         | 2.949        | 1.991    | 0.524    | 1.467    | 3.288      |
| (2) <i>FREN</i> | <i>No - CRRA=4.28</i>    | 3.952        | 1.864    | 0.449    | 1.415    | 3.012      |
|                 | <i>Yes - KSS</i>         | 3.919        | 1.868    | 0.451    | 1.416    | 3.020      |
| (3) <i>GHIP</i> | <i>No - CRRA=4.28</i>    | 3.589        | 1.919    | 0.477    | 1.442    | 3.106      |
|                 | <i>Yes - KSS</i>         | 3.429        | 1.934    | 0.487    | 1.447    | 3.150      |
| (4) <i>GRIP</i> | <i>No - CRRA=4.28</i>    | 4.022        | 1.856    | 0.445    | 1.411    | 2.994      |
|                 | <i>Yes - KSS</i>         | 4.016        | 1.857    | 0.446    | 1.411    | 2.996      |

Table 15: Homogeneous Vs. Heterogeneous Preferences: Equilibrium Allocations

|                           | <i>Case</i>     | $K^{IM}$     | $K^{CM}$ | <i>Prec. Saving (%)</i> | <i>Prec. Saving (<math>\Delta</math>)</i> |
|---------------------------|-----------------|--------------|----------|-------------------------|-------------------------------------------|
| <i>KSS vs. CRRA=4.28:</i> | (1) <i>FLIN</i> | 6.546, 6.298 | 5.590    | 17.08, 12.66            | 4.42                                      |
|                           | (2) <i>FREN</i> | 5.641, 5.617 | 5.501    | 2.55, 2.11              | 0.44                                      |
|                           | (3) <i>GHIP</i> | 6.093, 5.962 | 5.564    | 9.50, 7.15              | 2.35                                      |
|                           | (4) <i>GRIP</i> | 5.563, 5.559 | 5.494    | 1.25, 1.18              | 0.07                                      |
| <i>CP vs. CRRA=2.20:</i>  | (1) <i>FLIN</i> | 5.998, 5.937 | 5.645    | 6.24, 5.16              | 1.08                                      |
|                           | (2) <i>FREN</i> | 5.605, 5.603 | 5.555    | 0.90, 0.85              | 0.05                                      |
|                           | (3) <i>GHIP</i> | 5.815, 5.786 | 5.619    | 3.48, 2.98              | 0.50                                      |
|                           | (4) <i>GRIP</i> | 5.574, 5.574 | 5.549    | 0.45, 0.45              | 0.00                                      |

Table 16: Precautionary Savings: Heterogeneous Vs. Homogeneous Preferences

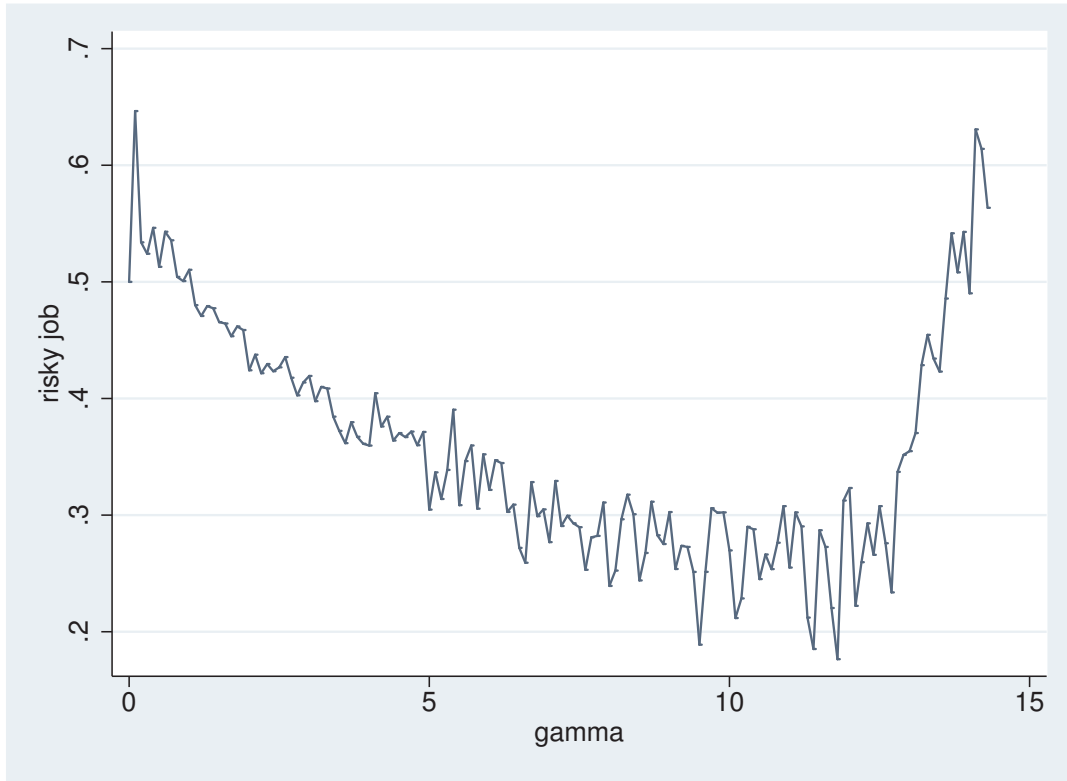


Figure 7: Percentage of agents in the riskier job, by the CRRA (Case ES1)

The wording of the initial PSID question is as follows:

Suppose you had a job that guaranteed you income for life equal to your current, total income. And that job was (your/your family's) only source of income. Then you are given the opportunity to take a new, and equally good, job with a 50-50 chance that it will double your income and spending power. But there is a 50-50 chance that it will cut your income and spending power by a third. Would you take the new job?