

Global Dynamics at the Zero Lower Bound*

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ABSTRACT

This article presents global solutions to standard New Keynesian models to show how economic dynamics change when the nominal interest rate is constrained at its zero lower bound (ZLB). We focus on the canonical New Keynesian model without capital, but we also study the model with capital, with and without investment adjustment costs. Our solution method emphasizes accuracy to capture the expectational effects of hitting the ZLB and returning to a positive interest rate. We find that the response to a technology shock has perverse consequences when the ZLB binds, even when a discount factor shock drives the interest rate to zero. Although we do not model the large scale asset purchases used by the Fed since 2009, our results suggest that the economy may have trouble recovering if the interest rate remains at zero. Given the perverse dynamics at the ZLB, we evaluate how monetary policy affects the likelihood of encountering the ZLB. We find that the probability of hitting the ZLB depends importantly on the monetary policy rule. A policy rule based on a dual mandate, such as the one proposed by Taylor (1993), is more likely to cause ZLB events when the central bank places greater emphasis on the output gap.

Keywords: Monetary Policy; Zero Lower Bound; Global Solution Method; Welfare

JEL Classifications: E31; E42; E58; E61

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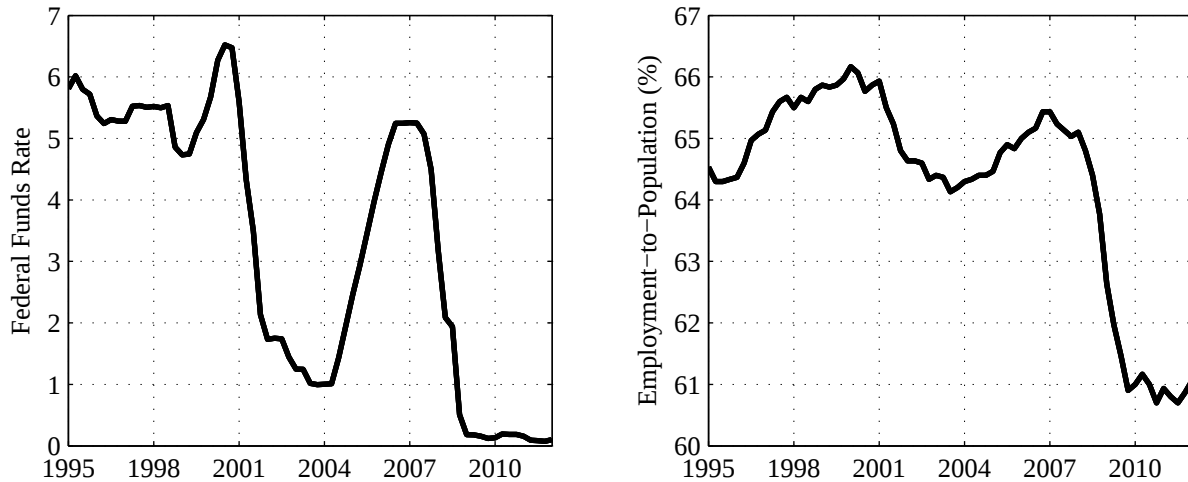


Figure 1: Federal funds rate (left panel) and employment-to-population percentage (right panel). Sources: Board of Governors of the Federal Reserve System and the U.S. Bureau of Labor Statistics.

1 INTRODUCTION

During the 2008 financial crisis, the Fed flooded the market with money, adding about \$600 billion in excess reserves to an economy that normally operates with about \$10 billion. This drove money market interest rates to their zero lower bound (ZLB). Four years after the crisis, the market remains saturated with money, money market interest rates remain near zero, and the economy is stagnant. The Fed has used two new unconventional monetary policy tools to keep money market interest rates at zero and lower longer-term interest rates. The first tool is large scale asset purchases. As of December 2012 these purchases have added over \$2 trillion to the Fed's balance sheet, increasing excess reserves to \$1.5 trillion. The second tool is "forward guidance," which signals to the public the Fed's expected policy rate path. The target has remained between 0 and 25 basis points since December 2008, but over time the Fed's policy statements have changed their language, generally to extend the length of time it expects to keep the rate unchanged.

Policymakers have been increasingly disappointed about the economy's failure to fully recover from the 2008-2009 recession. They continue to add to the stock of excess reserves, ensuring that the policy rate will remain at zero far into the future. Figure 1 shows U.S. data from 1995-2012 for the federal funds rate and the employment-to-population percentage. The federal funds rate (left panel) has varied between 6.5 percent and zero since 1995 and has been held below 25 basis points since the fourth quarter of 2008. During this period, the inflation rate has been at or below the Fed's long-run inflation objective, which led policymakers to shift their focus from the inflation target to the real economy. In public statements, the Fed has clearly articulated their commitment to employment stabilization, reacting to slow job growth during the recovery from the 2008-2009 recession. This is reflected in the low employment-to-population percentage (right panel). Weak labor market conditions have also been used as a justification for "forward guidance".

The Federal Reserve's statutory mandate for both price stability and full employment—known as the dual mandate—is somewhat unique for central banks around the world. Most central banks, other than the Fed, are formally instructed to promote price stability as their primary objective with the informal understanding that monetary policy can also assist in stabilizing output and employment. To achieve these policy objectives, many central banks target a nominal interest rate

and adjust that target to shifts in inflation relative to its target and to shifts in output relative to trend. These nominal interest rate rules are often referred to as Taylor rules, after Taylor (1993) found that a simple interest rate rule came close to replicating the Fed's behavior from 1987 to 1993. Whether or not a central bank has an explicit dual mandate, a monetary policy regime that utilizes a Taylor-type nominal interest rate rule behaves as if it has a dual mandate.

Our results suggest that promising to keep interest rates at zero may actually delay the recovery. We present global solutions to standard New Keynesian models to show how economic dynamics change when the nominal interest rate is pegged at its ZLB. First, we show that when the ZLB binds, positive technology shocks, which would normally aid the recovery, have perverse effects. At the ZLB, higher levels of technology lower employment and weaken aggregate demand, regardless of whether technology or discount factor shocks drive the interest rate to zero.¹

Given the perverse dynamics at the ZLB, we also evaluate how alternative monetary policy rules affect the likelihood of hitting the ZLB. We show how the dual mandate implicitly established in the Taylor rule affects the probability and frequency of ZLB events. Our results indicate that the probability of hitting the ZLB rises when the central bank places more emphasis on output stabilization and falls when there is more emphasis on price stability. Much of the work on the ZLB is done in models without capital. Capital accumulation is an important feature because it gives households another margin to smooth consumption following an exogenous shock. The presence of capital provides a savings channel that reduces the volatility of consumption and the real interest rate, which decreases the likelihood of ZLB events. Finally, we show that investment adjustment costs inhibit that savings mechanism and increase the probability of ZLB events.

Why do ZLB events matter? Friedman (1969), Kocherlakota (2005), and others argue that a nominal interest rate equal to zero is ideal because it promotes an optimal level of real cash balances. In the long run, that policy generates an inflation rate equal to the negative of the risk-free real interest rate, which is inconsistent with the Federal Reserve's 2 percent inflation rate target. Williams (2009) claims that central banks should 'embrace' the ZLB during periods of economic weakness, since it demonstrates that monetary policymakers will do everything possible to stimulate an underemployed economy. According to Williams (2009), the failure to hit the ZLB in past recessions was a sign of a suboptimal policy response to economic conditions.

In recent years, Japan has come the closest of any major industrialized country to embracing the ZLB. Specifically, the Bank of Japan has set its target interest rate near or slightly above zero since the mid-1990's. Since then, the Japanese economy has endured anemic economic growth and slight deflation. That experience has generated a significant amount of research on the effects of the Bank of Japan's policy [see, for example, Krugman (1998), Posen (1998), Hoshi and Kashyap (2000), Eggertsson and Woodford (2003), and Braun and Waki (2006)]. Many arguments against the ZLB are motivated, in part, by the recent Japanese experience. One set of arguments is based on the possibility of multiple equilibria at the ZLB. Schmitt-Grohé and Uribe (2012) show that when a central bank utilizes a Taylor-type policy rule, the consequences of hitting the ZLB may include moving to an undesirable low output/low inflation equilibrium.²

Summers (1991) argues that the inflation target should not be set to zero, but rather to some

¹There are some caveats to this pessimistic conclusion. First, we have not explicitly modeled the Fed's policy. The large scale asset purchase program seems to have kept deflation at bay. Modeling policy is high on our agenda for future research. Second, Wieland (2012) uses structural VAR evidence to argue that these perverse dynamics did not occur following shocks to the 2011 earthquake/tsunami in Japan or recent oil supply shocks.

²See Bullard (2010) for a summary of this argument and further references.

higher number precisely to avoid hitting the ZLB when conducting countercyclical policy. He argues that the central bank's ability to achieve its employment and output targets are constrained when the interest rate is pegged at zero. Reifschneider and Williams (2000), Chung et al. (2012), and Coibion et al. (2012) discuss the optimal policy when ZLB events are possible and provide analysis of the welfare losses during ZLB events. Chung et al. (2012) argue that the literature understates the probability of hitting the ZLB because past analyses have not taken proper account of model uncertainty, including uncertainty about the shock processes hitting the economy.

A practical criticism is that a low nominal interest rate target may be misinterpreted by households. Bullard (2010) notes that attempting to stimulate the economy by promising to keep the interest rate at zero may backfire as inflation expectations may fall rather than rise. Indeed, Del Negro et al. (2012) provide evidence that recent promises to maintain the ZLB for an extended period have been interpreted as a signal that the central bank believes the economic outlook has worsened.

A major contribution of this study is its examination of economic dynamics at the ZLB using global solutions to a nonlinear New Keynesian model.³ Most of the existing literature uses log-linearized New Keynesian models to study both the consequences of hitting the ZLB and the impact of monetary policy at the ZLB.⁴ However, using log-linearized models to analyze the effects of a ZLB event creates the potential for large approximation errors. For example, Braun et al. (2012) argue that log-linearized models often lead to incorrect inferences about an equilibrium's existence, uniqueness, and local dynamics. Moreover, they provide explicit examples of the mistakes resulting from log-linearized models evaluated at the ZLB.⁵ Our paper avoids these problems by using global nonlinear solutions methods [Richter et al. (2012)].

Our findings provide a compelling reason to avoid the ZLB that is directly relevant for output stabilization. While we do not examine the effects of recent unconventional policy in this paper, we do show that if the economy experiences a positive technology shock while at the ZLB, it has perverse effects on the economy. In all of our models, a positive technology shock depresses consumption and employment when the nominal interest rate is pegged at zero.

In this paper, the exogenous sources of variation are discount factor and technology shocks. When the Fed follows a Taylor rule, higher levels of technology lead to lower inflation and lower nominal interest rates. The effects of technology shocks are well-documented and understood in both New Classical and New Keynesian models when the ZLB does not bind. While no one believes interest rates fell to zero in 2008 due to a series of positive technology shocks, our main interest is to learn how the economy reacts to technology shocks when the ZLB binds.

This paper focuses sharply on the Fisher equation and draws a similar conclusion as Schmitt-Grohé and Uribe (2012). That is, the economy may not be able to recover until nominal interest rates are allowed to rise to a level consistent with the steady-state growth rates of inflation and technology. We use technology shocks to analyze equilibrium dynamics at the ZLB because it is important to understand the consequences of these shocks when the Fed promises to keep interest

³The paper closest to ours is Fernández-Villaverde et al. (2012), which uses different methods to generate global solutions to the New Keynesian model with capital and a larger set of shocks. It addresses a different set of issues.

⁴Within the policy arena, three influential papers that use linearized New Keynesian models to study the consequences of ZLB are Eggertsson and Woodford (2003), Gertler and Karadi (2011), and Werning (2012). Papers that address the likelihood of hitting the ZLB in linearized models include Reifschneider and Williams (2000), Chung et al. (2012), Gavin and Keen (2012), and Hatcher (2011).

⁵Braun et al. (2012) focus on the effect of labor taxes on the equilibrium level of labor and the size of fiscal multipliers and not on the impact of monetary policy.

rates at zero for several years. In our models, a positive technology shock that occurs when the nominal interest rate is zero pushes down the price level, consumption, and employment.

Section 2 briefly describes the alternative models. Section 3 describes the calibration and solution procedure, and sections 4 through 8 present the results. These sections report on the model solutions, the economic dynamics at the ZLB, the likelihood of hitting the ZLB, and the welfare consequences of ZLB events. Section 9 concludes.

2 ECONOMIC MODELS

This section presents three alternative models. The baseline specification is a New Keynesian model with Rotemberg (1982) price adjustment costs. Model 1 assumes stochastic processes for the discount factor and technology but does not include capital. Models 2 and 3 incorporate capital accumulation into Model 1, and Model 3 also includes investment adjustment costs. The main purpose of introducing Models 2 and 3 is to show how the model solutions and equilibrium dynamics change as the household's ability to save changes.

2.1 MODEL 1: BASELINE A representative household chooses sequences $\{c_t, n_t, b_t\}_{t=0}^{\infty}$ to maximize expected lifetime utility, given by,

$$E_0 \sum_{t=0}^{\infty} \tilde{\beta}_t \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\eta}}{1+\eta} \right\}, \quad (1)$$

where $1/\sigma$ is the intertemporal elasticity of substitution, $1/\eta$ is the Frisch elasticity of labor supply, c_t is consumption of the final good, n_t is labor hours, $\tilde{\beta}_0 \equiv 1$, and $\tilde{\beta}_t = \prod_{i=1}^t \beta_i$ for $t > 0$. β_i is a time-varying subjective discount factor that evolves according to

$$\beta_i = \beta(\beta_{i-1}/\beta)^{\rho_\beta} \exp(\varepsilon_{\beta,i}),$$

where β is the stationary discount factor, $0 \leq \rho_\beta < 1$, and $\varepsilon_{\beta,i} \sim N(0, \sigma_\beta^2)$.

The representative household's choices are constrained by

$$c_t + b_t = w_t n_t + r_{t-1} b_{t-1} / \pi_t + \tau_t,$$

where $\pi_t = p_t/p_{t-1}$ is the gross inflation rate, w_t is the real wage, τ_t is a lump-sum tax, b_t is a one-period real bond, and r_t is the gross nominal interest rate. Solving the household's utility maximization problem yields the following optimality conditions

$$w_t = \chi n_t^\eta c_t^\sigma, \quad (2)$$

$$1 = r_t E_t \{ \beta_{t+1} (c_t/c_{t+1})^\sigma / \pi_{t+1} \}. \quad (3)$$

The production sector consists of monopolistically competitive intermediate goods producing firms who produce a continuum of differentiated inputs and a representative final goods producing firm. Each firm $i \in [0, 1]$ in the intermediate goods sector produces a differentiated good, $y_t(i)$, with identical technologies given by $y_t(i) = z_t n_t(i)$, where $n_t(i)$ is the level of employment used by firm i . z_t represents the level of technology, which is common across firms and follows

$$z_t = \bar{z} (z_{t-1}/\bar{z})^{\rho_z} \exp(\varepsilon_{z,t}),$$

where $\bar{z}$ is steady-state technology, $0 \leq \rho_z < 1$, and $\varepsilon_{z,t} \sim \mathcal{N}(0, \sigma_z^2)$. Each intermediate firm chooses its labor supply to minimize its operating costs, $w_t n_t(i)$, subject to its production function.

Using a Dixit and Stiglitz (1977) aggregator, the representative final goods producer purchases $y_t(i)$ units from each intermediate firm to produce the final good, $y_t \equiv [\int_0^1 y_t(i)^{(\theta-1)/\theta} di]^{\theta/(\theta-1)}$, where $\theta > 1$ measures the elasticity of substitution between the intermediate goods. Maximizing profits for a given level of output yields the demand function for intermediate inputs given by $y_t(i) = (p_t(i)/p_t)^{-\theta} y_t$, where $p_t = [\int_0^1 p_t(i)^{1-\theta} di]^{1/(1-\theta)}$ is the price of the final good. Following Rotemberg (1982), each firm faces a cost to adjusting its price, which emphasizes the potentially negative effect that price changes can have on customer-firm relationships. Using the functional form in Ireland (1997), real profits of firm i are

$$d_t(i) = \left[\left(\frac{p_t(i)}{p_t} \right)^{1-\theta} - \Psi_t \left(\frac{p_t(i)}{p_t} \right)^{-\theta} - \frac{\varphi}{2} \left(\frac{p_t(i)}{\bar{\pi} p_{t-1}(i)} - 1 \right)^2 \right] y_t,$$

where $\varphi \geq 0$ determines the magnitude of the adjustment cost, Ψ_t is real marginal costs, and $\bar{\pi}$ is the steady-state gross inflation rate. Each intermediate goods producing firm chooses its price level, $p_t(i)$, to maximize the expected discounted present value of real profits $E_t \sum_{k=t}^{\infty} \lambda_{t,k} d_k(i)$, where $\lambda_{t,t} \equiv 1$, $\lambda_{t,t+1} = \beta_{t+1}(c_t/c_{t+1})^\sigma$, and $\lambda_{t,k} \equiv \prod_{j=t+1}^k \lambda_{j-1,j}$ is the stochastic discount factor between periods t and $k > t$. In a symmetric equilibrium, all intermediate goods producing firms make the same decisions and the optimality condition becomes

$$\varphi \left(\frac{\pi_t}{\bar{\pi}} - 1 \right) \frac{\pi_t}{\bar{\pi}} = (1 - \theta) + \theta \Psi_t + \varphi E_t \left[\lambda_{t,t+1} \left(\frac{\pi_{t+1}}{\bar{\pi}} - 1 \right) \frac{\pi_{t+1}}{\bar{\pi}} \frac{y_{t+1}}{y_t} \right].$$

In the absence of price adjustment costs (i.e. $\varphi = 0$), real marginal costs equal $(\theta - 1)/\theta$, which is equivalent to the inverse of the firm's markup of price over marginal cost.

Each period the fiscal authority finances a constant level of discretionary spending, $\bar{g}$, by levying lump-sum taxes. The monetary authority sets policy according to

$$r_t = \max\{1, \bar{r}(\pi_t/\pi^*)^{\phi_\pi} (y_t/\bar{y})^{\phi_y}\},$$

where π^* is the inflation rate target and ϕ_π and ϕ_y are the policy responses to inflation and output.

In this paper, the output gap is defined as the deviation of output from its steady state. We use this measure because we believe that policymakers, in the short-to-medium term, assume potential output grows at a relatively constant rate. Potential output measures are revised in the long run following incoming information about shocks, but the revisions occur well after the temporary economic effects from sticky prices have dissipated. In our model, a positive technology shock causes output to rise relative to its steady state and inflation to fall. For our baseline calibration, the downward movement in inflation dominates the higher output so the nominal interest rate declines.

Alternatively, the output gap can be defined as the difference between actual output and the level of output in the absence of nominal frictions. Under this definition of the output gap, a positive technology shock would result in a negative output gap because price frictions would prevent actual output from rising as much as it would in the flexible price economy. Thus, the downward pressure on the nominal interest rate coming from low inflation would be reinforced by the additional downward pressure coming from a negative output gap.

The aggregate resource constraint is given by $c_t + \bar{g} = [1 - \varphi(\pi_t/\bar{\pi} - 1)^2/2]y_t$. Equilibrium is characterized by the household's and firm's optimality conditions, the government's budget constraint, the bond market clearing condition ($b_t = 0$), and the aggregate resource constraint.

2.2 MODEL 2: BASELINE WITH CAPITAL Model 2 adds capital accumulation to Model 1, but assumes a constant discount factor. Assuming, $\beta_t = \beta$ for all t , the household chooses sequences $\{c_t, k_t, i_t, n_t, b_t\}_{t=0}^{\infty}$ to maximize (1) subject to

$$c_t + i_t + b_t = w_t n_t + r_t^k k_{t-1} + r_{t-1} b_{t-1} / \pi_t + \tau_t, \quad (4)$$

$$k_t = (1 - \delta) k_{t-1} + i_t, \quad (5)$$

where i_t is investment, k_t is the capital stock, and r_t^k is the real capital rental rate. The representative household's optimality conditions include (2), (3), and the consumption Euler equation, given by,

$$1 = \beta E_t \{ (c_t / c_{t+1})^\sigma (r_{t+1}^k + 1 - \delta) \}. \quad (6)$$

Each firm $i \in [0, 1]$ in the intermediate goods sector produces a differentiated good, $y_t(i)$, with identical technologies given by $y_t(i) = k_{t-1}(i)^\alpha n_t(i)^{(1-\alpha)}$, where $k_t(i)$ and $n_t(i)$ are the levels of capital and employment used by firm i . Each intermediate firm chooses capital and labor to minimize its operating costs, $r_t^k k_{t-1}(i) + w_t n_t(i)$, subject to its production function. The firm pricing equation remains unchanged, except that the definition of marginal costs changes. The aggregate resource constraint is now given by $c_t + i_t + \bar{g} = [1 - \varphi(\pi_t / \bar{\pi} - 1)^2 / 2] y_t$.

2.3 MODEL 3: BASELINE WITH CAPITAL AND INVESTMENT ADJUSTMENT COSTS Model 3 adds investment adjustment costs to Model 2. Capital now evolves according to

$$k_t = (1 - \delta) k_{t-1} + i_t \left[1 - \frac{\nu}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \right],$$

where ν measures the size of the adjustment costs. Optimality yields a new consumption Euler equation, which replaces (6) in Model 2, and an investment Euler equation, given by,

$$q_t = \beta E_t \left\{ (c_t / c_{t+1})^\sigma \left(r_{t+1}^k + q_{t+1} (1 - \delta) \right) \right\}$$

$$1 = q_t \left[1 - \frac{\nu}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 - \nu \frac{i_t}{i_{t-1}} \left(\frac{i_t}{i_{t-1}} - 1 \right) \right] + \beta \nu E_t \left\{ q_{t+1} \left(\frac{c_t}{c_{t+1}} \right)^\sigma \left(\frac{i_{t+1}}{i_t} \right)^2 \left(\frac{i_{t+1}}{i_t} - 1 \right) \right\},$$

where q_t is Tobin's q . The aggregate resource constraint is the same as in Model 2, except that both investment and output now include resources lost to investment adjustment costs.

3 CALIBRATION AND SOLUTION TECHNIQUE

The models in section 2 are calibrated at a quarterly frequency and the parameters are given in table 1. We set the coefficient of relative risk aversion equal to 1, implying log utility in consumption. The risk-free real interest rate is set equal to 4 percent annually, which implies a stationary quarterly discount factor, β , equal to 0.99. We set the persistence of the discount factor equal to 0.8 and the standard deviation of the shock equal to 0.0025. We follow Fernández-Villaverde et al. (2012) who chose these parameters so that a discount factor shock has a half life of about 3 quarters and an unconditional standard deviation of 0.42 percent. The Frisch elasticity of labor supply, $1/\eta$, is set to 3, which is consistent with Peterman (2012) who estimates that the Frisch elasticity lies between 2.9 and 3.1. The leisure preference parameter, χ , is calibrated so that steady-state labor

Constant of Relative Risk Aversion	σ	1	Inflation Target	π^*	1.005
Frisch Elasticity of Labor Supply	$1/\eta$	3	Inflation Coefficient: MP Rule	ϕ_π	1.50
Elasticity of Substitution between Goods	θ	6	Output Coefficient: MP Rule	ϕ_y	0.125
Steady State Government Spending Share	$\bar{g}/\bar{y}$	0.2	Stationary Technology	$\bar{z}$	1
Rotemberg Adjustment Cost Coefficient	φ	58.25	Technology Persistence	ρ_z	0.80
Leisure Preference Parameter	χ	4.507	Technology Shock Standard Deviation	σ_z	0.012
Capital Depreciation Rate	δ	0.025	Stationary Discount Factor	β	0.99
Cost Share of Capital	α	0.33	Discount Factor Persistence	ρ_β	0.80
Investment Adjustment Cost	ν	2.5	Discount Factor Standard Deviation	σ_β	0.0025

Table 1: Baseline calibration

equals $1/3$ of the available time. Capital's share of output, α , is set to 0.33 and the depreciation rate, δ , is set to 2.5 percent per quarter.

The baseline investment adjustment cost (IAC) parameter, ν , is set to 2.5, which is the value estimated by Christiano et al. (2005) in their benchmark model. This parameter value seems high and causes the model with IAC to behave like Model 1. We also report results with $\nu = 0.01$, which leads to a model that is similar to Model 2. However, even this low IAC parameter value causes a measurable change in the behavior of investment at the ZLB.

The steady-state price elasticity of demand for the intermediate good, θ , is set to 6, which corresponds to an average markup of price over marginal cost equal to 20 percent. The costly price adjustment parameter, φ , is set to 58.25, which is calibrated so that prices change on average once every four quarters. Although our qualitative results are robust to a flexible price model, we include nominal price frictions so that our results are comparable with much of the existing literature on the ZLB, which assumes nominal price rigidities.

In the policy sector, the steady-state gross inflation rate, $\bar{\pi}$, is set to 1.005, which implies an annual inflation rate target of 2 percent. The steady-state ratio of government spending to output is set to 20 percent. In our baseline case, the coefficients in the policy rule are consistent with Taylor (1993). That is, the coefficients on inflation and output are set to 1.5 and 0.125, respectively.⁶

The steady-state technology factor, $\bar{z}$, is normalized to 1. The likelihood of hitting the ZLB depends critically on the driving processes (σ_z^2 and ρ_z) for technology. When the values for σ_z^2 and ρ_z are small, the likelihood of hitting the ZLB is low. When we match the calibrated value for ρ_z typically used in quantitative New Keynesian and Neoclassical models, determinacy is not guaranteed on the entire state space of our model. A determinate solution requires us to compromise on the size of ρ_z as well as the steady-state real interest rate. It requires that the sizes of the coefficient on the output gap in the policy rule and the persistence and size of the technology shock are not too large.⁷ Thus, we set $\rho_z = 0.8$ and σ_z between 1 and 1.2 percent per quarter, depending on the model, which pushes the standard deviation of z_t toward values that are common in the literature.

When analyzing the effects of ZLB spells in a linear model, researchers typically assume a large preference or natural rate shock hits the economy to drive the nominal interest rate zero [Eggertsson and Woodford (2003), Jung et al. (2005), Erceg and Linde (2010), Christiano et al. (2011)]. The resulting ZLB event then lasts a predetermined duration.⁸ Once the ZLB event ends,

⁶ Note that our baseline value, $\phi_y = 0.125$, is equivalent to the value used in Taylor (1993) because he uses annualized rates for interest and inflation while we use quarterly rates.

⁷ The ZLB is equivalent to a fixed interest rate regime, and the solution is not determinate if there is very little expectation of returning to an interest rate rule that aggressively responds to inflation. Davig and Leeper (2007) discuss determinacy in fixed non-zero interest rate regimes (passive monetary/passive fiscal).

⁸ Erceg and Linde (2010) use an alternative linear solution method that endogenizes the duration of the ZLB as a

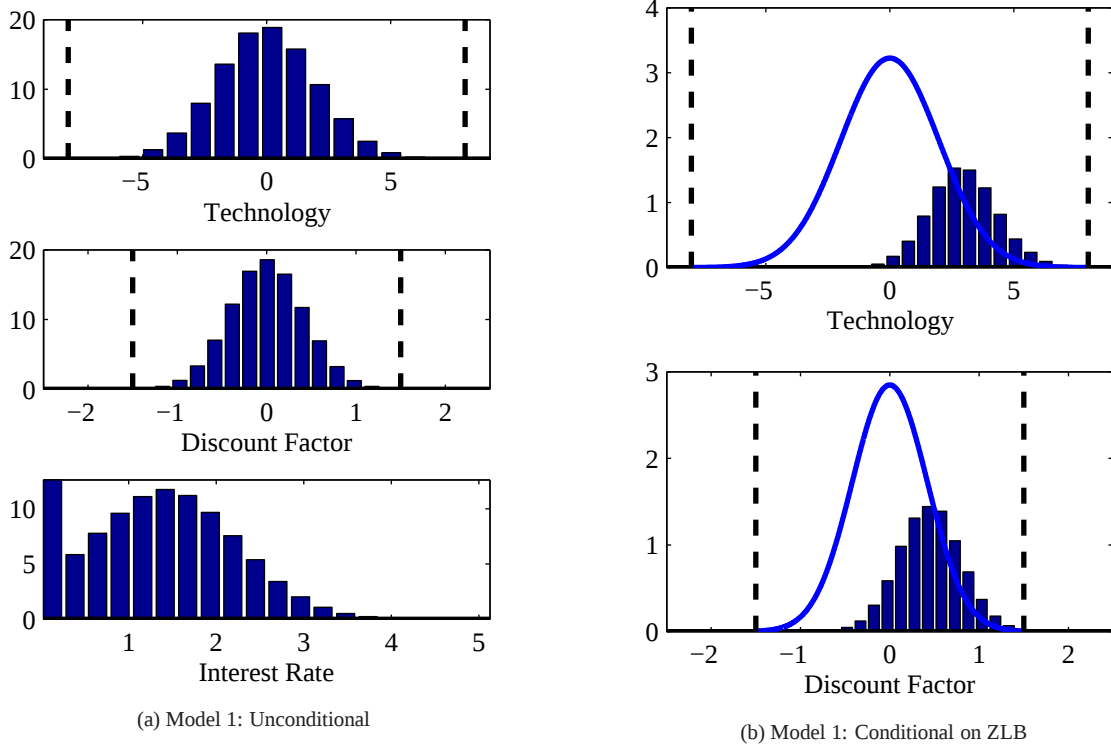


Figure 2: Model 1 distributions as a percentage of a 500,000 period simulation with parameters $\phi_y = 0.125$, $\phi_\pi = 1.50$, $\rho_z = 0.80$, $\sigma_z = 0.01189$, $\rho_\beta = 0.80$, and $\sigma_\beta = 0.0025$. State variables are given in percent deviations from steady state. The dashed lines are the bounds of the discretized state space. The solid blue lines are the theoretical unconditional distributions of the state variables scaled for comparison with the distributions conditional on the ZLB.

the nominal interest rate becomes positive and market participants do not expect it will return to zero. This type of experiment relies on unrealistically large shocks and arbitrarily fixes the frequency and duration of ZLB events, since expectations about exiting the ZLB are important. Our global nonlinear solution naturally captures these expectational effects and allows us to perform simulations without giving households perfect foresight about the end of the ZLB event. Additionally, Braun et al. (2012) and Fernández-Villaverde et al. (2012) show that linear solutions lead to significant approximation errors at the ZLB. These approximation errors overstate the effects of fiscal policy. While we do not model fiscal policy, Richter et al. (2012) show that the Euler equation errors from the global nonlinear solution are small near the ZLB. Using a global nonlinear solution removes the concern about approximation errors resulting from a linearized model.

The models are simulated using draws from the distributions for the discount factor and technology shocks. The state space is discretized to minimize extrapolation of the policy functions during the simulation. As an example, we plot the simulated distributions of the state variables for Model 1 in figure 2 and show that they are contained within the bounds of the state space. We simulate the model for 500,000 periods to obtain an accurate sample of ZLB events.

function of the government spending shock. However, the algorithm adds a sequence of current and future innovations so that the ZLB is satisfied conditional on the state. Performing a monte carlo simulation with such a method is infeasible, and therefore it too cannot be used to characterize the distribution of ZLB events. They also rely on a series of large preference shocks to hit the ZLB and there is no expectation to return after exiting.

Panel (a) shows the unconditional distributions of technology, the discount factor, and the nominal interest rate. The state space for technology lies within ± 8 percent of the steady state value, which is normalized to unity in our simulations. The state space of the discount factor lies between ± 1.5 percent of the steady state, which is equal to 0.99. Over these states, the net nominal interest rate is distributed over a range of 0 to 5 percent, with a large mass (12.6 percent of the simulated quarters) between 0 and 25 basis points. The steady state quarterly rate is 1.5 percent.

Panel (b) shows the distribution of the discount factor and technology conditional on the ZLB binding. Fernández-Villaverde et al. (2012) note that high levels of technology are associated with low interest rates. The reason is that for many monetary policy rules (including Taylor rules and fixed money growth rules), high levels of technology are associated with low inflation and low nominal interest rates. If technology is high enough and the central bank is following a Taylor rule, the nominal interest rate will hit its ZLB. Kiley (2003) uses U.S. data to show that periods of high labor productivity growth have been associated with relatively low inflation and shows that this result could be caused by the Fed's policy rule.

4 MODEL 1: STATES OF THE ECONOMY AND THE ZLB

This section shows policy functions that are based on the original Taylor (1993) specification with $\phi_\pi = 1.5$ and $\phi_y = 0.125$. We begin with a more thorough analysis of Model 1, which includes discount factor and technology shocks. All of the variables are given in percent deviations from their deterministic steady state, except inflation and the (net) interest rates, which are presented in levels. Price adjustment costs are measured as a percent of output.

Before discussing the results in detail, we reiterate that the persistence of the technology shock is set to 0.8, but the variance of the technology shock, σ_z^2 , is the largest possible value that permits a convergent solution. For Model 1, the standard deviation of the technology shock is 1.2 percent per quarter. Note that the allowable values for the persistence and volatility of technology shocks also depend on the value of ϕ_y . Fernández-Villaverde et al. (2012) choose $\phi_y = 0.25$ and $\rho_z = 0.9$, which are larger than our values, but they set $\sigma_z = 0.0025$, which is much smaller than our value.

Figure 3 shows three-dimensional contour plots of the non-predetermined variables over the entire state space, but these maps contain a lot of information and can be difficult to read. To obtain a clear picture of a subset of the model solutions, figure 4 shows the non-predetermined variables as a function of the technology state when the discount factor is fixed at its stationary value ($\beta = 0.99$); that is, we take a particular slice from the three-dimensional contour plot.

First consider the region of the state space where the ZLB does not bind; for Model 1 this includes states where technology ranges between 8 percent below and 3.6 percent above the steady state. A key feature of the model is that households smooth consumption and leisure over time. At low levels of technology, consumption is below its steady state, while inflation and the nominal interest rate are above their steady states. Households expect technology to rise as it returns to steady state. Therefore, consumption is expected to grow faster than the balanced growth rate and the *ex-ante* real interest rate is high relative to its steady state. Since consumption is low relative to its steady state, the household chooses to work more hours (reduce leisure), even with a low state of labor productivity (and a low real wage). The basic idea is that low technology reduces supply and increases the price of consuming (in terms of sacrificed leisure), which increases inflation.

With costly price adjustments, low labor productivity is associated with relatively high real marginal costs. High marginal costs are associated with high inflation and a relatively large loss of

output (about 1 percent). Price adjustment costs approach zero as inflation approaches its steady state and then rise as inflation falls below its steady state. In states with higher technology, we see higher consumption, lower inflation, a lower nominal interest rate, and less labor supplied. Households supply less labor since they want to increase leisure at higher levels of consumption.

Next consider the states where the ZLB binds (the shaded region in [figure 4](#)), which includes technology states that are more than 3.6 percent above the steady state. At high enough levels of technology, expected deflation drives the nominal interest rate to zero. With the nominal interest rate pegged at zero, inflation must decline to clear the bond market. The lower inflation is associated with a higher real interest rate, which induces the household to save more. However, in equilibrium, aggregate saving is zero, and the only way individuals will want to save less is with lower output and consumption. At even higher levels of technology, the real rate and leisure are even higher, but, again, consumption must fall to ensure aggregate saving is zero. Inflation will continue to fall, lowering real marginal costs, but raising price adjustment costs. Output is higher in states with higher technology, but the relative excess over consumption is lost to the costs of price adjustment because inflation is so far below target.

In [figure 5](#) we show the solutions across the technology space when the discount factor is held constant at 0.9997, which is the value where the ZLB begins to bind when technology is at its steady state. This is a second slice from the contour plots in [figure 3](#) at a higher discount factor. The main reason for showing this solution is to highlight that the perverse response of the economy to a positive technology shock at the ZLB does not depend on a high level of technology to drive the economy to its ZLB. The solutions from [figure 4](#) display the same properties as those in [figure 5](#). Looking at the highest discount factor shown in [figure 3](#) ($\beta_{t-1} = 1.005$), it is clear that the same dynamics continue to apply even when technology is below its steady state. Indeed, this is the area of the state space that is often considered in ZLB studies. If there was a very large discount factor shock as modeled by Fernández-Villaverde et al. (2012), Christiano et al. (2011), and Schmitt-Grohé and Uribe (2012), then we speculate that these same dynamics would appear.

5 MODEL 1: ECONOMIC DYNAMICS

The Taylor rule defines monetary policy in our baseline calibration. The short-run dynamics of the economy are distorted by price adjustment costs. However, this nominal friction is not necessary to generate the abrupt shift in dynamics at the ZLB. If we remove price adjustment costs, the impulse responses to a technology shock change quantitatively, but not qualitatively. All else equal, the likelihood of hitting the ZLB actually increases if prices are flexible, because inflation and nominal interest rates are more variable. In Model 1, the dynamics are driven by the household's attempt to smooth consumption and leisure and the two exogenous stochastic AR(1) processes.

Only looking at the baseline calibration does not reveal the extent to which the response of inflation to real marginal costs depends on the monetary policy rule. As [section 7](#) will make clear, changes in the monetary policy rule dramatically alters model dynamics and the likelihood of ZLB events. If monetary policy optimally eliminates the distortions caused by nominal frictions, then real marginal costs will have no effect on inflation. Instead, inflation is determined solely by expected inflation, which is pinned down by monetary policy.

The standard deviation of the technology shock in Model 1 is 1.2 percent per quarter. In [figure 6](#), we compare the average impulse responses to 4 consecutive 1-standard deviation technology shocks under two cases—the baseline case (dashed line), which is initialized at the stochastic

steady state, and the ZLB case (solid line), which is initialized with the discount factor equal to 0.9997, the value where the ZLB binds when technology is at its steady state. The values at the stochastic steady state are shown with horizontal dash-dotted lines. With $\rho_a = 0.8$, four consecutive one standard deviation shocks increase technology to 3.5 percent above its steady state.

In the baseline case, this series of shocks drives the nominal interest rate to zero after the fourth shock (period 3). At that point, looking back to [figure 4](#), we see that output and consumption are about 2 percent above their steady state values. Consumption reaches its peak with the arrival of the fourth shock. During the first 4 periods, consumption growth is rising and the real interest rate is falling because, following each shock, households expect consumption to recede to its steady state. Following the Taylor rule, the interest rate falls faster than the real rate and inflation must decline to clear the bond market. With every shock, households experience an increase in wealth. Even though real wages are rising with the marginal product of labor, households want to consume more goods and more leisure. As inflation falls below the steady state, some output is lost to the costs of price adjustment. Since there are no other shocks following this series of four, the interest rate immediately rises and the ZLB event lasts only one quarter. The economy then returns to steady state, which occurs relatively rapidly in a model without capital accumulation.

In the ZLB case (solid line), the nominal interest rate is fixed at zero for 7 quarters (quarters 0 through 6). Initially, consumption and output are well below their steady state values. The rapid convergence of output, consumption, and the real wage toward their steady state values is due to the fact that from the beginning of the experiment, the discount factor is converging to its stationary value, even as technology shocks continue to hit the economy. It is the fall in the discount factor that causes consumption, output, and inflation to converge toward their respective steady states. If some force, or series of shocks, causes the discount factor to remain near 1, then the recovery will not begin until the technology shocks decay. In this case, technology shocks can have perverse effects, even if technology is below its steady state.

6 MODELS 2 AND 3: STATES OF THE ECONOMY AND THE ZLB

To this point, we have only considered a model without capital. This section analyzes the effect of adding capital accumulation to Model 1, which gives the household another margin for smoothing consumption. Our primary finding in section is that many of the qualitative results from Model 1 still hold. Of course, the dynamics of output are different because output now includes consumption plus investment spending (as well as the fixed government share). New Keynesian models that include capital often include adjustment costs to either capital or investment. In this section we look at our basic version without IAC, Model 2, and two specifications with adjustment costs (Model 3), one parameterized to match the ‘consensus’ model ($\nu = 2.5$) and one with very small costs ($\nu = 0.01$). A secondary result of this section is to show that the consensus assumption about IAC makes the model behave very much like the model without capital, implying much more volatility for consumption relative to output than occurs in the data.⁹ Because our focus in this section is more narrow, we show only the alternative model solutions.

Once again, we focus on a particular slice of the complete model solution. [Figure 7](#) shows the non-predetermined variables as a function of the technology state when capital is at its steady state value. In general, the patterns for consumption, inflation and the interest rate are qualitatively

⁹See Christiano et al. (2011), figure 4 and the accompanying discussion. In their model, consumption responds more sharply to a discount factor shock than either output or investment when they include IAC.

similar to the model without capital, however consumption is less volatile. Away from the ZLB, when agents are able to invest in capital, higher technology increases the rental rate of capital. This spurs investment, which increases the capital stock, smoothes consumption, and raises labor, a key difference from the model without capital. Consumption, labor, output, the real wage, capital, investment, and the rental rate of capital are all higher at higher levels of technology until the ZLB binds. It is interesting to note that the ex-ante real interest rate stops rising in states that are high, but not high enough to encounter the ZLB. In these states, the household expects consumption growth to begin slowing and, as we move to states with slightly higher technology, we encounter the pattern that was evident in Model 1; that is, the real interest rate is lower at higher levels of technology because people expect the future growth rate of consumption to fall faster. Moreover, as we saw in Model 1, the pattern reverses when the ZLB is hit.

At levels of technology where the ZLB binds, inflation is lower and the real interest rate is higher. The mechanism which distorts the economy is essentially the same as in the case without capital. In Model 2, households trying to save more also sharply cut investment. Falling demand for both consumption and investment leads to a lower demand for labor. Falling wages and rental rates lead firms to cut prices despite the higher price adjustment costs. However, the consumption decline is modest. Most of the larger decline in output at higher technology levels is accounted for by lower investment and by higher price adjustment costs. Such a rapid adjustment in investment is at odds with the data and has led economists to add IAC to models with capital.

In Model 3, we first set $\nu = 2.5$, which is the benchmark value estimated in Christiano et al. (2005). [Figure 8](#) plots the policy functions for the non-predetermined variables. Comparing the outcome with earlier models shows that the model with IAC looks more like the model without capital than the model with capital. In comparison with the baseline model, the biggest difference is that the nominal interest rate hits its ZLB at a lower level of technology than in the model without capital. The ex-ante real rate falls as technology rises, similar to Model 1.

At the ZLB, agents would again like to save more as the real interest rate rises. In this model, they can save by investing, but the costs of adjusting investment reduce the volatility of investment so much that consumption is almost as volatile as total output. Also, investment appears to adjust less than consumption as the economy moves to higher levels of technology.

From the perspective of the consumer who wants to smooth consumption or the business cycle analyst who is trying to account for the relative variability of consumption to output, the consensus IAC parameter of 2.5 is too high. It shuts down the consumption smoothing mechanism provided by the capital accumulation process. As we lower the value of this parameter, we find that it takes a large decline to reduce the relative volatility of consumption to a level seen in U.S. data. We set $\nu = 0.01$, which approximates the results in Model 2. [Figure 9](#) plots the policy functions for the non-predetermined variables. Even these small costs substantially moderate the reaction of investment to technology shocks at the ZLB. Another difference between Model 2 and 3 is in the solution for the ex-ante real interest rate. In Model 2, the rate generally rises until technology approaches levels associated with the ZLB. With $\nu = 0.01$ in Model 3, we see the same tendency for the ex-ante real interest to fall as the ZLB states approach, but the rate is nearly invariant with respect to technology until that point. These results show the general equilibrium solution can be very sensitive to small frictions.

7 THE LIKELIHOOD OF HITTING THE ZLB

This section examines the likelihood of ZLB events in Models 1 and 2. The results are not strictly comparable because they are based on different assumptions about the shocks. Our main result is that a policymaker can reduce the chances of hitting the ZLB by de-emphasizing the dual mandate. This can be done by lowering the weight on the output gap relative to the weight on inflation.

7.1 SENSITIVITY TO WEIGHT ON THE OUTPUT GAP Table 2 shows the effect of reducing the weight on output while holding the weight on inflation constant at $\phi_\pi = 1.5$. We begin with the original Taylor (1993) specification, $\phi_y = 0.125$, and reduce this coefficient by increments of 0.025. In Model 1, the probability of hitting the ZLB is 8.6 percent when $\phi_y = 0.125$ and falls to 4.6 percent when $\phi_y = 0$. When $\phi_y = 0.125$, the longest ZLB event is 21 quarters, but it only falls to 20 quarters when $\phi_y \leq 0.1$. The presence of sticky prices creates a trade-off between the volatility of output and inflation. We find that reducing the weight on the output gap raises output volatility by about 20 percent and reduces inflation volatility by 19 percent.

In Model 2, capital accumulation makes interest rates less volatile and reduces the likelihood of hitting the ZLB, which is only about 2 percent even though the standard deviation of the technology shock is 1 percent per quarter. Once again, reducing the weight on the output gap reduces the likelihood of ZLB events. If the weight is as low as 0.05, then the ZLB never binds in any of these simulations. The longest episode at the ZLB is 10 quarters. Reducing the weight from 0.125 to 0 raises the standard deviation of output by 50 percent and reduces the standard deviation of inflation by about 71 percent.

7.2 SENSITIVITY TO WEIGHT ON THE INFLATION GAP Table 3 reports the results when we fix $\phi_y = 0.125$ and change the weight on the inflation gap in each of the models. As the weight on the inflation gap increases to 3, the longest and average ZLB events are more sensitive to raising the weight on inflation than to lowering the weight on the output gap. In Model 1, the probability of hitting the ZLB for this specification is 8.6 percent when $\phi_\pi = 1.5$ and falls to 2.8 percent when $\phi_\pi = 2$. The longest event is 21 quarters and falls to 15 quarters when $\phi_\pi = 3$. In Model 1, the average ZLB event is longer and more sensitive to the parameter on the inflation gap than in the models with capital. Also, raising the weight on the inflation gap raises the volatility of output by about 37 percent and reduces inflation variability by 62 percent.

When $\phi_\pi \geq 2$, the ZLB never binds in any of the simulations in Model 2. However, the longest ZLB event is 10 quarters. Increasing ϕ_π from 1.5 to 3.0 raises output volatility by about 21 percent and reduces the standard deviation of inflation by 73 percent.

7.3 THE VOLATILITY TRADE-OFF Figure 10 shows the Taylor (1979) trade-offs between output and inflation volatility when we reduce the likelihood of hitting the ZLB by lowering the weight on output or increasing the weight on inflation. For each model, we begin with baseline specification. In one case, shown with the solid line, we reduce the weight on the output gap from 0.125 to 0. In the other case, shown with a dashed line, we increase the weight on the inflation gap from 1.5 to 3. In both cases, reducing the volatility of inflation increases the volatility of output in either model. Thus, there is a trade-off in all cases. The top panel shows the trade-offs in Model 1 without capital. The bottom panel shows the results for Model 2. The investment component of output is highly volatile. The higher volatility of output in Model 2 solely reflects the composition effect of adding investment to consumption and government spending in computing GDP. As noted earlier,

consumption is smoother in the model with capital. Reducing the weight on output appears to be an inefficient way to avoid the ZLB. There is more output volatility for a given decline in inflation volatility when the weight on output falls and the weight on inflation is fixed at its baseline (solid line) than when the weight on inflation is fixed and the weight on output falls (dashed line).

8 WELFARE

Finally, we show how welfare changes as we de-emphasize the dual mandate. Define remaining lifetime utility in period n under policy k as

$$E_n W(c_t^k, n_t^k) \equiv E_n \sum_{t=n}^{T-1} \beta^{t-n} u(c_t^k, n_t^k).$$

We use the measure of welfare in Schmitt-Grohé and Uribe (2007) to quantify the percent of consumption goods required to equate utility between two policies. The welfare cost associated with any policy is the fraction of consumption goods a household must give up under policy 1 to be indifferent between policies 1 and 2. Specifically, we solve for a λ to satisfy

$$E_n W(c_t^2, n_t^2) = E_n W((1 - \lambda)c_t^1, n_t^1).$$

Hence, after solving for λ ,

$$\lambda = 1 - \exp \left\{ \frac{1 - \beta}{1 - \beta^{T-n}} (E_n W(c_t^2, n_t^2) - E_n W(c_t^1, n_t^1)) \right\}.$$

In [table 4](#), we show the welfare gains from de-emphasizing the dual mandate. Columns 1-3 show the results when we hold $\phi_\pi = 1.5$ and decrease the weight on ϕ_y to zero by 0.025 increments. The measure, $\bar{\lambda}_j(\%)$, represents the percentage of consumption goods that must be forgone in the baseline parameterization (first row) to equate utility to the alternative parameterization. Thus a negative (positive) number is a welfare gain (loss) in model j . Calculations are based on an average of 1000 simulations, each 2500 quarters long.

The first column lists the value of ϕ_y and the next two columns show the results for Model 1 and Model 2. For Model 1, the highest welfare is achieved with no weight on the output gap. For Model 2, with the inflation parameter fixed at 1.5, the highest utility is achieved when we reduced the weight by a factor of 5, from 0.125 to 0.025. Columns 4-6 de-emphasize the dual mandate by increasing the weight on inflation. In every case, increasing the weight leads to higher utility. These results are not surprising because the key distortion in this model is the nominal price rigidity. Any policy that reduces this distortion raises welfare.¹⁰

9 CONCLUSION

We calculate global solutions to standard New Keynesian models with and without capital to study dynamics at the ZLB. To avoid the problems associated with the approximation error in log-linearized models, we solve for the global nonlinear solution. We use a technology shock as

¹⁰This result is well known in the literature and, to our knowledge, first reported in the context of a New Keynesian model by King and Wolman (1996).

the driving force in the model because technology shocks are an important source of aggregate fluctuations in the behavior of a wide variety of dynamic macroeconomic models. We include a discount factor shock to analyze the effects of technology shocks when the ZLB binds at low technology levels.

The key result in all of our models is that a positive technology shock can lead to deflationary pressures, job loss, and falling consumption if it occurs when the nominal interest rate is pegged at zero, which is one potential reason to adopt policies that reduce the likelihood of hitting the ZLB and to avoid policies that extend the length of a ZLB episode. We find that positive technology shocks can have perverse effects if a temporarily high technology level is the cause of being at the zero lower bound. Whether this will occur when other shocks cause the ZLB to bind depends on whether the source of the ZLB binding continues to hold after the technology shock occurs.

We show that a policymaker can reduce the chances of hitting the ZLB by de-emphasizing the dual mandate. This is accomplished by lowering the weight on the output gap relative to the weight on inflation. We study the sensitivity of our results to the policy parameters, ϕ_π and ϕ_y . While larger and more persistent shocks increase the likelihood of ZLB events, the choice of policy parameters can reduce the likelihood, even under rather extreme assumptions about the shocks.

Placing more weight on output increases the likelihood of hitting the ZLB, because it leads to higher volatility in both inflation and interest rates. Putting more weight on inflation stabilizes inflation and the nominal interest rate close to their steady states values and breaks the negative correspondence between technology and inflation. Our results indicate that putting more weight on the inflation target gives the central bank the flexibility to smooth output. Whether monetary policy can smooth output depends on the frictions and distortions that define the tradeoff. Whether it should do so or not depends on how it affects consumption and welfare. Our results show that the same policies that reduce the likelihood of hitting the ZLB also raise welfare.

Christiano et al. (2011) use a model similar to our Model 1 and argue that dynamics at the ZLB do not depend on how the ZLB was reached. The “forward guidance” of the Fed is a promise to keep the rate near zero regardless of the reason why the economy has such low interest rates. The Fed continued to extend the expected time that the ZLB will bind by adding over a trillion dollars in bank excess reserves in 2011. It has adopted new language, new procedures, and new promises to continue adding bank reserves, all meant to commit the Fed to hold money market rates at zero for the next few years. One of the important lessons we take from our study is that the dynamics following any shock depend on the initial conditions and expectations about future interest rates and inflation. The Fed has adopted policies and communication strategies to pin expectations at 2-percent for the indefinite future and nominal interest rates at zero for the next few years. This outcome is inconsistent with equilibrium in our basic model.

An interesting question for future research is whether technology shocks would have perverse effects if the ZLB were binding because the economy was saturated with excess reserves. We have not explicitly modeled the unconventional monetary policies the Fed continues to employ. But, as far as we know, no one has done so in a DSGE model with global solutions. In reality, the ZLB began to bind in 2008:Q4 because the Fed flooded the market with excess bank reserves. When Lehman Brothers declared bankruptcy, the realization that AIG did not have the reserves to pay off on Lehman credits caused a panic and bank run on repurchase agreements. To prevent a potential worldwide collapse in financial markets, the Fed added \$600 billion in excess reserves by issuing short-term loans and buying short-term assets. These actions stopped the panic, but policymakers continued to be concerned that the banking system was overleveraged and worldwide

financial collapse was still possible. To shore up the banking system and guarantee liquidity to financial markets, the Fed rolled over the short-term credit that it had extended in the aftermath of the Lehman bankruptcy with its first large-scale purchase of long-term bonds—known in the financial press as QE1. It also began paying 25 basis points on excess reserves. It now uses a combination of large scale asset purchases plus forward guidance to hold the interest rate at zero. Would technology shocks have perverse effects if the reason that the ZLB was binding was due to monetary policy? This paper represents a step toward developing a model to answer that question; Searching for a more definitive answer is the subject of ongoing research.

REFERENCES

- BRAUN, R. A., L. M. KÖRBER, AND Y. WAKI (2012): “Some Unpleasant Properties of Log-Linearized Solutions When the Nominal Rate is Zero,” Federal Reserve Bank of Atlanta Working Paper 2012-5a.
- BRAUN, R. A. AND Y. WAKI (2006): “Monetary Policy During Japan’s Lost Decade,” *The Japanese Economic Review*, 57, 324–344.
- BULLARD, J. (2010): “Seven Faces of ‘The Peril’,” *Federal Reserve Bank of St. Louis*, 92(5), 339–352.
- CHRISTIANO, L., M. EICHENBAUM, AND S. REBELO (2011): “When Is the Government Spending Multiplier Large?” *Journal of Political Economy*, 119, 78–121.
- CHRISTIANO, L. J., M. EICHENBAUM, AND C. L. EVANS (2005): “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy,” *Journal of Political Economy*, 113, 1–45.
- CHUNG, H., J.-P. LAFORTE, D. REIFSCHNEIDER, AND J. C. WILLIAMS (2012): “Have We Underestimated the Likelihood and Severity of Zero Lower Bound Events?” *Journal of Money, Credit and Banking*, 44(S1), 47–87.
- COIBION, O., Y. GORODNICHENKO, AND J. F. WIELAND (2012): “The Optimal Inflation Rate in New Keynesian Models: Should Central Banks Raise Their Inflation Targets in Light of the ZLB?” *The Review of Economic Studies*, forthcoming.
- DAVIG, T. AND E. M. LEEPER (2007): “Generalizing the Taylor Principle,” *American Economic Review*, 97, 607–635.
- DEL NEGRO, M., M. GIANNONI, AND C. PATTERSON (2012): “The Forward Guidance Puzzle,” Federal Reserve Bank of New York Staff Report 574.
- DIXIT, A. K. AND J. E. STIGLITZ (1977): “Monopolistic Competition and Optimum Product Diversity,” *American Economic Review*, 67, 297–308.
- EGGERTSSON, G. AND M. WOODFORD (2003): “The Zero Bound on Interest Rates and Optimal Monetary Policy,” *Brookings Papers on Economic Activity*, 34(1), 139–235.
- ERCEG, C. J. AND J. LINDE (2010): “Is There a Fiscal Free Lunch in a Liquidity Trap?” *International Finance Discussion Paper* 1003.
- FERNÁNDEZ-VILLAYERDE, J., G. GORDON, P. A. GUERRÓN-QUINTANA, AND J. RUBIO-RAMÍREZ (2012): “Nonlinear Adventures at the Zero Lower Bound,” NBER Working Paper 18058.
- FRIEDMAN, M. (1969): *The Optimum Quantity of Money—And Other Essays*, Aldine Publishing Company, Chicago.
- GAVIN, W. T. AND B. D. KEEN (2012): “The Zero Lower Bound and the Dual Mandate,” Federal Reserve Bank of St. Louis Working Paper, 2012-026.

- GERTLER, M. AND P. KARADI (2011): “A Model of Unconventional Monetary Policy,” *Journal of Monetary Economics*, 58, 17–34.
- HATCHER, M. C. (2011): “Inflation versus Price-level Targeting and the Zero Lower Bound: Stochastic Simulations from the Smets-Wouters US model,” Cardiff Business School Working Paper E2011/24.
- HOSHI, T. AND A. KASHYAP (2000): “The Japanese Banking Crisis: Where Did It Come From and How Will It End?” in *NBER Macroeconomics Annual 1999, Volume 14*, MIT Press, Cambridge, 129–212.
- IRELAND, P. N. (1997): “A Small, Structural, Quarterly Model for Monetary Policy Evaluation,” *Carnegie-Rochester Conference Series on Public Policy*, 47, 83–108.
- JUNG, T., Y. TERANISHI, AND T. WATANABE (2005): “Optimal Monetary Policy at the Zero-Interest-Rate Bound,” *Journal of Money, Credit and Banking*, 37, 813–35.
- KILEY, M. T. (2003): “Why Is Inflation Low When Productivity Growth Is High?” *Economic Inquiry*, 41, 392–406.
- KING, R. G. AND A. L. WOLMAN (1996): “Inflation targeting in a St. Louis model of the 21st century,” *Federal Reserve Bank St. Louis Review*, 78(6), 83–107.
- KOCHERLAKOTA, N. R. (2005): “Optimal Monetary Policy: What We Know and What We Don’t Know,” *International Economic Review*, 46(2), 715–729.
- KRUGMAN, P. R. (1998): “It’s Baaack: Japan’s Slump and the Return of the Liquidity Trap,” *Brookings Papers on Economic Activity*, 29, 137–206.
- PETERMAN, W. B. (2012): “Reconciling Micro and Macro Estimates of the Frish Labor Supply Elasticity,” Manuscript, Federal Reserve Board of Governors.
- POSEN, A. (1998): *Restoring Japan’s Economic Growth*, Peterson Institute.
- REIFSCHNEIDER, D. AND J. C. WILLIAMS (2000): “Three Lessons for Monetary Policy in a Low-Inflation Era,” *Journal of Money, Credit, and Banking*, 32, 936–966.
- RICHTER, A. W., N. A. THROCKMORTON, AND T. B. WALKER (2012): “Accuracy, Speed and Robustness of Policy Function Iteration,” Manuscript, Indiana University.
- ROTEMBERG, J. J. (1982): “Sticky Prices in the United States,” *Journal of Political Economy*, 90, 1187–1211.
- SCHMITT-GROHÉ, S. AND M. URIBE (2007): “Optimal Simple and Implementable Monetary and Fiscal Rules,” *Journal of Monetary Economics*, 54, 1702–1725.
- (2012): “The Making Of A Great Contraction With A Liquidity Trap and A Jobless Recovery,” NBER Working Paper 18544.

- SUMMERS, L. (1991): “How Should Long-Term Monetary Policy Be Determined? Panel Discussion,” *Journal of Money, Credit and Banking*, 23, 625–31.
- TAYLOR, J. B. (1979): “Estimation and Control of a Macroeconomic Model with Rational Expectations,” *Econometrica*, 47, 1267–86.
- (1993): “Discretion versus Policy Rules in Practice,” *Carnegie-Rochester Conference Series on Public Policy*, 39(1), 195–214.
- WERNING, I. (2012): “Managing a Liquidity Trap: Monetary and Fiscal Policy,” NBER Working Paper 17344.
- WIELAND, J. (2012): “Are Negative Supply Shocks Expansionary at the Zero Lower Bound? Inflation Expectations and Financial Frictions in Sticky-Price Models,” Manuscript, UC Berkeley.
- WILLIAMS, J. C. (2009): “Heeding Daedalus: Optimal Inflation and the Zero Lower Bound,” in *Brookings Papers on Economic Activity, Fall*, Brookings Institute.

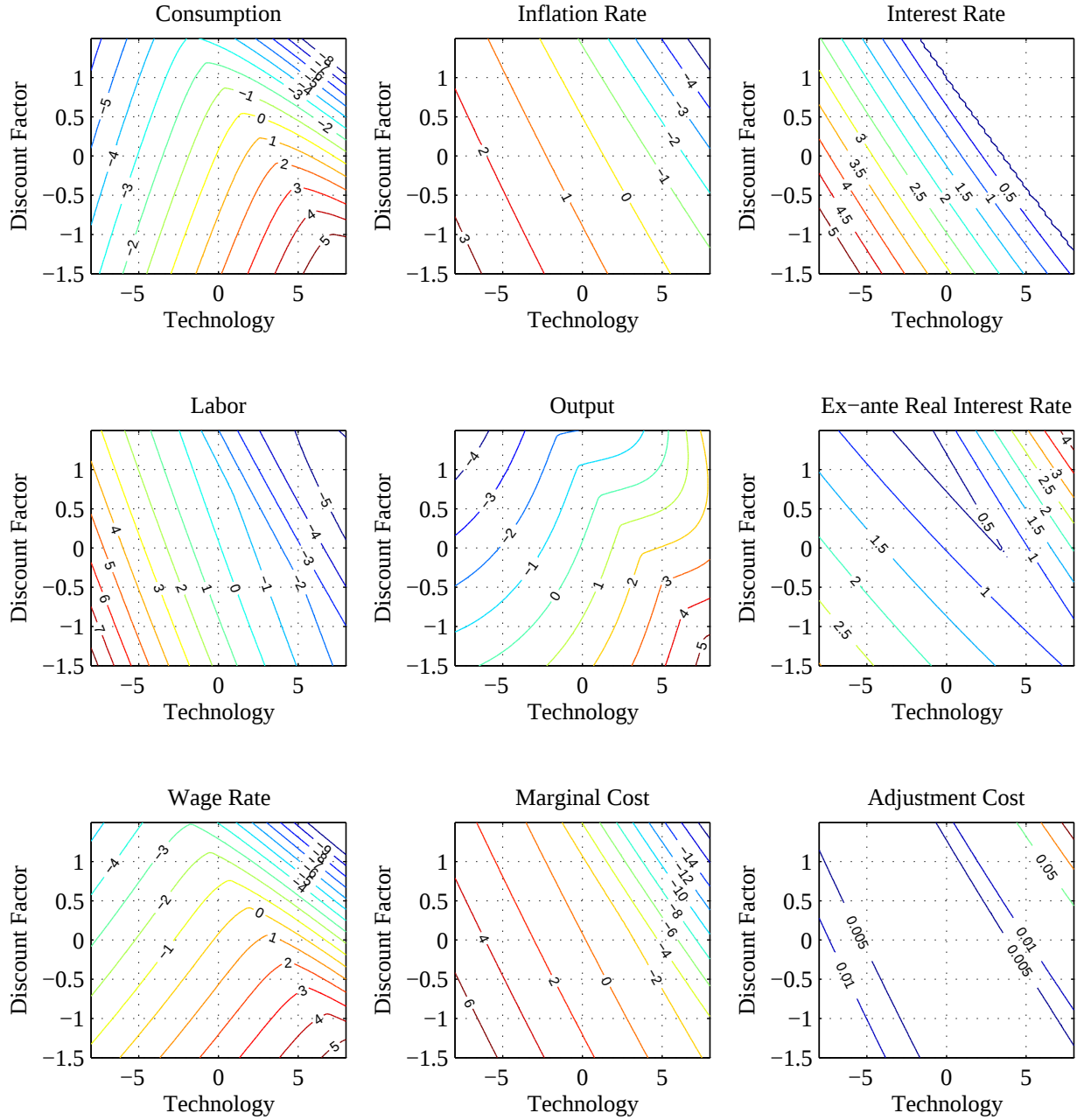


Figure 3: Model 1 non-predetermined variables as a function of technology and the discount factor. All of the variables are given in percent deviations from their deterministic steady state, except inflation and the (net) interest rates, which are presented in levels. Adjustment costs are measured as a percent of output.

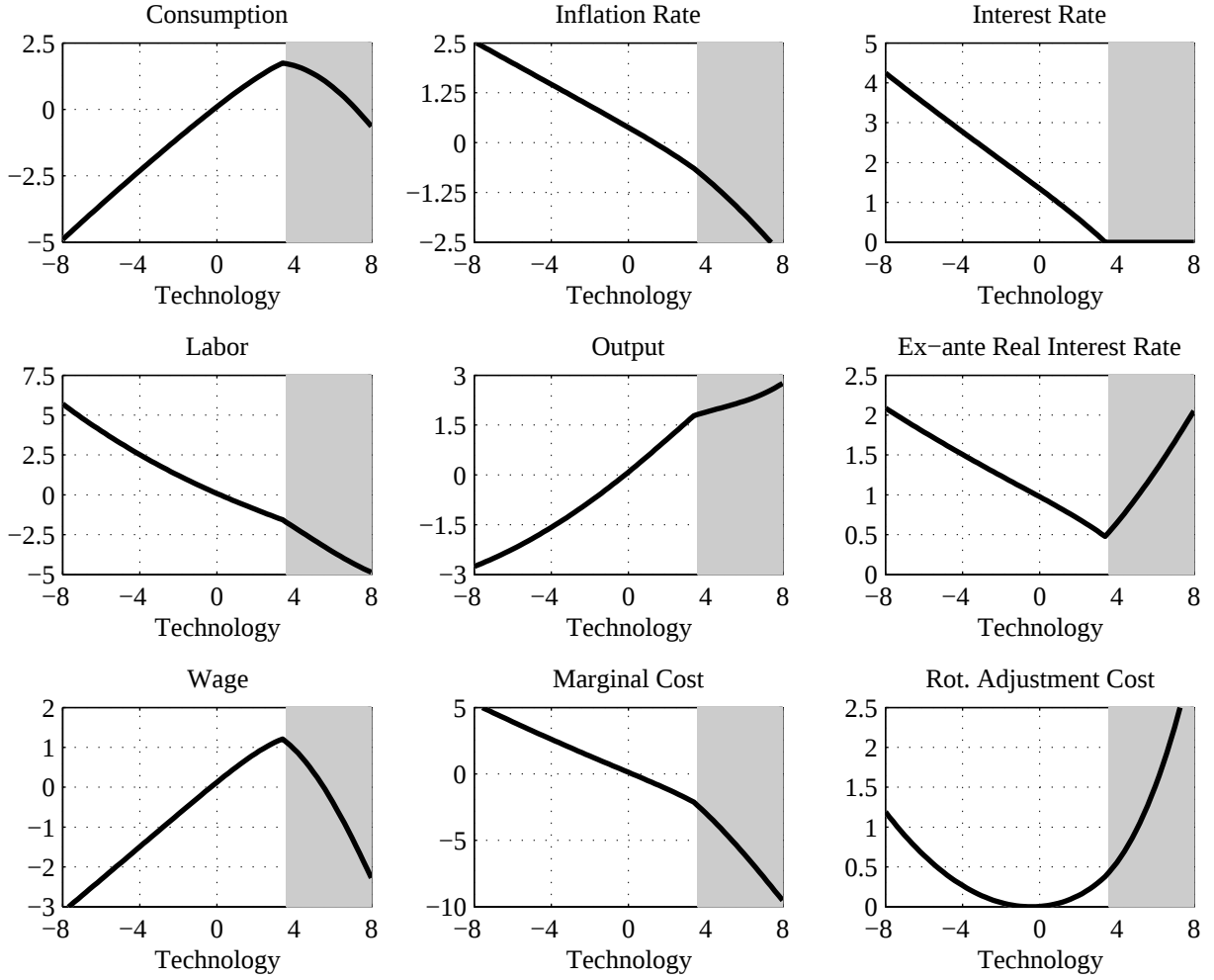


Figure 4: Model 1 non-predetermined variables as a function of technology. All of the variables are given in percent deviations from their deterministic steady state, except inflation and the (net) interest rates, which are presented in levels. Adjustment costs are measured as a percent of output. The discount factor state (β_0) is fixed at its deterministic steady state value. The shaded region indicates where the ZLB binds.

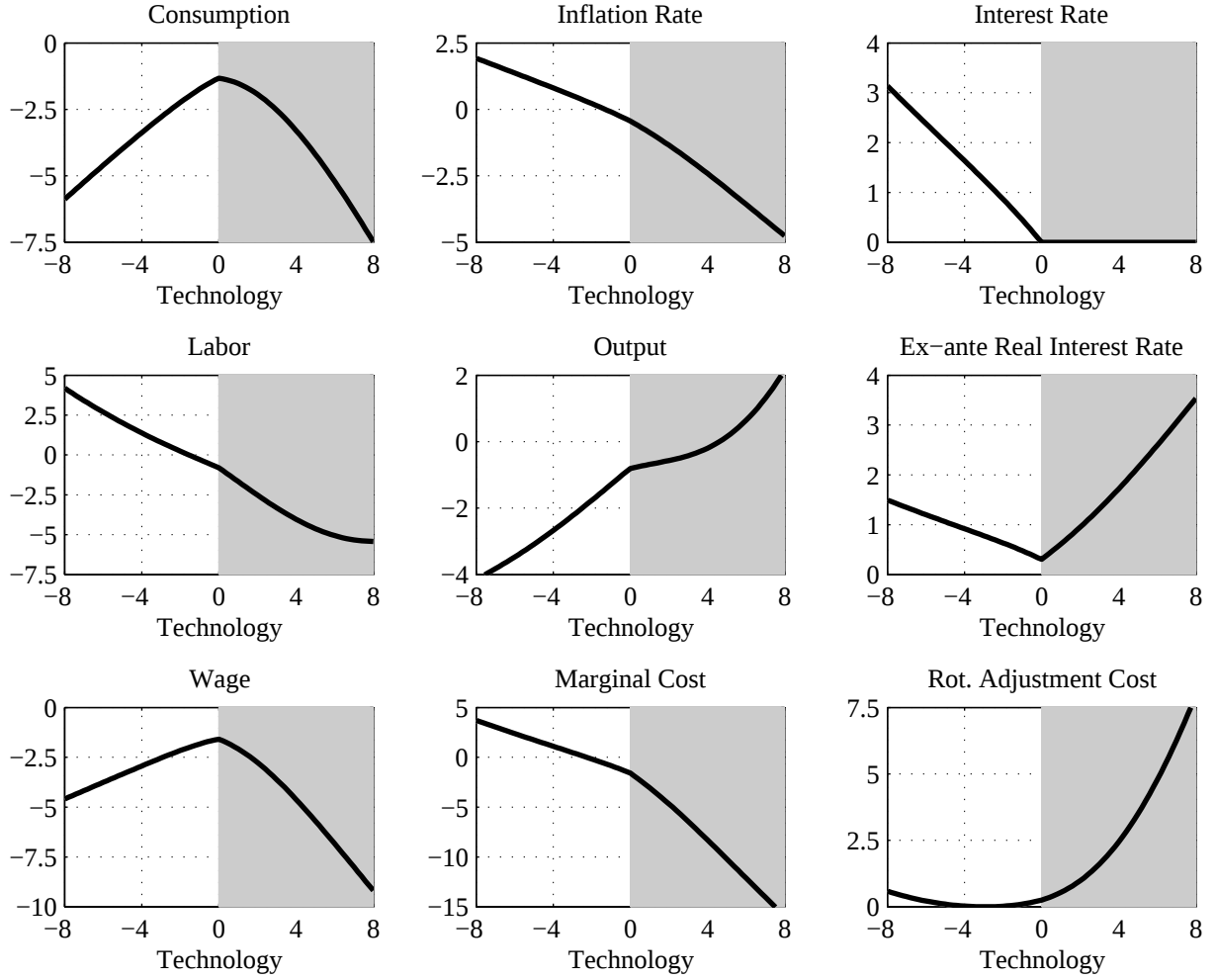


Figure 5: Model 1 non-predetermined variables as a function of technology. All of the variables are given in percent deviations from their deterministic steady state, except inflation and the (net) interest rates, which are presented in levels. Adjustment costs are measured as a percent of output. The discount factor state ($\beta_0 = 0.9997$) is fixed at the minimum value where the ZLB binds, given technology is at its steady state. The shaded region indicates where the ZLB binds and begins where technology is at its steady state value.

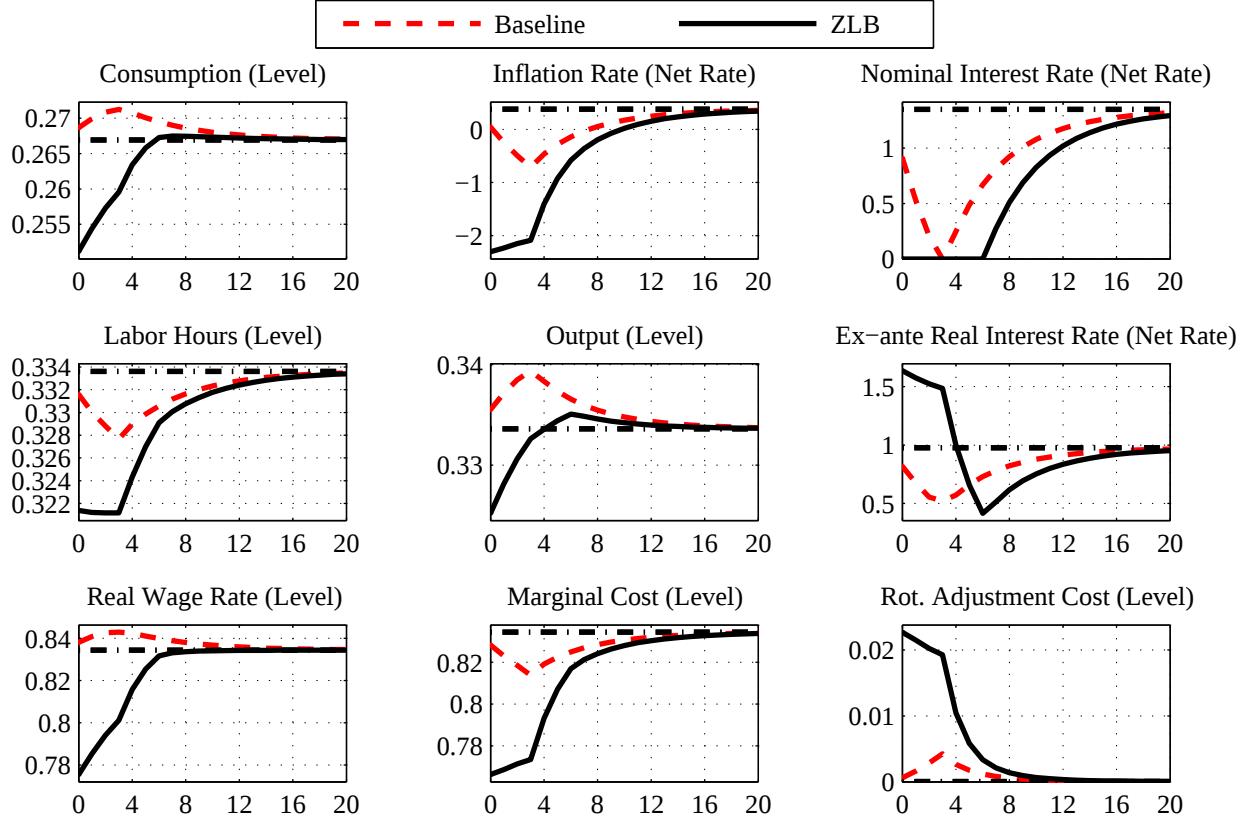


Figure 6: Model 1 impulse responses to 4 consecutive 1 standard deviation technology shocks. The baseline case (dashed line), where the ZLB does not initially bind, is initialized at the stochastic steady state. The case where the ZLB initially binds (solid line) is initialized at the average discount factor in the periods where the ZLB binds (absent of technology shocks) in a 500,000 period simulation. The dash-dotted line is the long-run stochastic steady state.

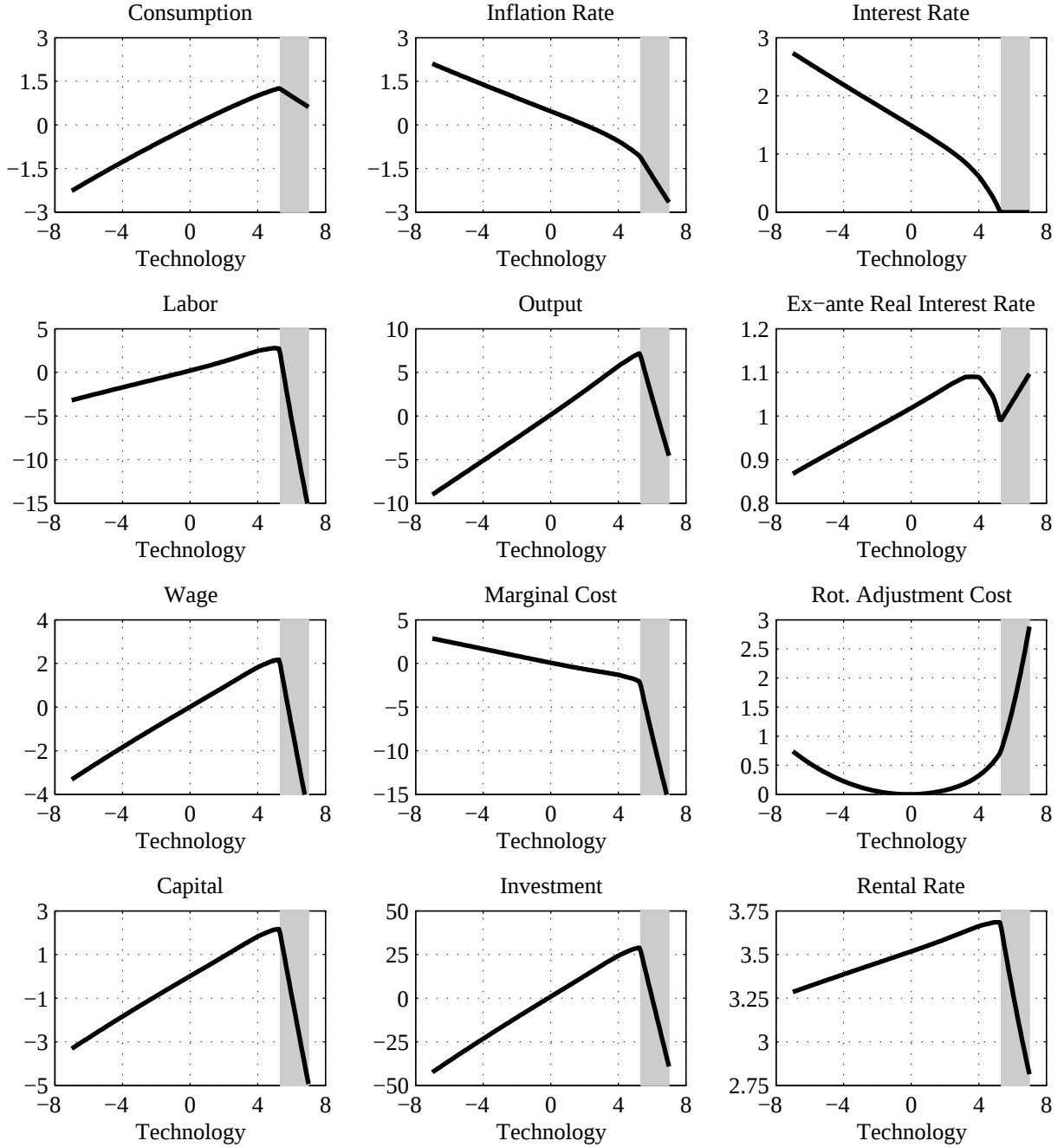


Figure 7: Model 2 non-predetermined variables as a function of technology. All of the variables are given in percent deviations from their deterministic steady state, except inflation and the (net) interest rates, which are presented in levels. Adjustment costs are measured as a percent of output. The capital stock state (k_{-1}) is fixed at its deterministic steady state value. The shaded region indicates where the ZLB binds.

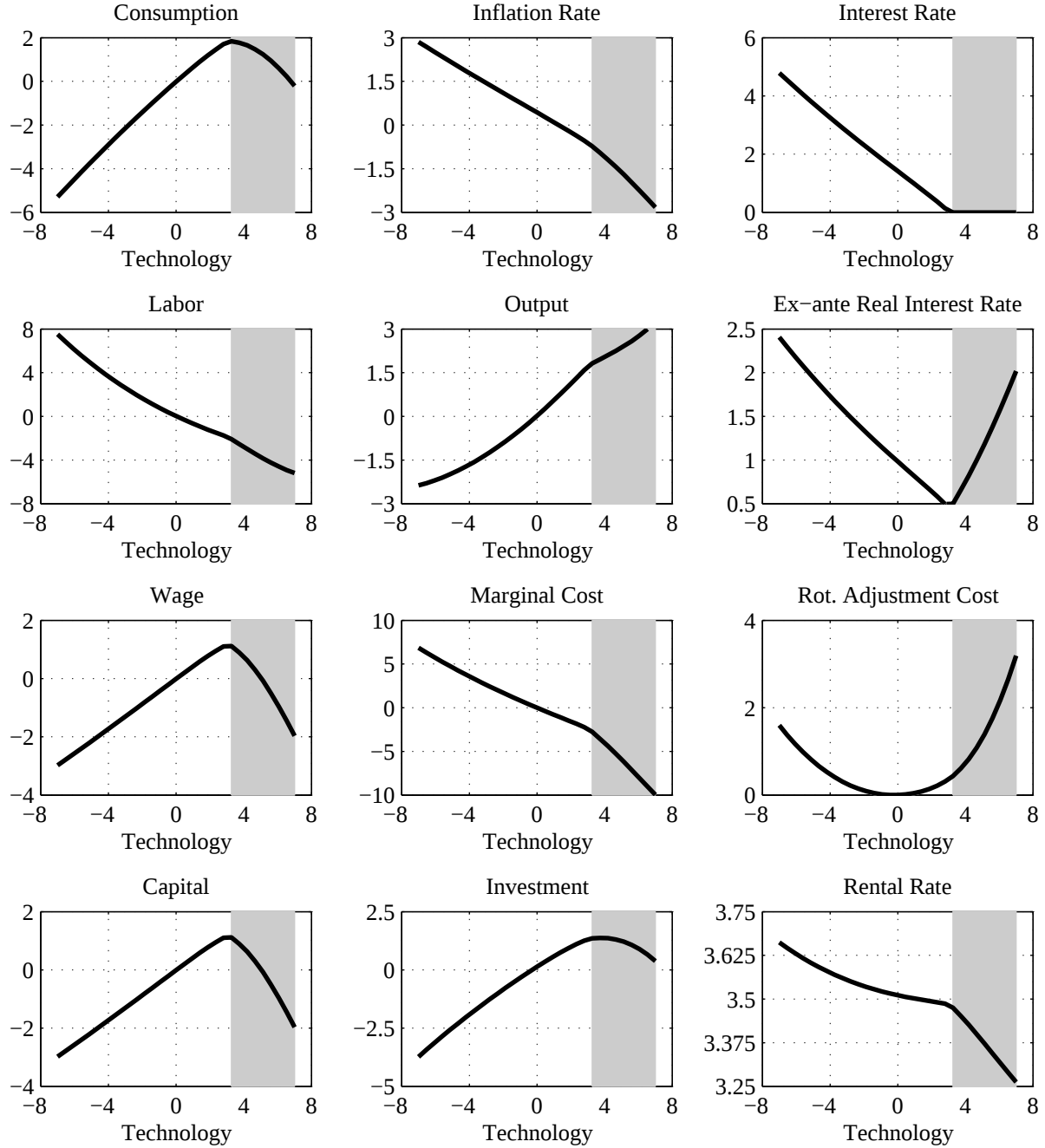


Figure 8: Model 3 ($\nu = 2.5$) non-predetermined variables as a function of technology. All of the variables are given in percent deviations from their deterministic steady state, except inflation and the (net) interest rates, which are presented in levels. Adjustment costs are measured as a percent of output. The capital stock and investment states (k_{-1}, i_{-1}) are fixed at their deterministic steady state value. The shaded region indicates where the ZLB binds.

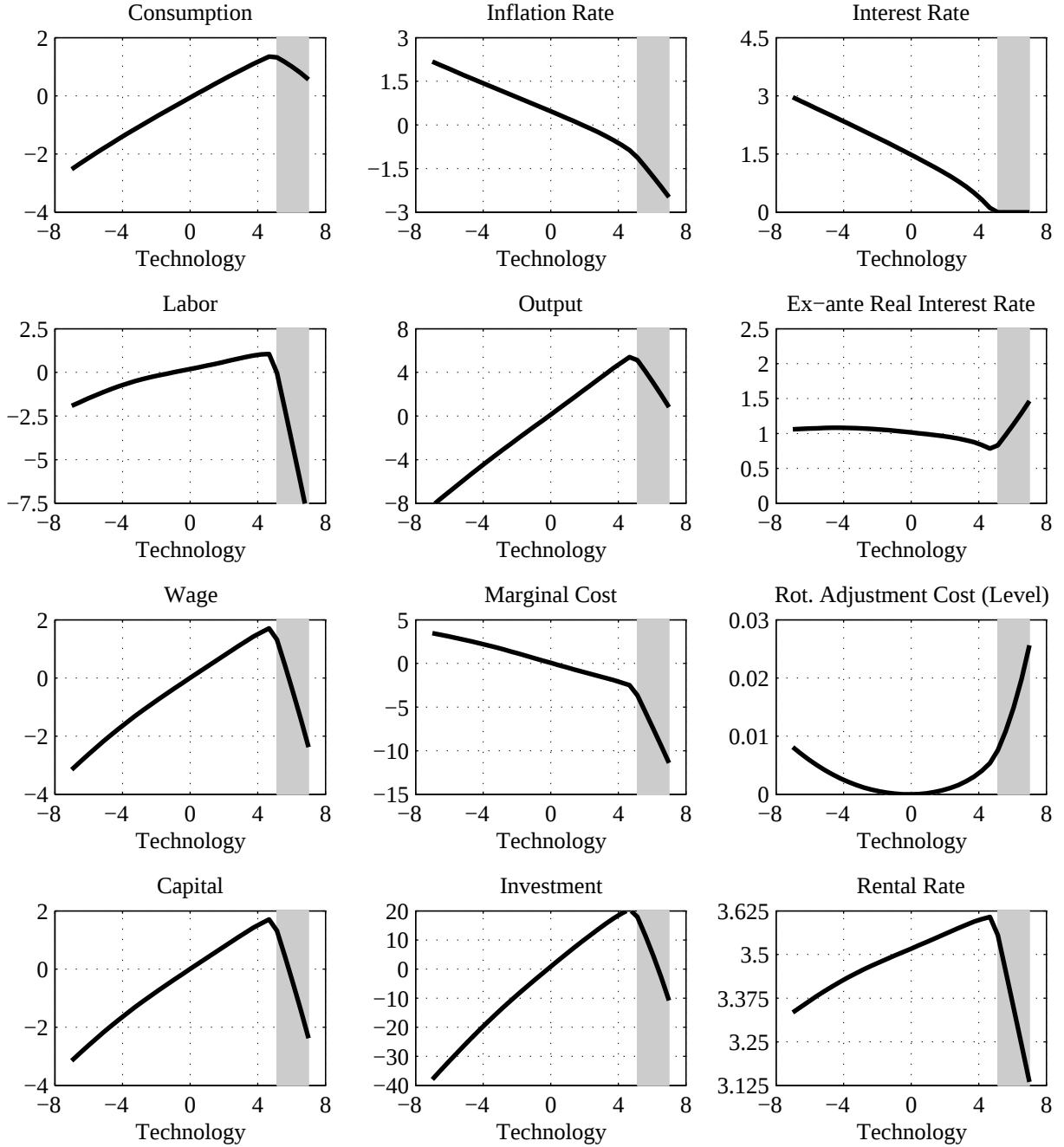
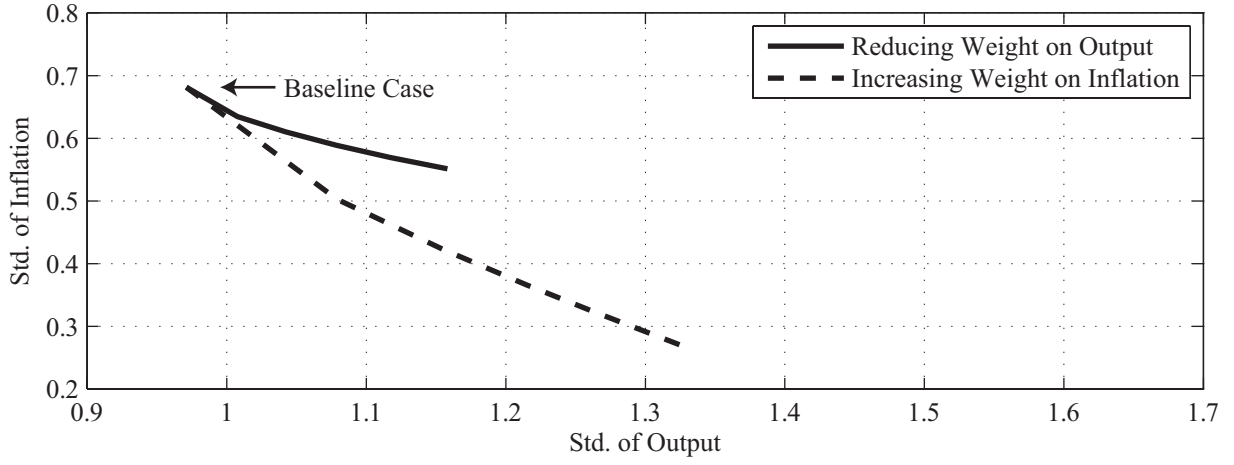
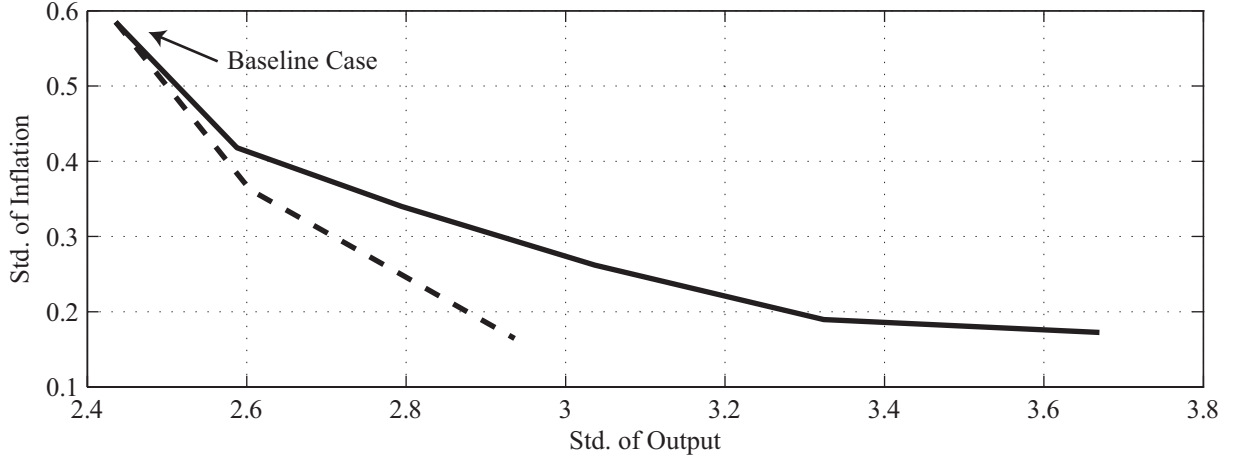


Figure 9: Model 3 ($\nu = .01$) non-predetermined variables as a function of technology. All of the variables are given in percent deviations from their deterministic steady state, except inflation and the (net) interest rates, which are presented in levels. Adjustment costs are measured as a percent of output. The capital stock and investment states (k_{-1} , i_{-1}) are fixed at their deterministic steady state value. The shaded region indicates where the ZLB binds.



(a) Model 1: No capital, technology and discount factor shocks



(b) Model 2: Capital, only technology shocks

 Figure 10: Comparison of the tradeoffs between output and inflation volatility as a function of the Taylor rule parameters. $\phi_y = 0.125$ and $\phi_\pi = 1.5$ in the baseline case.

ϕ_y	ZLB Binds		ZLB Spells		Std. Dev. (% of mean)	
	% of nodes	% of quarters	Average	Longest	Output	Inflation
0.125	28.72	8.594	2.22	21	0.97	0.68
0.100	26.76	6.675	2.11	20	1.01	0.63
0.075	25.96	5.944	2.06	20	1.04	0.61
0.050	25.29	5.416	2.03	20	1.08	0.59
0.025	24.66	4.989	2.01	20	1.12	0.57
0.000	24.13	4.632	1.98	20	1.16	0.55

(a) Model 1: No capital, technology and discount factor shocks. $\phi_\pi = 1.50$, $\rho_z = 0.80$, $\sigma_z = 0.01189$, $\rho_\beta = 0.80$, and $\sigma_\beta = 0.0025$.

0.125	21.49	2.326	1.57	10	2.44	0.59
0.100	8.55	0.113	1.41	5	2.59	0.42
0.075	1.68	0.003	1.70	3	2.79	0.34
0.050	0.00	0.000	0.00	0	3.04	0.26
0.025	0.00	0.000	0.00	0	3.32	0.19
0.000	0.00	0.000	0.00	0	3.67	0.17

(b) Model 2: Capital, only technology shocks. $\phi_\pi = 1.50$, $\rho_z = 0.80$, and $\sigma_z = 0.01089$.

Table 2: Volatility implications of a dual mandate

ϕ_π	ZLB Binds		ZLB Spells		Std. Dev. (% of mean)	
	% of nodes	% of quarters	Average	Longest	Output	Inflation
1.50	28.72	8.594	2.22	21	0.97	0.68
1.75	23.12	3.758	1.91	20	1.08	0.50
2.00	21.38	2.808	1.83	16	1.16	0.42
2.25	20.26	2.281	1.78	16	1.21	0.37
2.50	19.43	1.938	1.74	16	1.26	0.32
3.00	18.29	1.530	1.69	15	1.33	0.26

(a) Model 1: No capital, technology and discount factor shocks. $\phi_y = 0.125$, $\rho_z = 0.80$, $\sigma_z = 0.01189$, $\rho_\beta = 0.80$, and $\sigma_\beta = 0.0025$.

1.50	21.49	2.326	1.57	10	2.44	0.59
1.75	2.33	0.004	2.00	3	2.60	0.36
2.00	0.00	0.000	0.00	0	2.72	0.29
2.25	0.00	0.000	0.00	0	2.80	0.24
2.50	0.00	0.000	0.00	0	2.86	0.21
3.00	0.00	0.000	0.00	0	2.94	0.16

(b) Model 2: Capital, only technology shocks. $\phi_y = 0.125$, $\rho_z = 0.80$, and $\sigma_z = 0.01089$.

Table 3: Volatility implications of alternative weights on the inflation gap

ϕ_y	$\bar{\lambda}_1(\%)$	$\bar{\lambda}_2(\%)$	ϕ_π	$\bar{\lambda}_1(\%)$	$\bar{\lambda}_2(\%)$
0.125	0	0	1.50	0	0
0.100	-0.0560	-0.0487	1.75	-0.1841	-0.0617
0.075	-0.0796	-0.0676	2.00	-0.2397	-0.0757
0.050	-0.0982	-0.0814	2.25	-0.2722	-0.0833
0.025	-0.1144	-0.0888	2.50	-0.2935	-0.0878
0.000	-0.1293	-0.0842	3.00	-0.3191	-0.0926

Table 4: Welfare implications of a dual mandate. The subscript represents the model number. The welfare measure represents the percentage of consumption goods that must be forgone in the baseline parameterization (given in the first row) to equate utility with the alternative parameterization. Thus a negative (positive) number is a welfare gain (loss). Calculations are based on an average of 1000 simulations, each 2500 quarters long.