Monetary Policy in Korea through the lens of Taylor Rule in DSGE model

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Abstract

This paper shows assessments on the monetary policy of Korea based on an estimated model. During the sample period of the inflation targeting scheme, the monetary policy discretion, which is the monetary policy shock after the historical decomposition of the model, has been mostly inflationary while it was reducing the volatility of output growth and thus countercyclical. 3% target rate could have been achieved when the monetary policy shock's standard deviation was approximately half of its posterior estimate. Various degree of monetary policy stance has been simulated with the sample period. An aggressive monetary policy towards inflation stabilization would have generally led to the average level of inflation rate closer to its target rate but at the cost of higher volatilities of the output growth.

JEL Classicification: E3, E5, C5

Keywords: Monetary Policy, DSGE, Bayesian Estimation, Korean Economy

1 Introduction

Korean economy has experienced high growth within shortest periods in the world economic history but at the expense of high volatilities of key macroeconomic variables such as GDP and inflation. This phenomenon made Korean economy as a role model in terms of growth among emerging countries but also brought challenges to economists to study business cycle properties of Korean economy. Lee (2009) documented trends and cycles of GDP and inflation of Korean economy with various statistical methods but concluded that it is difficult to find a consistent results on disentangling trends and cycles due to irregular noises such as oil crisis and financial crisis in 1997. However, after Bank of Korea undertook the inflation targeting scheme in 1999, the volatilities of GDP growth and inflation rates have substantially reduced. The standard deviation of GDP growth reduced from 0.045(1980:Q1-1999:Q4) to 0.027(2000:Q1-2012:Q2) and that of inflation rate from 0.058 to 0.008. This reduction may be attributed to inflation targeting scheme or simply to reduction of exogenous shocks or to both, but at any rate this empirical fact provides a good laboratory to study business cycle properties and effects of monetary policy of Korean economy with Dynamic Stochastic General Equilibrium frameworks that have been better known to fit more stabilized economies.

This paper asks what have been the driving forces of Korean economy after 2000 by investigating an estimated DSGE model and whether the monetary policy have been aggressive towards inflation stabilization. And given the estimated model, it also examines how the economy would have evolved under more aggressive monetary policy by performing counterfactual simulations. While the questions addressed in this paper are conventional in macroeconomics, there are a few notable contributions both to the literature which studies Korean economies and to the Korean policy circle in which the assessment of monetary policy after inflation targeting scheme is an important issue. First, the model in this paper specifies a balanced growth path which allows the model to fully explain the time series data without preprocessing of data. When DSGE models are brought to the data, macro time series are typically filtered either by Hodrick-Prescott filter or Band-Pass filter only to pay attention to business cycle movements. Or a unique trend[1] is defined in total factor productivity process that entails a common trend for all of macro time series implied by the model. However, since a growth has been a substantial factor for Korean economy that is hard to be distinguished from business cycle movements with statistical filters, preprocessing of data outside the model can be problematic. Additionally, a unique trend is hard to reconcile possibly different trends for consumption and investment in Korean data since investment has been more strongly correlated with international trade rather than the domestic demands. But as Greenwood et al. (1997) and Greenwood et al. (2000) argued, the investment specific technology can be an important

[1] for U.S. economy, Smets and Wouters (2005) and for Korean economy, Lee and Yeo (2008)

source of explaining both the trend and aggregation fluctuations. So this paper defines two types of common trends in the model, namely trend in total factor productivity and that in investment specific technology, that capture trends for GDP components such as consumption, investment and government expenditures.

Second, historical decompositions of both output and inflation have been reported in this paper that are useful for Korean policy circle. Previous works[2] mostly focused only on historical decompositions for GDP gap but historical decomposition for inflation has been missing. With this addition, monetary policy discretion which is referred as the monetary policy shock in the model can be historically traced whether it has been mostly expansionary or contractionary so far. Third, data from Korean National Income Account tables are constructed to be aligned with the observable variables implied by the model. The model in this paper includes consumption, investment and government expenditures, but Korean NIPA tables do not have readily available time series for these model's implied macro variables. For example, governments expenditures are constructed by summing government consumption and capital formation. And consumption will be a sum of private consumption of nondurables and services while investment is a sum of private consumption of durables and private capital formation. The reason why durable consumption is excluded from consumption but rather included in investment is that consumption implied by the model is excessively smooth series due to intertemporal smoothing mechanism implied by DSGE models with habit formation which is difficult to support empirically a volatile series such as durable consumptions. Nevertheless, modeling a durable good sector may be a good extension but for the purpose of this paper, this has been rather omitted. Lastly, the deflator for consumption series has been constructed with the envelope calculations from real and nominal consumption series of nondurable and services since each time series is deflated with chain weighted index[3].

While the paper attempts to bring the model to Korean data closest as possible, there is a limitation yet to be worked on. The model considered here is a closed economy model that only covers consumption, investment and government expenditures and excludes exports and imports. Korean economy in fact has a considerable share of export but a trend in exports - GDP ratio is difficult to mimic with currently developed DSGE models which have common trends. The works remain with an open economy model that incorporates different trends that explain growing share of exports for Korean economy but it is left for the future research agenda.

The paper proceeds as follows. Section 2 describes a Medium Scale New Keynesian model following Fernández-Villaverde et al. (2010). Section 3 explains the choice of Korean data, the econometric methodology and estimation results. Section 4 presents the historical decompo-

[2] For example, Park(2012)

[3] Korean GDP is based on Laspeyres chain weighted index with reference year 2005, while U.S. GDP uses Fisher chain weighted index

sitions of inflation and output, implications of monetary policy discretion and counterfactual simulations of the economy under various degrees of monetary policy stance. 5 concludes.

2 Medium Scale New Keynesian Model

2.1 Households Problem

There is a continuum of households in the economy indexed by i which maximizes the lifetime utility function.

$$E_0 \sum_{t=0}^{\infty} \beta^t d_t \left\{ \log\left(c_{it} - bc_{it-1}\right) - \varphi_t \psi \frac{l_{it}^{1+\gamma}}{1+\gamma} \right\}$$

where b is the parameter that controls habit persistence, d_t, is an intertemporal preference shock and φ_t is a labor supply(intratemporal) shock :

$$\begin{aligned} \log d_t &= \rho_d \log d_{t-1} + \sigma_d \varepsilon_{d,t} \text{ where } \varepsilon_{d,t} \sim N(0,1) \\ \log \varphi_t &= \rho_\varphi \log \varphi_{t-1} + \sigma_\varphi \varepsilon_{\varphi,t} \text{ where } \varepsilon_{\varphi,t} \sim N(0,1) \end{aligned}$$

The i^{th} household's budget constraint is given by :

$$\begin{aligned} & c_{it} + i_{it} + \frac{B_{it+1}}{p_t} + \int q_{t+1,t} a_{it+1} d\omega_{i,t+1|t} \\ = \; & w_{it} l_{it} + \left(r_t u_{i,t} - q_t a\left[u_{it}\right]\right) \bar{k}_{i,t} + R_{t-1} \frac{B_{it}}{p_t} + a_{it} + T_t + F_t \end{aligned}$$

where p_t is price level of final good, w_{jt} is the real wage. B_{it} is the risk free nominal bond holding position of the household with a gross return, R_t, next period. a_{it+1} is Arrow security purchased this period by the household for a certain state next period with price, $q_{t+1,t}$. Complete Arrow securities market is assumed that entails perfect risk sharing among households. r_t is the rental price of capital, $u_{jt} > 0$ is the intensity of use of capital, q_t is the price of capital, and thus $q_t a\left[u_{jt}\right]$ is the physical cost of use of capital in resource terms where

$$a[u] = \gamma_1 (u-1) + \gamma_2 (u-1)^2$$

Also, T_t is lumpsum transfer from the government and F_t is the profits earned from the ownership of firms. Here, we assume the household has technology that transforms the final good into investment good that faces this exogenous process. Thus, the investment good is

$$I_{it} = \xi_t i_{it}$$

ξ_t is an investment-specific technology shock and its inverse is interpreted as the relative price of investment good in final good unit. Its exogenous process is

$$\xi_t = \xi_{t-1} \exp\left(\Lambda_\xi + \sigma_\xi \varepsilon_{\xi,t}\right)$$

where

$$\varepsilon_{\xi,t} \sim N(0,1)$$

For stationarizing purpose, this process can be replaced with stationary variable $\mu_{\xi,t} \equiv \frac{\xi_t}{\xi_{t-1}}$ so that

$$\log \mu_{\xi,t} = \Lambda_\xi + \sigma_\xi \varepsilon_{\xi,t}$$

And this investment good is newly installed to capital stock and thus the capital stock[4] evolves with

$$\bar{k}_{it+1} = (1-\delta)\,\bar{k}_{it} + \left(1 - S\left(\frac{I_{it}}{I_{it-1}}\right)\right) I_{it}$$

where

$$S\left(\frac{I_{it}}{I_{it-1}}\right) = \frac{\kappa}{2}\left(\frac{I_{it}}{I_{it-1}} - \Lambda_I\right)^2$$

is a quadratic investment adjustment cost.

The resulting equilibrium conditions for household problem will be symmetric due to complete asset market, i.e. the complete set of state contingent Arrow securities entail perfect risk sharing, except for wage determination and labor supply. Calvo type of friction for wage setting creates heterogeneous conditions for household labor problem which is explained separately in the following.

Household labor problem Each household facing monopolistically competitive labor market supplies a differentiated labor to the labor packer with posted wage under Calvo friction. The labor packer combines the differentiated labor and sells homogenous labor to perfectly competitive factor market demanded by firms. Thus, the demand for each differentiated labor is dictated by the relative wage and homogenous labor demands due to Dixit-Stiglitz type of monopolistic competition. And household who reoptimizes the wage with probability, θ_w, solves the following problem. Household sets the current wage such that it maximizes the intertemporal flow of fixed wage revenues minus disutility of labor supply and labor supply is determined via labor demand function that is derived from labor packer problem[5]. Wage indexation is added for wage inflation persistence mechanism.

$$\max_{w_{jt}} E_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^\tau \left\{ -d_t \varphi_t \psi \frac{l_{jt+1}^{1+\gamma}}{1+\gamma} + \lambda_{jt+1} \prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} w_{jt} l_{jt+\tau} \right\}$$

[4] Here, we denote $\bar{k}_t$ as installed capital stock and k_t as capital service.

[5] Labor packer problem is identical to final good producer problem explained in the next section.

subject to

$$l_{jt+\tau} = \left(\prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} \frac{w_{jt}}{w_{t+\tau}} \right)^{-\eta} l_{t+\tau}^{d} \quad \forall j$$

λ_{jt+1} is marginal utility of wealth that is the lagrangian multiplier of household problem that converts the wage revenue into utility terms. η is the elasticity of substitution between differentiated labors and χ_w is the indexation parameter for wage setting when household is not allowed to reoptimize the wage but only to change following the aggregate inflation. The equilibrium conditions for this problem can be summarized by three equations with recursive formulations similar to intermediate good producer problem facing Calvo price setting. More details are illustrated in the appendix.

2.2 Firms

Final Good Producer Final good producer produces one final good in perfectly competitive market using intermediate good with following technology.

$$y_t^d = \left(\int_0^1 y_{jt}^{\frac{\varepsilon_t - 1}{\varepsilon_t}} di \right)^{\frac{\varepsilon_t}{\varepsilon_t - 1}}$$

where ε_t controls the elasticity of substitution between intermediated goods. An exogenous shock that follows ARMA(1,1) to this elasticity is introduced here, which is better known as a markup shock as in Smets and Wouters (2005).

$$\log\left(\frac{1}{\varepsilon_t - 1}\right) = (1 - \rho_p) \log\left(\frac{1}{\varepsilon - 1}\right) + \rho_p \log\left(\frac{1}{\varepsilon_t - 1}\right) - \mu_p \varepsilon_{\varepsilon,t-1} + \sigma_\varepsilon \varepsilon_{\varepsilon,t}$$

The problem of final good producer is

$$\max_{y_{jt}} p_t y_t^d - \int_0^1 p_{jt} y_{jt} dj$$

subject to

$$y_t^d = \left(\int_0^1 y_{jt}^{\frac{\varepsilon_t - 1}{\varepsilon_t}} di \right)^{\frac{\varepsilon_t}{\varepsilon_t - 1}}$$

gives input demand function

$$y_{jt} = \left(\frac{p_{jt}}{p_t} \right)^{-\varepsilon_t} y_t^d \qquad \forall j$$

where the aggregate price level is

$$p_t = \left(\int_0^1 p_{jt}^{1-\varepsilon_t} dj \right)^{\frac{1}{1-\varepsilon_t}}$$

Intermediate Good Producer Intermediate good producer's technology is a typical Cobb-Douglas function of capital and labor with productivity shock,

$$y_{jt} = A_t k_{jt}^{\alpha} l_{jt}^{1-\alpha} - \phi z_t$$

where k_{jt} and l_{jt} are capital services and homogenous labor. ϕ is fixed cost parameter calibrated either to zero or to guarantee zero profits in the economy at steady state. And total factor productivity A_t follows

$$A_t = A_{t-1} \exp\left(\Lambda_A + \sigma_A \varepsilon_{A,t}\right) \quad \varepsilon_{A,t} \sim N(0,1)$$

Since it has a growth term Λ_A, define $\mu_{A,t} \equiv \frac{A_t}{A_{t-1}}$ for stationarization purpose,

$$\log \mu_{A,t} = \Lambda_A + \sigma_A \varepsilon_{A,t}$$

Note that the balanced growth path is determined by both investment specific shock and total factor productivity shock, the growth term can be combined due to Cobb-Douglas production as follows. Define z_t,

$$z_t \equiv A_t^{\frac{1}{1-\alpha}} \xi_t^{\frac{\alpha}{1-\alpha}}$$

This term can be useful to transform most of endogenous variables in the economy that are affected by growth into stationary variables. And also define $\mu_{z,t} \equiv \frac{z_t}{z_{t-1}}$

$$\log \frac{\mu_{z,t}}{\Lambda_z} = \frac{1}{1-\alpha} \log \frac{\mu_{A,t}}{\Lambda_A} + \frac{\alpha}{1-\alpha} \log \frac{\mu_{\xi,t}}{\Lambda_\xi}$$

and the balanced growth term is

$$\Lambda_z \equiv \Lambda_A^{\frac{1}{1-\alpha}} \Lambda_\xi^{\frac{\alpha}{1-\alpha}}$$

Firms are competitive in factor markets where they confront rents, w_t and r_t^k, from l_{jt}^d and k_{jt}^d. Thus, the firm solves the static cost minimization problem,

$$\min_{l_{jt}^d, k_{jt}} w_t l_{jt}^d + r_t k_{jt}$$

subject to the production

$$y_{jt} = A_t k_{jt}^{\alpha} l_{jt}^{1-\alpha} - \phi z_t$$

The resulting equilibrium conditions for this problem give factor prices equal to marginal productivity of factors and also imply industry wide marginal cost function which is used in the following Calvo price setting problem.

Intermediate good producer price decision Similar to wage setting, when the firm gets to reoptimize the price with probability θ_p, it chooses the the price such that it maximizes the revenue minus cost of production facing demand function for differentiated goods.

$$\max_{p_{it}} E_t \sum_{\tau=0}^{\infty} (\beta\theta_p)^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} \left\{ \left(\prod_{s=1}^{\tau} \Pi_{t+s-1}^{\chi} \frac{p_{it}}{p_{t+\tau}} - mc_{t+\tau} \right) y_{it+\tau} \right\}$$

subject to

$$y_{it+\tau} = \left(\prod_{s=1}^{\tau} \Pi_{t+s-1}^{\chi} \frac{p_{it}}{p_{t+\tau}} \right)^{-\varepsilon_t} y_{t+\tau}^{d} \quad \forall i$$

where λ_t is a weight for each period flow of profits since firms are owned by households and χ is the indexation parameter for inflation persistence. The equilibrium conditions are presented in appendix.

2.3 Government

Government sets the nominal interest rates according to the Taylor rule :

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left(\left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\Pi} \left(\frac{\frac{y_t^d}{y_{t-1}^d}}{\Lambda_{y^d}} \right)^{\gamma_y} \right)^{1-\gamma_R} \exp(m_t)$$

through open market operations that are financed to yield lump-sum transfers T_t and government expenditures g_t such that the deficit are equal to zero. :

$$T_t + g_t z_t = \frac{\int_0^1 B_{it+1} di}{p_t} - R_{t-1} \frac{\int_0^1 B_{it} di}{p_t}$$

Π represents the target levels of inflation (equal to inflation in the steady state), R steady state gross return of capital, and Λ_{y^d} the steady state gross growth rate of y_t^d. The term m_t is a random shock to monetary policy that follows $m_t = \sigma_m \varepsilon_{mt}$ where ε_{mt} is distributed according to $N(0,1)$. And the government expenditure is entered with z_t multiplied to keep the stationarity of government expenditure share of output and it is modeled with following exogenous process,

$$\log g_t = (1-\rho_g) \log g + \rho_g \log g_{t-1} + \sigma_g \varepsilon_{gt}$$

Consequently, the household aggregate budget constraint is reduced to

$$c_t + \frac{1}{\xi_t} I_t + g_t z_t = w_t l_t + (r_t u_t - q_t a[u_t]) \bar{k}_t + \Omega_t$$

2.4 Aggregation

The aggregate demand is

$$y_t^d = c_t + \frac{1}{\xi_t} I_t + g_t z_t + q_t a\left[u_t\right] \bar{k}_t$$

Calvo pricing produces price dispersion in the economy, thus

$$y_t^d \int_0^1 \left(\frac{p_{jt}}{p_t}\right)^{-\varepsilon_t} dj = A_t K_t^{\alpha} \left(l_t^d\right)^{1-\alpha} - \phi z_t$$

By defining $v_t^p \equiv \int_0^1 \left(\frac{p_{jt}}{p_t}\right)^{-\varepsilon} dj$, the price dispersion evolves as,

$$v_t^p = \theta_p \left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t}\right)^{-\varepsilon_t} v_{t-1}^p + (1-\theta_p)\, \Pi_t^{*-\varepsilon_t}$$

Thus, we have

$$y_t^d = \frac{A_t k_t^{\alpha} \left(l_t^d\right)^{1-\alpha} - \phi z_t}{v_t^p}$$

This condition implies that the aggregate demand[6] which is domestic absorption is distorted from the aggregate supply due to the price dispersion. Similarly, the wage dispersion can be expressed by defining $v_t^w \equiv \int_0^1 \left(\frac{w_{it}}{w_t}\right)^{-\eta} di$, then the labor supply and labor demand are related with wage dispersion

$$l_t^d = \frac{1}{v_t^w} l_t^s$$

where the wage dispersion variable evolves as

$$v_t^w = \theta_w \left(\frac{w_{t-1}}{w_t} \frac{\Pi_{t-1}^{\chi_w}}{\Pi_t}\right)^{-\eta} v_{t-1}^w + (1-\theta_w) \left(\Pi_t^{w^*}\right)^{-\eta}$$

Also in capital market where the capital intensity exists,

$$\int_0^1 k_{jt} dj = u_t \bar{k}_t$$

and thus

$$k_{jt} = u_t \bar{k}_t$$

[6] The following sections will use the terms, domestic expenditures and domestic output, interchangeably, but they will both refer to the aggregate demand of the economy rather than aggregate supply.

3 Empirical Analysis

3.1 Data

Table 1: Data Source

Variables	Measurement unit	Type	Freq.	Sample	Source	
Nominal gov't consumption expenditure	Bil.Won	SA	Y	2000 - 2011	ECOS	10.4.2.3
Nominal gross private fixed capital formation	Bil.Won	NSA	Y	2000 - 2011	ECOS	10.4.9.1
Nominal gross gov't fixed capital formation	Bil.Won	SA	Y	2000 - 2011	ECOS	10.4.9.1
Nominal consumption of non-durable goods	Bil.Won	SA	Y	2000 - 2011	ECOS	10.4.12.3
Nominal consumption of semi-durable goods	Bil.Won	SA	Y	2000 - 2011	ECOS	10.4.12.3
Nominal consumption of services	Bil.Won	SA	Y	2000 - 2011	ECOS	10.4.12.3
Nominal consumption of durable goods	Bil.Won	SA	Y	2000 - 2011	ECOS	10.4.12.3
Total population	Thousands of persons	NSA	Y	2000 - 2011	KOSIS	
Yields of treasury bonds(3-year)	Percent per annum	NSA	Q	2000:Q1-2012:Q2	ECOS	4.1.2
Nominal gov't consumption expenditure	Bil.Won	SA	Q	2000:Q1-2012:Q2	ECOS	10.4.2.1
Real gov't consumption expenditure	Bil.Won	SA	Q	2000:Q1-2012:Q2	ECOS	10.4.2.2
Nominal gross private fixed capital formation	Bil.Won	NSA	Q	2000:Q1-2012:Q2	ECOS	10.4.9.3
Nominal gross gov't fixed capital formation	Bil.Won	NSA	Q	2000:Q1-2012:Q2	ECOS	10.4.9.3
Real gross private fixed capital formation	Bil.Won	NSA	Q	2000:Q1-2012:Q2	ECOS	10.4.9.4
Real gross gover't fixed capital formation	Bil.Won	NSA	Q	2000:Q1-2012:Q2	ECOS	10.4.9.4
Nominal consumption of non-durable goods	Bil.Won	SA	Q	2000:Q1-2012:Q2	ECOS	10.4.12.1
Nominal consumption of semi-durable goods	Bil.Won	SA	Q	2000:Q1-2012:Q2	ECOS	10.4.12.1
Nominal consumption of services	Bil.Won	SA	Q	2000:Q1-2012:Q2	ECOS	10.4.12.1
Nominal consumption of durable goods	Bil.Won	SA	Q	2000:Q1-2012:Q2	ECOS	10.4.12.1
Real consumption of non-durable goods	Bil.Won	SA	Q	2000:Q1-2012:Q2	ECOS	10.4.12.2
Real consumption of semi-durable goods	Bil.Won	SA	Q	2000:Q1-2012:Q2	ECOS	10.4.12.2
Real consumption of services	Bil.Won	SA	Q	2000:Q1-2012:Q2	ECOS	10.4.12.2
Real consumption of durable goods	Bil.Won	SA	Q	2000:Q1-2012:Q2	ECOS	10.4.12.2
Nominal total wage	Won	NSA	Q	2000:Q1-2012:Q2	KOSIS	
Total working hours	Hours	NSA	Q	2000:Q1-2012:Q2	KOSIS	
Total employed persons	Thousands of persons	NSA	Q	2000:Q1-2012:Q2	KOSIS	

Data are imported from Economic Statistics System of Bank of Korea(hereafter, ECOS) and Korean Statistical Information Service of Statistics Korea(hereafter, KOSIS). Domestic expenditures such as consumption, capital formation and government spendings are from ECOS while labor related indices are from KOSIS. First, since the model is practically equivalent to a representative agent's problem[7], the variables in the model are in terms of per capita. Time series that are related to domestic demands are divided by population level. But since data for population is only available at annual frequency, population for quarterly frequency has been constructed by linear interpolation. Second, since the consumption series as mentioned above are constructed by consumption of nondurable, semidurable and services and the model's inflation rate is consumption deflator[8], the deflator for consumption of nondurable, semidurable

[7]In fact, there is heterogeneity within labor supply and intermediate goods. But due to Calvo price stickiness and Dixit-Stiglitz type of aggregation, the equilibrium conditions that fully describe the economy are in aggregate variables only.

[8]The numerarie considered in the model is consumption good

and services is necessary but this series is not available directly from the database. Since Korean real expenditures and corresponding deflators are reported in database with Laspeyres chain weighted method, the consumption deflator of the interest is derived under this chain weighted scheme[9]. Investment series is a sum of durable consumption and private fixed capital formation. However, because the model has an investment specific trend that deviates the investment from the numerarie, the investment series are deflated with its own deflator rather than consumption deflator after the investment deflator has been backed out by the same scheme as consumption deflator. Government spending is a sum of government consumption and government capital formation and domestic expenditures is sum of consumption, investment and government spending according to aggregate resource constraint in the model. Third, the implied wage rate in the model is an hourly real wage rate for an average household. The model lacks unemployment and therefore assumes all the households are employed, the corresponding data are computed to purport an hourly wage rate per employee deflated by consumption deflator. Fourth, the risk free interest rate in the model is a policy rate by the monetary authority and at the same time a rate which a household faces for saving and borrowing. In reality, there is a gap between the policy rate such as call rate in Korea and the interest rate that households face. But due to the limitation of the model that does not specify either financial friction or risk premium that household may bear, treasury bond rate has been adopted to serve as a proxy[10]. Lastly, all of constructed series are transformed into quarterly growth rates except risk free interest rate to coincide with observable variables in the model. And thus the observable vector is log of interest rate, inflation rate, growth rates of real wage, real domestic output, real consumption, real investment and real government spendings and detrended by each own trend defined by the model.

$$\begin{bmatrix} \hat{R}_t \\ \hat{\pi}_t \\ \hat{\gamma}_t^w \\ \hat{\gamma}_t^y \\ \hat{\gamma}_t^c \\ \hat{\gamma}_t^I \\ \hat{\gamma}_t^g \end{bmatrix} = \begin{bmatrix} \log R_t \\ \log \pi_t \\ \log \gamma_t^w \\ \log \gamma_t^y \\ \log \gamma_t^c \\ \log \gamma_t^I \\ \log \gamma_t^g \end{bmatrix} - \begin{bmatrix} \log R_{ss} \\ \log \Pi \\ \log \Lambda_w \\ \log \Lambda_y \\ \log \Lambda_c \\ \log \Lambda_I \\ \log \Lambda_g \end{bmatrix}$$

where $\Lambda_z = \Lambda_w = \Lambda_y = \Lambda_c = \Lambda_g$.

[9]See Whelan (2002) for constructing real time series and corresponding deflators under chain weighted indices. And this recovery requires annual series of nominal expenditures.

[10]CD rate is another popular measure for risk free interest rate but recent news in Korea has made researchers doubtful of its credibility

3.2 Econometric Methodology

Once the equilibrium conditions are stationarized with trend components, z_t, the system is approximated up to first order using the perturbation method[11] around the deterministic steady state. Based on this linearized system, structural parameters of the model are brought to estimation using Bayesian technique[12] which is a large body of research programs in macroeconomics literature nowadays. In the numerical algorithm, Random Walk Markov chain Monte Carlo method is adopted for the proposal value of parameters. The proposal density in each chain is based on the inverse of Hessian of likelihood function based on initial values. And the initial values are the posterior modes produced by Simulated Annealing method[13]. The posterior estimates reported below are based on the second half[14] of two million draws from Markov chain Monte Carlo.

A small subset of structural parameters which are mostly hard to be identified has been calibrated as Table 2. The discount factor, β, is calibrated to imply the inverse of the longrun real interest rate, i.e. the average inflation rate over the average nominal interest[15]. And the fraction of government spending in domestic demand is 0.1963 which is the average ratio in data as well. Rest of parameters follow values common in the literature.

Table 2: Fixed Parameters

β	δ	κ	ϕ	α	γ_2	$\frac{g}{y}$
0.9964	0.025	10	0	0.3	0.001	0.1963

3.3 Estimation Results

Prior and posterior distributions of structural parameters are reported in Table 3. Parameters that determine the trend such as Λ_A, Λ_ξ and Π should be aligned with data closest as possible to the average trends of data. This is, in fact, argued by Del Negro and Schorfheide (2008) to set prior distributions of which means are consistent with average trends in the sample. So prior distributions associated with Λ_A, Λ_ξ and Π are set such that their means are data trends and their variances are set to minimize the deviations that can potentially create inefficiency in Markov chains of the estimation procedures. Rest of priors are mostly set to be neutral to let data speak for it as much as possible, while their boundaries are determined within economic reasonings which are mostly common in DSGE estimations. One minor exception is the prior standard deviations of structural shocks which was set broader than most of Bayesian

[11] Perturbation method up to first order approximation is equivalent to log-linearization of the system. The computational exercise is done through MATLAB R2011b, and DSGE approximation and solution techniques are imported from Schmitt-Grohé and Uribe (2004). Codes are available from the author upon request.

[12] For more detailed summary of Bayesian estimation, see An and Schorfheide (2007).

[13] See Andreasen (2010) for this method in the context of DSGE models

[14] The overall chains were stabilized and the convergence properties were reasonable after million draws

[15] In data, the average interest rate is 4.8% per annum and the inflation rate was 3.29% per annum.

estimation literature that targets U.S. data which is considered to be more stable than Korea.

Table 3: Prior and Posterior Distributions

	Prior distribution			Posterior distribution		
	Distr.	Mean	St. Dev.	Mode	Mean	St. Dev.
b	Beta	0.706	0.107		0.981	0.006
γ	Normal	1.000	0.250		0.639	0.163
ψ	Normal	9.000	3.000		8.958	0.067
ε	Normal	10.000	0.750		9.054	0.148
η	Normal	10.000	0.750		8.637	0.082
θ_p	Beta	0.500	0.090		0.746	0.029
χ	Beta	0.500	0.151		0.810	0.076
θ_w	Beta	0.500	0.090		0.725	0.022
χ_w	Beta	0.500	0.151		0.417	0.097
γ_r	Beta	0.750	0.094		0.844	0.036
γ_y	Normal	0.500	0.050		0.194	0.031
γ_π	Normal	1.500	0.125		1.877	0.085
$100\,(\Pi-1)$	Gamma	0.832	0.071		0.852	0.075
$100\log(\Lambda_\xi)$	Normal	0.333	0.025		0.320	0.026
$100\log(\Lambda_A)$	Normal	0.317	0.025		0.301	0.021
ρ_d	Beta	0.500	0.151		0.333	0.086
ρ_φ	Beta	0.500	0.151		0.259	0.052
ρ_ε	Beta	0.500	0.151		0.408	0.144
μ_p	Beta	0.500	0.151		0.472	0.104
ρ_g	Beta	0.500	0.151		0.266	0.105
σ_d	InvGamma	0.1	4.000		0.3342	0.0873
σ_φ	InvGamma	0.1	4.000		5.3517	2.7567
σ_ξ	InvGamma	0.1	4.000		0.3585	0.0470
σ_A	InvGamma	0.1	4.000		0.1407	0.0162
σ_m	InvGamma	0.1	4.000		0.0034	0.0006
σ_ε	InvGamma	0.1	4.000		1.6011	0.2607
σ_g	InvGamma	0.1	4.000		0.1268	0.0144

Posterior estimates with trends imply that annual inflation rate on average in the sample was 3.39% and average growth rate of domestic output, $\Lambda_z = \Lambda_A^{\frac{1}{1-\alpha}} \Lambda_k^{\frac{\alpha}{1-\alpha}}$, is 2.27%, while in data inflation rate was 3.24% and growth rate of domestic expenditures was 2.32%[16]. And the trend for investment growth, $\Lambda_I = \Lambda_A^{\frac{1}{1-\alpha}} \Lambda_k^{\frac{\alpha}{1}}$, is estimated to be 3.55% which is lower then the sample average, 3.71%. Two important estimates that relate directly to monetary policy implications need to be highlighted before going to the next section. First parameter is the Calvo probability that stands for price stickiness of nominal friction because the stickiness determines the length of propagation of inflation and real variables in response to monetary policy shock. The posterior estimate on Calvo probability, θ_p, is 0.746 which is relatively lower than U.S. estimates[17]. The average price duration based on this estimate imply 11.8 months.

[16]One might think that this may be lower than observed growth rate of real GDP for Korea but note that it is per capita output growth and it does not have net export whose contribution towards growth might have been substantial in Korea.

[17]See Smets and Wouters (2005) and Del Negro and Schorfheide (2008).

Second parameter of our attention is γ_π which explains the monetary policy stance towards inflation stabilization. According to the posterior estimate, 1.877, the monetary authority was fairly aggressive towards the inflation stabilization. However, the influence of the prior distribution for this particular parameter should not go unnoticed since it was deliberately set tight for Markov chain to remain within a determinancy area especially to avoid values lower than one. Otherwise, Markov chain can go near one to stay for a long time and thus leading to deteriorate the acceptance ratio. Also, Markov chains for other parameters cannot reach to their potential modes of distributions when they are stuck to certain values in which γ_π is near its lower bound. Another explanation is an influence of the sample period that includes the global financial crisis in 2008. In 2008, there was a big negative hit on Korean economy and thus the monetary policy was extremely expansionary in response to negative gaps of both inflation and output growth. This aggressive behavior of the monetary policy in Korea may have affected this parameter to be quite aggressive, for γ_π represents the aggressiveness towards not only high inflation but also low inflation due to its symmetry. Lastly, the estimated inflation trend also implies the target rate of the monetary policy since the model does not differentiate its trend and target rate. Bank of Korea has declared the target rate of inflation to be 3% except for the year of 2001. But the resulting estimate on the inflation trend is not only higher than the sample average but also even higher than the declared inflation target rate of Bank of Korea. This inflation trend estimate together with high γ_π implies that Bank of Korea has been aggressive towards inflation stabilization while it seems to be targeting higher inflation rate on average than 3% and thus systematically inflationary.

Hence, it is difficult to evaluate the monetary policy stance solely by one parameter, γ_π. But the overall evaluation can be further extended by structural shocks decompositions which can demonstrate historical behaviors of the monetary policy as well. Moreover, counterfactual simulations under higher or lower values for this parameter can be useful experiments in a relative sense, for the resulting outcome of variables of interest such as inflation and output can be investigated for a possible improvement.

4 Policy Analysis

4.1 Impulse Response Function

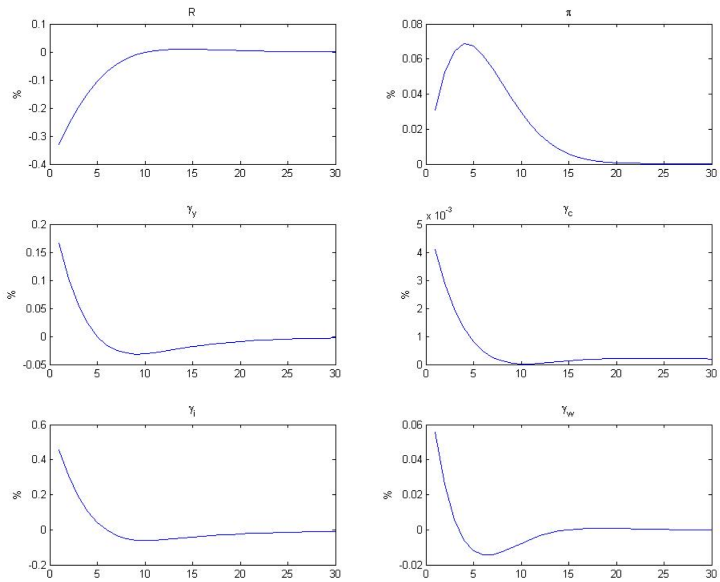


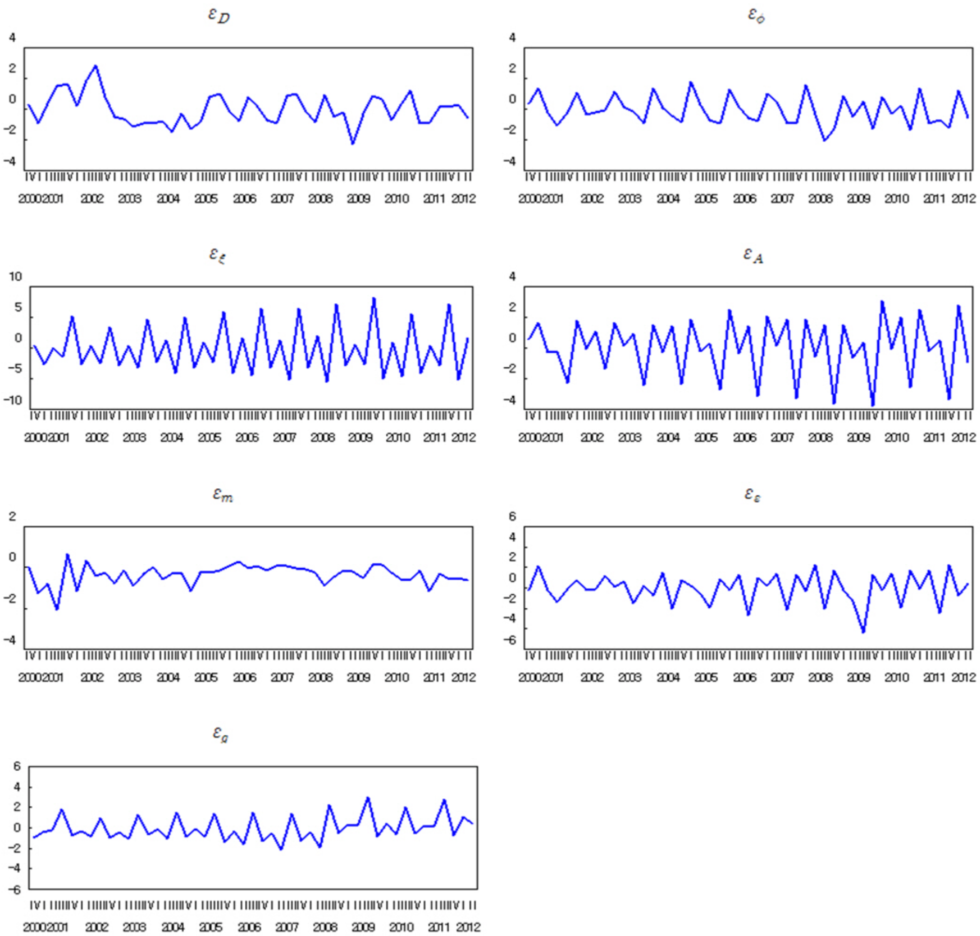
Figure 1: Monetary Policy Shock Impulse Response Function

As shown in Figure 1, the impulse response functions for six key variables are triggered by one unit of standard deviation of expansionary monetary shock. The unit in the y-axis is a percentage deviation from its own steady state. After the shock, the interest rate goes down by $33bp$ instantly and returns to its steady state after ten quarters. The inflation's impulse response function is hump shaped due to price stickiness. Its rate increases by 0.031% initially and peaks after four quarters reaching 0.068%, nearly more than double of the initial level. And the effect is dissipated after sixteen quarters. The domestic output responds by 0.17% instantly and quickly drops within a year. While consumption due to its smoothing mechanism only goes up by less than 0.004%, the investment increases quite drastically by 0.46%. Therefore, the monetary policy shock results in a higher growth rate of domestic output instantly which is mainly due to favorable investment incentives since consumption response is almost tacit.

4.2 Historical Shock Decomposition

Before computing shocks' contributions towards inflation and output, structural shocks of the estimated model can be retrieved by Kalman backward filter(or Kalman smooth filter). Those historical shocks are shown in Figure 2. Most of shocks fluctuate around zero but the monetary

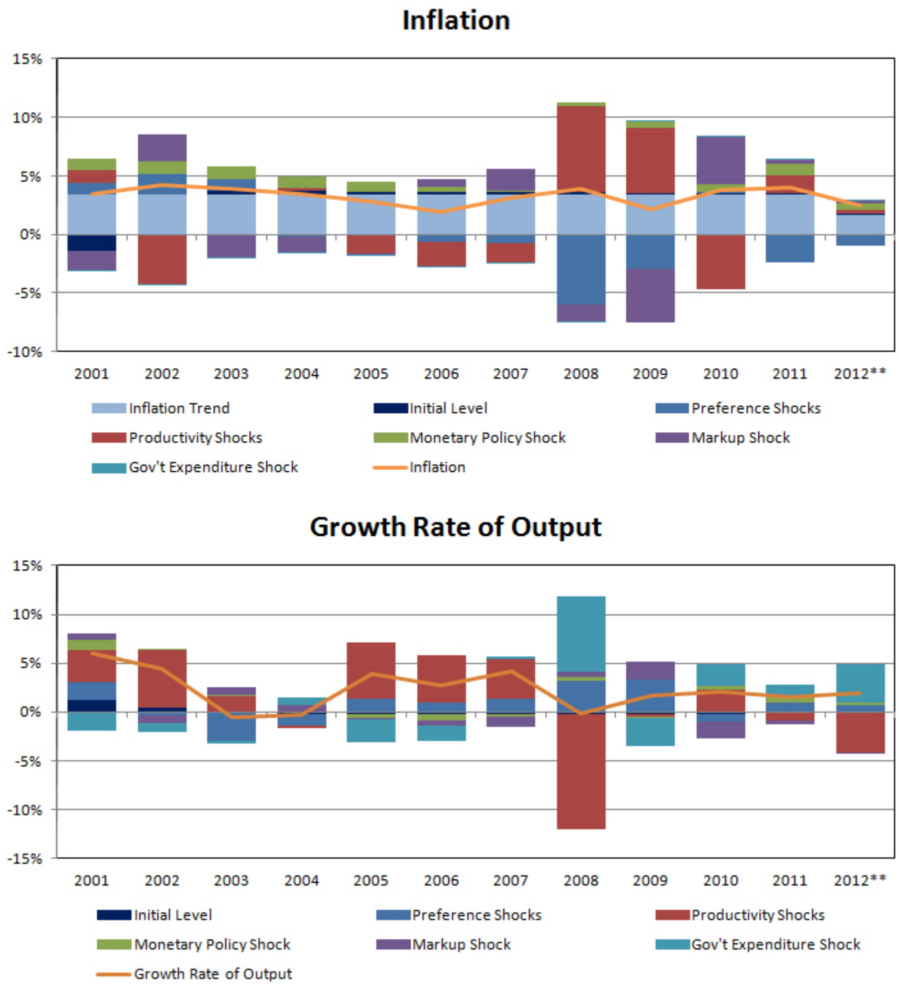
Figure 2: Estimated Structural Shocks



policy shock demonstrates asymmetric movements that seem to be below zero on average. This implies that the monetary policy was more or less expansionary on average during the sample period. This fact becomes even more clear with the following decomposition analysis.

Figure 3 demonstrates annualized series with having each year's observations decomposed into contributions by each historical shock with an exception of the measurement errors for they are trivially small. And since the level of predetermined state variables at the beginning of the sample are not decomposed by structural shocks, the effect of this initial level has been isolated. Figure 3 only covers from 2001 rather than 2000 because this effect is more stabilized after a year so that most of fluctuations are attributed to structural shocks. A trend component for inflation is constant over the sample periods since it is 3.39% per annum implied by the estimated model with the exception of year 2012 due to shorter periods of observations. For domestic expenditures, the trend effects are inherent within total factor productivity and investment specific technology shocks which are indicated by productivity shocks in the figure.

Figure 3: Historical Decomposition



** : Growth rate for Q1:Q2

A large negative contribution of these productivity shocks in 2008 clearly shows the global financial crisis while there was a massive government spending to promote the economy by then. The credit crunch in 2003 is demonstrated by low domestic output mainly due to preference shocks, particularly from the intertemporal preference shock[18], which represents the household's consumption behavior which was largely affected by this crisis.

Most conspicuous result from the historical decomposition is the contribution of monetary policy shock towards the inflation. This shock has been inflationary mostly over the sample periods except for the year of 2007. On the other hand, the monetary policy shock seems to be countercyclical with respect to output. In order to see the exact influence of this monetary policy discretion on inflation and output over the sample, empirical moments have been computed when this shock has been eliminated. Second column of Table 4 shows means and

[18]Figure 2 shows that in 2003 the intertemporal preference shock stay below zero for an extended period.

volatilities of inflation and output under no monetary shock. While the volatility of inflation do not change much, the average inflation is 0.7% lower. For output, the average growth rate reduces by 0.12% and its volatility increases by 0.07%. Thus, during the sample periods, the monetary policy discretion has promoted the growth rate of output on average and also has been countercyclical to stabilize its volatility but at the cost of higher level of inflation than the declared target rate. Then, the question is how much the monetary policy discretion has a room for achieving its goal of 3% while output volatility is kept at minimum. Third column of Table 4 shows the moments of simulated inflation and output, if the degree of standard deviation of monetary shock was relatively more conservative than the estimated one. The average inflation rate reaches roughly 3% when the standard deviation of monetary policy shock was half of its estimate. And the negative impact on average growth rate of output and its volatility is reduced from the case without the monetary policy discretion. Thus, in order to establish the target rate of 3% the monetary policy discretion should have been relatively more abstinent than the estimated monetary policy shock.

Table 4: Simulation with and without Monetary Policy shock

	Fitted Value	No Monetary Shock	.5 times Monetary Policy Shock
π	3.31	2.61	2.96
	(2.10)	(2.07)	(2.08)
γ^y	2.35	2.23	2.29
	(3.24)	(3.31)	(3.27)

Note : The figures in parentheses indicate standard deviations. (%per annum)

4.3 Counterfactual Simulation

How aggressive was the monetary policy stance towards inflation and domestic expenditures stabilization? This question can be answered by counterfactual simulations with Taylor rule parameters ranging below and above the posterior estimates while keeping all other parameters and structural shocks as posterior estimates. Table 5 shows means and volatilities of the counterfactual simulations under various degree of monetary policy stance.

A lower value, without loss of generality 1.2, for γ_π, stands for less aggressive monetary policy stance to inflation stabilization while higher values, 2.5 and 3.0, for a more aggressive stance. For inflation, the overall volatilities do not show disparities with a degree above the estimated $\hat{\gamma}_\pi$. On the other hand when the monetary policy is more aggressive towards output stabilization(γ_y=0.3) and less aggressive to inflation(γ_π=1.2), the inflation volatility increases. But the level of inflation is uniformly kept lower with any degree of γ_y. This is an inevitable outcome due to the inclusion of the global financial crisis in the sample. During this period, an aggressive monetary policy was conducted, if not, much lower inflation would have occurred resulting in lower level of average inflation. For the output growth, the volatilities range from 3.07 to 6.52. The volatility tends to be greater when γ_π is either higher or lower than its posterior estimate. Also it is lower when γ_y is higher except for the case where γ_π=1.2. In summary, there seems to be no dominant case which yields lower volatilities of both variables and thus there is a relationship of tradeoff for monetary policy stance towards two objectives.

Table 5: Counterfactual Simulations

Moments for Inflation (π) (%per annum)

	$\gamma_\pi = 1.20$	$\hat{\gamma}_\pi = 1.87$	$\gamma_\pi = 2.50$	$\gamma_\pi = 3.00$
$\gamma_y = 0.10$	3.08	3.26	3.21	3.18
	(2.05)	(2.08)	(2.09)	(2.11)
$\hat{\gamma}_y = 0.19$	3.17	**3.31**	3.24	3.21
	(2.11)	(**2.10**)	(2.09)	(2.10)
$\gamma_y = 0.30$	3.27	3.36	3.27	3.23
	(2.23)	(2.12)	(2.10)	(2.10)

Moments for Output Growth (γ^y) (%per annum)

	$\gamma_\pi = 1.20$	$\hat{\gamma}_\pi = 1.87$	$\gamma_\pi = 2.50$	$\gamma_\pi = 3.00$
$\gamma_y = 0.10$	2.44	2.35	2.35	2.36
	(3.40)	(3.60)	(5.20)	(6.52)
$\hat{\gamma}_y = 0.19$	2.45	**2.35**	2.35	2.36
	(3.88)	(**3.24**)	(4.61)	(5.89)
$\gamma_y = 0.30$	2.46	2.35	2.35	2.36
	(4.57)	(3.07)	(4.03)	(5.25)

Note : The figures in parentheses indicate standard deviations from its own trends.

5 Conclusion

This paper has estimated Korean economy with a medium scale New Keynesian model which has various frictions and shocks. The paper also attempts to bring the model to the data without preprocessing of data so that the model fully explains trends and cycles without applying detrending filters separately to each macro time series. As for estimation results, the inflation trend which also implies the target rate of monetary policy is estimated to be higher than the actual target rate Bank of Korea has announced. But despite of higher target rate the posterior estimate on the monetary policy stance towards inflation gap shows fairly aggressive stance probably mainly due to the inclusion of 2008 in the sample. Under the estimated model, in response to a monetary policy shock the growth rates of domestic expenditures increase over 0.17% mostly due to investment incentives but returns close to the steady state within a year. Consumption increases only a little due to the smoothing. And the inflation rate rises in a hump shaped fashion with peaking at 0.068% one year after. The next results of this paper demonstrated the historical decompositions of Korean economy over the sample period. After the inflation targeting was announced by Bank of Korea, the monetary shock has been mostly inflationary and thus yielding much higher inflation rate than it would have been without the shock. Although, the monetary discretion had a role to reduce the volatility of the output growth, the simulation with a more conservative monetary policy discretion could have achieved the target rate, 3%, of inflation. Lastly, the counterfactual simulations were performed under various combinations of Taylor rule parameters. More aggressive monetary policy stance towards inflation gap does improve on level of the inflation rate by reducing the gap from Bank of Korea's target rate. But this is at the cost of increasing the volatility of the growth rate of domestic expenditures which implies an existence of tradeoff between two objectives. Thus, this could lead to a next research agenda by defining an optimal criterion to derive the optimal monetary policy that can provide a optimal weights on inflation stabilization and output growth stabilization.

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6 Model Appendix

6.1 Households Problem

The lagrangian problem for household is presented with choice of $c_{it}, B_{it}, u_{it}, \bar{k}_{it+1}, i_{it}, I_{it}, a_{it+1,t}, w_{it}, l_{it}$ to maximize

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} d_t \left\{ \log\left(c_{it} - bc_{it-1}\right) - \varphi_t \psi \frac{l_{it}^{1+\gamma}}{1+\gamma} \right\} \\ -\lambda_{it} \left\{ \begin{array}{c} c_{it} + i_{it} + \frac{B_{it+1}}{p_t} + \int q_{t+1,t} a_{it+1,t} d\omega_{i,t+1|t} \\ -w_{it} l_{it} - \left(r_t u_{i,t} - q_t a\left[u_{it}\right]\right) \bar{k}_{i,t} - R_{t-1} \frac{B_{it}}{p_t} - a_{it} - T_t - F_t \\ -q_{it}\left[\bar{k}_{it+1} - (1-\delta)\bar{k}_{it} - F\left(I_{it}, I_{it-1}\right)\right] \\ -\zeta_{it}\left[I_{it} - \xi_t i_{it}\right] \end{array} \right\} \end{array} \right]$$

where

$$F\left(I_{it}, I_{it-1}\right) \equiv \left(1 - S\left(\frac{I_{it}}{I_{it-1}}\right)\right) I_{it}$$

and

$$\begin{aligned} F_{1t} &= 1 - S\left(\frac{I_{it}}{I_{it-1}}\right) - S'\left(\frac{I_{it}}{I_{it-1}}\right)\frac{I_{it}}{I_{it-1}} \\ F_{2t+1} &= S'\left(\frac{I_{it+1}}{I_{it}}\right)\left(\frac{I_{it+1}}{I_{it}}\right)^2 \end{aligned}$$

And households will determine w_{it} and l_{it} by maximizing relevant part of the lagrangian under Calvo wage setting which will be characterized separately.

Household Conditions FOCs of the above problem with respect to $c_{it}, B_{it}, u_{it}, \bar{k}_{it+1}, i_{it}, I_{it}, a_{it+1|t}$ are

$$d_t \left(c_{it} - bc_{it-1}\right)^{-1} - b\beta E_t d_{t+1}\left(c_{it+1} - bc_{it}\right)^{-1} = \lambda_{it}$$

$$\lambda_{it} = \beta E_t \lambda_{it+1} \frac{R_t}{\Pi_{t+1}}$$

$$r_t = q_t a'\left[u_{it}\right]$$

$$\lambda_{it} q_{it} = \beta E_t \left\{\lambda_{it+1}\left[(1-\delta) q_{it+1} + r_{t+1} u_{it+1} - q_{t+1} a\left(u_{it+1}\right)\right]\right\}$$

$$1 = \zeta_{it} \xi_t$$

$$\lambda_{it} \zeta_{it} = \lambda_{it} q_{it} F_{1,t} + \beta E_t \lambda_{it+1} q_{it+1} F_{2,t+1}$$

$$\lambda_{it+1} q_{t+1,t} = \lambda_{it}$$

Symmetric Equilibrium Since we consider a symmetric equilibrium due to complete asset market so that $c_{it} = c_t, B_{it} = B_t, \lambda_{it} = \lambda_t, u_{it} = u_t, q_{it} = q_t, \zeta_{it} = \zeta_t, i_{it} = i_t, I_{it} = I_t, \bar{k}_{it} =$

$\bar{k}_t, a_{it+1|t} = a_{t+1|t}$. After substituting $\zeta_t = \frac{1}{\xi_t}$ and rearranging,

$$d_t \left(c_t - bc_{t-1}\right)^{-1} - b\beta E_t d_{t+1} \left(c_{t+1} - bc_t\right)^{-1} = \lambda_t$$

$$\lambda_t = \beta E_t \lambda_{t+1} \frac{R_t}{\Pi_{t+1}}$$

$$r_t = q_t a' \left[u_t\right]$$

$$\lambda_t q_t = \beta E_t \left\{\lambda_{t+1} \left[\left(1-\delta\right) q_{t+1} + r_{t+1} u_{t+1} - q_{t+1} a \left(u_{t+1}\right)\right]\right\}$$

$$1 = q_t \xi_t F_{1,t} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \xi_{t+1} \frac{1}{\mu_{\xi,t+1}} F_{2,t+1}$$

Household labor problem Calvo wage problem for houshold

$$\max_{w_{jt}} E_t \sum_{\tau=0}^{\infty} \left(\beta\theta_w\right)^{\tau} \left\{ -d_t \varphi_t \psi \frac{l_{jt+1}^{1+\gamma}}{1+\gamma} + \lambda_{jt+1} \prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} w_{jt} l_{jt+\tau} \right\}$$

subject to

$$l_{jt+\tau} = \left(\prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} \frac{w_{jt}}{w_{t+\tau}} \right)^{-\eta} l_{t+\tau}^{d} \quad \forall j$$

This gives the law of motion

$$f_t = \frac{\eta - 1}{\eta} \left(w_t^*\right)^{1-\eta} \lambda_t w_t^{\eta} l_t^d + \beta\theta_w E_t \left(\frac{\Pi_t^{\chi_w}}{\Pi_{t+1}}\right)^{1-\eta} \left(\frac{w_{t+1}^*}{w_t^*}\right)^{\eta-1} f_{t+1}$$

$$f_t = \psi d_t \varphi_t \left(\frac{w_t}{w_t^*}\right)^{\eta(1+\gamma)} \left(l_t^d\right)^{1+\gamma} + \beta\theta_w E_t \left(\frac{\Pi_t^{\chi_w}}{\Pi_{t+1}}\right)^{-\eta(1+\gamma)} \left(\frac{w_{t+1}^*}{w_t^*}\right)^{\eta(1+\gamma)} f_{t+1}$$

The real wage index evolves :

$$w_t^{1-\eta} = \theta_w \left(\frac{\Pi_{t-1}^{\chi_w}}{\Pi_t}\right)^{1-\eta} w_{t-1}^{1-\eta} + \left(1-\theta_w\right) w_t^{*1-\eta}$$

which can be rewritten

$$1 = \theta_w \left(\frac{\Pi_{t-1}^{\chi_w}}{\Pi_t}\right)^{1-\eta} \left(\frac{w_{t-1}}{w_t}\right)^{1-\eta} + \left(1-\theta_w\right) \left(\Pi_t^{*w}\right)^{1-\eta}$$

where

$$\Pi_t^{w^*} = \frac{w_t^*}{w_t}$$

6.2 Firms

Final Good Producer Final good producer produces one final good in perfectly competitive market using intermediate good and yields the demand function for differentiated good.

$$y_{jt} = \left(\frac{p_{jt}}{p_t}\right)^{-\varepsilon_t} y_t^d \qquad \forall j$$

where the aggregate price level is

$$p_t = \left(\int_0^1 p_{jt}^{1-\varepsilon_t} dj\right)^{\frac{1}{1-\varepsilon_t}}$$

Intermediate Good Producer The static cost minimization problem,

$$\min_{l_{jt}^d, k_{jt}} w_t l_{jt}^d + r_t k_{jt}$$

subject to the production

$$y_{jt} = A_t k_{jt}^{\alpha} l_{jt}^{1-\alpha} - \phi z_t$$

Assuming interior solution, FOCs are

$$\begin{aligned} w_t &= \varrho (1-\alpha) A_t k_{jt}^{\alpha} \left(l_{jt}^d\right)^{-\alpha} \\ r_t &= \varrho \alpha A_t k_{jt}^{1-\alpha} \left(l_{jt}^d\right)^{1-\alpha} \end{aligned}$$

where ϱ is the Lagrangian multiplier. Then, the real marginal cost is mc_t by setting $A_t k_{jt}^{\alpha} \left(l_{jt}^d\right)^{1-\alpha} = 1$. Note that mc_t is not firm specific since

$$\begin{aligned} 1 &= A_t K_{jt}^{\alpha} \left(l_{jt}^d\right)^{1-\alpha} = A_t \left(\frac{K_{jt}}{l_{jt}^d}\right)^{\alpha} l_{jt}^d \\ &= A_t \left(\frac{w_t}{r_t}\frac{\alpha}{1-\alpha}\right)^{\alpha} l_{jt}^d \\ l_{jt}^d &= \frac{\left(\frac{\alpha}{1-\alpha}\frac{w_t}{r_t}\right)^{-\alpha}}{A_t} \end{aligned}$$

$$\begin{aligned} mc_t &= \left(\frac{1}{1-\alpha}\right) w_t l_{jt}^d \\ &= \left(\frac{1}{1-\alpha}\right) w_t \frac{\left(\frac{\alpha}{1-\alpha}\frac{w_t}{r_t}\right)^{-\alpha}}{A_t} \\ &= \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \frac{w_t^{1-\alpha} (r_t)^{\alpha}}{A_t} \end{aligned}$$

Intermediate good producer price decision Calvo Pricing decision

$$\max_{p_{it}} E_t \sum_{\tau=0}^{\infty} (\beta\theta_p)^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} \left\{ \left(\prod_{s=1}^{\tau} \Pi_{t+s-1}^{\chi} \frac{p_{it}}{p_{t+\tau}} - mc_{t+\tau} \right) y_{it+\tau} \right\}$$

subject to

$$y_{it+\tau} = \left(\prod_{s=1}^{\tau} \Pi_{t+s-1}^{\chi} \frac{p_{it}}{p_{t+\tau}} \right)^{-\varepsilon_t} y_{t+\tau}^{d} \quad \forall i$$

The law of motion

$$g_t^1 = \lambda_t mc_t y_t^d + \beta\theta_p E_t \left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}} \right)^{-\varepsilon_t} g_{t+1}^1$$

$$g_t^2 = \lambda_t \Pi_t^* y_t^d + \beta\theta_p E_t \left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}} \right)^{1-\varepsilon_t} \left(\frac{\Pi_t^*}{\Pi_{t+1}^*} \right) g_{t+1}^2$$

$$\varepsilon_t g_t^1 = (\varepsilon_t - 1)\, g_t^2$$

Price level evolves

$$1 = \theta_p \left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t} \right)^{1-\varepsilon_t} + (1-\theta_p)\, \Pi_t^{*1-\varepsilon_t}$$

6.3 Government

Government sets the nominal interest rates according to the Taylor rule :

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\gamma_R} \left(\left(\frac{\Pi_t}{\Pi} \right)^{\gamma_\Pi} \left(\frac{\frac{y_t^d}{y_{t-1}^d}}{\Lambda_{y^d}} \right)^{\gamma_y} \right)^{1-\gamma_R} \exp(m_t)$$

through open market operations. We assume balanced government budget constraint every period :

$$T_t + g_t z_t = \frac{\int_0^1 B_{it+1} di}{p_t} - R_{t-1} \frac{\int_0^1 B_{it} di}{p_t}$$

6.4 Aggregation

The aggregate demand is

$$y_t^d = c_t + \frac{1}{\xi_t} I_t + g_t z_t + q_t a\, [u_t]\, \bar{k}_t$$

Price dispersion evolves as

$$v_t^p = \theta_p \left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t} \right)^{-\varepsilon_t} v_{t-1}^p + (1-\theta_p)\, \Pi_t^{*-\varepsilon_t}$$

and thus

$$y_t^d = \frac{A_t k_t^{\alpha} \left(l_t^d\right)^{1-\alpha} - \phi z_t}{v_t^p}$$

Similarly, labor supply is

$$l_t^d = \frac{1}{v_t^w} l_t^s$$

and wage dispersions evolves as

$$v_t^w = \theta_w \left(\frac{w_{t-1}}{w_t}\frac{\Pi_{t-1}^{\chi_w}}{\Pi_t}\right)^{-\eta} v_{t-1}^w + (1-\theta_w)\left(\Pi_t^{w^*}\right)^{-\eta}$$

And capital market clears

$$k_{jt} = u_t \bar{k}_t$$

6.5 Equilibrium Conditions

- Intermediate good producer

$$\frac{u_t \bar{k}_t}{l_t^d} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t}$$

$$mc_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\frac{w_t^{1-\alpha}(r_t)^{\alpha}}{A_t}$$

$$g_t^1 = \lambda_t mc_t y_t^d + \beta\theta_p E_t \left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}}\right)^{-\varepsilon_t} g_{t+1}^1$$

$$g_t^2 = \lambda_t \Pi_t^* y_t^d + \beta\theta_p E_t \left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}}\right)^{1-\varepsilon_t}\left(\frac{\Pi_t^*}{\Pi_{t+1}^*}\right) g_{t+1}^2$$

$$\varepsilon_t g_t^1 = (\varepsilon_t - 1)\, g_t^2$$

$$1 = \theta_p \left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t}\right)^{1-\varepsilon_t} + (1-\theta_p)\,\Pi_t^{*1-\varepsilon_t}$$

- Households

$$d_t (c_t - bc_{t-1})^{-1} - b\beta E_t d_{t+1}(c_{t+1} - bc_t)^{-1} = \lambda_t$$

$$\lambda_t = \beta E_t \lambda_{t+1}\frac{R_t}{\Pi_{t+1}}$$

$$r_t = q_t a'[u_t]$$

$$\lambda_t q_t = \beta E_t\left\{\lambda_{t+1}\left[(1-\delta)\, q_{t+1} + r_{t+1}u_{t+1} - q_{t+1}a(u_{t+1})\right]\right\}$$

$$1 = q_t \xi_t F_{1,t} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} q_{t+1}\xi_{t+1}\frac{1}{\mu_{\xi,t+1}} F_{2,t+1}$$

$$f_t = \frac{\eta-1}{\eta}(w_t^*)^{1-\eta}\lambda_t w_t^{\eta} l_t^d + \beta\theta_w E_t \left(\frac{\Pi_t^{\chi_w}}{\Pi_{t+1}}\right)^{1-\eta}\left(\frac{w_{t+1}^*}{w_t^*}\right)^{\eta-1} f_{t+1}$$

$$f_t = \psi d_t \varphi_t \left(\frac{w_t}{w_t^*}\right)^{\eta(1+\gamma)} \left(l_t^d\right)^{1+\gamma} + \beta\theta_w E_t \left(\frac{\Pi_t^{\chi_w}}{\Pi_{t+1}}\right)^{-\eta(1+\gamma)} \left(\frac{w_{t+1}^*}{w_t^*}\right)^{\eta(1+\gamma)} f_{t+1}$$

$$1 = \theta_w \left(\frac{\Pi_{t-1}^{\chi_w}}{\Pi_t}\right)^{1-\eta} \left(\frac{w_{t-1}}{w_t}\right)^{1-\eta} + (1-\theta_w)\left(\Pi_t^{*w}\right)^{1-\eta}$$

$$\Pi_t^{w^*} = \frac{w_t^*}{w_t}$$

- Government

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left(\left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\Pi} \left(\frac{\frac{y_t^d}{y_{t-1}^d}}{\Lambda_{y^d}}\right)^{\gamma_y}\right)^{1-\gamma_R} \exp(m_t)$$

- Aggregation

$$y_t^d = c_t + \frac{1}{\xi_t} I_t + g_t z_t + q_t a[u_t]\bar{k}_t$$

$$y_t^d = \frac{A_t k_t^\alpha \left(l_t^d\right)^{1-\alpha} - \phi z_t}{v_t^p}$$

$$v_t^p = \theta_p \left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t}\right)^{-\varepsilon_t} v_{t-1}^p + (1-\theta_p)\Pi_t^{*-\varepsilon_t}$$

$$v_t^w = \theta_w \left(\frac{w_{t-1}}{w_t}\frac{\Pi_{t-1}^{\chi_w}}{\Pi_t}\right)^{-\eta} v_{t-1}^w + (1-\theta_w)\left(\Pi_t^{w^*}\right)^{-\eta}$$

$$\bar{k}_{t+1} = (1-\delta)\bar{k}_t + \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t$$

- Exogenous Process

$$A_t = A_{t-1}\exp\left(\Lambda_A + \sigma_A \varepsilon_{A,t}\right)$$

$$\xi_t = \xi_{t-1}\exp\left(\Lambda_\xi + \sigma_\xi \varepsilon_{\xi t}\right)$$

$$\log d_t = \rho_d \log d_{t-1} + \sigma_d \varepsilon_{d,t}$$

$$\log \varphi_t = \rho_\varphi \log \varphi_{t-1} + \sigma_\varphi \varepsilon_{\varphi,t}$$

$$\log \frac{g_t}{g} = \rho_g \log \frac{g_{t-1}}{g} + \sigma_g \varepsilon_{g,t}$$

$$\log\left(\frac{1}{\varepsilon_t - 1}\right) = (1-\rho_p)\log\left(\frac{1}{\varepsilon_t - 1}\right) + \rho_p \log\left(\frac{1}{\varepsilon_t - 1}\right) - \mu_p \varepsilon_{\varepsilon,t-1} + \sigma_\varepsilon \varepsilon_{\varepsilon,t}$$

$$m_t = \sigma_m \varepsilon_{mt}$$

- Definition for growth term

$$z_t = A_t^{\frac{1}{1-\alpha}} \xi_t^{\frac{\alpha}{1-\alpha}}$$

6.6 Stationary Equilibrium Conditions

Preliminaries

- Stationary variables.
  - $\tilde{c}_t = \frac{c_t}{z_t}, \tilde{w}_t = \frac{w_t}{z_t}, \tilde{w}_t^* = \frac{w_t^*}{z_t}, \tilde{\imath}_t = \frac{I_t}{z_t \xi_t}, \tilde{y}_t^d = \frac{y_t^d}{z_t}$
  - $\tilde{r}_t = r_t \xi_t, \tilde{k}_{t+1} = \frac{\bar{k}_{t+1}}{z_t \xi_t}, \tilde{q}_t = q_t \xi_t, \tilde{\lambda}_t = \lambda_t z_t,$

Stationarize I

- Intermediate good producer

$$\frac{u_t \bar{k}_t}{l_t^d z_{t-1} \xi_{t-1}} \frac{z_{t-1} \xi_{t-1}}{z_t \xi_t} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t} \frac{1}{z_t \xi_t}$$

$$mc_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \frac{\left(\frac{w_t}{z_t}\right)^{1-\alpha} (r_t \xi_t)^{\alpha}}{A_t} \frac{z_t^{1-\alpha}}{\xi_t^{\alpha}}$$

$$g_t^1 = \lambda_t z_t mc_t \frac{y_t^d}{z_t} + \beta \theta_p E_t \left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}}\right)^{-\varepsilon_t} g_{t+1}^1$$

$$g_t^2 = \lambda_t z_t \Pi_t^* \frac{y_t^d}{z_t} + \beta \theta_p E_t \left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}}\right)^{1-\varepsilon_t} \left(\frac{\Pi_t^*}{\Pi_{t+1}^*}\right) g_{t+1}^2$$

$$\varepsilon_t g_t^1 = (\varepsilon_t - 1) g_t^2$$

$$1 = \theta_p \left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t}\right)^{1-\varepsilon_t} + (1-\theta_p) \Pi_t^{*1-\varepsilon_t}$$

- Households

$$d_t \left(\frac{c_t}{z_t} - b \frac{c_{t-1}}{z_{t-1}} \frac{z_{t-1}}{z_t}\right)^{-1} - b \beta E_t d_{t+1} \left(\frac{c_{t+1}}{z_{t+1}} \frac{z_{t+1}}{z_t} - b \frac{c_t}{z_t}\right)^{-1} = \lambda_t z_t$$

$$\lambda_t z_t = \beta E_t \lambda_{t+1} z_{t+1} \frac{z_t}{z_{t+1}} \frac{R_t}{\Pi_{t+1}}$$

$$r_t \xi_t = q_t \xi_t a' [u_t]$$

$$\lambda_t z_t q_t \xi_t = \beta E_t \left\{ \lambda_{t+1} z_{t+1} \frac{z_t}{z_{t+1}} \left[\begin{array}{c} (1-\delta) q_{t+1} \xi_{t+1} \frac{\xi_t}{\xi_{t+1}} + r_{t+1} \xi_{t+1} \frac{\xi_t}{\xi_{t+1}} u_{t+1} \\ -q_{t+1} \xi_{t+1} \frac{\xi_t}{\xi_{t+1}} a(u_{t+1}) \end{array} \right] \right\}$$

$$1 = q_t \xi_t F_{1,t} + \beta E_t \frac{\lambda_{t+1} z_{t+1}}{\lambda_t z_t} \frac{z_t}{z_{t+1}} q_{t+1} \xi_{t+1} \frac{1}{\mu_{\xi,t+1}} F_{2,t+1}$$

where

$$F_{1t} = 1 - S\left(\frac{\tilde{\imath}_t}{\tilde{\imath}_{t-1}}\frac{z_t\xi_t}{z_{t-1}\xi_{t-1}}\right) - S'\left(\frac{\tilde{\imath}_t}{\tilde{\imath}_{t-1}}\frac{z_t\xi_t}{z_{t-1}\xi_{t-1}}\right)\frac{\tilde{\imath}_t}{\tilde{\imath}_{t-1}}\frac{z_t\xi_t}{z_{t-1}\xi_{t-1}}$$

$$F_{2t+1} = S'\left(\frac{\tilde{\imath}_{t+1}}{\tilde{\imath}_t}\frac{z_{t+1}\xi_{t+1}}{z_t\xi_t}\right)\left(\frac{\tilde{\imath}_{t+1}}{\tilde{\imath}_t}\frac{z_{t+1}\xi_{t+1}}{z_t\xi_t}\right)^2$$

$$f_t = \frac{\eta-1}{\eta}\left(\frac{w_t^*}{z_t}\right)^{1-\eta}\lambda_t z_t\left(\frac{w_t}{z_t}\right)^{\eta} l_t^d + \beta\theta_w E_t\left(\frac{\Pi_t^{\chi_w}}{\Pi_{t+1}}\right)^{1-\eta}\left(\frac{w_{t+1}^* z_t}{w_t^* z_{t+1}}\frac{z_{t+1}}{z_t}\right)^{\eta-1} f_{t+1}$$

$$f_t = \psi d_t\varphi_t\left(\frac{w_t z_t}{w_t^* z_t}\right)^{\eta(1+\gamma)}\left(l_t^d\right)^{1+\gamma} + \beta\theta_w E_t\left(\frac{\Pi_t^{\chi_w}}{\Pi_{t+1}}\right)^{-\eta(1+\gamma)}\left(\frac{w_{t+1}^* z_t}{w_t^* z_{t+1}}\frac{z_{t+1}}{z_t}\right)^{\eta(1+\gamma)} f_{t+1}$$

$$1 = \theta_w\left(\frac{\Pi_{t-1}^{\chi_w}}{\Pi_t}\right)^{1-\eta}\left(\frac{\frac{w_{t-1}}{z_{t-1}}}{\frac{w_t}{z_t}}\frac{z_{t-1}}{z_t}\right)^{1-\eta} + (1-\theta_w)\left(\Pi_t^{*w}\right)^{1-\eta}$$

$$\Pi_t^{w^*} = \frac{w_t^*}{w_t}$$

- Government

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R}\left(\left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\Pi}\left(\frac{\frac{y_t^d}{y_{t-1}^d}\frac{z_{t-1}}{z_t}\frac{z_t}{z_{t-1}}}{\Lambda_{y^d}}\right)^{\gamma_y}\right)^{1-\gamma_R}\exp\left(m_t\right)$$

- Aggregation

$$\frac{y_t^d}{z_t} = \frac{c_t}{z_t} + \frac{1}{z_t\xi_t}I_t + \frac{g_t z_t}{z_t} + \frac{q_t\xi_t a\left[u_t\right]\bar{k}_t}{z_{t-1}\xi_{t-1}}\frac{z_{t-1}\xi_{t-1}}{z_t\xi_t}$$

$$\frac{y_t^d}{z_t} = \frac{\frac{A_t}{z_t}\left(\frac{k_t}{\xi_{t-1}z_{t-1}}\right)^{\alpha}\left(\frac{\xi_{t-1}z_{t-1}}{\xi_t z_t}\right)^{\alpha}\left(\xi_t z_t\right)^{\alpha}\left(l_t^d\right)^{1-\alpha} - \phi}{v_t^p}$$

$$v_t^p = \theta_p\left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t}\right)^{-\varepsilon_t} v_{t-1}^p + (1-\theta_p)\,\Pi_t^{*-\varepsilon_t}$$

$$v_t^w = \theta_w\left(\frac{w_{t-1}}{w_t}\frac{z_t}{z_{t-1}}\frac{z_{t-1}}{z_t}\frac{\Pi_{t-1}^{\chi_w}}{\Pi_t}\right)^{-\eta} v_{t-1}^w + (1-\theta_w)\left(\Pi_t^{w^*}\right)^{-\eta}$$

$$\frac{\bar{k}_{t+1}}{z_t\xi_t} = (1-\delta)\frac{\bar{k}_t}{z_{t-1}\xi_{t-1}}\frac{z_{t-1}\xi_{t-1}}{z_t\xi_t} + \left(1 - S\left(\frac{\tilde{\imath}_t}{\tilde{\imath}_{t-1}}\frac{z_t\xi_t}{z_{t-1}\xi_{t-1}}\right)\right)\frac{i_t}{z_t\xi_t}$$

- Exogenous Process

$$A_t = A_{t-1}\exp\left(\Lambda_A + \sigma_A\varepsilon_{A,t}\right)$$

$$\xi_t = \xi_{t-1}\exp\left(\Lambda_\xi + \sigma_\xi\varepsilon_{\xi t}\right)$$

$$\log d_t = \rho_d\log d_{t-1} + \sigma_d\varepsilon_{d,t}$$

$$\log\varphi_t = \rho_\varphi\log\varphi_{t-1} + \sigma_\varphi\varepsilon_{\varphi,t}$$

$$\log \frac{g_t}{g} = \rho_g \log \frac{g_{t-1}}{g} + \sigma_g \varepsilon_{g,t}$$

$$\log\left(\frac{1}{\varepsilon_t - 1}\right) = (1-\rho_p)\log\left(\frac{1}{\varepsilon_t - 1}\right) + \rho_p \log\left(\frac{1}{\varepsilon_t - 1}\right) - \mu_p \varepsilon_{\varepsilon,t-1} + \sigma_\varepsilon \varepsilon_{\varepsilon,t}$$

$$m_t = \sigma_m \varepsilon_{mt}$$

- Definition for growth term

$$z_t = A_t^{\frac{1}{1-\alpha}} \xi_t^{\frac{\alpha}{1-\alpha}}$$

Stationarize II

- Intermediate good producer

$$\frac{u_t \tilde{k}_t}{l_t^d} \frac{1}{\mu_{z,t}\mu_{\xi,t}} = \frac{\alpha}{1-\alpha} \frac{\tilde{w}_t}{\tilde{r}_t}$$

$$mc_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} (\tilde{w}_t)^{1-\alpha} (\tilde{r}_t)^{\alpha}$$

$$g_t^1 = \tilde{\lambda}_t mc_t \tilde{y}_t^d + \beta\theta_p E_t \left(\frac{\Pi_t^\chi}{\Pi_{t+1}}\right)^{-\varepsilon_t} g_{t+1}^1$$

$$g_t^2 = \tilde{\lambda}_t \Pi_t^* \tilde{y}_t^d + \beta\theta_p E_t \left(\frac{\Pi_t^\chi}{\Pi_{t+1}}\right)^{1-\varepsilon_t} \left(\frac{\Pi_t^*}{\Pi_{t+1}^*}\right) g_{t+1}^2$$

$$\varepsilon_t g_t^1 = (\varepsilon_t - 1) g_t^2$$

$$1 = \theta_p \left(\frac{\Pi_{t-1}^\chi}{\Pi_t}\right)^{1-\varepsilon_t} + (1-\theta_p) \Pi_t^{*1-\varepsilon_t}$$

- Households

$$d_t \left(\tilde{c}_t - b\tilde{c}_{t-1}\frac{1}{\mu_{z,t}}\right)^{-1} - b\beta E_t d_{t+1} (\tilde{c}_{t+1}\mu_{z,t+1} - b\tilde{c}_t)^{-1} = \tilde{\lambda}_t$$

$$\tilde{\lambda}_t = \beta E_t \tilde{\lambda}_{t+1} \frac{1}{\mu_{z,t+1}} \frac{R_t}{\Pi_{t+1}}$$

$$\tilde{r}_t = \tilde{q}_t a' [u_t]$$

$$\tilde{\lambda}_t \tilde{q}_t = \beta E_t \left\{ \frac{\tilde{\lambda}_{t+1}}{\mu_{\xi,t+1}\mu_{z,t+1}} \begin{bmatrix} (1-\delta)\tilde{q}_{t+1} + \tilde{r}_{t+1} u_{t+1} \\ -\tilde{q}_{t+1} a(u_{t+1}) \end{bmatrix} \right\}$$

$$1 = \tilde{q}_t F_{1,t} + \beta E_t \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t \mu_{z,t+1}} \frac{\tilde{q}_{t+1}}{\mu_{\xi,t+1}} F_{2,t+1}$$

where

$$
\begin{aligned}
F_{1t} &= 1 - S\left(\frac{\tilde{\imath}_t}{\tilde{\imath}_{t-1}}\mu_{z,t}\mu_{\xi,t}\right) - S'\left(\frac{\tilde{\imath}_t}{\tilde{\imath}_{t-1}}\mu_{z,t}\mu_{\xi,t}\right)\frac{\tilde{\imath}_t}{\tilde{\imath}_{t-1}}\mu_{z,t}\mu_{\xi,t} \\
F_{2t+1} &= S'\left(\frac{\tilde{\imath}_{t+1}}{\tilde{\imath}_t}\mu_{z,t+1}\mu_{\xi,t+1}\right)\left(\frac{\tilde{\imath}_{t+1}}{\tilde{\imath}_t}\mu_{z,t+1}\mu_{\xi,t+1}\right)^2
\end{aligned}
$$

$$f_t = \frac{\eta-1}{\eta}\left(\tilde{w}_t^*\right)^{1-\eta}\tilde{\lambda}_t\left(\tilde{w}_t\right)^{\eta} l_t^d + \beta\theta_w E_t\left(\frac{\Pi_t^{\chi_w}}{\Pi_{t+1}}\frac{\tilde{w}_t^*}{\tilde{w}_{t+1}^*}\frac{1}{\mu_{z,t+1}}\right)^{1-\eta} f_{t+1}$$

$$f_t = \psi d_t \varphi_t\left(\frac{\tilde{w}_t}{\tilde{w}_t^*}\right)^{\eta(1+\gamma)}\left(l_t^d\right)^{1+\gamma} + \beta\theta_w E_t\left(\frac{\Pi_t^{\chi_w}}{\Pi_{t+1}}\frac{\tilde{w}_t^*}{\tilde{w}_{t+1}^*}\frac{1}{\mu_{z,t+1}}\right)^{-\eta(1+\gamma)} f_{t+1}$$

$$1 = \theta_w\left(\frac{\Pi_{t-1}^{\chi_w}}{\Pi_t}\right)^{1-\eta}\left(\frac{\tilde{w}_{t-1}}{\tilde{w}_t}\frac{1}{\mu_{z,t}}\right)^{1-\eta} + \left(1-\theta_w\right)\left(\Pi_t^{*w}\right)^{1-\eta}$$

$$\Pi_t^{w^*} = \frac{\tilde{w}_t^*}{\tilde{w}_t}$$

- Government

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R}\left(\left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\Pi}\left(\frac{\frac{\tilde{y}_t^d}{\tilde{y}_{t-1}^d}\mu_{z,t}}{\Lambda_{y^d}}\right)^{\gamma_y}\right)^{1-\gamma_R}\exp\left(m_t\right)$$

- Aggregation

$$\tilde{y}_t^d = \tilde{c}_t + \tilde{\imath}_t + g_t + \frac{\tilde{q}_t a\left[u_t\right]\tilde{k}_t}{\mu_{z,t}\mu_{\xi,t}}$$

$$\tilde{y}_t^d = \frac{\frac{\mu_{A,t}}{\mu_{z,t}}\left(u_t\tilde{k}_t\right)^{\alpha}\left(l_t^d\right)^{1-\alpha} - \phi}{v_t^p}$$

$$v_t^p = \theta_p\left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t}\right)^{-\varepsilon_t} v_{t-1}^p + \left(1-\theta_p\right)\Pi_t^{*-\varepsilon_t}$$

$$v_t^w = \theta_w\left(\frac{\tilde{w}_{t-1}}{\tilde{w}_t\mu_{z,t}}\frac{\Pi_{t-1}^{\chi_w}}{\Pi_t}\right)^{-\eta} v_{t-1}^w + \left(1-\theta_w\right)\left(\Pi_t^{w^*}\right)^{-\eta}$$

$$\tilde{k}_{t+1} = (1-\delta)\frac{\tilde{k}_t}{\mu_{z,t}\mu_{\xi,t}} + \left(1 - S\left(\frac{\tilde{\imath}_t}{\tilde{\imath}_{t-1}}\mu_{z,t}\mu_{\xi,t}\right)\right)\tilde{\imath}_t$$

- Exogenous Process

$$\log\mu_{A,t} = \Lambda_A + \sigma_A\varepsilon_{A,t}$$

$$\log\mu_{\xi,t} = \Lambda_\xi + \sigma_A\varepsilon_{\xi,t}$$

$$\log d_t = \rho_d\log d_{t-1} + \sigma_d\varepsilon_{d,t}$$

$$\log\varphi_t = \rho_\varphi\log\varphi_{t-1} + \sigma_\varphi\varepsilon_{\varphi,t}$$

$$\log \frac{g_t}{g} = \rho_g \log \frac{g_{t-1}}{g} + \sigma_g \varepsilon_{g,t}$$

$$\log\left(\frac{1}{\varepsilon_t - 1}\right) = (1-\rho_p)\log\left(\frac{1}{\varepsilon_t - 1}\right) + \rho_p \log\left(\frac{1}{\varepsilon_t - 1}\right) - \mu_p \varepsilon_{\varepsilon,t-1} + \sigma_\varepsilon \varepsilon_{\varepsilon,t}$$

$$m_t = \sigma_m \varepsilon_{mt}$$

- Definition for growth term

$$\log \frac{\mu_{z,t}}{\exp(\Lambda_z)} = \frac{1}{1-\alpha}\log\frac{\mu_{A,t}}{\exp(\Lambda_A)} + \frac{\alpha}{1-\alpha}\log\frac{\mu_{\xi,t}}{\exp(\Lambda_\xi)}$$

6.7 Steady State

Equilibrium Conditions Let $\mu^z \equiv \Lambda_z = \Lambda_A^{\frac{1}{1-\alpha}} \Lambda_k^{\frac{\alpha}{1-\alpha}}$ where $\mu^A \equiv \Lambda_A$ and $\mu^k \equiv \Lambda_k$. Given the definitions, the mean growth rate of the economy is $\Lambda_c = \Lambda_x = \Lambda_w = \Lambda_{w^*} = \Lambda_{y^d} = \Lambda_z$. $\tilde{u} = 1$ at steady state.

- Households satisfy

$$\left(\tilde{c} - \frac{b\tilde{c}}{\Lambda_z}\right)^{-1} - b\beta\left(\tilde{c}\Lambda_z - b\tilde{c}\right)^{-1} = \tilde{\lambda}$$

$$1 = \frac{\beta}{\Lambda_z \Lambda_k}\left(\tilde{r} + 1 - \delta\right)$$

$$\beta \frac{1}{\Lambda_z}\frac{R}{\Pi} = 1$$

$$\tilde{r} = \gamma_1$$

$$1 = \tilde{q}$$

$$f = \frac{\eta - 1}{\eta}\left(\tilde{w}^*\right)^{1-\eta}\left(\Pi^{*w}\right)^{-\eta}\tilde{\lambda}\left(\tilde{w}^*\right)^{\eta} l^d + \beta\theta_w\left(\frac{\Pi^{\chi_w}}{\Pi\Lambda_z}\right)^{1-\eta} f$$

$$f = \psi\left(\Pi^{*w}\right)^{-\eta(1+\gamma)}\left(l^d\right)^{1+\gamma} + \beta\theta_w\left(\frac{\Pi^{\chi_w}}{\Pi\Lambda_z}\right)^{-\eta(1+\gamma)} f$$

- Firms that can change prices set them to satisfy (4 eqs)

$$g^1 = \tilde{\lambda} mc \tilde{y}^d + \beta\theta_p\left(\frac{\Pi^\chi}{\Pi}\right)^{-\varepsilon} g^1$$

$$g^2 = \tilde{\lambda}\Pi^* \tilde{y}^d + \beta\theta_p\left(\frac{\Pi^\chi}{\Pi}\right)^{1-\varepsilon} g^2$$

$$\varepsilon g^1 = (\varepsilon - 1) g^2$$

where

$$\Pi^{*w} = \frac{\tilde{w}^*}{\tilde{w}}$$

- They rent inputs to satisfy their static minimization problem(2 eqs)

$$\frac{\tilde{k}}{l^d} = \frac{\alpha}{\alpha - 1} \frac{\tilde{w}}{\tilde{r}} \Lambda_z \Lambda_k$$
$$mc = \left(\frac{1}{1-\alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^{\alpha} (\tilde{w})^{1-\alpha} \tilde{r}^{\alpha}$$

- The wages evolve

$$1 = \theta_w \left(\Pi^{\chi_w - 1} \right)^{1-\eta} (\Lambda_z)^{-1+\eta} + (1-\theta_w) (\Pi^{*w})^{1-\eta}$$

- The price level evolve

$$1 = \theta_p \left(\Pi^{\chi - 1} \right)^{1-\varepsilon} + (1-\theta_p) \Pi^{*1-\varepsilon}$$

- Markets clear

$$\tilde{y}^d = \tilde{c} + \tilde{x} + g$$
$$v^p \tilde{y}^d = \frac{\Lambda_A}{\Lambda_z} \tilde{k}^{\alpha} \left(l^d \right)^{1-\alpha} - \phi$$

where

$$\begin{aligned} v^p &= \theta_p \left(\Pi^{\chi-1} \right)^{-\varepsilon} v^p + (1-\theta_p) \Pi^{*-\varepsilon} \\ v^w &= \theta_w \left(\frac{\Pi^{\chi_w - 1}}{\Lambda_z} \right)^{-\eta} v^w + (1-\theta_w) (\Pi^{*w})^{-\eta} \end{aligned}$$

and

$$\tilde{k} = (1-\delta) \frac{\tilde{k}}{\Lambda_z \Lambda_\xi} + \tilde{\imath}$$

- Exogeous processes evolve (6 eqs)

$$\begin{aligned} d &= 1 \\ \varphi &= 1 \\ \mu^k &= \Lambda_k \\ \mu^A &= \Lambda_A \\ m &= 0 \end{aligned}$$

where

$$\Lambda_z = \Lambda_A^{\frac{1}{1-\alpha}} \Lambda_k^{\frac{\alpha}{1-\alpha}}$$

Steady State computation

- Fixed Parameters

Table 1 : Fixed Parameters

β	δ	ϕ	α	γ_2	$\frac{g}{y}$
0.9964	0.025	0	0.3	0.001	0.1963

- Estimated parameters :

$$\{b, \gamma, \psi, \kappa, \theta_p, \chi_p, \theta_w, \chi_w, \Lambda_k, \Lambda_A, \gamma_R, \gamma_y, \gamma_\Pi, \Pi, \rho_d, \rho_\varphi, \rho_g, \sigma_K, \sigma_a, \sigma_g, \sigma_d, \sigma_\varphi, \sigma_M\}$$

- Free parameters : set $u = 1$ so that $\gamma_1 = \tilde{r}$.
- Parameters related to exogenous processes : $d = 1, \varphi = 1$, $m = 0$.
- Growth terms

$$\Lambda_z = \Lambda_A^{\frac{1}{1-\alpha}} \Lambda_k^{\frac{\alpha}{1-\alpha}}$$
$$\Lambda_y = \Lambda_z$$
$$\Lambda_x = \Lambda_z \Lambda_k$$

- Interest rate

$$\tilde{r} = \frac{\Lambda_z \Lambda_k}{\beta} - 1 + \delta$$
$$\gamma_1 = \tilde{r}$$
$$R = \frac{\Pi \Lambda_z}{\beta}$$

- Prices

$$\Pi^* = \left(\frac{1 - \theta_p \Pi^{-(1-\varepsilon)(1-\chi)}}{1 - \theta_p} \right)^{\frac{1}{1-\varepsilon}}$$
$$mc = \frac{\varepsilon - 1}{\varepsilon} \frac{1 - \beta\theta_p \Pi^{(1-\chi)\varepsilon}}{1 - \beta\theta_p \Pi^{-(1-\chi)(1-\varepsilon)}} \Pi^*$$

$$v^p = \frac{1 - \theta_p}{1 - \theta_p \Pi^{(1-\chi)\varepsilon}} \Pi^{*-\varepsilon}$$

- Wages

$$\Pi^{*w} = \left(\frac{1 - \theta_w \left(\Pi\right)^{(1-\eta)(\chi_w - 1)} \left(\Lambda_z\right)^{-1+\eta}}{1 - \theta_w} \right)^{\frac{1}{1-\eta}}$$

$$\tilde{w} = (1-\alpha) \left(mc \left(\frac{\alpha}{\tilde{r}} \right)^{\alpha} \right)^{\frac{1}{1-\alpha}}$$

$$\tilde{w}^* = \tilde{w} \Pi^{w^*}$$

$$v^w = \frac{1 - \theta_w}{1 - \theta_w \Pi^{(1-\chi_w)\eta} \left(\Lambda_z\right)^{\eta}} \left(\Pi^{*w}\right)^{-\eta}$$

- Capital/labor ratio

$$\frac{\tilde{k}}{l^d} = \frac{\alpha}{\alpha - 1} \frac{\tilde{w}}{\tilde{r}} \Lambda_z \Lambda_k$$

- Assuming $\phi = 0$ or ϕ satisfying the zero profits at steady state

$$\frac{\tilde{y}^d}{l^d} = \frac{\frac{\Lambda_A}{\Lambda_z} \left(\frac{\tilde{k}}{l^d} \right)^{\alpha} - \phi}{v^p}$$

$$\frac{\tilde{k}}{l^d} \left(1 - \frac{1-\delta}{\Lambda_z \Lambda_\xi} \right) = \frac{\tilde{\imath}}{l^d}$$

$$\frac{\tilde{\imath}}{l^d} = \frac{\tilde{k}}{l^d} \left(\frac{\Lambda_z \Lambda_\xi - 1 + \delta}{\Lambda_z \Lambda_\xi} \right)$$

$$\frac{\tilde{c}}{l^d} = \frac{\tilde{y}^d}{l^d} - \frac{\tilde{\imath}}{l^d} - \frac{g}{y}$$

- Labor demand

$$\begin{aligned}
\tilde{\lambda} l^d &= \left(\frac{\tilde{c}}{l^d} - \frac{b\tilde{c}}{\Lambda_z l^d} \right)^{-1} - b\beta \left(\frac{\tilde{c}}{l^d} \Lambda_z - b \frac{\tilde{c}}{l^d} \right)^{-1} \\
&= \left(\frac{\tilde{c}}{l^d} \right)^{-1} \left(1 - \frac{b}{\Lambda_z} \beta \right) \left(1 - \frac{b}{\Lambda_z} \right)^{-1} \\
&= \left(\frac{\tilde{c}}{l^d} \right)^{-1} \frac{(\Lambda_z - b\beta)}{(\Lambda_z - b)}
\end{aligned}$$

$$f = \frac{\frac{\eta - 1}{\eta} \tilde{w}^* \left(\Pi^{*w}\right)^{-\eta} \tilde{\lambda} l^d}{\left(1 - \beta \theta_w \left(\frac{\Pi^{\chi_w}}{\Pi \Lambda_z} \right)^{1-\eta} \right)}$$

$$l^d = \left[\frac{f \left(1 - \beta \theta_w \left(\frac{\Pi^{\chi_w}}{\Pi \Lambda_z} \right)^{-\eta(1+\gamma)} \right)}{\psi \left(\Pi^{*w}\right)^{-\eta(1+\gamma)}} \right]^{\frac{1}{1+\gamma}}$$