

Bargaining with Existing Workers, Over-hiring of Firms, and Labor Market Fluctuations

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Abstract

This paper investigates the effects of intra-firm bargaining on the standard real business cycle (RBC) search and matching model by explicitly considering the outside option of a firm in the bargaining with a new worker. In this paper, the outside option of a firm in the bargaining with a new worker is bargaining with existing workers (i.e., intra-firm bargaining). According to Stole and Zwiebel (1996), under the assumption that the firm facing diminishing marginal productivity of labor and its workers cannot commit to future wages, intra-firm bargaining gives the firm an incentive to hire an excessive amount of workers in order to drive wages of workers down. (i.e., the firm “over-hires” workers.) We show the outside option of a firm is crucial for the bargaining with a new worker in the sense that how much the firm pays existing workers, when the match breaks down, determines how many workers it over-hires. In this paper, wages in the outside option of a firm depend on the stochastic bargaining weight of existing workers (i.e., a bargaining shock), which can be identified through labor share data from U.S. Bargaining shocks affect the degree of over-hiring behavior of a firm, and in turn, provide another source to explain the behavior of labor markets in business cycles. The inclusion of intra-firm bargaining and bargaining shocks improves the capacity of the standard RBC search and matching model, especially in labor markets. Our calibrated model generates more volatile total hours, employment, hours per worker while labor share overshoots in response to productivity shocks. In particular, the volatility of employment in the model is similar to the actual U.S. data. Furthermore, the model provides a theory about the overshooting property of labor share, which can be explained by the general equilibrium effects of time-varying bargaining weights of existing workers and over-hiring behavior of the firm.

Keywords: Business Cycles, Intra-firm Bargaining, Over-hiring, Bargaining Shocks

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1 Introduction

This paper investigates the effects of intra-firm bargaining on the standard real business cycle (RBC) search and matching model by explicitly considering the outside option of a firm in the bargaining with a new worker. In this paper, the outside option of a firm in the bargaining with a new worker is bargaining with existing workers (i.e., intra-firm bargaining).¹ According to Stole and Zwiebel (1996), under the assumption that the firm facing diminishing marginal productivity of labor and its workers cannot commit to future wages, intra-firm bargaining gives the firm an incentive to hire an excessive amount of workers in order to drive wages of workers down. (i.e., the firm “over-hires” workers.) We show the outside option of a firm is crucial for the bargaining with a new worker in the sense that how much the firm pays existing workers, when the match breaks down, determines how many workers it over-hires. In this paper, wages in the outside option of a firm depend on the stochastic bargaining weight of existing workers (i.e., a bargaining shock), which can be identified through labor share data from U.S. Bargaining shocks affect the degree of over-hiring behavior of a firm, and in turn, provide another source to explain the behavior of labor markets in business cycles. The inclusion of intra-firm bargaining and bargaining shocks improves the capacity of the standard RBC search and matching model, especially in labor markets. Our calibrated model generates more volatile total hours, employment, hours per worker while labor share overshoots in response to productivity shocks. In particular, the volatility of employment in the model is similar to the actual U.S. data. Furthermore, the model provides a theory about the overshooting property of labor share, which can be explained by the general equilibrium effects of time-varying bargaining weights of existing workers and over-hiring behavior of the firm.

Stole and Zwiebel (1996) reveal that, under the assumption that the firm facing diminishing marginal productivity of labor and its workers cannot commit to future wages, intra-firm bargaining leads to no rent for employees by the over-hiring behavior of a firm in partial equilibrium. The key idea of Stole and Zwiebel (1996) is that firms have an incentive to over-hire workers. By hiring more workers, the firm is able to decrease the marginal productivity of labor of existing workers, which reduces the wages of existing workers in partial equilibrium. With this strategic incentive, the firm over-hires workers up to the point where workers their receive reservation wages or outside options. In this regard, Krause and Lubik (2007) incorporate intra-firm bargaining into a simple RBC search

¹In this paper, we will use intra-firm bargaining and the outside option of a firm in the bargaining with a new worker interchangeably.

and matching model without capital and intensive margins. They show that the aggregate effects of intra-firm bargaining are negligible because of general equilibrium effects. Over-hiring behavior of the firm leads to an increase in outside options of workers because of an increase in vacancies posted, and in turn, results in an increase in equilibrium wages in general equilibrium.

In this paper, the result of the bargaining with a new worker crucially depends on the outside options of a firm, which have not been examined thoroughly in the literature. More specifically, wages in the outside option of a firm determine the degree and consequences of over-hiring. If the match with a new worker breaks down, the outside option of a firm is produced with existing workers. Wages in the outside option of a firm would be paid to existing workers when the match with a new worker is not successful. The important thing is how much the firm pays existing workers when the match breaks down. The natural way to pin down these wages is by bargaining with existing workers. Depending on the bargaining weight of existing workers, these wages as well as the degree of over-hiring vary. In this paper, we assume the bargaining weight of existing workers varies stochastically, which can be identified through labor share data from U.S. By incorporating intra-firm bargaining and bargaining shocks into a standard RBC search and matching model with capital and intensive margins, our model generates higher volatility of total hours, employment, and hours per worker. We include intensive margins for two reasons. First, labor share is important for identifying bargaining shocks, and for the labor share in the model to be consistent with actual data, we need to include intensive margins. Second, bargaining shocks directly affect intensive margins because changing the bargaining weight of existing workers affects the relative usefulness of intensive and extensive margins for a firm.

This paper is related to several studies which can be classified into three groups. First, our model is based on Andolfatto (1996). His model embeds search and matching framework into an otherwise standard RBC model, and has both extensive margins and intensive margins. By incorporating search and matching framework in labor markets, the model improves the standard RBC model along several dimensions. However, the volatility of labor market variables is still far lower than that of actual data. The Andolfatto model also has highly pro-cyclical real wages and labor productivity, which have weakly pro-cyclical counterparts in actual data. Several papers have addressed these problems. Nakajima (2012) analyzes several volatility problems by explicitly distinguishing between leisure and unemployment benefits for the outside options of households. This distinction is consistent with the calibration proposed by Hagendon and Manovskii (2008). However, the main focus of Nakajima (2012) is unemployment and vacancies than employment and hours per worker, which are our main interest.

Cheron and Langot (2004) address the second failure of Andolfatto (1996) by using non-separable preference between consumptions and leisure such that the outside options of households can move counter-cyclically. This proposal results in less pro-cyclical real wages and labor productivity. However, this paper is not interested in the volatility of labor market variables in general.

The second branch of papers related to our study is literature on intra-firm bargaining and its applications to business cycle dynamics.² Stole and Zwiebel (1996) claim that if production technology exhibits diminishing marginal productivity of labor, the firm has an incentive to over-hire workers strategically, and in turn, extracts all the rent existing workers have. As a result of over-hiring, wages decrease in partial equilibrium. Krause and Lubik (2007) (KL, hereafter) incorporate intra-firm bargaining into a business cycle model with search frictions in labor markets to evaluate the quantitative effects of intra-firm bargaining on business cycle dynamics. They find that the aggregate effect of intra-firm bargaining is negligible in business cycle dynamics considering the general equilibrium effects as discussed before. Table 1 summarizes business cycle moments for the modified KL model³ The performance of KL model is almost the same as the Andolfatto model, and both models perform poorly in replicating moments along several dimensions. In contrast to KL, this paper explicitly considers the bargaining weight of existing workers when the match with a new worker fails. In addition, by assuming the bargaining weight of existing workers varies stochastically, the time-varying over-hiring behavior of the firm provides a new margin to increase the volatility of labor market variables. Later, it turns out that Andolfatto and KL are two extreme cases where bargaining weights of existing workers are fixed at 1, and 0, respectively, in the baseline model.

This paper is also related to papers studying labor share. Recently, Rios-Rull and Santaella-Llopis (2010) document several properties of labor share dynamics based on U.S. data. In particular, they propose a redistributive shock that can be identified by using labor share data in U.S. and point out the importance of the dynamic property of labor share (overshooting). They showed that labor share overshoots in response to productivity shocks, and the dynamic overshooting response of labor share drastically dampens the role of productivity shocks on labor markets due to huge wealth effects. Our model also generates an overshooting property of labor share, but total hours, employment, and hours per worker are still more volatile than the Andolfatto model. Different from Rios-Rull

²Acemoglu and Hawkins (2006), Elsby and Michaels (2008), Hawkins (2011), and Fujita and Nakajima (2009) also study the labor market fluctuations with intra-firm bargaining, but the main focus of these papers is unemployment and vacancies than employment and hours per worker.

³We add intra-firm bargaining to the Andolfatto model rather than to the KL model to assure fair comparison of the two models.

<i>Variable</i> (x)	$\sigma_x \% \left(\frac{\sigma_x}{\sigma_{Output}} \right)$			$\rho(x, Output)$			$\rho(x_t, x_{t-1})$		
	Data	Andolfatto	KL	Data	Andolfatto	KL	Data	Andolfatto	KL
Output	1.54 (1.00)	1.30 (1.00)	1.31 (1.00)	-	-	-	0.86	0.82	0.82
Total Hours	1.38 (0.90)	0.70 (0.54)	0.70 (0.53)	0.85	0.92	0.92	0.88	0.91	0.91
Employment	1.00 (0.65)	0.68 (0.52)	0.69 (0.53)	0.81	0.78	0.78	0.91	0.89	0.88
Hours per Worker	0.49 (0.32)	0.19 (0.15)	0.19 (0.15)	0.74	0.55	0.54	0.56	0.58	0.58
Wages	0.91 (0.59)	0.62 (0.48)	0.64 (0.49)	0.34	0.94	0.94	0.69	0.65	0.64
Labor Productivity	0.82 (0.53)	0.72 (0.55)	0.72 (0.55)	0.45	0.92	0.92	0.57	0.62	0.62
Labor Share	0.74 (0.48)	0.10 (0.08)	0.10 (0.08)	-0.08	-0.72	-0.71	0.78	0.51	0.50
Vacancies	13.23 (8.59)	3.65 (2.81)	3.69 (2.82)	0.90	0.80	0.80	0.91	0.54	0.54

1) All data are in logs and filtered using the HP filter with a smoothing parameter of 1600.

Table 1: Business Cycle Moments in Data and Models over 1960:Q1-2012:Q1

and Santaaulalia-Llopis (2010), the search and matching framework weakens wealth effects from the overshooting of labor share, and over-hiring behavior of the firm offsets the huge reduction of total hours.

The main contribution of this paper is as follows. First, we incorporating intra-firm bargaining into the Andolfatto model, which has both extensive and intensive margins. To the best of our knowledge, we first study the effect of the bargaining weight of existing workers on intra-firm bargaining, which is the outside option of a firm in the bargaining with a new worker, and assume the bargaining weight of existing workers varies stochastically. The bargaining weight of existing workers can be time-varying because when labor markets are tighter, mostly in booms, existing workers are more valuable as the firm will have difficulty finding new workers. However, when the labor market is less tight, mostly in recessions, existing workers become less attractive to firms, which could easily find new workers. This reason makes the bargaining weight of existing workers possibly pro-cyclical. Another possible explanation could be related to the entry and exit of firms. In booms, several firms compete with a specific firm because of higher entry rates of new firms and lower exit rates of existing firms. These situations reduce the monopolistic or bargaining power of the firm over existing workers. However, during recessions, the opposite happens. Given this explanation, the bargaining weight of existing workers moves pro-cyclically based on the pro-cyclical entry and the counter-cyclical exit rates. In this regard, by introducing stochastic bargaining weights of existing workers (or bargaining shocks), we show that the over-hiring behavior of a firm is an important factor in the behavior of labor markets over business cycles, especially in the volatility of total hours, employment, hours per worker, and labor share.

Second, we identify bargaining shocks by using labor share data. We provide theories about how

the bargaining weights of workers are related to the movement of labor share in U.S. Also, our model generates an overshooting property of labor share, but the effect of productivity shocks are still significant on labor markets in contrast to the prediction of Rios-Rull and Santaella-Llopis (2010) in which the effect of productivity shocks is dampened when labor share overshoots because of huge wealth effects from the overshooting property. In contrast to their model, the baseline model has a search and matching framework, and the nature of this framework weakens wealth effects resulting from the overshooting of labor share. On top of these differences, the over-hiring behavior of a firm offsets the huge reduction of total hours in booms.

The remainder of the paper is structured as follows. Section II introduces the baseline model with intra-firm bargaining and bargaining shocks. Section III discusses the calibration of the baseline model. Section IV shows quantitative analysis of the model. Section V discusses the robustness of the baseline model. Finally, Section VI concludes and proposes the further research.

2 Model

We develop a model based on a standard RBC search and matching model (the Andolfatto model). The main difference between this paper and the Andolfatto model is the outside option of a firm in the bargaining with a new worker. We explicitly considers the outside option of a firm when the firm bargains with a new worker. The outside option of a firm is bargaining with existing workers (i.e., intra-firm bargaining) and producing with them. The issue with bargaining with existing workers is the wages the firm pays. In this paper, these wages depend on the bargaining weight of existing workers. If the bargaining weight of existing workers is high, then existing workers will receive higher wages and if the bargaining weight of existing workers is low, then they will receive lower wages. Note that these wages are not realized if the match with a new worker is successful, but they still affect the equilibrium wages. In this paper, matches are always successful because the surplus of a new match is always positive. Therefore, wages bargained with existing workers would not be realized in equilibrium. Furthermore, we assume the bargaining weight of existing workers stochastically evolves. Except for intra-firm bargaining and bargaining shocks, the baseline model is similar to Andolfatto (1996), Cheron and Langot (2004), and Choi and Rios-Rull (2009).

2.1 Matching

We assume that the period in the model is a quarter. The timing of our model is as follows: (1) shocks are realized, (2) wages and hours per worker are bargained over with new workers, (3) if matches are not successful, the firm bargains wages with existing workers (4) workers are matched with the firm (5) production takes place and the firm post vacancies, (6) separations occur.

In this paper, labor markets are frictional, so the unemployed search for jobs and firms post vacancies to hire workers. The number of matches is determined by constant returns to scale matching function $M = M(V, 1 - N)$, which depends on the total number of vacancies, V , and the total number of the unemployed, $U \equiv 1 - N$. For later use, we define $\theta = V/(1 - N)$ as market tightness in labor markets. Also, we define the job-finding rate $\Psi(\theta) \equiv M/(1 - N) = M(\theta, 1)$ and the job-filling rate $\Phi(\theta) \equiv M/V = M(1, 1/\theta)$. Finally, we assume that workers are separated at the exogenous and constant rate $\chi \in (0, 1)$. Therefore, we have the following law of motion of total employment.

$$N' = (1 - \chi)N + M(V, 1 - N)$$

2.2 Household

There is a continuum of identical and infinitely lived households of measure one. The measure of members in each household is also normalized to 1. The aggregate states in this economy are given by $S = \{z, \nu; K, N\}$, where z is the aggregate productivity and ν is the bargaining weight of existing workers, which varies stochastically. K is the aggregate capital stock, N is total employment. The individual state variables of the household are $s_H = \{a, n\}$, where a is the amount of assets they hold and n is the measure of the employed in household. We can write the household problem as follows:

$$\begin{aligned} \Omega(S, s_H) &= \max_{c, a'} u(c) + nv(1 - h(S, s_H)) + (1 - n)v(1) + \beta E[\Omega(S', s'_H)] & (1) \\ s.t. & \\ c + a' + T(S) &= w(S, s_H)h(S, s_H)n + (1 - n)b + (1 + r(S))a + \Pi(S) \\ n' &= (1 - \chi)n + \Psi(S)(1 - n) \\ S' &= G(S) \\ c &\geq 0 \end{aligned}$$

where $u(c)$ is utility from consumption, a is the assets household holds, $v(\cdot)$ is utility from leisure, $T(S)$ is the lump-sum tax, $\Pi(S) = F(z, k, nh) - w(S, s_F)h(S, s_F)n - (r(S) + \delta)k - \kappa v$ is the dividend which will be defined in the firm's problem. $\Psi(S) = M/(1 - N)$ is the job-finding rate and $G(S)$ is the law of motion of aggregate state variables. Household takes wages $w(S, s_H)$ and hours per worker $h(S, s_H)$ as given. They are jointly determined via Nash bargaining.

The household consumes (c), accumulate assets (a) which they rent to a firm, and supplies labor. The n fraction of members in each household is matched with the firm and employed. And the $1 - n$ fraction of members is unemployed, searches for jobs, and they collect unemployment benefits (b) from the government. We assume that there is no search cost, and so every member who is not employed searches for the job.⁴ We also assume that there is a perfect insurance for unemployment within the household as noted in Andolfatto (1996).⁵ As a result, every member receives the same consumption level. Note that this implies unemployed members are better off than those that are employed since they receive the same consumption level but the unemployed enjoy a full amount of leisure.⁶

The first order conditions of household's problem give⁷

$$E \left[\beta \frac{u'_c}{u_c} (1 + r') \right] = 1 \quad (2)$$

This is a standard Euler equation for the household.

2.3 Firm

There exists a representative firm. The firm produces goods using a constant returns to scale production technology $F(z, k, nh)$, where z is the aggregate productivity. Given the aggregate state S , and the individual state variable of the firm $s_F = \{n\}$, we can write firm's recursive problem as

⁴In this sense, u in our model is the non-employed. We don't distinguish between the unemployed and the non-employed like Andolfatto (1996). Since the measure of the unemployment rate in model and data are inconsistent, we do not report any statistics regarding unemployment in this paper.

⁵Separable utility functions over consumption and leisure satisfy this assumption.

⁶We can relax this assumption. As noted in Cheron and Langot (2004), Nakajima (2012), if we use non-separable utility functions over consumption and leisure, the employed receive higher levels of consumption than the unemployed. Consequently, the employed are better off in equilibrium. If we use non-separable utility functions, the performance of the model would be better, especially for labor productivity and real wages. However, we don't use these utility functions because we prefer to setup the baseline model in a more parsimonious way.

⁷We will drop state variables for simple notations.

follows:

$$\begin{aligned}
J(S, s_F) &= \max_{v, k} \Pi(S) + E[m'(S, S') J(S', s'_F)] \\
&= \max_{v, k} F(z, k, nh) - w(S, s_F) h(S, s_F) n - (r + \delta) k - \kappa v + E[m'(S, S') J(S', s'_F)] \\
s.t. \\
n' &= (1 - \chi) n + \Phi(S) v \\
S' &= G(S)
\end{aligned} \tag{3}$$

where $m'(S, S') = \beta u_c(c(S')) / u_c(c(S))$ is the stochastic discount factor, κ is the cost of posting vacancies, and $\Phi(S) = M/V$ is the job-filling rate. Again, $G(S)$ is the law of motion of aggregate state variables. Firms hire the labor and rent the capital from the households, and post vacancies to hire more workers in the next period. Firms also take wages $w(S, s_F)$ and hours per worker $h(S, s_F)$ as given. They are jointly determined via Nash bargaining. From the first order conditions, we have two equilibrium conditions.

$$r = F_k - \delta \tag{4}$$

$$\kappa = \Phi E[m' J'_n] \tag{5}$$

The first condition is an equation for the equilibrium rental rate. The second equation is a job creation condition, which implies firms post vacancies up to the point where the marginal cost of posting vacancies equals to the value of an additional worker discounted by the probability firms meet a new worker.

2.4 The Bargaining with a New Worker

As stated before, wages, w , and hours per workers, h , are jointly determined via Nash bargaining between the worker and the firm each period. Formally, Nash bargaining problem can be written as follows:

$$(w, h) = \arg \max_{w, h} \left(\frac{\Omega_n}{u_c} \right)^\mu (J_n)^{1-\mu} \tag{6}$$

where μ is the bargaining weight of a new worker, V_n/u_c is the value of an additional worker for the household in terms of consumption units, and J_n is the value of an additional worker for the firm. The

only difference between the bargaining problem in this paper and the standard Nash bargaining is the outside option of the firm, which is defined in the value of an additional worker for the firm, J_n .

2.4.1 The Value of an Additional Worker for the Household

From the value function of the household, we have

$$\frac{\Omega_n}{u_c} = \frac{v(1-h) - v(1)}{u_c} + wh - b + (1 - \chi - \Psi) E \left[\beta \frac{\Omega'_n}{u_c} \right] \quad (7)$$

We can rewrite the value of an additional worker for the household by manipulating several terms.

$$\frac{\Omega_n}{u_c} = \left[\frac{u(c)}{u_c} + wh + \frac{v(1-h)}{u_c} + (1 - \chi) E \left[\beta \frac{\Omega'_n}{u_c} \right] \right] - \left[\frac{u(c)}{u_c} + b + \frac{v(1)}{u_c} + \Psi E \left[\beta \frac{\Omega'_n}{u_c} \right] \right] \quad (8)$$

Note that the first bracket is the value of working which includes utility from consumption, the wage bill, utility from leisure, and the continuation value of the worker given the worker is not separated at the end of the period. The second bracket is the outside option of the worker which consists of utility from consumption, unemployment benefits, utility from leisure, and the value of being matched with another job next period.

2.4.2 The Value of an Additional Worker for the Firm

The value of an additional worker for the firm is not trivial because the outside option of the firm can be defined in various ways. The outside option of the firm in the bargaining with a new worker is bargaining wages with existing workers (i.e., intra-firm bargaining) and producing with them. Let w^{ew} be the wages bargained over between the firm and existing workers when the match with a new worker breaks down⁸. Then, we can define the value of Δ additional workers for the firm as follows:

$$\begin{aligned} J_{+\Delta} - J &\equiv \left[F(z, k, (n + \Delta)h) - w_{+\Delta} (n + \Delta)h + (1 - \chi) E \left[\beta \frac{u'_c}{u_c} J'_{+\Delta} \right] \right] \\ &\quad - \left[F(z, k, nh) - w^{ew} nh + (1 - \chi) E \left[\beta \frac{u'_c}{u_c} J' \right] \right] \end{aligned} \quad (9)$$

⁸As we mention before, these wages, w^{ew} , would not be realized in equilibrium. These wages show up in the outside option of the firm, but the match with a new worker is always successful because the match surplus is always positive in this paper. Consequently, these wages are not realized in equilibrium, but they still affect equilibrium wages.

where $J_{+\Delta}$ and $w_{+\Delta}$ are the value of the firm and wages when the firm hires Δ more workers. The first bracket is the value of the firm when the firm hires Δ more workers, which includes the level of output less wage bills with workers including newly hired workers, and the continuation value of the firm given workers are not separated at the end of the period. The second bracket is the outside option of the firm, which consists of the level of output less wage bills with existing workers, and the continuation value of the firm given workers are not separated at the end of the period.

Proposition 1. *If $w^{ew} = w_{+\Delta}$, then the value of an additional worker for the firm reduces to*

$$J_n \equiv \frac{\partial J}{\partial n} = \frac{\partial F(z, k, nh)}{\partial n} - wh + (1 - \chi) E \left[\beta \frac{u'_c}{u_c} J'_n \right] \quad (10)$$

Proof. See Appendix. $\square$

This is the standard value of an additional worker for the firm in literature where wages are determined via the standard Nash bargaining as in Cheron and Langot (2004), and Choi and Rios-Rull (2009). Also, note that this equation can be directly derived by differentiating the firm's value function J with respect to n , under the assumption that wages are not a function of n when the firm bargains. $w^{ew} = w_{+\Delta}$ means the firm pays existing workers the same wages as the firm would have paid a new worker.

Proposition 2. *If $w^{ew} = w$, then the value of an additional worker for the firm reduces to*

$$J_n \equiv \frac{\partial J}{\partial n} = \frac{\partial F(z, k, nh)}{\partial n} - wh - \frac{\partial w}{\partial n} nh + (1 - \chi) E \left[\beta \frac{u'_c}{u_c} J'_n \right] \quad (11)$$

Proof. See Appendix. $\square$

This is the value of an additional worker for the firm when wages are determined via intra-firm bargaining as in Krause and Lubik (2007), and Acemoglu and Hawkins (2012). Note that this equation can be directly derived by differentiating the firm's value function J with respect to n , under the assumption that wages are a function of n when the firm bargains. $w^{ew} = w$ means the firm pays existing workers current wages when the match with a new worker fails. Also, note that the term $\frac{\partial w}{\partial n}$

captures the strategic incentive to over-hire workers, which will be turned out to be negative later. The negative $\frac{\partial w}{\partial n}$ implies that the firm has an incentive to hire more workers to decrease wages of existing workers by decreasing marginal productivity of labor of existing workers. As a result of over-hiring behavior, the firm can extract the rent existing workers have. In this regard, a new worker is more valuable to the firm than existing workers given the aggregate productivity, z , since hiring a new worker reduces wages of all existing workers as discussed in Krause and Lubik (2007). For this reason, $w_{+\Delta}$ is always greater than w given the aggregate productivity, z . This implies existing workers always prefer to receive higher wages $w_{+\Delta}$, and the firm always prefers to pay the lower wage w when the match with a new worker breaks down. Put it differently, existing worker prefer $w_{+\Delta}$ to w , and the firm prefer w to $w_{+\Delta}$ since over-hiring minimizes the surplus of existing workers and maximizes the surplus of the firm. If $w^{ew} = w$, the firm fully over-hires workers. On the other hand, if $w^{ew} = w_{+\Delta}$, we have $\frac{\partial w}{\partial n} = 0$, which implies the firm do not over-hire at all.

Now we assume wages, w^{ew} , are determined based on the bargaining weight of existing workers, ν . More specifically, we assume $w^{ew} \equiv \nu w_{+\Delta} + (1 - \nu)w$. For example, if existing workers have higher bargaining weights, they receive wages more close to $w_{+\Delta}$, and if they have lower bargaining weights, they receive wages more close to w .

Proposition 3. *If $w^{ew} \equiv \nu w_{+\Delta} + (1 - \nu)w$, then the value of an additional worker for the firm reduces to*

$$J_n \equiv \frac{\partial J}{\partial n} = \frac{\partial F(z, k, nh)}{\partial n} - wh - (1 - \nu) \frac{\partial w}{\partial n} nh + (1 - \chi) E \left[\beta \frac{u'_c}{u_c} J'_n \right] \quad (12)$$

Proof. See Appendix. $\square$

By construction, if $\nu = 1$, then *Proposition 3* reduces to *Proposition 1 (standard bargaining)*, and if $\nu = 0$, then *Proposition 3* reduces to *Proposition 2 (intra-firm bargaining)*. The equation (12) shows the bargaining weight of existing workers determines the degree of over-hiring, which is represented by the term $(1 - \nu) \frac{\partial w}{\partial n}$. Put it differently, w^{ew} , wages bargained over between the firm and existing workers when the match with a new worker breaks down, determines how many workers the firm over-hires. We can show that this term is always negative for any $\nu > 0$ later. The firm has a strategic incentive to over-hire workers, which results in lower wages in partial equilibrium, but as noted in Krause and Lubik (2007), over-hiring increases labor market tightness due to increase in firm's posting of vacancies

and outside options of workers in general equilibrium. As a result, equilibrium wages increase than standard bargaining case. The bottom line here is the bargaining weight of existing workers determines how much the firm over-hires workers and how much wages increase in the equilibrium.

2.4.3 Stochastic Bargaining Weight of Existing Workers, ν

The bargaining weight of existing workers, ν , can be time-varying because when labor markets are tighter, mostly in booms, existing workers are more valuable as the firm will have difficulty finding new workers. However, when the labor market is less tight, mostly in recessions, existing workers become less attractive to firms, which could easily find new workers. This reason makes the bargaining weight of existing workers possibly pro-cyclical. Another possible explanation could be related to the entry and exit of firms over business cycles, which are abstracted from in this paper. In booms, several firms compete with a specific firm because of higher entry rates of new firms and lower exit rates of existing firms. These situations reduce the monopolistic or bargaining power of the firm over existing workers. However, during recessions, the opposite happens. Given this explanation, the bargaining weight of existing workers moves pro-cyclically based on the pro-cyclical entry and the counter-cyclical exit rates.

Since the baseline model does not have any endogenous mechanism to generate time-varying bargaining weights of existing workers, we will assume $\nu \in [0, 1]$ varies stochastically and call innovations to ν bargaining shocks. We will show, in the calibration section, that bargaining shocks can be identified by using labor share data from U.S. once we have the solution to the first order differential equation from the wage bill equation. It is worth mentioning the reason why we set a fixed bargaining weight for new workers, but we allow the bargaining weight of existing workers to vary over time. In the robustness section, we show the time-varying bargaining weight of a new worker is quantitatively not an important factor given constructed shock series of bargaining weight of a new worker by using labor share data. Furthermore, estimated innovations to bargaining weight of a new worker are negatively correlated with innovations to productivity shocks. This implies in recessions the unemployed get positive innovations to the bargaining weight, which seems implausible. We will discuss it more in the robustness section.

2.4.4 The Bargaining with a New Worker

Now, we turn to the bargaining problem which is the same as standard Nash bargaining given the value of an additional worker for the household and the firm

$$\frac{\Omega_n}{u_c} = \frac{v(1-h) - v(1)}{u_c} + wh - b + (1 - \chi - \Psi) E \left[\beta \frac{\Omega'_n}{u_c} \right] \quad (13)$$

$$J_n = \frac{\partial F(z, k, nh)}{\partial n} - wh - (1 - \nu) \frac{\partial w}{\partial n} nh + (1 - \chi) E \left[\beta \frac{u'_c}{u_c} J'_n \right] \quad (14)$$

Given the bargaining weight of a new worker, $\mu \in [0, 1]$, and the bargaining weight of existing workers, $\nu \in [0, 1]$, wages and hours per worker are determined via the following standard bargaining problem

$$(w, h) = \arg \max_{w, h} \left(\frac{\Omega_n}{u_c} \right)^\mu (J_n)^{1-\mu} \quad (15)$$

The first order conditions give

$$\mu J_n = (1 - \mu) \left(\frac{\Omega_n}{u_c} \right) \quad (16)$$

and

$$\frac{v(1-h)(1-h)}{u_c} = F_{nh} - (1 - \nu) \frac{\partial w}{\partial n} n \quad (17)$$

where $F_{nh} = \frac{\partial F(z, k, nh)}{\partial (nh)}$ ⁹. The first condition is a sharing rule between the firm and the worker. The second condition is an intra-temporal condition for hours per worker. Note that we have an additional term, $-(1 - \nu) \frac{\partial w}{\partial n} n$ in equation (17). $\frac{\partial w}{\partial n}$ can be calculated by solving the first order differential equation, which will be defined from the wage bill equation shortly. Given the sharing rule and the definition of V_n/u_c and J_n , we have the following wage bill equation.

$$wh = \mu \left(F_n - (1 - \nu) \frac{\partial w}{\partial n} nh + \frac{V}{1 - N} \kappa \right) + (1 - \mu) \left(\frac{v(1) - v(1-h)}{u_c} + b \right) \quad (18)$$

This is similar to the wage bill equation as in Cheron and Langot (2004) and Choi and Rios-Rull (2009) except for the second term in the right hand side, $(1 - \nu) \frac{\partial w}{\partial n} nh$, which captures the degree of over-

⁹Following Andolfatto (1996), it is assumed that each worker is so small such that $F_{nh} = \frac{\partial F(z, k, nh)}{\partial (nh)}$ is taken as given by both the worker and the firm during the bargaining.

hiring behavior of the firm. We can rewrite the wage bill equation as the first order differential equation with respect to wages w . Assuming a Cobb-Douglas production function, $F(z, k, nh) = e^z k^\alpha (nh)^{1-\alpha}$, the solution to the first order differential equation is given as

$$w = \mu \left(\frac{(1-\alpha)}{1-\mu(1-\nu)\alpha} e^z k^\alpha h^{-\alpha} n^{-\alpha} + \frac{V}{1-N} \kappa h^{-1} \right) + (1-\mu) \left(\frac{v(1)-v(1-h)}{u_c} + b \right) h^{-1} \quad (19)$$

From equation (19), we can show

$$\frac{\partial w}{\partial n} = -\frac{\mu\alpha(1-\alpha)}{1-\mu(1-\nu)\alpha} e^z k^\alpha h^{-\alpha} n^{-\alpha-1} < 0 \quad (20)$$

$$(1-\nu) \frac{\partial w}{\partial n} = -\frac{\mu(1-\nu)\alpha(1-\alpha)}{1-\mu(1-\nu)\alpha} e^z k^\alpha h^{-\alpha} n^{-\alpha-1} < 0 \quad (21)$$

The firm has an incentive to hire more workers to decrease marginal product of existing workers which lowers wages of existing workers. The term $(1-\nu) \frac{\partial w}{\partial n}$ measures how much the firm over-hires workers and how much wages of existing workers drop as a result of over-hiring. Note that if the firm has all the bargaining power ($\nu = 0$), the firm fully over-hires workers. As the bargaining weight of existing workers, ν , increases, the absolute value of $(1-\nu) \frac{\partial w}{\partial n}$ decreases, which implies the degree of over-hiring declines. If existing workers have all the bargaining power ($\nu = 1$), the firm cannot over-hire workers. Also, note that equation (20) and (21) show only partial equilibrium effects. Over-hiring behavior of firms increases aggregate vacancies posted and market tightness which in turn, increases outside options of workers. Consequently, equilibrium wages could increase if general equilibrium effects dominate partial equilibrium effects as noted in Krause and Lubik (2007).

Using the equation (21), we can rewrite two important conditions as follows:

$$\frac{v(1-h)(1-h)}{u_c} = \frac{(1-\alpha)}{1-\mu(1-\nu)\alpha} e^z k^\alpha (nh)^{-\alpha} \quad (22)$$

$$wh = \mu \left(\frac{(1-\alpha)}{1-\mu(1-\nu)\alpha} e^z k^\alpha h^{1-\alpha} n^{-\alpha} + \frac{V}{1-N} \kappa \right) + (1-\mu) \left(\frac{v(1-h)-v(1)}{u_c} + b \right) \quad (23)$$

Stochastic bargaining weights of existing workers, ν , show up in the equations for both intensive and extensive margins. This implies bargaining shocks possibly increase the volatility of both margins. Intuitively, when the bargaining weight of existing workers decreases, the degree of over-hiring increases which in turn raises equilibrium wages. Therefore, the firm increases hours per worker as well. If $\nu = 1$,

we have the conditions in standard Nash bargaining.

$$\frac{v_{(1-h)}(1-h)}{u_c} = (1-\alpha) e^z k^\alpha (nh)^{-\alpha} \quad (24)$$

$$wh = \mu \left((1-\alpha) e^z k^\alpha h^{1-\alpha} n^{-\alpha} + \frac{V}{1-N} \kappa \right) + (1-\mu) \left(\frac{v(1-h) - v(1)}{u_c} + b \right) \quad (25)$$

If $\nu = 0$, we have similar conditions in intra-firm bargaining literature as we expect.

$$\frac{v_{(1-h)}(1-h)}{u_c} = \frac{(1-\alpha)}{1-\mu\alpha} e^z k^\alpha (nh)^{-\alpha} \quad (26)$$

$$wh = \mu \left(\frac{(1-\alpha)}{1-\mu\alpha} e^z k^\alpha h^{1-\alpha} n^{-\alpha} + \frac{V}{1-N} \kappa \right) + (1-\mu) \left(\frac{v(1-h) - v(1)}{u_c} + b \right) \quad (27)$$

2.5 Government

The government simply raises revenue in order to pay out unemployment benefits b to unemployed members within the household. Therefore, the government budget constraint is

$$T(S) = (1-n)b \quad (28)$$

2.6 Equilibrium

A *recursive competitive equilibrium* is a set of functions; the household's value function $\Omega(S, s_H)$, the household's policy functions $c(S, s_H)$, $a'(S, s_H)$, the firm's value function $J(S, s_F)$, the firm's policy functions $v(S, s_f)$, $k(S, s_f)$, aggregate prices $r(S)$, $m(S, S')$, taxes $T(S)$, dividends $\Pi(S)$, and the law of motion for aggregate state variables $G(S)$ such that

- (1) Household's policy functions solve the household's problem
- (2) Firm's policy functions solve the firm's problem
- (3) $m(S, S') = \beta u(c(S'))/u_c(c(S))$
- (4) Wages and hours per worker $(w(S), h(S))$ are the solution to the bargaining problem

- (5) All markets clear
- (6) The government budget constraint is balanced
- (7) The law of motion $G(S)$ is consistent with individual decisions

3 Calibration

First, we define the matching function and the aggregate production function to be

$$F(z, k, nh) = e^z k (nh)^{1-\alpha} \quad (29)$$

$$M = \omega V^\psi (1 - N)^{1-\psi} \quad (30)$$

where $\alpha \in (0, 1)$, $\psi \in (0, 1)$. We specify the household's utility function as follows

$$u(c) = \log(c) \quad (31)$$

$$v(1-h) = \phi_e \frac{(1-h)^{1-\eta}}{1-\eta} \quad (32)$$

Including the parameters in the functions defined above, we have 19 parameters to be calibrated. Parameters can be categorized into three groups based on the way to calibrate them. The first set of parameters are predetermined parameters outside the model. The second set of parameters are parameters for shock processes, which will be estimated from constructed shock processes from U.S. data. The last group of parameters are parameters to be determined in the model by using steady state conditions and relevant targets.

3.1 Predetermined Parameters (5)

We basically follow Andolfatto (1996) for the discount factor $\beta = 0.99$, the separation rate $\chi = 0.15$, the depreciation rate $\delta = 0.025$, the Cobb Douglas parameter for capital $\alpha = 0.36$, and the coefficient for vacancies in the matching function $\psi = 0.60$. Note that since the labor market is not competitive in this paper, we cannot use labor share data to calibrate α . Table 2 summarizes predetermined parameters.

Parameters	Description	Value	Source
β	Discount factor	0.99	Annual rate of return 4%
χ	Separation rate	0.15	Andolfatto (1996)
δ	Depreciation rate	0.025	Andolfatto (1996)
α	Cobb Douglas parameter for capital	0.36	Andolfatto (1996)
ψ	Coefficient for vacancies in matching function	0.60	Free parameter, Andolfatto (1996)

Table 2: Predetermined parameters

3.2 Parameters for Shock Processes (7)

Productivity shocks can be constructed as a series of the measure Solow residual. From the aggregate production function, we have

$$\hat{z}_t = \hat{y}_t - \alpha \hat{k}_t - (1 - \alpha) \hat{n}_t - (1 - \alpha) \hat{h}_t \quad (33)$$

where hats denote log-deviations from a linear trend for each variable over the period 1960:Q1-2012:Q1.

We normalize $\bar{z} = 1$.

For bargaining shocks, we can use the solution to the first order differential equation we solved before

$$\frac{\partial w}{\partial n} = -\frac{\mu\alpha(1-\alpha)}{1-\mu(1-\nu)\alpha} z k^\alpha h^{-\alpha} n^{-\alpha-1} \quad (34)$$

$$\frac{\partial w}{\partial n} \frac{n}{w} = -\frac{\mu\alpha(1-\alpha)}{1-\mu(1-\nu)\alpha} \frac{y}{nhw} = -\frac{\mu\alpha(1-\alpha)}{1-\mu(1-\theta)\alpha} \frac{1}{labor\ share} \quad (35)$$

$$labor\ share = \frac{\mu\alpha(1-\alpha)}{1-\mu(1-\nu)\alpha} \frac{1}{(-\epsilon_{w,n})} \quad (36)$$

where $\epsilon_{w,n} \equiv \frac{\partial w}{\partial n} \frac{n}{w}$.¹⁰ We assume the elasticity $\epsilon_{w,n}$ does not move much around the steady-state value $\overline{\epsilon_{w,n}} \equiv \frac{\partial w}{\partial n} \frac{n}{w} = -\frac{\mu\alpha(1-\alpha)}{1-\mu(1-\nu)\alpha} \frac{\bar{y}}{\bar{n}h\bar{w}}$, and we will show this assumption is innocuous in the quantitative analysis section. From equation (36), we can construct series of v_t given series of the labor share data from U.S.

$$(labor\ share)_t = \frac{\mu\alpha(1-\alpha)}{1-\mu(1-\nu_t)\alpha} \frac{1}{(-\overline{\epsilon_{w,n}})} \quad (37)$$

$$\nu_t = 1 - \left(\frac{1}{\mu\alpha} - \frac{(1-\alpha)}{(labor\ share)_t (-\overline{\epsilon_{w,n}})} \right) \quad (38)$$

¹⁰Note that $\epsilon_{w,n} < 0$.

Parameters	Value	Remarks
ρ_{zz}	0.9653	P-value = 0.000
$\rho_{\nu z}$	0.0059	P-value = 0.095
ρ_{zv}	-0.5177	P-value = 0.000
ρ_{vv}	0.9249	P-value = 0.000
σ_z	0.0070	-
σ_ν	0.0402	-
$\sigma_{z\nu}$	0.0001	$\rho(\varepsilon_z, \varepsilon_\nu) = 0.4233$

Table 3: Shock processes

The series of labor share are constructed from U.S. data. The detail can be found in the data appendix. Given signs of parameters and $\overline{\epsilon_{w,n}} < 0$, there exists the negative relationship between bargaining shocks ν_t and labor share. This implies the higher labor share is related to the lower bargaining weight of existing workers. This can be explained by general equilibrium effects of over-hiring behavior. If the bargaining weight of existing workers is low, the firm over-hires workers more, which in turn, increases equilibrium wages and hours because as firms post more vacancies, market tightness and outside options of workers increases in general equilibrium. Consequently, the labor share increases. Equation (39) shows the negative relationship more clearly.¹¹

$$\frac{\partial (labor\ share)}{\partial \nu} = -\frac{(\mu\alpha)^2 (1-\alpha)}{(1-\mu(1-\nu)\alpha)^2 (-\overline{\epsilon_{w,n}})} < 0 \quad (39)$$

This mechanism could be also consistent with weakly counter-cyclical or acyclic movements of labor share in U.S data. For example, if bargaining weight of existing workers decreases in recessions, labor share increases because of an increase in the over-hiring.

Based on several information criteria such as FPE, AIC, HQIC, and SBIC, we specify VAR(1) system for detrended shock series $\hat{z}, \hat{\nu}$ to estimate shock processes.

$$\begin{pmatrix} \hat{z}' \\ \hat{\nu}' \end{pmatrix} = \begin{pmatrix} \rho_{zz} & \rho_{\nu z} \\ \rho_{zv} & \rho_{\nu\nu} \end{pmatrix} \begin{pmatrix} \hat{z} \\ \hat{\nu} \end{pmatrix} + \begin{pmatrix} \varepsilon'_z \\ \varepsilon'_\nu \end{pmatrix} \quad (40)$$

$$\begin{pmatrix} \varepsilon_z \\ \varepsilon_\nu \end{pmatrix} \sim N \left(0, \begin{pmatrix} \sigma_{zz}^2 & \sigma_{z\nu} \\ \sigma_{z\nu} & \sigma_{\nu\nu}^2 \end{pmatrix} \right) \quad (41)$$

Table 3 summarizes parameters estimated using VAR(1) system above. All coefficient parameters except for $\rho_{\nu z}$ are significant. Note that we have $\rho_{zv} = -0.5177$ which means today's productivity

¹¹Note that $\overline{\epsilon_{w,n}} < 0$.



Figure 1: Time Series of the Bargaining Weight of Existing Workers, $\hat{\nu}_t$, and innovations ε_ν

shocks decrease tomorrow's bargaining weight of existing workers and increase tomorrow's degree of over-hiring. This makes total hours, employment and hours per workers more volatile. Also, we have a quite high correlation between innovations to productivity shocks and innovations to bargaining shocks, $\rho(\varepsilon_z, \varepsilon_\nu) = 0.4233$, which again justifies the assumption that the bargaining weight of existing workers, ν , varies stochastically as discussed before, in the sense that innovations to the bargaining weight of existing workers is pro-cyclical. The first panel of Figure 1 plots detrended shock series $\hat{\nu}_t$ constructed by using the equation (40) and the second panel plots innovations ε_ν . The shaded areas represent the NBER recession dates. Figure 1 shows the bargaining weight of existing workers increases mostly in booms, and decreases near recessions as we expected. Also, estimated innovations ε_ν have positive values more frequently in booms, and have negative values more frequently in recessions, which confirms a high correlation between innovations to productivity shocks and innovations to bargaining shocks, $\rho(\varepsilon_z, \varepsilon_\nu) = 0.4233$.

3.3 Parameters Determined Using Targets (7)

We choose the remaining 7 parameters using equilibrium conditions in the steady state and targets from the literature and data from U.S. over 1960:Q1-2012:Q1. The targets we used are summarized in Tables 3. First, we set Frisch elasticity of hours for employed to 0.50, the steady state job-filling rate to 0.90 as in Andolfatto (1996), which is common across the literature. According to Silva and

Target	Value	Source
Frisch elasticity of hours for those employed	0.50	Andolfatto (1996)
Steady-state employment to population ratio	0.60	Data (1960:Q1-2012:Q1)
Steady-state hours per worker ¹²	0.39	Data (1960:Q1-2012:Q1)
Steady-state job-filling rate	0.90	Andolfatto (1996)
Vacancy expenditure to output ratio ¹³	0.0218	Silva & Toledo (2009)
Replacement ratio	0.40	Shimer (2005)
Bargaining power of a new worker	μ	Model ¹⁴

Table 4: Targets

Parameters	Description	Baseline
η	Curvature parameter for leisure	3.0940
ϕ_e	Scale parameter for leisure	0.8854
κ	Cost of posting vacancies	0.1905
ω	Matching efficiency	0.5156
b	Unemployment Benefits	0.3950
μ	Bargaining power of a new worker	0.5605
$\bar{\nu}$	Bargaining power of existing workers	0.5605

Table 5: Parameters determined using targets

Toledo (2009), the average cost of time spent hiring one worker is approximately 3.6%-4.3% of total labor costs. We take the target the mid point of those range, 3.9%, which gives vacancy expenditure to output ratio $\frac{\kappa v}{y} = 0.0218$.¹⁵ We use 40 percent as the value of unemployment benefits following Shimer (2005). In Shimer, this value implicitly includes the value of leisure, but in this paper we explicitly consider the leisure in the utility function, so unemployment benefits, b , are purely unemployment benefits as in Nakajima (2012). Targets and parameters determined using these targets are listed in Table 4 and Table 5 respectively. We set the mean value of the bargaining shock, $\bar{\nu}$, to be 0.5605 which is the same as the bargaining power of a new worker, μ , calibrated in the model.¹⁶ Since this parameter is a free parameter and there is no clear way to pin down this parameter, we assume $\bar{\nu} = \mu$. As we will discuss in the robustness section later, lower values of $\bar{\nu}$ generates more volatile labor market variables. However, we set $\bar{\nu} = \mu = 0.5605$ which gives almost the least volatility among $\bar{\nu} \in (0, 1)$ in the baseline model. In this regard, we think the choice of $\bar{\nu} = 0.5605$ is innocuous and parsimonious. Also, note that calibrated value for the bargaining weight of a new worker, μ , is 0.5605,

¹⁵This value is calculated based on job-filling rate $\Phi = 0.90$ and labor share = 0.62.

¹⁶The mean of the bargaining weight of existing workers, $\bar{\nu}$, and the bargaining weight of a new worker, μ , are jointly determined in the steady state.

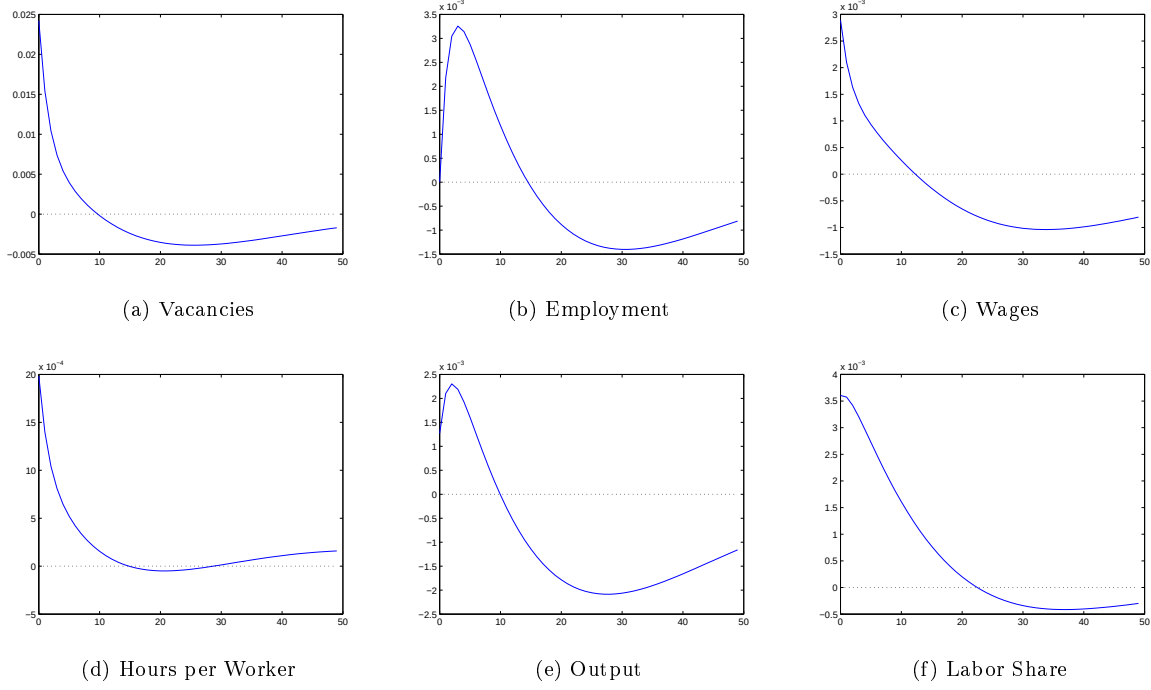


Figure 2: IRFs to the Negative One Standard Deviation Bargaining Shock

which guarantees quantitative results of the baseline model are not a direct result from a low value of μ as noted in Hagendon and Manovskii (2008).

4 Quantitative Analysis

4.1 Impulse Response Functions of Negative Bargaining Shocks

Figure 2 shows the impulse response of key labor market variables to the negative one standard deviation bargaining shock. When negative bargaining shocks hit the economy, the bargaining weight of existing workers instantly decreases. Since bargaining positions of existing workers weaken, a firm is able to over-hire workers more actively via intra-firm bargaining. Therefore, the firm instantly posts more vacancies, and employment increases one period later due to the nature of search frictions. Since over-hiring lowers marginal productivity of existing workers, wages decrease in partial equilibrium. However, higher vacancies posted raise the market tightness and outside options of workers, which in turn, increases wages and offsets the partial equilibrium effects. Consequently, wages increase in

	$\sigma_x \%$ $\left(\frac{\sigma_x}{\sigma_{Output}} \right)$			$\rho(x, Output)$			$\rho(x_t, x_{t-1})$		
Variable (x)	Data	Baseline	Andolfatto	Data	Baseline	Andolfatto	Data	Baseline	Andolfatto
Output	1.54 (1.00)	1.27 (1.00)	1.30 (1.00)	-	-	-	0.86	0.84	0.82
Total Hours	1.38 (0.90)	0.88 (0.70)	0.70 (0.54)	0.85	0.73	0.92	0.88	0.92	0.91
Employment	1.00 (0.65)	0.81 (0.64)	0.68 (0.52)	0.81	0.70	0.78	0.91	0.90	0.89
Hours per Worker	0.49 (0.32)	0.23 (0.18)	0.19 (0.15)	0.74	0.33	0.55	0.56	0.53	0.58
Wages	0.91 (0.59)	0.63 (0.50)	0.62 (0.48)	0.34	0.91	0.94	0.69	0.68	0.65
Labor Productivity	0.82 (0.53)	0.86 (0.68)	0.72 (0.55)	0.45	0.72	0.92	0.57	0.70	0.62
Labor Share	0.74 (0.48)	0.66 (0.52)	0.10 (0.08)	-0.08	-0.08	-0.72	0.78	0.77	0.51
Vacancies	13.23 (8.59)	4.07 (3.20)	3.65 (2.81)	0.90	0.78	0.80	0.91	0.57	0.54

Table 6: Business Cycle Moments in Data and Models over 1960:Q1-2012:Q1

general equilibrium as we can see panel (c) in Figure 2. As wages go up, hours per worker increase and higher degree of over-hiring behavior yields higher outputs in the equilibrium. Higher employment, hours per worker, and wages results in an increase in labor share by offsetting an increase in outputs.

4.2 Business Cycle Moments

Table 6 summarizes quantitative results of the baseline model. We compare our baseline model to the Andolfatto model to see what gains and what shortcomings the inclusion of intra-firm bargaining and bargaining shocks gives. Again, all data are in log and HP filtered. First of all, the baseline model generates a high (relative) volatility of employment, 0.64, which almost close to the actual U.S. data, 0.65. This is a remarkable success and the main contribution in this paper. Since employment is very volatile, total hours is much volatile than the Andolfatto model. Hours per worker and vacancies are slightly more volatile than Andolfatto, but the differences are small. The moments for labor share are almost similar to the actual U.S. data. This result might be a direct result of the identification strategy for bargaining shocks from labor share data. However, the moments for labor share in the model, along with the overshooting property we will discuss shortly, justify the assumption for the identification of bargaining shocks; $\epsilon_{w,n} \equiv \frac{\partial w}{\partial n} \frac{n}{w}$ does not move much around the steady state.

The main mechanism generates more volatile labor market variables is that the impact of productivity shocks is amplified by the firm's over-hiring behavior in addition to the impact of each shock. Recall that the estimated parameter for ρ_{zv} is -0.5177, which means that as the productivity shocks today negatively affect the bargaining shocks tomorrow, and as the bargaining weight of existing workers drops, the firm will excessively over-hire workers. This dynamic interaction between productivity

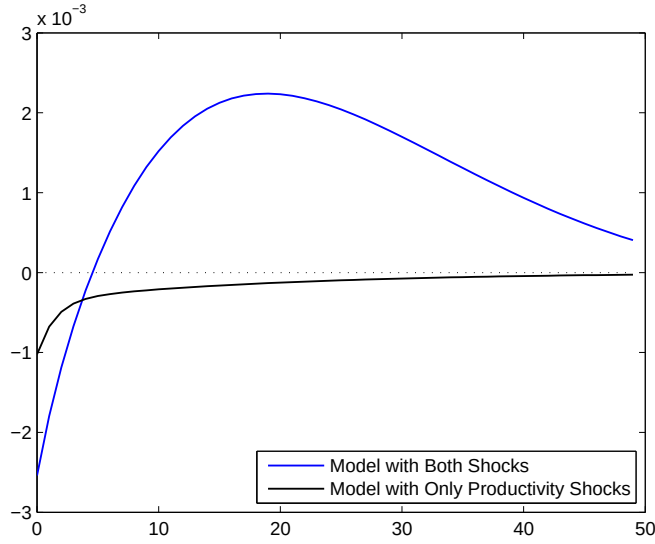


Figure 3: Impulse Response Function of Labor Share to Productivity Innovation

shocks and bargaining shocks amplifies the volatility of labor market variables, especially employment.

We now consider shortcomings of the baseline model relative to Andolfatto. The baseline model generates the higher volatility of labor productivity, weak pro-cyclicality of total hours, employment and hours per worker. Also, labor productivity is more persistent and vacancies are less persistent than the Andolfatto model and actual U.S. data. Despite of these shortcomings, the baseline model performs better than Andolfatto model in general. This result mainly comes from time-varying bargaining weights of existing workers and over-hiring behavior of the firm.

4.3 Implication on Labor Share

Rios-Rull and Santaella-Llopis (2010) first document the overshooting property of labor share. They showed that labor share overshoots in response to productivity shocks, and the dynamic overshooting response of labor share drastically dampens the role of productivity shocks on labor markets due to huge wealth effects. Figure 3 shows the overshooting of labor share in the baseline model. If we abstract from bargaining shocks, the model no longer generates the overshooting of labor share. The reason the model with bargaining shocks features the overshooting of labor share might be a direct result of the identification strategy of bargaining shocks. Again, the fact that labor share overshoots in the baseline model, along with other moments for labor share are almost the same as those in actual

<i>Variable (x)</i>	$\sigma_x \%$ $\left(\frac{\sigma_x}{\sigma_{Output}} \right)$			$\rho(x, Output)$			$\rho(x_t, x_{t-1})$		
	Both	Only z	Only ν	Both	Only z	Only ν	Both	Only z	Only ν
Output	1.27 (1.00)	1.30 (1.00)	0.41 (1.00)	-	-	-	0.84	0.82	0.91
Total Hours	0.88 (0.70)	0.70 (0.54)	0.65 (1.58)	0.73	0.91	0.99	0.92	0.91	0.91
Employment	0.81 (0.64)	0.69 (0.53)	0.58 (1.41)	0.70	0.78	0.96	0.90	0.88	0.88
Hours per Worker	0.23 (0.18)	0.19 (0.15)	0.20 (0.49)	0.33	0.54	0.45	0.53	0.58	0.57
Wages	0.63 (0.50)	0.62 (0.48)	0.41 (1.00)	0.91	0.94	0.66	0.68	0.65	0.60
Labor Productivity	0.86 (0.68)	0.72 (0.55)	0.24 (0.59)	0.72	0.92	-0.96	0.70	0.62	0.91
Labor Share	0.66 (0.52)	0.10 (0.08)	0.61 (1.49)	-0.08	-0.71	0.83	0.77	0.50	0.76
Vacancies	4.07 (3.20)	3.67 (2.82)	3.10 (7.56)	0.78	0.80	0.49	0.57	0.54	0.54

Table 7: Business Cycle Moments in Models with different shocks

data, justifies the assumption we pose to identify bargaining shocks; $\epsilon_{w,n} \equiv \frac{\partial w}{\partial n} \frac{n}{w}$ does not move much around the steady state.

More importantly, the baseline model generates the overshooting property of labor share, but the effect of productivity shocks is still significant on labor markets in contrast to the prediction of Rios-Rull and Santaaulalia-Llopis (2010) in which the effect of productivity shocks is dampened when labor share overshoots because of huge wealth effects from the overshooting property. In contrast to their model, the baseline model has a search and matching framework, and the nature of this framework weakens wealth effects resulting from the overshooting of labor share. On top of these differences, the over-hiring behavior of a firm offsets the huge reduction of total hours in booms. In response to positive productivity shocks output instantly increases, but employment does not increase because of search frictions, which cause an instant drop in labor share. As the productivity shocks today negatively affect the bargaining shocks tomorrow, $\rho_{zv} = -0.5177$, and as the bargaining weight of existing workers drops, the firm will excessively over-hire workers. Consequently, employment, wages, and hours per worker will increase because of the general equilibrium effects of over-hiring by offsetting an increase in outputs. This increase explains the overshooting of labor share in response to positive productivity shocks.

4.4 The Role of Productivity Shocks and Bargaining Shocks

Now we consider how productivity shocks and bargaining shocks differently affect the model predictions. When the economy has only productivity shocks, z , the model predictions are almost the same as the Andolfatto model. Comparing to the baseline model which has both shocks, the volatility of employment, labor share, and vacancies is dampened, but correlations between labor market variables

<i>Variable</i>	<i>z</i>	<i>ν</i>
Output	91.07	8.93
Total Hours	56.61	43.39
Employment	66.25	33.75
Hours per Worker	8.15	91.85
Wages	72.64	27.36
Labor Productivity	91.85	8.15
Labor Share	35.64	64.36
Vacancies	59.04	40.96

Table 8: Variance Decomposition (in percent)

and outputs get close to the actual data. Auto-correlations are almost the same as the baseline case.

When the economy has only bargaining shocks, the volatility of outputs significantly drops, which means bargaining shocks cannot be the main driving source of output fluctuations. On the other hand, the volatility of total hours, employment, hours per worker remarkably increases, which is far beyond the volatility in the baseline model. Also, total hours, employment, hours per worker, and labor shares are strongly pro-cyclical. However, auto-correlations are almost the same as the baseline case.

Table 8 shows the variance decomposition. Bargaining shocks have a substantial impact on the volatility of total hours, employment, hours per worker, vacancies, and labor share. Bargaining shocks determine how many workers the firm over-hires and how much labor market variables move. As discussed in the previous section, over-hiring increases employment, hour per worker, and wages because of the general equilibrium effect. While bargaining shocks play a remarkable role in the labor markets, productivity shocks seem to be still the main driving force of business cycles given productivity shocks account for about 91% of output fluctuations. This result is also consistent with the finding in moments in Table 7.

5 Robustness

5.1 Stochastic Bargaining Weight of a New Worker, μ

Now we assume the bargaining weight of a new worker varies stochastically while the bargaining weight of existing workers is fixed at $\nu = \bar{\nu} = \bar{\mu}$. Again, we identify series of μ_t by using the solution to the

	$\sigma_x \%$ $\left(\frac{\sigma_x}{\sigma_{Output}} \right)$		$\rho(x, Output)$		$\rho(x_t, x_{t-1})$	
<i>Variable (x)</i>	Shock on μ	Andolfatto	Shock on μ	Andolfatto	Shock on μ	Andolfatto
Output	1.40 (1.00)	1.30 (1.00)	-	-	0.85	0.82
Total Hours	0.80 (0.57)	0.70 (0.54)	0.93	0.92	0.92	0.91
Employment	0.76 (0.54)	0.68 (0.52)	0.82	0.78	0.89	0.89
Hours per Worker	0.19 (0.14)	0.19 (0.15)	0.57	0.55	0.59	0.58
Wages	0.62 (0.44)	0.62 (0.48)	0.94	0.94	0.68	0.65
Labor Productivity	0.73 (0.52)	0.72 (0.55)	0.91	0.92	0.63	0.62
Labor Share	0.15 (0.11)	0.10 (0.08)	-0.56	-0.72	0.56	0.51
Vacancies	4.00 (2.86)	3.65 (2.81)	0.76	0.80	0.55	0.54

Table 9: Business Cycle Moments in Model with Shocks on μ

first order differential equation, and series of the labor share data from U.S.

$$(labor\ share)_t = \frac{\mu_t \alpha (1 - \alpha)}{\mu_t (1 - \bar{\nu}) \alpha} \frac{1}{(-\bar{\epsilon}_{w,n})} \quad (42)$$

$$\mu_t = \frac{1}{(1 - \bar{\nu}) \alpha + \frac{\alpha(1-\alpha)}{(labor\ share)_t (-\bar{\epsilon}_{w,n})}} \quad (43)$$

where $\epsilon_{w,n} \equiv \frac{\partial w}{\partial n} \frac{n}{w}$. Again, we assume the elasticity $\epsilon_{w,n}$ does not move much around the steady-state value $\bar{\epsilon}_{w,n} \equiv \frac{\partial w}{\partial n} \frac{n}{w} = -\frac{\mu \alpha (1 - \alpha)}{1 - \mu (1 - \bar{\nu}) \alpha} \frac{\bar{y}}{\bar{n} h \bar{w}}$. Table 9 shows the comparison of business cycle moments. Stochastic bargaining weight of a new worker cannot quantitatively improve the Andolfatto model, even moments for labor share which is used for identifying shock series μ_t .¹⁷

5.2 Different values for $\bar{\nu}$

We now simulate the baseline model with different values for $\bar{\nu} = 0.2$ (a low value), 0.5605 (a middle value and the calibrated value for the baseline model), and 0.9 (a high value). Table 10 shows business cycle moments for each case. If we set $\bar{\nu} = 0.2$, then volatility of employment and hours per workers significantly increases than the baseline calibration case, $\bar{\nu} = 0.5605$. However, if we set $\bar{\nu} = 0.9$, then moments are almost the same as those of the baseline calibration case, $\bar{\nu} = 0.5605$. Mechanically, low values of $\bar{\nu}$ increase the volatility of total hours, employment, hours per worker, and vacancies. $\bar{\nu}$ is a free parameter in this paper and there is no clear way to pin down this parameter. However, the choice of $\bar{\nu} = 0.5605$ in the baseline model seems innocuous and parsimonious in the sense that we think setting the same values for the mean of bargaining weights of existing workers and bargaining

¹⁷This results do not change with different values of $\bar{\nu}$

<i>Variable</i> (x)	$\sigma_x \%$ $\left(\frac{\sigma_x}{\sigma_{Output}} \right)$			$\rho(x, Output)$			$\rho(x_t, x_{t-1})$		
	$\bar{\nu} = 0.2$	$\bar{\nu} = 0.5605$	$\bar{\nu} = 0.9$	$\bar{\nu} = 0.2$	$\bar{\nu} = 0.5605$	$\bar{\nu} = 0.9$	$\bar{\nu} = 0.2$	$\bar{\nu} = 0.5605$	$\bar{\nu} = 0.9$
Output	1.34 (1.00)	1.27 (1.00)	1.26 (1.00)	-	-	-	0.86	0.84	0.84
Total Hours	1.17 (0.87)	0.88 (0.70)	0.87 (0.69)	0.73	0.73	0.73	0.92	0.92	0.92
Employment	1.02 (0.76)	0.81 (0.64)	0.80 (0.63)	0.72	0.70	0.70	0.91	0.90	0.90
Hours per Worker	0.33 (0.25)	0.23 (0.18)	0.22 (0.17)	0.36	0.33	0.33	0.54	0.53	0.53
Wages	0.75 (0.56)	0.63 (0.50)	0.62 (0.49)	0.86	0.91	0.91	0.68	0.68	0.68
Labor Productivity	0.93 (0.69)	0.86 (0.68)	0.86 (0.68)	0.51	0.72	0.73	0.72	0.70	0.70
Labor Share	0.97 (0.72)	0.66 (0.52)	0.65 (0.52)	0.17	-0.08	-0.10	0.76	0.77	0.76
Vacancies	5.03 (3.75)	4.07 (3.20)	4.03 (3.19)	0.75	0.78	0.78	0.57	0.57	0.57

Table 10: Business Cycle Moments in the Model with different values for $\bar{\nu}$

weights of new workers, $\bar{\nu} = \mu$, is a reasonable given there is no information on $\bar{\nu}$, and $\bar{\nu} = 0.5605$ yields the least volatility of labor market variables among $\bar{\nu} \in (0, 1)$.

6 Conclusion

This paper investigates the effects of intra-firm bargaining on the standard RBC search and matching model by explicitly considering the outside option of a firm in the bargaining with a new worker. In this paper, the outside option of a firm in the bargaining with a new worker is bargaining with existing workers (i.e., intra-firm bargaining). We show the outside option of a firm is crucial for the bargaining with a new worker in the sense that how much the firm pays existing workers, when the match breaks down, determines how many workers it over-hires. In this paper, wages in the outside option of a firm depend on the stochastic bargaining weight of existing workers, which can be identified through labor share data from U.S. Bargaining shocks affect the degree of over-hiring behavior of a firm, and in turn, provide another source to explain the behavior of labor markets in business cycles.

The inclusion of intra-firm bargaining and bargaining shocks improves the capacity of the standard RBC search and matching model, especially in labor markets. Our calibrated model generates more volatile total hours, employment, hours per worker. In particular, the volatility of employment in the model is similar to the actual U.S. data. The main mechanism generates more volatile labor market variables is that the impact of productivity shocks is amplified by the firm's over-hiring behavior in addition to the impact of each shock. As the productivity shocks today negatively affect the bargaining shocks tomorrow, and as the bargaining weight of existing workers drops, the firm will excessively over-hire workers. This dynamic interaction between productivity shocks and bargaining shocks amplifies

the volatility of labor market variables, especially employment.

Moreover, the baseline model generates the overshooting property of labor share, but the effect of productivity shocks is still significant on labor markets in contrast to the prediction of Rios-Rull and Santaaulalia-Llopis (2010) in which the effect of productivity shocks is dampened when labor share overshoots because of huge wealth effects from the overshooting property. In contrast to their model, the baseline model has a search and matching framework, and the nature of this framework weakens wealth effects resulting from the overshooting of labor share. On top of these differences, the over-hiring behavior of a firm offsets the huge reduction of total hours in booms. In response to positive productivity shocks output instantly increases, but employment does not increase because of search frictions, which cause an instant drop in labor share. As the productivity shocks today negatively affect the bargaining shocks tomorrow, and as the bargaining weight of existing workers drops, the firm will excessively over-hire workers. Consequently, employment, wages, and hours per worker will increase because of the general equilibrium effects of over-hiring by offsetting an increase in outputs. This increase explains the overshooting of labor share in response to positive productivity shocks.

In this paper, we assume the bargaining weight of existing workers to be exogenous. Our quantitative results show that the time-varying bargaining weight of existing workers is an important margin to understand the fluctuations of total hours, employment, hours per workers, and labor share, and the overshooting property. However, this paper abstracts from the endogenous mechanism for the time-varying bargaining weight of existing workers. Therefore, coming up with an endogenous mechanism for bargaining shocks would be worthwhile for future research. One possible theory could be related to the entry and exit of firms over business cycles. In booms, several firms compete with a specific firm because of higher entry rates of new firms and lower exit rates of existing firms, which reduce the monopolistic or bargaining power of firms over existing workers. However, during recessions, the opposite happens. By incorporating the entry and exit decision of firms, we might be able to explain the endogenous movements of the bargaining weight of existing workers. Moreover, this paper does not focus on unemployment because the baseline model treats the unemployed and the non-employed who are out of the labor force similarly, and the measure of unemployment is inconsistent with the data. In this regard, we could extend the baseline model by distinguishing between the unemployed and the non-employed to obtain a proper measure of unemployment.

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Appendix

Proofs of Propositions

Proof of Proposition 1

After plugging $w^{ew} = w_{+\Delta}$ into (9), we divide the both sides by Δ . By taking the limit,

$$\begin{aligned}
\lim_{\Delta \rightarrow 0} \frac{J_{+\Delta} - J}{\Delta} &= \lim_{\Delta \rightarrow 0} \frac{[F(z, k, (n + \Delta)h) - F(z, k, nh)]}{\Delta} - \lim_{\Delta \rightarrow 0} \frac{[w_{+\Delta}(n + \Delta)h - w_{+\Delta}nh]}{\Delta} \\
&\quad + (1 - \chi) E \left[\beta \frac{u'_c}{u_c} \lim_{\Delta \rightarrow 0} \frac{J'_{+\Delta} - J'}{\Delta} \right] \\
\frac{\partial J}{\partial n} &= \frac{\partial F(z, k, nh)}{\partial n} - \lim_{\Delta \rightarrow 0} \frac{[(n + \Delta - n)w_{+\Delta}h]}{\Delta} + (1 - \chi) E \left[\beta \frac{u'_c}{u_c} \left(\frac{\partial J'}{\partial n'} \right) \right] \\
&= \frac{\partial F(z, k, nh)}{\partial n} - \left(\lim_{\Delta \rightarrow 0} \frac{n + \Delta - n}{\Delta} \right) \left(\lim_{\Delta \rightarrow 0} w_{+\Delta}h \right) + (1 - \chi) E \left[\beta \frac{u'_c}{u_c} \left(\frac{\partial J'}{\partial n'} \right) \right] \\
J_n \equiv \frac{\partial J}{\partial n} &= \frac{\partial F(z, k, nh)}{\partial n} - wh + (1 - \chi) E \left[\beta \frac{u'_c}{u_c} J'_n \right]
\end{aligned}$$

Proof of Proposition 2

After plugging $w^{ew} = w$ into (9), we divide the both sides by Δ . By taking the limit,

$$\begin{aligned}
\lim_{\Delta \rightarrow 0} \frac{J_{+\Delta} - J}{\Delta} &= \lim_{\Delta \rightarrow 0} \frac{[F(z, k, (n + \Delta)h) - F(z, k, nh)]}{\Delta} - \lim_{\Delta \rightarrow 0} \frac{[w_{+\Delta}(n + \Delta)h - wnh]}{\Delta} \\
&\quad + (1 - \chi) E \left[\beta \frac{u'_c}{u_c} \lim_{\Delta \rightarrow 0} \frac{J'_{+\Delta} - J'}{\Delta} \right] \\
\frac{\partial J}{\partial n} &= \frac{\partial F(z, k, nh)}{\partial n} - \lim_{\Delta \rightarrow 0} \frac{[w_{+\Delta}(n + \Delta)h - w_{+\Delta}nh + w_{+\Delta}nh - wnh]}{\Delta} + (1 - \chi) E \left[\beta \frac{u'_c}{u_c} \left(\frac{\partial J'}{\partial n'} \right) \right] \\
&= \frac{\partial F(z, k, nh)}{\partial n} - \lim_{\Delta \rightarrow 0} \frac{[w_{+\Delta}(n + \Delta)h - w_{+\Delta}nh]}{\Delta} - \lim_{\Delta \rightarrow 0} \frac{[w_{+\Delta}nh - wnh]}{\Delta} \\
&\quad + (1 - \chi) E \left[\beta \frac{u'_c}{u_c} \left(\frac{\partial J'}{\partial n'} \right) \right] \\
&= \frac{\partial F(z, k, nh)}{\partial n} - wh - \lim_{\Delta \rightarrow 0} \left(\frac{w_{+\Delta} - w}{\Delta} \right) nh + (1 - \chi) E \left[\beta \frac{u'_c}{u_c} J'_n \right] \\
J_n \equiv \frac{\partial J}{\partial n} &= \frac{\partial F(z, k, nh)}{\partial n} - wh - \frac{\partial w}{\partial n} nh + (1 - \chi) E \left[\beta \frac{u'_c}{u_c} J'_n \right]
\end{aligned}$$

Proof of Proposition 3

After plugging $w^{ew} = \nu w_{+\Delta} + (1 - \nu) w$ into (9), we divide the both sides by Δ . By taking the limit,

$$\begin{aligned}
\lim_{\Delta \rightarrow 0} \frac{J_{+\Delta} - J}{\Delta} &= \lim_{\Delta \rightarrow 0} \frac{[F(z, k, (n + \Delta) h) - F(z, k, nh)]}{\Delta} - \lim_{\Delta \rightarrow 0} \frac{[w_{+\Delta} (n + \Delta) h - w^{ew} nh]}{\Delta} \\
&\quad + (1 - \chi) E \left[\beta \frac{u'_c}{u_c} \lim_{\Delta \rightarrow 0} \frac{J'_{+\Delta} - J'}{\Delta} \right] \\
\frac{\partial J}{\partial n} &= \frac{\partial F(z, k, nh)}{\partial n} - \lim_{\Delta \rightarrow 0} \frac{[w_{+\Delta} (n + \Delta) h - (\nu w_{+\Delta} + (1 - \nu) w) nh]}{\Delta} \\
&\quad + (1 - \chi) E \left[\beta \frac{u'_c}{u_c} \left(\frac{\partial J'}{\partial n'} \right) \right] \\
&= \frac{\partial F(z, k, nh)}{\partial n} - \lim_{\Delta \rightarrow 0} \frac{[\nu w_{+\Delta} (n + \Delta) h + (1 - \nu) w_{+\Delta} (n + \Delta) h - (\nu w_{+\Delta} + (1 - \nu) w) nh]}{\Delta} \\
&\quad + (1 - \chi) E \left[\beta \frac{u'_c}{u_c} \left(\frac{\partial J'}{\partial n'} \right) \right] \\
&= \frac{\partial F(z, k, nh)}{\partial n} - \lim_{\Delta \rightarrow 0} \frac{\nu (w_{+\Delta} (n + \Delta) h - w_{+\Delta} nh)}{\Delta} - \lim_{\Delta \rightarrow 0} \frac{(1 - \nu) (w_{+\Delta} (n + \Delta) h - w nh)}{\Delta} \\
&\quad + (1 - \chi) E \left[\beta \frac{u'_c}{u_c} J'_n \right] \\
&= \frac{\partial F(z, k, nh)}{\partial n} - \nu wh - (1 - \nu) \left(wh + \frac{\partial w}{\partial n} nh \right) + (1 - \chi) E \left[\beta \frac{u'_c}{u_c} J'_n \right] \\
J_n \equiv \frac{\partial J}{\partial n} &= \frac{\partial F(z, k, nh)}{\partial n} - wh - (1 - \nu) \frac{\partial w}{\partial n} nh + (1 - \chi) E \left[\beta \frac{u'_c}{u_c} J'_n \right]
\end{aligned}$$

Derivation of the Equilibrium Conditions

Household solves the following dynamic programming problem.

$$\begin{aligned}
\Omega(S, s_H) &= \max_{c, a'} u(c) + \nu v(1 - h(S, s_H)) + (1 - \nu) v(1) + \beta E[\Omega(S', s'_H)] \\
s.t. & \\
c + a' + T(S) &= w(S, s_H) h(S, s_H) n + (1 - n) b + (1 + r) a + \Pi(S) \\
n' &= (1 - \chi) n + \Psi(S) (1 - n) \\
S' &= G(S) \\
c &\geq 0
\end{aligned}$$

Let λ_H and μ_H be the Lagrange multiplier on budget constraint, and law of motion for employment

respectively. Then we have the following first order conditions.

$$\begin{aligned} u_c &= \lambda_H \\ E[\beta\Omega'_a] &= \lambda_H \end{aligned}$$

From the envelope condition with respect to a , we get

$$\Omega_a = (1+r)\lambda_H$$

Taking a derivative with respect to n' , we have

$$\mu_H = E[\beta\Omega'_n]$$

By combining equations above, we get the standard Euler equation.

$$E\left[\beta\frac{u'_c}{u_c}(1+r')\right] = 1$$

Now, firms solve the following problem.

$$\begin{aligned} J(S, s_H) &= \max_{v,k} \Pi(S) + E[m'(S, S') J(S', s'_F)] \\ &= \max_{v,k} F(z, k, nh) - w(S, s_F) h(S, s_F) n - (r+\delta)k - \kappa v + E[m'(S, S') J(S', s'_F)] \\ s.t. \\ n' &= (1-\chi)n + \Phi(S)v \\ S' &= G(S) \end{aligned}$$

where $m'(S, S') = \beta u(c(S'))/u(c(S))$ is the stochastic discount factor and $\Phi(S) = M/V$ is the job-filling rate.

Let μ_F be the Lagrange multipliers on law of motion of employment. Then, we have the following first order conditions for firms.

$$\begin{aligned} \kappa &= \mu_F \Phi(S) \\ r+\delta &= F_k \end{aligned}$$

Taking a derivative with respect to n' , we have

$$E[m' J'_n] = \mu_F$$

By combining equations above, we have an equation for the rental rate and a job creation condition.

$$\begin{aligned} r &= F_k - \delta \\ \kappa &= \Phi E [m' J'_n] \end{aligned}$$

Derivation of Intra-firm Bargaining Problem

The values of an additional worker for the household and the firm are given as

$$\begin{aligned} \frac{\Omega_n}{u_c} &= \frac{v(1-h) - v(1)}{u_c} + wh - b + (1 - \chi - \Psi) E \left[\beta \frac{\Omega'_n}{u_c} \right] \\ J_n &= \frac{\partial F(z, k, nh)}{\partial n} - wh - (1 - \nu) \frac{\partial w}{\partial n} \frac{n}{w} wh + (1 - \chi) E \left[\beta \frac{u'_c}{u_c} J'_n \right] \\ &= \frac{\partial F(z, k, nh)}{\partial (nh)} h - wh - (1 - \nu) \frac{\partial w}{\partial n} \frac{n}{w} wh + (1 - \chi) E \left[\beta \frac{u'_c}{u_c} J'_n \right] \end{aligned}$$

Given the bargaining weight of the new worker, $\mu \in [0, 1]$, and the bargaining weight of existing worker, $\mu \in [0, 1]$, wages and hours per worker are jointly determined by the following standard Nash bargaining problem

$$(w, h) = \arg \max_{w, h} \left(\frac{\Omega_n}{u_c} \right)^\mu (J_n)^{1-\mu}$$

The first order conditions are derived as follows

$$\mu J_n = (1 - \mu) \left(\frac{\Omega_n}{u_c} \right)$$

and

$$\begin{aligned} \mu J_n \left(-\frac{v_{(1-h)}(1-h)}{u_c} + w \right) &= (1 - \mu) \left(\frac{\Omega_n}{u_c} \right) \left(-F_{nh} + (1 - \nu) \frac{\partial w}{\partial n} n + w \right) \\ \mu J_n \left(-\frac{v_{(1-h)}(1-h)}{u_c} + w \right) &= \mu J_n \left(-F_{nh} + (1 - \nu) \frac{\partial w}{\partial n} n + w \right) \\ \frac{v_{(1-h)}(1-h)}{u_c} &= F_{nh} - (1 - \nu) \frac{\partial w}{\partial n} n \end{aligned}$$

Where $F_{nh} = \frac{\partial F(z, k, nh)}{\partial (nh)}$.¹⁸ Given the sharing rule and the definition of V_n/u_c and J_n , we have the

¹⁸Following Andolfatto (1996), it is assumed that each worker is so small such that $F_{nh} = \frac{\partial F(z, k, nh)}{\partial (nh)}$ is taken as given by both the worker and the firm during the bargaining.

following wage bill equation.

$$\begin{aligned}
\mu J_n &= (1 - \mu) \left(\frac{\Omega_n}{u_c} \right) \\
\mu \left(F_n - (1 - \nu) \frac{\partial w}{\partial n} n h - w h + (1 - \chi) E \left[\beta \frac{u'_c}{u_c} J'_n \right] \right) &= (1 - \mu) \left(\frac{v(1 - h) - v(1)}{u_c} + w h - b + (1 - \chi - \Psi) \beta E \left[\frac{\Omega'_n}{u_c} \right] \right) \\
w h &= \mu \left(F_n - (1 - \nu) \frac{\partial w}{\partial n} n h \right) + (1 - \mu) \left(\frac{v(1) - v(1 - h)}{u_c} + b \right) + \mu (1 - \chi) E \left[\beta \frac{u'_c}{u_c} J'_n \right] \\
&\quad - (1 - \mu) (1 - \chi - \Psi) \beta E \left[\frac{\Omega'_n}{u_c} \right] \\
&= \mu \left(F_n - (1 - \nu) \frac{\partial w}{\partial n} n h + \Psi E \left[\beta \frac{u'_c}{u_c} J'_n \right] \right) + (1 - \mu) \left(\frac{v(1) - v(1 - h)}{u_c} + b \right) \\
&= \mu \left(F_n - (1 - \nu) \frac{\partial w}{\partial n} n h + \frac{V}{1 - N} \kappa \right) + (1 - \mu) \left(\frac{v(1) - v(1 - h)}{u_c} + b \right)
\end{aligned}$$

Derivation of the Solution to the first order differential equation w.r.t wages

The sharing rule, the intra-temporal condition, and the wage bill are given as

$$\begin{aligned}
\mu J_n &= (1 - \mu) \left(\frac{\Omega_n}{u_c} \right) \\
\phi_e (1 - h)^{-\eta} c &= (1 - \alpha) z k^\alpha (n h)^{-\alpha} - (1 - \nu) \frac{\partial w}{\partial n} n \\
w h &= \mu \left((1 - \alpha) z k^\alpha h^{1 - \alpha} n^{-\alpha} - (1 - \nu) \frac{\partial w}{\partial n} n h + \frac{V}{1 - N} \kappa \right) + (1 - \mu) \left(\frac{v(1) - v(1 - h)}{u_c} + b \right)
\end{aligned}$$

We can rewrite the wage bill as the first order differential equation like

$$\begin{aligned}
\mu (1 - \nu) n h \frac{\partial w}{\partial n} + h w &= \mu \left((1 - \alpha) z k^\alpha h^{1 - \alpha} n^{-\alpha} + \frac{V}{1 - N} \kappa \right) + (1 - \mu) \left(\frac{v(1) - v(1 - h)}{u_c} + b \right) \\
\frac{\partial w}{\partial n} + \frac{1}{\mu (1 - \nu) n} w &= \frac{1}{\mu (1 - \nu) n h} \left(\mu \left((1 - \alpha) z k^\alpha h^{1 - \alpha} n^{-\alpha} + \frac{V}{1 - N} \kappa \right) + (1 - \mu) \left(\frac{v(1) - v(1 - h)}{u_c} + b \right) \right)
\end{aligned}$$

So, the integrating factor is

$$e^{\int \left(\frac{1}{\mu (1 - \nu) n} \right) dn} = e^{\frac{1}{\mu (1 - \nu)} \ln(n)} = n^{\frac{1}{\mu (1 - \nu)}}$$

Multiplying both sides by $n^{\frac{1}{\mu (1 - \nu)}}$ and integrating both sides, we have

$$\begin{aligned}
w &= n^{-\frac{1}{\mu(1-\nu)}} \int n^{\frac{1}{\mu(1-\nu)}} \frac{1}{\mu(1-\nu)nh} \left[\mu \left((1-\alpha) z k^\alpha h^{1-\alpha} n^{-\alpha} + \frac{V}{1-N} \kappa \right) + (1-\mu) \left(\frac{v(1)-v(1-h)}{u_c} + b \right) \right] dn \\
&\quad + D n^{-\frac{1}{\mu(1-\theta)}} \\
&= n^{-\frac{1}{\mu(1-\nu)}} \left(\frac{(1-\alpha)}{-(1-\nu)\alpha + \frac{1}{\mu}} z k^\alpha h^{-\alpha} n^{-\alpha + \frac{1}{\mu(1-\nu)}} + \frac{1}{\mu} \left(\frac{V}{1-N} \kappa + \frac{(1-\mu)}{\mu} \left(\frac{v(1)-v(1-h)}{u_c} + b \right) \right) h^{-1} n^{\frac{1}{\mu(1-\nu)}} \right) \\
&\quad + D n^{-\frac{1}{\mu(1-\theta)}} \\
&= \frac{\mu(1-\alpha)}{1-\mu(1-\nu)\alpha} z k^\alpha h^{-\alpha} n^{-\alpha} + \mu \left(\frac{V}{1-N} \kappa + \frac{(1-\mu)}{\mu} \left(\frac{v(1)-v(1-h)}{u_c} + b \right) \right) h^{-1} + D n^{-\frac{1}{\mu(1-\nu)}}
\end{aligned}$$

where D is a constant of integration of the homogeneous equation. Assuming total wage bill wnh has to remain finite as employment becomes small as in Hawkins (2011) or alternatively by assuming $\lim_{n \rightarrow 0} wnh = 0$ as in Cahuc, Maroque, and Wasmer (2008), we have $D = 0$. From the equation above, we also have

$$\frac{\partial w}{\partial n} = -\frac{\mu\alpha(1-\alpha)}{1-\mu(1-\nu)\alpha} z k^\alpha h^{-\alpha} n^{-\alpha-1} < 0$$

Using this, we have

$$\begin{aligned}
\phi_e(1-h)^{-\eta} c &= (1-\alpha) z k^\alpha (nh)^{-\alpha} - (1-\nu) \frac{\partial w}{\partial n} n \\
\phi_e(1-h)^{-\eta} c &= (1-\alpha) z k^\alpha (nh)^{-\alpha} + \frac{\mu(1-\nu)\alpha(1-\alpha)}{1-\mu(1-\nu)\alpha} z k^\alpha h^{-\alpha} n^{-\alpha} \\
&= \frac{(1-\alpha)}{1-\mu(1-\nu)\alpha} z k^\alpha (nh)^{-\alpha}
\end{aligned}$$

Equilibrium Conditions

The equilibrium of the model is characterized by the following conditions under functional forms specified in the calibration section.

$$\begin{aligned}
E \left[\beta \frac{C}{C'} (1+r') \right] &= 1 \\
r &= F_k - \delta = \alpha \frac{Y}{K} - \delta \\
N' &= (1-\chi)N + \omega V^\psi (1-N)^{1-\psi} \\
\Phi &= \omega V^{\psi-1} (1-N)^{1-\psi} \\
Y &= C + I + \kappa V
\end{aligned}$$

$$\begin{aligned}
I &= K' - (1 - \delta) K \\
Y &= F(z, K, Nh) = e^z K^\alpha (Nh)^{1-\alpha} \\
\frac{\kappa}{\Phi} &= E \left[\beta \frac{C}{C'} \left[\frac{(1-\alpha)}{1-\mu(1-\nu)\alpha} \frac{Y'}{N'} - w' h' + (1-\chi) \frac{\kappa}{\Phi'} \right] \right] \\
\phi_e (1-h)^{-\eta} C &= \frac{(1-\alpha)}{1-\mu(1-\nu)\alpha} \frac{Y}{Nh} \\
wh &= \mu \left(\frac{(1-\alpha)}{1-\mu(1-\nu)\alpha} F_n + \frac{V}{1-N} \kappa \right) + (1-\mu) \left(\left(\phi_e \frac{1}{1-\eta} - \phi_e \frac{(1-h)^{1-\eta}}{1-\eta} \right) C + b \right)
\end{aligned}$$

Data Appendix

Raw Data

1. Employment, Average Weekly Hours Worked, Population: Bureau of Labor Statistics (BLS).
2. Real GDP, GDP, Compensation of Employees, Proprietors Income, GDP deflator: National Income and Product Accounts (NIPA) published by the Bureau of Economics Analysis. (BEA)
3. Vacancies: Conference Board's Help Wanted Index and the Composite Help Wanted Index by Barnichon (2010)
4. Consumption of Fixed Capital, Capital Expenditure in non-financial non-corporate business: Flow of Funds

Constructed Data Used in Models

1. $\text{Employment} = \frac{\text{Employment}}{\text{Population}}$
2. $\text{Hours per Worker} = \frac{\text{Average Weekly Hours Worked}}{20 \times 5}$
3. $\text{Total Hours} = \text{Employment} \times \text{Hours per Worker}$
4. $\text{Labor Share} = \frac{\text{Compensation of Employees}}{\text{GDP-Proprietors Income}}$
5. $\text{Real Wage} = \frac{\text{Labor Share} \times \text{Real GDP}}{\text{Total Hours}}$
6. $\text{Labor Productivity} = \frac{\text{Real GDP}}{\text{Total Hours}}$
7. Vacancies = Conference Board's Help Wanted Index and the Composite Help Wanted Index by Barnichon (2010)
8. Investment = Capital Expenditure deflated by GDP deflator
9. Depreciation = Consumption of Fixed Capital deflated by GDP deflator
10. Capital Stock is constructed by the perpetual inventory method using follow law of motion

$$k_{t+1} = k_t + \text{Investment} - \text{Depreciation}$$

Initial capital stock is chosen so that the capital-output ratio does not display any trend over the period 1960Q1-2012Q1.