

A Theory of Targeted Search

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Abstract

We develop a model of matching where participants have finite information processing capacity. The equilibrium of our model covers the middle ground between the equilibria of random matching and the directed search literatures and reproduces them as limiting cases. Our theory of targeted search generates a unique equilibrium which is generally inefficient.

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1 Introduction

Search and matching models have played a pivotal role in macroeconomic literature. They have been instrumental tools for explaining allocations of people and goods across markets and have been useful to study the positive and normative properties of these allocations.

Two classes of allocations have emerged from the matching literature. On the one hand, a strand of the literature starting from Butters (1977), Hall (1979) and Pissarides (1979) assumes that a large set of people (or goods) are assigned to each other by luck. This outcome is known as random matching. On the other hand, the directed search literature starting from Moen (1997) predicts that the process of competitive search allows each person to perfectly identify her best match and pair with it in equilibrium. We refer to this outcome as optimal assignment.

Nevertheless, markets where one or both sides search for each other rarely display such a level of randomness or precision. In fact, in most markets - from labor and marriage markets to health care markets- people might not be paired in a completely random fashion, nor are they paired with their best match.

This paper proposes a model of this middle ground. To our knowledge, this is the first paper that produces an outcome in between random matching and optimal assignment.¹

Building on the rational inattention theory of Sims (2003), we identify costly information processing as the modeling device that can encompass situations in which the outcome of search need not be random nor perfect. Instead, when information processing is costly, people are able to only partially identify their best matches. They optimally choose to target those prospective matches which are expected to render a higher payoff. This is why we call it a theory of *targeted* search. Our theory reproduces the outcome of the random matching model when information costs tend to infinity. Likewise, optimal assignment is an outcome of our model when the information-processing constraint is not binding.

Our model features a large number of heterogenous participants with limited ability to process information on one or both sides of the market. First, we characterize the outcome of the search process when one side of the

¹To our knowledge, the only other paper which attempts to develop a model in this middle ground is Butters (1977). Since then, most models have been built using either random matching, or directed search, or combinations of these two approaches.

market is actively looking for the other side while the latter idly waits to be contacted. Then, we characterize the outcome of the search process when both sides of the market actively look for each other. In both cases, a match is formed if it is mutually beneficial and the surplus from the match is split between the two parties.

For each case, our analysis consists of two parts. First, we construct a matching market equilibrium and show conditions for its existence and uniqueness. We provide conditions under which the equilibrium strategies are computable in close form. Second, we compare the equilibria of the decentralized economy to the constrained efficient allocation. It emerges that, in equilibrium, information-processing constraints lead to (1) lower search effort than is socially optimal; (2) a lower number of matches than is socially optimal.

The role of information-processing constraints in our model is threefold. First, they generate endogenous delays in matching, as search involves balancing the cost and the precision of information about prospective matches.

Second, they result in a partially targeted distribution of attention. This distribution places a higher probability on matches which promise a higher return. By setting information costs to extreme values we can make search perfectly targeted or random.

Third, the information constraint gives rise to a novel source of inefficiency in the search process. For the search effort to be socially optimal the side of the market that has reasons to target its search should appropriate the whole surplus of the prospective match. If only one side is searching, this is feasible although very unlikely. If both sides are searching, then the socially optimal outcome cannot be attained in equilibrium.

This novel prediction of our model is a consequence of complementarities between search strategies of market participants. The complementarity comes from the fact that if one person targets another, that other has an incentive to reciprocate. These complementarities are absent in a random matching model, whereas in the optimal assignment model they are very strong and lead to multiplicity of equilibria. The information-processing constraint makes the search strategies of market participants less complementary and eliminates the multiplicity of equilibria. The same complementarity makes the equilibrium constrained inefficient.

With endogenous information selection as the driving force of matching patterns, our model is well suited to study a host of real-life matching markets where people typically have limited time and cognitive ability to process

information. Roth and Sotomayor (1990) and Sönmez and Ünver (2010) provide examples of such markets for both two-sided and one-sided search. Moreover, for many markets equilibrium outcomes are neither pure random matching nor optimal assignment as documented by the empirical literature.² Our model can be a useful tool for analyzing these markets.

From a theoretical standpoint, the paper contributes to the search and matching literature by providing a framework that produces equilibrium outcomes in between random matching and directed search.

We contribute to the literature on the microfoundations of search. Like Lagos (2000) we generate a search friction as an endogenous outcome of the model. We also find that using an exogenous aggregate matching function has different policy implications compared to the case when it is derived from first principles. However, the source of the inefficiency is different in our model.

Finally, the paper contributes to the literature on directed search and coordination frictions. The directed search paradigm generally predicts efficient equilibrium outcomes.³ In contrast, in our targeted search model there seems to be no *market* mechanism that can implement the constrained efficient allocation.

The paper proceeds as follows: Section 2 outlines the theoretical framework. We first discuss a one-sided search model and then we move on to a model of two-sided search. For each model we show conditions under which there exists a unique equilibrium and establish that equilibria in both models are socially suboptimal. Section 3 discusses extensions of the model and its applicability to different markets. Section 4 concludes. Extensions of the model are described in the Appendix.

2 Theoretical Framework

We consider an environment where a number of participants are searching for a match. We model the search process building on elements of information

²An extensive albeit necessarily non-exhaustive list of examples from the empirical literature is in Section 3.

³This paradigm was developed by Moen (1997) and has been extended in many directions. Among others, Shi (2002) considers the case of heterogeneity, Shimer (2005) accomodates coordination failures, and Kircher (2009) incorporates simultaneous search. There are a few exceptions which do generate inefficient outcomes, such as Shimer and Smith (2000) and Anderson and Smith (2010).

theory and the rational inattention literature. We assume that each agent can choose how much information to gather about potential matches. Given that information processing is costly, agents optimally choose a search strategy: a distribution of attention over all possible counterparties.⁴

When agents have infinite information processing capacity, i.e. costs of acquiring information are zero, they can perfectly identify mutual best matches, and the outcome of the model is an equilibrium of the classical assignment model. In this case each agent's attention is focused on a single counterparty, and the optimal strategy is infinitely precise. Likewise, when agents have no information processing capacity, i.e. costs of acquiring information are infinite, their attention is distributed uniformly over all possible matches, and the equilibrium outcome is random matching.

Our framework represents the middle ground connecting these two polar cases. When information processing capacity is finite, i.e. we assume that agents have finite costs of acquiring information, agents choose how to optimally distribute their attention.

Even though our mechanism applies to many different markets, here we make use of the labor market as an example. We defer the discussion of applications to other markets until Section 3.

In the next two subsections we describe the model. We start by considering the case of one-sided search. This case applies to markets where participants on one side of the market actively search, while participants on the other side are passively waiting. We then move to the two-sided search case, where both sides search actively. In each case we show conditions for existence and uniqueness of the equilibrium, characterize the equilibrium in closed form, and check whether the equilibrium is efficient. We find that the two cases we consider have remarkably different welfare implications.

2.1 One-sided matching model

We assume that there are N workers indexed by $x \in \{1, \dots, N\}$, who are actively searching, and M firms indexed by $y \in \{1, \dots, M\}$, which are waiting for applications. Firms will accept an applicant with probability $q(y)$. A match between worker x and firm y generates a surplus $f(x, y)$. If a firm and

⁴Equivalently, we could describe agents as receiving costly signals about potential matches and choosing not only the precision of these signals, but the whole probability distribution.

a worker match, the surplus is split between them by unilateral bargaining in such a way that the worker gets a wage $w(x, y)$ and the firm gets a profit $\pi(x, y)$. The surplus, wage and profit are known to all potential participants of each match. Since negative payoffs lead to de facto zero payoffs due to absence of a match, without loss of generality, we can assume that all payoffs are non-negative:

$$f(x, y) \geq 0, \quad \pi(x, y) \geq 0, \quad w(x, y) \geq 0.$$

The strategy of a worker, denoted $p(x, y)$, represents the probability of worker x applying to firm y . It is also the worker's distribution of attention. We assume that each worker can rationally choose his strategy while facing a trade-off between a higher payoff and a higher cost of processing information.

A worker's cost of searching is given by $c(x, \kappa(x))$. This cost is a function of the amount of information processed by a worker measured in bits, $\kappa(x)$. Each worker makes a single draw from the chosen distribution $p(x, y)$ to determine where to send an application.

Worker x chooses a strategy $p(x, y)$ to maximize his expected income flow:

$$Y(x) = \sum_{y=1}^M w(x, y) p(x, y) q(y) - c(x, \kappa(x)) \rightarrow \max_{p(x, y)}$$

We normalize the outside option of the worker to zero. The worker receives his expected wage in a match with firm y conditional on matching with that firm. He also incurs a search cost, which depends on the information processing capacity defined as follows:

$$\kappa(x) = \sum_{y=1}^M p(x, y) \log_2 \frac{p(x, y)}{1/M} \quad (1)$$

where the worker's strategy must satisfy $\sum_{y=1}^M p(x, y) = 1$ and $p(x, y) \geq 0$

for all y . Our definition of information, $\kappa(x)$, represents the relative entropy between a uniform prior $\{1/M\}$ over firms and the posterior strategy, $p(x, y)$. Shannon's relative entropy can be interpreted as the reduction of uncertainty that the worker can achieve by choosing his distribution of attention. This definition is a special case of Shannon's channel capacity when information

structure is the only choice variable. Thus, our assumption is a special case of a uniformly accepted definition of information tailored to our problem.

Definition 1 *A matching equilibrium of the one-sided matching model is a set of strategies of workers, $\{p(x, y)\}_{x=1}^N$, which solve their optimization problems.*

Theorem 1 *If the cost functions are non-decreasing and convex, the one-sided matching model has a unique equilibrium.*

Proof. The payoffs of all workers are continuous in their strategies. They are also concave in these strategies when cost functions are (weakly) increasing and convex in information capacities. Hence, each problem has a unique solution. ■

When in addition the cost functions are differentiable, it is easy to verify that first-order conditions are necessary and sufficient conditions for equilibrium.⁵ Rearranging the first order conditions for the worker, we obtain:

$$p^*(x, y) = \exp \left(\frac{w(x, y) q(y)}{\frac{1}{\ln 2} \frac{\partial c(x, \kappa)}{\partial \kappa} \Big|_{p^*}} \right) / \sum_{y'=1}^M \exp \left(\frac{w(x, y') q(y')}{\frac{1}{\ln 2} \frac{\partial c(x, \kappa)}{\partial \kappa} \Big|_{p^*}} \right). \quad (2)$$

Note that the assumptions we use to prove uniqueness are by no means restrictive. The assumption that cost functions are non-decreasing and convex is a natural one. Most of the literature on information processing assumes that either the cost function is linear, or there is a capacity constraint on processing information, which implies a vertical cost function after a certain amount of information has been processed. Our assumption incorporates both of these as special cases.

⁵Taking derivatives of the Lagrangian function corresponding to the problem of worker x , we obtain for all y :

$$w(x, y) q(y) - \frac{\partial c(x, \kappa)}{\partial \kappa} \Big|_{p^*} \frac{1}{\ln 2} \left(\ln \frac{p^*(x, y)}{1/M} + 1 \right) = \lambda(x)$$

We can invert this first-order condition to characterize the optimal strategy:

$$p^*(x, y) = \frac{1}{M} \exp \left(\frac{w(x, y) q(y)}{\frac{1}{\ln 2} \frac{\partial c(x, \kappa)}{\partial \kappa} \Big|_{p^*}} - \frac{\lambda(x)}{\frac{1}{\ln 2} \frac{\partial c(x, \kappa)}{\partial \kappa} \Big|_{p^*}} - 1 \right)$$

The equilibrium condition (2) has an intuitive interpretation. It predicts that the higher is the worker's expected gain from matching with a firm, the higher is the probability of applying to that firm. Thus, firms are naturally sorted in each worker's strategy by probabilities of applying to those firms.

It is instructive to understand the properties of the equilibrium for two limiting cases. First, consider the case when marginal costs of processing information go to zero. In this case, the strategies become more and more focused. In the limit, each worker places a unit probability on a single firm. Second, consider the opposite case when marginal costs go to infinity. In this case, the difference between probabilities of applying to different firms shrinks. In the limit, optimal strategies of workers approach a uniform distribution.

Our model generates a continuum of possible outcomes in between these two special cases. For intermediate values of costs the search strategies of workers are distributed among all firms and optimally skewed towards their best matches.

Efficiency In order to evaluate the efficiency of the equilibrium we compare the solution to the decentralized problem to a social planner's solution. We assume that the social planner maximizes the total surplus of the economy, which is a utilitarian welfare function. In order to achieve a social optimum, the planner can choose workers' strategies. If no costs of processing information were present, the planner would always choose to match each worker with the job that produces the highest surplus. The socially optimal strategies of workers would be infinitely precise.

To study the constrained efficient allocation we impose upon the social planner the same information processing constraints that we place on the workers. Thus, the social planner maximizes the following welfare function:

$$\sum_{x=1}^N \sum_{y=1}^M f(x, y) p(x, y) q(y) - \sum_{x=1}^N c(x, \kappa(x))$$

subject to the information constraint (1) and to the constraint that the $p(x, y)$'s are well-defined probability distributions. Under the assumption of increasing convex cost functions, the social welfare function is concave in the strategies of workers. Hence, first-order conditions are sufficient conditions for a maximum. Rearranging and substituting out Lagrange multipliers, we arrive at the following characterization of the social planner's allocation:

$$p^o(x, y) = \exp \left(\frac{f(x, y) q(y)}{\frac{1}{\ln 2} \frac{\partial c(x, \kappa)}{\partial \kappa} \Big|_{p^o}} \right) / \sum_{y'=1}^M \exp \left(\frac{f(x, y') q(y)}{\frac{1}{\ln 2} \frac{\partial c(x, \kappa)}{\partial \kappa} \Big|_{p^o}} \right). \quad (3)$$

The first observation to make is that the structure of the social planner's solution is very similar to the structure of the decentralized equilibrium. Second, from the workers' perspective, the only difference between the centralized and decentralized equilibrium strategies is that the probability of applying to a firm depends on the social gain from a match rather than on the private gain. Thus, it is socially optimal to consider the whole expected surplus when determining the socially optimal strategies, while in the decentralized equilibrium workers only consider their private gains.

To decentralize the socially optimal outcome the planner needs to give all of the surplus to the workers, $w(x, y) = f(x, y)$, effectively assigning them a bargaining power of 1. Note that, if the planner could choose the probability that a firm accepts a worker, $q(y)$, he would also set it to 1.

The only special cases, when the outcome is always efficient are the limiting cases discussed earlier. When costs of information are absent, the equilibrium of the model is socially optimal. When costs of information are very high, the random matching outcome is the best possible outcome. For all intermediate values of costs, the decentralized equilibrium is socially efficient contingent on the worker having all the bargaining power.

2.2 Two-sided matching model

Now assume that both workers and firms search for each other. Similarly to strategies of workers $p(x, y)$, we denote strategies of firms $q(y, x)$. They represent probabilities of worker x applying to firm y and firm y considering application of worker x respectively. We also refer to them as allocations of attention of workers and firms. We assume that both workers and firms can rationally choose these strategies given potential gains and costs of search. We denote costs of search of workers and firms, $c_w(x, \kappa_w(x))$ and $c_f(y, \kappa_f(y))$, respectively. As before, these costs are functions of information capacities, denoted $\kappa_w(x)$ for workers and $\kappa_f(y)$ for firms.

We denote $m_w(x, y)$ the equilibrium matching rate faced by worker x when applying to firm y . Similarly, we denote $m_f(y, x)$ the equilibrium matching rate faced by firm y when considering worker x . These rates are

assumed to be common knowledge to participating parties. We model worker applications and firm acceptances as random draws from the chosen distributions $p(x, y)$ and $q(y, x)$.

We restrict ourselves to a one shot model. That is, each worker sends an application to a single firm, and each firm accepts an application from a single worker. A match is formed between worker x and firm y if and only if: 1) according to the worker's random draw from $p(x, y)$, worker x applies to firm y ; 2) according to the firm's random draw from $q(y, x)$, firm y accepts the application from worker x ; and 3) their payoffs are non-negative. Without loss of generality we restrict payoffs to be non-negative.

Worker x chooses his strategy $p(x, y)$ to maximize his expected income flow:

$$Y_w(x) = \sum_{y=1}^M w(x, y) m_w(x, y) p(x, y) - c_w(x, \kappa_w(x)) \rightarrow \max_{p(x, y)}.$$

We normalize the outside options of the worker and the firm to zero. The worker gets his expected wage from a match with firm y conditional on matching with that firm. He also incurs a search cost, which depends on the amount of information he processed, defined as follows:

$$\kappa_w(x) = \sum_{y=1}^M p(x, y) \log_2 \frac{p(x, y)}{1/M}, \quad (4)$$

where the worker's strategy must satisfy $\sum_{y=1}^M p(x, y) = 1$ and $p(x, y) \geq 0$ for all y .

Similarly, firm y chooses her strategy $q(y, x)$ to maximize her expected income flow:

$$Y_f(y) = \sum_{x=1}^N \pi(x, y) m_f(y, x) q(y, x) - c_f(y, \kappa_f(y)) \rightarrow \max_{q(y, x)}.$$

The firm profits from a match with worker x conditional on matching with that worker and pays the cost of search. The search cost on the firm's side also depends on the amount of information processed:

$$\kappa_f(y) = \sum_{x=1}^N q(y, x) \log_2 \frac{q(y, x)}{1/N}, \quad (5)$$

where the firm's strategy must satisfy $\sum_{x=1}^N q(y, x) dx = 1$ and $q(y, x) \geq 0$ for all x .

Definition 2 A matching equilibrium is a set of strategies of workers, $\{p(x, y)\}_{x=1}^N$, and firms, $\{q(y, x)\}_{y=1}^M$, and matching rates $\{m_f(y, x)\}_{x,y=1}^{N,M}$ and $\{m_w(x, y)\}_{x,y=1}^{N,M}$ such that:

- 1) strategies solve problems of the workers and the firms;
- 2) matching rates satisfy equilibrium conditions:

$$m_f(y, x) = p(x, y), \quad m_w(x, y) = q(y, x). \quad (6)$$

Theorem 2 A (Nash) equilibrium of the matching model exists.

Proof. Note that if we substitute the matching rates (6) into the payoffs of workers and firms we can express the model as a normal-form game. The equilibrium of the matching model can be interpreted as a standard Nash equilibrium. All the results for lattices described by Vives (1990) apply to it. Since cross-derivatives of objective functions in our case are all non-negative, this game is supermodular. Hence there exists a Nash equilibrium. ■

When cost functions are non-decreasing and convex, it is easy to verify that first-order conditions are necessary and sufficient conditions for equilibrium. Rearranging the first order conditions for the worker and the firm, we obtain:

$$\begin{aligned} p^*(x, y) &= \exp \left(\frac{w(x, y) q^*(y, x)}{\frac{1}{\ln 2} \frac{\partial c_w(\kappa_w)}{\partial \kappa_w} \Big|_{p^*}} \right) / \sum_{y'=1}^M \exp \left(\frac{w(x, y') q^*(y', x)}{\frac{1}{\ln 2} \frac{\partial c_w(\kappa_w)}{\partial \kappa_w} \Big|_{p^*}} \right), \\ q^*(y, x) &= \exp \left(\frac{\pi(x, y) p^*(x, y)}{\frac{1}{\ln 2} \frac{\partial c_f(\kappa_f)}{\partial \kappa_f} \Big|_{q^*}} \right) / \sum_{y'=1}^M \exp \left(\frac{\pi(x', y) p^*(x', y)}{\frac{1}{\ln 2} \frac{\partial c_f(\kappa_f)}{\partial \kappa_f} \Big|_{q^*}} \right). \end{aligned} \quad (7)$$

These sufficient conditions for equilibrium cast the optimal strategy of worker x and firm y in the form of a best response to optimal strategies of firms and workers respectively.

Theorem 3 *The equilibrium of the model is unique, if*

a) cost functions are non-decreasing and convex;

$$b) \left. \frac{\partial c_w(\kappa_w)}{\partial \kappa_w} \right|_{p^*(x,y)} > w(x,y) p^*(x,y);$$

$$c) \left. \frac{\partial c_f(\kappa_f)}{\partial \kappa_f} \right|_{q^*(y,x)} > \pi(x,y) q^*(y,x).$$

Proof. The payoffs of all firms and workers are continuous in their strategies. They are also concave in these strategies when cost functions are (weakly) increasing and convex in information capacities. "Diagonal dominance" conditions (b) and (c) guarantee that the Hessian of the game is negative definite along the equilibrium path. Then, by the generalized Poincare-Hopf index theorem of Simsek, Ozdaglar and Acemoglu (2007), the equilibrium is unique. ■

Note that the assumptions we make to prove uniqueness are by no means restrictive. The assumption that cost functions are non-decreasing and convex is a natural one. The additional "Diagonal dominance" conditions in our case can be interpreted as implying that the marginal cost of information processing should be sufficiently high for the equilibrium to be unique. If these conditions don't hold, then there can be multiple equilibria. This is a well-known outcome of the assignment model, which is a special case of our model under zero marginal information costs.

Note also, that by the nature of the index theorem used in the proof, it is enough to check diagonal dominance conditions locally in the neighborhood of the equilibrium. There is no requirement for them to hold globally. This suggests a simple way of finding equilibria of our model in most interesting cases. We first need to find one solution to the first-order conditions (7) and then check that diagonal dominance conditions are satisfied.

Equilibrium conditions (7) have an intuitive interpretation. They predict that the higher the worker's private gain from matching with a firm, the higher the probability of applying to that firm. Similarly, the higher the probability that a firm considers a particular worker, the higher the probability that that worker applies to the firm. Overall, workers place higher probabilities on applications to firms which give them higher expected private gains.

Thus, firms are naturally sorted in each worker's strategy by the probability of applying to each firm. The strategies of firms have the same properties due to the symmetry of the problem. In equilibrium, a firm's strategy is a best response to the strategies of workers, and a worker's strategy is a best response to the strategies of firms.

Now, consider the properties of equilibria for two limiting cases. First, as the marginal costs of processing information go to zero, application and consideration strategies become more and more precise. In the limit, in every equilibrium each worker places a unit probability on a particular firm, and that firm responds with a unit probability of considering that worker. Each equilibrium of this kind implements a stable matching of the classical assignment problem.

Second, consider the opposite case when marginal costs go to infinity. In this case, optimal strategies of firms and workers approach a uniform distribution. This unique equilibrium implements the standard uniform random matching assumption extensively used in the literature. Thus, the assignment model and the random matching model are special cases of our model, when costs of information are either very low or very high.

Efficiency To study the welfare implications of the two-sided model we now consider the social planner's problem. As before, we assume that the social planner maximizes a utilitarian welfare function. To study the constrained efficient allocation we impose upon the social planner the same information processing constraints that we place on workers and firms. In order to achieve a social optimum, the planner maximizes the following welfare function by choosing search strategies for workers and firms:

$$\sum_{x=1}^N \sum_{y=1}^M f(x, y) p(x, y) q(y, x) - \sum_{x=1}^N c_w(x, \kappa_w(x)) - \sum_{y=1}^M c_f(y, \kappa_f(y))$$

subject to information constraints (4-5) and to the constraints that $p(x, y)$ and $q(y, x)$ are well-defined probability distributions.

Under the assumption of increasing convex cost functions, the social welfare function is concave in the strategies of workers and firms. Hence, first-order conditions are sufficient conditions for a maximum. Rearranging and substituting out Lagrange multipliers, we arrive at the following characterization of the social planner's allocation:

$$\begin{aligned}
p^o(x, y) &= \exp \left(\frac{f(x, y) q^o(y, x)}{\frac{1}{\ln 2} \frac{\partial c_w(\kappa_w)}{\partial \kappa_w} \Big|_{p^o}} \right) / \sum_{y'=1}^M \exp \left(\frac{f(x, y') q^o(y', x)}{\frac{1}{\ln 2} \frac{\partial c_w(\kappa_w)}{\partial \kappa_w} \Big|_{p^o}} \right), \\
q^o(y, x) &= \exp \left(\frac{f(x, y) p^o(x, y)}{\frac{1}{\ln 2} \frac{\partial c_f(\kappa_f)}{\partial \kappa_f} \Big|_{q^o}} \right) / \sum_{y'=1}^M \exp \left(\frac{f(x', y) p^o(x', y)}{\frac{1}{\ln 2} \frac{\partial c_f(\kappa_f)}{\partial \kappa_f} \Big|_{q^o}} \right). \quad (8)
\end{aligned}$$

The structure of the social planner's solution is very similar to the structure of decentralized equilibrium. From the workers' perspective, the only difference between the centralized and decentralized equilibrium strategies is that the probability of applying to a firm depends on the social gain from a match rather than on the private gain. Notice the same difference holds from the perspective of the firm. Thus, it is socially optimal for both firms and workers to consider the total surplus, while in the decentralized equilibrium they only consider their private gains.

Since the social gain is always the sum of private gains, there is no feasible way of splitting the surplus such that it implements the social optimum. When information costs are finite and positive, a socially optimal equilibrium has to satisfy the following conditions simultaneously:

$$\pi(x, y) = f(x, y), \quad w(x, y) = f(x, y).$$

These optimality conditions can only hold in equilibrium if the surplus is zero, as private gains have to add up to the total surplus, $\pi(x, y) + w(x, y) = f(x, y)$. Therefore, we have just proven the following theorem:

Theorem 4 *The equilibrium is socially inefficient for any split of the surplus if all of the following hold:*

- 1) *the equilibrium is unique;*
- 2) *$f(x, y) > 0$ for some (x, y) ;*
- 3) *$f(x, y) \neq f(x, y')$ for some x, y and y' ;*
- 4) *$f(x, y) \neq f(x', y)$ for some x, y and x' ;*
- 5) $0 < \frac{\partial c_w(\kappa_w)}{\partial \kappa_w} \Big|_{p^*} < \infty$;
- 6) $0 < \frac{\partial c_f(\kappa_f)}{\partial \kappa_f} \Big|_{q^*} < \infty$.

The first two conditions are self-explanatory; the case when all potential matches yield zero surplus is a trivial case of no gains from matching. Conditions 5 and 6 state that marginal costs of information have to be finite and positive in the neighbourhood of equilibrium. When costs of information are zero, the best equilibrium of the assignment model is socially optimal. When costs of information are very high, the random matching outcome is the best possible outcome. For all intermediate values of costs the decentralized equilibrium is socially inefficient.

Conditions 3 and 4 together require heterogeneity to be two-sided. If heterogeneity is one-sided, i.e. condition 3 or condition 4 is violated, then the allocation of attention towards the homogeneous side of the market will be uniform. In this case, search becomes one-sided and equilibrium allocations are efficient contingent on one side having all the bargaining power.

One notable property of the equilibrium is that, by considering only fractions of the total surplus in choosing their strategies, workers and firms place lower probabilities on applying to their best matches. This implies that in equilibrium attention of workers and firms is more dispersed and the number of matches is lower than is socially optimal. The main reason for this inefficiency is the reduction in strategic complementarities.

To illustrate these complementarities consider the case of a firm, which chooses its strategy under the assumption that all workers implement socially optimal application strategies. Because the firm only considers its private gains from matches with workers, the firm's optimal response would be to pay less attention to the best workers, than is socially optimal. In a second step, taking as given these strategies of firms, workers will be dis-incentivized not only by the fact that they consider fractions of the total gains from a match, but also by the fact that firms pay less attention to them than it is socially optimal. These complementary dis-incentives will lower the probabilities of applying to their best matches for all workers. Iterating in this way on strategies of workers and firms, at each step we get a reduction in the probability of applying to the best match.

The negative externalities of considering only private gains by workers and firms reinforce each other through strategic best responses of workers to firms and firms to workers. Thus, we have uncovered a major source of inefficiency in the matching process. Information processing constraints weaken strategic complementarities between strategies of workers and firms. By the same token they reduce synergies from cooperation and enhance negative externalities in search efforts. As a consequence of these negative externalities,

firms and workers fail to fully internalize the gains from coordination.

The inefficiency that arises in the two-sided model can in principle be corrected by a central planner. This can be done by promising both workers and firms that they will get the whole surplus and then collecting lump-sum taxes from both sides of the market to cover the costs of the program. Nevertheless, in order to do so, the planner himself would need to acquire extensive knowledge about the distribution of the surplus, which is costly. We leave this direction of research for future work.

3 Extensions and applications

In this section we start by describing possible extensions of the model. Then we discuss how different markets can be modeled using the different versions of our framework. We point out the relationship between different properties of markets and the corresponding choice of model.

3.1 Extensions

The model presented in Section 2 is revealing but parsimonious. We assumed that each participant sends or considers only one application. If the target of that application does not reciprocate, then the participant is unmatched forever. Furthermore, if we double the number of participants on both sides of the market, the number of matches barely increases.

In the Appendix we extend our model to a continuous-time framework with a continuum of workers and firms, multiple applications and a more realistic meeting protocol. The general model allows market participants to choose how many counterparties to contact and fully accounts for the possibility of repeated interactions. The meeting protocol modulates congestion and can lead to constant returns to scale. We show the assumptions that are necessary to derive a constant-returns-to-scale matching function and to solve the model in close form.

All the qualitative results from Section 2 carry through to this richer environment. We show that under similar conditions the equilibrium of the general model is unique and generally inefficient.

3.2 Applications

Modeling search frictions with information-processing constraints might be a useful representation of many types of markets where the equilibrium outcome is neither random matching nor mutual best matches as postulated by the classical assignment model. We can categorize these markets according to the interaction of six main factors.

The first factor is the number of participants on each side of the market. The larger the number of participants the higher is the effective cost of information. An asymmetry in the number of participants will determine if search is one-sided or two-sided. For instance, if the number of participants on one side of the market is restricted to one or two, then the cost of search for the other side is relatively low, and search is one-sided. Grocery shopping is a one-sided search effort, while search in the commercial loans market is two-sided.

The second factor is the degree of heterogeneity among participants. If participants on one side of the market are equally valuable to participants on the other side, then all matches are equally beneficial. In this case, only the homogeneous side of the market will actively search. However, if agents on both sides of the market are heterogeneous, search is two-sided. An example of a homogeneous good leading to one-sided search is the consumer electricity market.

The third factor is whether both demand per buyer and supply per seller are limited. If a product can be produced by the same supplier in unrestricted quantities, then it can satisfy any demand and has no incentive to actively search for customers. In this case, search is one-sided. Credit cards are an example of unlimited supply, while the marriage market has symmetric limitations. Restaurants can accomodate a finite number of eaters which places them in the middle of the spectrum.

The fourth factor which affects search is the period of time for which a potential match remains beneficial, i.e. the durability of the surplus. When the possibility of a match remains in place for a long period of time, this gives both sides of the market more time to collect information and makes the necessity to match less urgent. Durability effectively lowers the costs of information processing. The housing rental market is an example of durable surplus. The typical search period is long enough to find out all the possible options. While the market for human organs is a case where a delay makes

a match obsolete.⁶

The fifth factor is the structure of information flow. Our model applies to markets where information flow is unrestricted on either side of the market. The only restriction posed is on the capacity of participants to process information about the other side. This assumption is compatible with markets where participants on one side of the market endogenously choose to process more information than on the other. For instance, a student applying for public school in the U.S. may choose to process more information about underchosen schools in his district than his peers to increase the odds of being selected.⁷ This endogenously chosen information advantage agrees with the set-up of our model.

In contrast, such an assumption makes our model not directly applicable to markets where asymmetric information arises from ex-ante restrictions on information flow on either side of the market. For instance, markets for used goods and mortgage loans⁸ are better captured by models of private information. However, our model can be nested into models of search with asymmetric information.⁹

The sixth factor is the degree of centralization in the market. By centralization we mean a situation when an organization or a platform facilitates search by structuring the information flow and setting the rules for interaction. Our model describes markets where the degree of centralization is fairly low. This structure encompasses a number of markets ranging from labor markets to education and health care.¹⁰ In contrast, our model does not directly apply to markets where the degree of centralization is fairly high as in the case of football bowls, college admissions, market for physicians, and two-sided platform markets.¹¹ Specifically, in two-sided market mod-

⁶See Roth, Sönmez and Ünver(2007) for the case of kidney exchange.

⁷See Schneider, Teske and Marschall (2000).

⁸Used goods markets are described in, e.g. Lewis (2011), while mortgage loans are documented in Woodward and Hall (2010).

⁹Such as the work of Guerrieri, Shimer and Wright (2010) and Guerrieri and Shimer (2012).

¹⁰For instance, the efficiency of the labor market for new economists is discussed in Coles, Cawley, Levine, Niederle, Roth, Siegfried (2010). For the case of the education market see, e.g., Ballou (2010). A study of the medicare advantage program is conducted by Brown, Duggan, Kuziemko and Wooldston (2011).

¹¹See Frechette, Roth and Ünver(2010) for the case of football bowls. College admissions are studied by Roth and Sotomayor (1989), the market for physicians by, e.g., Roth and Peranson (1999). Two-sided and multi-sided market models have been developed by

els the platform acts both as a coordination device and as a mechanism of surplus transfers. Our model can be used to study the optimal degree of centralization and the social efficiency of pricing schemes in these markets. We leave the study of the optimal design of centralization in two-sided search environments for future research.

While most of the matching markets we mention above would not be well described by either random matching or classical assignment, they rest comfortably within the predictions of our model of targeted search.

4 Conclusion

We presented a model of matching where participants have finite information-processing capacity. We established that such a model delivers an outcome that lies in between the findings of random matching models and optimal assignment models. Furthermore, if we assume information-processing costs to be high (infinite in the limit) the outcome of our model is observationally equivalent to that of a random matching model whereas if information-processing costs are zero, then our model mimicks the outcomes of an optimal assignment model.

The novel predictions of our model are a consequence of complementarities between search strategies of participants on both sides of the market. Consider for instance the labor market. If a worker knows that a firm is interested in him, he would actively seek out that firm. This complementarity is absent in a random matching model, whereas in the optimal assignment model it is very strong and leads to multiplicity of equilibria. In our model, the information-processing constraint makes the search strategies of market participants less complementary and eliminates the multiplicity of equilibria. However, the complementarity makes the unique equilibrium inefficient.¹²

As long as the Hosios condition holds, the random matching equilibrium is unique and efficient while in an optimal assignment model efficiency is achieved by refinement. Our model of targeted search fills the middle ground between these two cases. It generates a unique equilibrium which is generally inefficient.

Rochet and Tirole (2003) and Weyl (2010).

¹²The optimal way of correcting the inefficiency depends on whether search is one-sided or two-sided. We leave a detailed analysis of mechanisms which could correct the inefficiency for future research.

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5 Appendix Extended model

5.1 Primitives

Let worker and firm types be continuously distributed on compact measurable sets X and Y . Let there be a measure $u(x)$ of workers of each type $x \in X$ and a measure $v(y)$ of firms of each type $y \in Y$. Workers and firms search for each other in order to match. Like before, a match between a worker of type x and firm y generates a surplus $f(x, y)$. If a firm and a worker match, the surplus is split between the worker and the firm in a such a way that the worker gets a wage $w(x, y)$ and the firm gets a profit $\pi(x, y)$. The surplus, wage and profit conditional on types are common knowledge.

We assume that the worker and the firm face relatively general search costs of the forms:

$$c_w(\alpha(x), \kappa(x)) = \chi_w(x) \frac{\alpha(x)^{k_w(x)}}{k_w(x)} + \alpha(x) \theta_w(x) \kappa_w(x)$$

$$c_f(\gamma(y), \kappa(y)) = \chi_f(y) \frac{\gamma(y)^{k_f(y)}}{k_f(y)} + \gamma(y) \theta_f(y) \kappa_f(y)$$

Each search cost has two components. The first component represents a convex cost of processing applications, which depends only on the numbers of applications, $\alpha(x)$ and $\gamma(y)$. The second component is the cost of processing information. It is proportional to the number of applications. Following notation of the one-shot model, $\kappa_w(x)$ and $\kappa_f(y)$ are amounts of information per application processed by firms and workers. Both are measured in bits. Agent-specific parameters, denoted $\theta_w(x)$ and $\theta_f(y)$, stand for marginal costs of processing information in dollars per bit.

Denote the equilibrium matching rate faced by the worker of type x when applying to a firm of type y as $m_w(x, y)$. Similarly, we denote the matching rate faced by firm y when considering worker of type x as $m_f(y, x)$. The worker maximizes his expected income flow:

$$Y_w(x) = \int_Y w(x, y) m_w(x, y) p(x, y) \alpha(x) dy - c_w(\alpha(x), \kappa_w(x))$$

with respect to his search intensity $\alpha(x)$ and allocation of attention $p(x, y)$. The worker x gets his expected wage conditional on matching with a firm of

type y net of the cost of search. The search cost depends on the amount of information processed by the worker, defined as follows:

$$\kappa_w(x) = \int_Y p(x, y) \log_2 \frac{p(x, y)}{v(y) / \int_Y v(y) dy} dy \quad (9)$$

where $p(x, y)$ is a probability distribution, which satisfies the usual assumptions:

$$\int_Y p(x, y) dy = 1, \quad p(x, y) \geq 0. \quad (10)$$

The firm also maximizes her expected income flow:

$$Y_f(y) = \int_X \pi(x, y) m_f(y, x) q(y, x) \gamma(y) dx - c_f(\gamma(y), \kappa_f(y))$$

with respect to her search intensity $\gamma(y)$ and allocation of attention $q(y, x)$. Firm y gets a profit conditional on matching with a worker of type x net of the cost of search. The search cost depends on the amount of information processed by the worker, defined as follows:

$$\kappa_f(y) = \int_X q(y, x) \log_2 \frac{q(y, x)}{u(x) / \int_X u(x) dx} dx \quad (11)$$

where $q(y, x)$ is a probability distribution, which also satisfies the usual assumptions:

$$\int_X q(y, x) dx = 1, \quad q(y, x) \geq 0. \quad (12)$$

5.2 Meeting protocol

We extend the telephone line meeting protocol of Stevens (2007) to allow for two-sided heterogeneity as shown in Figure 1. We assume that out of the stock of $u(x)$ workers of type x , $\alpha(x)u(x)$ are sending applications, while the rest are enjoying leisure/waiting. The expected number of applications sent by worker of type x to firm of type y is $p(x, y)\alpha(x)u(x)$.

Out of the stock of $v(y)$ firms of type y , $v_p(y)$ spend time processing applications. Before knowing the type of worker they are facing, firms choose applications from which types of workers to pay attention to, and how quickly

to respond. Upon receiving an application from worker of type x , the firm processes on average $\gamma(y)$ applications and accepts the application with probability $q(y, x)$.

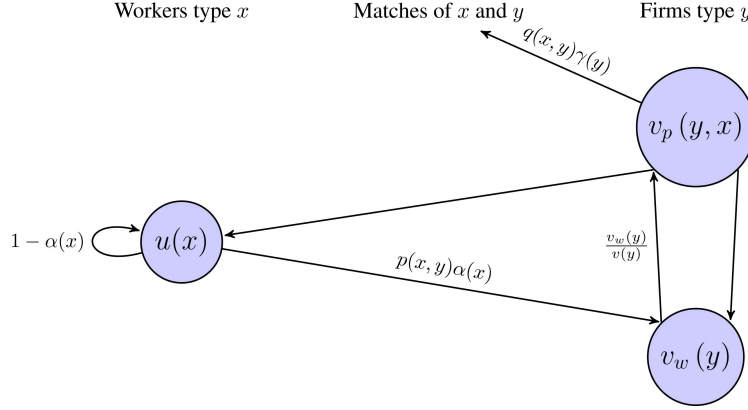


Figure 1. Meeting Protocol

We denote $v_p(x, y)$ the stock of firms of type y processing applications from workers of type x . A fraction $\gamma(y)$ of them transition to the waiting state per period. The total outflow to the waiting state is $\gamma(y) v_p(y)$. Those firms that accepted the application hire the worker and are replaced by a copy of them in the waiting pool. Those which rejected the application start waiting for another application to arrive. In a stationary equilibrium, the inflow of firms into the processing pool equals the outflow:

$$u_s(x, y) \frac{v_w(y)}{v(y)} = \gamma(y) v_p(x, y).$$

Using the accounting identity for the number of firms of type y , we can solve for the numbers of firms in each state. Then, the equilibrium number of matches for each pair of types equals:

$$m(x, y) = \gamma(y) v_p(x, y) q(y, x) = \frac{p(x, y) \alpha(x) u(x) q(y, x) \gamma(y) v(y)}{\int_X (v(y) \gamma(y) q(y, x') + p(x', y) \alpha(x') u(x')) dx'}$$

The personal meeting rates arising from this meeting protocol are computed as follows:

$$\mu_w(x, y) = \frac{m(x, y)}{q(y, x) p(x, y) \alpha(x) \gamma(y) u(x)} \frac{1}{u(x)} \quad \mu_f(y, x) = \frac{m(x, y)}{q(y, x) p(x, y) \alpha(x) \gamma(y) v(y)} \frac{1}{v(y)}$$

5.3 Equilibrium

Definition 3 *An equilibrium matching process is a set of strategies of workers $\{p(x, y), \alpha(x)\}$ and firms $\{q(y, x), \gamma(y)\}$, matching rates $m_f(y, x)$ and $m_w(x, y)$ such that:*

- 1) *strategies solve problems of the workers and firms;*
- 2) *matching rates satisfy steady-state equilibrium conditions:*

$$m_w(x, y) = q(y, x) \gamma(y) \mu_w(x, y), \quad m_f(y, x) = p(x, y) \alpha(x) \mu_f(x, y).$$

We can simplify the definition of equilibrium and cast it into a Bayesian Nash equilibrium by redefining strategies of firms and workers. We introduce the following notation:

$$\hat{p}(x, y) = \alpha(x) p(x, y), \quad \hat{q}(y, x) = \gamma(y) q(y, x)$$

Utilizing this notation, the workers' and firms' problems can be rewritten as an unconstrained maximization problems with payoffs:

$$Y_x(\hat{p}_x, \hat{q}) = \left[\int_Y w(x, y) \mu_w(x, y) \hat{q}(y, x) \hat{p}(x, y) dy - \chi_w(x) \frac{(\int_Y \hat{p}(x, y) dy)^{k_w(x)}}{k_w(x)} \right. \\ \left. - \frac{\theta_w(x)}{\ln 2} \int_Y \hat{p}(x, y) \ln \frac{\hat{p}(x, y) / \int_Y \hat{p}(x, y') dy'}{v(y) / \int_Y v(y') dy'} dy \right]$$

$$Y_y(\hat{q}_y, \hat{p}) = \left[\int_X \pi(x, y) \mu_f(x, y) \hat{p}(x, y) \hat{q}(y, x) dx - \chi_f(y) \frac{(\int_X \hat{q}(y, x) dx)^{k_f(y)}}{k_f(y)} \right. \\ \left. - \frac{\theta_f(y)}{\ln 2} \int_X \hat{q}(y, x) \ln \frac{\hat{q}(y, x) / \int_X \hat{q}(y, x') dx'}{u(x) / \int_X u(x') dx'} dx \right]$$

where equilibrium meeting rates are taken as given. These payoffs can be analyzed and optimized using standard techniques borrowed from the calculus of variations. We leave out the technical details and proofs for now. We introduce the following assumptions:

- A1.** Type sets $x \in X$ and $y \in Y$ are compact.
- A2.** Action sets $\hat{p}_x \in [0, P]$ and $\hat{q}_y \in [0, Q]$ are compact, i.e. P and Q are finite.
- A3.** $w(x, y) \mu_w(x, y) \geq 0$ and $\pi(x, y) \mu_f(x, y) \geq 0$ for all x and y .

- A4.** Costs parameters $\theta_w(x)$, $\theta_f(y)$, $\chi_w(x)$, $\chi_f(x)$ are non-negative. Costs of applications are convex: $k_w(x) \geq 1$, $k_f(y) \geq 1$.
- A5.** "Diagonal dominance" conditions are satisfied along the equilibrium path:

$$\left| \frac{\partial^2 Y_x(\hat{p}_x, \hat{q}_y)}{\partial \hat{p}_x \partial \hat{p}_x} \right|_{\hat{p}_x^*, \hat{q}_y^*} > \left| \frac{\partial^2 Y_x(\hat{p}_x, \hat{q}_y)}{\partial \hat{p}_x \partial \hat{q}_y} \right|_{\hat{p}_x^*, \hat{q}_y^*}$$

$$\left| \frac{\partial^2 Y_y(\hat{q}_y, \hat{p}_x)}{\partial \hat{q}_y \partial \hat{q}_y} \right|_{\hat{p}_x^*, \hat{q}_y^*} > \left| \frac{\partial^2 Y_y(\hat{q}_y, \hat{p}_x)}{\partial \hat{q}_y \partial \hat{p}_x} \right|_{\hat{p}_x^*, \hat{q}_y^*}$$

Assumptions A1-A2 postulate that types and actions lie on compact domains, while Assumption A3 states that matching is profitable for both parties. Assumption A4 requires information processing costs to be non-negative. This assumption is important for uniqueness of equilibrium since information-processing constraints lower the perceived degree of complementarities between search efforts of workers and firms. Finally, A.5 guarantees that we have a contraction mapping of the best response functions.

Theorem 5 *Under assumptions A1, A2 and A3 Nash Equilibria exist.*

Proof. The proof is achieved in three steps and follows Vives (1990):

- (a) The set of all measurable functions mapping a compact set into a compact set is a lattice under the natural ordering.
- (b) The game is supermodular since the cross-derivatives of the objective functions are all non-negative.

$$\frac{\partial^2 Y_x(\hat{p}_x, \hat{q}_y)}{\partial \hat{p}_x \partial \hat{q}_y} = w(x, y) \mu_w(x, y) \quad \frac{\partial^2 Y_y(\hat{q}_y, \hat{p}_x)}{\partial \hat{q}_y \partial \hat{p}_x} = \pi(x, y) \mu_f(x, y)$$

- (c) In a supermodular game on a lattice Nash equilibria exist. ■

Lemma 1 *Under A1 and A2 Y_x and Y_y are continuous in $\hat{p}_x$ and $\hat{q}_y$ respectively.*

Proof. All the integrands are continuously differentiable with respect to strategies, and all the integrals are taken over compact sets. ■

Lemma 2 *Under assumptions A1, A2 and A4 Y_x and Y_y are concave in $\hat{p}_x$ and $\hat{q}_y$ respectively.*

Proof. Using the previous lemma, it remains to verify that the second variational derivatives are everywhere non-positive. That is indeed the case under assumption A4. ■

Theorem 6 *Under A.1, A.2, A.3, A.4 the first-order conditions are necessary and sufficient conditions for equilibrium.*

Proof. This theorem is a direct consequence of the previous two lemmas and assumption A3. ■

Theorem 7 *Under assumptions A.1, A.2, A.3, A.4, A.5 the matching process has a unique Nash equilibrium.*

Proof. Diagonal dominance conditions guarantee that the Hessian of the game is negative definite along the equilibrium path. It follows from lemmas 1 and 2 that the payoff functionals are continuous and concave. Then, the generalized Poincare-Hopf index theorem of Acemoglu, Simsek and Ozdaglar (2010) implies that the equilibrium is unique. ■

The first-order conditions can be simplified and rewritten using the original notation to yield distributions of attention and search intensities for both firms and workers. We only report the necessary and sufficient conditions here:

$$p^*(x, y) = \frac{v(y) \exp\left(\frac{\ln 2}{\theta_w(x)} g_w(x, y)\right)}{\int_Y v(y') \exp\left(\frac{\ln 2}{\theta_w(x)} g_w(x, y')\right) dy'}$$

$$\alpha^*(x) = \left[\frac{1}{\ln 2} \frac{\theta_w(x)}{\chi_w(x)} \ln \frac{\int_Y v(y) \exp\left(\frac{\ln 2}{\theta_w(x)} g_w(x, y)\right) dy}{\int_Y v(y) dy} \right]^{\frac{1}{k_w(x)-1}}$$

$$q^*(y, x) = \frac{u(x) \exp\left(\frac{\ln 2}{\theta_f(y)} g_f(x, y)\right)}{\int_X u(x') \exp\left(\frac{\ln 2}{\theta_f(y)} g_f(x', y)\right) dx'}$$

$$\gamma^*(y) = \left[\frac{1}{\ln 2} \frac{\theta_f(y)}{\chi_f(y)} \ln \frac{\int_X u(x) \exp\left(\frac{\ln 2}{\theta_f(y)} g_f(x, y)\right) dx}{\int_X u(x) dx} \right]^{\frac{1}{k_f(y)-1}}$$

where private gains of workers and firms are defined as follows:

$$g_w(x, y) = w(x, y) \mu_w(x, y) q^*(y, x) \gamma^*(y)$$

$$g_f(y, x) = \pi(x, y) \mu_f(x, y) p^*(x, y) \alpha^*(x)$$

Like in the one shot model, equilibrium allocations of attention have an intuitive interpretation. The higher agents' expected private gains from matching with each other, the higher the probabilities of applying/processing applications. Firms and workers are naturally ordered in probabilities of allocating attention to each other. In equilibrium, firms' strategies are best responses to strategies of workers, and workers' strategies are best responses to strategies of firms. The strategies of firms and workers have similar properties due to the symmetry of the problem.

The rich structure of heterogeneity in costs, surpluses and types is fully taken into account by all agents in the model. Relatively unrestrictive conditions for uniqueness allow us to accomodate a rich class of matching models with different structures of fundamentals. Each element of this rich structure of fundamentals potentially has an impact on matching rates between all type pairs, which in turn affect the number and quality of matches in equilibrium. Therefore, this model can be extremely useful for understanding the consequences of heterogeneity for the aggregate matching function.

Note, that neither existence nor uniqueness of equilibrium relies on supermodularity of the surplus function. Therefore, assortative matching (in expected terms) needs not be an equilibrium outcome of the model. Thus, our model can generate a rich structure of equilibrium outcomes and has a potential to speak to the rich empirical literature on the determinants of wages.

5.4 The social planner's problem

Similarly to the one-shot model, we assume that the social planner maximizes the total surplus of the economy subject to the the same constraints

that we place on workers and firms in equilibrium. Note that under the aforementioned assumptions the resulting conditions for social optimality are the same as for equilibrium, except social gains are defined as follows:

$$g_w^o(x, y) = f(x, y) \mu_w(x, y) q^o(y, x) \gamma^o(y) - \phi(y),$$

$$g_f^o(y, x) = f(x, y) \mu_f(x, y) p^o(x, y) \alpha^o(x) - \phi(y),$$

where $\phi(y)$ is a constant which only depends on the firm types.

Theorem 8 *The equilibrium is socially inefficient under assumptions A1-A5 and if all of the following hold:*

- 1) $0 < \theta_w(x) < \infty$
- 2) $0 < \theta_f(y) < \infty$
- 3) $f(x, y) > 0$ for some (x, y) .

The proof has a similar intuition to the one shot model. It is not feasible to achieve the social optimum, because to do that the planner needs to promise private gains which violate the resource constraint. This result is crucial for understanding the magnitude of potential inefficiencies in the matching process. For that it is useful to compute the aggregate number of equilibrium matches. Note that in this framework the matching rate, an analog of the matching function, can be computed as:

$$M = \int_X \int_Y \frac{q(y, x) p(x, y) \alpha(x) u(x) \gamma(y) v(y)}{v(y) \gamma(y) + \int_X p(x', y) \alpha(x') u(x') dx'} dx dy$$

5.5 Simplifying assumptions

To facilitate quantitative explorations of the properties of equilibrium outcomes and the size of inefficiency we make several auxiliary assumptions.

A6 Workers and firms are distributed uniformly: $u(x) = U$, $v(y) = V$.

A7 Workers are identical: $\theta_w(x) = \theta_w$, $\chi_w(x) = \chi_w$, $k_w(x) = k_w$.

Firms are identical: $\theta_f(x) = \theta_f$, $\chi_f(x) = \chi_f$, $k_f(y) = k_f$.

A8 Workers and firms are placed on connected unit intervals:

$$X = [0, 1], \quad Y = [0, 1].$$

A9 Match surplus and Nash bargaining weights depend on distance, $d(x, y)$, only:

$$f(x, y) = f(d(x, y)), \quad w(d) = \beta(d) f(d), \quad \pi(d) = (1 - \beta(d)) f(d).$$

$$\text{where } d(x, y) = \min\{|x - y|, 1 - x + y, 1 - y + x\} \in [0, \tfrac{1}{2}].$$

Thus, we place workers and firms on connected unit intervals and define the surplus of each match as a function of the distance between types. Firm and worker types are symmetric. Symmetry and uniformity simplify the analysis substantially. Conditional on assumptions A6-A9, all match-specific variables become distance-specific, all firm- or worker-specific variables lose this dependence. Therefore, the solution to the model can be rewritten as follows:

$$\begin{aligned} \alpha^* &= \left[\frac{1}{\ln 2} \frac{\theta_w}{\chi_w} \ln 2 \int_0^{\frac{1}{2}} \exp \left(\frac{\ln 2}{\theta_w} \frac{V\gamma^*}{V\gamma^* + \alpha^*U} w(d) q^*(d) \right) dd \right]^{\frac{1}{k_w - 1}} \\ \gamma^* &= \left[\frac{1}{\ln 2} \frac{\theta_f}{\chi_f} \ln 2 \int_0^{\frac{1}{2}} \exp \left(\frac{\ln 2}{\theta_f} \frac{\alpha^*U}{V\gamma^* + \alpha^*U} \pi(d) p^*(d) \right) dd \right]^{\frac{1}{k_f - 1}} \\ p^*(d) &= \frac{\exp \left(\frac{\ln 2}{\theta_w} \frac{V\gamma^*}{V\gamma^* + \alpha^*U} w(d) q^*(d) \right)}{2 \int_0^{\frac{1}{2}} \exp \left(\frac{\ln 2}{\theta_w} \frac{V\gamma^*}{V\gamma^* + \alpha^*U} w(d') q^*(d') \right) dd'} \\ q^*(d) &= \frac{\exp \left(\frac{\ln 2}{\theta_f} \frac{\alpha^*U}{V\gamma^* + \alpha^*U} \pi(d) p^*(d) \right)}{2 \int_0^{\frac{1}{2}} \exp \left(\frac{\ln 2}{\theta_f} \frac{\alpha^*U}{V\gamma^* + \alpha^*U} \pi(d') p^*(d') \right) dd'} \end{aligned}$$

Socially optimal allocations are similar, with the exception that private gains $w(d)$ and $\pi(d)$ are replaced by social gains, $f(d)$. Therefore, it is straightforward to see that no bargaining weights can help achieve the socially optimal allocation, unless $w(d) = \pi(d) = f(d)$, which is not feasible. The matching rate in this case equals:

$$M = 2 \frac{\alpha U \gamma V}{\alpha U + \gamma V} \int_0^{\frac{1}{2}} p(d) q(d) dd$$

The matching function takes the form of a constant-returns-to-scale matching function with a constant elasticity of substitution between unemployed workers and vacant firms. In practice, it can be approximated by a CES or Cobb-Douglas function. Parameters of this function are fully endogenous. They are determined exclusively by the distribution of surplus and by costs of search. The solution to this matching process is easily computable using standard optimization algorithms. It also allows for a closed-form solution under additional assumptions, which we describe next.

5.6 Closed-Form Solution

We proceed to a closed-form solution by adding assumptions that cost functions, numbers of workers and bargaining powers are also symmetric:

$$\mathbf{A10} \quad \chi_f = \chi_w = \chi, \quad \theta_w = \theta_f = \theta, \quad k_w = k_f = k, \\ U = V \text{ and } \pi(d) = w(d) = \frac{1}{2}f(d).$$

In this case, the solution is symmetric with $p(d) = q(d)$ and $\alpha = \gamma$, and can be solved in closed-form:

$$p^*(d) = \frac{1}{A^*} \exp \left(-W \left(-\frac{1}{A^*} \frac{\ln 2}{4\theta} f(d) \right) \right) \quad \alpha^* = \left[\frac{\theta}{\ln 2} \frac{1}{\chi} \ln A^* \right]^{\frac{1}{k-1}}$$

where $W(y)$ is the real branch of the Lambert-W function, defined as the solution to $y = We^W$ for $W(y) \geq -1$, and A^* is a normalizing constant, which makes sure that the distribution of attention integrates to one. The planner's allocation has a similar form with both workers and firms assuming they will get the whole surplus instead of a half. Assuming the existence of an upper bound, F , on the surplus function, the equilibrium is unique if:

$$\theta \geq \theta_0 = \frac{Fe \ln 2}{8 \int_0^{\frac{1}{2}} \exp \left(-W \left(-\frac{f(d)}{Fe} \right) \right) dd}$$

Constraints on costs of information illustrate that a high enough cost is necessary to weaken the strategic complementarity between strategies of workers and firms. The intuition behind the lower bounds is that, for $\theta < \theta_0$, the marginal cost is smaller than the marginal benefit of information:

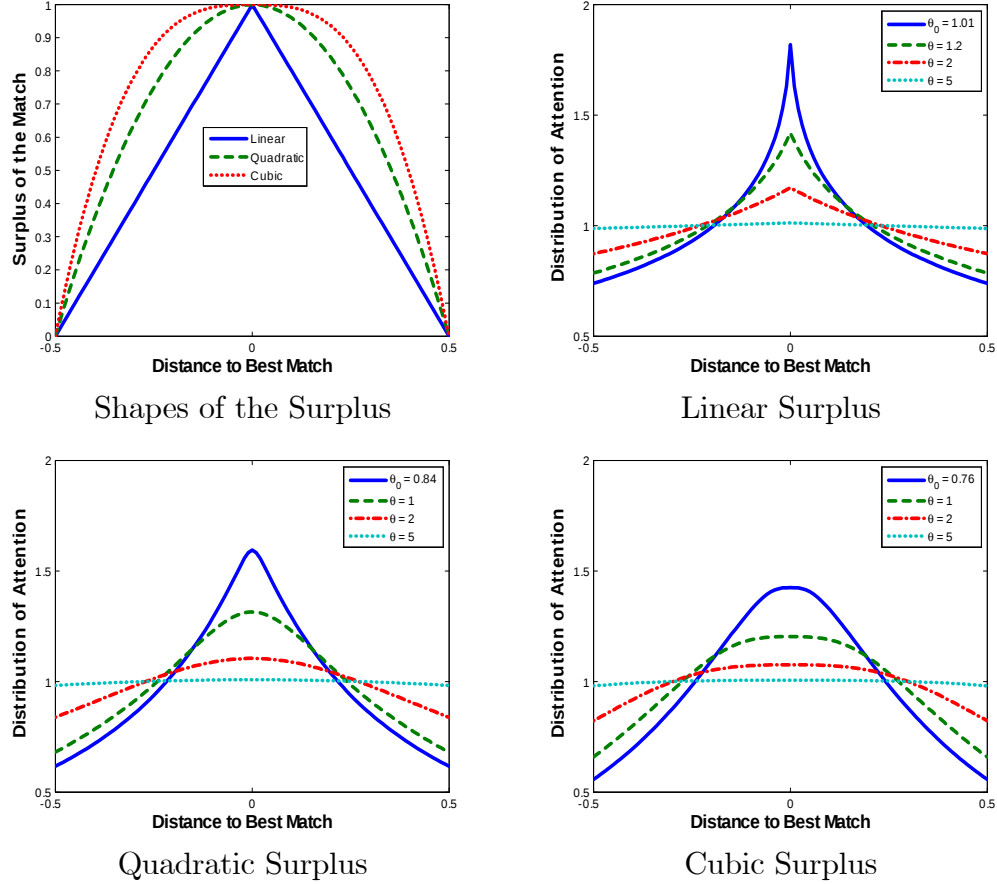


Figure 2. Shapes of the Surplus and Distributions of Attention

$$\frac{F \ln 2}{A \ 4\theta} > \frac{1}{e} > \frac{\ln p(d)}{p(d)}$$

For lower costs of information, the strategic complementarities dominate. One solution to the problem in this case is the solution to the assignment model, characterized by infinitely precise strategies described by the Dirac-delta function, $p(d) = \delta(d)$. There is a multiplicity of other infinitely precise strategies which are also equilibria.

In Figure 2 we plot distributions of attention for three shapes of the surplus $f(d) = 1 - (2d)^p$ for different values of costs above their limiting values. For different values of curvature, $p = \{1, 2, 3\}$, the limiting values

of costs for equilibria to be unique equal $\theta_0 = \{1.00, 0.83, 0.75\} * \ln 2$. The matching rate in these cases also has a closed-form solution:

$$M = U\alpha^* \int_0^{\frac{1}{2}} (p^*(d))^2 dd$$

For each of the aforementioned surplus functions the matching function is strictly decreasing in the cost of information as illustrated in the Figure 3 for the case $k \rightarrow \infty$. Figure 3 helps quantify losses in efficiency due to existence of strategic complementarities. For the symmetric economy the efficient outcome is equivalent to the equilibrium outcome under the assumption that cost of information is reduced in half. Figure 3 shows that, for intermediate values of costs, the number of lost matches in equilibrium can reach 50% compared to the social planner's allocation.

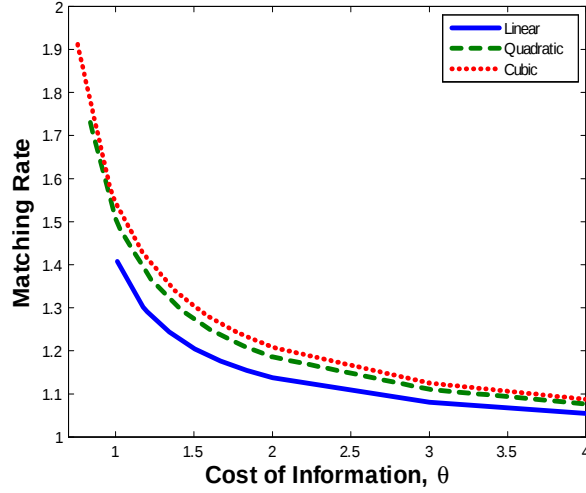


Figure 3. Matching Efficiency