

The Market for Used Capital: Endogenous Irreversibility and Reallocation over the Business Cycle

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Abstract

This paper explains the procyclicality of capital reallocation documented by Eisfeldt and Rampini (2006) and Cui (2012) by endogenising the resale price of capital in a dynamic general equilibrium model with heterogeneous firms hit by aggregate and idiosyncratic productivity shocks. I build a simple theory of endogenous investment irreversibility by assuming that used investment goods are imperfect substitutes for newly produced ones because of firm-level capital specificity. This creates a downward sloping demand for used capital that shifts with aggregate shocks. In recessions, the wedge between the price of new investment goods and the resale price becomes larger, so that the option value of holding capital for unproductive firms rises and they optimally choose to sell less capital to productive firms, inducing an amplification mechanism on total output and measured Total Factor Productivity.

Keywords: Capital reallocation, Heterogeneous firms, Investment irreversibility.

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1 Introduction

Empirical evidence from firm-level data shows that reallocation of physical capital across U.S. firms is strongly procyclical (Eisfeldt and Rampini, 2006, and more recently Cui, 2012). According to Compustat data, firms' total sales of plants, property and equipment and acquisitions, which account for roughly 30% of total capital expenditures, are significantly lower during all recessions since the 1980's than during expansions. Clearly, the reallocation of production inputs from less productive to more productive parts of the economy can increase aggregate productivity, so that the procyclicality in the data suggests the presence of a potentially important amplification mechanism for exogenous shocks. In a recession, this reallocative process is often seen as crucial for the onset of a strong recovery, consistently with the idea of creative destruction. This could be of great relevance for the current recovery. For instance, an External Member of the Monetary Policy Committee of the Bank of England has recently attributed low labour productivity in the slow recovery post-2008 in the U.K. to a lack of capital reallocation across sectors (Broadbent, 2012). These observations suggest that explaining the cyclical behaviour of capital reallocation may have very important consequences for our understanding of business cycles and the dynamics of aggregate Total Factor Productivity.

This paper explains the procyclicality of capital reallocation by endogenising the resale price of capital in a model with heterogeneous firms hit by idiosyncratic and aggregate productivity shocks. Because of some degree of capital specificity at the firm level, new and used investment goods are assumed to be imperfect substitutes. As a consequence, when the price of used capital goods falls with respect to the price of new capital, investing firms do not demand only used capital, because part of the investment needs to be done using newly produced investment goods that are specifically tailored to the investing firm's product line. For example, when a firm acquires another firm's production plant, some further investment is necessary in order to make the plant ready to operate for the acquiring firm, and this investment has to be done using newly produced specific investment goods. Under this assumption, firms are willing to choose a higher ratio of used to new investment goods only if the price of used capital falls, leading to a downward sloping demand for used capital, that will shift with aggregate shocks.

After a negative aggregate productivity shock, higher supply and lower demand of used

capital lead to a decrease in the price to clear the market. For investing firms, this creates an amplification of the shock because irreversibility is now more binding: Not only is the marginal product of capital lower, but also the future expected value of undepreciated capital has decreased. On the other hand, disinvesting firms realise that selling capital at this time is less convenient as they get a lower price than in good times. This fact dampens their incentive to downsize in a recession and leads to a decrease in reallocation, consistently with the above-mentioned empirical evidence.

Importantly, I assume that used capital is useless for consumers. As in standard models of investment, the output good can be used as consumption or new investment. However, after it has been installed as physical capital by a firm, it becomes an altogether different good, that has no direct value for consumers and some (endogenously determined) value for other firms. As a consequence, the price of used capital must clear a market where supply is given by disinvesting firms and demand comes from investing firms only.

A key feature of the model is a time-varying wedge between the price of new investment goods and their resale price. In booms this wedge is small, while in recessions it becomes larger. As is well known from previous work on investment irreversibility, this wedge creates inaction regions in the state space, where firms optimally wait before taking any investment or disinvestment action and just let their capital depreciate. In this model, the endogenous dynamics of the difference between the price of new and used investment goods implies that the size of this inaction region is countercyclical. This can potentially explain the freezing of investment/disinvestment activity at the beginning of a recession that other authors have attributed for example to uncertainty shocks (e.g. Bloom et al., 2007) and may lead to procyclical multipliers for investment subsidies, as in bad times more firms are in the inaction region and would be less responsive to this kind of stimulus. Furthermore, the time-varying option value of waiting induced by the movements in the inaction regions interacts with the asymmetric duration of booms and recessions. On average, recessions last less than booms and this makes it even less attractive for unproductive firm to reallocate capital in bad times, as they expect to be able to reinvest relatively soon.

By endogenising investment irreversibility, the assumption of imperfect substitutability between new and used capital can lead to an empirically plausible correlation between reallocation and output after aggregate T.F.P shocks, in contrast to previous models of

investment with heterogeneous firms. In fact, procyclical capital reallocation seems to be at odds with a standard one-sector model of the economy with production heterogeneity and T.F.P. shocks. In such an economy, after a bad aggregate shock, more capital is being reallocated. Many firms want to dispose of some of their capital. Who can purchase this used capital? Either consumers or other (more productive) firms, as they are all are willing to buy it in exchange for a fixed quantity of the output good. Even though total demand for the output good will fall, consumption goods, new investment goods and used investment goods are all perfect substitutes, implying that the demand for used capital is perfectly elastic, so that a shift in the supply must imply a shift in the same direction for the quantity (and value) traded, inducing more reallocation after bad productivity shocks and less reallocation in booms.

This puzzle has led recent work by other authors to emphasise the combined role of time-varying collateral constraints and partial irreversibility of investment (Khan and Thomas, 2011, Cui, 2012) in explaining the cyclical patterns of capital reallocation presented by Eisfeldt and Rampini (2006). In the tradition of models of investment irreversibility, these authors assume that the relative price of new capital in terms of output is 1, while the resale price of capital is some exogenous constant value less than 1. Importantly, they distinguish critically between two possible sources of business cycle shocks: T.F.P. shocks and financial shocks, i.e. exogenous tightening of collateral constraints. According to Cui (2012), negative T.F.P. shocks induce more reallocation (consistently with the intuition developed above for a one-sector frictionless model), while negative financial shocks lead to less reallocation, consistently with the procyclicality found in the data. Intuitively, a tightening of the borrowing constraint does not affect the productivity of the capital stock, so that it does not induce a direct incentive to disinvest. At the same time it decreases wages because firms can hire less labour, so that now there is an even stronger incentive to hold on to the existing capital stock and use output to pay back debt and deleverage instead of selling part of the capital stock at a discount.

This paper aims to improve on two features of this literature. First, capital reallocation has decreased in all U.S. recessions since the 1980's and it seems hard to argue that all of them are due to exogenous credit shocks and forced deleveraging. Hence, it seems promising to try and find a way to reconcile procyclical reallocation with more standard

T.F.P. shocks.¹ Second, in the data, disinvestment is often independent of firm liquidation, i.e. firms sell part of their capital without necessarily going out of business. However, in Cui (2012), firms hold on to their capital until the point where they optimally choose to exit. In contrast to this, this paper explains disinvestment as a downsizing activity of unproductive firms that continue their operations and sell part of their capital stock to more productive units, consistently with empirical evidence reported in Maksimovic and Phillips (2001).

Another strand of literature starting with Eisfeldt (2004) and continuing more recently with Kurlat (2012) points to the role of asymmetric information in the quality of used capital. For instance, in the latter paper capital is of good or bad quality and this is private information to its owner. Firms can only finance new investment by selling old capital. When they are hit by a positive aggregate shocks, they sell a fraction of their capital to finance investment, decreasing the fraction of lemons in the market, hence increasing the equilibrium price of used capital as well as the quantity traded. This induces procyclical reallocation. While it seems very reasonable to imagine that markets for used capital goods are affected by lemons problems, it is not entirely clear that lemons problem would necessarily get worse in recessions in a more general setting where firms are allowed to borrow or issue equity. One could imagine a case where negative productivity shocks lead to the liquidation of a larger than average fraction of good quality assets, hence increasing average quality and resale price.

Differently from the existing literature, the key assumption in the present paper is the imperfect substitutability between new and used capital in firms' investment technology. There are several reasons to believe that new and used capital are not perfect substitutes. First and most importantly, a significant part of the physical capital stock is to some extent specific at the firm level, as some machines and equipment need to be tailored specifically to the product line of the firm that operates them. Producing using machines previously used by competitors may be an option, but it creates additional costs so that the mix between new and used will in general depend on the relative price of used capital. In

¹Clearly, this does not mean denying the very important role of financial frictions in business cycles! In fact, the introduction of a collateral constraint in the present model could lead to endogenous tightening of borrowing capacity after negative productivity shocks, although this is left for future work.

other words, firms may be willing to increase the ratio of used to new capital in their investment only when used capital becomes relatively cheaper. On the other hand, Eisfeldt and Rampini (2007) suggest that used capital may be more easily available, especially for those types of equipment that require long production times. In this case, when firms want to expand quickly, they may resort to some used capital even when it becomes relatively more expensive than on average. These observations seem to suggest that the demand of used and new capital may be quite different from the behaviour implied by a model of perfect substitutability, where even a very minor change in the relative price would lead to corner solution in favour of one or the other kind. Furthermore, the assumption of firm-level capital specificity is gaining ground also in different areas of macroeconomics. For example, Altig et al. (2011) show how it helps reconcile macro-evidence on inflation persistence with micro-evidence on the frequency of price adjustment.

Finally, note that the idea of generating a procyclical resale price of used assets is consistent with the prediction of models of firesales (e.g. Shleifer and Vishny, 1992), where selling assets is harder when aggregate conditions are bad, because potential buyers are also facing hard times. Sectoral evidence for aircraft transactions in Pulvino (1998), for instance, suggests that indeed aggregate shocks lead to lower prices obtained by distressed sellers.

The rest of the paper is organised as follows. Section 2 presents the key assumptions of the paper in a simple deterministic model that allows for some analytical solutions. Section 3 presents a more quantitative general equilibrium model with idiosyncratic and aggregate shocks. Section 4 concludes by suggesting three extensions of this work.

2 A simple deterministic model

This section presents a model where two firms switch between two different levels of idiosyncratic productivity every period, while aggregate productivity also moves over time deterministically, switching between two different values every two periods. This very simple model contains some of the key features that will arise in the quantitative model with uncertainty presented in the next section. In particular, it allows to present the assumption of imperfect substitutability between new and used capital in a simple framework, where

its implications are more transparent, thanks to the possibility of obtaining some analytical solutions.

Both firms are owned by a risk-neutral representative consumer with discount factor $\beta \in (0, 1)$. At each period t , firm $i \in \{1, 2\}$ produces a homogeneous output good using a decreasing returns to scale technology:

$$y_{it} = z_t s_{it} k_{it}^\alpha, \quad (1)$$

with $\alpha \in (0, 1)$. Idiosyncratic productivity is $s_{it} \in \{s^l, s^h\}$ and aggregate productivity is $z_t \in \{z^l, z^h\}$, with $s^l < s^h$ and $z^l < z^h$. The deterministic path of productivity of firm 1 is $\{s^l, s^h, s^l, s^h, \dots\}$, while that of firm 2 is $\{s^h, s^l, s^h, s^l, \dots\}$, so that at each point in time one firm has high productivity and one firm has low productivity. The path of aggregate productivity is $\{z^l, z^l, z^h, z^h, z^l, z^l, \dots\}$ so that booms and recessions last for two periods each. By assumption, the variance of aggregate productivity is less than the variance of idiosyncratic productivity.

When firms have low idiosyncratic productivity, they know that they will have higher productivity in the following period, hence they want to invest. Conversely, when they have high idiosyncratic productivity, they disinvest in the expectation of a lower future value of s .² The assumption of perfect negative autocorrelation of idiosyncratic productivity is obviously extremely unrealistic and will be abandoned in the full model, but it is convenient in order to insure that the market for used capital is open in every period in this simple model with two firms only. An investing firm purchases a bundle of newly produced capital and used capital, which is sold by the disinvesting firm. While the relative price of new capital in terms of output is fixed at 1, the price of used capital is q and is endogenously determined by market clearing in the used capital market. The degree of substitutability between new and used capital is generally assumed to be perfect in standard models of investment. However, there are several reasons to believe that this substitutability may not be perfect in reality. On the one hand, clearly some part of the capital stock needs to be idiosyncratic to the specific product of firm, so that using a high proportion of competitors' machines and equipment may be optimal only if those come at a discount. When a firm

²Theoretically, they could just let their capital depreciate. This imposes some restrictions on the values of s^l , s^h , z^l and z^h , given other parameters, to make sure that they indeed want to disinvest by more than depreciation.

purchases a used plant, typically a further investment is needed to make the plant ready to produce for the product lines of the acquiring firm, so that in this sense one could even rationalise some degree of complementarity between new and used investment. On the other hand, used capital may have the advantage of being more easily and quickly available, especially for those types of machines that require long production time (this is the assumption in Eisefeldt and Rampini, 2006). In this paper, I make the simplifying assumption that the substitutability at the time of the investment is governed by a Constant Elasticity of Substitution (C.E.S.) aggregator and that after the new investment has been installed it becomes completely idiosyncratic to the firm that purchased it so that after one period it is no longer possible to distinguish its origin. While this is clearly a big simplification, it is meant to capture the idea that all capital, after it has been installed and used by a firm, becomes a different kind of commodity with respect to the output good. At the same time this assumption preserves computational tractability by reducing the dimensionality of the state vector in the more complete version of the model with uncertainty. A similar C.E.S. investment technology is often used in international real business cycle applications, where investment is done partly using home produced capital goods and partly with foreign capital goods (e.g. Backus et al., 1994).

It is worth noting that in equilibrium the cost of one unit of used capital will always be strictly less than the cost of one unit of new capital. To make sure that this is the case, one could assume that if used capital ever became more expensive than new capital, investing firms would just buy new capital instead of using the C.E.S. aggregator above. That could not be an equilibrium, as disinvesting firm would supply a positive amount of used capital at that price, so that this extra assumption rules out the possibility that $q + c > 1$ in equilibrium. However, even assuming a simple C.E.S. isoquant this event never happens for a reasonable calibration of the productivity process (also in the full model presented in the next section), so that this further assumption, although realistic, is in fact irrelevant. In the Appendix, I illustrate this technical assumption in figure 13 and then briefly discuss this issue.

Furthermore, I assume that used capital can only be sold to another firm and it does not have direct value for the consumer, so that the price q has to clear the market for used capital among firms only. This is in contrast to the existing literature on partial irreversibility,

where typically new capital is bought at a price equal to 1 and sold indistinguishably to either consumers or other firms at some exogenously fixed price $q < 1$. While it may be true that some types of capital like cars and computers could be of some use to consumers after having been used in production by firms, for most kinds of equipment this is phisically close to impossible.

Formally, the investment technology is given by

$$k' - (1 - \delta)k = [a^{\frac{1}{\epsilon}} (i_{new})^{\frac{\epsilon-1}{\epsilon}} + (1 - a)^{\frac{1}{\epsilon}} (i_{used})^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon}{\epsilon-1}} \quad (2)$$

where future capital is denoted with a prime, δ is depreciation, a is a parameter that will determine the steady-state ratio between new and used capital and ϵ is the elasticity of substitution between new and used investment.

The associated C.E.S. price index of a bundle of investment goods is

$$Q = [a + (1 - a)(q + c)^{1-\epsilon}]^{\frac{1}{1-\epsilon}}, \quad (3)$$

where c is a constant reallocation cost that needs to be incurred when purchasing a unit of used capital, at price q . It may be worth noting that the limit of this technology for $\epsilon \rightarrow \infty$ delivers a model of constant partial irreversibility, where new and old capital are perfect substitutes and used capital is sold at fixed price $q = 1 - c$.

In general, however, when the elasticity of substitution is finite, the price of used capital will vary over time. More precisely, in equilibrium, it will be a function of the aggregate state of the economy. In this simple model, the state is fully described by just one variable that defines the location of the economy in the 4-period cycle of aggregate productivity. Formally, let us define the position of the economy in this cycle as $j \in \{1, 2, 3, 4\}$, with transition law $j' = j + 1$ if $j < 4$ and $j' = 1$ if $j = 4$. For the sake of convenience and without loss of generality, let us also define a mapping from the space of j into that of z , so that the cycle of z_t is as follows: $z_t = z^l$ for $j = 1, 2$ and $z_t = z^h$ for $j = 3, 4$. Then, the aggregate state is fully described by j , as this provides information on z as well.³

³In principle, the distribution of firms over individual states should be a state variable as well. However, the economy in this deterministic model converges to a constant equilibrium distribution that depends only on j and henceforth I will only focus on this stationary equilibrium path. The dynamics of firms' distribution will become a more interesting issue in the full model with uncertainty.

By assumption, both firms have perfect foresight. The problem of firm i can then be described by two value functions, one for investing and the other for disinvesting. For each of the two possible actions, these equations define the maximised present discounted value of profits starting with capital stock k , idiosyncratic productivity s and aggregate state j . The value of an investing firm is

$$V^i(k, s, j) = \max_{k' \geq (1-\delta)k} zsk^\alpha - Q[k' - (1-\delta)k] + \beta V(k', s', j') \quad (4)$$

and the value of a disinvesting firm is

$$V^d(k, s, j) = \max_{k' < (1-\delta)k} zsk^\alpha - q[k' - (1-\delta)k] + \beta V(k', s', j'). \quad (5)$$

At each period, a firm chooses optimally whether to invest or disinvest, so that $V(k, s, j) = \max \{V^i(k, s, j), V^d(k, s, j)\}$. Given the simple alternating pattern of the idiosyncratic component of productivity, provided that the difference between s^l and s^h is sufficiently large given other parameters, all choices are in the interiors of the constraint sets, so that along the equilibrium path a firm with future low productivity always disinvests and a firm with future high productivity always invests. In this case, first order conditions for future capital combined with standard envelope conditions imply that the level of capital chosen by an investing firm k^h and that chosen by a disinvesting firm k^l are functions of j that satisfy

$$Q(j) = \beta [\alpha z' s^h k^h(j)^{\alpha-1} + (1-\delta) q(j')] \quad (6)$$

and

$$q(j) = \beta [\alpha z' s^l k^l(j)^{\alpha-1} + (1-\delta) Q(j')]. \quad (7)$$

Equation (6) defines the optimal investment choice. Its marginal cost Q is equated to the marginal benefit, defined as the future discounted marginal product of capital plus the future discounted value of the undepreciated capital, which will be sold as an investing firm turns into a disinvesting firm in the following period. It is already clear from this equation that q and its dynamics introduce an element of endogenous irreversibility of investment in the model. At this point, it may be worth anticipating a key feature of the equilibrium. The price of used capital q will be positively correlated with future aggregate productivity, while the price index of investment goods Q will be less volatile, being an average of $(q + c)$ and

a constant. Now, looking at equation (6) we can see that the endogeneity of irreversibility will amplify the responses of investment to changes in aggregate productivity. Going from a boom to a recession, not only the investing firm anticipates a future lower marginal product of capital, but also a lower future resale price of undepreciated capital.

On the other hand, equation (7) describes the choice of the disinvesting firm. The marginal benefit of selling one unit of capital is given by its price q , while the marginal cost is the foregone future marginal product and the future marginal value of undepreciated capital. Note that in this case future capital is evaluated at price Q as a disinvesting firm turns into an investing firm in the following period. From this equation, it is clear that the endogenous movements in q will dampen disinvestment responses to changes in aggregate productivity. Going from a boom to a recession, a lower future expected productivity would induce more disinvestment, but at the same time a low q acts in the opposite direction giving an incentive to sell less capital.

Combining equation (6) at j and equation (7) at $j - 1$ we can derive total investment at j , which is

$$k^h(j) - (1 - \delta)k^l(j-1) = \left[\frac{\alpha\beta z' s^h}{Q(j) - \beta(1 - \delta)q(j')} \right]^{\frac{1}{1-\alpha}} - (1 - \delta) \left[\frac{\alpha\beta z s^l}{q(j-1) - \beta(1 - \delta)Q(j)} \right]^{\frac{1}{1-\alpha}} \quad (8)$$

Given total investment, the investing firm decides how much new capital and how much used capital to purchase in order to minimise the expenditure $i_{new} + (q + c)i_{used}$. The first order condition of this standard C.E.S. expenditure minimisation problem is

$$\frac{i_{used}(j)}{i_{new}(j)} = \frac{1 - a}{a} (q(j) + c)^{-\epsilon}, \quad (9)$$

so that investment in used capital will be

$$i_{used}(j) = \frac{\frac{1-a}{a} (q(j) + c)^{-\epsilon} Q(j)}{\left[1 + \frac{1-a}{a} (q(j) + c)^{1-\epsilon} \right]} [k^h(j) - (1 - \delta)k^l(j-1)]. \quad (10)$$

Now, using equation (7) at j and equation (6) at $j - 1$, we can obtain total disinvestment at j :

$$(1 - \delta)k^h(j-1) - k^l(j) = (1 - \delta) \left[\frac{\alpha\beta z s^h}{Q(j-1) - \beta(1 - \delta)q(j)} \right]^{\frac{1}{1-\alpha}} - \left[\frac{\alpha\beta z' s^l}{q(j) - \beta(1 - \delta)Q(j')} \right]^{\frac{1}{1-\alpha}} \quad (11)$$

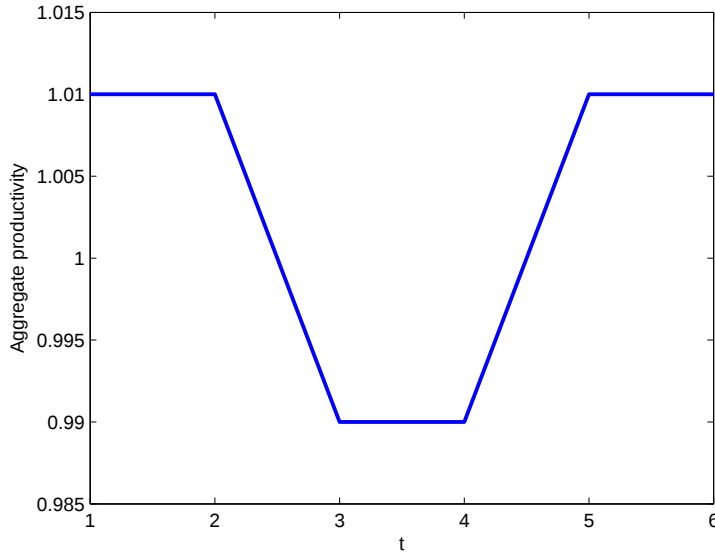
Finally, the equilibrium price function must be such that investment in used capital equals disinvestment for all j . This function can be easily found by solving the non-linear system formed by the market clearing conditions obtained equating (10) and (11) for all j . Given a set of parameters, reported in the Appendix, I can solve this system and simulate the model. In equilibrium, to a history of aggregate productivity $\{z^l, z^l, z^h, z^h\}$ correspond sequences of chosen capital $\{k^h(1), k^l(2), k^h(3), k^l(4)\}$ for one firm and $\{k^l(1), k^h(2), k^l(3), k^h(4)\}$ for the other firm. Figure 1 plots a sequence of aggregate productivity z , in which a boom is followed by a recession and then another boom. Given the deterministic nature of this model, firms fully anticipate the cyclical changes in z , so that from the point of view of investment decisions, the boom starts in the last period of the recession, and the recession in the last period of the boom. Hence the key periods to focus on in order to understand the dynamics of the model are period 2 and 4 in the following figures.

Figure 2 plots the optimal capital choices of the two firms, which jump every period from a low to a high level of capital following the path of their idiosyncratic productivity. Figure 3 plots the price of used capital q and the price index of investment goods Q . Clearly, q is more volatile and more strongly procyclical than Q . This is the key difference between this model and a standard model of partial irreversibility. In a model with constant q , during a recession the investing firm would decrease investment demand and the disinvesting firm would increase its supply of used capital because of the decrease in the marginal product of capital. This implies that if the market has to clear between the two firms, then a larger fraction of investment must be done with used capital.

However, in this model, substitutability between new and used capital is not perfect, so that large shifts in the ratio between new and used capital are inefficient. Hence, for the market to clear there must be a decrease in the relative price of used capital. This implies that there is now a new force opposing the direct effect of the marginal product of capital for disinvesting firms. In a recession the resale price of capital is lower than in booms, so that it may be optimal to sell a smaller amount of capital. In other words, the standard channel is a shift in the supply for used capital. This model is adding a movement along this supply curve, due to the fact that a downward sloping demand is also shifting inwards. As a consequence, depending on the elasticity of substitution ϵ and the size of these shifts, the

movement in the resale price q may be sufficient to deliver a procyclical amount (and value) of capital being reallocated, as can be seen in figure 4, where reallocation is lowest in period 2 and highest in period 4. In particular, the solid blue line is the value of reallocation, the red dashed line is the quantity of capital being reallocated, while the black dashed-dotted line with opposite cyclicality is the reallocation that would emerge in a model with fixed resale price equal to the average q . As it can be seen, endogenising q allows to obtain the right sign in the response of reallocation to aggregate productivity.

Figure 1: One-period-ahead aggregate productivity



3 A General Equilibrium model with idiosyncratic and aggregate productivity shocks

3.1 Model

Let us now move to a more quantitative model with a continuum of firms $i \in [0, 1]$ that receive idiosyncratic and aggregate productivity shocks. An infinitely lived representative consumer owns all the firms in the economy, consumes the output good and supplies labour.

Figure 2: Individual capital levels

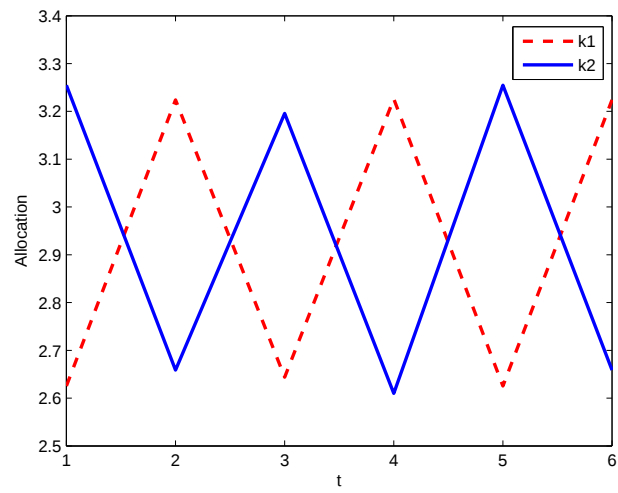


Figure 3: Prices of investment goods

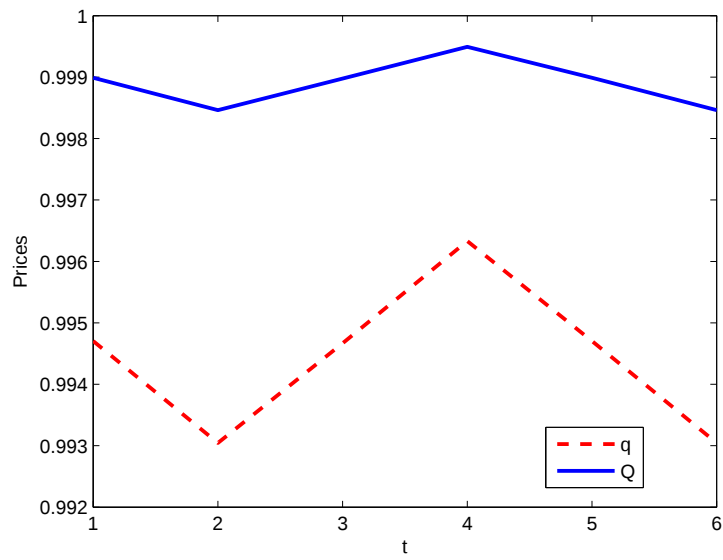
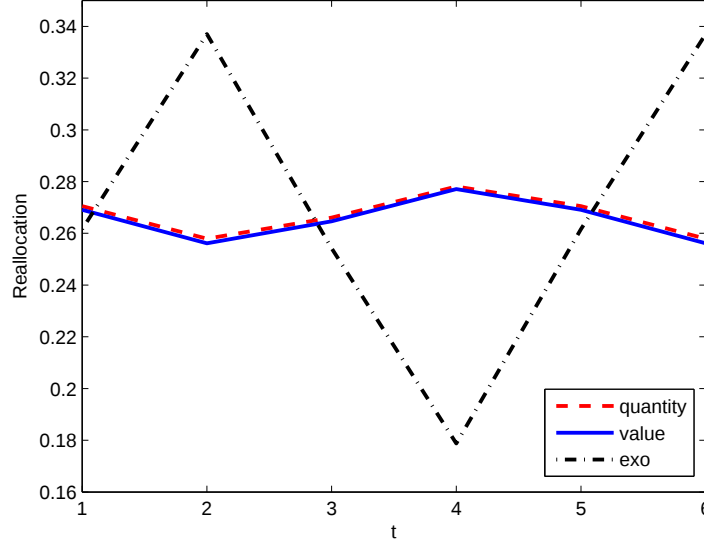


Figure 4: Reallocation



Her preferences are described by the utility function

$$E_0 \sum_{t=0}^{\infty} [\log(c_t) - \chi n_t] \quad (12)$$

where c_t is consumption and n_t are hours worked. As in Khan and Thomas (2008), the indivisible labour preferences assumption of Hansen (1985) and Rogerson (1988) simplifies the computation of the equilibrium significantly by letting the stochastic discount factor be a function of current and future wage only.

The consumer's budget constraint is

$$c_t = w_t n_t + \pi_t \quad (13)$$

where π_t are aggregate profits.⁴

The infinite-elasticity labour supply schedule is implicitly defined by the first order condition that equates the marginal rate of substitution between hours and consumption

⁴Alternatively one could write the budget constraint including the consumer's choice of buying and selling shares in all firms. In equilibrium, his portfolio would have to coincide with the distribution of firms in the economy and stock prices would be given by firms' value functions below. The distinction between these two formulations is immaterial in terms of competitive equilibrium allocations and prices.

to the wage w_t

$$\chi c_t = w_t \quad (14)$$

which together with log-utility from consumption implies that the stochastic discount factor is:

$$\beta \frac{c_t}{c_{t+1}} = \beta \frac{w_t}{w_{t+1}}. \quad (15)$$

From now on I will only need to solve the firms' optimisation problem, using equation (15) to price the future value of profits in terms of consumer's utility and insuring that wage and aggregate consumption implied by goods market clearing are consistent with the consumer intratemporal first order condition (14).

Let us consider the firms' optimisation problem. At each period t , productivity of firm i is the product of an aggregate component $z_t \in \{z^l, z^h\}$ that follows a Markov chain with transition matrix P_z and an idiosyncratic component $s_{it} \in \{s^l, s^h\}$ with Markov transition matrix P_s . Both stochastic processes are assumed to have positive autocorrelation. At time t each firm i produces a homogenous output good with technology

$$y_{it} = z_t s_{it} k_{it}^\alpha n_{it}^\nu \quad (16)$$

with $\alpha + \nu < 1$ and chooses current labour demand and the future level of capital in order to maximise its value for the consumer taking prices q_t and w_t as given. By assuming a flexible labour market with no adjustment costs, I can separate the labour demand choice from the investment decision in a very convenient way. Let us first describe the intratemporal labour decision and then derive the implied return on capital and then move on to the intertemporal investment problem.

Labour demand is such that the marginal product of labour equates the wage:

$$n_{it} = \left(\frac{\nu z_t s_{it} k_{it}^\alpha}{w_t} \right)^{\frac{1}{1-\nu}} \quad (17)$$

Hence, it is easy to derive an expression for output net of the wage bill as a function of the shocks, current capital level and the wage:

$$y_{it} - w_t n_{it} = A(w_t) z_t^\theta s_{it}^\theta k_{it}^{\alpha\theta}, \quad (18)$$

where $A(w_t) = \left[\left(\frac{\nu}{w_t} \right)^{\frac{\nu}{1-\nu}} - w_t \left(\frac{\nu}{w_t} \right)^{\frac{1}{1-\nu}} \right]$, and $\theta = 1/(1-\nu)$. This transformation of the production function is what firms look at in order to evaluate the return on investment

in physical capital. In other words, the flexible labour demand decision is incorporated in their expectations as they know that at every period they will be free to reoptimise their required labour input.

Let m be the distribution of firms over individual capital level and idiosyncratic productivity. Both the price of used capital and the wage will depend on it, so that this distribution is now a state variable with its own law of motion.

$$m' = \Gamma(m, z) \quad (19)$$

After observing the state (k, s, z, m) , each firm decides whether to invest or disinvest, and by how much.⁵ The value of an investing firm is now

$$V^i(k, s, z, m) = \max_{k' \geq (1-\delta)k} A(w) z^\theta s^\theta k^{\alpha\theta} - Q[k' - (1-\delta)k] + \beta E \left\{ \frac{w}{w'} V(k', s', z', m') | s, z \right\} \quad (20)$$

and the value of a disinvesting firm is

$$V^d(k, s, z, m) = \max_{k' < (1-\delta)k} A(w) z^\theta s^\theta k^{\alpha\theta} - q[k' - (1-\delta)k] + \beta E \left\{ \frac{w}{w'} V(k', s', z', m') | s, z \right\} \quad (21)$$

At the beginning of each period, the discrete choice between investment and disinvestment gives $V(k, s, z, m) = \max \{V^i(k, s, z, m), V^d(k, s, z, m)\}$. Note that these Bellman equations implicitly define the value of the firm as the present discounted value of profits (i.e. output net of the wage bill and investment expenditure), evaluated using the representative consumer's discount factor, obtained in equation (15).

Market clearing in the used capital market needs to be imposed in an analogous way to the deterministic model. Investing firms purchase some new capital and some used capital solving a standard C.E.S. expenditure minimisation problem, with solution in equation (9) and market clearing implies that total investment in used capital equals total disinvestment.

⁵If idiosyncratic productivity s is extremely persistent, I assume that all firms need to sell a small fraction of their capital used capital, regardless of whether they want to invest or disinvest in order to ensure market clearing in the used capital market at all periods. To clarify this, it may be useful to think of the limit with constant s , in which firms would only invest to replace their depreciated capital, so that the supply of used capital would approach 0. However, quantitatively for the calibrated process for s below this assumption is not needed.

Let us now define a competitive equilibrium and then illustrate the properties of the policy functions.

Definition 1. *A recursive competitive equilibrium is defined as a set of functions $m, \Gamma, w, q, Q, \pi, C, N, V^i, V^d, V, n, k', i, i_{new}, i_{used}, d$ that solve the household's and firms' optimisation problems and clear markets for output good, labour and used capital:*

- *Consumption $C(z, m)$ and labour supply $N(z, m)$ solve the consumer's problem of maximising (12) subject to (13)*
- *Firms labour demand $n(k, s, z, m)$ satisfies equation (17)*
- *The value functions V^i, V^d and V satisfy the Bellman equations (20), (21) and $V(k, s, z, m) = \max \{V^i(k, s, z, m), V^d(k, s, z, m)\}$*
- *For investing firms, i.e. firms such that $V^i(k, s, z, m) \geq V^d(k, s, z, m)$ the policy function $k'(k, s, z, m)$ solves (20), investment is $i(k, s, z, m) = k'(k, s, z, m) - (1 - \delta)k$ and is allocated to new and used investment goods according to the C.E.S expenditure minimisation first order condition:*

$$\frac{i_{used}(k, s, z, m)}{i_{new}(k, s, z, m)} = \frac{1 - a}{a} (q(z, m) + c)^{-\epsilon}$$

- *For disinvesting firms, i.e. $V^i(k, s, z, m) < V^d(k, s, z, m)$, the policy function $k'(k, s, z, m)$ solves (21) and disinvestment is $d(k, s, z, m) = (1 - \delta)k - k'(k, s, z, m)$*
- *Aggregate profits are given by $\pi(z, m) = z \int s k^\alpha n^\nu dm(k, s) - w(z, m)N(z, m)$*

$$-Q(z, m) \int i(k, s, z, m) dm(k, s) + q \int d(k, s, z, m) dm(k, s)$$

- *The market for the output good clears:*

$$C(z, m) = z \int s k^\alpha n^\nu dm(k, s) - Q(z, m) \int i(k, s, z, m) dm(k, s) + q \int d(k, s, z, m) dm(k, s)$$

- *The labour market clears: $N(z, m) = \int n(k, s, z, m) dm(k, s)$*

- The market for used capital clears:

$$\int d(k, s, z, m) dm(k, s) = \int i_{used}(k, s, z, m) dm(k, s)$$

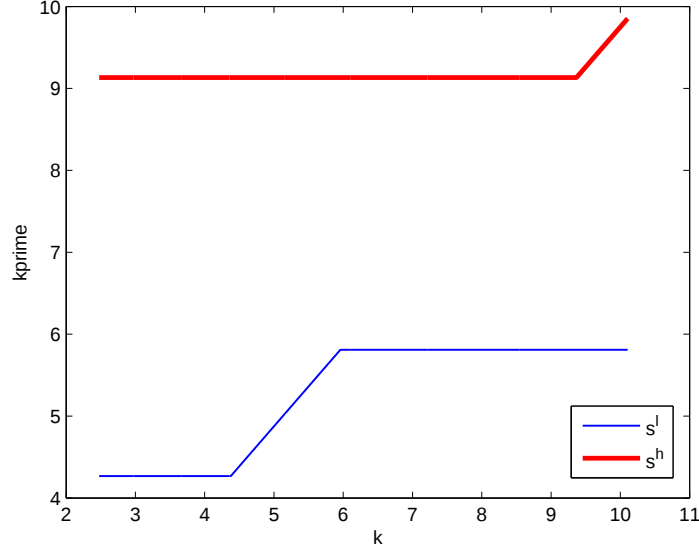
- The price functions $q(z, m)$ and $Q(z, m)$ satisfy equation (3)
- The transition function Γ defines the evolution of the distribution of firms m according to the policy function k' and the Markov transition matrix P_s

As in more standard models of exogenous partial irreversibility, the wedge between the price of investment goods Q and the resale price q generates inaction areas, where firms optimally let their capital depreciate without taking any action. As $q < Q$, it is always the case that the capital level that solves (20) without the inequality constraint of positive investment, call it $k^i(k, s, z, m)$, is always strictly greater than the capital level that solves (21) without the inequality constraint of positive disinvestment, call it $k^d(k, s, z, m)$. It follows that the policy function for future capital will be:

$$k'(k, s, z, m) = \begin{cases} k^i(k, s, z, m), & k \leq k^i(k, s, z, m)/(1 - \delta) \\ (1 - \delta)k, & k^i(k, s, z, m)/(1 - \delta) < k \leq k^d(k, s, z, m)/(1 - \delta) \\ k^d(k, s, z, m), & k > k^d(k, s, z, m)/(1 - \delta). \end{cases}$$

Figure 5 illustrates the steady-state policy function for future capital for firms with low productivity (thin blue line) and high productivity (thick red line) under the parametrisation reported below in table 1. The variable on the x-axis is the current capital level, while on the y-axis I plot next period capital. In a world without resale frictions, this picture would consist of only two horizontal lines, one at higher level for s^h and one at a lower level for s^l and firms would jump from one level to another depending on s and regardless of their initial size k , given that there are no adjustment costs. However, partial irreversibility induces disinvesting firms not to sell the whole amount of capital needed to jump to the bottom part of the blue solid line, because they expect to need to reinvest in the future, either to repurchase depreciated capital or to jump back up to the high level if they receive a positive shock in the next period and they would clearly incur a loss due to the fact they would repurchase capital at a higher price than the one obtained for their disinvestment. In other words, the wedge between the price paid for investment and the price received

Figure 5: Policy function for future capital level



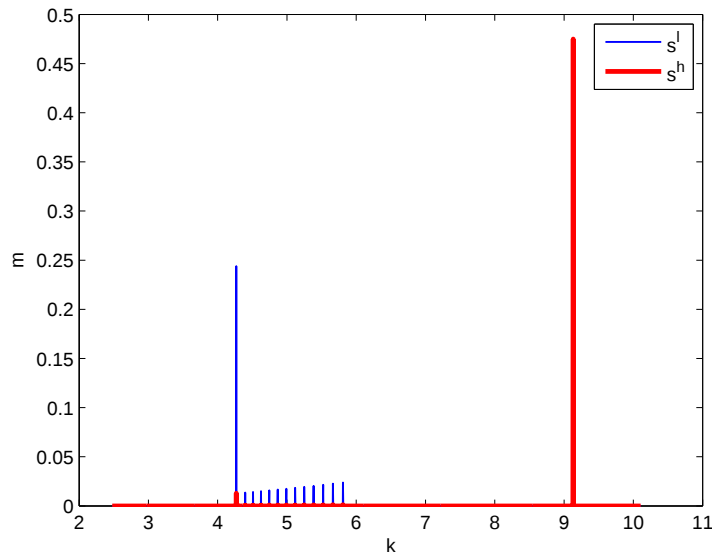
for disinvestment creates an option value from waiting and hence an inaction region where firms optimally wait before taking any action and just let their capital depreciate in the hope for a high productivity shock. The inaction region for low productivity firms is the upward sloping part of the blue solid line.

Steady-state firm level dynamics are as follows. As soon as firms get a high idiosyncratic shock, they jump to the horizontal part of the thick red line. They stay there as long as they are high productivity. As soon as they get a bad shock that brings them to s^l , they sell part of their capital and jump down to the upper part of the thin blue line. Then, as long as they have productivity s^l they move down left along this line until they reach the bottom horizontal part, where they stay until a further positive shock. Hence, on the market for used capital, supply comes from the firms that have a high level of capital and get a bad idiosyncratic shock, whereas demand comes from firms of all sizes that obtain a positive shock, plus the smallest firms with bad productivity that invest to keep their size constant.

These firm-level dynamics give rise to the stationary distribution plotted in figure 6, where the x-axis is again k and the y-axis is the mass of firms m . The thick red line

with high mass on the right hand side of the picture represents firms with productivity s^h . Moving towards the left, the thin blue lines represent the masses of firms with productivity s^l . Gradually, the mass decreases as some of the firms with those sizes receive a positive shock and only the remaining fraction let their capital depreciate for one more period. At the left of the picture, there is a mass of low productivity firms that just rebuy their depreciated capital and keep the same small size until they get a good shock.

Figure 6: Stationary distribution of firms



3.2 Calibration, solution and simulation

Table 1 reports the choice of parameter values. As much as possible, these choices reflect the attempt to stay as close as possible to previous work on firm heterogeneity and investment. A period coincides with a quarter. Parameters β , χ and δ are standard values to match quarterly interest rate, hours worked and investment to capital ratio. The production function is calibrated using the same capital and labour shares as in Khan and Thomas (2011) and the two-value stochastic process for the idiosyncratic shock is parametrised in order to be close to Cui (2012) for comparison purposes. The two-state aggregate productivity is parametrised in order to match the average duration of booms and recessions. This is in

contrast to the models mentioned above, as they estimate AR(1) processes for T.F.P.. The reason for this choice is that the asymmetry in the duration between booms and recessions may play a role in the decisions of firms about the timing and amount of disinvestment, especially in a model where the resale price is a function of aggregate market conditions.

The investment technology is defined by two parameters: a and ϵ . The first parameter is chosen to match the steady-state ratio of used capital to total capital purchased by investing firms. The target chosen is a ratio of 30%, which is an upper bound of the estimates found by Eisfeldt and Rampini (2007), in order to take into account that smaller firms out of their sample are likely to buy a higher ratio of used capital.

The elasticity of substitution ϵ is a key parameter in the model. When $\epsilon \rightarrow \infty$, the model features constant partial irreversibility. Hence, as it has been shown by Cui (2012), the impulse response functions give more reallocation after negative aggregate T.F.P. shocks and less reallocation after positive shocks. Clearly, this is in contrast with the data that this paper aims to rationalise. While Cui (2012) draws the conclusion that the right source of business cycles are exogenous shocks to financing constraints, here I suggest that procyclical reallocation can be reconciled with aggregate T.F.P. shocks as long as the elasticity of substitution between new and used capital is finite and hence the resale price is endogenous. Now, the question is how far from infinity does ϵ need to be in order to get an empirically plausible correlation between output and the reallocation series?

Lacking a proper calibration for ϵ at this stage, I illustrate the results obtained with a value such that new and used investment goods are relatively good substitutes, and the price of used capital oscillates very little over the business cycle, in order to show that even a small departure from the constant resale price assumption is sufficient to change the sign of the response of reallocation to aggregate T.F.P. shocks. Hence I set $\epsilon = 5$ and $c = .01$ in my parametrisation so that the resale price moves around .98, in a ballpark of choices made in previous work (for example Khan and Thomas set their constant resale price equal to .95). Further ongoing work is aimed at grounding the choices of values for these parameters in empirical moments of investment data.

Given a set of parameter values, the model can be solved by approximating the distribution m with its first moment as in Krusell and Smith (1998) and Khan and Thomas (2011). Agents perceive a law of motion for aggregate capital $\log(K') = \hat{\phi}_0 + \hat{\phi}_1 \log(K) + \hat{\phi}_2 z + \eta$

Parameter	Value
β	.99
δ	.025
χ	2.15
α	.27
ν	.6
a	.7
ϵ	5
c	.01
s^L, s^H	.94, 1.06
p_s^l	.95
p_s^{hh}	.95
z^L, z^H	.995, 1.005
p_z^l	.7
p_z^{hh}	.95

Table 1: Parameter values in the general equilibrium model

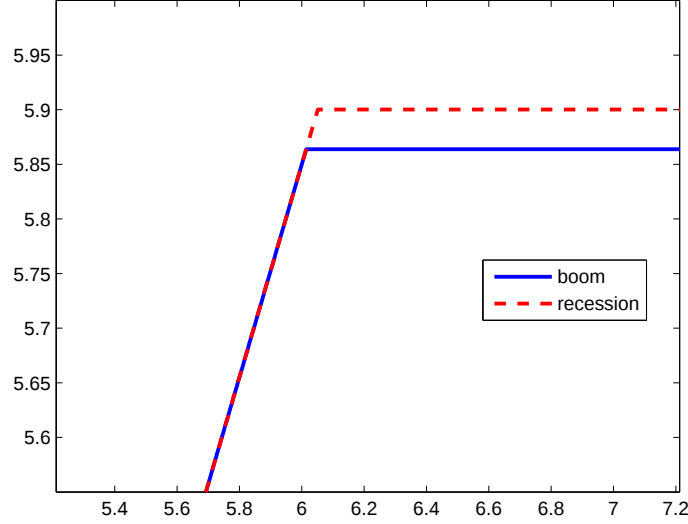
and price functions $\hat{q}(K, z)$, $\hat{w}(K, z)$. Given these perceptions, I solve the individual firm's problem by value function iteration and obtain the policy functions. Then, I simulate a continuum of firms using the non-stochastic simulation method of Young (2010) and update the price functions by imposing market clearing in the used capital market and in the labour market along the simulation. Finally, I update the law of motion for aggregate capital using standard regression methods.

Before showing simulation plots, it may be worth illustrating the main mechanism of the model by looking at the policy function for low idiosyncratic productivity firms in booms and recessions plotted in figure 7. The solid blue line is the policy function corresponding to high aggregate productivity and the dashed red line is the policy function corresponding to low aggregate productivity. First, let us analyse the horizontal part of the policy function, representing the target level of capital for firm that go from high to low idiosyncratic

productivity. In a model without endogenous irreversibility, the target level would be higher in a boom than in a recession, following the direct effect of the marginal product of capital. However, endogenous irreversibility means that there is a change in the resale price that dampens that effect and can even overturn it, as illustrated in the figure: Given that the resale price is lower, disinvesting firms have an incentive to sell less capital in the recession and hence choose a higher target level than in booms. Furthermore, following a bad aggregate shock the inaction region is wider, as the kink moves to the right. This is because the higher wedge between the price of new investment goods and the resale price is larger in a recession, increasing the option value of inaction. This feature of the model resembles the behaviour of a model with non-convex adjustment costs and uncertainty shocks (e.g. Bloom et al., 2007). In that case, the freezing of investment activity associated with a widening of the inaction region is driven by exogenous increases in uncertainty. In this model, the same behaviour happens in response to first order productivity shocks, via the endogenous reaction of the resale price of capital, without resorting to changes in second order moments. Interestingly, the time-varying wedge between investing and disinvesting price may have important policy implications. For example, investing subsidies are likely to have procyclical multipliers in this setting as more firms are in the inaction region in recessions and are thus likely to be less responsive to this kind of stimulus.

Let us now move to the aggregate dynamic properties of the model by showing a short representative sequence taken out of simulation. As can be seen in figure 8, the exogenous sequence of aggregate productivity induces a recession of average duration, lasting three quarters. In the first period of the recession (period 8 in the figures), the price of used capital falls (figure 9), although quantitatively it moves by just a fraction of a percentage point, and then increases again at the beginning of the new expansion, in period 11. Note that the price overshoots in the switching periods. Intuitively, this is because at the beginning of a recession there is a large amount of capital that disinvesting firms would like to get rid of and viceversa the beginning of an expansion leads to high demand for used (as well as new) capital in order to move to higher target levels for the capital stock associated with booms. The small movements in q are sufficient to generate relatively large procyclical movements in the reallocation series, shown in figure 10, clearly both in terms of value and in terms of physical quantities, given the small fluctuations in the price. These results suggest

Figure 7: Low productivity: Inaction regions are wider in recessions

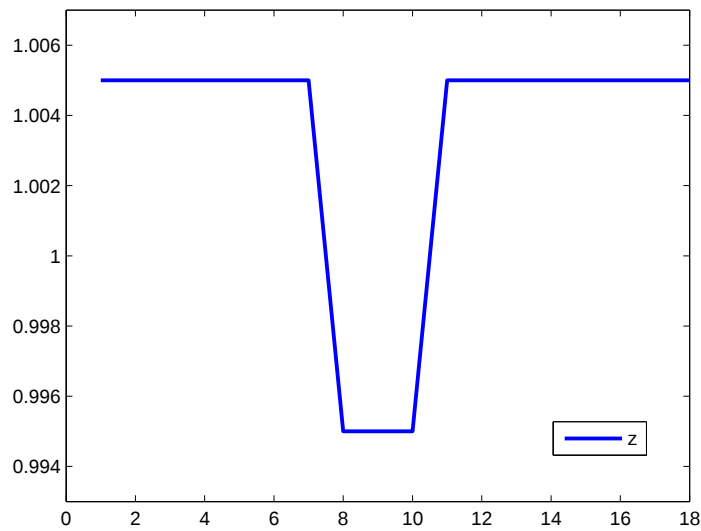


that endogenous irreversibility is potentially a very powerful mechanism in shaping firms' disinvestment decision. Interestingly, in the simulated sample the correlation between the reallocation series and aggregate output is 0.93, not too far from the empirical correlation of 0.85 found in Compustat data by Cui (2012).

Figure 11 shows the path of aggregate output and figure 12 looks at the component of measured aggregate T.F.P. that is due to the allocation of inputs in the economy. Measured T.F.P., call it Z_t , is what an econometrician would compute by assuming an aggregate production function $Y_t = Z_t K_t^\alpha N_t^\nu$. Part of it is clearly due to the exogenous component z_t , while the rest is due to how efficiently capital and labour are allocated across the heterogeneous productive units in the economy. To compute this second endogenous component I evaluate the expression $\frac{Y_t}{z_t K_t^\alpha N_t^\nu} - 1$, hence depurating Z_t of the exogenous shock process. During recessions, reallocation decreases and firms with idiosyncratic productivity s^l are in a sense 'too large', with respect to a model where the resale price is constant, which not only implies that capital is less productively used, but also employment is 'too high' in these relative less productive firms, as labour demand is an increasing function of a firm's capital stock. This endogenous amplification mechanism on the aggregate productivity

of the economy is illustrated in figure 12.⁶ Interestingly, note that even after the exogenous component of T.F.P. has recovered, the endogenous component due to reallocation stays below its average expansion value for a number of periods. This implication of the model seems consistent with the patterns observed in the current slow recovery with low productivity in the U.K., that Broadbent (2012) attributes precisely to insufficient capital reallocation.

Figure 8: Exogenous aggregate productivity



4 Conclusion

This paper suggests that the procyclicality of capital reallocation can be rationalised in a model where new and used investment goods are imperfect substitutes and hence the resale price of capital is endogenous to aggregate productivity shocks, leading to countercyclical irreversibility. Ongoing research is addressing the empirical question of calibrating the key

⁶The reason why this expression is positive in general is Jensen's inequality. After taking into account optimal factor choices, output at the firm level is a convex function of s . Hence aggregate output is higher than the output of a firm with average idiosyncratic productivity $\bar{s} = 1$.

Figure 9: Resale price of capital

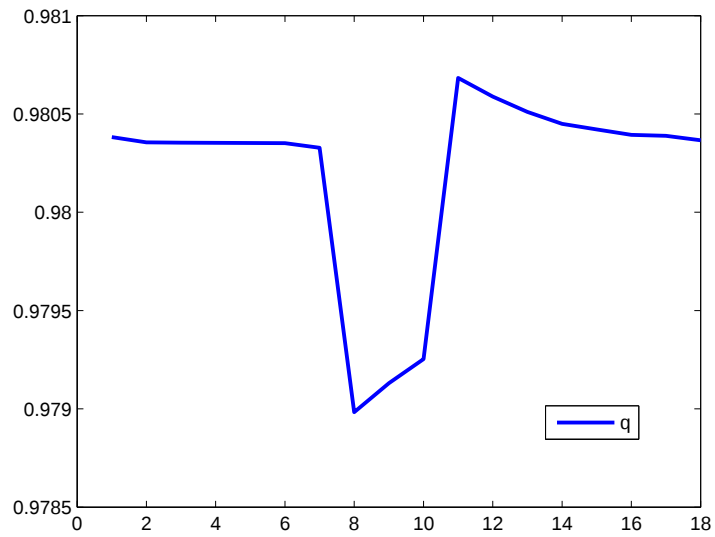


Figure 10: Reallocation

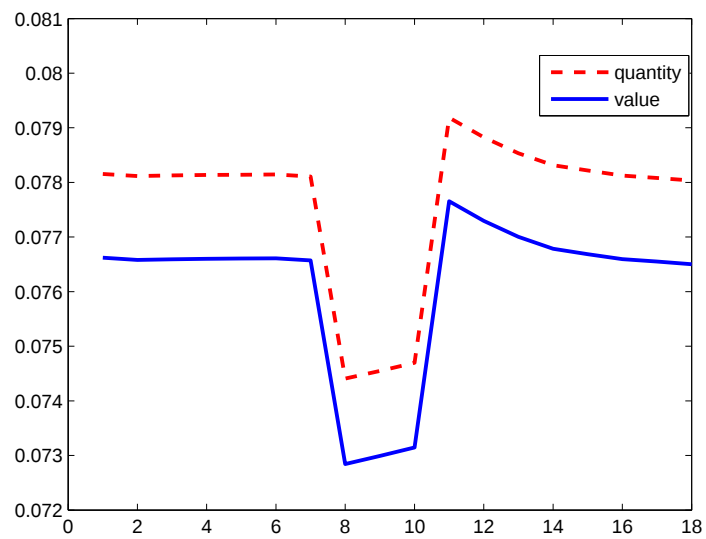


Figure 11: Output

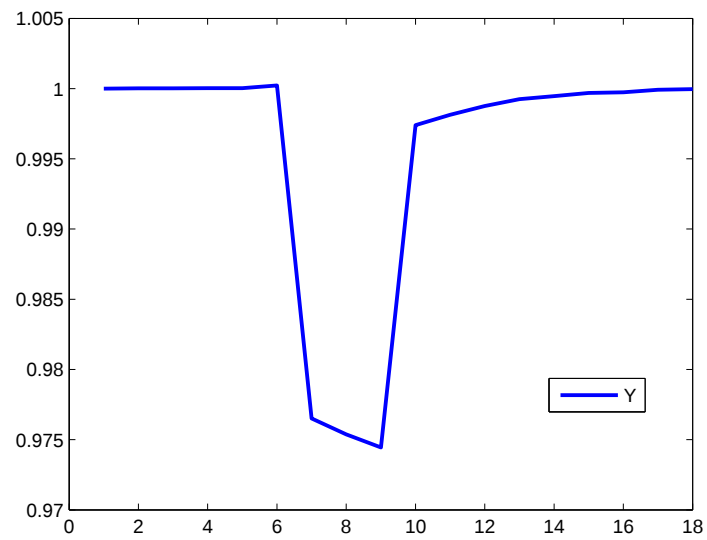
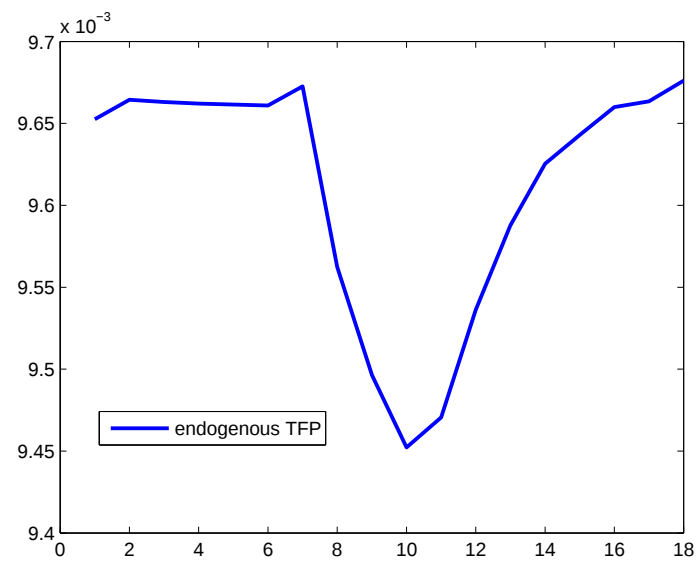


Figure 12: Allocative component of aggregate T.F.P.



parameter of the model, that is the elasticity of substitution between new and used capital, but the simulations presented show that even with a relatively small departure from a model with a constant resale price, it is possible to obtain a positive sign in the response of reallocation to aggregate T.F.P. shocks, suggesting that endogenous irreversibility is a potentially powerful channel in driving firm investment and disinvestment dynamics.

The model is well suited to address three further questions in future work. First, what kind of policies can facilitate capital reallocation and get the economy out of a recession faster? A key feature of this model is that the wedge between the price of new investment and the resale price of capital becomes higher in recessions, leading to wider inaction regions so that more firms optimally choose to wait before taking any investment or disinvestment decision. In this case, investment subsidies are likely to have procyclical multipliers, as the high option value of inaction would make firms less responsive to this stimulus in bad times. Potentially, interventions in the secondary market for capital are likely to be more successful.

Second, this model assumes perfect capital markets, but it is clear that financing constraints play an important role in shaping investment dynamics. Introducing a collateral constraint that ties the borrowing capacity to the resale value of a firm's capital is likely to further amplify the mechanism described in the paper. This has important implications for the question on the source of business cycles. Previous work has suggested that procyclical capital reallocation is evidence in favour of exogenous credit shocks (Cui, 2012). An extension of the present model with collateral constraints could potentially generate an endogenous tightening of collateral constraints after a negative T.F.P. shock hence reconciling both the cyclicity of reallocation and that of credit availability with productivity shocks.

Finally, U.S. plant level data suggest that while entry is strongly procyclical, exit is almost acyclical (Lee and Mukoyama, 2012). This evidence on exit is to some extent a puzzle for models with productivity shocks where the exit decision is driven by a fixed cost of production. In such models, after a bad aggregate T.F.P. shock, more firms optimally decide to liquidate their capital and exit. Endogenous irreversibility leading to a procyclical resale price of capital seems to be a promising explanation for this puzzle. In a recession, on the one hand the value of staying in business falls, so that more firms would like to exit, but on the other hand also the value of exit falls, as it depends of the resale price of capital,

so that overall the incentive to liquidate is dampened.

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Appendix

Figure 13: C.E.S. investment isoquants

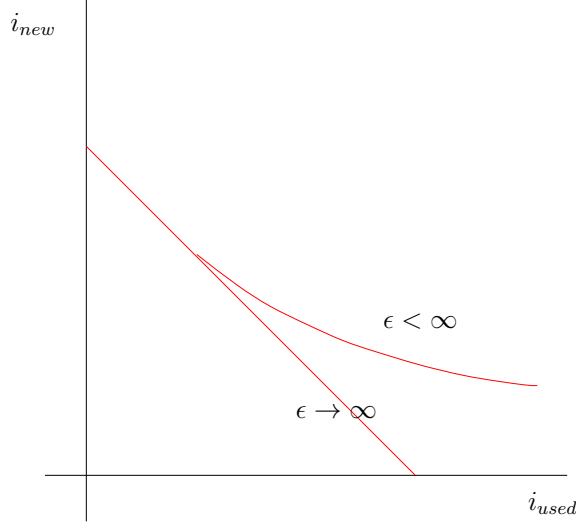


Figure 13 illustrates the fact that the imperfect substitutability in this model does not necessarily need to be symmetric. The straight line has slope -1 and implicitly represents the investment isoquant in a standard one-sector model with perfect substitutability. As long as the cost of one unit of used capital $q + c$ is below 1, the isoquant is the curved line representing imperfect substitutability, implying that investing firms choose an interior solution instead of jumping to the South-East corner solution where only used capital is demanded. However, one could imagine that if the cost of used capital ever happened to be above 1, then firms would jump to the other corner solution of the linear isoquant, where only new investment goods are demanded. Of course, this could not be an equilibrium because at that price there would be positive supply for used capital by disinvesting firms. Hence, assuming that the investment isoquant is C.E.S. up to the point where the slope of the isoquant is -1 and then becomes linear rules out the possibility that the price of used investment goods is higher than the price of new ones. However, in the region of the state space that is relevant for all simulations shown in the paper used capital is always strictly cheaper than new capital, so that it is indifferent whether one assumes just a C.E.S. investment technology or this suitably defined mixed linear-C.E.S. technology.

Parameter	Value
β	.96
α	.3
δ	.1
a	.7
ϵ	20
c	.002
z^l	.99
z^h	1.01
s^l	.9
s^h	1.1

Table 2: Parameters in the simple deterministic model