Efficient Entry in Competitive Search with Nonrival Meetings and Asymmetric Information
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Abstract

In this paper, we consider the efficiency of entry in a model of competitive search. By "competitive search" we mean that we analyze a large market in which buyers (or sellers) can direct their search based on the terms of trade that are posted (with commitment) by their counterparts on the other side of the market. We consider in particular entry on the side of the market on which the terms of trade are advertised. We generalize this literature on efficiency entry on competitive search in two directions. First, we allow for many-on-one meetings; e.g., a seller may interact with two or more buyers at the same time. Second, we allow for asymmetric information; e.g., a seller may not know how much the buyers she is interacting with value her good.

1 Introduction

In this paper, we consider the efficiency of entry in a model of competitive search. By "competitive search" we mean that we analyze a large market in which buyers (or sellers) can direct their search based on the terms of trade that are posted (with commitment) by their counterparts on the other side of the market. We consider in particular entry on the side of the market on which the terms of trade are advertised.

Moen (1997) and Shimer (1996) demonstrate the efficiency of entry in a competitive search market with two special features. First, meetings between buyers and sellers are assumed to be one-on-one ("rival" in the terminology used by Eeckhout and Kircher 2010). Second, there is complete information "within" any buyer-seller match. This literature shows how competition in posted wages among firms in a labor market with search frictions can implement the Hosios (1990) condition, i.e., the condition required for the constrained-efficient level of vacancy creation in the Diamond-Mortensen-Pissarides model (Pissarides 2000).

We generalize this literature on efficient entry in competitive search in two directions. First, we allow for many-on-one ("nonrival") meetings; e.g., a seller may interact with two or more buyers at the same time. Second, we allow for asymmetric information; e.g., a seller may not know how much the buyers she in interacting with value her good. With a rival meeting technology and complete information, the only relevant mechanism for selling a good is price posting. With nonrival meeting and asymmetric informatino, there are other mechanisms, especially auctions, to consider.

When goods are auctioned, buyers extract information rent from sellers. One might therefore expect that the equilibrium level of seller entry into a competitive search market would be less than the level that a social planner would choose. There is, however, a counterbalancing force, namely, that when a seller enters the market, she "steals" buyers from the sellers who were already there, and so potentially reduces the surpluses associated with these sellers. Our main result is that when the meeting technology is nonrival and when sellers are free to choose their preferred selling mechanisms, these two external effects – information rent versus business stealing – exactly offset each other, leading to the socially efficient level of seller entry.

2 The Basic Model

2.1 Environment

We consider a market with B buyers and S sellers with $B, S \to \infty$. We let $B/S = \theta$ denote "market tightness." In this market, each buyer wants to purchase one unit; each seller has one unit of the good for sale. Every seller posts and commits to a selling mechanism, and each buyer, after observing all posted mechanisms, chooses one seller from whom he will attempt to buy the good. The meeting technology is purely nonrival – the fact that one or more buyers choose to visit a particular seller does not make it more difficult for any other buyer to visit that seller. Finally, as is standard in the directed search literature, we assume that buyers cannot coordinate their visiting strategies.

We model asymmetric information in this market in a particular way. Specifically, we assume that buyer valuations for the good are distributed as

$X \sim F(x)$, a continuous distribution function with corresponding density, $f(x)$. We normalize the range of X to $[0, 1]$. Buyer draws of valuations are independently and identically distributed and are private information; i.e., we are considering a model of "independent private values." We consider two cases. In the first, buyers know their valuations *ex ante*, i.e., before deciding which seller to visit. One might, for example, imagine sellers offering vacation packages. Buyers differ in their willingness to pay for this good – some are very eager to go on vacation and are willing to pay a high price if necessary; others are happy to go on vacation but only if they can do so at a low enough price. In the second case, buyers learn their valuations *ex post*, i.e., only after choosing a particular seller. This is the "inspection good" case, and in this case we treat buyer valuations as idiosyncratic match-specific random variables. Consider, for example, buyers looking to purchase in a "vertically homogeneous" segment of the housing market. Even though the houses in this market may look *ex ante* identical, a buyer, upon visiting a particular house, will have a "personal, idiosyncratic" reaction – some buyers like houses with brightly colored wallpaper while others prefer a more subdued decor, etc.

On the seller side, we also consider two cases. First, we consider the simpler case of homogeneous sellers. In this case, we assume that all sellers have the same reservation value, which we normalize to zero. In the second case, we allow for seller heterogeneity. A fraction q of the sellers have reservation value $s > 0$ while the remaining sellers have reservation value zero. We interpret seller type as motivation – the sellers with the higher reservation value are "relaxed" about selling their good while those with reservation value zero are "desperate" (or, as is often seen in housing ads, "motivated"). Seller type is private information; q is common knowledge. We first consider the simpler case of homogenous sellers.

Our analysis of the two cases – buyers learn their valuations *ex ante* versus buyers learn their valuations *ex post* – draws heavily on Peters and Severinov (1997), who analyze "competitive matching equilibrium" in large markets for these two cases. The Peters and Severinov treatment of the *ex ante* case builds on work by McAfee (1993); their treatment of the *ex post* case extends Wolinsky (1988). This analysis is done taking market tightness as given and is a building block for the main part of our paper, namely, the analysis of seller entry in competitive search equilibrium.

2.2 Competitive Search Equilibrium – The Ex Post Case

Peters and Severinov (1997) consider competition in a large market in which sellers post reserve prices for second-price auctions. They show that in the limit that the symmetric equilibrium reserve price solves

$$\max_{r,\xi} \Pi(r,\xi) \text{ subject to } V(r,\xi) \geq \overline{V},$$

where

$$\Pi(r,\xi) = \xi \int_0^1 v(x) e^{-\xi(1-F(x))} f(x) dx$$

is the expected payoff that a seller can expect if she posts reserve price r, inducing a Poisson arrival rate of buyers equal to ξ, and

$$V(r,\xi) = \int_0^1 (1-F(x)) e^{-\xi(1-F(x))} dx$$

is the expected payoff for a buyer who chooses this seller. Here

$$v(x) = x - \frac{1-F(x)}{f(x)}$$

is the "virtual valuation function" and $\overline{V}$ is the expected payoff available to buyers elsewhere in the market. The constraint, $V(r,\xi) \geq \overline{V}$, expresses the idea that the "market utility property" applies in the competing auctions environment.

Peters and Severinov (1997, p.156) argue that the reserve price does not fall to zero in competitive search equilibrium. However, this turns out to be incorrect as we show in an earlier paper (Albrecht, Gautier and Vroman 2012). In the unique symmetric equilibrium, all sellers post $r = 0$ and buyers randomize their visits across sellers, i.e., $\xi = \theta$. The equilibrium allocation is thus constrained efficient – "constrained" in the sense that a social planner cannot choose which buyers to allocate to which sellers. Each seller who is visited by one or more buyers sells her good to the buyer with the highest valuation. In addition, given the constraint that buyers cannot coordinate their strategies, buyer randomization across sellers maximizes the expected number of transactions.

The intuition for this result is as follows. A seller chooses a reserve price to maximize her expected payoff subject to the constraint that the expected payoff for each buyer entering her auction should be no less than is available from other sellers. A seller's expected payoff is the total surplus generated by her auction minus the expected suplus of each buyer who participates in her auction. However, in a large market, expected buyer surplus is fixed by the market utility property, so it is in each seller's interest to choose an efficient mechanism. In fact, a large market is not strictly speaking needed for this result. Levin and Smith (1994) consider a single seller facing N potential buyers, each with a common outside option. Each buyer chooses to visit this seller with probability q (each buyer pays a cost if he participates in the auction, so q is endogenously determined), and the seller takes into account that reducing r increases q. Their result – endogenizing buyer entry drives r to zero, even though there is only one seller in the market – is generated by the fixed outside option, which is playing the same role as "market utility" does in the large market case.

We can also augment this analysis by allowing a seller to charge a fee for participating in her auction. Extending the Peters and Severinov (1997) approach, the problem for each seller is one of choosing ϕ, r and ξ to maximize

$$\phi\xi + \Pi(r, \xi) \text{ subject to } -\phi + V(r, \xi) \geq \overline{V},$$

where ϕ is the entrance fee. The seller maximand reflects the fact that the seller can expect ξ participants in her auction, each paying an entrance fee of ϕ; the constraint on the buyer side reflects the fact that a buyer who participates in this seller's auction has to pay the entrance fee. A straightforward extension of Albrecht, Gautier and Vroman (2012) shows that both ϕ and r are zero in the symmetric competitive search equilibrium and, again, that buyers randomize their visits across sellers, i.e., $\xi = \theta$. It is interesting to constrast this result with the one shown in Albrecht and Jovanovic (1986), which considers a similar model but with a rival meeting technology. In their competitive search equilibrium, sellers post positive "contact fees."

The fact that both reserve prices and and entrance fees are zero in a large market with a nonrival meeting technology means that not only is efficiency ensured in competitive search equilibrium but also that the division of total surplus between buyers and sellers is determined. This result is related to one derived in Gorbenko and Malenko (2011). They consider competition in "securities auctions" in which sellers auction off the right to develop projects for a combination of cash and a share of the profits. Getting buyers to pledge a profit share is a way for sellers to recapture some of the information rent associated with buyer private information. Proposition 4 in Gorbenko and Malenko (2011) shows that as the number of buyers and sellers in the market gets large, all-cash auctions are posted in the competitive search equilibrium.

2.3 Competitive Search Equilbrium – The Ex Ante Case

Before we consider seller entry in the *ex post* case, we will characterize the equilibrium in competing auctions for the case in which buyers know their valuations before choosing which seller to visit. It turns out that the competitive search equilibrium is the same as in the *ex post* case, but interestingly, the argument is quite different. Competition in selling mechanisms in the *ex ante* case was first considered in McAfee (1993). Our treatment follows Peters and Severinov (1997) and Peters (forthcoming) closely.

McAfee (1993) makes the following argument to show that all sellers post $r = 0$ in the *ex ante* case. Suppose buyers know their valuations before deciding which seller to visit. Suppose further that all sellers post postive reserve prices so that buyers with low valuations are shut out of the market. Now consider the seller posting the lowest reserve price, say $r' > 0$. If this seller deviates to $r = 0$, she captures the entire market between zero and r'. What is more interesting is that the deviant seller doesn't lose any buyers (in expectation) with valuations above r'. The reason is that buyers with valuations above r' don't care about competition from buyers with valuations below r'. More precisely, a buyer with valuation $x > r'$ knows that the lowest price he can possibly pay if he wins the auction is r' irrespective of whether this seller is posting $r = r'$ or $r = 0$. Buyers with valuations above r' continue to allocate themselves across all sellers (including the deviant) exactly as they would have absent the deviation. In short, Bertrand competition drives the equilibrium reserve price to zero in the *ex ante* case. Of course, once we know that every seller posts $r = 0$ in the competitive search equilibrium, it is straightforward to show that competition among sellers precludes entrance fees, just as in the *ex post* case.

3 Seller Entry – The Homogenous-Seller Case

The above characterization of competitive search equilibrium was done taking $\theta = B/S$ as given. What happens when we allow for free entry of sellers into the market? We suppose that there is an entry cost (e.g., an advertising cost) of A for sellers to enter the market. The free-entry conditon is

$$\Pi(0, \theta) - A = 0. \tag{1}$$

This condition holds in both cases, i.e., irrespective of whether buyers learn their valuations *ex ante* or *ex post*.

We now address the question of whether the level of seller entry is efficient in the free-entry competitive search equilibrium. The social planner problem can be posed as follows. For fixed B, choose S to maximize

$$BV(0,\theta) + S\,(\Pi(0,\theta) - A)$$

or equivalently, since B is fixed, choose θ to maximize

$$V(0,\theta) + \frac{\Pi(0,\theta) - A}{\theta}.$$

That is, the social planner wants to choose market tightness to maximize total market surplus, expressed on a per-buyer basis. The solution to this problem is characterized by

$$V_\theta(0,\theta) + \frac{\Pi_\theta(0,\theta)\theta - (\Pi(0,\theta) - A)}{\theta^2},$$

and this holds in free-entry equilibrium since (i) $\Pi(0,\theta) - A = 0$ and (ii) $\Pi_\theta(0,\theta) + \theta V_\theta(0,\theta) = 0$. The former condition follows by free entry; the latter is demonstrated in Albrecht, Gautier and Vroman (2012). In short, we have shown:

Proposition 1 *In the homogeneous-seller version of the model, seller entry is constrained efficient in free-entry competitive search equilibrium.*

The intuition for this result is as follows. The condition that is required for efficiency of seller entry is that the marginal entrant should receive an expected payoff equal to the expected increase in market surplus that is generated by her entry. The marginal seller's expected payoff equals the expected value of the second highest valuation among the buyers participating in her auction (or zero if fewer than two buyers choose to participate). From the social planner's point of view, this seller's entry creates an expected increase in market surplus equal to the expected value of the highest valuation among buyers who participate in her auction (or zero if no buyers visit this seller). The difference between the expected highest and second highest valuations is an information rent and would seem to suggest that sellers do not have the correct incentives to enter the market. There is, however, a counterbalancing business-stealing effect. When a seller enters the market, the buyers (if any) who participate in her auction are drawn from the auctions of incumbent sellers, so there is a decrease in surplus at the incumbent sellers' auctions. The key to our result is that the information-rent effect and the business-stealing effect exactly offset each other in competitive search equilibrium.

To understand why these two effects are exactly offsetting, it is useful to digress to consider the payoffs for buyers and seller in the marginal seller's auction. Suppose exactly n buyers visit this seller. Then her expected payoff is $E[Y_{n-1}]$, where Y_{n-1} is the $(n-1)^{st}$ order statistic. Using standard results from order statistics (see Milgrom and Weber 1982 for the application to auction theory)

$$E[Y_{n-1}] = E[Y_n] - n\int_0^1 F(x)^{n-1}(1-F(x))dx,$$

where

$$E[Y_n] = \int_0^1 xnf(x)F(x)^{n-1}dx$$

is the expected value of the highest valuation drawn among these n buyers. For fixed n, the difference, $E[Y_n]-E[Y_{n-1}]$, is the information rent that goes to the winning bidder. However, since buyers are randomizing their visits across all sellers (including the marginal entrant), the number of buyers visiting any one seller is a Poisson random variable with parameter θ, and we need to take this into account in the computation of buyer and seller payoffs. We have

$$\begin{aligned}
\Pi(0,\theta) &= \sum_{n=0}^{\infty}\frac{e^{-\theta}\theta^n}{n!}E[Y_{n-1}] \\
&= \sum_{n=0}^{\infty}\frac{e^{-\theta}\theta^n}{n!}\left(\int_0^1 xnf(x)F(x)^{n-1}dx - n\int_0^1 F(x)^{n-1}(1-F(x))dx\right) \\
&= 1-\int_0^1 e^{-\theta(1-F(x))}dx - \theta\int_0^1 (1-F(x))e^{-\theta(1-F(x))}dx \qquad (2) \\
&= \theta\int_0^1\left(x-\frac{1-F(x)}{f(x)}\right)e^{-\theta(1-F(x))}f(x)dx. \qquad (3)
\end{aligned}$$

The last line follows from an integration by parts:

$$\theta\int_0^1 xe^{-\theta(1-F(x))}f(x)dx = 1-\int_0^1 e^{-\theta(1-F(x))}dx.$$

Equation (3) is the expression for expected seller payoff that is given in Peters and Severinov (1997) (and that we used above), albeit derived from a different perspective.

Since second-price auctions with zero reserve prices are efficient, the total expected surplus is divided between the seller and the buyers who participate in her auction. That is,

$$\Pi(0,\theta) + \theta V(0,\theta) = 1 - \int_0^1 e^{-\theta(1-F(x))}dx, \tag{4}$$

which in turn implies that

$$V(0,\theta) = \int_0^1 (1 - F(x))\, e^{-\theta(1-F(x))}dx, \tag{5}$$

which, again, is the expression for each buyer's expected payoff given in Peters and Severinov (1997). Finally, using equations (4) and (5), it is straightforward to verify that

$$\Pi_\theta(0,\theta) + \theta V_\theta(0,\theta) = 0,$$

as we argued earlier.

Returning now to the intuition for the efficiency of seller entry, we have shown that (i) the total surplus associated with the marginal entrant's auction is $1 - \int_0^1 e^{-\theta(1-F(x))}dx$ and (ii) the difference between this total surplus and what the seller can expect to receive (the information rent) is $\theta \int_0^1 (1-F(x))e^{-\theta(1-F(x))}dx = \theta V(0,\theta)$. Written in this way, it is clear that information-rent and business-stealing effects exactly cancel each other. On average, the new entrant can expect to attract θ buyers, and each of these buyers would have made an expected contribution of $V(0,\theta)$ to the surplus associated with some other seller's auction had the marginal seller not entered the market. That is, the business-stealing loss caused by the marginal seller's entry equals $\theta V(0,\theta)$.

4 Heterogeneous Sellers - The Ex Post Case

Suppose a fraction q of the sellers in the market are "relaxed" with reservation value $s > 0$, while a fraction $1-q$ are "desperate" with reservation value zero. Although seller type is private information, it is striaghtforward to show that in the unique Perfect Bayesian Equilibrium, the desperate sellers post $r = 0$ while the relaxed sellers post $r = s$. (See Albrecht, Gautier and Vroman 2013 for the argument in a more complicated scenario.) Relative to the homogeneous-seller case, buyers need to decide which seller type to visit. In the *ex post* case (i.e., when buyers are *ex ante* identical), every buyer mixes between the two seller types with the same probability. In symmetric equilibrium, the probability, h, with which any one buyer chooses a relaxed seller has to be optimal given that other buyers use the same mixing probability.

The arrival rates to the two seller types are determined by the buyer mixing probability, h. Suppose each buyer visits a relaxed seller with probability h. Then the arrival rate to relaxed sellers is $\dfrac{hB}{qS} \to (\dfrac{h}{q})\theta \equiv \theta_H$. The corresponding arrival rate to desperate sellers is $\dfrac{(1-h)B}{(1-q)S} \to (\dfrac{1-h}{1-q})\theta \equiv \theta_L$. The Buyer Optimality Copndition, i.e., the condition that the buyer mixing probability has to satisfy is

$$V(0,\theta_L) \geq V(0,\theta_H) \text{ with equality if } h > 0. \tag{6}$$

Let B, L and H be the measures of buyers, desperate sellers and relaxed sellers in the market. Taking B and L as given, we want to endogenize H. Here we are implicitly assuming that all desperate sellers have already entered the market – if that were not the case, then the question of how many relaxed sellers would choose to enter would be moot. The free-entry condition for relaxed sellers is

$$\Pi^H(s,\theta_H) - (A+s) \leq 0,$$

with equality if any relaxed sellers enter. The extra notation (the superscript H) reflects the fact that a relaxed seller who fails to sell her good retains her reservation value, s. To be more explicit, the free-entry condition for relaxed sellers is

$$s + \theta_H \int_s^1 (v(x) - s)e^{-\theta_H(1-F(x))} f(x)dx - (A+s) \leq 0, \tag{7}$$

again with equality if there is any entry of relaxed sellers.

Now consider the problem of efficient entry by relaxed sellers. The social planner chooses θ_L, θ_H and h to maximize the sum of surpluses generated by the two seller types. The expected surplus associated with a desperate seller is

$$1 - \int_0^1 e^{-\theta_L(1-F(x))}dx$$

while that associated with a relaxed seller is

$$1 - \int_s^1 e^{-\theta_H(1-F(x))}dx.$$

The social planner problem is thus one of choosing θ_L, θ_H and h to maximize

$$\frac{L}{B}\left(1-\int_0^1 e^{-\theta_L(1-F(x))}dx\right)+\frac{H}{B}\left(1-\int_s^1 e^{-\theta_H(1-F(x))}dx\right).$$

We can rewrite this problem in terms of q and h as follows. Define $\phi = B/L$ and note that

$$\begin{aligned}\frac{H}{B} &= \frac{L}{B}\frac{H}{L}=\frac{1}{\phi}\left(\frac{q}{1-q}\right)\\ \theta_L &= (1-h)B/L=(1-h)\phi\\ \theta_H &= hB/H = h\left(\frac{1-q}{q}\right)\phi.\end{aligned}$$

The social planner problem is then one of choosing q and h to maximize

$$\frac{1}{\phi}\left(1-\int_0^1 e^{-(1-h)\phi(1-F(x))}dx\right)+\frac{1}{\phi}\left(\frac{q}{1-q}\right)\left(1-\int_s^1 e^{-h\left(\frac{1-q}{q}\right)\phi(1-F(x))}dx\right).$$

The first-order condtion with respect to h is

$$\begin{aligned}\int_0^1(1-F(x))e^{-(1-h)\phi(1-F(x))}dx &= \int_s^1(1-F(x))e^{-h\left(\frac{1-q}{q}\right)\phi(1-F(x))}dx;\ \text{i.e.,}\\ \int_0^1(1-F(x))e^{-\theta_L(1-F(x))}dx &= \int_s^1(1-F(x))e^{-\theta_H(1-F(x))}dx.\end{aligned}$$

This is precisely the Buyer Optimality Condition (6). That is, for any $q>0$, the social planner value for the mixing probability, h, is the same as the value chosen by buyers in equilibrium. Given any mix of seller types in the market, the expected queue lengths faced by desperate and relaxed sellers are efficient.

Next, the first order condition with respect to q – here we allow for the possibility that the social planner may want no relaxed types in the market – is

$$\frac{1}{\phi}\left(\frac{1}{(1-q)^2}\right)\left(1-\int_s^1 e^{-h\left(\frac{1-q}{q}\right)\phi(1-F(x))}dx-(A+s)\right)\leq\frac{1}{\phi}\left(\frac{q}{1-q}\right)\int_s^1\frac{h\phi}{q^2}(1-F(x))e^{-h\left(\frac{1-q}{q}\right)\phi(1-F(x))}dx$$

with equality if $h>0$. Equivlalently, this first-order condition can be written as

$$1-\int_s^1 e^{-\theta_H(1-F(x))}dx-\theta_H\int_s^1(1-F(x))e^{-\theta_H(1-F(x))}dx\leq A+s \text{ with equality if } q>0.$$

Note that

$$1 - \int_s^1 e^{-\theta_H(1-F(x))} dx = s + \theta_H \int_s^1 (x-s) e^{-\theta_H(1-F(x))} f(x) dx$$

by integrating the right-hand side by parts ($u = x - s$; $dv = \theta_H e^{-\theta_H(1-F(x))} f(x)$). The social planner's first-order condition can therefore be written as

$$s + \theta_H \int_s^1 (x-s) e^{-\theta_H(1-F(x))} f(x) dx - \theta_H \int_s^1 (1-F(x)) e^{-\theta_H(1-F(x))} dx \leq A + s$$

with equality if $\theta_H > 0$. This is precisely the free-entry equilibrium condition (inequality 7), so the equilibrium and social planner values of q are the same.

We have thus shown:

Proposition 2 *In the heterogeneous-seller version of the model, when buyers draw their valuations ex post, seller entry is constrained efficient in free-entry competitive search equilibrium.*

5 Heterogeneous Sellers – The Ex Ante Case

We next consider the case in which buyers learn their valuations before choosing which seller to visit. The Perfect Bayesian Equilibrium in this case also entails sorting by sellers – desperate sellers post $r = 0$ while relaxed sellers post $r = s$. In contrast to the *ex post* case, however, buyers sort themselves based on their valuations. Buyers with low valuations ($x < x^*$) randomize their visits across sellers posting the zero reserve price while those with high valuations ($x \geq x^*$) randomize their visits across all sellers. Thus in the *ex ante* case, buyer sorting determines both expected queue lengths and the expected distributions of valuations at the two seller types.

The search strategy of low valuation buyers is intuitively obvious. For a buyer with $x \leq s$, there is no point in visiting a seller posting $r = s$ since even if he were the winning bidder, his payoff would be negative. Essentially the same is true for buyers with valuations only slightly greater than s – these buyers are better off visiting a desperate seller, even though there is more competition there. However, why do buyers with $x \geq x^*$ randomize across *all* sellers? Consider the buyer with valuation x^*, i.e., the buyer with the lowest valuation such that he is indifferent between visiting a desperate versus a relaxed seller. Given the assumed behavior of buyers with higher valuations, the buyer with valuation x^* is equally likely to be the high bidder in a relaxed seller's auction as he is in a desperate seller's auction. The threshold value

x^* is then determined by equating the expected payoffs in the two auctions, conditional on being the high bidder. That is, by definition, the buyer with valuation x^* is indifferent between visiting the two seller types. The same is true for buyers with valuations above x^*. If buyers with valuations above x^* randomize their visits across all sellers, then a buyer with valuation $x > x^*$ is equally likely to win either seller type's auction. Finally, given that expected payoffs conditional on winning are the same for the buyer with valuation x^* and given that buyers with valuations between x^* and x randomize their visits across all sellers, the expected payoff, conditional on winning, for the buyer with valuation x is the same whether he chooses an auction with reserve price zero or one with reserve price s.

To characterize x^* explicitly, we can reason as follows. Suppose buyer x^* visits a seller posting $r = s$. Then, conditional on winning, his payoff is $x^* - s$. Suppose, alternatively, that buyer x^* visits a seller posting $r = 0$. Suppose n buyers with valuations below x^* visit this seller. Then, conditional on winning, buyer x^* has an expected payoff of $x^* - E[Y_n]$, where Y_n is the highest valuation drawn by these n other buyers. The density of valuations across buyers with $x < x^*$ is $f(x)/F(x^*)$; thus

$$E[Y_n] = x^* - \int_0^{x^*} \left(\frac{F(x)}{F(x^*)} \right)^n dx.$$

Buyers with valuations below x^* arrive at rate $\frac{BF(x^*)}{L} \to \phi F(x^*)$; hence, conditional on being the high bidder, buyer x^* can expect a payoff of

$$\sum_{n=0}^{\infty} \frac{e^{-\phi F(x^*)} \left(\phi F(x^*)\right)^n}{n!} E[Y_n] = x^* - \int_0^{x^*} e^{-\phi(F(x^*) - F(x))} dx.$$

The cutoff threshold x^* is thus characterized by

$$s = \int_0^{x^*} e^{-\phi(F(x^*) - F(x))} dx. \tag{8}$$

This cutoff valuation describes the sorting behavior of buyers.

We also need the free-entry condition for relaxed sellers when buyers draw their valuations *ex ante*. Only buyers with valuations of x^* and above visit these sellers, and these buyers arrive at rate $\theta(1 - F(x^*))$. The density and cdf of valuations among those who visit relaxed sellers are

$$\begin{aligned} f_H(x) &= \frac{f(x)}{1 - F(x^*)} \text{ for } x^* \leq x \leq 1 \\ F_H(x) &= \frac{F(x) - F(x^*)}{1 - F(x^*)} \text{ for } x^* \leq x \leq 1. \end{aligned}$$

The expected payoff for a relaxed seller, including the entry cost and foregone opportunity cost associated with staying out of the market, is therefore

$$s+\theta(1-F(x^*))\int_{x^*}^{1}(v(x)-s)\,e^{-\theta(1-F(x^*))(1-F_H(x))}dx-(A+s),$$

where

$$v(x)=x-\frac{1-F_H(x)}{f_H(x)}.$$

Using $1-F_H(x)=\dfrac{1-F(x)}{1-F(x^*)}$, the free-entry condition for sellers in the *ex ante* case can thus be written as

$$\theta\int_{x^*}^{1}\left(x-\frac{1-F(x)}{f(x)}-s\right)e^{-\theta(1-F(x))}dx-A=0. \tag{9}$$

Equations (8) and (9) describe free-entry equilibrium in the heterogeneous-seller case in which buyers draw their valuations *ex ante*.

The final step is to consider the social planner problem. The planner chooses an allocation of buyer types between desperate and relaxed sellers; i.e., the planner chooses a cutoff valuation x^*, and a level of entry by relaxed sellers. As in the *ex post* case, if we take B and L as given (with $B/L\equiv\phi$), then the level of entry, H, by relaxd sellers can be expressed in terms of $q=H/(H+L)$.

Let S_L be the expected surplus per desperate seller in the market; similarly, let S_H be the expected surplus per relaxed seller in the market. The social planner's objective can be expressed on a per-buyer basis as

$$\frac{L}{B}S_L+\frac{H}{B}\left(S_H-(s+A)\right)=\frac{1}{\phi}S_L+\frac{1}{\phi}\left(\frac{q}{1-q}\right)\left(S_H-(s+A)\right).$$

The next step is to develop expressions for S_L and S_H. We start with S_H since this term is a bit simpler. Buyers with valuations of x^* and above visit Type-H sellers at rate θ; buyers with valuations below x^* do not visit these sellers. Suppose n buyers visit a Type-H seller. The expected surplus associated with this buyer is then $E\max[Y_n,s]$, where Y_n is the highest valuation among the n visitors. Since $x^*>s$, $E\max[Y_n,s]=s$ only if $n=0$, an event that occurs with probability $e^{-\theta(1-F(x^*))}$. We thus need to compute $E[Y_n]$ for $n=1,2,..$Using the expressions for $f_H(x)$ and $F_H(x)$ given above,

$$E[Y_n]=\int_{x^*}^{1}ynf_H(x)F_H(x)^{n-1}dy=1-\int_{x^*}^{1}\left(\frac{F(y)-F(x^*)}{1-F(x^*)}\right)^n dy.$$

Thus,

$$\begin{aligned} S_H &= se^{-\theta(1-F(x^*))} + \sum_{n=1}^{\infty} \frac{e^{-\theta(1-F(x^*))}\left(\theta(1-F(x^*))\right)^n}{n!}\left(1 - \int_{x^*}^{1}\left(\frac{F(y)-F(x^*)}{1-F(x^*)}\right)^n dy\right) \\ &= 1-(x^*-s)e^{-\theta(1-F(x^*))} - \int_{x^*}^{1} e^{-\theta(1-F(y))}dy. \end{aligned}$$

Finally, note that $\theta = B/(L+H) = (1-q)\phi$; i.e.,

$$S_H = 1-(x^*-s)e^{-(1-q)\phi(1-F(x^*))} - \int_{x^*}^{1} e^{-(1-q)\phi(1-F(y))}dy.$$

The expected surplus per desperate seller in the market is computed in a similar manner. Buyers with $x < x^*$ visit these sellers with probability one; buyers with $x \geq x^*$ visit with probability $1-q$ (because these buyers randomize their visits across all sellers). So, the density of valuations across desperate sellers is

$$f_L(x) = \begin{cases} \frac{f(x)}{1-q(1-F(x^*))} & \text{for } 0 \leq x < x^* \\ \frac{(1-q)f(x)}{1-q(1-F(x^*))} & \text{for } x^* \leq x \leq 1 \end{cases}$$

with corresponding distribution function

$$F_L(x) = \begin{cases} \frac{F(x)}{1-q(1-F(x^*))} & \text{for } 0 \leq x < x^* \\ \frac{(1-q)F(x)+qF(x^*)}{1-q(1-F(x^*))} & \text{for } x^* \leq x \leq 1 \end{cases}.$$

Conditional on receiving n visitors, the expected surplus generated in an auction held by a desperate seller is

$$E[Y_n] = \int_0^1 xnf_L(x)F_L(x)^{n-1}dx,$$

which, after considerable algebra, can be written as

$$E[Y_n] = 1 - \int_0^{x^*}\left(\frac{F(y)}{1-q(1-F(x^*))}\right)^n dy - \int_{x^*}^{1}\left(\frac{(1-q)F(y)+qF(x^*)}{1-q(1-F(x^*))}\right)^n dy.$$

Finally, the arrival rate of buyers to desperate sellers can be computed as follows. There are $BF(x^*)$ buyers with $x < x^*$, all of whom visit desperate sellers, and there are $B(1-F(x^*))$ buyers with $x \geq x^*$, a fraction $1-q$ of whom visit desperate sellers. The expected number of buyers per desperate seller is thus $\dfrac{B\left(F(x^*)+(1-q)(1-F(x^*))\right)}{L}$, so the arrival rate of buyers to any one desperate seller is $\phi(1-q(1-F(x^*)))$. We can therefore compute S_L as follows:

$$\begin{aligned} S_L &= \sum_{n=1}^{\infty} \frac{e^{-\phi(1-q(1-F(x^*)))}\left(\phi(1-q(1-F(x^*)))\right)^n}{n!} E[Y_n] \\ &= 1-\int_0^{x^*} e^{-\phi(1-F(y)-q(1-F(x^*)))}dy - \int_{x^*}^1 e^{-\phi(1-q)(1-F(y))}dy. \end{aligned}$$

Proposition 3 *In the heterogeneous-seller version of the model, when buyers draw their valuations ex ante, seller entry is constrained efficient in free-entry competitive search equilibrium.*

Proof. To be completed. ■

6 Conclusion

The constrained efficiency of competitive search equilibrium is well understood when meetings between buyers and sellers take place on a one-on-one basis. However, in many situations, e.g., in standard auction settings, it is more appropriate to assume a nonrival meeting technology, i.e., many-on-one meetings. These are situations in which buyers differ in terms of how much they value the good that is being offered for sale and in which these valuations are private information. In this paper, we have shown that the constrained efficiency of competitive search equilibrium continues to hold in this richer environment.

We show that competitive search equilibrium is constrained efficient in both a short-run and in a long-run sense. In the short run, i.e., taking market tightness as given, competition drives sellers to post efficient mechanisms. In a setting in which sellers post second-price auctions, this means that competition drives the symmetric equilibrium reserve price down to the common seller reservation value so that "no surplus is left on the table." We show that sellers post efficient mechanisms whether buyers learn their valuations before or after choosing which seller to visit and whether sellers are homogeneous or heterogeneous with respect to their reservation values. Our short-run efficiency results are mostly drawn from Peters and Severinov (1997) and from our earlier work (Albrecht, Gautier and Vroman 2012).

Our main contribution in this paper is to show that competitive search equilibrium is constrained efficient in the long-run sense, i.e., allowing for endogenous market tightness, even when meetings are nonrival and there is asymmetric information within meetings. To get efficiency, sellers who could potentially enter the market need the correct incentives. The payoff expected by the marginal entrant should equal her expected contribution to market surplus. If we look only at the auction that the marginal entrant posts, it seems that sellers have too little incentive to enter the market. The expected contribution to market surplus from this auction is the expectation of the highest valuation drawn by buyers who participate in that auction, while the seller's expected payoff is the expectation of the second-highest valuation across buyers in her auction. This difference between what the winning buyer expects to get and what the seller expects to receive is an "information rent," and it is the existence of this information rent that makes it seem at first glance that there will not be enough seller entry. This argument, however, neglects the fact that the buyers who participate in the marginal entrant's auction would have participated in some other seller's auction had the last seller not entered the market. That is, the marginal seller creates a business-stealing externality by her entry. Our contribution, therefore, can be understood as showing that the information-rent and business-stealing effects exactly offset each other in competitive search equilibrium, thus generating the efficient level of seller entry.

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