Life Cycle Uncertainty and Portfolio Choice Puzzles

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Abstract

The standard theory of household portfolio choice is hard to reconcile with the following facts. (i) Despite a high rate of returns the average household holds a low share of risky assets (equity premium puzzle). (ii) The share of risky assets increases in age. (iii) The share of risky assets is disproportionately larger for richer households. We show that a simple life-cycle model with learning about earnings ability can successfully address all three puzzles. Young workers, on average asset poor, face larger uncertainty in their lifetime labor income because they do have perfect knowledge of their ability in the market. They hedge this risk in human capital by investing in relatively safe financial assets. As earnings ability is gradually revealed over time, they take more risk in financial investment. When the labor income risks are calibrated to those observed in the Panel Study of Income Dynamics, our model with learning reproduces the investment profile we see in the Survey of Consumer Finances.

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1 Introduction

Three stylized facts have been documented by Haliassos and Michaelides (2002) among others. (i) Despite a high rate of returns the average household holds a low share of risky assets (equity premium puzzle). (ii) The share of risky assets increases in age. (iii) The share of risky assets is disproportionately larger for richer households. The standard life-cycle model of household portfolio choice is hard to reconcile with these facts. The standard theory predicts that a household with a constant relative risk aversion should invest aggressively when young and gradually move towards a safer portfolio. Moreover, the share of risky investment should be negatively related to the level of wealth.

We show that a model where agents learn their ability through labor market experience can help us understand all three puzzles. Young (on average) wealth-poor workers, unaware of their ability, face a large degree of life-time labor income uncertainty. This risk in human capital discourages the investment in risky financial assets. As people get older they tilt their portfolio towards stocks partially because most of their type uncertainty is resolved and also because they have a shorter time horizon of risky labor income. We quantitatively evaluate the interaction between these two sources of risk- human capital and financial investment in a simple life-cycle portfolio choice model that features (i) imperfect information about earnings ability (ii) learning about ability through successive labor income realizations iii) two savings instruments- a bond and a stock and (iv) a limited amount that households can borrow.

We calibrate the parameters of income process to match the empirical distribution of earnings we observe in the data. The ability component in our model corresponds to the deterministic components of individual earnings while labor income risk arises from stochastic fluctuations in the idiosyncratic productivity component of earnings. More specifically, we calibrate the income process to match the life-time path of cross-sectional variance in log-hourly earnings as reported in Storesletten et al. (2004). For example, the ability distribution is calibrated to match the cross-sectional variation of earnings among the youngest cohort. The average earnings profile adopted from the labor-efficiency profile in Hansen (1993). The (uninsurable) stochastic income shock process is chosen to match the variance of log earnings over the life-cycle. We set the initial amount of prior uncertainty about ability (initial prior distribution) equal to the population moment (actual distribution). We also assume that workers discount some of their information and use a learning mechanism which allows for various speed of learning. We consider this environment more realistic than a framework where agents update their priors quickly and resolve their uncertainty with only a few observations.

Our model reproduces the investment patterns in the Survey of Consumer Finances (SCF). (1) The benchmark model reproduces an average risky share of 0.48 very close to the value of 0.47 in the data (defined in detail in Section 2) with a constant relative risk aversion of 5, much lower than values usually employed in the literature. (2) The risky share increases with age at the similar rate we see in the data. In our model, on average investors increase their risky share by 0.85 percentage points every year whereas investors increase their risky share by 1.45 percentage points in the SCF. (3) Our model also reproduces the positive correlation between the risky share and financial wealth. In the data, the risky share is 42%, 46%, and 50%, respectively, for the 2nd, 3rd, and 4th quintile of wealth distribution. In our model these numbers are 40%, 44%, and 48%. Our results illustrates that there is a quantitatively important link between labor market uncertainty and financial investment decisions over the life-cycle. We find that the a slow speed of learning is crucial to generate these patterns.

Our work contributes to existing studies on the life-cycle portfolio profiles. The closest paper to ours is Cocco, Gomes and Maenhout (2005) who analyze the portfolio choice allocation along the life cycle. Our work extends their analysis in two important directions. First, we introduce imperfect information about labor earnings. This implies that perceived labor earnings are riskier than what the data might suggest, especially for younger investors. Second, we test our model over a wide range of portfolio statistics most notably the correlation between risky share and financial assets. We do so both within and across age groups. Another closely related paper is Gomes and Michaelides (2005) who show among other that heterogeneity in risk aversion and Epstein-Zin preferences is not enough to explain the empirical risky share age pattern. Wachter and Yogo (2010) also analyze life-cycle portfolio profile as we do. They match the investment profile in the data using nonhomothetic utility and decreasing relative risk aversion whereas we match the data with slow learning about earnings ability. Finally, Ball (2008) examines the effect of social security system on household portfolio choice and finds that the above mentioned puzzles are robust to the generosity of the social security system.

Our paper also distinguishes itself from the literature focusing on the covariance between labor income risk and stock returns. For example, Benzoni, Collin-Dufresne and Goldstein (2007) show how at longer time horizons labor income and stock market returns are likely to move together. As a result, stocks are riskier for young workers than for old. Storesletten, Telmer and Yaron (2004) show that if if labor income is perfectly correlated with stock returns the a model of portfolio choice can generate a hump shaped investment profile. Lynch and Tan (2010) also show that the countercyclical variation in volatility of labor income growth, which plays an important role in discouraging the stock holdings motive for low-wealth young households. Huggett and Kaplan (2012) decompose human capital on safe and risky components and find that human capital and stock returns have a small positive correlation.

Our paper is also closely closely to Guvenen (2007). The author examines the implications of an imperfect information model on life cycle consumption behavior. We go a step further and ask whether gradual learning about one's type can explain the allocation of savings between risky and riskless assets.

The paper is organized as follows. In Section 2, based on the extensive data from the SCF we document and report stylized facts on household portfolio profiles. In particular, we provide a detailed statistics across age and wealth groups. In Section 3 we present a simple 3-period life-cycle model to illustrate the important link between labor income risks and financial investment over the life-cycle. Section 4 develops the fully-specified calibrated model with learning about ability. In Section 5 we evaluate the quantitative performance of the model where parameters are calibrated to be consistent with the individual income process estimated in the literature. Section 6 performs sensitivity analysis and Section 7 concludes.

2 Life-Cycle Profile of Household Portfolio

Based on the 1998 Survey of Consumer Finances (SCF) we document some stylized facts on the life-cycle profile of households' portfolios. The SCF provide detailed information on the households' characteristics and their investment decisions. The survey is conducted every three years and we use data from 1998 survey.[1] For our purpose, we categorize assets into two classes, safe versus risky (explained below). Several facts emerges from the 1998 SCF data:

1. *Participation*: Just a little over half (56.9%) of population participate in investing in risky assets. This participation rate shows a hump shape over the life cycle with its peak around the average retirement age.

2. *Conditional Risky Share*: For households who invest in any form of risky assets, on average they allocate about half (47%) of their financial wealth into risky assets. This conditional risky ratio increases monotonically over the life cycle.

3. *Unconditional Risky Share*: The participation and conditional risky share combined, the unconditional risky share also exhibits a hump shape over the life cycle.

[1] Ameriks and Zeldes (2004) use the available SCF studies from 1983-1998. They find that both the unconditional and the conditional share weakly increase with age (or exhibit a hump shaped) if time effects are controlled for but increase strongly with age if they control for cohort effects.

4. *Wealth Correlation*: Wealthier people tend to allocate a larger fraction of their savings towards risky assets.

The first step to understand households' portfolios is to group assets in the data by their degree of investment risk. To be consistent with our model where households face a choice between risk-free and risky investment, we classify assets in the SCF into two categories, namely "safe" and "risky" assets. Some assets can be easily classified to one or the other. For example, checking, savings, and money market accounts are safe investment while direct holdings of stocks are risky. However, some assets (e.g., mutual funds and retirement accounts) are invested in a bundle of risk-free and risky instruments. Fortunately, the SCF provides some information on how these accounts are invested. The respondents are asked not only how much money they have in each account but also about the way these money are invested. If the respondent reports that most of the money in that accounts are in bonds, money market, or other risk-free instruments, we classify them as safe investment. If they report that the money is invested in some form of stocks, we categorize them as risky investment. If they report that the account involves investments in both risk-free and risky instruments, we assign the half of the money into each category.

More specifically, the assets considered as safe are checking accounts, savings accounts, money market accounts, certificates of deposit, cash value of life insurance, U.S. government or state bonds, mutual funds invested in tax-free bonds or government backed bonds, trusts and annuities invested in bonds, money market accounts, or life insurance. The assets consider as risky are stocks, brokerage accounts, mortgage-backed bonds, foreign and corporate bonds, mutual funds invested in stock funds, trusts and annuities invested in stocks or real estate and pension plans that are a thrift, profit sharing or stock purchase plan are considered risky. As part of the risky investment we also include the "share value of businesses owned but not actively managed excluding ownership of publicly traded stocks." That is we exclude the share value of *actively managed* business from the benchmark definition.

Table 1: **Basic Accounts**

Type of Account	Average($)	Participation
Checking account	3,483$	86.9%
Savings account	4,746$	60.2%
Savings bond (safe)	5,916$	22.4%
Life insurance	8,955$	31.7%
Retirement Accounts (safe)	12,103$	38.1%
Total safe assets	64,488$	99.8%
Stocks	32,898$	20.6%
Trust (risky)	6,121$	1.2%
Mutual funds (risky)	12,574$	16.3%
Retirement Accounts (risky)	26,376$	45.2%
Total risky assets	89,403$	56.7%
Total financial assets	153,891$	100%

Table 1 summarizes the average amount (in '98 dollars) held in specific accounts as well as the participation rate (the fraction of households who have a positive amount in that account). Here, we restrict the sample to the households who have at least a positive amount of assets. (Thus, we only look at households' gross assets.) The participation rates clearly show a strong preference towards safe assets. While 86.9% of people hold a checking account and 60.2% hold a savings account, only 20.6% directly own stocks. Nearly everyone (99.8%) owns some form of safe assets while only 56.7% save in risky assets. Even for households who do invest in risky assets, the fraction of investment allocated to risky assets (the risky share of financial assets) is less than half (46.9%).

Risky Share by Age

We now investigate the risky share across age and wealth groups. Figure 1 shows the three types of portfolio choice over the life cycle: participation rate and, conditional and unconditional risky shares. The dotted line represents the average rate in each age group. The solid line represents the average of 5 year range (e.g., 21-25, 26-30, etc.). The the participation rate (the fraction of households who participate in risky investment) exhibits a hump shape over the life cycle with its peak around the average retirement age. It increases from

24.7% in the ages of 21-25, to 55.2% in ages 31-35, to its peak of 67.3% in age 51-55 and then decreases to 48.4% in ages 61-66.

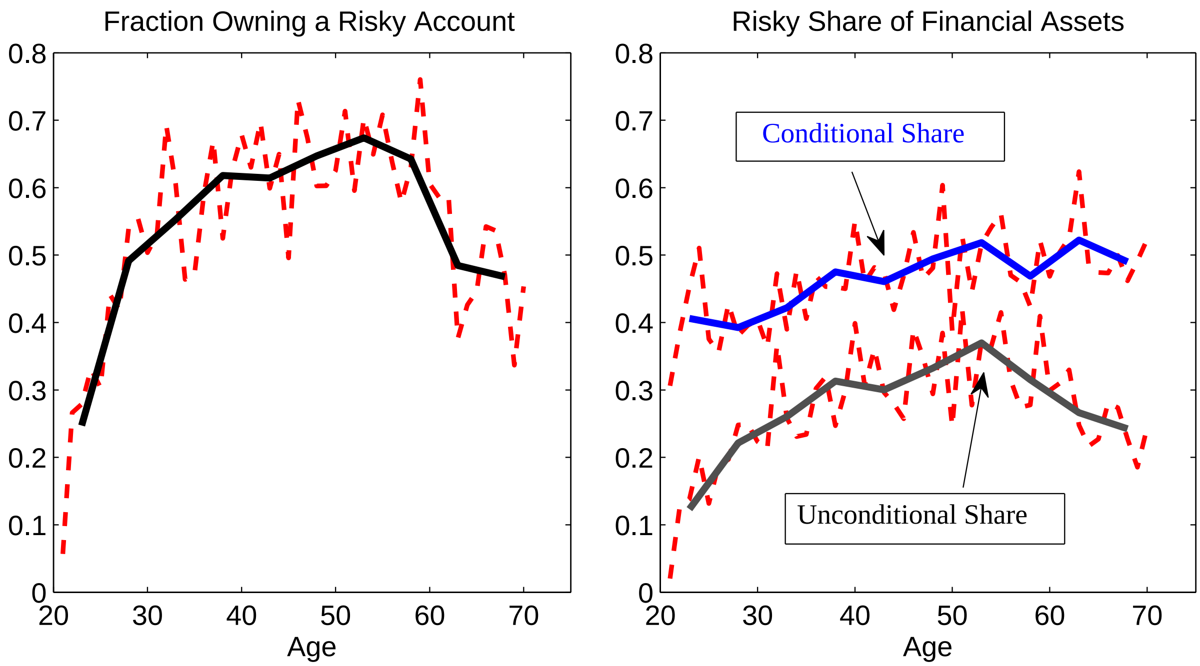

Figure 1: *Left Panel:* Fraction owing a risky account across ages. *Right Panel:* Risky share of financial assets across ages: unconditional sample and sample conditional on owing a risky account.

The right panel the conditional and unconditional risky share. The conditional risky share is the ratio of risky assets in total financial wealth for those who participate in risky investment. The conditional risky share monotonically increases over the life cycle. It increases from 40.6% in the age 21-25 to 46.0% in the age 41-45 to 52.2% in age 61-65. The unconditional risky share (combining both participation and conditional risky share), also exhibits a hump shape. It rises from a mere 12.3% in ages 21-25 to its peak of 36.9% in ages 51-55, and then decreases to 26.6% in age 61-65. In sum, these life-cycle patterns of risky share clearly suggest that younger investors are reluctant to take risks in financial assets despite longer investment horizon.

Risky Share by Wealth

We turn our attention to the relationship between the risky share and wealth (total financial assets). Table 2 reports the average risky share (both conditional and unconditional) across wealth quintile groups. Both measures of risky share shows a strong positive correlation with the level of wealth: Wealthier households take more risks. The unconditional risky share increases almost linearly from 4.8% at the 1st quintile to 33.9% at the 3rd and 64.6% at 5th fifth. The conditional risky share also increases (to a lesser degree than the participation) from 37.9% at the 1st quintile to 46.0% at the 3rd and 66.1% at the 5th. While we do not report here, the participation rate also monotonically increases with the wealth level. In the 5th quintile of wealth, almost everyone (97%) participate in risky investment.

Table 2: **Portfolio Choice and Financial Assets**

Wealth Quintile	Risky Share	
	Unconditional	Conditional
1st	4.8%	37.9%
2nd	23.5%	42.0%
3rd	33.9%	46.0%
4th	46.8%	50.7%
5th	64.6%	66.1%
Average	26.6%	46.9%

Since older people are on average wealthier, one might suspect that the age effect solely drives this positive correlation between the risky share and wealth. To see this we plot the average conditional risky share for every wealth quintile *within* age group in Figure 2. While it is not always monotonic (perhaps due to a small cell size), the figure clearly shows the positive correlation between the risky share and wealth for each age group we consider.

Robustness: Actively managed businesses and homeownership

So far, we have excluded two types of assets from the calculation of household wealth: (i) investment in the owned business and (ii) home. To see whether the life-cycle portfolio pattern above is robust to the inclusion of these assets, we report the (conditional) risky

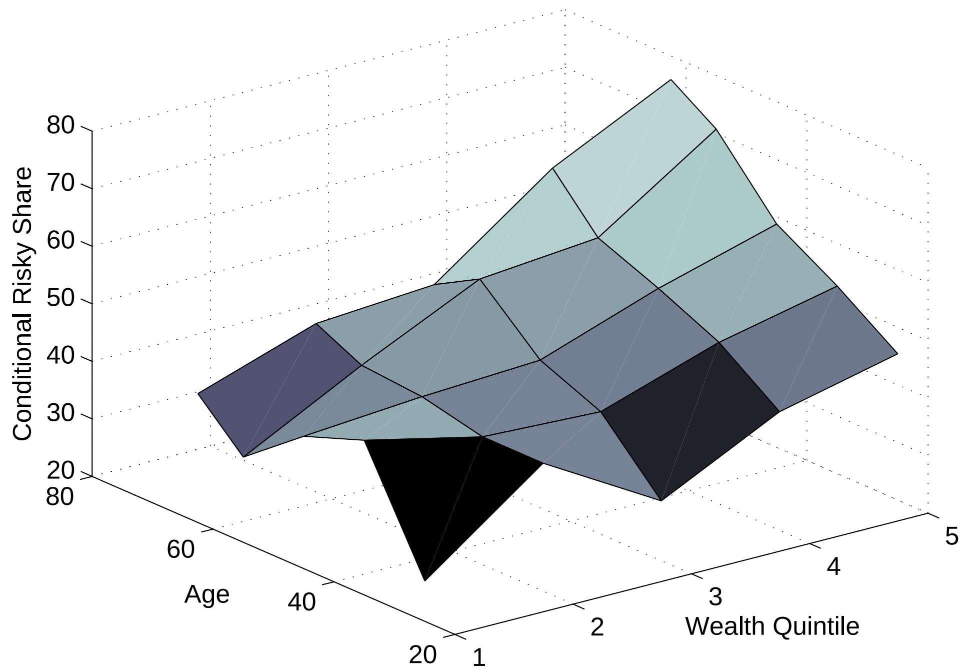

Figure 2: Risky share across wealth quintile for every age group.

shares including these assets in the households financial wealth in Table 3. The first column ("Benchmark") reflects the risky shares by age shown in Figure 1.

In the second column ("Business Included") we include the share value of actively managed businesses as part of the risky assets. In particular, we calculate the amount invested in a business as the net worth of investor's share in the business. We add the amount owed to the investor by the business and subtract money owed by the investor. When actively managed business values are included, the average risky share increases to 51.3% (from 46.7% according to our benchmark measure). More importantly, the increasing risky share in age is unaffected. It increases from 42% in age 21-30 to 56% in age 60s.

In the 3rd column ("Housing Included") we include the net worth of house(s). The net worth of house(s) is the sum of the house(s) value minus the amount borrowed as well as other lines of credit or loans the investor may have. We also add any investment in real estate like vacation houses. With house net worth included, the average risky share

Table 3: **Portfolio Choice, Share Value of Business and Homeownership**

Age Group	Conditional Risky Share of Financial Assets						
				Benchmark definition			
	Benchmark	Business	Housing	No business	Business	Rents H	Owns H
21-30	39.9%	43.5%	60.9%	40.0%	37.8	41.4%	40.2%
31-40	44.8%	51.1%	67.9%	45.5%	40.0%	38.8%	46.5%
41-50	47.7%	53.6%	71.9%	47.2%	51.2%	44.0%	48.8%
51-60	49.3%	54.1%	75.1%	49.2%	54.9%	–	50.4%
61-65	52.2%	56.7%	77.5%	51.7%	55.8%	–	53.9%
Average	46.9%	51.3%	71.1%	46.6%	48.9%	41.1%	48.3%

increases significantly to 71.1%. However, the increasing pattern of risky share in age is, again, unaffected. It increases from 61% in age 20s to 77% in 60s.

Next, we compare risky shares of financial wealth (our benchmark measure) of the households who actively invested in their own business to those not involved in their own business. These numbers are shown in next two columns ("No Business vs. Business"). First, the average risky shares of financial wealth are not so different from each other (46.6% vs. 48.9%). Second, the increasing pattern of risky share in age is not affected by this distinction.

Finally, we compare risky shares of financial wealth (our benchmark measure) of the home owners to those of renters ("Renters vs. Home Owners"). There many reasons to suspect that home-ownership might explain financial behavior. (1) Homeownership might crowd out stock holdings because it increases households' exposure to risk and illiquidity. Since young people have a high housing wealth to wealth ratio they are more likely to be affected by this channel. However as Cocco (2005) documents in the the cross-sectional data mortgage and stock shares are actually positively correlated suggesting that housing cannot explain why young people refuse to invest heavily in stocks. (2) Housing and stock market returns might be correlated. Cocco (2005) uses PSID data to show that this is not the case. Using our simple exercise we find that the average risky share is higher as they are richer than renters (48.3% vs. 41.1%). The risky share increases with age in both home owners and renters.

3 Simple Portfolio-Choice Theory

In this section, using a simple model we illustrate the basic economic mechanism. We examine how the portfolio choice (risky share) is influenced by the time horizon, labor income risk, and the level of financial wealth over the life cycle. (We provide additional intuition based on this simple model in the Appendix with graphical explanations.)

3.1 Model

The agent lives for $t = 1, .., T$ periods. Each period she receives income y_t which is an i.i.d random variable with probability function $f(y_t)$. Preferences are given by

$$U = E \sum_{t=1}^{T} \beta^{t-1} \frac{{c_t}^{1-\gamma}}{1-\gamma}$$

where γ is the coefficient of risk aversion. There are two financial instruments available for saving. A (risk-free) bond b_t pays a fixed gross return R after one period. A stock s_t pays a stohastic (gross) return of $R_s = R + \mu + \eta$ where μ is the risk premium and η is the innovation to excess return distributed according to $N(0, \sigma_\eta^2)$. The probability function associated with η is denoted by $\pi(\eta)$.

The agent divides the current output between consumption c and savings $b' + s'$. It is convenient to collapse total wealth into a single state variable $W = bR + sR_s$. Borrowing is not allowed for each investment ($b \geq 0$ and $s \geq 0$). The problem can be written recursively when the next period value is denoted with a superscript ($'$).

$$V_j(W, y) = \max_{c, s', b'} \quad \left\{ \frac{c^{1-\gamma}}{1-\gamma} + \beta \int_{\eta'} \int_{y'} V_{j+1}(W', y') df(y') d\pi(\eta') \right\}$$

$$\text{s.t.} \quad c + s' + b' = y + W$$

$$c \geq 0, \quad s' \geq 0, \quad b' \geq 0$$

Case 1: No labor income.

Under CRRA preferences, with no labor income, agent divides future assets with a constant share and the risky share is given by the so-called Samuelson (1969) rule:

$$\frac{s'}{s' + b'} \approx \frac{1}{\gamma} \frac{\mu}{\sigma_\eta^2}$$

The risky share 1) increases in the risk premium (μ), 2) decreases in the risk aversion (γ), and 3) decreases in the volatility of stock returns (σ_η). The total financial assets (wealth), W, and time horizon (age), j, are irrelevant for the portfolio decision[2]. If there is no labor income the investor behaves every period the same way.

Case 2: A two-period model.

Based on 2 period model, we first illustrate how labor income uncertainty affects the risky share in household portfolio and explain its correlation with wealth. The time horizon effect is discussed below in a 3-period model.

Human capital generates a stream of labor incomes. If labor income is stochastic, then human capital is equivalent to the implicit holding of a risky asset. If labor income is deterministic then human capital is viewed as a risk-free asset. Investors take the labor risk into account when making financial decisions. For example, investors who face high

[2] However, many financial planners advice people with longer investment horizon (i.e. younger people) to increase their risk exposure. As Ameriks and Zeldes (2004) note their approach stands on two observations: a) Over time stocks provide better returns than bonds. b) With a longer time horizon you have more time to weather the ups and downs of the stock market. Hence the longer the investors participate in the stock market the closer the ex-post stock return will be to the ex-ante expectation. However this argument seems to be missing the effect of time horizon on the variance of *total returns* which will be increasing over time. With CRRA preferences and i.i.d stock returns the two effects cancel out and the risky share is constant over time.

earnings risk are less willing to take risk in their financial decisions.

Now consider a worker in period 1 who expects with certainty a given amount of money in period 2 for example, 50,000$. If the worker has very little cash in hand in period 1 for example, 1000$ then she will place most of it if not all of it in the stock market since next period's total income is for the most part safe. If the same worker had more cash in hand in period 1 for example, 25,000$ then she would be more conservative in her portfolio choice since wealth is a larger part of her period's 2 total income. Things change if the worker faces large labor income risk in period 2. For example, she expects to earn 50,000$ but there is some positive probability of unemployment. Now the worker with little cash in hand (1000$) will likely place most of it if not all of it in a risk-free bond since next period's total income carries significant risk. If she has more cash (25,000$) she can afford taking some risk since her savings will be a significant part of period's 2 total income.

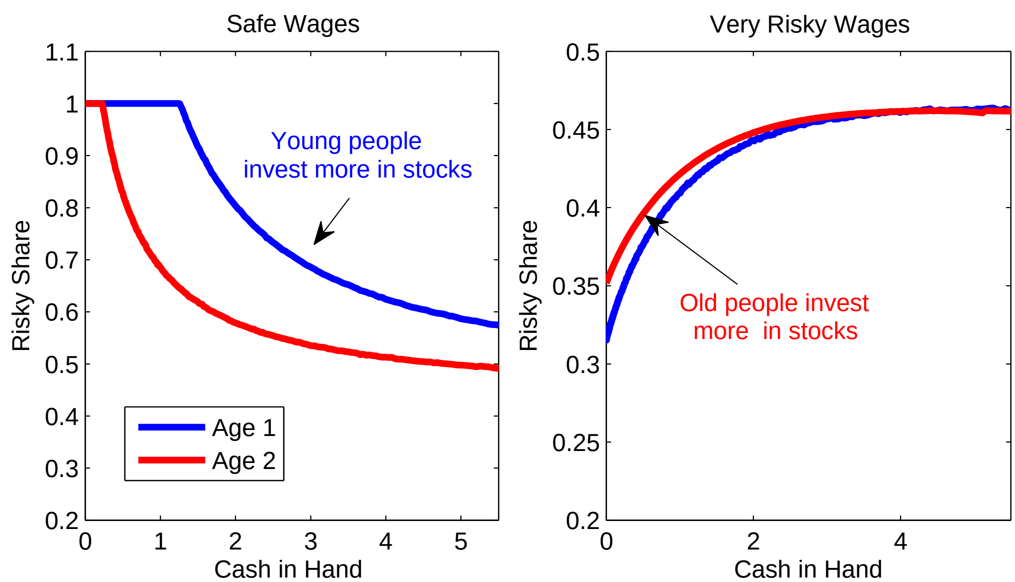

Figure 3: Risky Share, Wealth and Time Horizon for different amounts of labor income risk.

Figure 3 illustrates these patterns. We show how the risky share ($\frac{s'}{s'+b'}$) changes in total cash in hand ($W+y$) for two cases of labor income stream: deterministic, somewhat risky, and very risky. The policy functions at age 2 are similar to the ones we would get if we had solved a 2-period problem. If there is no wage risk case the risky share decreases in the amount of cash in hand. A worker with little financial wealth allocates 100% of his savings into stocks. As her financial wealth increases, the risky share decreases. When the labor income is somewhat risky, the risky share is smaller and less than 100% even for a worker with little financial wealth. However, the risky share is still decreasing (and converges to the same ratio with our previous case) as the wealth increases, opposite to what we see in the SCF. When the labor income is very risky, the risky share is increasing in wealth, consistent with the pattern we see in the SCF.

Case 3: A 3-period model: time horizon effect

We now examine the time horizon (life-cycle) effect in portfolio choice in a 3-period model. The 1st period corresponds to "Young" and the second to "Old." The time horizon distinguishes the young from old in one important way: the length of future wage payments. In present discounted value terms young people have a larger value of human capital. This allows them to take larger financial risk. Consider a worker who will retire next year and who decides to invest 10,000$. She will probably invest most of her money in bonds since her future total income (excluding some benefit) depends heavily on her financial assets. Now consider a worker just starting her career who undertakes the same amount of investment. Then she will likely allocate a larger amount to stocks since she can use her future payments to cover potential losses. However if wage payments are volatile a longer horizon will likely discourage risky investment. In Figure 3 we plot the risky share rule for the young and old workers. With low labor income uncertainty, young workers invest more heavily on risky assets for given amount of cash in hand. If labor income risk increases the young still take more risk albeit not significantly more than the old.

4 Life-Cycle Model

4.1 Basic Economic Environment

We develop a life-cycle model that will be calibrated to study quantitative implications. Earnings consists of deterministic ("ability") and stochastic ("shocks") components. Workers enter the labor market with different level of earnings ability. It is assumed that workers do not have a perfect information about their ability. They have a prior about their ability. Only the total earnings are observed. Given the existence of stochastic component, the prior about ability is updated through the successive realization of incomes over the life. This uncertainty about their ability is resolved over time.

For simplicity, we abstract from the general equilibrium aspect (by assuming constant average rates of returns to both stocks and bond) and the labor supply decision. Given the rates of returns to investments and the stochastic process of labor incomes, the model generates a distribution of portfolio allocation.[3]

Demographics The economy is populated by a continuum workers with total measure of one. Each worker enters the labor market at age $j = 1$ and lives for J periods. There is no population growth. At age j_R all agents have to retire (mandatory retirement) and receive a social security benefit amount ss which is financed from taxing labor income at rate τ_{ss}.

Preferences Agents derive utility from consumption c_j. Preferences are given by a time separable utility function:

$$U = E \sum_{j=1}^{J} \beta^{j-1} \frac{{c_j}^{1-\gamma}}{1-\gamma} \tag{1}$$

where β is the discount factor and γ captures both the intertemporal elasticity of substitu-

[3]We also develop a version of the model with perfect information where workers know their ability level when they enter the labor market. Comparing the perfect information model, can evaluate the quantitative importance of imperfect information for portfolio choice.

tion and the risk aversion[4].

Earnings Profile Workers face different earnings profiles due to differences in ability and/or shocks. We assume that the log earnings of a worker with age j is:

$$y_j = a + z_j + x_j + \varepsilon. \tag{2}$$

where log earnings consists of determinstic and stochastic components. The deterministic component consists of ability and age profile. The ability level (an intercept in age profile), a, is distributed across the population by $a \sim N(0, \sigma_a^2)$). We assume that all workers face a common age profile (usually a hump shape) in age z_j.[5] The stochastic component consists of persistent and transitory component. The persistent component, x, follows a AR(1) process:

$$x_j = \rho x_{j-1} + \nu_j, \quad \text{with} \quad \nu_j \sim \text{ i.i.d. } N(0, \sigma_\nu^2) \tag{3}$$

where the transition probability is represented by as a finite state Markov chain $\Gamma(x_j|x_{j-1})$. In our benchmark case, we assume a random walk ($\rho = 1$). The transitory component follows $\varepsilon_j \sim$ i.i.d. $N(0, \sigma_\varepsilon^2)$ where the probability distribution is denoted by $f(\varepsilon)$.

Savings We assume that capital markets are incomplete in two senses: (i) agents cannot borrow and also cannot fully insure against labor income shocks and (ii) they can only (partially) self-insure using two savings instruments. A risk-free bond b (paying gross return R in consumption units) and a stock s (paying $R_s = R + \mu + \eta$). We assume that investors can adjust their financial portfolio without any cost. Workers save for reasons in this model

[4]The literature has focused on alternative preferences to address the portfolio choice puzzles. For example Gomes and Michaelides (2005) use Epstein-Zin preferences with heterogeneity in both risk aversion and intertemporal elasticity of substitution, Gomes and Michaelides (2005) use habit formation and Wachter and Yogo (2008) use nonhomothetic preferences. Our approach is to use standard preferences with constant relative risk aversion. This way we can highlight the potential of labor income risk and learning to solve the portfolio choice puzzles.

[5]In Section 6 we also consider a case with heterogeneity in the growth profile.

economy— (i) to prepare for the retirement (life-cycle savings) and (ii) to insure against future labor income shocks (precautionary savings).

Social Security We assume that the government runs a balanced budget social security system to provide income for retirees. The government tax the labor income with rate rate τ_{ss} to finance the constant retirement benefit ss per retire.

Worker's Problem: Perfect Information It is useful to first present the worker's problem where he has a perfect information about his ability. As in the simple illustrative example, we collapse bond and stock holdings in one state variable $W = bR + sR_s$, total financial wealth. Then the j-period value function, denoted by $V_j(W, a, x, \varepsilon)$, can be expressed as:

$$V_j(W,a,x,\varepsilon) = \max_{c,s',b'} \left\{ u(c) + \beta \int_{\eta'} \int_{x'} \int_{\varepsilon'} V_{j+1}(W', a, x', \varepsilon') df(\varepsilon') d\Gamma(x'|x) d\pi(\eta') \right\} \tag{4}$$

$$\text{s.t.} \quad c + s' + b' = (1-\tau_{ss})e^{y_j} + ss\{j \geq j_R\} + W \tag{5}$$

$$y_j = a + z_j + x_j + \varepsilon. \tag{6}$$

Workers' Problem: Imperfect Information Now consider that workers do not have a perfect information about their ability. Since only the total earnings are observed, workers cannot completely separate a from y_j.

Given the normality assumption, the agent's prior belief about the ability is summarized by $\{\mu_a, \sigma_a^2\}$. Similarly, the agent's prior belief about the AR(1) shock component is summarized by $\{\mu_x, \sigma_x^2\}$. The beliefs over the cross correlation of the components is given by σ_{ax}. We express the priors as:

$$\mathbf{M}_{j|j-1} = \begin{bmatrix} \mu_a \\ \mu_x \end{bmatrix}_{j|j-1} \qquad \mathbf{V}_{j|j-1} = \begin{bmatrix} \sigma_a^2 & \sigma_{ax} \\ \sigma_{ax} & \sigma_x^2 \end{bmatrix}_{j|j-1} . \tag{7}$$

where the subscript $j|j-1$ denotes prior beliefs about a variable at age j formed before y_j is realized. Similarly, the subscript $j|j$ denotes posterior beliefs about the variable that incorporates the signal y_j. We assume that agents update their priors according to a "constant gain" learning mechanism. Abstracting from the life-cycle component, z_j, to make the notation easy, the posterior means are given by:

$$\mathbf{M}_{j|j} = \mathbf{M}_{j|j-1} + \begin{bmatrix} \frac{\sigma_a^2+\sigma_{ax}}{\sigma_a^2+2\sigma_{ax}+\sigma_x^2+\sigma_\varepsilon^2} \\ \frac{\sigma_x^2+\sigma_{ax}}{\sigma_a^2+2\sigma_{ax}+\sigma_x^2+\sigma_\varepsilon^2} \end{bmatrix} (y_j - \mathbf{H}_j'\mathbf{M}_{j|j-1}). \tag{8}$$

$$\mathbf{V}_{j|j} = \mathbf{V}_{j|j-1} - \lambda \begin{bmatrix} \frac{\sigma_a^2+\sigma_{ax}}{\sigma_a^2+2\sigma_{ax}+\sigma_x^2+\sigma_\varepsilon^2} \\ \frac{\sigma_x^2+\sigma_{ax}}{\sigma_a^2+2\sigma_{ax}+\sigma_x^2+\sigma_\varepsilon^2} \end{bmatrix} \mathbf{H}_j'\mathbf{V}_{j|j-1} \tag{9}$$

The parameter λ represents the agents' updating technology. Lower λ implies that prior beliefs and not new information matter more in shaping the posterior variance. In this case the agent's learning about her type will be relatively slow Note also (1) that the prior variance does not depend explicitly on the particular realization of incomes (y_j) and (2) that prior variance decreases over time. The latter element makes the uncertainty of labor income for older workers smaller than that of younger workers. Using the above formula, the next period's income follows the conditional distribution function:

$$F(y_{j+1}|y_j) = N(\mathbf{H}_{j+1}'\mathbf{M}_{j+1|j}, \mathbf{H}_{j+1}'\mathbf{V}_{j+1|j}\mathbf{H}_{j+1} + \varepsilon_j) \tag{10}$$

where $\mathbf{H}$ is a (2×1) unit vector. The value function of a worker at age j is:

$$V_j(W, y, \mathbf{M}_{j|j-1}) = \max_{c,s',b'} \left\{ u(c) + \beta \int_{\eta'} \int_{y'} V_{j+1}(W', y', \mathbf{M}_{j+1|j}) dF_j(y'|y) d\pi(\eta') \right\} \tag{11}$$

$$\text{s.t.} \quad c + s' + b' \ = (1-\tau_{ss})e^{y_j} + ss\{j \geq j_R\} + \ W \tag{12}$$

where the state variables include wealth, W, the realized income y, and the prior about the mean of the distribution ($\mathbf{M}_{j|j-1}$). Note that the prior for the second moment ($\mathbf{V}_{j|j-1}$) is not explicitly included in the value function since age (j) is a sufficient statistic due to our assumption that agents enter the labor market with the same prior.

5 Quantitative Analysis

5.1 Calibration

We calibrate the model to study its quantitative implications on household portfolio profiles. There are four set of parameters: (i) life cycle parameters $\{j_R, J\}$, (ii) preferences $\{\gamma, \beta\}$, (iii) asset market structure $\{R, \mu, \sigma_\eta^2\}$, (iv) labor income process $\{z_j, \rho, \sigma_a^2, \sigma_\varepsilon^2, \sigma_\nu^2, \mathbf{V}_{1|0}\}$, and (v) the social security system $\{\tau_{ss}, ss\}$. Unless stated otherwise, we keep the parameter values constant across the models. The model period is one year. The agent lives for a total of $J = 60$ periods. This life-cycle framework corresponds to ages 21-80. In our model, agents retire at the age of 65, $j_R = 45$ when they receive the social security benefit, ss. Each year the social security benefit is financed by taxing labor income with a rate $\tau_{ss} = 12\%$.

The relative risk aversion, γ, is assumed to be 5. We examine how sensitive our results are for different values of risk aversion in Section 6. The discount factor, β, is calibrated to match a wealth-to-income ratio of 3, the value commonly targeted in the literature. Matching the average asset holdings is key to generate a realistic average share of risky savings. As mentioned in a number of papers (Storesletten et al. (2004), Ball (2008)) if the average asset holdings is very high, income risk becomes unimportant since workers can effectively self-insure against them. As illustrated by simple examples in Section 3, investors with sufficiently large assets will invest the same amount regardless of labor income risk. Hence matching the cross-sectional distribution of asset holdings is important for the quantitative analysis of portfolio choice.

The risk-free rate is set to $R = 1.02$ based on the average real rate of US 3-month treasury bills in the post war period. Following Gomes and Michaelides (2005) we set the equity premium equal to $\mu = 4\%$. As common in the literature this value is lower than its long-run historical average because it takes into account lower forward looking estimates. The standard deviation of the innovations to the risky asset is set at $\sigma_\eta = 18\%$ again based on Gomes and Michaelides (2005)[6]. We assume that the stock returns are orthogonal to labor income risks. The empirical evidence on the correlation between labor income risk and stock market returns is mixed. While Davis and Willen (2000) find a positive correlation, Campbell et al. (2001) find a positive correlation only for specific population groups.

We calibrate the parameters of income process to target the cross-sectional variance in log-hourly earnings across age cohorts reported in Storesletten et al. (2004). The cross sectional variation in earnings among the youngest cohort is attributed to a distribution of abilities, a. That is, $\sigma_a^2 = 21\%$ is chosen to match the variance of log earnings upon the labor market entry. We assume that there are two types (a_1, a_2) with equal mass ($a_1 = e^{-\sigma_a}$ and $a_2 = e^{\sigma_a}$). The common growth component (the average age profile), z_j, is taken from Hansen (1993). We assume that the persistent component of income shock follows a random walk ($\rho = 1$) and choose the variance of innovation ($\sigma_\nu^2 = 1.7\%$) to match the variance of log-earnings at age 60. The residual variation of earnings is explained by the variance of i.i.d. component ($\sigma_\varepsilon^2 = 6\%$). Finally, we assume that the initial uncertainty about ability (initial prior distribution) is the same as the actual distribution of a[7].

The parameter λ is set based on Edwards (1968) who documents that it takes two to five observations to induce an individual to change her priors. This means that people use between 20%-50% of new information when forming their beliefs. In our benchmark model we use a value of $\lambda = 0.3$. We discuss the significance of this parameter and other ways to motivate it in the conclusion.

[6]Jagannathan and Kocherlakota (1996) report that for the period between 1926 and 1990 the standard deviation of annual real returns in S&P stock price index were 21% as opposed to 4.4% in T-bills.

[7]This assumption implies that the agent does not have more information than the econometrician when entering the labor market. This can be seen as an upper bound for the amount of prior uncertainty. We confirm that the model performs similarly for smaller amounts of initial priors in Section 6.

5.2 Benchmark Results

We simulate the model and generate key statistics like the average risky share, the age-profile of the risky share and the correlation between risky share and financial assets. We discuss the potential of the two models to explain the empirical patterns discussed in Section 2. Empirically in spite of the high equity premium investors prefer to allocate a small fraction of their savings to stocks. The average portfolio share is **46.9%**. In the PIM there is a surprisingly large demand for stocks given realistic values of equity premium and stock returns. Investors allocate **81.6%** of their savings to stocks. This feature is directly related to the *equity premium puzzle*[8]. Once we allow for profile uncertainty (IIM) the demand for stocks decreases significantly in response to larger wage risk. The average share is now **48.6%**.

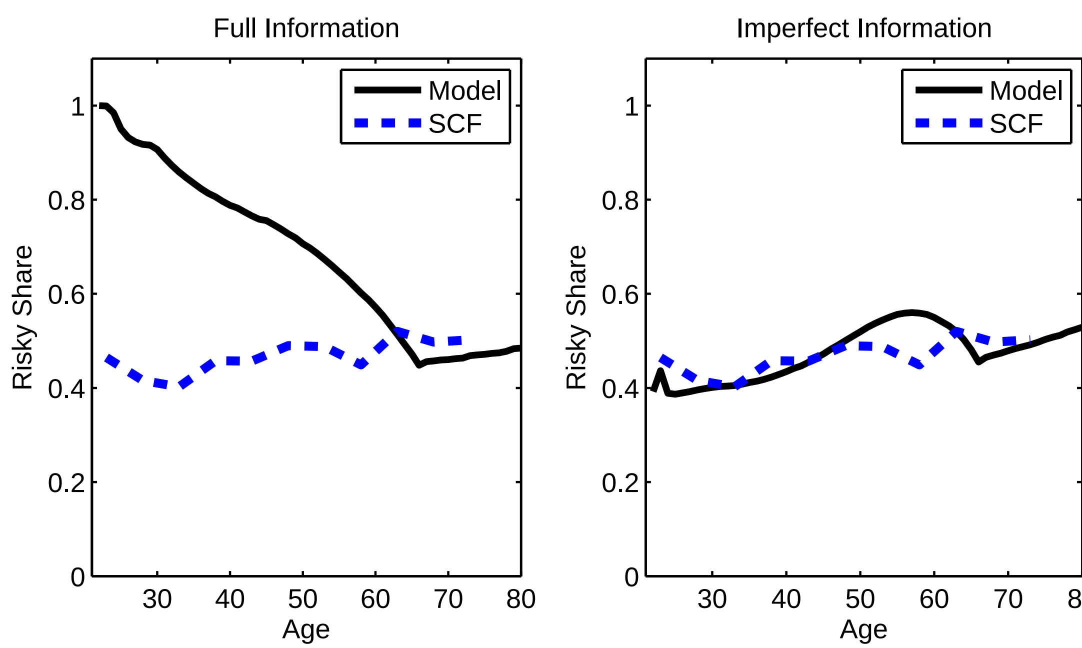

Figure 5: Age Profile of Conditional Risky Share: *Left Panel:* Perfect Information Model. *Right Panel:* Imperfect Information Model.

We next turn our attention to the age-profile of risky share. Cocco, Gomes and Maen-

[8]Mehra and Prescott (1986)showed how the covariance of aggregate consumption with stock returns which defines the riskiness of stock holdings cannot be justified by the high equity premium unless investors are extremely risk averse.

hout (2005) among others document the inability of portfolio choice model to reproduce the weakly increasing evolution of the risky share. In the data investors *increase* on average their risky share by **1.45** percentage points every year. In the PIM investors *decrease* on average their share by **0.75** percentage points. This can also be seen in the left panel of Figure 5. The underlying mechanism is the small risk exposure of younger cohorts due to a long lifetime of relatively stable wage payments. We find that imperfect information is able to correct for this feature and fit the data very well. In the IIM investors *increase* on average their risky share by **0.85** percentage points every year much closer to the data. Large labor income uncertainty discourages younger workers to take risk in the financial market. As seen in Figure 5 (right panel) the IIM can track life time investment behavior pretty closely.

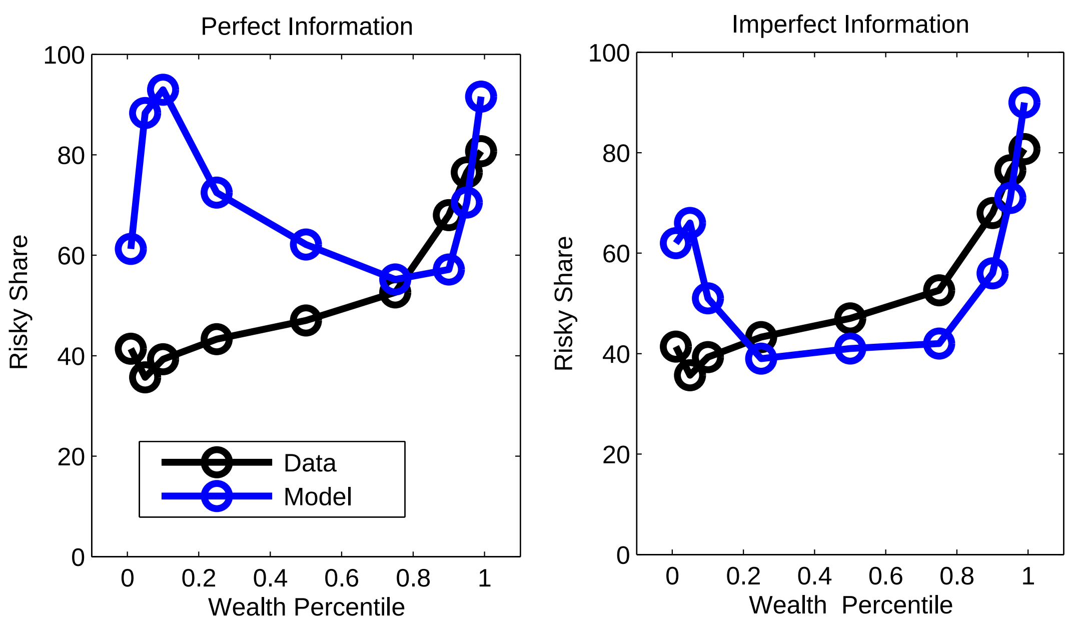

Figure 6: Risky Share by wealth groups: *Left Panel:* Perfect Information Model. *Right Panel:* Imperfect Information Model.

A second composition puzzle relates to the positive relation between the risky share and financial assets. In figure 6 we calculate the average risky share across wealth quintile in both models and compare it to the data. The IIM is able to reproduce the positive relation seen in the data.

6 Conclusion-Discussion

We build a model of portfolio choice with optimal learning about labor income to address the long standing puzzles namely, the low average share of risky assets, the positive relation between risky assets and financial assets and the increasing age profile of the risky share. Our model can fit the data closely as long as agents can learn their type relatively slow. Higher labor income risk reduces the willingness to take financial risk and decreases the average risky share. This is especially true for younger households who have little information about their ability. As households grow older their willingness to take risk increases partially due to the information update and also due to a shorter time horizon. As a next step of this research project we wish to introduce heterogeneity in growth components as a way to make learning slower.

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