

Demand for Liquidity and Welfare Cost of Inflation by Cohort and Age of Households*

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Abstract

Cross-sectional data show that money holding differs significantly over household consumption and age. Liquidity demand for money (i.e., money holding per dollar of consumption) decreases as household consumption increases. It also increases with household age conditional on the level of consumption. Observed age differences in money holdings contain not only age-specific information but also cohort-specific one. Using a life-cycle model, this paper disentangles these two effects on money demand and quantifies welfare gains of reducing the long-run inflation rate. We dynamically calibrate the model to micro data and macroeconomic conditions over time. We find that, although a large part of the observed cross-sectional age differences in money demand can be accounted for by some age effects, cohort effects play a non-negligible part, supporting a presence of financial innovation. In addition, changing inflation has significantly different impacts across household groups due to their heterogeneity in money holding. When inflation increases from the 2009 level to 10%, we find the aggregate welfare loss in consumption to be 1.34%. These losses are accrued mostly by generations that are currently alive and less by future cohorts. Finally, poorer households lose more than their rich peers.

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1 Introduction

The literature on money demand has mostly focused on aggregate money demand. In this paper we document money demand patterns across household's age and social class using cross-sectional data. Measuring money demand across age based on cross-sectional data raises a potential issue of whether observed differences are due to *age effects* or *cohort effects*. The potential existence of these two effects implies that an identification of age-specific money demand requires separating out the cohort effects from those that purely come from age. Although there is a growing literature documenting heterogeneity over money holdings of households, there has not been any study disentangling age and cohort effects of money demand. In addressing this issue, we develop an overlapping generations model with a choice of cash and credit purchases of consumption goods, where transaction costs of credit incorporate age and cohort dependence. In addition, an analysis of cohort effects (i.e., multiple generations born at different points in time) necessarily requires an incorporation of *time effects* on money demand from macroeconomic conditions such as inflation and interest rates. Hence, we calibrate the model dynamically over a long horizon where macroeconomic conditions change over time and use cross-sectional money demand patterns over two periods to disentangle the contributions of age and cohort effects to observed money demand patterns. We use the calibrated model to conduct analysis on welfare costs of inflation and interest rate elasticity of money.

We use the money-consumption ratio as a measure of money demand for transaction purposes. Cross-sectional data show that money-consumption ratios increase with household age and decrease with consumption. Regarding age-specific money demand, on one hand, age would make credit transaction more costly as potentially complex information processing is required for credit transactions (e.g., reading fine prints of a credit contract) than it would for money. On the other hand, currently old households would have faced more costly credit transaction technologies when they were young relative to those that currently young households face today due to financial innovation (i.e., the cohort effects). We introduce age- and cohort-dependent credit transaction costs in our model to allow flexibility in capturing these possibilities. Credit transactions face a fixed cost with respect to total consumption, increasing the use of credit relative to the use of money as the level of consumption rises. This fixed cost is age specific, giving the model flexibility to capture age variations of money demand. We also introduce a cohort (or generation) specific parameter in credit transaction

costs. Our identification strategy in separating the age and cohort effects is the following. We target two sets of moments: (i) money-consumption ratios by age given a time period and (ii) money-consumption ratios across two periods given age. These moments allow us to separately identify the age and cohort effects as the moment (i) contains information on both age and cohort effects and the moment (ii) only the cohort effects. We simultaneously calibrate the age and cohort parameters to match these moments from the model and the data.

The calibrated model replicates money demand variations across households in the data well. We find that age-dependent cost parameters are increasing with age as well as generating more elastic money demand with age. Calibrated cohort-dependent cost parameters imply that credit transactions have become less costly for generations who were born later, an evidence supporting financial innovation. In addition, the calibrated model allows us to disentangle how much each of the age and cohort effects contributes to the observed money demand. By conducting a hypothetical simulation without cohort effects, we find that about 12.5% to 21.5% of the observed money demand variations across age groups in 2009 are coming from the cohort effects (and the rest from the age effects).

We also conduct welfare analysis of inflation. When inflation is increased from the 2009 level of 1.92% to 10%, we find that aggregate welfare decreases by 1.34%, measured in consumption equivalence. Some part of these losses become additional seignorage revenues to the government. Since this amounts to 0.79%, the net loss to the economy is 0.55%. Our model of different household types can tell us who bears these losses. This loss is mostly accrued by generations that are alive in 2009, e.g., a loss of almost 2% of average consumption by 60 year-old households in 2009. On the social class dimension, poor households lose the most, 2.35% of their average consumption while their rich peers only lose 0.94%.

There is a long list of previous studies measuring the welfare cost of inflation. [Lucas \(2000\)](#) finds that when the annual inflation rate is reduced from 10 per cent to 0 per cent, the gain is slightly less than 1 per cent of real income. However, Lucas points out that “...Using aggregate evidence only, it may not be possible to estimate reliably the gains from reducing inflation further, ...”. That is, in order to accurately and reliably assess the cost of inflation, one needs to look into the cross-section evidence on the demand for money. [Mulligan and Sala-i Martin \(2000\)](#) document that 59 per cent of the U.S. households hold cash and checking accounts, and do not hold interest-bearing financial assets. This fact is interpreted as the existence of fixed cost for financial transactions. The fixed cost is one form of dead

weight loss in models of money demand. As the inflation rate decreases, households will hold more cash, and avoid paying the fixed cost of financial transactions. The aggregate dead weight loss is not necessarily equal to the area under the inverse aggregate money demand curve, except in extreme cases. Ignoring these fixed (adoption) costs will underestimate the welfare cost of higher inflation. [Attanasio, Guiso, and Jappelli \(2002\)](#) use a household level data set with much richer information on cash holding, cash transactions, and usage of ATM cards. They estimate the demand for cash and for interest-bearing assets, and find that the welfare cost of inflation varies considerably within the population but is small (0.1 per cent of consumption or less). [Alvarez and Lippi \(2009\)](#) make important contributes to the analysis of money demand using micro data. They extend cash-inventory management models of [Baumol \(1952\)](#) and [Tobin \(1956\)](#) to incorporate precautionary cash holdings due to uncertainty with respect to a chance to withdraw cash without cost. They use Italian household survey data with detailed information on cash management to estimate the model. They use the estimated model for quantitatively analyzing the elasticity of money demand as well as the welfare cost of inflation. Also related to our study is [Ragot \(2010\)](#) that finds the distribution of money across households is more similar to the distribution of financial assets than to that of consumption. It shows that transaction frictions in the financial market explains 85% of total money demand.

Another important contribution to measuring the welfare cost of inflation is given by [Erosa and Ventura \(2002\)](#). They extend the Aiyagari model to include the cash-in-advance constraint and study the welfare distribution of changing inflation rates. On the other hand, [Chiu and Molico \(2010\)](#) calibrate a search-theoretical model of demand for money, and find that the welfare cost of inflation is 40 per cent smaller than that in complete market representative-agent models, such as [Lucas \(2000\)](#). However, the existing studies often ignore the age profile of money holding. One of existing studies that analyze age profiles of money is [Heer and Maussner \(2007\)](#). They document that the money-age profile is hump-shaped in the U.S., and money is only weakly correlated with income and wealth.

In the next section, we use Canadian data and document how money holding varies with age and consumption. A life-cycle model of demand for money is developed in Section 3, and Section 4 discuss the model calibration. In Section 5, we disentangle age and cohort effects. Section 6 evaluates welfare from changing inflation. Finally, Section 7 concludes the paper.

2 Money demand in Canadian households

Our measure of money demand for transaction purposes is the ratio of money holdings to consumption. Data availability on both household money holdings and consumption expenditures is limited. In Canada, the Canadian Financial Monitor (CFM) by Ipsos Reid contains information on household income, expenditure and balance sheets for the 2008-2010 period. CFM also covers the period 1999-2007 but without the information on consumption.¹ We focus on the CFM data for most of our analysis, however, we additionally use another household survey data set, the Survey of Household Spending (SHS) by Statistics Canada, for consumption information prior to 2008. The SHS provides detailed annual consumption information of households since 1999.²

We analyze money holdings and consumption, where our definition of money includes balances on checking and low-interest rate savings accounts. Since a focus of this paper is to analyze effects of inflation on households, a broad measure of liquid assets that are used for transactions and whose values are sensitive to inflation is used.³

We define consumption as the household's sum of gross monthly spending on non-durable goods, services, and durable goods. Excluded from this consumption definition are the property tax and housing services from owned and rented residential properties. Some durable spending is at an annual frequency, including the purchase of a vehicle, home appliances and electronics, home furnishings, vacation trips, home renovation. We divide them by twelve to obtain the monthly average spending on these goods and services.

Figure 1 displays average money-consumption ratios over the 1999-2009 period, where we calculate the average money holdings from the CFM and the average consumption from the SHS. Household money demand for transaction purpose appears to vary over time. Furthermore, Figure 2 shows that this variation of money demand is mostly driven by money holdings rather than by consumption. If financial innovation that lowers the cost of credit transactions was the main factor driving money demand, we would observe a secular decrease in money demand. Hence, other effects are likely at work, including time effects such as the movements in inflation, a cost of holding money.

¹See Appendix for more detailed description of CFM.

²The SHS does not contain information on household assets. See Appendix for more detailed description of SHS.

³Data on “cash in wallet” are also available in the CFM but only from 2009. Hence, we do not include it in our analysis. It does not qualitatively impact our study as cash holdings are relatively small compared to bank account balances.

2.1 Money demand by age

In order to observe money-consumption ratios over age, we group households by the age of the household head into six categories: less than or equal to 35 years old, 36-45 years old, 46-55 years old, 56-65 years old, 66-75 years old, and 76-85 years old. Given an age group, we further make five sub-groups based on consumption quintile. Money-consumption ratios are calculated by taking the ratio of the average money holding and the average consumption of a given group. Using the 2008-2010 CFM data, Figure 3 displays the three stylized facts regarding money-consumption ratios, where the values on the horizontal axis are relative to the overall average consumption. We observe that (i) money holding per dollar of consumption (i.e., money-consumption ratio) decreases with consumption; (ii) money-consumption ratio increases with age conditional on consumption; and (iii) age differences in money-consumption ratio shrink as consumption increases.

The first fact has been documented in previous studies (see [Erosa and Ventura \(2002\)](#)). It suggests that, as household consumption increases, the fraction of consumption purchased with money becomes smaller, implying that non-cash payment methods become more important as consumption increases.⁴ What has not been studied previously is the age aspect of money holding, i.e., the second and the third stylized facts.

The use of cross-sectional data raises an issue in interpreting what may be causing these observations. For example, the observed age differences in money demand may be a result of both age effects and cohort effects. All else equal (i.e., the same person in a non-changing environment over time), age effects imply a change in money demand due to aging, e.g., higher cost of credit transactions that require processing complex information such as a credit contract with fine prints. Alternatively, cohort effects imply a change in money demand due to the experience of different generations of households, e.g., financial innovation. Currently old households have lived their lives under a path of historical financial technologies. The experience that currently young households will have growing up to be the same age as the currently old households will be quite different and under a path of future financial technologies. A dynamic nature of these cohort effects requires money demand information beyond what cross-sectional data can provide.

Finally and in addition to age and cohort effects, macroeconomic environments are changing over time such as inflation, interest rates and taxes. These conditions capture time-specific

⁴[Erosa and Ventura \(2002\)](#) replicate this fact by assuming that the transaction cost of using credit displays economies of scale as in [Dotsey and Ireland \(1996\)](#).

macroeconomic effects (i.e., time effects) on money demand and interact with age and cohort effects. In analyzing welfare cost of inflation in aggregate as well as by type of households it is important to disentangle these effects as underlying fundamentals affecting the magnitude of these effects are changing over time. For example, age distributions change over time, affecting the magnitude of age effects in aggregate. In addition, there will always be new cohorts being born who will face new financial technology. A structural model that allows these three effects is necessary to separately identify the effects for the analysis of welfare cost of inflation. In the next section, we describe such a model.

3 Model

In this section, we present an over-lapping generation model of money holding. There is no uncertainty. There is a continuum of households in the economy. Each household lives for I periods and belongs to one of J social classes, where i indexes the age and j their class. In addition, let h index a cohort of households. They supply labour exogenously given a fixed labour endowment. In each period, households use money and credit to purchase consumption good.⁵ The fraction of consumption purchased with money is subject to a cash-in-advance constraint. Purchasing by credit requires a fixed transaction cost.

A trade-off between the use of money and credit is the following. On one hand, holding money is subject to inflation tax as inflation reduces the real value of money. Using credit, on the other hand, involves a direct cost. The credit transaction technology is a function of the fraction of consumption purchased with credit, s , and specified as:⁶

$$\gamma_{ih}(s) = \int_0^s \gamma_i \cdot \eta^h \cdot \left(\frac{x}{1-x} \right)^{\theta_i} dx, \quad (1)$$

where $\gamma_i > 0$, $\theta_i \geq 1$ and $\eta > 0$.⁷ This function is convex and strictly increasing in s for all $s \in [0, 1)$. It is independent of the level of consumption. Thus, the credit technology exhibits increasing returns to scale: the credit transaction cost per unit of consumption decreases with consumption. This assumption helps generate the first stylized fact: the money-consumption ratio decreases with consumption. We allow both γ_i and θ_i to vary

⁵There is only one type of good which can be purchased by money or credit.

⁶This specification is similar to those used in [Dotsey and Ireland \(1996\)](#) and [Erosa and Ventura \(2002\)](#) except that we introduce age specific parameters and a parameter capturing cohort effects.

⁷The assumption, $\theta_j \geq 1$, guarantees that money holdings increase with consumption at a decreasing rate, i.e., stylized fact (i) discussed in Section 2.1. See a discussion below with Equation 4 below.

with age. The empirical evidence shown in the previous section suggests that the demand for money is higher for older households than for their younger counterparts. One possible reason is that old households face more difficulty processing complex financial information associated with a use the credit technology.⁸ Age-specific parameters, γ_i and θ_i , allow the model to capture these age effects. Another possible explanation is that currently old households have not experienced the same level of financial technologies growing up that later generations would, leading the currently old to use less financial-technology intensive money as a means of payment. In this explanation, currently young households with their experience in later financial technology will use less money and more credit as a means of payment in their old age than do the currently old households. Hence, this cohort effect may explain the cross-sectional age differences in money demand. This possibility will be captured by η in the cost function (1), where h is an integer indexing the cohort. Thus, we assume that cohort effects take a form of secular changes in financial innovation.

Given this cost function, each household makes money/credit payment decisions, consumption/savings decisions, and decisions on money/non-money asset portfolios in savings. Let c_{hij} be consumption, a_{hij} be the real asset, and m_{hij} be real money holding of the agent of cohort h , age i and class j . The agent's optimization problem is given by:

$$\max_{\{c_{hij}, s_{hij}, a_{h,i,j+1}, m_{h,i,j+1}\}} \sum_{i=1}^I \beta_{ij} \frac{c_{hij}^{1-\sigma} - 1}{1-\sigma} \quad (2)$$

subject to

$$c_{hij}(1 - s_{hij}) \leq m_{hij}, \quad (3)$$

$$c_{hij} + q_t \gamma(s_{hij}) + a_{h,i,j+1} + (1 + \pi_t) m_{h,i,j+1} \leq [1 + r_t(1 - \tau_{a,t})] a_{hij} + m_{hij} + (1 - \tau_{z,t}) w_t z_{ij},$$

$$m_{h,1,j} = \underline{m},$$

where q_t is the price per unit of credit-transaction service, w_t the wage rate, r_t the interest rate, $\tau_{a,t}$ the tax rate on non-money asset income, and $\tau_{z,t}$ the tax rate on wage income, all at time t .⁹ Labor endowment, z_{ij} , is assumed to be independent of cohorts or time. Newborns are endowed with a small amount of initial money holdings, $\underline{m}$.

⁸In cash-inventory management literature, empirical evidence shows that older households pay fewer visits to bank branches than younger households. See Mulligan and Sala-i Martin (2000).

⁹Following Erosa and Ventura (2002), we assume $q_t = w_t$, implying that credit transaction costs are specified in terms of time costs.

Condition (3) is the cash-in-advance constraint. Given its current money holding, a household chooses consumption and a fraction of credit transaction to purchase it. Given no uncertainty, the cash-in-advance constraint holds with equality. Consumption can only increase above current money holdings if the household uses credit. We assume that discount factors differ by age and income type, which may reflect changes in family size and costs related to labor force participation.¹⁰

From the household's first-order necessary condition, we can obtain the optimal money-consumption ratio as follows:

$$\frac{m_{hij}}{c_{hij}} = 1 - s_{hij} = \frac{1}{1 + \left[\widetilde{R}_t c_{hij} / (q_t \gamma_i \eta^h) \right]^{1/\theta_i}}, \quad (4)$$

where $\widetilde{R}_t$ is the after-tax nominal interest rate, $\widetilde{R}_t = (1 + \pi_t)[1 + r_t(1 - \tau_{a,t})] - 1$, where π_t is the inflation rate. If $\theta_i \geq 1$, the equality involving the two end terms in (4) implies that the demand for money increases with consumption. The second equality in (4) also implies that the rate of increase in money will be less than that of consumption. Hence, the money-consumption ratio declines as consumption rises.

All three sets of parameters, γ_i 's, θ_i 's and η , dictate interest rate elasticity of money demand, as the interest rate elasticity of money demand is given by

$$\frac{\partial(m_{hij}/c_{hij})}{\partial \widetilde{R}_t} \frac{\widetilde{R}_t}{m_{hij}/c_{hij}} = -\frac{1}{\theta_i} \cdot \left[1 + \left(\frac{q_t \gamma_i \eta^h}{\widetilde{R}_t c_{hij}} \right)^{1/\theta_i} \right]^{-1}. \quad (5)$$

From Equation 5, we observe that the nominal interest rate elasticity of money demand is negative. The interest rate elasticity increases (in absolute terms) with consumption, meaning the demand for money is more elastic as consumption becomes larger. In addition, the elasticity decreases as γ_i or η increases. It also decreases as θ_i increases, given $\theta_i \geq 1$.

There is a government that balance its budget every period. The government has an exogenous spending G_t , which is financed by revenues from inflation tax, capital income tax and labour income tax, as follows:

$$G_t = (1 + \pi_t)M_{t+1} - M_t + \tau_{a,t}r_t A_t + \tau_{z,t}w_t Z, \quad (6)$$

¹⁰In calibration, heterogeneous β 's allow us to capture consumption variations across household types, especially given that there is no idiosyncratic uncertainty in the model.

where M_t is the aggregate demand for money in real terms, A_t is the aggregate assets of all households at time t , and Z is the total labour supply. The aggregate variables are defined as:

$$M_t = \sum_{ij}^{IJ} m_{hijt}, \quad (7)$$

$$A_t = \sum_{ij}^{IJ} a_{hijt}, \text{ and} \quad (8)$$

$$Z = \sum_{ij}^{IJ} z_{ij}, \quad (9)$$

where the cohort index h changes by i given t .¹¹ The first two terms in the government budget equation, (6), represent seigneurage. This specification is motivated by our assumptions that money supply just satisfies its demand by households and the central bank perfectly controls inflation.

Finally, a Cobb-Douglas production technology, $F(K_t, Z) = K_t^\alpha Z^{1-\alpha}$, exists with capital and labour as inputs and produces output which is traded in a global market. The real interest rate, r_t , is exogenously determined in the global capital market. Labour input is assumed to be non-tradable and only domestically supplied. In equilibrium, these assumptions imply time-varying wage rates that are consistent with the rest of the economy.

Exogenous macroeconomic variables are $\widetilde{R}_t$, π_t and τ_{at} . Real interest rates, r_t , are implied by $\widetilde{R}_t$ and π_t . An equilibrium of our economy is defined by a set of variables, $\{\tau_{zt}, K_t, M_t, A_t, Z, c_{hij}, s_{hij}, a_{h,i,j+1}, m_{h,i,j+1}, w_t\} \forall h, i, j, t$, such that

- Each cohort of households, h , optimally choose c_{hij} , s_{hij} , $a_{h,i,j+1}$ and $m_{h,i,j+1}$ by solving (2);
- τ_{zt} is set to balance the government budget, (6);
- Conditions (7), (8) and (9) hold;
- K_t is implied by $r_t = \alpha \left(\frac{Z}{K_t} \right)^{1-\alpha}$; and
- Wage rates are determined by $w_t = (1 - \alpha) \left(\frac{K_t}{Z} \right)^\alpha$.

¹¹Given time t , age and cohort correspond one to one.

4 Calibration

A focus of our calibration strategy will be on identifying three effects on money-consumption ratios (i.e., money demand): age, cohort and time. Two assumptions allow us to separately identify the three effects. First, we assume that age and cohort effects are embodied in the credit transaction technology as specified in Equation (1). Specifically, we assume that cohort effects display secular changes in the technology across cohorts.¹² Second, changes in money demand due to changing macroeconomic conditions over time (i.e., $\widetilde{R}_t$, π_t , τ_{at} and τ_{zt}) capture the time effects. Although parameters are calibrated to simultaneously match all target moments together, we discuss specific parameters and their corresponding target moments to help clarify how we identify the three effects.

A model period is 10 years. We set $I = 7$ (seven age groups) and $J = 5$ (five social classes).¹³ For calibration and the following analysis, we focus on the six oldest age groups ($i = 2, \dots, 7$), excluding the newborns as their money holdings are imposed exogenously to $\underline{m}$. Although the main cross-sectional data period for calibration is 2009, in order to capture age and cohort effects, calibration requires a long period of simulation where a new cohort is born and an oldest cohort dies every period and multiple cohorts exist cross sectionally at each point in time. The model economy is in a stationary state prior to 1939, implying no cohort or time effects. During this period, we set $\eta^h = \eta$ and macroeconomic variables (i.e., $\widetilde{R}_t$, π_t , τ_{at} and τ_{zt}) to take the same values as in 1939. Starting in 1949, macroeconomic variables evolve as in historical data (except for τ_{zt} which is used to balance the government budget) and η will take the calibrated value. The simulation lasts for 24 periods (i.e., 24 cohorts) until the year 2179.¹⁴ The long simulation period is necessary to capture potential impact of cohort and time effects on household decisions through a life cycle. For example, a 80-year old household in 2009 was 20 years of age in 1949, and hence, her decisions since 1949 are reflected in her asset portfolio in 2009.

There are three sets of parameters to calibrate in the credit transaction technology: γ_i 's, θ_i 's and η . The first two capture the age effects and η captures the cohort effects. We choose $\frac{1}{5} \sum_{j=1}^5 \frac{m_{ij}}{c_{ij}}$, the average money-consumption ratio for age group i in 2009,¹⁵ as a target

¹²For example, every new cohort faces proportionally lower cost than that faced by the cohort born one period earlier.

¹³Five social classes are based on consumption quintile similar to the discussion in Section 2.1.

¹⁴Values of macroeconomic variables post-2009 are kept at their 2009 levels except for τ_{zt} which is solved every period to balance the government budget.

¹⁵We use 2008-2010 three-year data to calculate money-consumption ratios to average out shorter-term

for corresponding γ_i .¹⁶ In addition, θ_i is calibrated to match $\frac{1}{4} \sum_{j=2}^5 \frac{m_{ij}/c_{ij} - m_{ij-1}/c_{ij-1}}{c_{ij} - c_{ij-1}}$, the average slope of the money-consumption ratio curve over consumption for age group i from Figure 3. These moments are derived from cross-sectional observations in 2009. They contain information on both age and cohort effects. Thus, η is simultaneously calibrated to match a moment that does not involve age effects. We choose a change in the average money-consumption ratio of all households between 1999 and 2009, which averages out any age-specific effects. This moment however contains information on both cohort and time effects. As time effects on money demand are simultaneously captured from changing macroeconomic conditions, what η absorbs is the cohort effects.

The model allows household discount factor to be type specific. Age-specific discount factors allow us to capture hump-shaped consumption profiles over the life cycle. Such consumption profiles can be attributable to changes in household size and in costs of participating in the labour force. In addition, heterogeneous discount factor across social types allows us to match the observed consumption dispersion within an age group. In total, we have 42 parameters and 42 moments that we calibrate by solving the model for an equilibrium: 29 β_{ij} 's, 6 γ_i 's, 6 θ_i 's and η .¹⁷

Other parameters are chosen without solving the model. The parameter dictating the household's inter-temporal elasticity of substitution, σ , is set to 2. We set the capital depreciation rate to be 7 per cent. The share of labour in the aggregate production is 0.65. Labour endowments of households are determined to replicate the age-profile in labour earnings from the 2008-2010 CFM with an adjustment that their present value of life-cycle endowments is the same as that of consumption. This assures that the model will be able to capture the observed consumption dispersions in the data.¹⁸ Resulting endowment profiles are shown in Figure 6. Values of macroeconomic variables are also presented in the upper panel of Table 1. Inflation rates are the 10-year average of annual changes in CPI in Canada. Nominal interest rates are the prime rates. Government expenditures as a percentage of GDP are obtained from the national accounts. Capital income tax rates are from Mendoza, Razin, and Tesar (1994). We set the rate at 0.4 for all periods as their estimates do not vary greatly over time. The lower panel of Table 1 shows the labour income tax rates obtained in the equilibrium given the calibrated parameter values.

fluctuations.

¹⁶The cohort index h correspond one-to-one to the age index i in a specific time period.

¹⁷ $\beta_{i=4,j=3}$ is normalized to 0.8.

¹⁸The focus of our paper is the portfolio allocation of these endowments and not the endowments themselves.

Table 1: Macroeconomic variables

	1939	1949	1959	1969	1979	1989	1999	2009
Exogenous variables:								
Annual inflation (%)	2.37	4.00	1.75	5.60	8.11	3.36	2.02	1.92
Annual nominal interest rate (%)	4.96	4.50	5.60	7.53	12.50	9.70	5.50	4.11
Government expenditure (% of GDP)	26	26	26	27	26	25	22	24
Capital income tax rate	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
Labour income tax rate	0.25	0.42	0.25	0.38	0.29	0.23	0.24	0.30

4.1 Calibration results

Figure 7 shows the parameter values for β_{ij} 's. Given a social class, the discount factors are monotonically declining in age except for the first two age groups. Table 2 shows the rest of the parameters calibrated by solving the model equilibrium. γ_i is monotonically increasing in age and θ_i monotonically declining. This implies that, given a level of consumption and a fraction of consumption purchased by credit, the increase in γ_i with age (while holding θ_i fixed) increases the cost, and the decrease in θ_i with age increases the nominal-interest-rate elasticity of money demand (while holding γ_i fixed).¹⁹ The target moment for η is the ratio of average money-consumption ratios in 1999 and 2009, $\frac{\bar{m}_{1999}}{\bar{c}_{1999}} / \frac{\bar{m}_{2009}}{\bar{c}_{2009}}$. The value of calibrated η suggests that the cost of credit transactions declines by about 18% for each new cohort every 10 years.

Figure 4 displays the model outcome of money-consumption ratios by age in 2009. Figure 5 separately shows each age group and compares the model generated results to the data. The calibrated model results match the data in the cross-sectional variation of money demand by household type fairly well.

5 Disentangling age and cohort effects

Our model and calibration allow us disentangle the age and cohort effects into the parameters of the model: γ_i , θ_i and η . We can quantitatively simulate the model to disentangle

¹⁹The second statement involving θ_i can be shown using Equation (5).

Table 2: Calibration results

Parameter	Value	Target	Data	Model
γ_2	0.0044	$\frac{1}{5} \sum_j \left(\frac{m}{c}\right)_{2,j}$	0.1455	0.1384
γ_3	0.0167	$\frac{1}{5} \sum_j \left(\frac{m}{c}\right)_{3,j}$	0.1747	0.1777
γ_4	0.0417	$\frac{1}{5} \sum_j \left(\frac{m}{c}\right)_{4,j}$	0.2304	0.2405
γ_5	0.0461	$\frac{1}{5} \sum_j \left(\frac{m}{c}\right)_{5,j}$	0.2869	0.3005
γ_6	0.1104	$\frac{1}{5} \sum_j \left(\frac{m}{c}\right)_{6,j}$	0.4117	0.4313
γ_7	0.1284	$\frac{1}{5} \sum_j \left(\frac{m}{c}\right)_{7,j}$	0.6069	0.5877
θ_2	1.9073	$\frac{1}{4} \sum_j \Delta \left(\frac{m}{c}\right)_{2,j} / \Delta c_{2,j}$	-0.1021	-0.0995
θ_3	1.6672	$\frac{1}{4} \sum_j \Delta \left(\frac{m}{c}\right)_{3,j} / \Delta c_{3,j}$	-0.1231	-0.1262
θ_4	1.4857	$\frac{1}{4} \sum_j \Delta \left(\frac{m}{c}\right)_{4,j} / \Delta c_{4,j}$	-0.1970	-0.2050
θ_5	1.4661	$\frac{1}{4} \sum_j \Delta \left(\frac{m}{c}\right)_{5,j} / \Delta c_{5,j}$	-0.2557	-0.2673
θ_6	1.2169	$\frac{1}{4} \sum_j \Delta \left(\frac{m}{c}\right)_{6,j} / \Delta c_{6,j}$	-0.4615	-0.4780
θ_7	1.2000	$\frac{1}{4} \sum_j \Delta \left(\frac{m}{c}\right)_{7,j} / \Delta c_{7,j}$	-0.9359	-0.9367
η	0.8150	$\frac{\overline{m}_{1999}}{\overline{c}_{1999}} / \frac{\overline{m}_{2009}}{\overline{c}_{2009}}$	0.96	0.9608

these effects from the cross-sectional observations of money demand. We set $h = 1$, the cohort index for the oldest cohort alive in 2009, for all cohorts and simulate the model. All other parameters remain as in the calibration result. This simulation gives a hypothetical money-consumption ratio by household type for 2009 that take out the cohort effects.²⁰ Figure 8 shows the 2009 money-consumption ratios obtained from this simulation. The observed differences between curves of different ages are the age effects. We find that overall age effects are still preserved after separating out the cohort effects, although money-consumption ratios became close between some age groups. Figure 9 compares the results to those obtained in the calibration, Figure 5. We can observe by comparing the two graphs that taking out cohort effects shrink age differences in money-demand. In Figure 9, given an age group, the differences in the two money-consumption curves are the magnitude of cohort effects in money demand. The bottom-right panel shows that there is no cohort effect for households aged 75 and above. This is by default as their cohort index h has not changed in this experiment. For other age groups, we observe noticeable differences in two curves.

²⁰Alternatively, the experiment asks: how would households in 2009 behave if they were to have faced the credit transaction technology that the oldest cohort in 2009 has experienced?

Table 3: Differences in money-consumption ratios relative to age >75 households, with and without cohort effects, %

	<35	36-45	46-55	56-65	66-75	>75
With cohort effects						
Poor	-0.807	-0.749	-0.632	-0.554	-0.362	0.000
Poor-Middle	-0.781	-0.733	-0.642	-0.547	-0.322	0.000
Middle	-0.753	-0.703	-0.608	-0.537	-0.223	0.000
Middle-Rich	-0.712	-0.661	-0.563	-0.498	-0.328	0.000
Rich	-0.634	-0.605	-0.537	-0.441	-0.255	0.000
Average	-0.737	-0.690	-0.596	-0.515	-0.298	0.000
Without cohort effects						
Poor	-0.681	-0.584	-0.451	-0.403	-0.257	0.000
Poor-Middle	-0.639	-0.554	-0.439	-0.394	-0.218	0.000
Middle	-0.590	-0.499	-0.388	-0.364	-0.077	0.000
Middle-Rich	-0.527	-0.432	-0.320	-0.316	-0.188	0.000
Rich	-0.412	-0.337	-0.310	-0.244	-0.126	0.000
Average	-0.570	-0.481	-0.382	-0.344	-0.173	0.000
Contribution from cohort effects						
Difference	0.168	0.209	0.215	0.171	0.125	0.000

Table 3 helps us quantitatively measure the contribution of cohort effects to the observed cross-sectional age differences in money demand in 2009. The table shows the percentage differences in money-consumption ratios from that of age > 75 households, given a social class. The upper panel is with cohort effects and the lower panel without them. We notice that the differences are smaller (in absolute value) in the lower panel than those in the upper panel, implying that cohort effects partly contribute to between-age differences in money demand. The bottom row shows the differences of the averages from the two panels. The interpretation of these numbers are the following. The contribution of cohort effects given age < 35 is 0.168. This means that about 17% of the cross-sectional age differences in money-demand between age < 35 and > 75 are accounted for by the cohort effects. Hence, we find that cohort effects contribute to a range of 12.5% to 21.5% of the observed age differences in money demand in 2009. As cohort effects come from a secular reduction in credit transaction costs, the results also speak to the effects of financial innovation. Finally, while our results show that cohort

effects are present in the observed cross-sectional differences in money demand of households across age, age effects account for large parts of the observed differences in 2009.

6 Welfare cost of inflation

In the calibrated economy, inflation rates in 2009 and onwards are 1.92%. We calculate changes in aggregate welfare and its distribution when inflation increases from 1.92% in 2009 to 10% in 2019 and onwards. Changes in welfare are measured as the consumption equivalent variation. Let λ_{hj} be the consumption equivalent variation for cohort h and social class j . Let V_{hj}^0 be the life time utility of the households of type hj ,

$$V_{hj}^0 = \sum_{i=2}^7 \beta_{ij} u(c_{hij}^0),$$

where c_{hij}^0 is the consumption obtained in the calibrated model. The welfare measure for household hj is obtained from the following equation,

$$\sum_{i=2}^7 \beta_{ij} u(c_{hij} + \lambda_{hj}) = V_{hj}^0,$$

where c_{hij} is the consumption under the new inflation rates. The welfare measure for household hj is then calculated as

$$W_{hj} = \frac{6\lambda_{hj}}{\sum_{i=2}^7 c_{hij}^0} \times 100.$$

From W_{hj} , the cohort-specific and the class-specific welfare measures are, respectively:

$$W_h = \frac{6 \cdot \sum_{j=1}^5 \lambda_{hj}}{\sum_{i=2}^7 \sum_{j=1}^5 c_{hij}^0} \times 100, \text{ and} \quad (10)$$

$$W_j = \frac{6 \cdot \sum_{h=h_{1949}}^{h_{2069}} \lambda_{hj}}{\sum_{h=h_{1949}}^{h_{2069}} \sum_{i=2}^7 c_{hij}^0} \times 100, \quad (11)$$

where the last cohort for welfare calculations is set to h_{2069} , the cohort born in 2069. Finally, the aggregate welfare measure is obtained from aggregating W_{hj} as follows:

$$W = \frac{6 \cdot \sum_{h=h_{1949}}^{h_{2069}} \sum_{j=1}^5 \lambda_{hj}}{\sum_{h=h_{1949}}^{h_{2069}} \sum_{i=2}^7 \sum_{j=1}^5 c_{hij}^0} \times 100.$$

Table 4: Welfare cost of inflation in 2009, %

Aggregate Welfare Effects							
Total	Seignorage	Net effects					
1.34	0.79	0.55					
Welfare Effects by Cohort, Age in 2009							
80-y.o.	70-y.o.	60-y.o.	50-y.o.	40-y.o.	30-y.o.	20-y.o.	10-y.o.
0.00	1.80	1.98	1.87	1.81	1.64	1.49	1.30
Welfare Effects by Social Class							
Poor	Poor-Middle	Middle	Middle-Rich	Rich			
2.35	1.80	1.71	1.30	0.94			

The welfare analysis is done under the assumption that the fiscal authority does not respond to inflation changes.²¹ Table 4 shows the results of welfare analysis. Positive values imply welfare losses. The upper panel summarizes the aggregate welfare findings. When inflation increases from 1.92% in 2009 to 10% in 2019, aggregate welfare decreases by 1.34%. Of this measure, 0.79% goes to an increase in seignorage and not lost to the economy. Hence, the net welfare loss for the economy is 0.55%. These numbers are fairly inline with those obtained by [Erosa and Ventura \(2002\)](#).

In addition, the middle panel shows welfare changes by cohort based on Equation 10. Older households in 2009 tend to suffer more than their younger counterparts. For example, Consumption of a household of age 60 years in 2009 decreases by almost 2% on average, while that of 10 years old only decreases by 1.3%. The bottom panel summarizes welfare losses by social class of households using Equation 11. The poor tend to suffer the most as they rely relatively more on money for transactions than their richer peers do.

7 Conclusions

The paper documents money holding heterogeneity over age and consumption levels using cross-sectional data. We find that older households hold more money conditional on

²¹Alternatively, fiscal authority could respond to an increase in tax revenues from higher seignorage by reducing capital or labour income taxes.

consumption. The observed cross-sectional age differences in money demand contains both age-specific and cohort-specific effects. In order to separately identify these effects, an overlapping generation model is developed and dynamically calibrated to match the observed heterogeneity in money holding over two data periods. The calibrated model shows that, while the age effects are the major factor in accounting for the cross-sectional age differences, the cohort effects are also present and account for up to 20% of the observed differences. We also use the calibrated model to conduct welfare analysis for the aggregate economy as well as for the welfare distribution across households. When inflation increase from 1.92% in 2009 to 10%, we find a welfare change that is consistent with the literature, and in addition find that large differences in welfare cost across households. Currently alive generations as well as the poor households absorb a larger portion of the welfare losses.

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Appendix - Data

The Canadian Financial Monitor (CFM) Ipsos Reid Canada collects detailed household balance sheet and income. The survey has a sample size of approximately 12,000 households per year responding to a mail-in questionnaire. The Survey of Household Spending (SHS) is a cross-sectional survey conducted by Statistics Canada, and provides information on household spending and dwelling characteristics. The effective sample size (i.e., the number of respondents to the survey) of the SHS varies each year, ranging from 14,000 to 17,000 households. The survey data are collected via personal interviews.

Information on money is obtained from the CFM and is defined to consist of the sum of the household's current balances in checking accounts and low-interest-rate saving accounts in all financial institutions. For the period 2008-2010, the CFM contains consumption information. We define consumption to consist of the household's sum of gross monthly spending on non-durable goods, services and durable goods, but excludes the property tax and housing services from owned and rented residential properties. Specifically, it includes the following items: hydro bills, other utilities, insurance premiums, domestic and child care services/school, groceries including beverages, food and beverages at/from restaurants/clubs/bars, snacks and beverage from convenience stores, recreation, health services, automobile maintenance/gas, public and other transportation, clothing/footwear, gifts or donations, health and beauty aids/personal grooming, a new or used automobile/RV/motorcycle/boat/truck, home appliances and electronics, home furnishing, and vacation/trip.

From the SHS which contains more detailed information on consumption than the CFM, we choose items that are comparable to those from the CFM. They are: food, maintenance, repairs and replacements, homeowners' insurance premiums, traveller accommodation, household operation, household furnishings and equipment, clothing, transportation, health care, personal care, recreation, reading materials, education, tobacco and alcohol, life insurance premiums, annuity contracts and transfers to RRIFs, employment insurance premiums, and gifts of money and contributions.

Figures

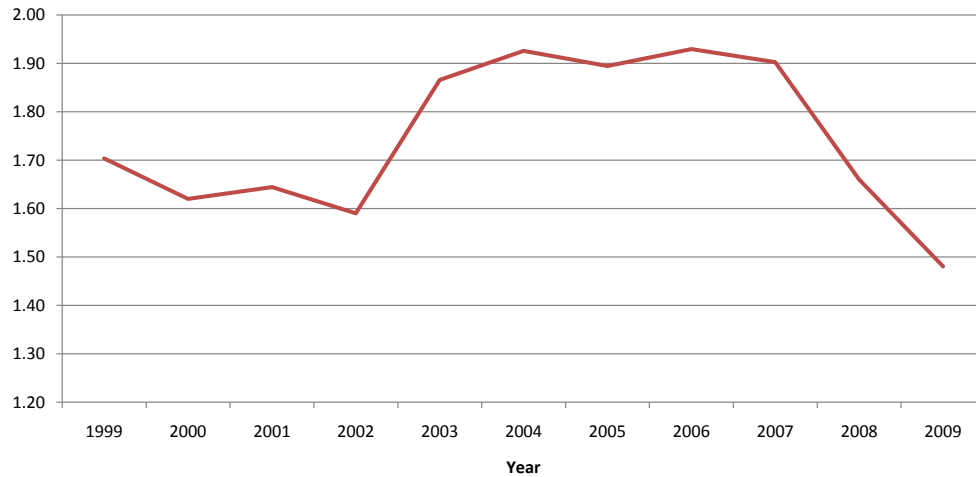


Figure 1: Average Money-Consumption Ratios (Data source: CFM and SFS 1999-2010)

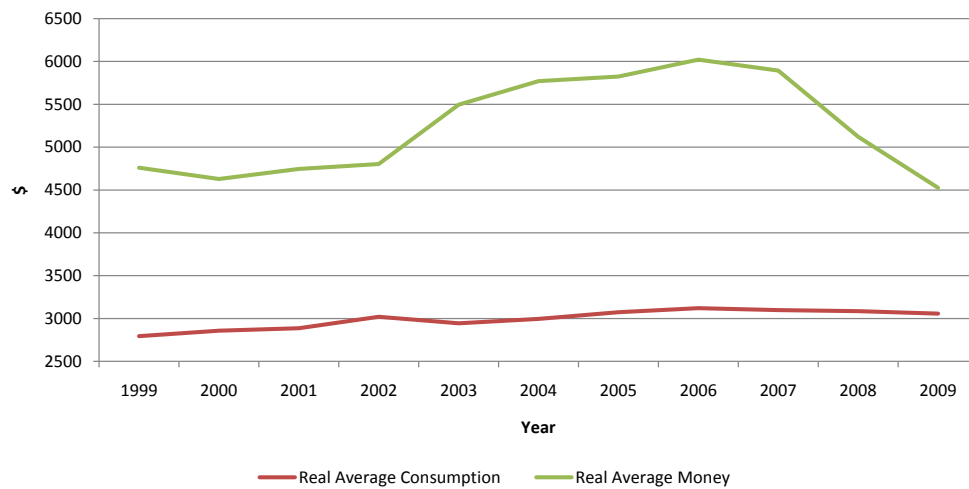


Figure 2: Average Real Money and Consumption (Data source: CFM and SFS 1999-2010)

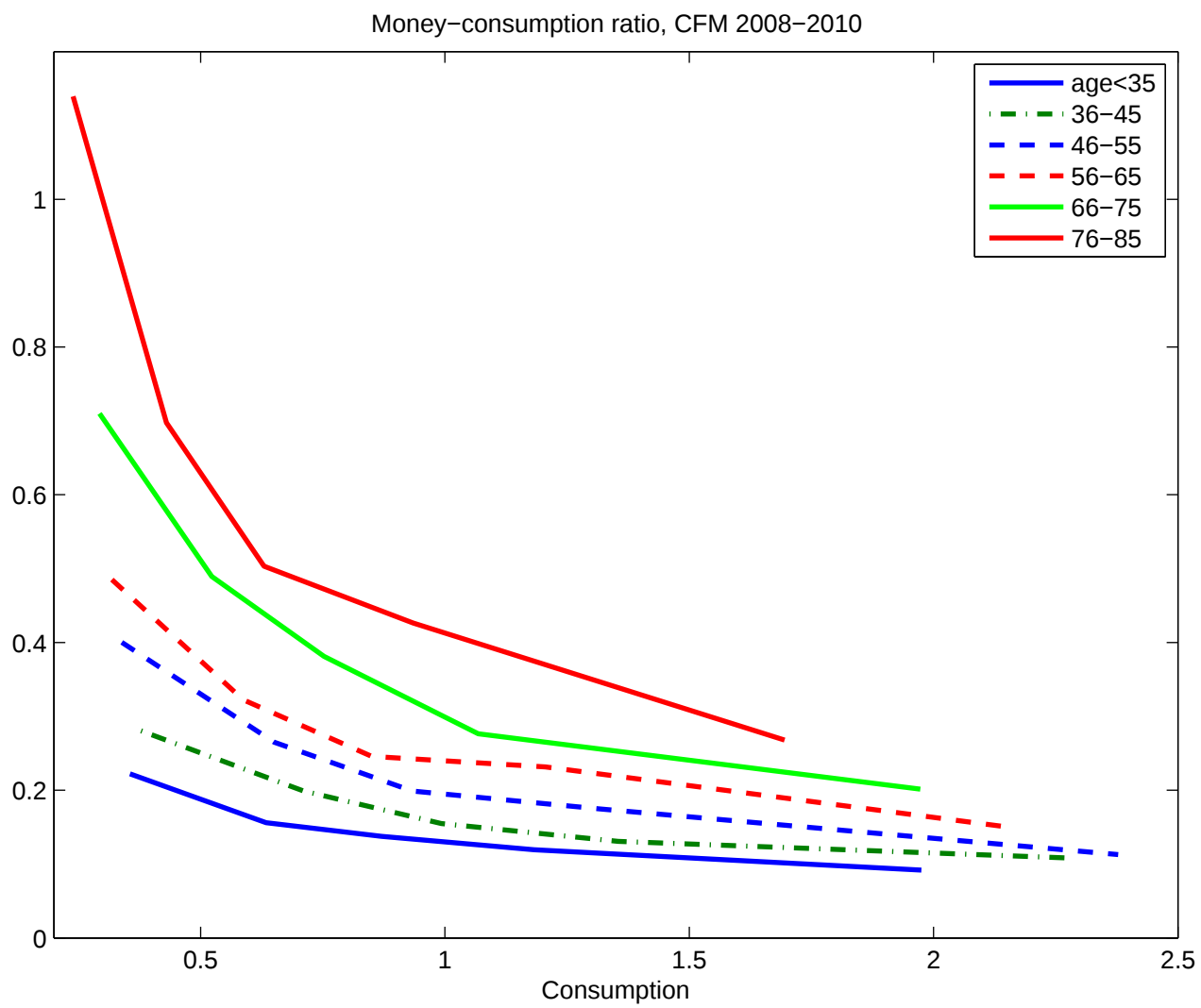


Figure 3: Money-consumption ratio by age (Data source: CFM 2008-2010)

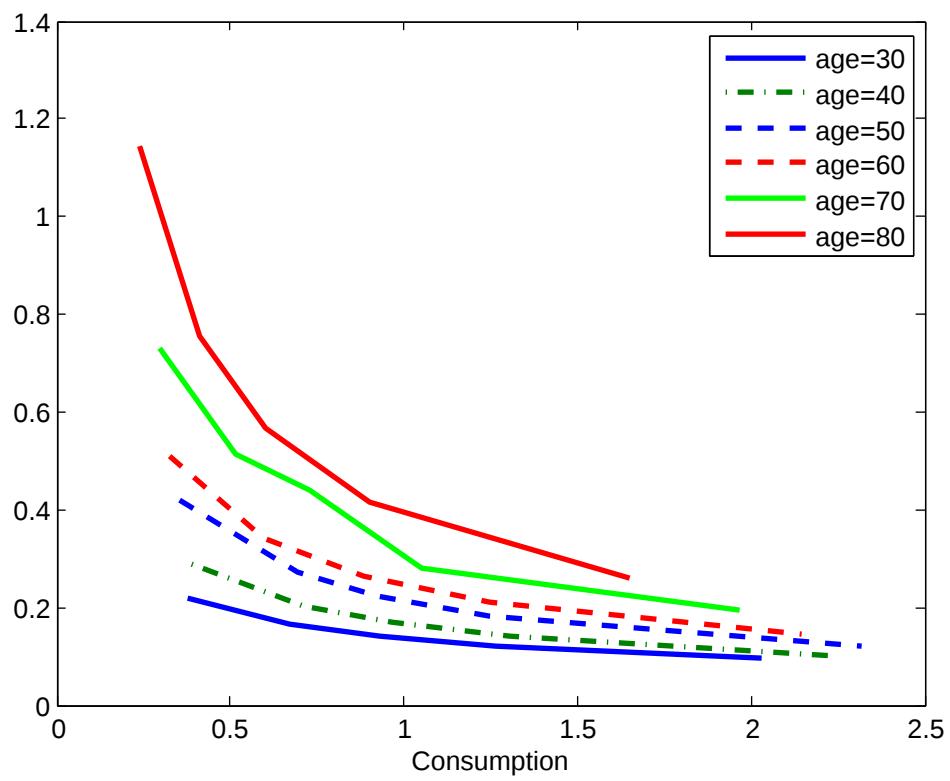


Figure 4: Calibrated money-consumption ratio by age

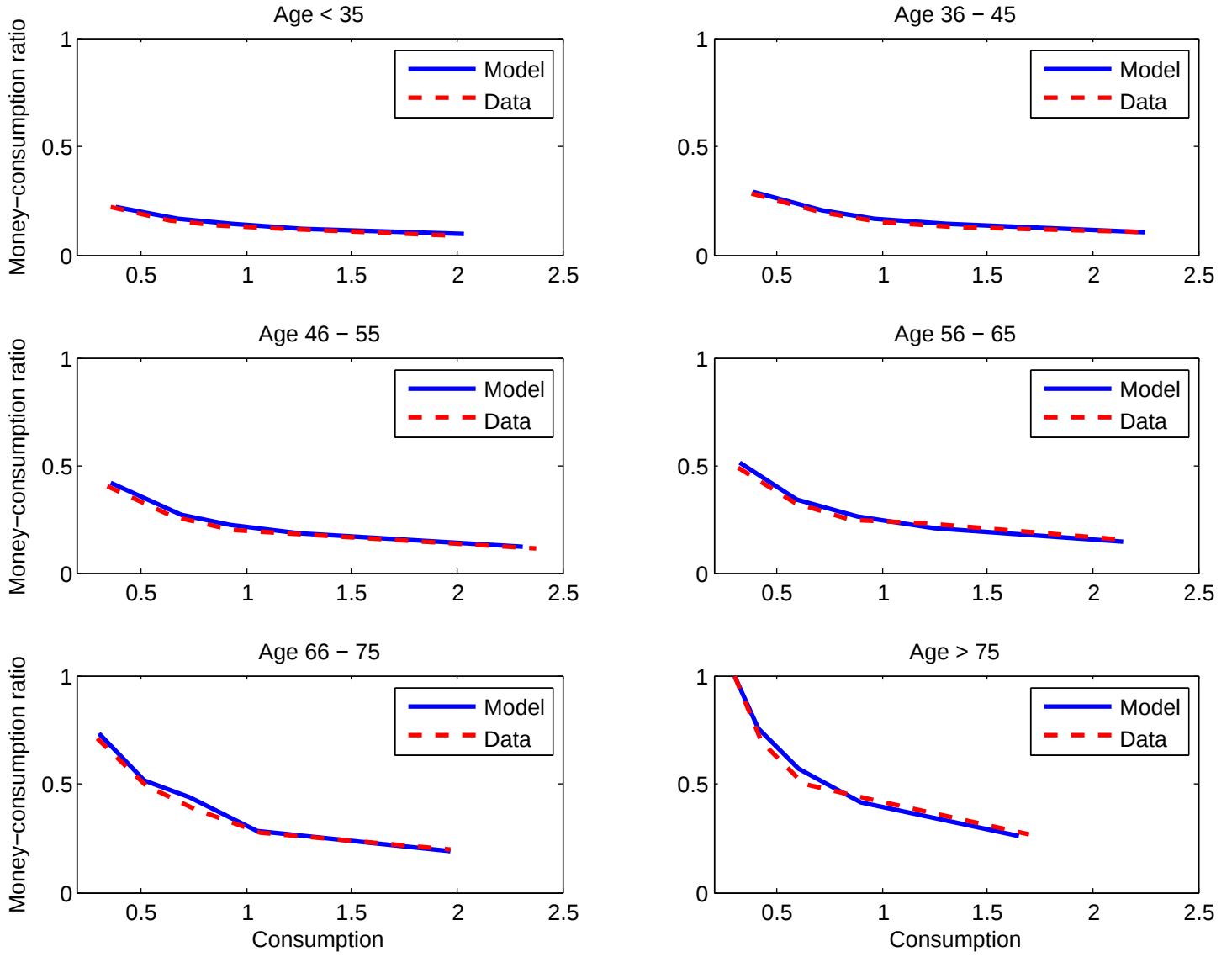


Figure 5: Comparison of data and calibrated money-consumption ratios

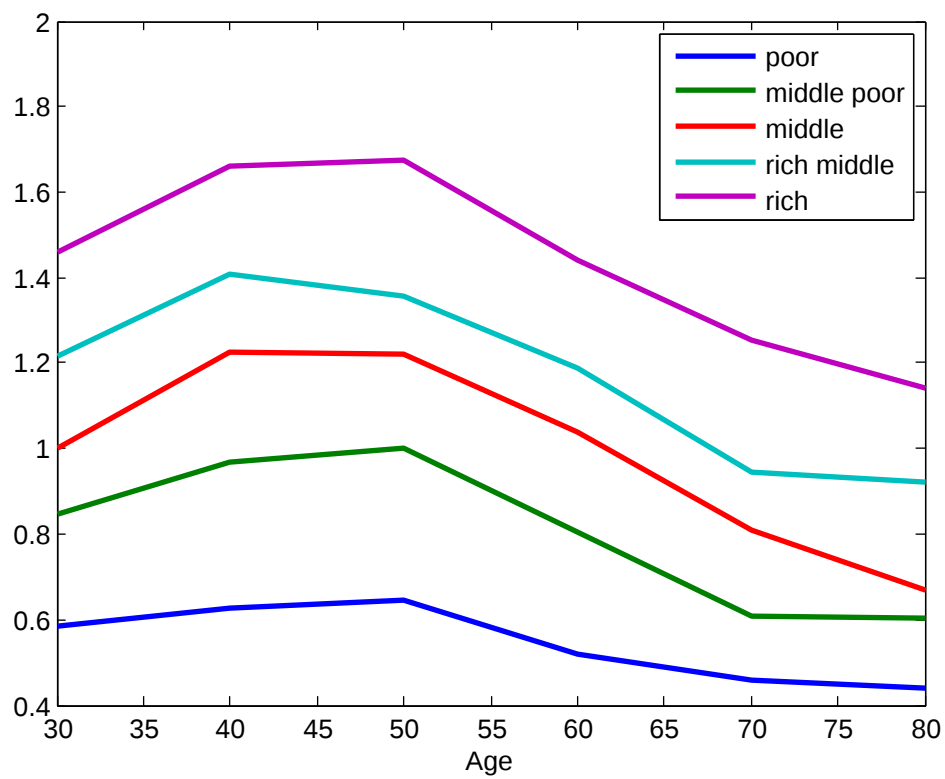


Figure 6: Labour endowments by household type

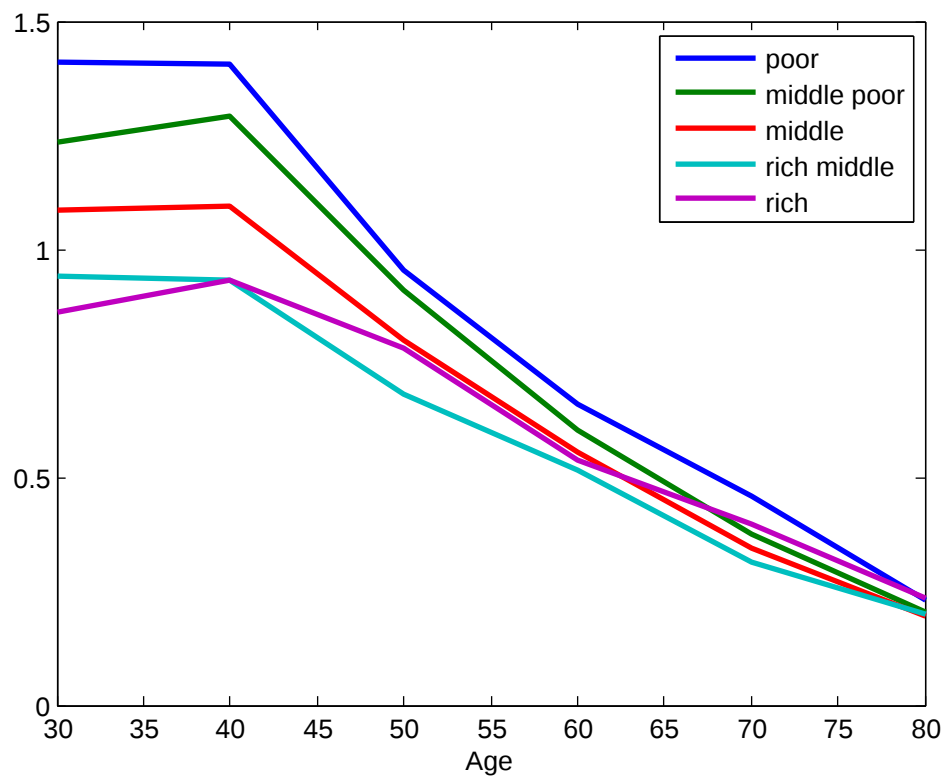


Figure 7: Calibration: discount factor

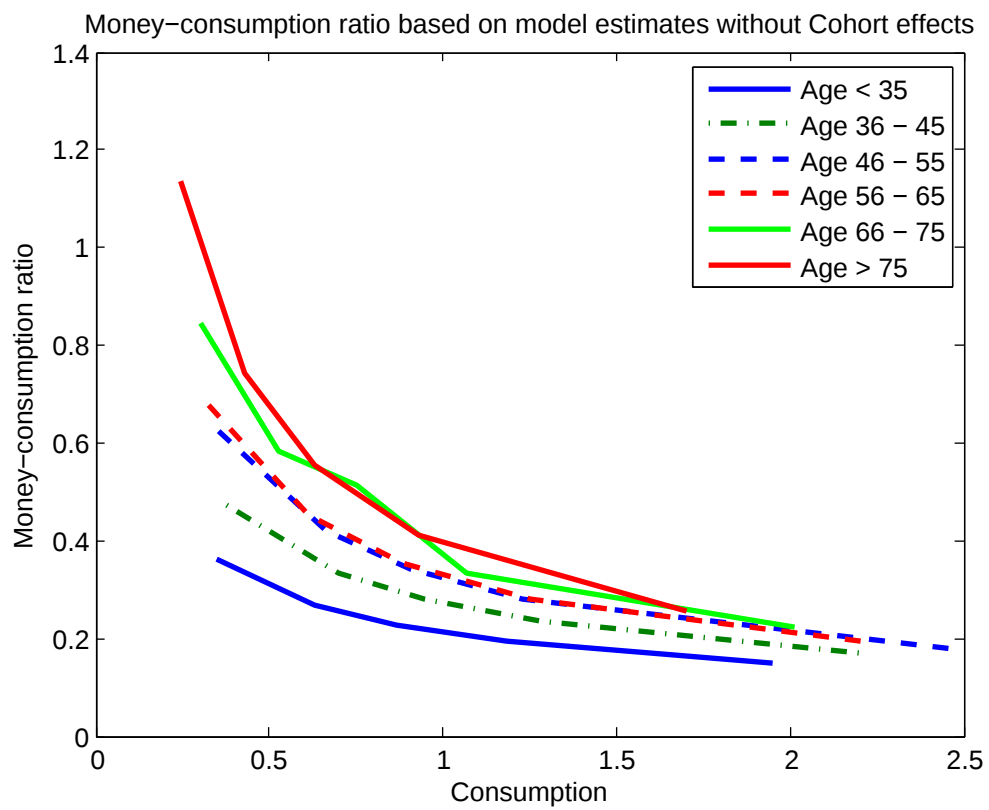


Figure 8: Money-consumption ratios with only age effects

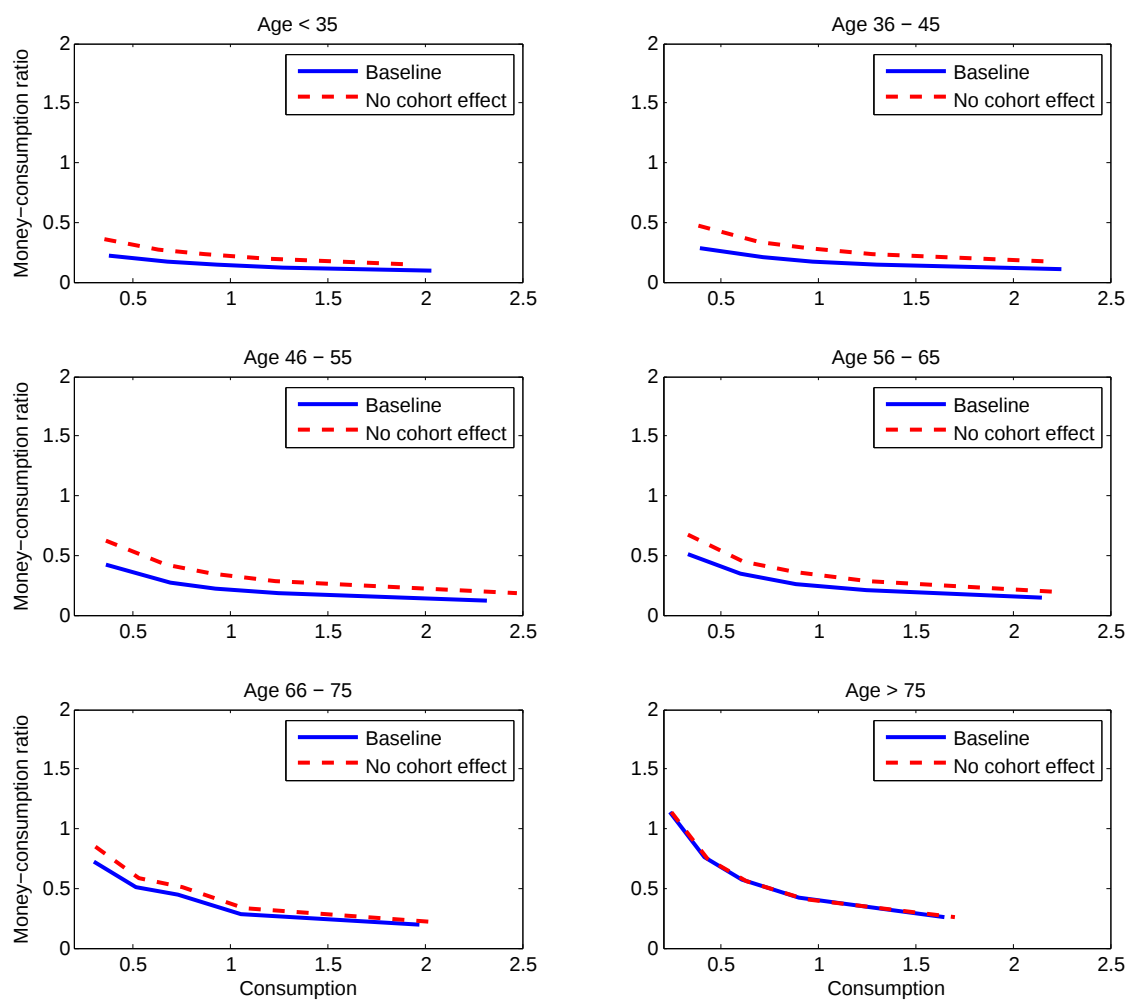


Figure 9: Money-consumption ratios with and without cohort effects