

Regional Mismatch and Labor Reallocation in an Equilibrium Model of Migration

Plamen Nenov*

Norwegian Business School (BI)

December, 2012

Abstract

This paper examines the importance of regional labor reallocation for unemployment and studies the link between inter-regional mobility and reallocation. I construct a multi-region economy with segmented labor markets, endogenously determined regional house prices, and limited worker mobility, and structurally estimate it using state level data for the U.S. The estimated model can account for the comovements of relative house prices and unemployment with gross out- and in-migration observed in a panel of U.S. states. The estimation reveals large reallocation barriers across local labor markets in the form of imperfect directed migration, relative house price differences compensating for labor market differences, and a high moving cost for unemployed workers. These reallocation barriers dampen the response of individual migration decisions to regional labor market differences and contribute to regional mismatch and to shifts in the U.S. Beveridge curve. I apply this framework to understand the reallocation effects of the recent housing bust in the U.S.

JEL Codes: E24, J61, J64, R23

Keywords: regional reallocation, mobility, mismatch, housing market, indirect inference

*email: plamen.nenov@bi.no. This paper subsumes much of the analysis contained in a previous paper titled "Labor Market and Regional Reallocation Effects of Housing Busts". I want to thank Daron Acemoglu, George-Marios Angeletos and Ricardo Caballero for invaluable guidance and support. I also want to thank Drago Bergholt, Hilde Bjornland, JB Doyle, Antonio Fatas, Laura Feiveson, Francesco Furlanetto, Jonathan Goldberg, Marcus Hagedorn, Tom-Reiel Heggedal, Felipe Iachan, Luigi Iovino, Amir Kermani, Franc Klassen, Espen Moen, John Muellbauer, Amine Ouazad, Michael Peters, Fabien Postel-Vinay, Stefanie Stantcheva, Kjetil Storesletten, Tommy Sveen, Ivan Werning, Bill Wheaton, Geoffrey Woglom, seminar participants at MIT, UC Louvain, University of Amsterdam, University of Cologne, Aarhus University, Norges Bank, Stockholm University, University of Copenhagen, ECB, Norwegian Business School BI, the Federal Reserve Board, Paris School of Economics, INSEAD, Oxford, University of Exeter, University of Bristol, the Bank of Finland, the Cologne Workshop in Macroeconomics, the 2012 Urban Economics Association Meeting, and University of Oslo (ESOP). All remaining errors are mine.

1 Introduction

The recent U.S. recession has been characterized by a divergence in regional economic fortunes. In particular, in 2010, unemployment dispersion across metropolitan statistical areas (MSAs) was 2.3% compared to around 1.1% in the early 2000s, including the 2001 recession.¹ At the same time, internal migration in the U.S. fell in the aftermath of the recession, with inter-state migration at an all time low of around 1.4% in 2009-2010 compared to 2% in the early 2000s.² Parallel to the recession, the U.S. experienced a housing market slump - substantial house price declines across housing markets. A common hypothesis for the decline in mobility in the recession involves the distortion that this housing bust may have created in households migration decisions.³ This “housing bust mobility hypothesis” has raised concerns about the implications of a possible reduction in individual mobility for the performance of the labor market.⁴

Motivated by these issues, this paper examines two related questions. First, it seeks to understand the importance of regional labor reallocation, the net flow of workers from relatively under-performing to relatively better performing regional labor markets, for unemployment. Secondly, it studies the degree to which inter-regional mobility in the U.S. leads to reallocation. For this purpose, I construct and estimate a multi-region equilibrium model with segmented labor markets and limited mobility, which captures important features of mobility patterns in the U.S.. One particular application of this framework, which I explore, concerns the reallocation and unemployment effects of a distortion (a tax) in individual migration decisions. This allows me to address the likely quantitative relevance of the housing bust mobility hypothesis for unemployment and the labor market.

I start by showing a set of reduced-form empirical facts, using a panel of U.S. states, about the link between state characteristics and gross migration flows. In particular, the unemployment rate in a state, relative to the national level, correlates positively with migration out of the state and negatively with migration into the state, controlling for relative house prices. More surprisingly, relative house prices correlate positively with out-migration and negatively with in-migration, controlling for relative unemployment. These comovements contain important information for the set of models that can generate the observed data. In particular, they exclude the classical spatial equilibrium framework with frictionless mobility as a possible data generating model. I

¹These figures are for labor force weighted unemployment dispersion at the MSA level. Looking at a weighted average of state unemployment dispersion by occupation for 10 broad occupation groups, the respective numbers are 2.3% and 1.2%. See Section 6 for a precise definition of these measures.

²These figures are based on the CPS adjusted data of Kaplan and Schulhofer-Wohl (2012a).

³Section 2 summarizes several possible mechanisms through which a housing market slump may lead to reduced mobility.

⁴A representative example of this particular concern is the following quote from the Economist: “Homeowners that are reluctant to default but unable to sell at a loss are left stuck where they are. This throws sand in the gears of America’s famously fluid labour market. (The Economist (2010))”.

then proceed to study an equilibrium model that can account for these comovements.

The model economy I consider consists of a continuum of islands (regions) with labor markets characterized by search and matching frictions, competitive housing markets and local labor market shocks. Workers reside across the regions of this economy and migrate out of a region due to local labor and housing market conditions and for idiosyncratic reasons. Their migration decision is a combination of directed and random migration, i.e. with a certain probability, workers migrate to regions offering the most favorable labor and housing market conditions or to any region of the economy. In addition, workers incur a fixed cost of moving, with a subset of workers facing a higher moving cost.⁵ Each region is endowed with a fixed supply of durable housing that workers value for its housing services and rent in a competitive rental market. A downward sloping demand for housing services by workers leads to an endogenous response of regional house prices to changes in the state of labor markets and in regional populations.

I first characterize stationary equilibria for a basic version of the model. The characterization results allow me to show that the model can generate the positive comovements of relative unemployment and house prices with gross out-migration and negative comovement with in-migration observed in the panel of U.S. states. Three features of the model are important for this result. First, durable housing and a downward sloping housing demand by workers lead to a difference in house prices across regions with different populations. Combining these with idiosyncratic regional preferences of workers, which act to smooth out-migration flows, creates a dependence of regional house prices on the history of labor market shocks. Consequently, it leads to interesting house price dynamics and a rich equilibrium distribution of regional house prices. The resulting equilibrium house price heterogeneity drives the positive comovement between out-migration and house prices controlling for unemployment, since regions with lower house prices, other things equal, are more attractive to potential emigrant workers. Limited mobility, due to idiosyncratic regional preferences of workers, also leads to the comovement between out-migration and unemployment. Lastly, a non-zero probability of directed migration implies that regions with better performing labor markets and lower populations, and consequently, house prices, have larger population inflows leading to the negative comovement between house prices and unemployment with in-migration.

I proceed to estimate the model using the method of indirect inference and the observed reduced-form relations between migration rates and state characteristics. I estimate 9 structural parameters, including parameters for the stochastic process of labor market shocks, housing utility parameters and a set of migration parameters including the probability of directed migration, the migration cost, and the dispersion in idiosyncratic regional preferences. The model can match well the reduced-form comovements and aggregate moments used in the estimation, suggesting that it

⁵This heterogeneity in moving costs allows me to look at the aggregate effects of distortions in individual migration decisions, while maintaining a rich regional heterogeneity.

contains the relevant features of individual migration decisions that ultimately generate these aggregate patterns. It also performs well when comparing untargeted moments of the observed data and of data generated by the estimated model. For example, the model produces high persistence in house prices as observed in the data.⁶

The estimation reveals a low probability of directed migration, a high mobility cost for unemployed workers, and an important effect of relative house prices on migration decisions.⁷ Each of these features contribute to low regional labor reallocation and a dampened response of individual migration decisions to regional labor market differences. As a result the estimated model can explain a large part of the relation between regional unemployment dispersion and shifts in the Beveridge curve. In particular, I show that data on vacancy and unemployment rates generated by the model, given shocks to aggregate unemployment and unemployment dispersion, can produce the comovements from standard U.S. Beveridge curve regressions augmented with measures of regional unemployment dispersion. Consequently, the resulting shifts in the model generated Beveridge curve are consistent with shifts of the U.S. Beveridge curve. This indicates that labor reallocation barriers are important for regional mismatch, the co-existence of relatively tight and slack regional labor markets that workers try to find jobs in. Therefore, regional mismatch, created by the combination of reallocation barriers and shocks that affect regional labor markets asymmetrically, is important for shifts of the Beveridge curve.

I finally examine the likely magnitude of the reallocation effects of the housing bust in the Great Recession by simulating the labor market consequences of the recession and then looking at several counterfactual scenarios. Even with a large effect on aggregate *gross* migration, the housing bust would have had a very limited effect on *net* migration and from there on aggregate unemployment. The reason for this discrepancy is the reallocation barriers mentioned above, which limit net migration and labor reallocation. These factors strongly affect how migration is mapped into regional reallocation and net flows. In particular, relaxing these barriers produces a much larger response of unemployment to any mobility distortions.

Related literature. This paper spans several strands of literature. The theoretical model is close in spirit to previous work on regional reallocation (Lucas and Prescott (1974) and more recently Alvarez and Shimer (2011), Carrillo-Tudela and Visschers (2011), Lkhagvasuren (2011), Coen-Pirani (2010), and Shimer (2007)). In these papers, dispersion in economic conditions across islands induces worker reallocation, which has implications for aggregate unemployment. However, these papers do not investigate the effects of the housing market on mobility. In contrast, my

⁶This persistence has been a puzzle for recent literature examining regional house prices in a frictionless spatial equilibrium framework (Gleaser, Gyourko, Morales, and Nathanson (2012)).

⁷The estimated model is also consistent with housing market slumps potentially affecting individual migration decisions.

model features housing markets with endogenously determined house prices that respond to labor market shocks and population adjustments and affect worker migration decisions. This paper also structurally estimates an equilibrium model of regional reallocation of this type.⁸

The paper is closely related to studies of the effect of homeownership and the housing market on mobility and the labor market (Oswald (1996), Head and Lloyd-Ellis (2010), Van Nieuwerburgh and Weill (2010), Davis, Fisher, and Veracierto (2010)). Head and Lloyd-Ellis (2010) investigate the impact of home ownership on mobility and aggregate unemployment and build a theory that accounts for the reduced mobility of homeowners versus renters observed in micro-data, and explore its aggregate implications. The authors find that the mobility distortions of homeownership translate into a small effect on aggregate unemployment, which is similar to the labor market effect of reduced mobility that I find. Van Nieuwerburgh and Weill (2010) document a secular increase in dispersion in regional house prices and wages and propose a model with frictionless mobility that explains and quantitatively matches this increase. The focus of my paper on the implications of regional business cycles and limited mobility for regional house price dynamics and gross migration in a multi-region equilibrium model complements these two studies.

The paper studies a model that nests both directed and random inter-regional migration. The issue of directed versus random migration is related to issues of directed versus random labor market search (Godoy and Moen (2012)). Examining the quantitative implications of a model with regional wage rigidity connects this paper to the growing literature on rigid wages in search and matching models of aggregate unemployment (Hall (2005), Shimer (2010), Gertler and Trigari (2009)).

Recent works that also deal with the labor reallocation implications of the housing bust include Sterk (2010), Sahin, Song, Topa, and Violante (2011), and Karahan and Rhee (2011).⁹ Sterk (2010) builds a single region DSGE model with financial frictions and a reduced-form migration decision by workers to study the impact of a housing bust on unemployment and the Beveridge curve. He finds a substantial effect of the housing bust for the recent shift in the Beveridge curve. In contrast, I find that shifts in the Beveridge curve arise from regional mismatch and regional reallocation barriers independently of any possible migration distortion effects of the recent housing bust, which would have had a limited impact on unemployment and the Beveridge curve. Sahin, Song, Topa, and Violante (2011) also address whether the recent housing market disruptions may have contributed to the contemporaneous shift in the Beveridge curve. Similarly to the conclusion of this paper, they find that the housing bust would have had a very limited effect on the Beveridge

⁸In recent work Mangum (2012) estimates a discrete choice model of reallocation with a finite number of locations. Given these feature that model differs conceptually from the regional reallocation models mentioned above and from the model studied in the present paper.

⁹This is not an exhaustive list of recent work on this topic but includes papers that are close in some dimensions to this paper.

curve. However, the reason for their conclusion is the limited importance of regional mismatch that they argue for. In this paper, I find that even if regional mismatch may be an important shifter of the Beveridge curve, reallocation barriers dampen any migration distortion effect of the housing bust. Karahan and Rhee (2011) model one mechanism through which large house price declines, combined with imperfections in mortgage markets, may lead to an individual migration distortion, and study its implications in a model with two regions. The aggregate unemployment and net migration effects of a housing market slump that the authors uncover in their model calibration is larger than what I find here. This discrepancy can be explained by the different migration decision parameters for the probability of directed migration and the cost of moving used in their calibration and the estimated parameters of my model. Additionally, my framework allows for a separation of any migration distortion effects due to financial market imperfections or other factors, from changes in relative house prices, which are combined by construction in their model.

The rest of the paper is organized as follows. Section 2 presents the motivating empirical facts. In Section 3, I set up the basic model. Section 4, provides a characterization of stationary equilibria and shows how the model can account for the comovement of state gross flows and labor and housing market conditions. Section 5 contains a brief description of the quantitative model and describes the estimation procedure. Finally, section 6 contains the estimation results, shows how the model accounts for shifts in the Beveridge curve and contains the Great Recession simulation. Section 7 provides brief concluding remarks. Additionally, the Appendix contains data description, proofs of results omitted from the main text, as well as details of the quantitative model and computational procedures used for solving the model and the estimation.

2 Empirical Facts

I present a set of motivating empirical facts that relate state characteristics and gross migration flows. These comovements provide insights about the determinants of state gross migration flows and important information for the set of models that can generate the observed data. Constructing and estimating a model that is consistent with these facts will be the focus of the rest of the paper. Such a model will capture important features of mobility patterns in the U.S., especially with respect to individual migration responses to local business cycles. These particular aspects of migration decisions are important for understanding how mobility maps into regional labor reallocation due to diverging labor market conditions.

I look at an unbalanced panel of annual household state out- and in-migration rates for U.S. states and the District of Columbia, from 1993-2008.¹⁰ I focus on the following regression:

¹⁰Appendix A contains information on how I construct all the relevant variables as well as details on the data

$$y_{st} = \alpha_s + \zeta_t + \mathbf{x}_{st}' \gamma + \epsilon_{st} \quad (1)$$

where y_{st} is the (log of the) gross out- or in-migration rate for state s and year t , α_s and ζ_t are state and year fixed effects and $\mathbf{x}_{st}$ is a vector of state characteristics. In particular, $\mathbf{x}$ contains the state unemployment rate, and state house price level relative to the national level. In addition, I include a variable that proxies for a local housing market slump (beyond changes in relative house prices). A good candidate, given the recent sharp increase in the fraction of households with negative housing equity, parallel to the broad deterioration in U.S. housing market conditions, is the fraction of households with limited housing equity within a given state, which I denote by neg_{st} .¹¹

To construct this variable, I use information on a sample of households from the Consumer Expenditure Survey (CEX) to compute housing equity levels, E , for a household's primary residence. I then construct state level estimates, $\widehat{neg}_{st}$, for a state s and year t , of the fraction of households with housing equity, $E < c$, where c is a given equity cutoff. I consider three cutoff levels for the variable neg_{st} , $c = 0$, $c = -0.05$ and $c = 0.05$.¹²

I also include a measure of the relative wage as the log of the ratio of the state average hourly wage in manufacturing to the national counterpart, as well as the log of state income per capita relative to national income. I control for mortgage credit conditions proxied by the average debt-to-value ratio and the home ownership rate. Lastly, I control for demographic effects by including a set of age distribution variables.

Table 1 presents three salient reduced-form results. First of all, higher unemployment in a state relative to the national level correlates with increased migration out of the state and decreased migration into the state, controlling for relative house prices. More specifically, a ten percent

sources. Table 11 in Appendix A contains summary statistics for the data used.

¹¹There are several economic mechanisms that can lead to a mobility distortion of housing market slumps. First, households with negative equity may face pecuniary or non-pecuniary default penalties. Empirical studies dealing with homeowner default decisions uncover such default penalties (Bhutta, Dokko, and Shan (2010), Guiso, Sapienza, and Zingales (2009) and Foote, Gerardi, and Willen (2008)). Secondly, the combination of low housing wealth, together with a downpayment requirement on a new house, would also affect mobility compared to a frictionless mortgage financing arrangement (Stein (1995), Ortalo-Magne and Rady (2006)). Alternative mechanisms involve a decrease in housing market liquidity due to search frictions (Novy-Marx (2009)), potentially compounded with a general unwillingness of movers to buy a new property before selling their old house (Anenberg and Bayer (2012)), or loss aversion (Engelhardt (2003)).

¹²Appendix A contains details on how I compute equity levels. There are two potential econometric issues with using an estimate of neg_{st} rather than the true population value. First, there may be a potential misclassification of households. Misclassification does not appear to be an important issue, as I show in Appendix A. Secondly, there can be an errors-in-variables problem and attenuation bias in the coefficient estimates. However, using information on the sampling variance of $\widehat{neg}_{st}$, one can construct an estimator that is not subject to such bias and hence obtain unbiased estimates of these coefficients. In particular, I estimate the coefficients of interest for equation (1) using a two-step GMM estimator similar to (Erickson and Whited (2002)). Appendix A provides details on this estimator. Table 12 in Appendix A shows estimation results without errors-in-variables correction.

Table 1: Estimation results

Dep. Variable:	out-migration rate, $100 \ln(out)$			in-migration rate, $100 \ln(in)$		
relative unemployment $100 \ln(\frac{u_s}{u})$	0.163*** (0.039)	0.159*** (0.038)	0.172*** (0.040)	-0.225*** (0.036)	-0.224*** (0.036)	-0.226*** (0.035)
relative house price $100 \ln(\frac{p_s}{p})$	0.263*** (0.069)	0.261*** (0.071)	0.251*** (0.069)	-0.160*** (0.049)	-0.163*** (0.049)	-0.158*** (0.048)
households with equity < 0, (%)	-0.693* (0.359)			0.169 (0.464)		
households with equity < -0.05, (%)		-0.532* (0.282)			0.331 (0.468)	
households with equity < 0.05, (%)			-0.812** (0.392)			0.136 (0.366)
N	606	606	606	606	606	606

Notes: Robust standard errors with clustering on state in parenthesis; * = significant at 10%; ** = significant at 5%; *** = significant at 1%. Source: Own calculations from BLS, IRS, CE, FMHPI, and US Census Bureau. See Appendix A for detailed description. Relative house price is constructed from the CMHPI index and median house value by state from the 2000 Census. Relative unemployment is the log of the unemployment rate for that state and year divided by national unemployment rate for that year. Housing equity is defined as $E = \frac{V-D}{V}$ where D is the total debt balance outstanding on all mortgages and home equity loans that a property collateralizes and V is the subjective property value. Households with housing equity < c is the fraction of the population who are homeowners with $E < c$. Additional controls include: state and year fixed effects, relative wage rate as the log of the average hourly average manufacturing wage for that state and year divided by the national average hourly manufacturing wage for that year; relative income as the log of income per capita for that state and year divided by national income per capita for that year; average debt-to-value ratio, defined as the average of $\frac{D}{V}$; home ownership rate, defined as the percent of households in the state that own a house; state demographics that include the age distribution of the population with 6 age bins and the median age

increase in state relative unemployment is associated with an increase in out-migration of around 1.7% and a decrease in in-migration of 2.3%. Overall, these estimates point to labor market conditions being an important driver of inter-regional mobility and, conversely, migration serving as an adjustment mechanism in response to local labor market shocks (Blanchard and Katz (1992)). Note, that both the out- and in-migration margins co-move with changes in unemployment, which restricts the set of models of migration that can account for them as opposed to accounting for net migration responses only. For example, models with no directed migration, though potentially able to account for net migration cannot generate the observed in- and out-migration patterns.

Secondly, and more surprisingly, higher relative house prices correlate with increased out-migration and decreased in-migration, controlling for relative unemployment. The comovement of relative house prices with out- and in-migration shows that even after controlling for labor market conditions, house prices are associated with variation in migration.¹³ One way to rationalize this effect is that states with high relative house prices are for some reason “too expensive”, so the expensive housing induces more individuals to migrate out of these states and fewer individuals to migrate in. However, formalizing such an argument in an equilibrium framework is not straightforward.

A natural first step for explaining this pattern is the classical spatial equilibrium framework with frictionless mobility (Rosen (1979), Roback (1982)), where equilibrium house prices reflect differences in regional characteristics (either the state of the local labor market or local amenities). However, there are two issues with applying this framework. First of all, it would predict that this comovement should disappear after controlling for variation in labor market conditions. Such shocks are the natural candidate for regional characteristics that vary at annual frequency, since regional amenities tend to be either time-invariant or vary at a much lower frequency. Secondly, it would predict the reverse comovements in response to shocks to regional characteristics. This is because in that framework a region has a high house price, because more people move into it and fewer people move out of it, given the region’s current characteristics.

The classical spatial equilibrium framework is unable to produce this comovement because it is, in some sense, static. Since house prices merely respond to the underlying shocks to regional characteristics, given frictionless migration, they reflect the dynamic behavior of the stochastic process of shocks. Therefore, there is no possibility for internal propagation of shocks in such a framework. While this may be fine for explaining low frequency (secular) inter-regional regularities, it obviously runs into troubles when looking at higher frequency patterns. This points to the spatial equilibrium framework lacking some (arguably more realistic) features of individual migration decisions and housing markets, that would propagate higher frequency shocks. Therefore, as with

¹³Note that Saks and Wozniak (2011) document similar observations as side results for state-to-state and MSA-to-MSA migration flows. Such comovements appear very robust in the U.S. at least when using IRS data. Jackman and Savouri (1992) obtain similar results for the U.K.

the comovements of relative unemployment, comovements of relative house prices with migration flows provide important model restrictions, especially for models that require realistic migration flows in response to local business cycles as is the case in this paper. In particular, in Section 3 I show that limited inter-regional mobility and a durable housing stock can produce this comovement in a multi-region equilibrium model of migration.

Lastly, a local housing market slump, proxied by the fraction of households with limited housing equity, correlates with a decrease in migration out of that state but is not correlated with migration into the state. I show in Section 6 that this pattern is consistent with a higher mobility cost of some individuals, which ultimately leads to these comovements.¹⁴

I now turn to a model that will be able to account for the facts relating gross migration rates and relative unemployment and house prices. The model will be rich enough to be structurally estimable and to match these comovements. The estimated model will provide important insights on the link between migration and reallocation and on the degree of regional labor reallocation in the U.S. economy.

3 Basic Model of Regional Reallocation

I start by introducing a basic version of the model of regional reallocation that I will be estimating. The aim of this section is to provide intuition for the mechanisms in the model. I consider a discrete time economy with an infinite number of periods $t = 0, 1, 2, \dots$. The economy consists of a unit measure of islands or regions. The economy is populated by a measure L of infinitely lived workers, distributed across regions of the economy. Similarly to Van Nieuwerburgh and Weill (2010), workers have separable flow utility for consumption and housing. The flow utility function is then $\tilde{u}(c, h) = c + v(h)$, where $v(\cdot)$ is strictly increasing, twice continuously differentiable, concave, and satisfies Inada conditions. Workers can supply 1 unit of labor. As explained below workers differ in their mobility costs across regions. The initial measure of workers in each region j is given by $l_{-1}^j \leq L$, with $\int l_{-1}^j dj = L$. The end-of-period or post-migration measure of workers in a region j at time t is given by $l_t^j \in [0, \bar{L}]$, $\bar{L} \geq L$.¹⁵

3.1 Regional labor markets, job creation, and destruction

In each region there is a representative firm that can open job vacancies at a per-period cost of k and recruit workers. For the basic model I consider in this section, I assume that jobs remain

¹⁴As I discuss in that section, this pattern is therefore consistent with a potential mobility distortion of housing market slumps. Furthermore, having a model that delivers this pattern, implies that it would be informative for the aggregate effects of such a distortion.

¹⁵The upper bound of $\bar{L}$ is a technical restriction necessary for showing the analytical results. It can be thought of as a limit on the space available within a region.

productive for one period only.¹⁶ Each job in region j has the capacity for the production of a_t^j units of the consumption good at time t if it can hire a worker. Regional productivity a_t^j can have two possible realizations $\bar{a}$ or $\underline{a}$ ($\bar{a} \geq \underline{a}$) and follows a Markov chain with persistence $\rho \geq \frac{1}{2}$. Furthermore, given the stationary distribution for this Markov chain, at any time t , one half of regions have $a_t^j = \bar{a}$ and the other half have $a_t^j = \underline{a}$. I refer to the former as (relatively) “booming” regions and to the latter as (relatively) “depressed” regions.¹⁷

The labor market of each region is characterized by a search and matching friction as in the standard Diamond-Mortensen-Pissarides framework (Pissarides (2000)). In particular, in a region j at time t , after migration, a measure l_t^j of workers and measure v_t^j of vacancies try to match with each other. Matching is described by a standard regional reduced-form constant returns to scale (CRS) matching function $m^j(l_t^j, v_t^j)$ giving the total number of regional matches per period. I assume that matching functions are identical across regions, i.e. $m^j(l_t^j, v_t^j) = m(l_t^j, v_t^j) = l_t^j \cdot m\left(1, \frac{v_t^j}{l_t^j}\right)$. Letting $\theta_t^j = \frac{v_t^j}{l_t^j}$ be the regional labor market tightness and defining $\mu(\theta) = m(1, \theta)$, we have that $m(l_t^j, v_t^j) = l_t^j \mu(\theta_t^j)$. This translates into a job finding probability for a worker in a given period of $\mu(\theta_t^j)$ and a job filling probability for a vacancy of $\frac{\mu(\theta_t^j)}{\theta_t^j}$. Additionally, I assume that $\lim_{\theta \rightarrow 0} \frac{\mu(\theta)}{\theta} = 1$ and $\lim_{\theta \rightarrow \infty} \frac{\mu(\theta)}{\theta} = 0$. Workers that are matched obtain wage w , and those that remain unmatched after the matching phase in a given period receive a period payoff of e . The total measure of unemployed in region j is then given by $(1 - \mu(\theta_t^j)) l_t^j$ and the unemployment rate is simply $U_t^j = (1 - \mu(\theta_t^j))$.¹⁸

3.2 Job creation decisions and wage determination

I allow for wages to be determined either by Nash bargaining or to be rigid as in Hall (2005). The particular wage determination rule does not affect the results of this section, so I focus on a model with the more standard assumption of Nash bargaining. However, as I discuss in Section 5, the process of wage determination does matter for the quantitative model.

Given one period job length, the vacancy posting decision of the representative firm is straightforward. In particular, the firm opens vacant jobs until the cost of opening a vacancy equals the

¹⁶The reason for this is analytical tractability, since under it there will be no agent heterogeneity in terms of the idiosyncratic employment state at the time of the migration decision. This leads to only one relevant endogenous state variable that affects migration decisions, which makes equilibrium characterization possible. However, in Section 5, where I look at the quantitative effect of the housing bust in the recent recession, I work with the more general set-up with stochastic job destruction.

¹⁷These local productivity shocks are reduced-form labor market shocks that result in regional variations in unemployment and unemployment differences across regions.

¹⁸Note that I set θ to be the market tightness since this is a standard notation in the literature. In Section 5, I will refer to market tightness as z and instead use θ as the vector of structural parameters to be estimated.

expected payoff from posting a vacancy, or:

$$k = \frac{\mu(\theta_t^j)}{\theta_t^j} (a_t^j - w_t^j) \quad (2)$$

where w_t^j is the wage rate in region j at time t . Letting workers' bargaining power be $\eta \in [0, 1)$, the wage rate is:

$$w_t^j = e + \eta (a_t^j - e) \quad (3)$$

where e is the outside option. Hence, equations (2) and (3) pin down the labor market tightness in a region as a function of a_t^j , so that one can write $\theta(a)$ for a given region productivity a . Note that labor market tightness is a function only of regional productivity and not of the labor force in a region. This is because the representative firm's production technology has constant returns.

3.3 Regional housing markets

Every region j , has a fixed supply of housing H (constant across regions). Housing is perfectly durable and does not depreciate.¹⁹ The housing stock is owned by a competitive sector of real estate firms that trade it in a competitive housing market and also rent it to workers residing in the region in a competitive rental market. Note that given the frictionless housing and credit markets, home ownership rates are indeterminate in the model so that one can equivalently consider a specification of all workers owning housing rather than renting it. Both specifications lead to identical results.

The regional rent is given by r_t^j and the price of a unit of housing is p_t^j . Real estate firms are equally owned by all workers in the economy and hence also discount future payoffs by β .²⁰ Therefore, a real estate firm solves the following problem:

$$\max_{\tilde{h}_t} \left\{ r_t^j \tilde{h}_t - p_t^j (\tilde{h}_t - \tilde{h}_{t-1}) + \beta E_t [p_{t+1}^j] \tilde{h}_t \right\},$$

where $\tilde{h}_{t-1}$ and $\tilde{h}_t$ denote the firm's ownership of housing in periods $t-1$ and t , respectively. This gives the no arbitrage relationship for the price of housing:

$$p_t^j = r_t^j + \beta E_t [p_{t+1}^j] \quad (4)$$

¹⁹The assumption of a durable housing stock is similar to the assumption in Glaeser and Gyourko (2005). However, while these authors explore the amplification effects of durable housing on labor market shocks in a static context, in this model I consider its dynamic implications.

²⁰Note that for notational convenience I do not include the profit rebates from real estate firms into the worker's problem below as those enter additively and are equal across workers.

This, together with a transversality condition on p_t^j , $\lim_{T \rightarrow \infty} \beta^T E_t [p_{t+T}^j] = 0 \forall t, j$, implies that

$$p_t^j = E_t \left[\sum_{s=0}^{\infty} \beta^s r_{t+s}^j \right] \quad (5)$$

3.4 Regional migration

Workers have an idiosyncratic region preference ϵ for regions they currently reside in. At the beginning of each period a worker draws a new ϵ from a continuous distribution F_ϵ with $E[\epsilon] = 0$ and support over $[-B, B]$ for some $B > 0$. After observing his match quality, the worker decides whether to move to a different region. Moving is instantaneous and entails a fixed cost of $c \in \{c_0, c_1\}$, depending on the worker type, as I describe below. After moving, the worker draws an $\epsilon \sim F_\epsilon$ for the new region.²¹

I assume that worker migration is a combination of directed and random migration. In particular with probability $\lambda > 0$ a worker migrates to regions that offer the best labor and housing market conditions, while with probability $1 - \lambda$ the worker is equally likely to migrate to any region in the economy. Setting $\lambda = 1$ would produce a migration pattern with perfect directed migration. Section 6 provides several interpretations for the random migration component, in the context of the estimation results.

I allow for worker heterogeneity in terms of the fixed costs of moving. In particular, I assume that $c_0 > c_1$. The reason for including some agent heterogeneity in mobility costs is that it will allow me to look at the aggregate effects of an individual mobility distortion (a tax on moving) that need not affect all individuals uniformly. For example, such a set-up allows me to easily examine the implications of a possible housing bust mobility distortion for migration flows and for labor reallocation and unemployment (see Section 6), while maintaining the rich regional heterogeneity that is central for the analysis in this paper. I also assume that c_0 is sufficiently high, so that these workers remain completely immobile, which allows me to focus on steady state equilibria only. Therefore, for the rest of the paper, I refer to workers with a high migration cost as immobile workers and to workers with a lower migration cost as mobile workers.

3.5 Timing

The timing within a period is as follows:²²

²¹I deliberately choose to model the regional preference shocks as i.i.d. rather than include some persistence, since a persistent regional preference shock essentially acts as a higher fixed cost of moving c , so that one can include its effect on migration in a higher c . See Bayer and Juessen (2012) for a discussion of the dynamic selection effect due to persistent regional preference shocks, which is observationally equivalent to a higher mobility cost.

²²Note that changing the model slightly, so that with some probability workers receive a job offer from the region they have chosen to migrate to prior to their actual migration decision will not change the qualitative results of

1. Agents observe the realization of regional productivity a ;
2. Workers draw region specific payoffs and make migration decisions;
3. Housing and rental markets opens;
4. Firms make job creation decisions for new jobs;
5. Matching of workers and jobs;
6. Production occurs and wages are paid;

4 Equilibrium

I will be focusing on stationary symmetric recursive equilibria, in which each region j is fully characterized by a vector of state variables $X_t^j = (a_t^j, l_{0,t-1}^j, l_{1,t-1}^j)$ and there is an invariant distribution of workers over booming and depressed regions. The relevant state vector contains the current period productivity a_t^j , as well as the beginning-of-period distribution of workers with different migration costs, $l_{0,t-1}^j$ and $l_{1,t-1}^j$, respectively. I denote the beginning-of-period distributions of workers in booming and depressed regions by, $\bar{\nu}^*$ and $\underline{\nu}^*$.

A stationary symmetric recursive equilibrium will then be defined by laws of motion for the endogenous state variable, $l_1'(X_t^j)$, and for population distributions $\bar{\Xi}(\bar{\nu}, \underline{\nu})$, $\Xi(\bar{\nu}, \underline{\nu})$, and by functions, $\theta(a_t^j)$, $w(a_t^j)$, $p(X_t^j)$, $r(X_t^j)$ giving regional market tightness and wages and regional house prices and rents as a function of the regional state such that (i) workers migration decisions are optimal given the laws of motions for a and the endogenous state variables, (ii) the law of motion for the endogenous state variable are consistent with workers migration decisions and with population constancy in the economy; (iii) the invariant distributions $\bar{\nu}^*$ and $\underline{\nu}^*$ satisfy $\bar{\nu}^* = \bar{\Xi}(\bar{\nu}^*, \underline{\nu}^*)$, and $\underline{\nu}^* = \Xi(\bar{\nu}^*, \underline{\nu}^*)$ and (iv) there is rental and housing market clearing.²³

4.1 Worker housing choices and migration decisions

I next turn to characterizing the migration decision of the mobile workers in the economy (workers with mobility cost c_1).²⁴ Let $V^1(X)$ be a worker's end-of-period value given the regional state X .

the model as this is isomorphic to the current set-up but with a more efficient regional labor market matching technology. It will not change the results of the quantitative model in Section 5 significantly if that probability is low. Identifying such a probability would not be possible from aggregated migration data and would instead require looking at high frequency individual level data on joint job search and migration decisions. This is beyond the scope of the current paper.

²³The complete definition of a symmetric stationary recursive equilibrium is given in Appendix B and follows this broad definition.

²⁴Workers with mobility cost c_0 will face a similar problem, which I do not consider here since I am focusing on economies in which they are immobile.

Then

$$\bar{V} = \lambda \cdot \max_{\tilde{x}} \{V^1(\tilde{x})\} + (1 - \lambda) \left(\int V^1(\bar{a}, l_0, l_1) d\bar{\nu}^* + \int V^1(\underline{a}, l_0, l_1) d\bar{\nu}^* \right) \quad (6)$$

is the migration value of the worker. The first term captures the value from directed migration, since with probability λ the worker moves to a region giving him the highest expected utility, and with probability $1 - \lambda$ the worker is equally likely to migrated to any region of the economy. We have that:

$$V^1(X) = \max_h \{v(h) - r(X) \cdot h\} + e + \mu(\theta(A))(w(A) - e) + \beta E_X [W^1(X')] \quad (7)$$

where

$$W^1(X) = \max_{\bar{\epsilon}} \left\{ F_{\epsilon}(\bar{\epsilon}) \bar{V} + (1 - F_{\epsilon}(\bar{\epsilon})) V^1(X) - F(\bar{\epsilon}) c_1 + \int_{\bar{\epsilon}} \epsilon dF_{\epsilon} \right\} \quad (8)$$

is the beginning-of-period value function for the worker. The interpretation of these value functions is straightforward. For V^1 , the first term captures the flow utility from housing, net of rental payment, as well as from consumption. The second is the expected value in the next period, which takes into account the migration option of the worker. In particular for a given region preference ϵ , a worker compares the value of staying in the region to the value of moving, net of the migration cost.

The worker's optimal housing consumption satisfies the first order condition

$$v'(h) = r(X) \quad (9)$$

This condition, combined with market clearing in the rental market

$$(l_0(X) + l_1(X)) h = H \quad (10)$$

pins down the equilibrium consumption of housing for each worker in the region, $h(X) = \frac{H}{l_0(X) + l_1(X)}$, and the regional rental rate:

$$r(X) = v' \left(\frac{H}{l_0(X) + l_1(X)} \right) \quad (11)$$

Given the structure of the problem, migration will follow a cutoff rule for ϵ , which I denote by $\bar{\epsilon}$, and which is given by:

$$\bar{\epsilon}(X) = \bar{V} - V^1(X) - c_1 \quad (12)$$

Then for $\epsilon < \bar{\epsilon}(X)$ the worker migrates and for $\epsilon > \bar{\epsilon}(X)$ the worker stays, which means that the

fraction of workers migrating from a region with state X is:

$$q(X) = Pr(\epsilon \leq \bar{\epsilon}(X)) = F_\epsilon(\bar{\epsilon}(X)) \quad (13)$$

which is also the ex ante probability of worker migration prior to realization of ϵ .

4.2 Laws of motion for the endogenous state variable

Given the worker migration decisions above, it follows that the end-of-period measure of workers with a low mobility cost in a given region j , $l_{1,t}^j$, is:

$$l_{1,t}^j = l_1'(X_t^j) \equiv (1 - q(X_t^j)) l_{1,t-1}^j + \Psi(X_t^j) \quad (14)$$

where $\Psi(X_t^j)$ is a function that gives the measure of workers migrating into region j at time t . In particular,

$$\begin{aligned} \Psi(X_t^j) &\geq (1 - \lambda) \cdot M \quad X_t^j \in \arg \max_x \{V^1(x)\} \\ \Psi(X_t^j) &= (1 - \lambda) \cdot M \quad o.w. \end{aligned}$$

where

$$M = \int q(\bar{a}, l_0, l_1) l_1 d\bar{\nu}^* + \int q(\underline{a}, l_0, l_1) l_1 d\underline{\nu}^* \quad (15)$$

is the aggregate measure of workers migrating in a given period. Note that due to partially directed migration, some regions will experience higher inflows than others. The exact form of $\Psi(X)$ for $X \in \arg \max_x \{V^1(x)\}$ and the set $\{\arg \max_x \{V^1(x)\}\}$ are determined in equilibrium.

4.3 Regional Dynamics

I now show some equilibrium characterization results, which provide insights into how the model works. I also show that the basic model can qualitatively account for Facts 2 and 3 from Section 2. I will show these results for an economy, in which each region has the same measure l_0 of high mobility cost workers, so I remove l_0 from the vector of state variables X . Also, for notational convenience, I use l to denote the beginning-of-period measure of mobile workers in a region in place of l_1 .²⁵

I first characterize regional dynamics in this economy and the link between migration and regional characteristics resulting from the equilibrium behavior of workers. A set of technical results in Appendix B, Lemmas 7, 8, and 9, which characterize the equilibrium properties of the worker value function, $V^1(a, l)$, and law of motion, $l_1'(a, l)$, allow for this. Here, I only summarize

²⁵I focus on equilibria in which both $V^1(a, l)$ and $l_1'(a, l)$ are continuous. Lemma 6 in Appendix B shows that having $V^1(a, l_1)$ and $l_1'(a, l_1)$ continuous is mutually consistent.

their implications. First of all, Lemma 7 implies that mobile workers weakly prefer regions with lower populations. This is a result of the congestion effect on housing rental markets that a higher regional population has. Secondly, the law of motion $l'_1(a, l)$ is increasing in l , so that regions with a high beginning-of-period population of mobile workers can never end up with lower populations after migration compared to regions with a low beginning-of-period population. The intuition for this result is related to the housing market again. Since post-migration populations determine housing rental rates, regions with a given labor market state can never lose more or gain more population than other regions with the same labor market state.

Lemma 8, in turn, shows that there are stable populations of mobile workers depending on regional labor productivity a , $\underline{l}^*$ for regions with productivity $\underline{a}$, and $\bar{l}^*$ for regions with productivity $\bar{a}$. Regions with productivity $\bar{a}$ and with $l < \bar{l}^*$ will be experiencing inflows that increase their population of mobile workers toward $\bar{l}^*$, or slow declines in population towards $\bar{l}^*$, if $l > \bar{l}^*$, and similarly for regions with productivity $\underline{a}$. Therefore, Lemma 8 describes how the in-migration function $\Psi(a, l)$ looks like in equilibrium.

Lastly, Lemma 9 shows that in equilibrium, regions with high productivity weakly dominate regions with low productivity in workers migration decisions for any given population l . This implies that, depending on parameter values, there can be two types of equilibria. The first type is a “partial compensation” equilibrium with, $\underline{l}^* \geq 0$ but $V^1(\underline{a}, l) < V^1(\bar{a}, l)$, $\forall l \geq \underline{l}^*$, while the second is a “full compensation” equilibrium with $\underline{l}^* \geq 0$ and $V^1(\underline{a}, \underline{l}^*) = V^1(\bar{a}, \tilde{l}(\bar{a}))$, where $\tilde{l}(\bar{a})$ is described in Lemma 1 below. In the first type of equilibrium, rental rate differences cannot compensate for labor market differences for any population of mobile workers, whereas in the second rental rate differences do compensate fully for labor market differences as long as populations in low productivity regions fall sufficiently.²⁶

Therefore, we can make the following observations about the dynamic evolution of regions in a stationary equilibrium of this economy:

Lemma 1. *The following hold in a stationary equilibrium of this economy*

1. *Regional populations of mobile workers lie in the set $[\underline{l}^*, \bar{l}^*]$;*
2. *Transitioning from depressed to booming, a region’s population of mobile workers increases to $\tilde{l}(\bar{a}) \leq \bar{l}^*$ and after that moves up towards $\bar{l}^*$, if $\tilde{l}(\bar{a}) < \bar{l}^*$, experiencing an increasing out-migration rate as the population of mobile workers increases towards $\bar{l}^*$;*
3. *A depressed region’s population of mobile workers moves down towards $\underline{l}^*$ experiencing a decreasing out-migration rate as the population of mobile workers declines towards $\underline{l}^*$;*

²⁶However, even if rental rate differences cannot compensate for labor market differences, with a positive probability of random migration, $\underline{l}^* > 0$.

Proof. See Appendix B. □

Therefore, idiosyncratic regional preferences, the mobility cost c_1 and partially directed migration lead to limited spatial arbitrage. Depressed regions lose population more slowly than in an economy with frictionless mobility, and booming regions gain population more slowly than in an economy with fully directed migration. Limited spatial arbitrage, in turn, creates a non-degenerate distribution of regions over populations. This, combined with durable housing and concave utility from housing services for the workers creates a dependence of regional house prices on the history of labor market shocks. The dependence of regional house prices on the history of labor market shocks also implies a rich regional distribution of house prices. This variation, leads to the observed comovements between regional house prices and regional out- and in-migration from Section 2 as I now show.

²⁷The regional evolution for “partial compensation” equilibria is similar to that of “full compensation” equilibria.

Proposition 2. *Let $out(a, l)$ and $in(a, l)$ be the out-migration and in-migration rates for a region described by (a, l) . Then:*

1. *$out(\bar{a}, l) \leq out(\underline{a}, l)$, $l \in [\underline{l}^*, \bar{l}^*]$ and $in(\bar{a}, l) \geq in(\underline{a}, l)$, $l \in [\underline{l}^*, \bar{l}^*]$ with the inequalities strict for some l .*
2. *$out(a, l)$ is increasing in l and $in(a, l)$ is decreasing in l for $l \in [\underline{l}^*, \bar{l}^*]$.*

Proof. See Appendix B □

The above result is intuitive: higher regional productivity, implies lower out-migration from that region and higher in-migration for a given population. On the other hand, for a given labor market state, a region with a smaller population experiences lower out-migration and higher in-migration. This effect is due to the difference in rental rates across such regions, with workers migrating out less from regions with lower rental rates and migrating more into such regions.

However, this result does not make it clear how house prices co-move with regional flows. In order to make this connection, I first characterize the behavior of house prices across regions. I defined two prices, the regional house price prior to workers' migration decisions, $\tilde{p}(a, l)$, and the house price after migration takes place, $p(a, l)$, i.e. $\tilde{p}$ is the regional house price in the beginning of a period, while p is the regional house price in the end of a period. We have the following result.

Proposition 3. *$p(a, l)$ is increasing in a and l for $l \in [\underline{l}^*, \bar{l}^*]$, and $\tilde{p}(a, l)$ is increasing in l for $l \in [\underline{l}^*, \bar{l}^*]$.*

Proof. See Appendix B □

The proposition establishes a tight link between $\tilde{p}$ and l . Therefore, defining $U(a, l) = (1 - \mu(\theta(a)))$ as the unemployment rate in region (a, l) , we immediately have the following result.

Proposition 4. *Consider a cross-sectional sample of J regions from the model economy. Let $out^j = out(a^j, l^j)$ and $in^j = in(a^j, l^j)$ be the out-migration and in-migration rates for region $j \in \{1, 2, \dots, J\}$. Also, let $U^j = U(a^j, l^j)$ be the unemployment rate and $\tilde{p}^j = \tilde{p}(a^j, l^j)$ be the beginning-of-period house price for a region $j \in \{1, 2, \dots, J\}$. Then:*

1. *for a given $\tilde{p}^j$, out^j is increasing in U^j and in^j is decreasing in U^j ;*
2. *for a given U^j , out^j is increasing in $\tilde{p}^j$ and in^j is decreasing in $\tilde{p}^j$.*

Proof. First, Proposition 3 implies that $\tilde{p}^j = \tilde{p}(a^j, l^j)$ is increasing in l^j for $l^j \in [\underline{l}^*, \bar{l}^*]$. Secondly, note that $U^j = U(a^j, l^j)$ is decreasing in a . Given these two observations, fact 1 follows from fact 1 in Proposition 2 and fact 2 follows from fact 2 in that Proposition as well. □

Proposition 4 establishes that the model can account for the comovements between relative unemployment and house prices and regional migration documented in Section 2. It implies that if one simulates data for many regions from the model and runs the panel regression from Section 2, one would obtain coefficient estimates with the same signs as in the data. The intuition for the result is that the unemployment rate U^j captures the variation in regional productivity a^j , while the house price $\tilde{p}^j$ captures the variation in l^j , the measure of mobile workers. In that sense, it is directly linked to the results from Proposition 2, however it casts it into comovements based on observable variables, like U^j and $\tilde{p}^j$ rather than the state variables a and l .

It is important to discuss, which features of the model drive these results since the information contained in these comovements will help identify the structural parameters underlying these particular model components when we move to estimation in the next section. First of all, the equilibrium house price heterogeneity arising from regional histories, drives the positive comovement between out-migration and house prices holding current labor market conditions fixed. Limited spatial arbitrage also leads to the comovement between out-migration and unemployment, controlling for house prices. Partially directed migration, on the other hand, implies that regions with booming labor markets and lower populations and consequently, house prices and rental rates, have larger population inflows leading to the negative comovement between house prices and unemployment with in-migration. Models with fully random (undirected) migration would have troubles accounting for the in-migration comovements. Additionally, as discussed in Section 2, models with frictionless regional mobility in a perfect spatial arbitrage setting, as in the classical spatial equilibrium framework would have problems accounting for the house price comovements.

5 Quantitative Model and Structural Estimation

The basic model, which I examined in Section 3, was useful in providing intuition for the main modeling assumptions and model mechanics but is not appropriate for establishing quantitative effects. Therefore, in this section I move to a richer model of regional reallocation, which I proceed to estimate structurally. I first briefly describe the dimensions along which the quantitative model differs from the basic model above. Appendix C contains a more detailed description of that model.

5.1 Model set-up

Rather than one period long jobs, the quantitative model has stochastic job destruction, which is the standard assumption of search and matching models of the labor market. In particular, at the end of each period, after production takes place, with probability $s \in (0, 1)$ a job becomes unproductive and is destroyed. This assumption is important for generating realistic employment

flows and *ex ante* employment heterogeneity among workers. I also assume that there's no on-the-job search and that only unemployed workers match to vacant jobs. Also, I expand the set of labor market states, which allows for richer regional dynamics.

With *ex ante* employment heterogeneity among workers, it also becomes necessary to specify migration decisions for both the employed and unemployed. I assume that unemployed workers suffer idiosyncratic region preference shocks of the form described in the previous section. Furthermore, unemployed workers that migrate remain unemployed upon moving and search for a job in their new region of residence. For employed workers, on the other hand, I assume that a fraction ϕ move out of a region in a given period.²⁸ Also, I assume that employed workers remain employed in their new region of residence, i.e. in a sense the job moves with the worker. Lastly, I assume that the out-migration of employed workers from a region equals the in-migration of employed workers into that region, so that the net migration of employed workers across regions is zero, and also the job is not destroyed upon the worker's move but rather the worker is immediately substituted for another. The reason for this last assumption is to simplify the job creation decision of firms since with it the migration of employed workers does not lead to job destruction.²⁹ Furthermore, even though employment-to-employment moves across regions may respond to differences in job finding probabilities across regions, their sensitivity would be substantially lower than the migration of unemployed workers.³⁰

Lastly, wages in the calibrated model are rigid in the sense of Hall (2005). As was first pointed out by Shimer (2005), the standard search model with Nash bargaining leads to a large response of wages to changes in labor productivity, unless the value of unemployment is very high (Hagedorn and Manovskii (2008)). The large sensitivity of the bargained wage implies that changes in labor productivity are mostly absorbed by changes in the wage resulting in small effects on the job finding probability and from there on unemployment. As a result, the standard search model has problems accounting for the volatility of unemployment, given the observed volatility in labor productivity. An analogous problem arises in the environment with regional labor markets that I consider. In particular, a model with Nash bargaining has troubles producing the observed state unemployment dispersion in the recent recession without setting an implausibly high variance for labor market shocks. The modification of the standard search model that Hall (2005) proposes is to

²⁸This constant migration fraction can be thought of as the outcome of idiosyncratic region preference shocks as well, with region preference shocks taking two values: a value of 0 with probability $1 - \phi$ and a value of $-Q$ with probability ϕ for some Q that is sufficiently large to induce employed workers to move regardless of the region's labor and housing market states.

²⁹In the case of non-zero net migration of employed workers there will be job destruction that will depend on the measure of employed workers in the region and hence job creation decisions will become dependent on the population of workers in the region rather than on the labor market state a only.

³⁰Attributing all net flows to the unemployed increases the importance of reallocation for unemployment since unemployed migrants are directly affected by differential job finding probabilities across regions, while employed migrants are affected only indirectly in the future states where their jobs are destroyed and they become unemployed.

have a rigid wage, arising, for example, from a social norm, which does not vary with the aggregate business cycle, thus breaking the strong link between productivity and the wage. Furthermore, the wage lies in the bargaining set of a worker-job pair for every value of productivity over the cycle and hence does not violate individual rationality³¹. This is the approach I adopt as well.³² A rigid wage also simplifies the computation of equilibrium since the firm's problem does not depend on the distribution of workers with different mobility costs, thus making labor market tightness independent of the regional distribution of workers, which would not be the case with a Nash bargained wage.³³

5.2 Estimation procedure

The model described above is too complicated to allow for an analytical solution. This necessitates a simulation-based estimation procedure for the model parameters. In particular, I will be using indirect inference (Smith (1993), Gourieroux, Monfort, and Renault (1993)). Indirect inference is a simulation based estimation method that uses an auxiliary model that links the observed data and the structural model parameters. The auxiliary model is chosen to capture features of the data, which are particularly informative about the parameters of the underlying model. Indirect inference aims to minimize the distance between the parameter estimates of the auxiliary model given the observed data and data generated by the underlying structural model.³⁴ More specifically, suppose that the parameters of the auxiliary model satisfy:

$$\bar{m} = m(X_N) = \arg \min_m Q(X_N, m) \quad (16)$$

where X_N is the observed data with N observations (the so called binding function (Gourieroux, Monfort, and Renault (1993))), and

$$m^i(\theta) = m(Y_N^i(\theta)) = \arg \min_m Q(Y_N^i(\theta), m) \quad (17)$$

³¹There is a large subsequent literature dealing with rigid wages in search models (see Gertler and Trigari (2009) and Shimer (2010)).

³²See Appendix C for the exact conditions.

³³The choice of Nash bargaining versus wage rigidity should not be relevant for the simulation results, since I am not interested in identifying the exact mechanism from economic shock to labor market outcome and treat regional labor market conditions as exogenous.

³⁴The simulated method of moments (SMM) is a particular special case of indirect inference, in which the parameters of the auxiliary model are moments of the data.

where Y_N^i is data generated by the i -th $i \in \{1, 2, \dots, S\}$ simulation of the structural model given parameter vector θ . Then the indirect inference estimator of θ , $\hat{\theta}$ satisfies:

$$\hat{\theta} = \arg \min_{\theta} \left(\bar{m} - \frac{1}{S} \sum_{i=1}^S m^i(\theta) \right)' W_N \left(\bar{m} - \frac{1}{S} \sum_{i=1}^S m^i(\theta) \right) \quad (18)$$

where W_N is an arbitrary positive definite weighting matrix. The optimal weighting matrix is given by the inverse of the variance-covariance matrix of $\bar{m} - \frac{1}{S} \sum_{i=1}^S m^i(\theta)$, which is proportional to the variance-covariance matrix of $\bar{m}$, Ω . Then for a fixed S , as N tends to infinity, the indirect inference estimator is asymptotically normal with variance $\text{var}(\hat{\theta}) = (1 + \frac{1}{S}) \left[\left(\frac{\partial m(\theta)}{\partial \theta} \right)' \Omega^{-1} \left(\frac{\partial m(\theta)}{\partial \theta'} \right) \right]^{-1}$.

5.3 Functional form assumptions and parameters

The quantitative model contains a large set of parameters and functions with unspecified functional form. Estimating a large parameter vector becomes computationally costly in the case of simulation-based estimation. More importantly, the model can explain only certain features of the data, so that estimation of some parameters may lead to bad identification. Therefore, I structurally estimate only a subset of the model parameters and calibrate the rest to values in datasets, different from the data I will use for estimation or values that are standard in the literature. Additionally I make a number of functional form assumptions for the unspecified functions.

Starting with the particular functional forms I work with, I assume that the flow utility function is $\tilde{u}(c, h) = c + v(h)$, where $v(h) = \kappa \frac{h^{1-\gamma}}{1-\gamma}$ for $\gamma \neq 1$ and $v(h) = \kappa \log(h)$ for $\gamma = 1$. Turning to the matching technology, I use a Cobb-Douglas matching function, $m(u, v) = M \cdot u^{1-\alpha} v^\alpha$, which implies that $\mu(z) = M \cdot z^\alpha$ for $z = \frac{v}{u}$. I assume that the labor productivity shock η is normally distributed with variance σ_η^2 . Similarly, I assume that regional preferences are drawn from a truncated normal distribution with zero mean and variance σ_ϵ^2 , where the domain is given by $[-B, B]$. The value of B has a small effect on the results as long as it is sufficiently large, so I set $B = 3\sigma_\epsilon$.

I set a time period in the model to correspond to a month. Also, a region in the model corresponds to a U.S. state.³⁵ I set the discount factor β to 0.995, which gives an annual discount rate of around 6%. For the flow benefit from unemployment I set e to 0.65, which lies between the values proposed by Shimer (2005) and Hagedorn and Manovskii (2008). The exogenous job destruction probability s is set to be 0.034, which is consistent with the rate used in Shimer (2005)

³⁵ Metropolitan statistical area (MSA) is the more natural counterpart of a region in the model, since according to the definition of an MSA (Office of Management and Budget (2009)) it constitutes a single labor market, so workers within an MSA do not need to move in order to search and be matched to a vacant job in the MSA. This matches well the specification of regions in the model, however, due to data limitations and the availability of housing equity data at the state level only (see Section 2), I work with state-level data.

and Hall (2005). I set the supply of housing in each region to $H = 1$ and normalize the average labor productivity to $A = 1$.³⁶ For the matching function, I estimate α and M from JOLTS using monthly data from December, 2000 to December, 2007.³⁷ The estimates I obtain for the matching function are $\alpha = 0.605$ and $M = 1$. The value of α obtained lies in the middle of the set of estimates reported by Petrongolo and Pissarides (2001).

Lastly, I work with a two point distribution of immobile workers across regions, with $l_0 \in L_0 = \{0, 0.04\}$ and $\Pr(l_0 = 0) = 0.5$. This distribution approximates the distribution of households with limited housing equity (equity $< 5\%$). Additionally, with per-period probability of $1 - \rho_{l_0} = 0.05$, a region switches between the two point masses on the distribution of immobile workers. This matches the annual auto-regressive coefficient of the fraction of households in limited housing equity by state of 0.55 observed in the data.³⁸ The reason for including immobile workers in the estimation stage is to show that the third reduced-form fact discussed in Section 2 is constant with increases in the migration costs for some individuals in the economy.

I summarize the predetermined parameters in Table 2.

Given this set of predetermined parameters, there are 9 model parameters that remain to be estimated. I denote these by the vector θ . In particular,

$$\theta = \{\rho, \sigma_\eta, k, \gamma, \kappa, \lambda, c_1, \sigma_\epsilon, \phi\} \in \Theta \quad (19)$$

where the parameter space Θ is a compact subset of a Cartesian product of positive real intervals. The parameters to be estimated are summarized in Table 3.

Lastly, the regional wage rate is set at the wage rate from a standard search and matching model with symmetric Nash bargaining and no regional productivity dispersion but otherwise parametrized as my model. This is similar to the approach from Hall (2005).

5.4 Data and Moments

The data I use for the estimation is annual state-level data between 1993-2008 for the 50 states and the District of Columbia. The choice of this time period and geographical level of aggregation

³⁶I also normalize the total population of workers in the economy to $L = 1$.

³⁷JOLTS contains information on total hires per month, which, when divided by the total stock of unemployed, gives the job finding probability $\mu(z)$. The value of z is similarly obtained as the total vacancies divided by the stock of unemployed. Note that I need an estimate for a matching function at a low level of aggregation but have only aggregate data. This would be worrying, when regional dispersion is high since mismatch would affect the estimated aggregate matching function, which will no longer correspond to the matching function at a lower level of aggregation (Barnichon and Figura (2011)). Therefore, I estimate the aggregate matching function for a time period when of low regional dispersion to ensure that estimate would be close to a matching function estimate at a lower level of aggregation.

³⁸Note that I assume that this does not affect the decision problems of workers, i.e. for workers these shocks are unanticipated.

Table 2: Calibrated Parameters

parameter	description	value
β	discount factor	0.995
e	flow unemployment payoff	0.65
s	job destruction probability	0.034
A	average regional productivity	1
$\mu(z)$	matching function	$z^{0.605}$
L_0	distribution of immobile workers, l_0	$\{0, 0.04\}$
$1 - \rho_{l_0}$	per-period probability of a switch in l_0	0.05

is not surprising given the dataset used in Section 2, since several of the arguments of the binding function used for the indirect inference procedure will be regression coefficients from the estimation in Section 2. In particular, I focus on the regression coefficients in the specifications with the fraction of households with equity $< 5\%$ for both the out and in-migration regressions, i.e. a total of 6 coefficients.

The reason for focusing on these particular coefficients is that those are important features of the data that the particular model I consider can account for, whereas by construction it is silent on the effect of the other controls, relative income or wages (there is no heterogeneity in skills or secular growth rates of regions), homeownership rates and debt-to-value ratios (owning and renting are perfect substitutes and there are no downpayment requirements or other financial frictions), state specific characteristics (regions are homogenous in their housing stock and matching functions) or aggregate shocks (in the case of the steady state equilibrium of the model). Partialling out all these other regressors as well as state and time fixed effects makes the observed data and model generated data comparable.

Note, that it is fine for the estimation procedure I use if these regression coefficients arise from reduced-form regressions rather than structural regressions, and if the auxiliary model is misspecified, as long as the underlying structural model I have can produce the structural relationship

Table 3: Estimated Parameters

parameter	description
ρ	persistence of labor productivity
σ_η	dispersion of labor market shocks
k	vacancy posting cost
γ	inverse elasticity of housing demand
κ	weight on utility from housing
λ	probability of directed migration
c_1	mobility cost
σ_ϵ	dispersion in regional preferences
ϕ	probability of migration of an employed worker

between variables that exists in the observed data. This particular feature is a very attractive property of indirect inference estimation.

Apart from these 6 “moments”, I also include 5 additional data parameters, which include the annual coefficient of auto-correlation in state unemployment after controlling for state and year fixed effects, that is the coefficient in the regression

$$u_{st} = \alpha_s + \zeta_t + \rho_u \cdot u_{st-1} + \epsilon_{st}, \quad (20)$$

the average aggregate unemployment rate, average labor force weighted unemployment dispersion:

$$\hat{\sigma}_t^u = \sqrt{\sum_{s=1}^n \frac{l_{st}}{l_t} (u_{st} - u_t)^2} \quad (21)$$

where l_{st} is the labor force in state s at time t and u_{st} is unemployment rate in state s at time t , average interstate migration rate:

$$q_t = \sum_s \frac{l_{st}}{l_t} out_{st} \quad (22)$$

where out_{st} is the gross out-migration rate from state s at time t , and net migration rate:

$$net_t = \frac{1}{2} \left| \frac{l_{it}}{l_t} (in_t - out_t) \right| \quad (23)$$

where in_{st} is the gross in-migration rate into state s at time t . Each of these is for the period 1993-2008 and is obtained after controlling for a linear time trend in the annual time series.³⁹

5.5 Identification and estimation

As long as the derivative matrix of $m(\theta)$ has full column rank, the parameter vector θ will be identified by the estimation procedure. Additionally, the particular choice of moments is guided by the need to use features of the data that are informative for the underlying model parameters, i.e. we want $m(\theta)$ to be sensitive to changes in θ . Proposition 4 in Section 4.3 and the discussion after it already touched on these issues. In particular, the coefficients on relative unemployment in the migration regressions, as well as average interstate migration and net migration rates are informative for the parameters in the model that drive migration patterns (λ , c_1 , σ_ϵ , ϕ). For example, a higher net migration rate corresponds to greater directed migration λ and/or higher dispersion of region preference shocks (σ_ϵ). The interstate migration rate is informative about the

³⁹Table 4 in the next section summarizes the 11 regression coefficients and aggregate moments that I use in the indirect inference procedure, their observed values, as well as their values at the identified model parameters.

region preference shocks and migration cost of unemployed workers, c_1 and σ_ϵ , and of employed workers, ϕ . The out-migration regression coefficient, in turn, is particularly informative about c_1 and σ_ϵ , since a higher value of c_1 for a given value of σ_ϵ leads to lower out-migration in response to an adverse labor market shock and higher unemployment in a region. The in-migration coefficient is informative about λ .

The house price coefficients in the migration regressions are informative for the parameters in the housing utility function (γ, κ) . A higher value of γ , implies more volatile regional house prices since house prices fall more sharply with population changes. Higher price variability leads to a higher magnitude of the regression coefficients on house prices. Higher values of κ imply that house prices do not have to fall by much to compensate workers for labor market differences, so given the migration process, the equilibrium cross-sectional dispersion in house prices is lower, which again affects the magnitude of the regression coefficients.

The auto-correlation of unemployment, unemployment rate and unemployment dispersion are informative for the productivity process parameters and cost of opening a vacancy (ρ, σ_η, k) . In particular, the first is informative about ρ , the second, about k , and the third, about σ_η .⁴⁰

Lastly, we need the variance-covariance matrix of $\bar{m}$ in order to form an optimal weighting matrix and compute standard errors for θ . In this particular case, this matrix corresponds to the variance-covariance matrix of a set of regression coefficients. I obtain them by estimating the migration regressions (1), and equation (20) in a system of equations. Similarly, I obtain the aggregate averages in a system of equations with a constant term and linear time trend. I assume that the two sets of regression coefficients are uncorrelated so that the variance-covariance matrix for $\bar{m}$ is block diagonal. This assumption is reasonable since the first set of estimates uses state-year variation, whereas the second set uses aggregate time-series variation.

The estimation procedure relies on a simulation of the steady state equilibrium of the model economy. Given a solution for the steady state equilibrium of the model economy, the estimation algorithm proceeds by generating data for 45 regions and 16 years and computing moments as in the data and checks the value of the criterion function. To solve for the steady state equilibrium, I discretize the state space over which the value functions of workers and the law of motion for the labor force are defined. Appendix C contains information on the numerical algorithm I use to simulate this discretized version of the model. The algorithm is standard apart from two points that are worth mentioning briefly. First of all, rather than simulating a nested fixed point model where, for example, for a given law of motion for the labor force, I solve for the worker value function until convergence, I adjust all endogenous objects simultaneously. This greatly speeds up the

⁴⁰Even though Proposition 4 suggests that some parameters in the model can be identified it is far from an identification result. Unfortunately, it is hard to derive such an identification result for this model. However, a numerical estimate of the derivative matrix of $m(\theta)$ shows that the objective is sensitive to changes in the parameter vector θ . The estimate of the derivative matrix is available upon request.

Table 4: Data and Model Moments

Description	Observed	Simulated
β_{neg}^{out}	-0.8123	-1.2132
β_u^{out}	0.1727	0.2713
β_p^{out}	0.251	0.2534
β_{neg}^{in}	0.1355	-0.5401
β_u^{in}	-0.226	-0.2413
β_p^{in}	-0.1577	-0.1936
ρ_u	0.7378	0.752
aggregate unemployment	5.3657	5.3016
unemployment dispersion	1.0679	1.0931
migration rate	3.013	2.9174
net migration rate	0.2794	0.2253
Criterion	28.227	

Notes: Calculations are based on state-level data between 1993-2008. Estimation is performed by Indirect Inference. The simulated moments are obtained from 50 simulated panel of 45 regions over 293 time periods (months) keeping the last 193 time periods only (16 years and 1 month). The first six moments are regression coefficients from out- and in-migration regressions. The seventh moment is auto-correlation coefficient for state unemployment. The last four moments are aggregate averages. Value of criterion function at minimizing parameter vector reported last.

computation of equilibrium, which is highly desirable for the estimation procedure but at the cost of more unstable convergence. Secondly, the law of motion for the labor force basically constitutes a continuum of housing market clearing conditions. Therefore, in the case of a discretized law of motion we have a large number of market clearing conditions that have to be adjusted in a tatonnement-like fashion. The particular adjustment process chosen influences greatly the speed and stability of convergence.

6 Estimation results and counterfactual experiments

I first presents the results of the estimation procedure. Table 4 compares the observed and simulated moments (regression coefficients and aggregate averages). The model is fairly good at matching the targeted moments observed in the data. There are two particular issues with the model's fit to be discussed. First of all, the model cannot reconcile perfectly the observed net migration rate and the sensitivities of out- and in-migration to relative unemployment. Matching a higher net migration rate is associated with higher sensitivities of out- and in-migration to

unemployment. The main reason for this is that a region in the model is more comparable to an MSA than a whole state. Since there are moves across MSAs but within states that the state level data does not pick up, this leads to weaker comovements between unemployment and migration rates than for data generated by the model.⁴¹

The second issue relates the behavior of house prices in the model. However, it is not possible to see this from a comparison of moments but becomes clear when one examines the parameter estimates, which I turn to next. Table 5 presents the parameter estimates and asymptotic standard errors. The most interesting parameter estimates are the parameters of the utility from housing (γ and κ) and the parameters guiding migration decisions (λ , c_1 , and σ_ϵ). First of all, the inverse elasticity of housing demand, γ is close to estimates obtained in micro-studies (Hanusheck and Quigley (1980)). On the other hand, the value of κ , leads to a substantially lower housing expenditure share (around 4%) than observed in the data (24% in Davis and Ortalo-Magne (2009)).⁴² The reason for this low estimate is the following: House price variability, cross-sectionally and over time, is important for the observed magnitudes of regression coefficient estimates in the migration regressions. A low level of housing expenditure share leads to a more dispersed population in the cross-section of regions in equilibrium and from there to more variable rental rates. Since house price variability is only driven by rental rate variability in the model, this allows the model to match the regression coefficient magnitudes observed in the data.⁴³

Turning back to Table 4, we see that there is a similar pattern in the coefficient estimates on households with limited housing equity from the data and the coefficient for immobile workers from the model, with the out-migration coefficient higher in magnitude than the in-migration coefficient.⁴⁴ Therefore, an asymmetric pattern between out- and in-migration as observed in the data is consistent with an increase in the mobility cost of some individuals. The implications of this observation are two-fold. First, if a housing market slump does distort the mobility decisions

⁴¹It could also be the case that the model cannot explain all of the net migration present in the data since it does not consider the effect of relative income for migration flows. Explaining the additional effects of relative income beyond the effects of relative unemployment is a promising venue for future research, which I discuss briefly in Section 7.

⁴²Estimating the model under the restriction of $\gamma = 2$ and $\kappa = 0.2355$ to match the inverse elasticity of housing demand from micro-studies and the expenditure share in the data, produces similar results for the other parameters. In particular, the probability of directed migration changes the most and equals $\lambda = 0.4924$, the mobility cost, $c_1 = 17.0921$ and $\sigma_\epsilon = 6.3193$.

⁴³This discrepancy in expenditure share estimates provides indirect support for the observation that regional house price variability both in the time series but also cross-sectionally is driven by variability in discount factors beside rental rate variability (Campbell, Davis, Gallin, and Martin (2009)). Therefore, including volatile discount factors of house prices can help match both rental rate and house price variability. However, this is beyond the scope of this paper.

⁴⁴Both coefficient estimates in the simulated model are within the 95% confidence intervals of the coefficient estimates from the data despite the in-migration coefficient having a different sign than in the data. The 95% confidence interval of β_{neg}^{out} for the equity cutoff of 5% is [-1.581, -0.0437] and for β_{neg}^{in} , it is [-0.582, 0.853]. The reason for the wrong sign is that this particular regression coefficient is not targeted by the optimization since it has a low weight in the objective function as it is very imprecisely estimated in the data.

Table 5: Parameter estimates

ρ	σ_η	k	γ	κ	λ	c_1	σ_ϵ	ϕ
0.9625	0.0054	0.6491	2.7734	0.0336	0.351	17.1775	6.3437	0.0026
(0.0033)	(0.0012)	(0.04)	(0.032)	(0.0264)	(0.0329)	(0.1301)	(0.092)	(0.0006)

Notes: Parameter estimates from Indirect Inference estimation. Estimation procedure chooses vector of parameter values to minimize the distance between moments observed in the data and moments generated by simulated data from the model. Standard errors derived from numerical derivatives in parenthesis.

of some individuals, this distortion will appear in aggregate migration data as such an asymmetric pattern. Secondly, modeling this distortion directly with heterogeneous moving costs still allows the estimated model to be consistent with such an effect. Therefore, if a housing bust increases mobility costs, then looking at the direct effect of a larger fraction of workers with high mobility costs in the estimated model, as I do in Section 6.2 below, will be informative about the aggregate effects of the housing bust through that channel.

The mobility cost estimates λ , c_1 , and σ_ϵ in Table 5 are of particular interest. First of all, the low estimate for the probability of directed migration has several interpretations. It implies that agents are not perfectly informed about regional characteristics when making their migration decisions. Secondly, agents may have unmodeled heterogeneous beliefs about the future evolution of specific regional labor and housing markets. Lastly, it implies that a substantial part of inter-regional moves are not related to labor or housing market differences and are the result of unmodeled preferences for other regions.

The estimated mobility cost c_1 is substantial. It equals around 2 years of wage income in the model. This is similar in magnitude to the estimates obtained in Kennan and Walker (2011). Lastly, the volatility of regional preferences σ_η is low compared to the mobility cost c_1 . This translates into a lower mobility rate and lower sensitivity of out-migration to mobility, respectively. A drop in the variability in preference shocks (higher quality of matches between individuals and locations) has been proposed as one explanation for the secular decline in mobility observed in the US (see Kaplan and Schulhofer-Wohl (2012b)).

Apart from looking at the discrepancy between the features of the data that the estimation has explicitly targeted, I also report a set of moments, which the estimation has not targeted. Table 6 presents the standard deviation for (the logarithm of) out-migration, in-migration, relative unemployment and relative house prices, as well as the fraction of households with limited housing equity/fraction of immobile workers in the observed data (after partialling out covariates and fixed effects that the model does not aim to explain, as discussed in Section 5.4) and in the estimated model. Similarly, table 7 shows the correlation coefficients between these variables in the data and

Table 6: Standard deviations of observed and simulated data from the estimated model

	Observed	Simulated
σ_{out}	5.542	5.443
σ_{in}	5.207	19.850
σ_{neg}	1.747	1.823
σ_u	10.554	18.676
σ_p	6.419	8.961

Notes: Standard deviations for (100 times) log of out-migration rate (σ_{out}), in-migration rate (σ_{in}), relative unemployment (σ_u) and relative house prices (σ_p), as well as fraction of households with housing equity < 5%/immobile workers (σ_{neg}), for the observed data (left column), after controlling for state and year fixed effects, relative wage, relative income-per-capita, home ownership rate, average debt-to-value and demographics and for data simulated by the model. The standard error for the data simulated by the model is averaged over 50 simulations.

in the model.

Both the standard deviations and correlation coefficients are fairly close in the estimated model and the observed data. The standard deviations of out-migration are nearly identical.⁴⁵ The variability in relative unemployment and house prices is slightly larger in the model than in the data. The model produces substantially more variable in-migration, which is due to the non-trivial fraction of directed migration that leads to jumps in the population. Jumps in population in the model occur in response to house price differences opening up between regions with similar labor market conditions. As a result house prices also jump with positive labor market shocks. However, house prices do adjust down smoothly when labor market conditions worsen. This creates high persistence in house price changes, and as a result, the coefficient of autocorrelation in the log of annual relative house prices in the model is very high, around 0.959. This is consistent with a high coefficient of autocorrelation of 0.818 observed in the data. The inability of a spatial equilibrium model with frictionless mobility to match the observed autocorrelation in house prices has been a puzzle in recent literature (Gleaser, Gyourko, Morales, and Nathanson (2012)).

Therefore, congestion effects in housing markets are not sufficient to produce a smooth responses of in-migration or house prices to labor market improvements. Smoothness in in-migration and in house price increases can arise either from a richer stochastic process for labor market shocks or from a technological constraint on the rate at which individuals can move into a region that does not arise through the housing market. For example a technological adjustment cost to in-migration will allow for that.

⁴⁵The standard deviations of fraction of households with limited housing equity/fraction of immobile workers are very close because of the way the distribution of immobile workers is calibrated.

Table 7: Correlation coefficients for observed (a) and simulated (b) data

	$\log(out)$	$\log(in)$	neg	$\ln(\frac{u_s}{u})$	$\ln(\frac{p_s}{p})$
$\log(out)$	1				
$\log(in)$	-0.3186	1			
neg	-0.0759	-0.0279	1		
$\ln(\frac{u_s}{u})$	0.2429	-0.4051	0.1084	1	
$\ln(\frac{p_s}{p})$	0.2316	-0.1274	-0.0427	-0.205	1

(a)

	$\log(out)$	$\log(in)$	neg	$\ln(\frac{u_s}{u})$	$\ln(\frac{p_s}{p})$
$\log(out)$	1				
$\log(in)$	-0.2464	1			
neg	-0.3625	-0.1094	1		
$\ln(\frac{u_s}{u})$	0.831	-0.213	0.0089	1	
$\ln(\frac{p_s}{p})$	0.0389	0.0277	-0.1064	-0.3366	1

(b)

Notes: Correlation coefficients for (100 times) log of out-migration rate (σ_{out}), in-migration rate (σ_{in}), relative unemployment (σ_u) and relative house prices (σ_p), as well as fraction of households with housing equity < 5%/immobile workers (σ_{neg}), for the observed data (left column), after controlling for state and year fixed effects, relative wage, relative income-per-capita, home ownership rate, average debt-to-value and demographics and for data simulated by the model. The standard error for the data simulated by the model is averaged over 50 simulations.

Examining the cross-correlation coefficients shows a similar picture, with some coefficients fairly close but with discrepancies in others. There is a negative correlation between out- and in-migration in the model and in the data with similar magnitude. Two coefficients that are noteworthy are the correlation between relative house prices and out- and in-migration. The reason for the low correlation is the low estimates κ , which as discussed above, leads to a low expenditure share on housing and a dampened response of migration flows to house prices. The negative correlation between relative house prices and the fraction of immobile workers in the model is somewhat puzzling. The reason for it is that even though the measure of immobile workers is exogenous, its fraction of the total population is endogenous and is lower when regional populations are higher, which is when house prices are higher. The zero correlation between relative unemployment and the fraction of immobile workers is more expected since unemployment rates are not related to populations given the constant returns to scale production function. In the data, there is a negative correlation since stagnant labor markets correlate with an increases in the fraction of households with limited housing equity. Overall, the fairly close match between untargeted moments in the data and the estimated model is encouraging for the results of the counterfactual simulations that I perform in the next sections.

The large mobility cost, low probability of directed migration and the effect of relative house price differences on migration decisions that I estimate constitute important reallocation barriers that may lead to regional mismatch, the co-existence of relatively tight and slack regional labor markets that unemployed workers look for jobs in, when regional economic fortunes diverge. I examine this possibility by showing that the model can account for the comovement between regional unemployment dispersion and shifts in the Beveridge curve observed in the data.⁴⁶

6.1 Beveridge curve shifts

In her study on the shifts of the U.S. Beveridge curve, Abraham (1987) conjectures that dispersion in regional economic conditions are associated with shifts in the curve. I confirm this by first showing that there is a positive comovement between regional unemployment dispersion and shifts in the Beveridge curve and then showing how my model can produce and explain this comovement.

I look at annual data for the U.S. and construct a synthetic vacancy rate using the Conference Board HWI and JOLTS (see Appendix A for details), as well as annual employment weighted state and MSA unemployment dispersion series. I estimate a standard Beveridge curve regression, adding a cubic time trend to control for secular shifts in the curve as well as for trends in the

⁴⁶Note that both discrepancies between the model and data discussed above would lead to lower identified reallocation barriers than are present in the data, since in the first case sensitivities to unemployment are higher in the data, whereas in the second the compensation of the low expenditure share from housing leads to low compensation of housing markets for labor market differences. Therefore, the counterfactual experiments below are performed with lower levels of reallocation barriers than those that are likely to exist in reality.

Table 8: Beveridge curve regressions with vacancy rate as dependent variable (a) and unemployment rate as dependent variable (b)

(a)				
Dep. Variable:	vacancy rate			
		v		$\ln(v)$
unemployment rate (u)	-0.203*** (0.0327)	-0.458*** (0.101)	-0.455 (0.311)	
State Unemployment Dispersion		1.146** (0.458)	1.146** (0.459)	0.246* (0.129)
u^2			-0.000255 (0.0217)	
$\ln(u)$				-0.742*** (0.172)
R^2	0.801	0.851	0.851	0.881

(b)				
Dep. Variable:	unemployment rate			
		u		$\ln(u)$
v	-2.294*** (0.415)	-1.044*** (0.224)	-4.081*** (1.286)	
State Unemployment Dispersion		3.177*** (0.237)	2.803*** (0.252)	0.475*** (0.0421)
v^2			0.411** (0.191)	
$\ln(v)$				-0.662*** (0.117)
R^2	0.801	0.851	0.851	0.881

Notes: Own calculations from BLS, Conference Board and Abraham (1987). Annual data from 1960-2010. Newey-West robust standard errors in parenthesis; * = significant at 10%; ** = significant at 5%; *** = significant at 1%. All regressions include a cubic time trend. v is national vacancy rate and u is national unemployment rate. Dispersion measure for unemployment is defined as $\hat{\sigma}_t^u = \sqrt{\sum_{i=1}^n \frac{e_{i,t}}{e_t} (u_{i,t} - u_t)^2}$ where $e_{i,t}$ is employment in state i at time t , e_t is national employment at time t , $u_{i,t}$ is unemployment rate state i at time t and u_t is the national unemployment rate.

Table 9: Beveridge curve regressions on model generated data

Dep. Variable	v	u
u	-0.5607	
v		-1.2338
Unemployment Dispersion	0.3496	0.7207

Notes: OLS regressions based on model generated aggregate time series on vacancy rate, unemployment rate and regional unemployment dispersion. Number of observations: 50. Data is derived by solving the model for different values of A and σ_η . Each observation is derived as the average of 12 monthly observations. Each 12 months a new set of values for A and σ_η is drawn from the sets $[0.995, 1.005]$ and $[0, 0.02]$, respectively. These shocks are assumed to be unanticipated. Unemployment dispersion is defined as $\hat{\sigma}_t^u = \sqrt{\sum_{i=1}^n \frac{l_{i,t}}{l_t} (u_{i,t} - u_t)^2}$.

vacancy or unemployment rates, and also include unemployment dispersion to the regression. The main regressions I run are of the form:

$$y_t = \alpha_0 + \alpha_1 x_t + \alpha_2 \hat{\sigma}_t^u + \sum_{i=1}^3 \alpha_{2+i} t^i + \epsilon_t \quad (24)$$

with $y_t = v_t$, the aggregate vacancy rate in year t and $x_t = u_t$, the aggregate unemployment rate, and vice versa. $\hat{\sigma}_t^u$ is regional unemployment dispersion either at the state or MSA level.⁴⁷

Table 8 shows the regression results for state unemployment dispersion.⁴⁸ Not surprisingly there is a very significant negative relation between vacancies and unemployment. More interestingly, there is also a positive relation between regional unemployment dispersion and vacancies, controlling for unemployment (or vice versa in the regressions where unemployment is the dependent variable).

This relation implies a positive comovement between state/MSA unemployment dispersion and shifts in the Beveridge curve. The recent recession and its aftermath are examples of this positive comovement as both state and MSA unemployment dispersion have increased significantly, and at the same time the Beveridge curve appears to have shifted out.⁴⁹

⁴⁷I also augment (24) with the square of unemployment rate and vacancy rate, respectively and also run a regression with the vacancy and unemployment rates in logs to control for non-linearities.

⁴⁸Table 13 contains the results for MSA unemployment dispersion.

⁴⁹The possibility of an outward shift in the curve has renewed interest among academics and policy makers about the importance of mismatch for the labor market. (Katz (2010) and Kocherlakota (2010)). Empirical work on the Beveridge curve and JOLTS data by Barnichon and Figura (2010) and Barnichon, Elsby, Hobijn, and Sahin (2010) provides additional evidence for the magnitude of the recent shift.

I now show how the model I study in this paper can generate this comovement. I simulate a sequence of (unanticipated) shocks to average aggregate labor productivity A and productivity dispersion σ_η and solve the model (at the estimated parameters) for these shocks, constructing a time series of aggregate vacancy and unemployment rates, and aggregate unemployment dispersion. I then run Beveridge curve regressions similar to (24) on this data. Table 9 shows the estimation results from these regressions. The coefficients on unemployment dispersion are both positive and statistically significant. They are particularly close in magnitude to the coefficients estimated for MSA unemployment dispersion, shown in Table 13 in Appendix A. Figure 2 shows visually the shifts in the Beveridge curve implied by the model generated data, as well as the US Beveridge curve between 1960 and 2010.⁵⁰

Therefore, the model can explain a large part of the shifts in the Beveridge curve that have not been due to secular trends. These results suggest that regional mismatch, due to the combination of limited regional labor reallocation and shocks that hit regional labor markets asymmetrically, is an important shifter of the Beveridge curve. Conversely, this implies that relaxing regional reallocation by partially or fully removing these barriers would have an important effect on aggregate and regional unemployment. However, changing the level of mobility without affecting its sensitivity to labor market shocks should not reduce regional mismatch much. These two observations are important for understanding the quantitative results I obtain in the next section when I vary mobility and reallocation barriers.

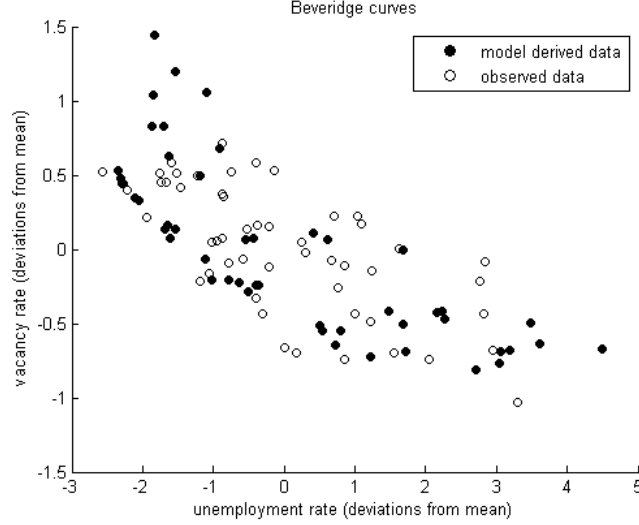
6.2 Great Recession simulation

I now turn to a simulation of the Great Recession. The main purpose of this exercise is two-fold. First of all, it gives a sense of the importance of reallocation barriers for the labor market that the structural estimation identified, and secondly it examines the likely quantitative magnitude of the housing bust mobility hypothesis, discussed in the Introduction, for regional reallocation and unemployment. I examine these effects by calibrating a permanent shock to average labor productivity A , productivity dispersion σ_η , and the measure of immobile workers across regions L_0 to match the average unemployment rate, average state unemployment dispersion by occupation and interstate migration rate for members of the labor force in 2010.⁵¹ These values are $u = 9.5\%$,

⁵⁰The average unemployment dispersion for the model generated series is around 2.5%, which is close to the long run mean of average unemployment dispersion by occupation from 1968-2010 of around 2.3%. See Section 6.2 for a precise definition.

⁵¹The assumption of a permanent shock is fine as long as shocks are fairly persistent at monthly frequency in the data. Examining average state unemployment dispersion by occupation gives a coefficient of autocorrelation at annual frequency of 0.86. Also, state unemployment dispersion data points to a regime switching process. . A test for multiple structural breaks (Bai and Perron (1998)) confirms this.

Figure 2: Beveridge curves for model generated and observed data.



Notes: Observed data is data on aggregate vacancy rate and unemployment rate between 1960 and 2010 after removing a cubic time trend. See Appendix A for details. Model generated data is derived by solving the model for different values of A and σ_η . Each observation is derived as the average of 12 monthly observations. Each 12 months a new set of values is drawn.. These shocks are assumed to be unanticipated. The total number of model derived observations is 50.

$\hat{\sigma}^u = 2.3\%$ and $q = 1.46$, respectively.⁵²

This particular approach provides an upper bound on any mobility and regional reallocation distortions that the housing bust may have caused and leads to half of the population in the model having a high mobility cost (i.e. being immobile). To calibrate these parameters I simulate a steady state equilibrium of the model (otherwise parametrized at the estimated parameters above). As in the structural estimation, I average the model derived moments over 50 simulations. I then conduct a series of counterfactual experiments, in which I reduce the fraction of immobile workers to its baseline level and also sequentially relax other reallocation barriers. As discussed in the previous section, there are several reallocation barriers that act in the model. First of all, imperfect directed migration has two effects on migration: It discourages migration since workers migration values

⁵²I take average state unemployment dispersion by occupation rather than state unemployment dispersion unconditionally, since the former is more informative about the magnitude of labor market disparities in the presence of costs to switching occupations. I derive the average state unemployment dispersion by occupation from CPS micro data by constructing unemployment rates by occupation, state and year with 10 general occupation groups. I then construct labor force weighted state unemployment dispersions by occupation o and year t of the form $\hat{\sigma}_{o,t}^u = \sqrt{\sum_{i=1}^n \frac{l_{i,o,t}}{l_t} (u_{i,o,t} - u_{o,t})^2}$. I then take a weighted average of these unemployment dispersion across occupations with weights given by the employment share in each occupation nationally. The value of $\hat{\sigma}^u = 2.3\%$ used is for 2010. The 10 occupation groups are managerial occupations, professional specialty occupations and technicians, sales occupations, administrative support and clerical occupations, service occupations, farming and forestry occupations, mechanics, repairers and precision production occupations, construction trades and extractive occupations, machine operators, and transportation and material moving occupations.

Table 10: Simulating the Great Recession and Counterfactual Experiments

Scenario	U_{agg}	$\hat{\sigma}^U$	migration rate	net migration rate	U_1	U_2
Great Recession ($l_0 = 0.5$)	9.49	2.301	1.46	0.17	11.27	7.79
Counterfactual: ($l_0 \in \{0, 0.04\}$)	9.43	2.276	2.91	0.26	11.25	7.75
Counterfactual: ($l_0 \in \{0, 0.04\}$, $\lambda = 1$)	9.39	2.251	2.99	0.43	11.15	7.75
Counterfactual: ($l_0 \in \{0, 0.04\}$, $\lambda = 1$, $\kappa \rightarrow 0$)	9.36	2.175	3.16	0.71	11.14	7.7
Counterfactual: ($l_0 \in \{0, 0.04\}$, $\lambda = 1$, $\kappa \rightarrow 0$, $c_1 = 13.22$)	9.12	2.136	3.94	1.39	11.03	7.7

Notes: Model simulation results. See text for details. All numbers are in percentage points. U_{agg} is the aggregate unemployment rate, $\hat{\sigma}^U$ is aggregate unemployment dispersion, and U_1 and U_2 are average unemployment rates for regions above and beyond the median unemployment, respectively. The first exercise calibrates, A , σ_η , and l_0 to match the aggregate unemployment rate, average interstate migration rate by occupation and migration rate for 2010 in a steady state equilibrium of the model. The remaining exercises use these parameters for A and σ_η but vary l_0 , λ , κ , and c_1 as shown in the table.

are reduced. It also decreases net migration. Since labor reallocation is the result of net migration, a lower probability of directed migration should translate into a lower level of labor reallocation. Secondly, utility from housing services and relative house price differences provide compensation for different labor market conditions across regions. A higher utility from housing thus reduced labor reallocation, as well. Lastly, the cost of migration for unemployed workers affects the sensitivity of migration to labor market differences, and hence, net migration flows.

The results for the Great Recession simulation and counterfactual experiments are shown in Table 10. Each scenario reports the aggregate unemployment rate, unemployment dispersion, inter-regional and net migration rates, as well as average unemployment in regions with unemployment above and below the median. As is evident from a comparison of the first and second row, the mobility distortion effect of the housing bust, although potentially substantial, has a very limited labor market effect. In particular, aggregate unemployment falls by only 6 basis points if the fraction of immobile workers is reduced to the baseline level. Inspecting the third and fourth columns reveals the reason for this. Although the effect on inter-regional migration is substantial, with regional mobility doubling, the effect on net migration is limited.

Therefore, given the estimated parameters, even a large gross migration effect due to a mobility distortion (from the housing bust or via any other channel) would correspond to a smaller effect on

net migration and from there on labor reallocation and unemployment. Hence, relaxing reallocation barriers in the model such as the probability of directed migration and the compensation effect of housing consumption for labor market differences, which affect net migration, would enhance the labor market impact of a reduction in worker mobility. For example, moving to fully directed migration (row 3) produces a small effect on gross migration but a large effect on net migration. Similarly, removing housing from the utility function, reduced gross migration slightly but the net migration response is large. The result of both is a larger effect on aggregate unemployment. Lastly, a reduction in the cost of moving, c_1 , by around 20%, increases net migration substantially as well and leads to an effect on aggregate unemployment of almost 0.4%.⁵³

To summarize, a mobility reduction of the housing bust may have affected gross migration substantially, but its effect on net migration and labor market reallocation would have been limited. More generally, reducing individual mobility will affect gross flows more than it would regional reallocation and unemployment. The reason for this is that the labor market differences produce low levels of reallocation, at least over short time horizons. This is due to a combination of reallocation barriers that the estimated model identifies, such as imperfect directed migration, relative house price differences that compensate for labor market differences and fixed costs of moving.

These observations also shed some light on the effects of policies aimed at promoting labor reallocation. Although the model is not constructed for a complete welfare analysis of different policies, since mobility distortions are modeled in a reduced-form way, it is still informative about their ultimate labor market effects. The most important insight is that on its own any policy that reduces mobility distortions will have a very limited effect on aggregate unemployment. Any such policy must be combined with policies aiming to improve labor reallocation more generally. Such policies include, for example, providing better information to workers about job vacancies and the state of labor markets in other regions, or providing housing subsidies for moving to regions with house prices above the national average or a tax on housing consumption for regions with house prices below the national average. Lastly, a moving subsidy that reduces moving costs will have a beneficial effect on reallocation, as well.

⁵³Note that, estimating the model to MSA data may lead to slightly larger effects since gross and net migration rates are higher across MSAs than across states, so the estimated probability of directed migration will be higher and the estimated mobility cost, lower. However, the effects will be of similar magnitude to the effects reported in Table 10. For example, in a previous version of this paper, I looked at the effects of reduced mobility in a model with only directed migration, no house price differences and a lower migration cost than the one estimated here and uncover an effect of 0.5%.

7 Concluding comments

This paper studies regional labor reallocation and its labor market consequences and evaluates the housing market mobility hypothesis for labor market performance as one particular application. I construct a multi-region equilibrium model with regional labor and housing markets and limited mobility, which I proceed to estimate structurally using state level data for the U.S.. The model can account for the comovements between unemployment, house prices and gross migration flows. It uncovers substantial reallocation barriers, which lead to large regional mismatch as evinced by the ability of the model to match shifts in the U.S. Beveridge curve.

Finally, I examine the likely magnitude of a labor market reallocation distortion of the recent U.S. housing market slump. The effects uncovered are small despite potentially large effects on aggregate migration, due to limited regional reallocation resulting from the high reallocation barriers that the model identifies. The small effects on unemployment do not imply that any mobility distortions of the housing bust were of little consequence. Even though net flows are the important margin for the response of unemployment, gross flows are important for productivity. For example, a firm that has to relocate an employee to an establishment in a different city may delay such a move because of housing market problems that the employee faces, which would ultimately be reflected in lower productivity.

The model with limited mobility that I study in this paper and the empirical methodology that I use have important applications beyond this particular example. For instance, the paper shows that limited mobility has implications for regional population and house price dynamics, which are not present in the benchmark spatial equilibrium model of frictionless mobility. Furthermore, the model with limited mobility produces a lower cross-sectional variability of rental rates compared to a model with frictionless mobility for the same level of the housing expenditure share. Therefore, limited mobility may be an important component of a model aiming to match the variation in rental rates given the observed housing expenditure share (Davis and Ortalo-Magne (2009)). Also, a model with limited mobility creates positive autocorrelation in house prices, which is a desirable property for models that try to explain regional house price dynamics (Gleaser, Gyourko, Morales, and Nathanson (2012)). More generally, such a model may prove important for understanding the nature and sources of house price volatility. Therefore, applying the model to explain the behavior of regional housing markets and the link between regional housing markets and labor market performance is an important topic for future research.

Additionally, allowing for both directed and random mobility in an estimable model and bringing it to the data provides an important insight into the migration decisions of individuals since existing models of regional reallocation assume either fully directed or fully random mobility. However, there are additional important determinants of migration decisions that the model has not

considered, which may explain part of the random migration component. Most importantly, the model is silent on the effects of differences in regional income on migration decisions. The estimation was performed on a model with wage rigidity so there was no cross-sectional variability in wages and variability in income per-capita came only through variability in unemployment. However, even allowing for wage flexibility, the model in its current form would produce cross sectional variations in unemployment, wages, and per-capita income with strong collinearity. The reason for this is that unemployment and wages (and hence income per-capita) in a model with a search and matching friction respond almost contemporaneously to cyclical labor market shocks and as a result co-move together very strongly. On the other hand, relative income per-capita has an asymmetric effect on gross migration flows (no effect on out-migration and strong effect on in-migration), even after controlling for relative unemployment. Therefore, enriching the model by allowing for differential growth rates in income across regions and shocks to income growth may help in explaining this additional comovement as well as the broader time series and cross sectional patterns in unemployment, house prices, and income per-capita.

Appendix A - Data Appendix

Data for panel regression

The data for the state panel regression results reported in Section 2 come from a number of sources:

1. Data on household state in- and out-migration rates I obtain from the IRS U.S. Population Migration Database, which is available up to 2008. The data is based on the year-to-year address changes of individual income tax returns. From that raw data, for each state the IRS computes the total number of tax returns, which approximates the number of households, that have migrated into and out of that state. From this data I compute state in- and out- migration rates for a given year as the ratio of the number of movers into or out of the state to sum of the number of movers (into and out of the state, respectively) and non-movers. Note that the period covered does not correspond exactly to the calendar year as it covers moves from April of a given year to April of the next year. However, I treat it as corresponding to the calendar year. The advantage of using the IRS data for tracking state migration patterns as opposed to, for example the Current Population Survey micro-data, is that it is continuously available from up to 2008, whereas CPS has a gap in 1995. Furthermore, the CPS is a survey with a limited sample of individuals so computing state inflow and outflow rates would introduce significant measurement error, which would be problematic for having precise estimates. On the other hand, the IRS data is not completely representative since it excludes the very poor and elderly. However, that should not create problems since the very poor are not homeowners and the elderly do not generally migrate for employment reasons. Also, the IRS data and looks at mobility of all households rather than mobility of individuals that are part

of the labor force.

2. Data on fraction of homeowners with limited housing equity by state I obtain from the Interview Survey of the Consumer Expenditure Survey (CE).⁵⁴ The Interview Survey of the CE is a quarterly survey of consumption patterns and expenditures of American consumers. The survey collects data on household characteristics, income, and major expenditures. The sample design is a rotary panel survey with data on each household available for 4 quarters. The survey includes questions on ownership of real estate, including subjective property valuation as well as principal balance outstanding on all mortgages and home equity credit lines that a specific property collateralizes. The household characteristics include information on homeownership as well as state of residence, although the state identifier is suppressed for several states. Data on households mortgage balance outstanding, which is necessary for constructing estimates of a household's housing equity, becomes available from 1988. However, household state identifiers are only available from 1993. Using this information for each unique homeowner in a given year I construct their housing equity E for their primary residence as $E = \frac{V-D}{V}$, where D is the total balance outstanding on all mortgages and home equity lines of credit that a property collateralizes and V is the value of the property. Note that I remove homeowners with $E < -2$, as those are homeowners that report either very low home values or do not report home values. Together with the state identifier for that homeowner, I can then construct an estimate for the fraction of homeowners housing in negative equity below a given cutoff c for each state for the given year. Multiplying by the homeownership rate for state s and year t , I get the estimated fraction of households with limited housing equity, $\widehat{neg}_{st}$, that I use in the reduced-form empirical analysis. I also use this data to construct estimates of average mortgage debt-to-value ratio $\frac{D}{V}$ by state and year.
3. Data on homeownership rates by state I obtain from the US Census Bureau's Housing Vacancies and Homeownership data.
4. Data on relative house prices I obtain by using the Freddie Mac House Price Index (FMHPI), formerly known as the Conventional Mortgage House Price Index (CMHPI) as well as the 2000 U.S. Census data on single family median home values by states and at the national level. In particular, the single family median house price in state s at time t , $p_{s,t} = p_{s,2000} \cdot \frac{FMHPI_{s,t}}{FMHPI_{s,2000}}$ and similarly for national house prices, p_t . Relative house price in state s and time t is then $\frac{p_{s,t}}{p_t}$.
5. Data on relative unemployment rate is constructed using data from the BLS LAUS database.
6. Data on relative wage rates is constructed using the BLS CES database. I use the average hourly manufacturing wage as it is the only wage series that spans my sample period 1993-2007.

⁵⁴There are 5 states that are not included because of missing observations on households housing wealth for all years between 1993 and 2008 -Mississippi, Montana, New Mexico, North Dakota, South Dakota, and Wyoming.

Table 11: Summary Statistics

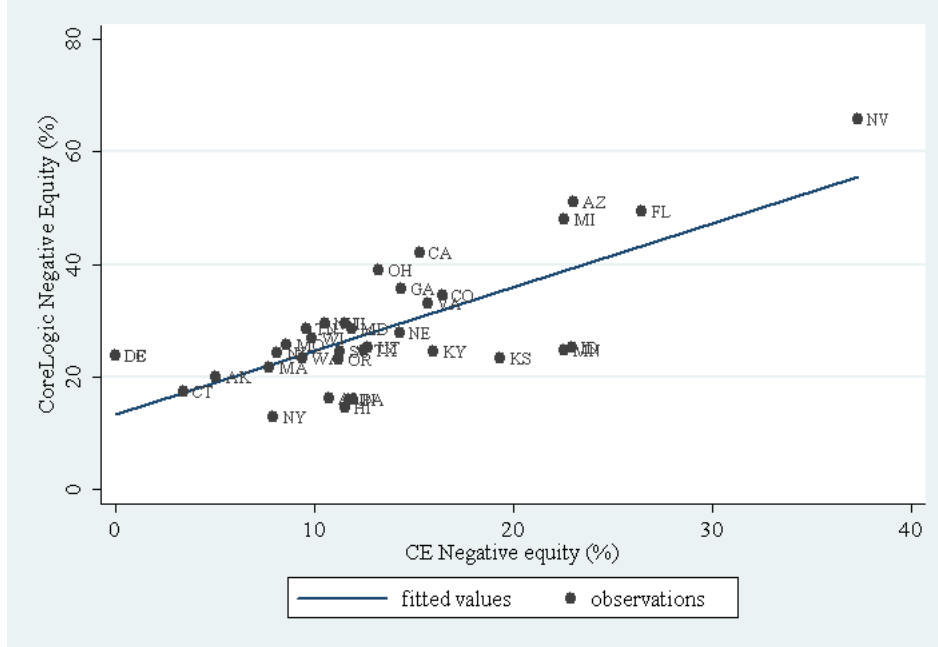
Variable	Observations	Mean	Std. Dev	Min	Max
out-migration rate (%)	625	3.494856	1.487469	1.851336	11.82
in-migration rate (%)	625	3.594642	1.647181	1.32	12.21448
households with equity < 0 (%)	625	3.230758	2.072744	0	15.19286
households with equity < -.05	625	2.689981	1.88775	0	15.19286
households with equity < .05	625	5.5426	3.053796	0	21.94524
relative house price	625	1.081658	0.439113	0.472	3.489
relative unemployment rate	625	0.951554	0.201718	0.492412	1.77963
relative wage rate	606	0.996155	0.106483	0.743845	1.34703
relative income	625	1.000815	0.153285	0.753323	1.698292
ave. debt-to-value ratio (%)	625	39.57029	7.779075	15.35	65.06
home ownership rate (%)	625	67.99328	6.86025	35.7	81.3
state population (0-14)	625	1411806	1435166	82140	7847912
state population (15-24)	625	936434.6	926224.5	57388	5509994
state population (25-34)	625	950746.8	980325.7	68702	5650931
state population (35-44)	625	1038544	1020040	80850	5592337
state population (45-64)	625	1506485	1432922	109978	8906679
state population (65+)	625	835971.2	792239.2	30980	4056084
median age	625	35.57696	2.167582	26.7	42

7. Data on relative income is constructed using annual data on state level income from the BEA and population data from the US Census.
8. Data on state demographics, including age distribution and median age come from the US Census. The 6 age bins are 0-14 years, 15-24 years, 25-34 years, 35-44 years, 45-64 years and 65+ years.

Addressing misclassification of households

As mentioned in Section 2, there may be potential misclassification problems of households with limited housing equity due to the use of subjective property value that would affect the estimates of neg_{st} (Ferreira, Gyourko, and Tracy (2010)). To address this, I compare my estimates of the fraction of households in negative equity (cutoff $c = 0$) by state with estimates from First American CoreLogic for 2009. The CoreLogic estimates are based on much more precisely measured house price data and hence are prone to much lower misclassification problems but are available for a much shorter time period (2008-2010) (see CoreLogic (2009) for details). Comparing the two series reveals a very high cross-sectional correlation of

Figure 3: Negative equity fractions from First American CoreLogic vs. CE Data



Notes: State level estimates from 2009 of fraction of mortgage holders in negative equity from Consumer Expenditure Survey versus First American CoreLogic. CE data is from own calculations. CoreLogic data is from CoreLogic (2009). Regression coefficient is 1.13 and intercept is 13.4. Correlation coefficient is $\rho = 0.731$.

$\rho = 0.731$. Hence, misclassification does not appear to be a problem for the state-level variation within a given year. Figure 3 plots the two series and a linear regression line. This also highlights the high comovement between the two series with any discrepancy likely due to classical measurement error. The only difference is that the CEX estimates predict lower values for all states

Addressing measurement error in $\widehat{neg}_{st}$

Since $\widehat{neg}_{st}$ is an estimate of the true fraction, neg_{st} , it contains estimation error. This implies that the relationship between neg_{st} and $\widehat{neg}_{st}$ can be represented by:

$$\widehat{neg}_{st} = neg_{st} + \nu_{st} \quad (25)$$

where ν_{st} is a random variable with mean zero and variance σ_ν^2 , distributed i.i.d. over s and t and independent of neg_{st} , $\forall s, t$. Here, σ_ν^2 is simply the sampling variance of $\widehat{neg}_{st}$. Therefore, I estimate σ_ν^2 by using an estimate of the sampling variance of $\widehat{neg}_{st}$.⁵⁵ More specifically, I construct estimates of the sampling variance for each observation $\widehat{neg}_{st}$, using a bootstrap procedure on the equity data for each state and year, and then take a weighted average of the observations to obtain $\widehat{\sigma}_\nu^2$. The weights I

⁵⁵Note that using information on the signal-to-noise ratio in a variable (reliability ratio) is one way to deal with errors-in-variables problems (Angrist and Krueger (1999)).

use are proportional to the number of observations for a given state and year, since an equally weighted average produces a sampling error estimate that is very high because of the large number of state-year observations with few observations of households.

The two-step GMM procedure I use is the following: In the first step I “partial out” all variables whose coefficients are not of particular interest (fixed effects and demographic variables). I then use the residuals for the dependent variable, $\tilde{Y}$, and remaining regressors, $\tilde{X}$, in a GMM estimator with moment conditions given by:

$$E[g(\tilde{x}_{st}, \tilde{y}_{st}, \beta, \Sigma_\nu)] = E\left[\tilde{y}_{st}\tilde{x}_{st} - \left(\tilde{x}_{st}\tilde{x}_{st}' - \Sigma_\nu\right)\beta\right] = 0 \quad (26)$$

where $\tilde{x}_{st}$ and β are column vectors of length equal to the number of regressors and Σ_ν is a covariance matrix for the measurement errors in the regressors, which in this case has only a single positive element of σ_ν^2 , corresponding to the measurement error variance of the regressor $\widehat{neg}_{st}$. It is important to note that partialling out in the presence of fixed effects leads to time-demeaned residuals. This necessitates a correction when computing consistent estimates of second order moments of the residuals. In particular, following Woodbridge (2002), Chapter 17, for the case of an unbalanced panel, the correction will be $\frac{\sum_s T_s - 1}{\sum_s T_s}$, where T_s is the number of observations in panel s . In the GMM estimation procedure, this correction is used in the moment condition above to multiply the measurement error variance Σ_ν . To see why this correction is needed, suppose that for the residuals, $\tilde{x}_{st}$ and $\tilde{y}_{st}$, from the “partialling-out” regression we have the relations $E[\tilde{x}_{st}\tilde{x}_{st}'] = E[\eta_{st}\eta_{st}'] + E[\nu_{st}\nu_{st}']$, where η_{st} are the latent regressors and ν_{st} is the vector of measurement errors, and $E[\tilde{y}_{st}\tilde{x}_{st}] = E[\eta_{st}\eta_{st}']\beta$. We have that the relationship between these residuals and the time-demeaned residuals, $\hat{x}_{st}$ and $\hat{y}_{st}$, is $E[\hat{x}_{st}\hat{x}_{st}'] = E[T_s - 1]E[\tilde{x}_{st}\tilde{x}_{st}] = E[T_s - 1]\left(E[\eta_{st}\eta_{st}'] + E[\nu_{st}\nu_{st}']\right)$ and $E[\hat{y}_{st}\hat{x}_{st}] = E[T_s - 1]E[\eta_{st}\eta_{st}']\beta$. Substituting for $E[\eta_{st}\eta_{st}']$ from the first relation into the second, we obtain that $E[\hat{y}_{st}\hat{x}_{st}] = \left(E[\hat{x}_{st}\hat{x}_{st}'] - E[T_s - 1]\Sigma_\nu\right)\beta$. Note that this correction is unnecessary when there is no errors-in-variables problem.

Data for the Beveridge curve regressions

I use the Conference Board Help Wanted Index and data from the BLS including the national unemployment rate (derived from the CPS), state and MSA unemployment rates and employment and labor force levels (derived from the Local Area Unemployment Statistics database), and the national vacancy rate (derived from JOLTS). Additionally I take the state unemployment dispersion data for the period 1960-1975 from Abraham (1987).

The Conference Board Help Wanted Index is the source for data on job vacancies prior to the introduction of JOLTS in December 2000. It tracks the number of help wanted advertisings in the newspapers of 51 cities, which are then aggregated into 9 Census Divisions and 1 National Index. There are several well known problems with using the HWI as a proxy for vacancies particularly for looking at shifts in the Beveridge curve. Most recently, there has been a downward secular trend in the HWI, which has been attributed to the more extensive use of online job advertising. To remedy this I construct a synthetic vacancy rate by regressing monthly vacancy data from JOLTS for the period December 2000 to December

2003 on data from the HWI and a constant, this is a period where the two series track each other well. I then use these coefficient estimates to construct a synthetic vacancy rate for the entire sample period, taking annual averages to get annual data for the period 1960 to 2000. For years 2001 to 2010 I use the JOLTS vacancy data. This approach, however, may be unsatisfactory since it does not take care of trends in the HWI in earlier years unrelated to the labor market Abraham (1987)), which get transferred directly to the synthetic vacancy index. To address this, I include a cubic trend in all regressions, which should account for these additional secular trends.

Table 12: State panel regressions.

Dep. Variable:	out-migration rate, 100 $\ln(out)$			in-migration rate, 100 $\ln(in)$		
relative unemployment 100 $\ln(\frac{u_s}{u})$	0.161*** (0.041)	0.159*** (0.040)	0.165*** (0.041)	-0.222*** (0.036)	-0.222*** (0.036)	-0.222*** (0.036)
relative house price 100 $\ln(\frac{p_s}{p})$	0.257*** (0.073)	0.257*** (0.074)	0.252*** (0.073)	-0.178*** (0.044)	-0.180*** (0.044)	-0.178*** (0.045)
households with equity < 0, (%)	-0.273** (0.126)			0.049 (0.185)		
households with equity < -0.05, (%)		-0.237* (0.118)			0.136 (0.208)	
households with equity < 0.05, (%)			-0.309** (0.126)			0.035 (0.134)
N	606	606	606	606	606	606

Notes: Robust standard errors with clustering on state in parenthesis; * = significant at 10%; ** = significant at 5%; *** = significant at 1%. Source: Own calculations from BLS, IRS, CE, FMHPI, and US Census Bureau. See Data Appendix for detailed description.

For the national unemployment rate data, I take annual averages of the monthly seasonally adjusted series. Similarly to the HWI there may be a secular trend in the unemployment rate because of compositional changes with the aging of the baby boomer generation (Shimer (1999)). Again a cubic trend in the Beveridge curve regression would account for such a trend as well.

Following Abraham (1987) and Lilien (1982), I construct employment-weighted state unemployment rate dispersion according to the formula: $\hat{\sigma}_t^x = \sqrt{\sum_{i=1}^n \frac{e_{i,t}}{e_t} (x_{i,t} - x_t)^2}$, where $e_{i,t}$ is employment in state i at time t , e_t is national employment at time t , $x_{i,t}$ is the realization in state i at time t and x_t is weighted average of state realizations weighted by employment weights. For the unemployment dispersion I use annual averages of monthly seasonally adjusted state unemployment rates as well as annual averages of seasonally adjusted monthly state employment levels. Finally, I construct MSA unemployment dispersion according to: $\hat{\sigma}_t^x = \sqrt{\sum_{i=1}^n \frac{e_{i,t}}{e_t} (x_{i,t} - x_t)^2}$ where $e_{i,t}$ is labor force in MSA i at time t , e_t is national labor

Table 13: Beveridge curve regressions with vacancy rate as dependent variable (a) and unemployment rate as dependent variable (b)

(a)				
Dep. Variable:	vacancy rate			
		v		$\ln(v)$
unemployment rate (u)	-0.447*** (0.0510)	-0.498*** (0.0380)	-0.965*** (0.170)	
MSA Unemployment Dispersion		0.533** (0.235)	0.528** (0.189)	0.0941 (0.0623)
u^2			0.0371** (0.0132)	
$\ln(u)$				-0.933*** (0.0664)
R^2	0.970	0.977	0.988	0.983

(b)				
Dep. Variable:	unemployment rate			
		u		$\ln(u)$
vacancy rate (v)	-2.036*** (0.165)	-1.841*** (0.0944)	-3.495** (1.201)	
MSA Unemployment Dispersion		1.287* (0.669)	0.954 (0.643)	0.135* (0.0680)
v^2			0.250 (0.173)	
$\ln(v)$				-1.001*** (0.0763)
R^2	0.970	0.982	0.984	0.983

Notes: Own calculations from BLS and Conference Board. Annual data from 1990-2010. Newey-West robust standard errors in parenthesis; * = significant at 10%; ** = significant at 5%; *** = significant at 1%. All regressions include a cubic time trend. v is national vacancy rate and u is national unemployment rate. Dispersion measure for variable x is defined as $\hat{\sigma}_t^x = \sqrt{\sum_{i=1}^n \frac{e_{i,t}}{e_t} (x_{i,t} - x_t)^2}$ where $e_{i,t}$ is labor force (employment) in MSA (sector) i at time t , e_t is national labor force (employment) at time t , $x_{i,t}$ is the realization in MSA/sector i at time t and x_t is weighted average of MSA (sector) realizations weighted by labor force (employment) weights.

force at time t , $x_{i,t}$ is the realization in MSA i at time t and x_t is weighted average of MSA realizations weighted by labor force weights.

Appendix B - Proofs and auxiliary results

This part of the appendix contains proofs of all results not contained in the body of the paper.

Law of motion for $\bar{\nu}_t$ and $\underline{\nu}_t$:

The laws of motion for the distributions $\bar{\nu}_t$ and $\underline{\nu}_t$, $\bar{\nu}_{t+1} = \bar{\Xi}(\bar{\nu}_t, \underline{\nu}_t)$, $\underline{\nu}_{t+1} = \Xi(\bar{\nu}_t, \underline{\nu}_t)$ are given by,

$$\begin{aligned} \bar{\nu}_{t+1}(l_0, l_1) &= \bar{\Xi}(\bar{\nu}_t, \underline{\nu}_t)(l_0, l_1) = \int \rho \cdot I \left\{ l_0, l_1 = l'_1(\bar{a}, l_0, l_{1,-1}) \right\} d\bar{\nu}_t(l_0, l_{1,-1}) + \\ &+ \int (1 - \rho) \cdot I \left\{ l_0, l_1 = l'_1(\underline{a}, l_0, l_{1,-1}) \right\} d\underline{\nu}_t(l_0, l_{1,-1}) \end{aligned} \quad (27)$$

and similarly,

$$\begin{aligned} \underline{\nu}_{t+1}(l_0, l_1) &= \Xi(\bar{\nu}_t, \underline{\nu}_t)(l_0, l_1) = \int \rho \cdot I \left\{ l_0, l_1 = l'_1(\underline{a}, l_0, l_{1,-1}) \right\} d\underline{\nu}_t(l_0, l_{1,-1}) + \\ &+ \int (1 - \rho) \cdot I \left\{ l_0, l_1 = l'_1(\bar{a}, l_0, l_{1,-1}) \right\} d\bar{\nu}_t(l_0, l_{1,-1}) \end{aligned} \quad (28)$$

Definition of stationary recursive equilibrium:

Definition 5. A symmetric stationary recursive equilibrium for the economy described above consists of market tightness $\theta(a)$, wages $w(a)$, worker value function $V^1(X)$, migration value $\bar{V}$, migration threshold $\bar{\epsilon}(X)$, regional house prices $p(X)$, regional rental rates $r(x)$, laws of motion, Γ , for X , and distributions $(\bar{\nu}^*, \underline{\nu}^*)$ such that:

1. $\theta(a)$ satisfies (2) given $w(a)$;
2. $w(a)$ satisfies (3) ;
3. $V^1(X)$ satisfies (7) given $\theta(a)$, $r(X)$, $\bar{V}$, and Γ ;
4. $\bar{V}$ satisfies (6) given $V^1(X)$;
5. $\bar{\epsilon}(X)$ satisfies (12) given $\bar{V}$ and $V^1(X)$;
6. $p(X)$ satisfies (4) given Γ ;

7. $r(X)$ satisfies (11) given Γ ;
8. Γ and satisfies 14 and law of motions for a given $\bar{\epsilon}(X)$;
9. $(\bar{\nu}^*, \underline{\nu}^*)$ are a fixed point of (27) and (28).
10. $\int l'_1(\bar{a}, l_1) d\bar{\nu}^* + \int l'_1(\underline{a}, l_1) d\underline{\nu}^* + l_0 = L$ (population constancy).
11. $(l_0 + l_1(X))h = H$ (rental market clearing)

Equilibrium characterization:

I make the following technical assumption:

Assumption: $\exists \bar{B} > 0$ s.t. $q(X) > 0, \forall X, \forall B \geq \bar{B}$.

This assumption means that I consider economies, in which there is gross out-migration out of any region.

Lemma 6. *Suppose that $l'_1(a, l)$ is a continuous function of l for $a \in \{\underline{a}, \bar{a}\}$. Then $V^1(a, l)$ is a bounded continuous function of l for $a \in \{\underline{a}, \bar{a}\}$. Conversely, suppose that $V^1(a, l)$ is a continuous function of l for $a \in \{\underline{a}, \bar{a}\}$. Then $l'_1(a, l)$ is a continuous function of l for $a \in \{\underline{a}, \bar{a}\}$.*

Proof. Let us define the operator

$$T[v(a, l)] = \max_{\{\bar{\epsilon}(a', l')\}_{a' \in \{\underline{a}, \bar{a}\}}} \left\{ \max_h \{v(h) - r(a, l) \cdot h\} + e + \mu((\theta(a))(w(a) - e) + \right. \\ \left. + \beta E_a \left[F_\epsilon(\bar{\epsilon}(a', l')) \bar{V} + (1 - F_\epsilon(\bar{\epsilon}(a', l'))) v(a', l') - F_\epsilon(\bar{\epsilon}(a', l')) c + \int_{\bar{\epsilon}(a', l')} \epsilon dF_\epsilon \right] \right\}$$

with $l' = l'_1(a, l)$ and a given $\bar{V}$. Note that $r(A, l) = v'(\frac{H}{l_0 + l'})$ is a continuous function of l' by assumptions on $v'()$ and $l'_1(A, l)$ is also a continuous function of l . Both are also clearly bounded. Hence T maps bounded continuous functions into bounded continuous functions. Furthermore, T satisfies Blackwell's sufficient conditions for a contraction. Hence, by the Contraction Mapping Theorem (Stokey and Lucas (1989)), T has a unique fixed point in the space of bounded continuous functions. Hence, $V^1(a, l)$, is a bounded continuous function.

Suppose now that $V^1(a, l)$ is continuous in l . Given this it follows immediately that $r(A, l)$ must be continuous in l and since $r = r(A, l) = v'(\frac{H}{l_0 + l'})$ is continuous in l' , it follows that l'_1 must be continuous in l . □

As mentioned in Section 4.3, I focus on equilibria, in which both $V^1(a, l)$ and $l'_1(a, l)$ are continuous. Given continuity of $V^1(a, l_1)$ and the compactness of the domain of X it follows immediately that the set $\arg \max_x \{V(x)\}$ is nonempty. I then have the following result.

Lemma 7. $V^1(a, l)$ is a non-increasing function of l , and $l'_1(a, l)$ is a non-decreasing function of l .

Proof. The basic idea behind showing this result is showing that, if $V^1(a, l)$ is a non-increasing function of l , then $l'_1(a, l)$ is a non-decreasing function of l and vice versa, and then showing that it is impossible for l'_1 to be strictly decreasing in l if $V^1(a, l)$ is strictly increasing in l , i.e. the only property that is mutually consistent and hence possible in equilibrium is for $V^1(a, l)$ to be a non-increasing function of l and l'_1 to be a non-decreasing function of l . I proceed to show this in three steps.

Step 1: I show that if $l'_1(a, l)$ is a non-decreasing function of l then $V^1(a, l)$, $a \in \{\underline{a}, \bar{a}\}$ is a non-increasing function of l . Define again the operator

$$T[v(a, l)] = \max_h \{v(h) - r(a, l) \cdot h\} + e + \mu((\theta(a))(w(a) - e) + \beta E_a [\omega(a', l')])$$

where

$$\omega(a, l) = \max_{\bar{\epsilon}} \left\{ F_{\epsilon}(\bar{\epsilon}) \bar{V} + (1 - F_{\epsilon}(\bar{\epsilon})) v(a, l) - F_{\epsilon}(\bar{\epsilon}) c_1 + \int_{\bar{\epsilon}} \epsilon dF_{\epsilon} \right\}$$

We have that l' is non-decreasing in l and hence $-r(a, l)$ is non-decreasing in l . Furthermore, for $l_1 < l_2$:

$$\begin{aligned} \omega(a, l_1) - \omega(a, l_2) &= \\ &= (1 - F_{\epsilon}(\bar{\epsilon}(a, l_1))) v(a, l_1) + \int_{\bar{\epsilon}(a, l_1)} \epsilon dF_{\epsilon} + F_{\epsilon}(\bar{\epsilon}(a, l_1)) (\bar{V} - c_1) - \\ &- (1 - F_{\epsilon}(\bar{\epsilon}(a, l_2))) v(a, l_2) + \int_{\bar{\epsilon}(a, l_2)} \epsilon dF_{\epsilon} + F_{\epsilon}(\bar{\epsilon}(a, l_2)) (\bar{V} - c_1) \geq \\ &\geq (1 - F_{\epsilon}(\bar{\epsilon}(a, l_2))) v(a, l_1) + \int_{\bar{\epsilon}(a, l_2)} \epsilon dF_{\epsilon} + F_{\epsilon}(\bar{\epsilon}(a, l_2)) (\bar{V} - c_1) - \\ &- (1 - F_{\epsilon}(\bar{\epsilon}(a, l_2))) v(a, l_2) + \int_{\bar{\epsilon}(a, l_2)} \epsilon dF_{\epsilon} + F_{\epsilon}(\bar{\epsilon}(a, l_2)) (\bar{V} - c_1) \\ &\geq (1 - F_{\epsilon}(\bar{\epsilon}(a, l_2))) (v(a, l_1) - v(a, l_2)) \end{aligned}$$

where the inequality comes from the fact that a different cutoff from the optimal, $\bar{\epsilon}(a, l_1)$, is used. Hence, if $v(a, l)$ is non-increasing in l then so is $\omega(a, l)$. Therefore, $\omega(a', l')$ is non-increasing in l . Then it follows that $T[V(a, l)]$ is non-increasing in l . Hence, T maps non-increasing functions to non-increasing functions. Therefore, its fixed point, $V^1(a, l)$, is non-increasing in l .

Step 2: I show that if $V^1(a, l)$, $a \in \{\underline{a}, \bar{a}\}$ is a non-increasing function of l then $l'_1(a, l)$ is a non-decreasing function of l . Suppose that $V(a, l)$ is non-increasing in l but $\exists l_1, l_2$ with $l_1 < l_2$ s.t. $l'_1 = l'_1(a, l_1) > l'_1(a, l_2) = l'_2$. Then

$$0 \leq V^1(a, l_1) - V^1(a, l_2) = -r(a, l_1) + r(a, l_2) + \beta E_a [W^1(a', l'_1) - W^1(a', l'_2)] < 0$$

since $-r(A, l_1) + r(A, l_2) < 0$ given that $l'_1 > l'_2$ and $E_a \left[W^1(a', l'_1) - W^1(a', l'_2) \right] < 0$ since $V^1(a, l)$ is non-increasing in l and $l'_1 > l'_2$. Hence, we have a contradiction, which is due to the assumption that $\exists l_1, l_2$ with $l_1 < l_2$ s.t. $l'_1 = l'_1(a, l_1) > l'_1(a, l_2) = l'_2$.

Step 3: I show that if $V^1(a, l)$ is strictly increasing in l then l'_1 is increasing in l . Let $l_1 < l_2$. Then $V(a, l_1) < V(a, l_2)$, $a \in \{\underline{a}, \bar{a}\}$ and so $\bar{\epsilon}(a, l_1) > \bar{\epsilon}(a, l_2)$. Hence, $l'_1(a, l_1) = (1 - F(\bar{\epsilon}(a, l_1)))l_1 + \Psi(a, l_1)$ and $l'_1(a, l_2) = (1 - F(\bar{\epsilon}(a, l_2)))l_2 + \Psi(a, l_2)$. Now, observe that $\Psi(a, l_1) \leq \Psi(a, l_2)$ and $(1 - F(\bar{\epsilon}(a, l_1)))l_1 < (1 - F(\bar{\epsilon}(a, l_2)))l_2$, which implies that $l'_1(a, l_1) < l'_1(a, l_2)$.

Steps 1 and 2 show that it is mutually consistent for V^1 and l'_1 to be non-increasing and non-decreasing in l , respectively. However, Step 3 shows that it is not mutually consistent for V^1 and l'_1 to be strictly increasing and strictly decreasing in l , respectively. Hence, in any equilibrium V^1 and l'_1 are non-increasing and non-decreasing, respectively. □

Next, I define:

$$\bar{l} = \begin{cases} \sup_l \left\{ l : (\bar{a}, l) \in \arg \max_{\bar{l}} \left\{ V^1(a, \bar{l}) \right\} \right\} & , \left\{ l : (\bar{a}, l) \in \arg \max_{\bar{l}} \left\{ V^1(a, \bar{l}) \right\} \right\} \neq \emptyset \\ 0 & , o.w. \end{cases} \quad (29)$$

and also $\bar{l}_{min}$ implicitly by:

$$(1 - q(\bar{a}, \bar{l}_{min})) \bar{l}_{min} + (1 - \lambda) \cdot M = \bar{l}_{min} \quad (30)$$

with

$$M = \int q(\bar{a}, l_{0,t-1}, l_{1,t-1}) l_{1,t-1} d\nu^* + \int q(\underline{a}, l_{0,t-1}, l_{1,t-1}) l_{1,t-1} d\nu^* \quad (31)$$

the average out-migration from regions in the economy, and similarly

$$\underline{l} = \begin{cases} \sup_l \left\{ l : (\underline{a}, l) \in \arg \max_{\underline{l}} \left\{ V^1(a, \underline{l}) \right\} \right\} & , \left\{ l : (\underline{a}, l) \in \arg \max_{\underline{l}} \left\{ V^1(a, \underline{l}) \right\} \right\} \neq \emptyset \\ 0 & , o.w. \end{cases} \quad (32)$$

and $\underline{l}_{min}$, defined similarly to $\bar{l}_{min}$. Note that $\bar{l}_{min}$ and $\underline{l}_{min}$ are unique.⁵⁶ Therefore, regions with populations of mobile workers above $\bar{l}$ and $\underline{l}$ can only attract workers that migrate in a non-directed way. Consider the case of $\bar{a}$ -regions (the case of $\underline{a}$ -regions is analogous). We have that net migration into regions with $l \geq \bar{l}$ is given by

$$n(a, l) = -q(a, l) \cdot l + (1 - \lambda) \cdot M \quad (33)$$

There are two possibilities for $n(a, \bar{l})$ depending on parameters. In the first case, $n(a, \bar{l}) \leq 0$ and so $\bar{l} \geq \bar{l}_{min}$. In the second, $n(a, \bar{l}) > 0$ and $\bar{l} < \bar{l}_{min}$. For the first case, it follows that $\bar{a}$ -regions with $l > \bar{l}$ must lose population in equilibrium, i.e. $l'_1(\bar{a}, l) < l$, for $l > \bar{l}$, which means that the function $l'_1(\bar{a}, l)$ has

⁵⁶These bounds always exist, given that $l_1 \leq L$.

a fixed point, which is unique whenever there are potential house price differences across regions, that is when $v(\cdot)$ is strictly concave. In the second case $l'_1(\bar{a}, l)$ also has a unique fixed point given by $\bar{l}_{min}$. I summarize these observations in the following result:

Lemma 8. *There exist $\bar{l}^*$ and $\underline{l}^*$ such that $l'_1(\bar{a}, \bar{l}^*) = \bar{l}^*$ and $l'_1(\underline{a}, \underline{l}^*) = \underline{l}^*$. They are unique if $v(\cdot)$ is strictly concave. Furthermore, $l'_1(\bar{a}, l)$ is constant for $l \in [0, \bar{l}]$ and $l'_1(\underline{a}, l)$ is constant for $l \in [0, \underline{l}]$ with $l'_1(\bar{a}, l) = \bar{l}^*$ and $l'_1(\underline{a}, l) = \underline{l}^*$.*

Proof. I show this for $a = \bar{a}$ since the other case is analogous. Consider first the case of $n(\bar{a}, \bar{l}) \leq 0$. Note that if $\{l : (\bar{a}, l) \in \arg \max_{\bar{l}} \{V^1(\bar{a}, \bar{l})\}\} \neq \emptyset$, then $\bar{l} \in \{l : (\bar{a}, l) \in \arg \max_{\bar{l}} \{V^1(\bar{a}, \bar{l})\}\}$ since $V^1(\bar{a}, l)$ is continuous in l . Hence, $\{l : (\bar{a}, l) \in \arg \max_{\bar{l}} \{V^1(\bar{a}, \bar{l})\}\}$ is a compact set. Now, l'_1 is continuous. Hence, by Brouwer's fixed point theorem, there exists a $\bar{l}^*$ such that $l'_1(\bar{a}, \bar{l}^*) = \bar{l}^*$ and $\bar{l}^* \leq \bar{l}$. If $\{l : (\bar{a}, l) \in \arg \max_{\bar{l}} \{V^1(\bar{a}, \bar{l})\}\} = \emptyset$ then $\bar{l} = 0$, and by the definition of $\bar{l}_{min}$, for $\bar{l}^* = \bar{l}_{min}$, $l'_1(\bar{a}, \bar{l}^*) = \bar{l}^*$. Noting that for $l > \bar{l}$, $l'_1(\bar{a}, l) < l$, it follows that no fixed point of $l'_1(\bar{a}, l)$ can be greater than $\bar{l}$.

It is straightforward to show that for $v(\cdot)$ strictly concave, $l'_1(\bar{a}, l)$ is constant for $l \in [0, \bar{l}]$ by using Lemma 7 and proceeding by contradiction. In particular, suppose that $l'_1(\bar{a}, l_1) > l'_1(\bar{a}, l_2)$ for some l_1, l_2 , such that $\bar{l} \geq l_1 > l_2$. However, this implies that $V^1(\bar{a}, l_1) < V^1(\bar{a}, l_2)$ and so $l_1 > \bar{l}$. This also implies that the fixed point has to be unique.

Consider the case of $n(\bar{a}, \bar{l}) > 0$. l'_1 is continuous and the set of populations $[0, \bar{L}]$ is compact, so l'_1 has a fixed point. One such fixed point is $\bar{l}^* = \bar{l}_{min}$, which is unique in the set $[\bar{l}, \bar{L}]$. Given that $n(\bar{a}, \bar{l}) > 0$ and the constancy of l'_1 for $l \leq \bar{l}$, it follows that l'_1 does not have a fixed point in the set $[0, \bar{l}]$ and so the fixed point is unique. □

Lemma 8 describes how $\Psi(a, l)$ looks like in equilibrium. In particular, it follows that

$$\Psi(\bar{a}, l) = \begin{cases} \tilde{l}(\bar{a}) - l + q(\bar{a}, l) \cdot l & , l < \bar{l} \\ (1 - \lambda) \cdot M & , o.w. \end{cases} \quad (34)$$

where $M = \int q(\bar{a}, l_{0,t-1}, l_{1,t-1}) l_{1,t-1} d\bar{\nu}^* + \int q(\underline{a}, l_{0,t-1}, l_{1,t-1}) l_{1,t-1} d\underline{\nu}^*$ with

$$\tilde{l}(\bar{a}) = (1 - q(\bar{a}, \bar{l})) \bar{l} + (1 - \lambda) \cdot M \quad (35)$$

and similarly for $\Psi(\underline{a}, l)$. Note also that $\tilde{l}(\bar{a}) = \bar{l}^*$ if $n(\bar{a}, \bar{l}) \leq 0$. Lastly, I compare mobile workers' value function for regions with the same populations but different productivities, a :

Lemma 9. $V^1(\bar{a}, l) \geq V^1(\underline{a}, l)$, $\forall l$

Proof. Suppose that $V^1(\underline{a}, l) > V^1(\bar{a}, l)$ for some l . Hence, $(\bar{a}, l) \notin \arg \max \{V^1(a, l)\}$ and hence by

equation Equation 14, $l'_1(\bar{a}, l) < l'_1(\underline{a}, l)$. Then,

$$\begin{aligned} V^1(\bar{a}, l) - V^1(\underline{a}, l) &= -r(\bar{a}, l) + d(\underline{a}, l) + \mu(\theta(\bar{a}))(w(\bar{a}) - e) - \\ &\quad - \mu(\theta(\underline{a}))(w(\underline{a}) - e) + \beta E[W^1(a', l') | (\bar{a}, l)] - \beta E[W^1(a', l') | (\underline{a}, l)] \end{aligned}$$

However, given that $l'_1(\bar{a}, l) < l'_1(\underline{a}, l)$, it follows that $W^1(a, l'_1(\bar{a}, l)) > W^1(a, l'_1(\underline{a}, l))$ for $a \in \{\underline{a}, \bar{a}\}$ from Lemma 7. Now, if $W^1(\bar{a}, l'_1(\bar{a}, l)) \geq W^1(\underline{a}, l'_1(\bar{a}, l))$ or $W^1(\bar{a}, l'_1(\underline{a}, l)) \geq W^1(\underline{a}, l'_1(\underline{a}, l))$, then $\beta E[W^1(a', l') | (\bar{a}, l)] - \beta E[W^1(a', l') | (\underline{a}, l)] > 0$ and we have a contradiction. Hence,

$$W^1(\bar{a}, l'_1(\bar{a}, l)) < W^1(\underline{a}, l'_1(\bar{a}, l))$$

and $W^1(\bar{a}, l'_1(\underline{a}, l)) < W^1(\underline{a}, l'_1(\underline{a}, l))$. If $W^1(\bar{a}, l'_1(\bar{a}, l)) \geq W^1(\underline{a}, l'_1(\underline{a}, l))$, then, again we will arrive at a contradiction. Hence, $W^1(\bar{a}, l'_1(\bar{a}, l)) < W^1(\underline{a}, l'_1(\underline{a}, l))$, which implies that $V^1(\bar{a}, l'_1(\bar{a}, l)) < V^1(\underline{a}, l'_1(\underline{a}, l))$.

Now, define, $\bar{l}^0 = l$, $\bar{l}^1 = l'_1(\bar{a}, \bar{l}^0)$, $\bar{l}^2 = l'_1(\bar{a}, \bar{l}^1)$, etc. and similarly $\underline{l}^0 = l$, $\underline{l}^1 = l'_1(\underline{a}, \underline{l}^0)$, $\underline{l}^2 = l'_1(\underline{a}, \underline{l}^1)$, etc. Hence, by induction we can show that $V(\bar{a}, \bar{l}^i) < V(\underline{a}, \underline{l}^i)$ for $i = 1, 2, \dots$. Now, clearly each sequence is either decreasing and bounded below by $\bar{l}^*$ and $\underline{l}^*$, respectively, or increasing and bounded above by $\bar{l}^*$ and $\underline{l}^*$, respectively. Hence, by continuity of $V^1(a, l)$ and from Lemma 8 it follows that the sequences converge to the unique fixed points of $l'_1(\bar{a}, l)$ and $l'_1(\underline{a}, l)$, $\bar{l}^*$ and $\underline{l}^*$, respectively and by continuity of V^1 then $V^1(\bar{a}, \bar{l}^*) < V^1(\underline{a}, \underline{l}^*)$, which is only possible if the set $\{l : (\bar{a}, l) \in \arg \max_{\bar{l}} \{V^1(\bar{a}, \bar{l})\}\}$ is empty, in which case $\bar{l}^* = \bar{l}_{min}$ and so $V^1(\bar{a}, \bar{l}_{min}) < V^1(\underline{a}, \underline{l})$. However, note that

$$\begin{aligned} V^1(\bar{a}, \bar{l}_{min}) &= \max_h \{v(h) - r(\bar{a}, \bar{l}_{min}) \cdot h\} + e + (\mu(\theta(\bar{a}))(w(\bar{a}) - e) + \beta E[W^1(a', 0) | \bar{a}]) \geq \\ &\geq \max_h \{v(h) - r(\bar{a}, \bar{l}_{min}) \cdot h\} + e + (\mu(\theta(\bar{a}))(w(\bar{a}) - e) + \\ &\quad \beta \cdot \left(V^1(\bar{a}, \bar{l}_{min}) + F_\epsilon(\bar{\epsilon}(\underline{a}, l)) \cdot \bar{\epsilon}(\underline{a}, l) + \int_{\bar{\epsilon}(\underline{a}, l)} \epsilon dF_\epsilon \right)) = \\ &\max_h \{v(h) - r(\bar{a}, \bar{l}_{min}) \cdot h\} + e + (\mu(\theta(\bar{a}))(w(\bar{a}) - e) + \\ &+ \beta \left(\max_h \{v(h) - r(\bar{a}, \bar{l}_{min}) \cdot h\} + e + (\mu(\theta(\bar{a}))(w(\bar{a}) - e) + \right. \\ &\quad \left. + \beta E[W^1(a', 0) | \bar{a}] + F_\epsilon(\bar{\epsilon}(\underline{a}, l)) \cdot \bar{\epsilon}(\underline{a}, l) + \int_{\bar{\epsilon}(\underline{a}, l)} \epsilon dF_\epsilon \right)) \geq \dots \geq \\ &\geq \sum_{j=0}^{\infty} \beta^j \left(\max_h \{v(h) - r(\bar{a}, \bar{l}_{min}) \cdot h\} + e + (\mu(\theta(\bar{a}))(w(\bar{a}) - e) \right) \\ &\quad + \sum_{j=1}^{\infty} \beta^j \left(F_\epsilon(\bar{\epsilon}(\underline{a}, l)) \cdot \bar{\epsilon}(\underline{a}, l) + \int_{\bar{\epsilon}(\underline{a}, l)} \epsilon dF_\epsilon \right) \end{aligned}$$

Similarly, one can show that

$$\begin{aligned}
V^1(\underline{a}, \underline{l}) &= \max_h \{v(h) - r(\underline{a}, \underline{l}) \cdot h\} + e + (\mu(\theta(\underline{a}))) (w(\underline{a}) - e) + \beta E[W^1(a', l) | \underline{a}] \leq \\
&\leq \max_h \{v(h) - r(\underline{a}, \underline{l}) \cdot h\} + e + (\mu(\theta(\underline{a}))) (w(\underline{a}) - e) + \\
&\quad + \beta \cdot \left(V^1(\underline{a}, \underline{l}) + F_\epsilon(\bar{\epsilon}(\underline{a}, l)) \cdot \bar{\epsilon}(\underline{a}, l) + \int_{\bar{\epsilon}(\underline{a}, l)} \epsilon dF_\epsilon \right) = \\
&\quad \max_h \{v(h) - r(\underline{a}, \underline{l}) \cdot h\} + e + (\mu(\theta(\underline{A}))) (w(\underline{A}) - e) + \\
&\quad + \beta \left(\max_h \{v(h) - r(\underline{a}, \underline{l}) \cdot h\} + e + (\mu(\theta(\underline{a}))) (w(\underline{a}) - e) + \right. \\
&\quad \left. + \beta E[W^1(a', l) | \underline{a}] + F_\epsilon(\bar{\epsilon}(\underline{a}, l)) \cdot \bar{\epsilon}(\underline{a}, l) + \int_{\bar{\epsilon}(\underline{a}, l)} \epsilon dF_\epsilon \right) \leq \dots \leq \\
&\leq \sum_{j=0}^{\infty} \beta^j \left(\max_h \{v(h) - r(\underline{a}, \underline{l}) \cdot h\} + e + (\mu(\theta(\underline{a}))) (w(\underline{a}) - e) \right) \\
&\quad + \sum_{j=1}^{\infty} \beta^j \left(F_\epsilon(\bar{\epsilon}(\underline{a}, l)) \cdot \bar{\epsilon}(\underline{a}, l) + \int_{\bar{\epsilon}(\underline{a}, l)} \epsilon dF_\epsilon \right)
\end{aligned}$$

Hence, $V^1(\underline{a}, \underline{l}) > V^1(\bar{a}, \bar{l}_{min})$ implies that

$$\begin{aligned}
&\sum_{j=0}^{\infty} \beta^j \left(\max_h \{v(h) - r(\underline{a}, \underline{l}) \cdot h\} + e + (\mu(\theta(\underline{a}))) (w(\underline{a}) - e) \right) > \\
&> \sum_{j=0}^{\infty} \beta^j \left(\max_h \{v(h) - r(\bar{a}, \bar{l}_{min}) \cdot h\} + e + (\mu(\theta(\bar{a}))) (w(\bar{a}) - e) \right)
\end{aligned}$$

Observing that $\bar{l}_{min} = \frac{(1-\lambda)M}{q(\underline{a}, \bar{l}_{min})} < \frac{(1-\lambda)M}{q(\underline{a}, \underline{l})} = \underline{l}$, we arrive at a contradiction since $r(\underline{a}, \underline{l}) > r(\bar{a}, 0)$. $\square$

Proof of Lemma 1

Results 1 through 3 follow directly from the implication of Lemmas 7, 8, and 9 for the law of motion for l'_1 . To show result 4, Lemma 8 implies that regions with productivity $\bar{a}$ move to a population of mobile workers of $\tilde{l}(\bar{a})$ and then move deterministically through a countably infinite set $\bar{\mathcal{L}}$. Furthermore, Lemma 9 implies that regions with $\bar{l}^*$ that experience a sequence of negative productivity shocks always move deterministically through a set $\underline{\mathcal{L}}$ of population levels, which is countably infinite in the case equilibria with $\bar{l}^* = l_{min}$ or finite in the case of equilibria with $\bar{l}^* > l_{min}$ since out-migration rates are always bounded below by $F(\bar{\epsilon}(\bar{a}, \bar{l})) > 0$ (the natural out-migration rate from regions $X \in \arg \max_{\tilde{X}} \{V^1(\tilde{X})\}$). Since regions with populations $l \notin \underline{\mathcal{L}} \cup \bar{\mathcal{L}}$ move to $\tilde{l}(\bar{a})$ with probability 1 but conditional on being in $\tilde{l}(\bar{a})$ they never move to population states outside $\underline{\mathcal{L}} \cup \bar{\mathcal{L}}$, it follows that all states $l \notin \underline{\mathcal{L}} \cup \bar{\mathcal{L}}$ are transient, i.e.

the ergodic set of the process governing regional evolutions is given by $\underline{\mathcal{L}} \cup \overline{\mathcal{L}}$. Therefore, the stationary distribution of populations across regions $\underline{\nu}^*$ and $\overline{\nu}^*$ are given by the set $\underline{\mathcal{L}} \cup \overline{\mathcal{L}}$.

Proof of Proposition 2

We have that $out(a, l) = \frac{q(a, l) \cdot l}{l + l_0}$ and similarly $in(a, l) = \frac{\Psi(a, l)}{l + l_0}$. To show Result 1, first of all, Lemma 9 implies that $\overline{V} - V^1(\bar{a}, l) \leq \overline{V} - V^1(\underline{a}, l)$, $\forall l$ and hence, $\bar{\epsilon}(\bar{a}, l) \leq \bar{\epsilon}(\underline{a}, l)$, which immediately implies that $out(\bar{a}, l) \leq out(\underline{a}, l)$. This inequality is of course strict for $l \in [\underline{l}, \bar{l}^*]$. Similarly, from (34) it follows that $in(\bar{a}, l) \geq in(\underline{a}, l)$. To show Result 2, Lemma 7 implies that $q(a, l)$ is increasing in l and strictly so for $a = \underline{a}$, which immediately implies that $out(a, l)$ is increasing in l . Turning to $in(a, l)$, note that $\Psi(a, l)$ is decreasing in l and hence so will $\frac{\Psi(a, l)}{l + l_0}$.

Proof of Proposition 3

We have that $p(a, l) = r(a, l) + \beta \cdot E_a[p(a', l'_1(a, l))]$. First of all note that $r(a, l) = v' \left(\frac{H}{l_0 + l'_1(a, l)} \right)$ is continuous in l by the continuity of $l'_1(a, l)$ and $v'(\cdot)$. Furthermore, it is bounded and increasing in (a, l) since $l'_1(a, l)$ is increasing in (a, l) by Lemma 7 and equation (34) and $v'(\cdot)$ is decreasing. Now, clearly the operator $T[f(a, l)] = v(a, l) + \beta \cdot E_a[f(a', l'_1(a, l))]$ maps bounded continuous functions into bounded continuous functions and, furthermore, satisfies Blackwell's sufficient conditions for a contraction.. Hence, by the Contraction Mapping Theorem, T has a unique fixed point in the space of bounded continuous functions. Therefore, since $r(a, l)$ and $l'_1(a, l)$ are increasing in (a, l) it follows that T maps increasing functions into increasing functions and therefore its unique fixed point is increasing.

To show the second part of the proposition, first of all, note that $\tilde{p}(a, \bar{l}^*) = p(\bar{a}, \bar{l}^*)$ and $\tilde{p}(a, \underline{l}^*) = p(\underline{a}, \underline{l}^*)$. Furthermore, $\tilde{p}(a, l) = p(\underline{a}, l'^{-1}_1(\underline{a}, l))$ for $l \in (\underline{l}^*, \bar{l}^*)$ given equation (34) and the inverse l'^{-1}_1 is well-defined. Furthermore, given that $l'_1(a, l)$ is increasing in l it follows that $l'^{-1}_1(\underline{a}, l)$ is increasing in l as well. Hence, by the properties of p it follows that $p(\underline{a}, l'^{-1}_1(\underline{a}, l))$ is increasing in l for $l \in (\underline{l}^*, \bar{l}^*)$ and hence so is $\tilde{p}(a, l)$. Now, noting that $p(\bar{a}, \bar{l}^*) \geq p(a, l)$ $l < \bar{l}^*$, it follows that $\tilde{p}(a, l)$ is increasing in l for $l \in [\underline{l}^*, \bar{l}^*]$.

Appendix C - Computational

Model for calibration

In this section I provide a description of the differences between the model used for estimation in Section 5 and the basic model from Section 3. I only include model assumptions that differ across the two models.

Regional labor markets

As before, I consider a discrete time economy with infinite number of periods $t = 0, 1, 2, \dots$. The economy consists of a unit measure of islands or regions. The economy is populated by a measure L of infinitely lived workers, residing in different regions who are risk neutral and derive utility from consumption as well as from housing services. Workers can supply 1 unit of labor and have a separable flow utility over consumption and housing services.

In each region there is a representative firm that can open job vacancies at a per-period cost of k and recruit workers. Each job in region j has the capacity for the production of a_t^j units of the consumption good at time t if it can hire a worker. Regional productivity a_t^j has bounded support over $[\underline{a}, \bar{a}]$ and follows a persistent stochastic process with persistence parameter ρ . In particular,

$$a_t^j = \begin{cases} a_{t-1}^j & , \rho \\ \max \left\{ \underline{a}, \min \left\{ \bar{a}, A + \eta_t^j \right\} \right\} & , 1 - \rho \end{cases} \quad (36)$$

where η_t^j is distributed i.i.d. with distribution function F_η , and with $E[\eta] = 0$.

The labor market of each region is characterized by a search and matching friction as in the standard Diamond-Mortensen-Pissarides framework (Pissarides (2000)). In particular, in a region j at time t , after migration, a measure u_t^j of workers and measure v_t^j of vacancies try to match with each other. Matching is described by a standard regional reduced-form constant returns to scale (CRS) matching function $m^j(u_t^j, v_t^j)$ with standard properties giving the total number of regional matches per period. I assume that matching functions are identical across regions, i.e. $m^j(u_t^j, v_t^j) = m(u_t^j, v_t^j) = u_t^j \cdot m(1, \frac{v_t^j}{u_t^j})$. Letting $\theta_t^j = \frac{v_t^j}{u_t^j}$ be the regional labor market tightness and defining $\mu(\theta) = m(1, \theta)$, we have that $m(u_t^j, v_t^j) = u_t^j \mu(\theta_t^j)$. This translates into a job finding probability for a worker in a given period of $\mu(\theta_t^j)$ and a job filling probability for a vacancy of $\frac{\mu(\theta_t^j)}{\theta_t^j}$. Additionally, I assume that $\lim_{\theta \rightarrow 0} \frac{\mu(\theta)}{\theta} = 1$ and $\lim_{\theta \rightarrow \infty} \frac{\mu(\theta)}{\theta} = 0$. Workers that are matched obtain wage w remain unmatched after the matching phase in a given period receive a period payoff of e .

Job creation decisions

Let $J(a)$ be the value from posting a vacant job in a region with productivity a . Then

$$J(a) = -k + \frac{\mu(\theta(a))}{\theta(a)} K(a) + \left(1 - \frac{\mu(\theta(a))}{\theta(a)}\right) \beta(1-s) E[J(a')] \quad (37)$$

Similarly, let $K(a)$ be the value from a matched job. Then

$$K(a) = a - w + \beta(1-s) \cdot E[K(a')] \quad (38)$$

where w is the wage rate paid. The firm is owned by the agents in the economy and discounts payoffs at

the discount rate of workers β . The firm opens vacant jobs until the cost of opening a vacancy equals the expected payoff from a filled vacancy, or:

$$J(a) = 0 = -k + \frac{\mu(\theta(a))}{\theta(a)} K(a)$$

or

$$K(a) = k \frac{\theta(a)}{\mu(\theta(a))}$$

Then, substituting in for $K(a)$, we get that $\theta(a)$ satisfies the equation:

$$\frac{\theta(a)}{\mu(\theta(a))} = \frac{a - w}{k} + \beta(1 - s)E \left[\frac{\theta(a')}{\mu(\theta(a'))} \right] \quad (39)$$

Regional housing market and migration decisions

The set-up for regional housing markets and worker migration decisions are identical as with the basic model with the only difference that only unemployed workers experience idiosyncratic region preference shocks. The migration decisions of employed workers were extensively described in Section 5.

Regional state variables

In contrast to the one-period job model, in this model each region j will be fully characterized by its current period productivity a_t^j plus the beginning-of-period measure of workers with different mobility costs $l_{h,t-1}^j$, $h \in \{0, 1\}$, and the beginning-of-period measure of unemployed workers, $u_{h,t-1}^j$, $h \in \{0, 1\}$. The four variables $X = (a_t^j, l_{0,t-1}^j, l_{1,t-1}^j, u_{1,t-1}^j)$ are relevant for the worker's problem, while $u_{0,t-1}^j$ is only relevant for determining regional and aggregate unemployment. Also, I denote by $\tilde{u}_{h,t}^j$ the post-migration measure of unemployed workers in region j , while $\tilde{l}_{h,t}^j$ is the end-of-period labor force in region j . The invariant distribution over regional characteristics X is denoted by ν^* .

Worker value functions

I define $V_U^1(X)$ to be the value function of an unemployed worker with low mobility costs. Similarly an employed worker's value function is given by $V_E^1(X)$. I do not show the value function of workers with high mobility costs as those are analogous. Then, letting

$$\bar{V}_U = \lambda \cdot \max_{\tilde{x}} \{V_U^1(\tilde{x})\} + (1 - \lambda) \int V_U^1(X) d\nu^* \quad (40)$$

$$\bar{V}_E = \int \frac{(l_1 - u_1)}{\int (l_1 - u_1) d\nu^*} V_E^1(X) d\nu^* \quad (41)$$

be the migration value of unemployed and employed, respectively, we have that:

$$V_U^1(X) = \mu(\theta(a)) V_E^1(X) + (1 - \mu(\theta(a))) \left(\max_h \{v(h) - r(X)h\} + e + \beta E_a [W_U^1(X')] \right) \quad (42)$$

and

$$V_E^1(X) = \max_h \{v(h) - r(X)h\} + w + \beta \cdot E_a [(1-s)W_E^1(X') + s \cdot W_U^1(X')] \quad (43)$$

where

$$W_U^1(X) = \max_{\bar{\epsilon}(X)} \left\{ F(\bar{\epsilon}(X)) \bar{V}_U + (1 - F(X)) V_U^1(X) - F(\bar{\epsilon}(X)) c_1 + \int_{\bar{\epsilon}(X)} \epsilon dF_\epsilon \right\} \quad (44)$$

and

$$W_E^1(X) = \phi \bar{V}_E + (1 - \phi) V_E^1(X) \quad (45)$$

The particular form for the migration value of an employed worker, (41), arises from the assumption that there is no net-migration of employed workers so the probability that an employed worker moves to a given region equals the fraction of employed workers is proportional to the measure of employed workers that move out of that region, which gives the probability weights multiplying $V_E^1(X)$.

Wage determination

I allow for wages to be rigid as in Hall (2005). Rather than through Nash bargaining, the wage rate in each region is fixed at w . However, w lies within the bargaining set of a worker-job match. In particular, at the bargaining stage, in region j , the outside option of a worker is $\tilde{V}_U^\chi(X) = e + \max_h \{v(h) - r(X)h\} + \beta E_a [W_U^\chi(x')]$, where $\chi \in \{0, 1\}$ is the worker's mobility cost type. Hence, the minimum wage a worker would accept, $\underline{w}^\chi(X)$, leaves him indifferent between employment and unemployment, i.e. it solves:

$$V_E^\chi(X, \underline{w}^\chi) = \tilde{V}_U^\chi(X) \quad (46)$$

or

$$\underline{w}^\chi(X) = e + \beta E [W^\chi(X')] - \beta \left[(1-s)E [W_E^\chi(X')] + sE [W_U^\chi(X')] \right] \quad (47)$$

which implies that

$$\underline{w}^\chi(X) = e + (1-s)\beta \left[E [W_U^\chi(X')] - E [W_E^\chi(X')] \right] \quad (48)$$

Similarly, the maximum wage a firm would accept, $\bar{w}(a)$, leaves it indifferent between employing the worker and keeping the job vacant, i.e. it solves:

$$K(a, \bar{w}) = 0 \quad (49)$$

or

$$\bar{w}(a) = a + \beta(1-s) \cdot E [K(a')] \quad (50)$$

Using (39), we get that:

$$\bar{w}(a) = a + k\beta(1-s)E \left[\frac{\theta(a')}{\mu(\theta(a'))} \right] \quad (51)$$

Given the properties of $\mu(\theta)$ it follows that $\bar{w}(a) \geq \bar{w}(\underline{a}) = \underline{a} + k\beta(1-s)E\left[\frac{\theta(a')}{\mu(\theta(a'))}\right]$. Therefore,

$$w \in \left[\max_{x, \chi} \{\underline{w}^\chi(X)\}, \bar{w}(\underline{a}) \right] \quad (52)$$

Note that with wages determined via Nash bargaining wages will be different across regions and across workers with different mobility costs, and the wage rate will be a weighted average of the two boundaries.

Laws of motion for regional unemployment and labor force

We have the following laws of motion:

$$\tilde{u}_{1,t}(X_t) = (1 - q(X_t)) u_{1,t} + \Phi(X_t) \quad (53)$$

$$\tilde{l}_{1,t}(X_t) = l_{1,t} - \phi(l_{1,t} - u_{1,t}) + \phi(l_{1,t} - u_{1,t}) - q(X_t)u_{1,t} + \Phi(X_t) \quad (54)$$

The laws of motion for workers with high migration costs are trivial since these workers do not move across regions. I describe $\Phi(X)$ below. The laws of motion for the invariant distribution ν^* follow directly from (53) and (54) as well as the law of motion for labor productivity (36). Additionally, we have a population constancy condition:

$$\int \tilde{l}_{1,t}(X_t) d\nu^* + \int l_0 d\nu^* = \int l_{1,t} - q(X_t)u_{1,t-1} + \Phi(X_t) d\nu^* + \int l_0 d\nu^* = L \quad (55)$$

Similarly to the basic model with one-period long jobs, the in-migration function for unemployed workers will have the form:

$$\Phi(X) = \begin{cases} \tilde{l}(a, l_0, u_1) - l_1 + q(X)u & , l_1 < \bar{l}(a, l_0, u_1) \\ (1 - \lambda) \cdot M & , o.w. \end{cases} \quad (56)$$

where

$$M = \int q(X) u_1 d\nu^*, \quad (57)$$

$\tilde{l}(a, l_0, u_1) = \bar{l}(a, l_0, u_1) - q(a, l_0, \bar{l}, u_1) u_1 + (1 - \lambda) M$, and

$$\bar{l}(a, l_0, u_1) = \begin{cases} \sup_l \{l : (a, l_0, l, u_1) \in \arg \max_x \{V^1(x)\}\} & , \{l : (a, l_0, l, u_1) \in \arg \max_x \{V^1(x)\}\} \neq \emptyset \\ 0 & , o.w. \end{cases} \quad (58)$$

and also that there is a unique fixed point $\bar{l}^*(a, l_0, u_1)$ that equals $\tilde{l}(a, l_0, u_1)$ if the net migration function $n(a, l_0, \bar{l}, u_1) \leq 0$ and $l_{min}(a, l_0, u_1)$ otherwise, where $l_{min}(a, l_0, u_1)$ solves:

$$l_{min} - q(a, l_0, l_{min}, u_1)u_1 + (1 - \lambda) M = l_{min} \quad (59)$$

Computational Algorithms

Here I describes the algorithms used for simulating the stationary equilibrium of the quantitative model. After that I describe the estimation algorithm.

Solving for steady state equilibrium

For a given parameter vector θ , I follow the following algorithm to solve for the steady state equilibrium:

1. Construct a grid $\mathcal{G}$ for a, l_0, l_1, u_0 and u_1 ;
2. Make an initial guess for all endogenous objects over $\mathcal{G}$,

$$\Psi_0 = \left[V_{U,0}^1(\cdot), V_{E,0}^1(\cdot), \bar{V}_{U,0}, \bar{V}_{E,0}, M_0, l'_{1,0}(\cdot), \nu_0^* \right];$$

3. Generate Ψ_i , $i > 0$ using Ψ_{i-1} . For the value functions $V_U^1(\cdot)$ and $V_E^1(\cdot)$ this entails using all members of Ψ_{i-1} to compute $V_{U,i}^1(\cdot)$ and $V_{E,i}^1(\cdot)$ according to equations (42) and (43). For $\bar{V}_{U,i}$ and $\bar{V}_{E,i}$ I use Ψ_{i-1} in equations (40) and (41). For M_i , I use equation (57). For ν_i^* , I use Ψ_{i-1} and the law of motion for a to simulate vectors of a, l_0, l_1, u_0 and u_1 for B regions over $T_0 + T_1$ time periods, but only use the last T_1 time periods. Lastly, for $l'_{1,i}(\cdot)$, I use $V_{U,i-1}^1(\cdot)$ to determine the best regions, i.e. states x , for which $x \in X = \arg \max_{\tilde{x}} \left\{ V_{U,i-1}^1(\tilde{x}) \right\}$. For $x \in X$ I increase $l'_1(x)$ by a factor $\psi^{inc}(\Psi_{i-1})$, whereas for $x \notin X$, I decrease by a factor $\psi^{dec}(\Psi_{i-1})$ if it's greater than 0 or keep it at 0 otherwise.
4. Check population constancy condition. If $|L_i - L| > \delta > 0$, adjust $\psi^{inc}(\Psi_i)$ and $\psi^{dec}(\Psi_i)$ depending on the sign of the deviation.
5. Check if $\|\Psi_i - \Psi_{i-1}\| < \epsilon$. If not, repeat steps 3. and 4.

The speed of convergence depends on the vector θ because of the adjustment in l'_1 which may be fast and stable or slower and exhibiting some cycles depending on θ . Experimentation shows that around 1000 iterations are sufficient for convergence.

Indirect inference estimation algorithm

I nest the algorithm for solving the equilibrium of the model into the indirect inference estimation algorithm:

1. Choose an initial value of θ , θ_0 and set objective to ∞ ;
2. Solve for equilibrium of model given θ_i according to the algorithm above;

3. Create $S = 50$ data sets with 45 regions over 293 time periods (months) keeping the last 193 (16 years + 1 month). For each data set estimate migration regressions by OLS as well as for auto-correlation of annual state unemployment rate. Additionally compute aggregate averages for unemployment, unemployment dispersion, migration and net migration rates to form $m^i(\theta_i)$;
4. Compute $\left(\bar{m} - \frac{1}{S} \sum_{i=1}^S m^i(\theta_i)\right)' W_N \left(\bar{m} - \frac{1}{S} \sum_{i=1}^S m^i(\theta_i)\right)$ and update θ_{i+1} based on θ_i and the value for the objective obtained;
5. Repeat steps 2 through 4 as many times as possible.

For the discretized grid I use a grid with 6 grid points for productivity a , 2 grid points for measure of immobile workers, l_0 , 81 grid points for l_1 , and 9 grid points for u_1 . For the minimization of the objective I use a simulated annealing algorithm. I start with a number of initial guesses θ_0 to ensure that the global minimum is attained.

References

- ABRAHAM, K. (1987): “Help-Wanted Advertising, Job Vacancies, and Unemployment,” *Brookings Papers on Economic Activity*, pp. 207–248.
- ALVAREZ, F., AND R. SHIMER (2011): “Search and Rest Unemployment,” *Econometrica*, 79(1), 75–122.
- ANENBERG, E., AND P. BAYER (2012): “Endogenous Sources of Cycling in Housing Markets: The Joint Buyer-Seller Problem,” mimeo.
- ANGRIST, J., AND A. KRUEGER (1999): “Empirical Strategies in Labor Economics,” *Handbook of Labor Economics*, Volume 3, Part A, 1277–1366.
- BAI, J., AND P. PERRON (1998): “Estimating and Testing Linear Models with Multiple Structural Changes,” *Econometrica*, 66:1, 47–78.
- BARNICHON, R., M. ELSBY, B. HOBIJN, AND A. SAHIN (2010): “Which Industries are Shifting the Beveridge Curve?,” Federal Reserve Bank of San Francisco Working Paper 2010-32.
- BARNICHON, R., AND A. FIGURA (2010): “What Drives Movements in the Unemployment Rate? A Decomposition of the Beveridge Curve,” Federal Reserve Board Working Paper 2010-48.
- (2011): “What Drives Matching Efficiency? A Tale of Composition and Dispersion,” Federal Reserve Board Working Paper 2011-10.

- BAYER, C., AND F. JUESSEN (2012): “On the Dynamics of Interstate Migration: Migration Costs and Self-Selection,” *Review of Economic Dynamics*, 15:3, 377–401.
- BHUTTA, N., J. DOKKO, AND H. SHAN (2010): “The Depth of Negative Equity and Mortgage Default Decisions,” Federal Reserve Board Working Paper, 2010-35.
- BLANCHARD, O., AND L. KATZ (1992): “Regional Evolutions,” *Brookings Papers on Economic Activity*, pp. 1–75.
- CAMPBELL, S., M. DAVIS, J. GALLIN, AND R. MARTIN (2009): “What Moves Housing Markets: A Variance Decomposition of the Rent-Price Ratio,” *Journal of Urban Economics*, 66:2, 90–102.
- CARRILLO-TUDELA, C., AND L. VISSCHERS (2011): “Unemployment and Endogenous Reallocation over the Business Cycle,” mimeo.
- COEN-PIRANI, D. (2010): “Understanding gross worker flows across U.S. states,” *Journal of Monetary Economics*, 57(7), 769–784.
- CORELOGIC (2009): “Summary of Second Quarter 2009 Negative Equity Data from First American CoreLogic,” available from http://www.loanperformance.com/infocenter/library/FACL%20Negative%20Equity_final_081309.pdf.
- DAVIS, M., J. FISHER, AND M. VERACIERTO (2010): “The Role of Housing in Labor Reallocation,” Federal Reserve Bank of Chicago Working Paper WP2010-18.
- DAVIS, M., AND F. ORTALO-MAGNE (2009): “Household Expenditures, Wages, Rents,” mimeo.
- ENGELHARDT, G. (2003): “Nominal loss aversion, housing equity constraints, and household mobility: evidence from the United States,” *Journal of Urban Economics*, 53(1), 171–195.
- ERICKSON, T., AND T. WHITED (2002): “Two-step GMM Estimation of the Errors-in-Variables Model Using Higher-Order Moments,” *Econometric Theory*, 18, 776–799.
- FERREIRA, F., J. GYOURKO, AND J. TRACY (2010): “Housing busts and household mobility,” *Journal of Urban Economics*, 68(1), 34–45.
- FOOTE, C., K. GERARDI, AND P. WILLEN (2008): “Negative equity and foreclosure: Theory and evidence,” *Journal of Urban Economics*, 64(2), 234–245.
- GERTLER, M., AND A. TRIGARI (2009): “Unemployment Fluctuations with Staggered Nash Wage Bargaining,” *Journal of Political Economy*, 117(1), 38–86.

- GLEASER, E., AND J. GYOURKO (2005): “Urban Decline and Durable Housing,” *Journal of Political Economy*, 113:2, 345–375.
- GLEASER, E., J. GYOURKO, E. MORALES, AND C. NATHANSON (2012): “Housing Dynamics,” mimeo.
- GODOY, A., AND E. MOEN (2012): “Mixed search,” mimeo.
- GOURIEROUX, C., A. MONFORT, AND E. RENAULT (1993): “Indirect Inference,” *Journal of Applied Econometrics*, 8, S85–S118.
- GUIO, L., P. SAPIENZA, AND L. ZINGALES (2009): “Moral and Social Constraints in Strategic Default on Mortgages,” EUI Working Paper 2009/27.
- HAGEDORN, M., AND I. MANOVSKII (2008): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited,” *American Economic Review*, 98(4), 1692–1706.
- HALL, R. (2005): “Employment Fluctuations with Equilibrium Wage Stickiness,” *American Economic Review*, 95(1), 50–65.
- HANUSHECK, E., AND J. QUIGLEY (1980): “What is the Price Elasticity of Housing Demand?,” *Review of Economics and Statistics*, 62:3, 449–454.
- HEAD, A., AND H. LLOYD-ELLIS (2010): “Housing Liquidity, Mobility and the Labour Market,” mimeo.
- JACKMAN, R., AND S. SAVOURI (1992): “Regional Migration in Britain: An Analysis of Gross Flows Using NHS Central Register Data,” *Economic Journal*, 102, 1433–1450.
- KAPLAN, G., AND S. SCHULHOFER-WOHL (2012a): “Interstate Migration Has Fallen Less Than You Think: Consequences of Hot Deck Imputation in the Current Population Survey,” *Demography*, DOI: 10.1007/s13524-012-0110-3.
- (2012b): “Understanding the Long-Run Decline in Interstate Migration,” Federal Reserve Bank of Minneapolis Working Paper 697.
- KARAHAN, F., AND S. RHEE (2011): “Housing and the Labor Market: The Role of Migration on Aggregate Unemployment,” mimeo.
- KATZ, L. (2010): “Long-Term Unemployment in the Great Recession,” Testimony for the Joint Economic Committee in the U.S. Congress.

- KENNAN, J., AND J. WALKER (2011): “The Effect of Expected Income on Individual Migration Decisions,” *Econometrica*, 79:1, 211–251.
- KOCHERLAKOTA, N. (2010): “Inside the FOMC,” available from <http://www.minneapolisfed.org/>.
- LILIEN, D. (1982): “Sectoral Shifts and Cyclical Unemployment,” *Journal of Political Economy*, 90(4), 777–793.
- LKHAGVASUREN, D. (2011): “Large Locational Differences in Unemployment Despite High Labor Mobility: Impact of Moving Cost on Aggregate Unemployment and Welfare,” mimeo.
- LUCAS, R., AND E. PRESCOTT (1974): “Equilibrium Search and Unemployment,” *Journal of Economic Theory*, 7, 188–209.
- MANGUM, K. (2012): “A Dynamic Model of Cities and Labor Markets,” mimeo.
- NOVY-MARX, R. (2009): “Hot and Cold Markets,” *Real Estate Economics*, 37:1, 1–22.
- OFFICE OF MANAGEMENT AND BUDGET (2009): “Office of Management and Budget Bulletin NO. 10-02,” .
- ORTALO-MAGNE, F., AND S. RADY (2006): “Housing Market Dynamics: On the Contribution of Income Shocks and Credit Constraints,” *Review of Economic Studies*, 73, 459–485.
- OSWALD, A. (1996): “A Conjecture on the Explanation for High Unemployment in the Industrialized Nations: Part 1,” Warwick Economic Research Paper 475.
- PETRONGOLO, B., AND C. PISSARIDES (2001): “Looking into the Black Box: A Survey of the Matching Function,” *Journal of Economic Literature*, 39(2), 390–431.
- PISSARIDES, C. (2000): *Equilibrium Unemployment Theory*. MIT Press.
- ROBACK, J. (1982): “Wages, rent, and the quality of life,” *Journal of Political Economy*, 90(2), 191–229.
- ROSEN, S. (1979): “Wage-based Indexes of Urban Quality of Life,” in *Current Issues in Urban Economics*, ed. by P. Miezkowski, and M. R. Straszheim. Johns Hopkins University Press.
- SAHIN, A., J. SONG, G. TOPA, AND G. VIOLANTE (2011): “Measuring Mismatch in the US Labor Market,” mimeo.
- SAKS, R., AND A. WOZNIAK (2011): “Labor Reallocation over the Business Cycle: New Evidence from Internal Migration,” *Journal of Labor Economics*, 29(4), 697–739.

- SHIMER, R. (1999): “Why is the U.S. Unemployment Rate So Much Lower?,” in *NBER Macroeconomics Annual 1998*, ed. by B. S. Bernanke, and J. Rotemberg.
- (2005): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies,” *American Economic Review*, 95(1), 25–49.
- (2007): “Mismatch,” *American Economic Review*, 97(4), 1074–1101.
- (2010): *Labor Markets and Business Cycles*. Princeton University Press.
- SMITH, A. (1993): “Estimating Nonlinear Time-series Models Using Simulated Vector Autoregressions,” *Journal of Applied Econometrics*, 8, S63–S84.
- STEIN, J. (1995): “Prices and Trading Volume in the Housing Market: a Model with Down-Payment Effects,” *Quarterly Journal of Economics*, 110, 379–406.
- STERK, V. (2010): “Home Equity, Mobility and Macroeconomic Fluctuations,” mimeo.
- STOKEY, N., AND R. LUCAS (1989): *Recursive Methods in Economic Dynamics*. Harvard University Press.
- THE ECONOMIST (2010): “Drowning or Waiving,” available from <http://www.economist.com/>.
- VAN NIEUWERBURGH, S., AND P.-O. WEILL (2010): “Why Has House Price Dispersion Gone Up?,” *Review of Economic Studies*, 77(4), 1567–1606.
- WOODBIDGE, J. (2002): *Econometric Analysis of Cross Section and Panel Data*. The MIT Press.