Wage dynamics in long-term contracts

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Abstract

Empirically many workers stay for years with the same employer. Long-term contracts offer insurance possibilities for these workers. However, these workers still experience frequent wage changes. This paper analyzes long-term contracts when workers freely leave the contract and move between employers (one-sided limited commitment). We examine if the theory can account for the observed wage dynamics and worker mobility and study its quantitative predictions for the insurability of productivity risk. Our contracting framework allows for job-to-job flows in equilibrium. This leads to much richer and realistic wage dynamics. We prove existence, monotonicity and differentiability of firm profits and provide first-order conditions that characterize the properties of the optimal contract. We demonstrate how the shape of the contract depends on the persistence and variance of the productivity process, on worker and firm risk, and on non-pecuniary utility components.

1 Introduction

A large fraction of workers stay with their employer for their entire work-life or change jobs only once or twice. Long-term contracts between employers and employees can be used to provide workers with insurance against income risk. The fact that even continuously

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employed workers at the same firm experience frequent and substantial income changes suggests that these risk-sharing possibilities are only partially lifted in the market.[1]

This paper analyzes long-term wage contracts in cases when workers do freely leave the contract and move between employers (one-sided limited commitment). The primary purpose is to understand to what extend limited commitment models can jointly account for the observed wage dynamics and job-mobility. The secondary purpose is to quantitatively study how competition between firms limit insurance possibilities in employment relationships when worker face persistent income risk. We extend the exiting insights of long-term contracts by studying the effects of productivity persistence and the different implications of worker and firm risk. We find that by smoothing wages relative to productivity, contracts make wages more persistent relative to productivity. Regarding the sources of risk, we find that contracts in the presence of worker risk and in the presence of firm risk are inverse to each other. While with worker risk, workers pay an insurance premium upfront to buy insurance in the presence of limited commitment, they receive an upfront payment in the presence of firm risk to induce default when productivity is low. Finally, our contracting framework allows for job-to-job flows in equilibrium. This leads to much richer and realistic wage dynamics. First, wages in our model are linked to productivity and profits so that wages on the job can also decrease. Second, there is default on the contract and job mobility.

In our model, firms fully commit to state-contingent wage contracts. Each firm faces a stochastic productivity process. Workers are risk averse and also face risk to their productivity. Competition among firms limits the amount of insurance that can be granted. Workers receive offers from competing firms and cannot commit to reject better offers (one-sided limited commitment). The outside option of agents is as in **?** endogenously determined by competition between firms. We establish qualitative properties of the optimal contract. We extend the setup of existing contracting models by adding a second dimension to each job-offer. This component captures non-pecuniary aspects of a particular job like moving cost, distance to family, or work-atmosphere, dimensions that have been recently stressed as important by **?** and **?**. Worker's acceptance decision depends both on the wage of the competing contract and on the non-pecuniary component. This ensures that workers will leave contracts with positive probability. By adding a second dimension to each job-offer our framework bridges the gap between the competitive market framework that constraints

[1] In section 2, we exemplify these facts using German microdata with employment histories of up to 30 years.

initial contracts to be competitive and the frictional search framework that creates a locally differentiated good due to the search friction. This allows us to jointly study labor mobility and risk-sharing on the job within a one-sided limited commitment framework.

We prove existence, monotonicity and differentiability of firm profits. The latter allows us to study the properties of the contract based on first-order conditions. The shape of the contract depends, qualitatively and quantitatively, on the persistence and variance of the underlying productivity process, on the shock type, and on the stochastic utility component. We find, that as long as there is a positive probability that the contract terminates profits and wages positively co-move. In the absence of further shocks wages tend to converge to a point where expected firm profits are zero. These findings contrast with the limited commitment contract explored in **?**. In their model insurance against negative shocks is provided and wages can only move up (*ratcheting up effect*). Firms need to front-load profits and will make losses in the long-run. Our framework captures these properties as a knife-edge case if productivity shocks are i.i.d. and average moving cost become sufficiently large such that job-to-job transitions converge to zero. If exit probabilities are positive, firms provide insurance to workers by transforming the productivity process into a more persistent but less volatile wage process. In the i.i.d. case, we show that up to first-order, the wage process follows persistent process with an autocorrelation coefficient inversely related to the average job-to-job transition rate.

Efficient contracting models imply that job-mobility decisions depend on the total surplus of the match, see **?** for a quantitative analysis. In contrast, limited commitment on the worker-side implies that a worker, when deciding to leave a contract, does not take into account the profits (losses) of the firm. With worker risk, shocks affect productivity in all matches. With firm risk, shocks only affect productivity in the current match. Given identical productivity realizations, the two cases are hard to distinguish using wage data alone. Only differences in the transition pattern and in the wage change at transitions can tell the source of the shocks apart.

The intuition is simple: Wages and productivity positively co-move. If shocks are worker-specific, quit decisions are negatively correlated with productivity because the worker can monetize high productivity elsewhere. If shocks are match-specific, quit decisions are positively correlated, because the worker is in a particularly good match. The resulting selection effect lowers average observed productivity of a match in the former case, and increases it in the latter. In the former case, the firm wants to provide insurance and front-loads profits. In the latter case, the firm wants to provide incentives to quit in bad states and back-loads profits, paying a high wage in the initial period when job-to-job-transitions are not distorted. As a result, we find quantitatively that persistent worker-specific shocks alone can reproduce the increase in the cross-sectional variance over the life-cycle for a cohort of workers and can account for productivity risk but lead to negative wage changes at transitions. Match-specific shocks alone can explain wage changes at transitions but do not generate much wage dispersion over time. Both shock types must be present to explain the data.

We provide a calibrated version of our model with worker and firm productivity risk. The model is quantitatively consistent with stylized features of the German labor market. In particular, the model generates the observed job-to-job transitions, generates the increase in wage dispersion for continuously employed worker, features positive but dispersed gains from job-to-job transitions, and generates frequent wage cuts on the job. We use the model to quantify the insurance possibilities that risk-neutral firms can grant to risk-averse workers in the presence of competitive pressure.

Why do we observe frequent wage changes on the job and how should wage-risk been optimally shared between (ex-post) heterogeneous workers and firms? The literature on spot markets relates wage payments to current productivity, either via a bargaining mechanism or competition, but does not deliver a theory of risk-sharing on the job. The related search literature that uses contract posting models with risk-averse agents (**?**, **?**) delivers a theory for the average wage-tenure profile in the presence of search frictions but mainly pays attention to wage changes between subsequent matches and residual wage inequality (**?**). The implications for risk-sharing arrangements within a given contract in the presence of productivity shocks have been largely ignored. The 'endogenous' incomplete market literature (**?** and **?**) has shown that in the presence of idiosyncratic worker-specific risk good-will can be built, negative productivity shocks can be insured and wages typically move upward whenever a positive shock hits.[2] The optimal contract prevents equilibrium default and all workers remain permanently at one firm. The resulting contract has some severe limitations though: it can not jointly explain wages and job-changes, it can not explain wage cuts on the job and does not generate the unit-root type behavior of wages observed in the data.[3]

[2]To study wage cuts on the job the literature refers to models with two-sided limited commitment, see **?**. While the legal framework allows worker to freely move (no slavery), which leads to a one-sided limited commitment problem, a firm can, in principal, be forced to fulfill the contract.

[3]More broadly, there are essentially three answers to the question provided in the literature: first, firms do not have to provide insurance because self-insurance via complete financial markets can work (a skeptical view on this is provided by the incomplete market literature); second, firms do not want to give insurance because they want to overcome moral hazard problems and use permanent shocks to provide incentives (see **?** for an analysis); or third, they can not provide insurance due to limited commitment problems. This paper explores the last alternative. Our finding should be interpreted as add-on relative to the aforementioned

The paper proceeds as follows: section 2 summarizes some empirical facts using German microdata, section 3 describes the model and proofs some properties, section 4 explores the properties of the contract, and section 5 provides and estimate of the productivity process that can explain the key facts outlined above. We conclude in section 6

2 Data

[TO BE DONE]

Model

Time is discrete. There is a continuum of risk-averse workers and of risk-neutral firms. Workers can not save or borrow, so consumption equals income. Workers utility from consumption is determined by an instantaneous utility function

$$u(c) = \begin{cases} \frac{c^{1-\gamma}}{1-\gamma} & \gamma \neq 1 \\ \log(c) & \gamma = 1 \end{cases} .$$

Workers discount future utility at rate $\beta \in (0,1)$, firms discount future profits at risk-free rate $R > 1$, and we assume $\beta R \leq 1$.

We restrict attention to matches between a single worker and a single firm. To produce, a firm must be matched to a worker. Each worker is endowed with a stochastic worker-specific skill component $x_w \in [\underline{x}_w, \overline{x}_w]$ and each match is endowed with a stochastic match-specific skill component $x_f \in [\underline{x}_f, \overline{x}_f]$. We use x to denote the joint skill state $x = (x_f, x_w)$ and $\overline{x} = (\overline{x}_f, \overline{x}_w)$ respectively $\underline{x} = (\underline{x}_f, \underline{x}_w)$ denote the highest and the lowest skill state. A match produces using a continuous, increasing production technology $y = f(x)$. The history of a match in period s of its existence is $h_s = \{x_1, x_2, \ldots, x_s\}$ and the space of all histories is denoted by $\mathcal{H}$. The worker and the firm sign a contract at match formation for all possible future histories. A contract is a mapping from histories to wage payments $w : \mathcal{H} \rightarrow \mathbb{R}_+$. We restrict the contracting space and require wage payments to be always positive.[4]

Within period timing is as follows: If the match has not terminated last period, the match produces and wages are paid following the contract. Firm's period profits are $y-w$. After the

channels.

[4] Allowing for payments from workers to firms seems of no relevance to us to explain observed wage dynamics.

production state, the match- and worker-specific state for the next period realize. Finally, the worker receives an outside offer with probability π_o from competing firms. We assume that competing firms all start with match-specific component $x_f = 1$. With every outside offer comes a random utility component ε. This non-pecuniary component captures characteristics of the new job like distance from home, workplace atmosphere, work schedule, or job health characteristics. The utility component ε is i.i.d. across workers and time and logistically distributed with mean μ and standard deviation $\frac{\pi\psi}{\sqrt{2}\beta}$. The worker can accept the outside offer or can stay in the contract with the current firm. Firms must always satisfy the contractual obligations, so there is one-sided limited commitment.

Recursive contracts

We restrict attention to recursive contracts. We formulate the contracting problem as a recursive problem using promised utility as a state variable (?). We adopt standard notation for this case and denote next period's variables using primes. The state of an agent is summarized by her level of promised utility V and the current productivity state x. The promise keeping constraint reads

$$V = u(w) + \sum_{x'} \left(\pi_o \beta \int_{\varepsilon'} \max \{V(x'), V_o(x'_o) + \varepsilon\} + (1 - \pi_o)\beta V(x') \right) \pi(x', x), \tag{1}$$

where $x'_o = (1, x'_w)$. Using the distributional assumptions on ε, the value of a wage w today and state-contingent continuation utilities $V(x')$ tomorrow is

$$
\begin{aligned}
V &= u(w) + (1-\pi_o)\sum_{x'} \beta V(x')\pi(x',x) \\
&\quad +\pi_o\Bigg(\sum_{x'}(1-q(V(x'),V_o(x'_o)))\beta V(x')\pi(x',x) \\
&\quad +\sum_{x'} q(V(x'),V_o(x'_o))(\beta V_o(x'_o)-\mu)\pi(x',x) \\
&\quad +\sum_{x'}\Psi(q(V(x'),V_o(x'_o)))\pi(x',x)\Bigg), \\
\Psi(q(V(x'),V_o(x'_o))) &= -\psi\Bigg((1-q(V(x'),V_o(x'_o)))\log(q(V(x'),V_o(x'_o))) \\
&\quad +q(V(x'),V_o(x'_o))\log(q(V(x'),V_o(x'_o)))\Bigg), \\
q(V(x'),V_o(x'_o)) &= \frac{\exp(\psi^{-1}\beta V_o(x'_o)-\psi^{-1}\mu)}{\exp(\psi^{-1}\beta V(x'))+\exp(\psi^{-1}\beta V_o(x'_o)-\psi^{-1}\mu)}, \qquad (2)
\end{aligned}
$$

where $q(V(x'), V_o(x'_o))$ denotes the offer-contingent probability that the offer is accepted. $\Psi(q(V(x'), V_o(x'_o)))$ denotes the *option value* of having the choice to decide between staying in the current match or accepting the outside offer.

The objective function of a firm given promised utility level V and productivity state x is

$$
\begin{aligned}
J(V,x) &= \max_{(w,\{V(x')\})\in\Gamma(V,x)} y - w + R^{-1}\Bigg(\pi_o\sum_{x'}((1-q(V(x'),V_o(x'_o)))J(V(x'),x')\,\pi(x',x) \\
&\quad +(1-\pi_o)\sum_{x'} J(V(x'),x')\pi(x',x)\Bigg) \qquad (3)
\end{aligned}
$$

with

$$
\begin{aligned}
\Gamma(V,x) &= \Bigg\{(w,\{V(x')\})\,\Bigg|\, w\geq 0, V = u(w)+(1-\pi_o)\sum_{x'}\beta V(x')\pi(x',x) \\
&\quad +\pi_o\Bigg(\sum_{x'}(1-q(V(x'),V_o(x'_o)))\beta V(x')\pi(x',x) \qquad (4) \\
&\quad +\sum_{x'} q(V(x'),V_o(x'_o))(\beta V_o(x'_o)-\mu)\pi(x',x)+\sum_{x'}\Psi(q(V(x'),V_o(x'_o)))\pi(x',x)\Bigg)\Bigg\}
\end{aligned}
$$

Given that skill states are bounded, we can derive an upper bound $\overline{V}$ for the domain of utility promises.[5] We construct the upper bound as the maximum utility a social planner could provide with the maximum amount of resources and add the worker's maximum option value of $\Psi(0.5)$ each period. The wage sequence in this case must satisfy $w' = (\beta R)^{\gamma} w$ and the resulting utility level is

$$\begin{aligned} \overline{V} &= \sum_{t=0}^{\infty} \beta^t (u(w_0(\beta R)^{\gamma t}) - \psi \log(0.5)) \\ w_0 &= f(\overline{x}) \frac{1 - \left(\beta R^{1-\frac{1}{\gamma}}\right)^{\gamma}}{1 - R^{-1}}. \end{aligned} \tag{5}$$

We use the upper bound $\overline{V}$ together with the restriction that wages must be positive to bound profits. The lower bound on wages immediately gives rise to the upper bound $\overline{J} = \frac{f(\overline{x})}{1-R^{-1}}$. The lower bound of profits $\underline{J}$ is attained if the firm provides the worker with $\overline{V}$ and has the lowest output level in each period. Given that the wage sequence to provide $\overline{V}$ has the same present value as $w = f(\bar{x})$ in every period, the lower bound on profits is $\underline{J} = \frac{f(\underline{x}) - f(\overline{x})}{1-R^{-1}}$.

3 Optimal contract

Profits in the model are bounded. Let B denote the set of bounded functions $J : \mathbb{R} \times [\underline{x}, \overline{x}] \to \mathbb{R}$ under the sup norm. We define the following operator $T : B \to B$

$$\begin{aligned} J_{k+1}(V, x) &= \max_{\{w, \{V'(x')\}\} \in \Gamma(V,x)} f(x) - w + R^{-1} \Bigg((1 - \pi_o) \sum_{x'} J(V(x', x')) \pi(x', x) \\ &\quad + \pi_o \sum_{x'} \Big((1 - q(V(x'), V_o(x'_o)) J_k(V'(x'), x') \Big) \pi(x', x) \Bigg) \end{aligned} \tag{6}$$

We apply Blackwell's sufficient conditions to show that the operator is a contraction mapping.

Proposition 1. *For an interest rate $R \geq 1$ the operator $T : B \to B$ constitutes a contraction mapping.*

Proposition 2. *The optimal solution $\{w(V, x), V'(x'; V, x)\}$ is a correspondence that is non-empty, compact-valued, and upper-hemi continuous in V.*

[5] We only need a loose upper bound on the utility promises to construct an upper bound on firm's profits in equilibrium.

We denote fixed point of T by $J(V, x)$.

Lemma 1. *$J(V, x)$ is a continuous, decreasing function of V.*

In the next step, we put the contracting problem in a competitive environment by imposing a free-entry condition on outside offers V_o. The free-entry condition requires that competing firms make zero profits in expectation when offering a contract. We define the following partial ordering $\geq$ on outside offers as usual

$$\hat{V}_o \geq V_o : \hat{V}_o(x) \geq V_o(x) \qquad \forall x$$

and we define the set of possible outside Ω as follows

$$\Omega := \left\{ V_o \in \mathbb{R}^n : V_o(x) \in [\underline{V}, \bar{V}] \qquad \forall x \right\}$$

Lemma 2. *The set Ω together with the ordering $\geq$ is a complete lattice.*

We define the following operator $P : \Omega \longrightarrow \Omega$

$$V_{o,k+1} = \{V_o : J_k(V_o, x) = 0 \forall x, J_k = TJ_k \text{given } V_{o,k}\}$$

Lemma 3. *The operator $P : \Omega \to \Omega$ is monotone.*

An equilibrium is the fixed point V_o^* to the operator $P : V \to V$. Given that the operator P is monotone, then it follows from Tarski's fixed point theorem that a fixed point V_o^* to P exists. This fixed point satisfies the free-entry condition ($J(V_o, x) = 0$) and is consistent with optimal behavior by firms given that $J(V, x)$ is a fixed point to $J = TJ$ for given outside options V_o.

4 Properties up to first-order

The contract has some important non-linear features which we will study below. However, the basic properties can best be visualized when looking at a first-order approximation. Assume that either only worker-specific or match-specific shocks exist, i.e. we assume that $x_f = 1$ and x_w follows an autoregressive AR(1) process with autocorrelation-coefficient ρ and variance σ^2 or vice versa. The results are obtained from a lineariziation around the outside option w_0 at the non-stochastic steady state (with respect to productivity but allowing for idiosyncratic job-to-job transitions). The basic system can be characterized by the state equations

$$\begin{aligned} \widehat{w}' &= f_w^w \widehat{w} + f_x^w \rho x + f_x^w \epsilon' \\ x' &= \rho x + \epsilon' \end{aligned} \tag{7}$$

where hat denotes deviation from steady state, f_w^w and f_x^w are constant coefficients given by:

$$\begin{aligned} f_w^w &= \frac{1}{1 - f_w^J q^{ss} \frac{\beta}{\psi}} \\ f_x^w &= \frac{q^{ss} \frac{\beta}{\psi} f_x^J}{1 - f_w^J q^{ss} \frac{\beta}{\psi}} \\ f_x^J &= \frac{1}{(1 - \frac{1}{R}(1 - q^{ss})\rho - \frac{1}{R}(1 - q^{ss})\rho f_w^J \frac{q^{ss}\frac{\beta}{\psi}}{1 - f_w^J q^{ss}\frac{\beta}{\psi}})} \\ q^{ss} \frac{\beta}{\psi} f_w^J &= -\frac{[q^{ss}\frac{\beta}{\psi} + \frac{1}{R}(1 - q^{ss}) - 1]}{2} - \sqrt{\frac{[q^{ss}\frac{\beta}{\psi} + \frac{1}{R}(1 - q^{ss}) - 1]^2}{4} + q^{ss}\frac{\beta}{\psi}} \\ q^{ss} &= \frac{1}{1 + e^{\frac{\mu}{\psi}}} \end{aligned} \tag{8}$$

Note that the state-contingency of the optimal contract induces a perfect correlation between the shocks in the two state equations.

We now summarize basic properties with respect to the variance and the autocorrelation of the wage behavior on the job.

Proposition 3. *The variance and the autocorrelation of wages on the job are given respectively by*

$$\begin{aligned} E\widehat{w}'^2 &= f_x^{w2} \frac{\sigma^2}{1 - \rho^2} \frac{\frac{1 + f_w^w \rho}{1 - f_w^w \rho}}{1 - f_w^{w2}} \\ \frac{E\widehat{w}'\widehat{w}}{E\widehat{w}'^2} &= f_w^w + \frac{1 - f_w^{w2}}{1 + \rho f_w^w}\rho \end{aligned} \tag{9}$$

Comparative statics shows that:

$$\frac{\partial \frac{E\widehat{w}'\widehat{w}}{E\widehat{w}'^2}}{\partial \psi}|_{q_{ss}} > 0, \frac{\partial E\frac{E\widehat{w}'\widehat{w}}{E\widehat{w}'^2}}{\partial \mu} > 0$$

The first order approximation highlights, how the contract is shaped by the presence of job-to-job transitions that occur in equilibrium: an i.) increase in q^{ss} due to a decline in the moving cost μ will reduce the autocorrelation of the wage-process and correspondingly increase its variance; ii.) an increase in idiosyncratic shocks ψ for a given q^{ss} will increase the autocorrelation of the wage-process and correspondingly decrease its variance; iii) in the limit, if $\rho \to 1$ the wage-process also converges to one; iv.) in case of idiosyncratic shocks $\rho = 0$ and if $\psi = 1, \beta = \frac{1}{R}$ and $R \to 1$ we have the direct link between the quitting rate and the autocorrelation $E\widehat{w}'^2 \simeq \sigma^2 \frac{(q^{ss})^2}{2\sqrt{q^{ss}}+q^{ss}}$ and $\frac{E\widehat{w}'\widehat{w}}{E\widehat{w}'^2} \simeq \frac{1}{1+\sqrt{q^{ss}}}$.
The last point highlights that the optimal contract transforms idiosyncratic productivity shocks into autocorrelated wage-shocks with coefficient $\frac{1}{1+\sqrt{q^{ss}}}$ but simultaneously lowers the variance of the resulting wage series by a proportional facor of $\frac{(q^{ss})^2}{2\sqrt{q^{ss}}+q^{ss}}$. The strength of the dampening effect depends positively on the liklihood of leaving the contract.

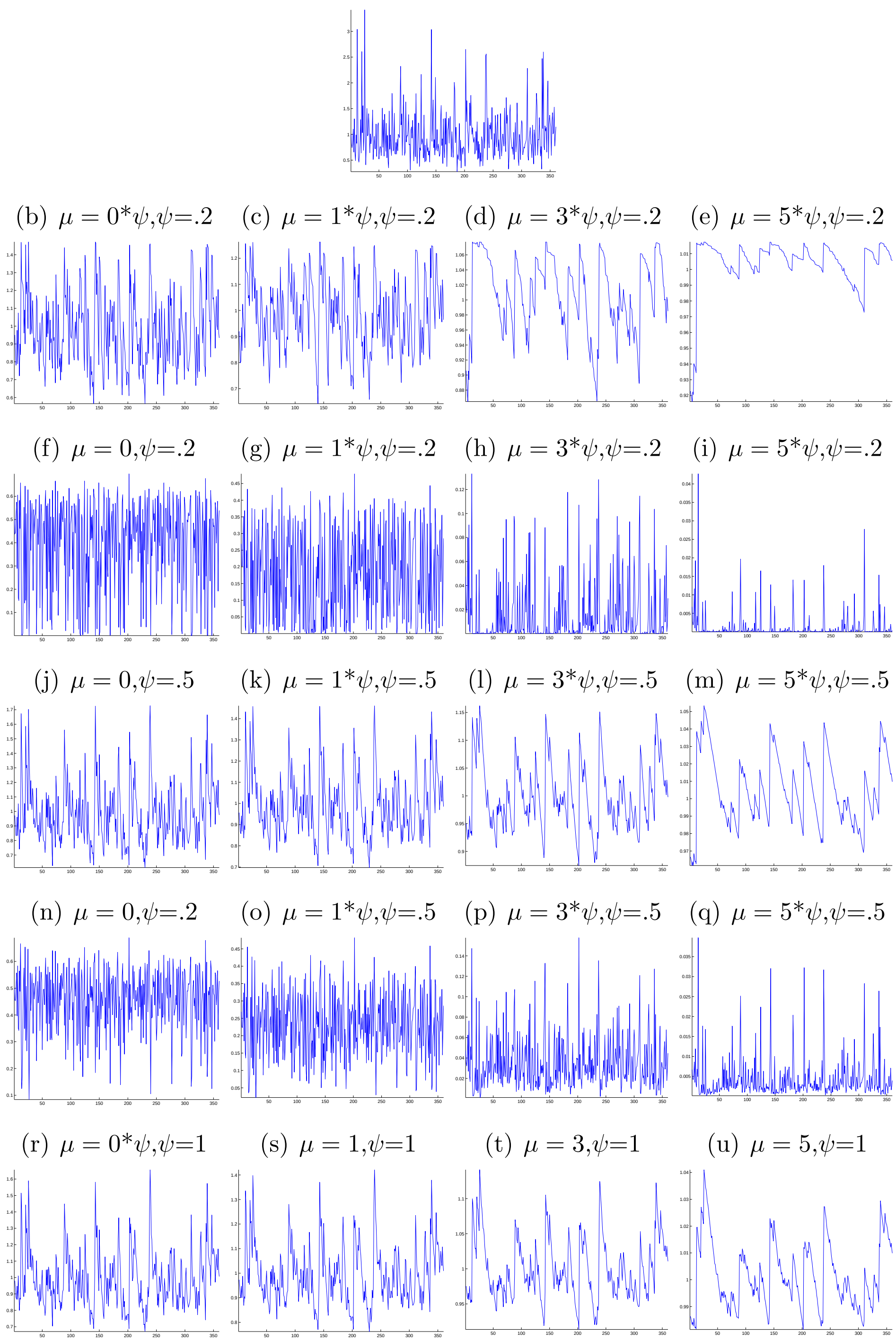
5 Quantitative Results

We turn next to properties of the contract based on the non-linear system. To compare to the literature we first study the shape of the contract if shocks are i.i.d. (either worker- or match-specific). In particular we highlight the role played by the two utility components μ and ψ. We then describe how the shape of the contract is changed, if we look at highly persistent autoregressive processes. In the next section we combine the two shock-processes and apply the model to German data.

5.1 The idiosyncratic case

To study the shape of the contract, we fix the seed of a random number generator, so conditional on not switching jobs, all contracts have the same productivity realization. Figure (1) for the i.i.d. worker-specific case and figrue (2) for the corresponding match-specific case report the shape of the contract as a function of a combination of μ and ψ.

Figure 1: Decomposition by mean and variance $\rho = .0$



(a) $\mu = 0^*\psi$,ψ=.2

(b) $\mu = 0^*\psi$,ψ=.2 (c) $\mu = 1^*\psi$,ψ=.2 (d) $\mu = 3^*\psi$,ψ=.2 (e) $\mu = 5^*\psi$,ψ=.2

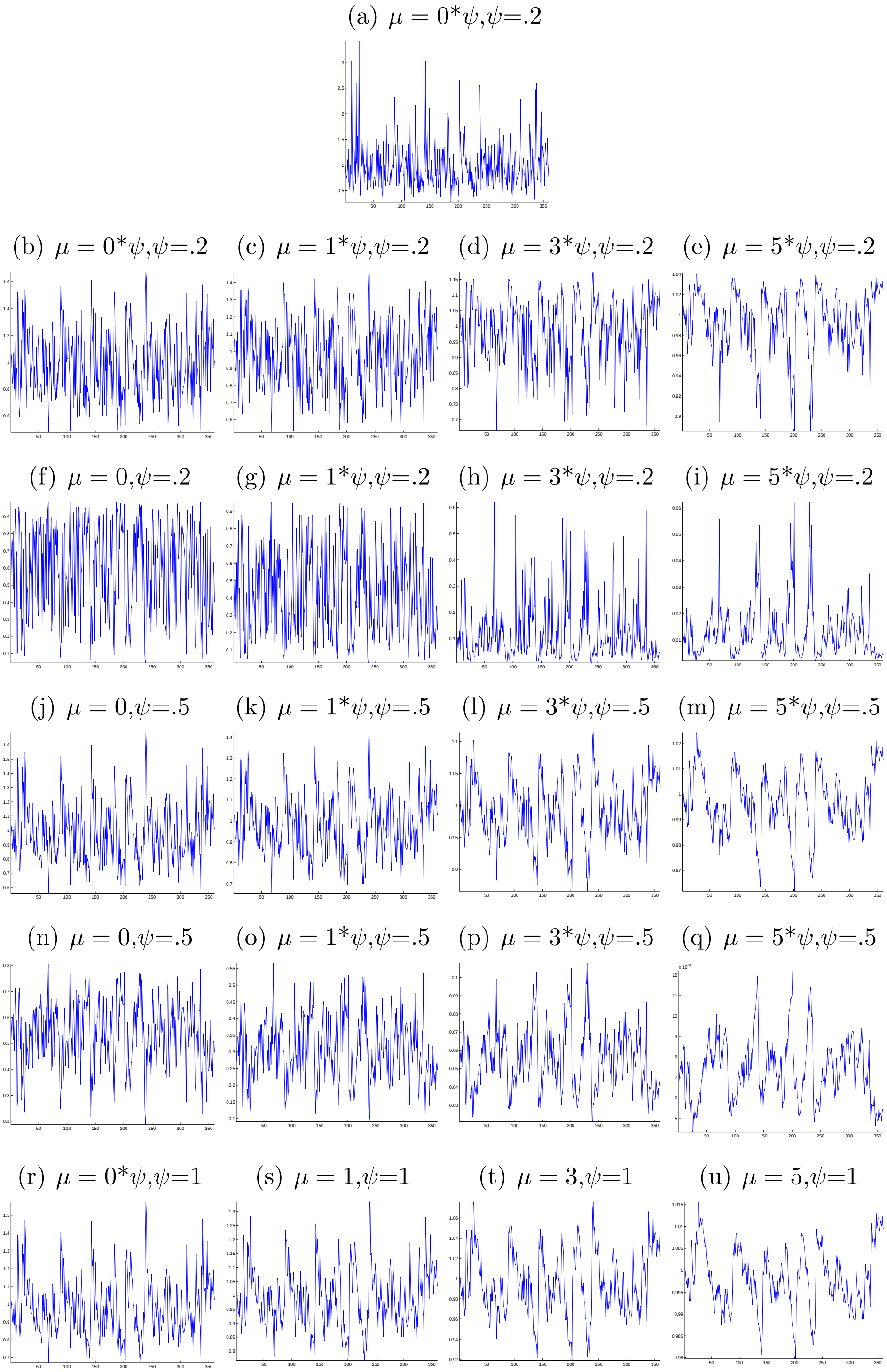
(f) $\mu = 0$,ψ=.2 (g) $\mu = 1^*\psi$,ψ=.2 (h) $\mu = 3^*\psi$,ψ=.2 (i) $\mu = 5^*\psi$,ψ=.2

(j) $\mu = 0$,ψ=.5 (k) $\mu = 1^*\psi$,ψ=.5 (l) $\mu = 3^*\psi$,ψ=.5 (m) $\mu = 5^*\psi$,ψ=.5

(n) $\mu = 0$,ψ=.2 (o) $\mu = 1^*\psi$,ψ=.5 (p) $\mu = 3^*\psi$,ψ=.5 (q) $\mu = 5^*\psi$,ψ=.5

(r) $\mu = 0^*\psi$,ψ=1 (s) $\mu = 1$,ψ=1 (t) $\mu = 3$,ψ=1 (u) $\mu = 5$,ψ=1

Figure 2: Decomposition by mean and variance $\rho = .0$, i.i.d.

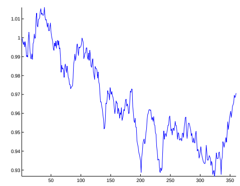


5.2 The persistent case

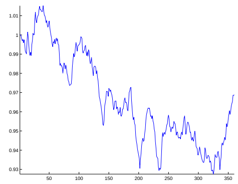
We turn next to the persistent case, approximating the unit-root type behavior by using an autocorrelated process with an autocorrelation coefficient $\rho = .996$.

Figure 3: Decomposition by mean and variance $\rho = .996$ Worker-Specific

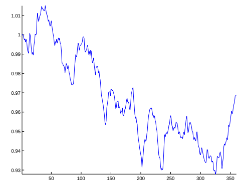
(a) $\mu = 0{*}\psi$,ψ=.2



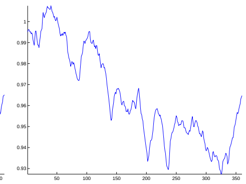
(b) $\mu = 0{*}\psi$,ψ=.2



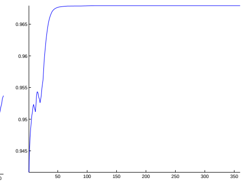
(c) $\mu = 1{*}\psi$,ψ=.2



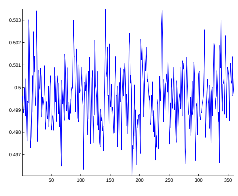
(d) $\mu = 3{*}\psi$,ψ=.2



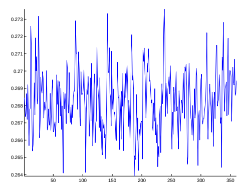
(e) $\mu = 5{*}\psi$,ψ=.2



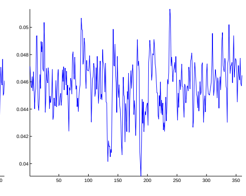
(f) $\mu = 0$,ψ=.2



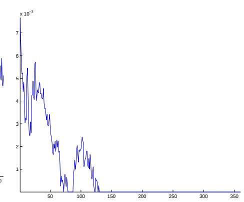
(g) $\mu = 1{*}\psi$,ψ=.2



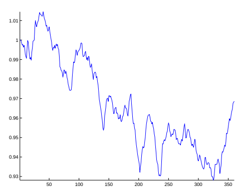
(h) $\mu = 3{*}\psi$,ψ=.2



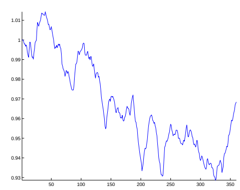
(i) $\mu = 5{*}\psi$,ψ=.2



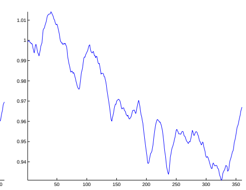
(j) $\mu = 0$,ψ=.5



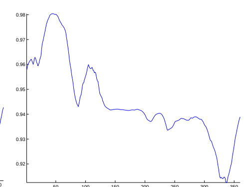
(k) $\mu = 1{*}\psi$,ψ=.5



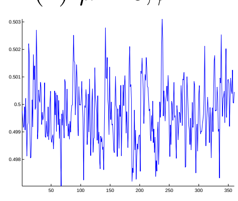
(l) $\mu = 3{*}\psi$,ψ=.5



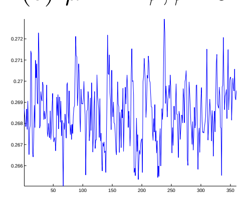
(m) $\mu = 5{*}\psi$,ψ=.5



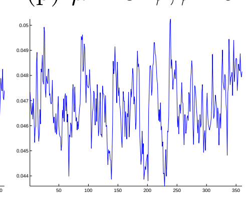
(n) $\mu = 0$,ψ=.2



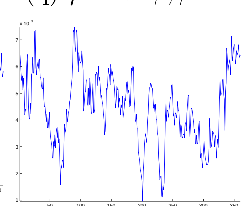
(o) $\mu = 1{*}\psi$,ψ=.5



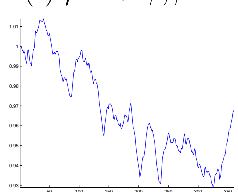
(p) $\mu = 3{*}\psi$,ψ=.5



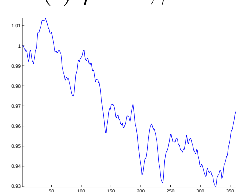
(q) $\mu = 5{*}\psi$,ψ=.5



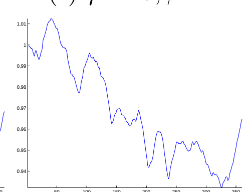
(r) $\mu = 0{*}\psi$,ψ=1



(s) $\mu = 1$,ψ=1



(t) $\mu = 3$,ψ=1



(u) $\mu = 5$,ψ=1

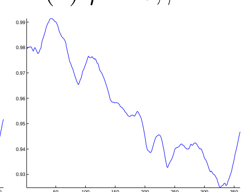
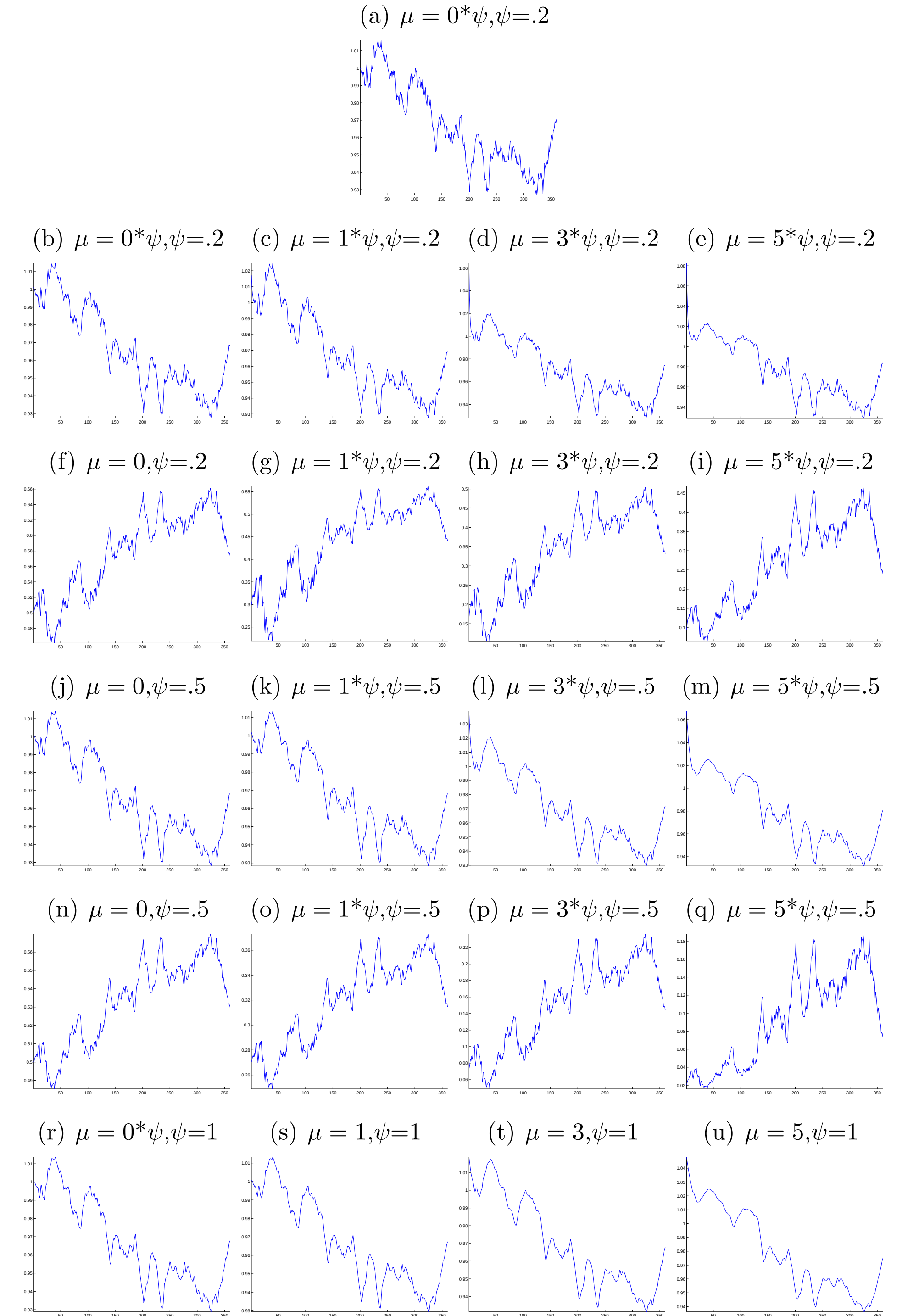


Figure 4: Decomposition by mean and variance $\rho = .996$ Match-Specific



(a) $\mu = 0{*}\psi$,ψ=.2

(b) $\mu = 0{*}\psi$,ψ=.2

(c) $\mu = 1{*}\psi$,ψ=.2

(d) $\mu = 3{*}\psi$,ψ=.2

(e) $\mu = 5{*}\psi$,ψ=.2

(f) $\mu = 0$,ψ=.2

(g) $\mu = 1{*}\psi$,ψ=.2

(h) $\mu = 3{*}\psi$,ψ=.2

(i) $\mu = 5{*}\psi$,ψ=.2

(j) $\mu = 0$,ψ=.5

(k) $\mu = 1{*}\psi$,ψ=.5

(l) $\mu = 3{*}\psi$,ψ=.5

(m) $\mu = 5{*}\psi$,ψ=.5

(n) $\mu = 0$,ψ=.5

(o) $\mu = 1{*}\psi$,ψ=.5

(p) $\mu = 3{*}\psi$,ψ=.5

(q) $\mu = 5{*}\psi$,ψ=.5

(r) $\mu = 0{*}\psi$,ψ=1

(s) $\mu = 1$,ψ=1

(t) $\mu = 3$,ψ=1

(u) $\mu = 5$,ψ=1

6 Conclusion

This paper analyzes limited commitment contracts with endogenous job-to-job transitions that occur in equilibrium.

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Appendix

Proof of proposition 1. We check Blackwell's sufficient conditions.

1. Monotonicity: Take $\hat{J}_k(V,x) \geq J_k(V,x)$. It follows

$$\begin{aligned}
\hat{J}_{k+1}(V,x) &= \max_{\{w,\{V'(x')\}\}\in\Gamma(V,x)} f(x) - w + R^{-1}\Bigg((1-\pi_o)\sum_{x'}\hat{J}_k(V'(x'),x')\pi(x',x) \\
&\quad +\pi_o\sum_{x'}\Bigg((1-q(V(x'),V_o(x'))\hat{J}_k(V'(x'),x')\Bigg)\pi(x',x)\Bigg) \\
&\geq \max_{\{w,\{V'(x')\}\}\in\Gamma(x,V)} f(x) - w + R^{-1}\Bigg((1-\pi_o)\sum_{x'}J_k(V'(x'),x')\pi(x'|x) \\
&\quad +\pi_o\sum_{x'}\Bigg((1-q(V(x'),V_o(x'))J_k(V'(x'),x')\Bigg)\pi(x'|x)\Bigg) \\
&\geq J_{k+1}(V,x)
\end{aligned}$$

   Monotonicity is satisfied.

2. Discounting: Take $\hat{J}_k(V,x) = J_k(V,x) + \Delta$. It follows

$$\begin{aligned}
\hat{J}_{k+1}(V,x) &= \max_{\{w,\{V'(x')\}\}\in\Gamma(x,V)} f(x) - w + R^{-1}\Bigg((1-\pi_o)\sum_{x'}\hat{J}_k(V'(x'),x')\pi(x',x) \\
&\quad +\pi_o\sum_{x'}\Bigg((1-q(V(x'),V_o(x')))\hat{J}_k(x',V'(x'))\Bigg)\pi(x',x)\Bigg) \\
&= \max_{\{w,\{V'(x')\}\}\in\Gamma(V,x)} f(x) - w + R^{-1}\Bigg((1-\pi_o)\sum_{x'}(J_k(V'(x'),x')+\Delta)\pi(x',x) \\
&\quad +\pi_o\sum_{x'}\Bigg((1-q(V(x'),V_o(x')))(J_k(x',V'(x'))+\Delta)\Bigg)\pi(x'|x)\Bigg) \\
&= \max_{\{w,\{V'(x')\}\}\in\Gamma(V,x)} f(x) - w + R^{-1}\Bigg((1-\pi_o)\sum_{x'}J_k(V'(x'),x')\pi(x',x) \\
&\quad +\pi_o\sum_{x'}\Bigg((1-q(V(x'),V_o(x'))J_k(V'(x'),x')\Bigg)\pi(x',x)\Bigg) \\
&\quad +R^{-1}\Delta\Bigg((1-\pi_o)+\sum_{x'}\pi_o\Bigg((1-q(V(x'))\Bigg)\pi(x'|x)\Bigg) \\
&< J_{k+1}(x,V) + R^{-1}\Delta
\end{aligned}$$

   Given that $q(V,V_o) \in (0,1)$ and from the assumption that $R > 1$ it follows that

$R^{-1} \leq 1$, discounting is satisfied.

The operator $T : B \to B$ is a contraction with modulus $\frac{1}{R}$. $\square$

Proof of proposition 2. We have to check the conditions so that the theorem of the maximum (Berge's theorem) can be applied.

1. $J(V, x)$ is continuous in V.
2. $\Gamma(V, x)$ is compact-valued and continuous in V.

1. Lemma 1.
2. $\Gamma(V, x)$ is compact-valued because the feasible set of promised utilities is compact. The promise-keeping constraint is continuous in V, so all conditions are satisfied.

$\square$

Proof of lemma 1. Pick V_1 and V_2 with $V_1 > V_2$. Denote the optimal solution at V_i by $\{w_i, \{V_i'\}\}$. Given that the optimal solution is in $\Gamma(V_i, x)$ it satisfies

$$\begin{aligned} V_i &= u(w_i) + (1-\pi_o)\sum_{x'}\beta V(x')\pi(x',x) + \pi_o\Bigg(\sum_{x'}(1-q(V_i'(x'),V_o(x_o')))\beta V_i'(x')\pi(x',x) \\ &\quad + \sum_{x'} q(V_i'(x'),V_o(x_o'))(\beta V_o(x_o') - \mu)\pi(x',x) + \sum_{x'}\Psi(q(V_i'(x'),V_o(x_o')))\pi(x',x)\Bigg) \end{aligned}$$

The right hand side is increasing in w and V', so that there exists a feasible solution $\{\hat{w}_1, \{V_1'\}\}$ such that

$$\begin{aligned} V_2 &= u(\hat{w}_1) + (1-\pi_o)\sum_{x'}\beta V(x')\pi(x',x) + \pi_o\Bigg(\sum_{x'}(1-q(V_1'(x'),V_o(x_o')))\beta V_1'(x')\pi(x',x) \\ &\quad + \sum_{x'} q(V_1'(x'),V_o(x_o'))(\beta V_o(x_o') - \mu)\pi(x',x) + \sum_{x'}\Psi(q(V_1'(x'),V_o(x_o')))\pi(x',x)\Bigg) \end{aligned}$$

and it follows directly that $J(V_2, x) \geq \hat{J}(V_2, x) \geq J(V_1, x)$ where $\hat{J}(V_2, x)$ denotes the profits resulting from $\{\hat{w}_1, \{V_1'\}\}$. $\square$

Proof of lemma 2. Given that all V_o are elements of a compact set in $\mathbb{R}$ and that $\geq$ is a partial ordering on Ω it follows directly that $(\Omega, \geq)$ is a complete lattice. $\square$

Proof of lemma 3. To prove this we first show that the fixed point of the operator T for given $V_{o,k}$ $J_k(V, V_{o,k})$ has the property that $J_k(V, V_o^1) \geq J_k(V, V_o^2)$ if $V_o^1 = V_o^2 + z$ with $z \in \mathbb{R}_+^n$. It follows then from the monotonicity of $J_k(V, V_o)$ in V that $V_{o,k+1}^1 \geq V_{o,k+1}^2$ which establishes the monotonicity of the operator P.

1. Denote by w_2 and $V_2(x')\forall x'$ the optimal strategy played at $v_{o,2}$. Rewrite the constraint as

$$v = u(w) + \beta V(x')\pi(x'|x) + \sum_{x'} \Bigg(\psi \log \left(\exp(\psi^{-1}\beta V_o(x') - \psi^{-1}\beta V(x')) + 1 \right) \Bigg) \pi(x'|x)$$

   Consider playing at $v_{o,1}$ the strategy $V_1(x') = V_2(x') + z$ and a wage strategy such that $u(w_2\gamma) - u(w_2) = -\beta \sum_{x'} z(x')\pi(x'|x)$. By properties of the utility function $\gamma < 1$. (with log, for example, we have $\gamma = e^{-\beta \sum_{x'} z(x')\pi(x'|x)} < 1$) Notice that this strategy delivers the same promised utility level to the agent and induces the same job-to-job transition probability q because $\psi^{-1}\beta V_o(x') + \beta z - \psi^{-1}\beta V(x') - \beta z$. Noticing that $\gamma(x) < 1\forall x$ and both sequences have the same probability path profits under strategy 1 must be larger $J_1(x, V) > J_2(x, V)$.

2. From proposition XX we know that $J_k(x, V)$ is decreasing in V so that it follows directly that $V_{o,k+1}^1 \geq V_{o,k+1}^2$ if $V_{o,k}^1 \geq V_{o,k}^2$.

$\square$