

On the Optimal Size of Public Employment*

Anna Carolina Dos Reis

Eduardo Zilberman

Department of Economics

Department of Economics

PUC-Rio

PUC-Rio

February 2013

Abstract

A public job can be seen as a source of insurance against income risk. Indeed, many public employees have job stability, which is compounded with a less volatile and more compressed wage distribution. Hence, by increasing its number of public employees, the government enhances the overall degree of insurance in the economy. In this paper, we introduce public employment in a standard incomplete markets model with overlapping generations. The aim is to explore the welfare gains or losses due to a larger government, accounting for this extra source of insurance. In a model economy calibrated to Brazil, we find that if the government relies on consumption taxes to balance its budget, the optimal size of public employment is nearly flat, ranging from 10 to 24 percent of the workforce. However, if the public employment is reduced from 22 to 10 percent, welfare losses due to a reduction in the degree of insurance are 4.5 percent, which are compensated by welfare gains due to level and inequality effects. Finally, if the wage distribution becomes even less volatile and more compressed, social welfare decreases.

Keywords: pubic employment, incomplete markets, optimal policy.

JEL Classification: D31, E24, H11.

*Dos Reis: annacsaba@gmail.com. Zilberman: zilberman@econ.puc-rio.br. We thank without implicating Vinícius Carrasco, Gustavo Gonzaga and Rodrigo R. Soares for insightful comments and discussions.

1 Introduction

In most countries, public sector jobs offer some advantages over private sector jobs. In particular, governments usually provide protection against dismissals for public workers. In Brazil, for instance, job stability is a right guaranteed by constitution for those that, after entering the public sector, have stayed at the job for at least three years.¹

In a similar vain, many empirical studies have found that wages in the public sector are more compressed and less dispersed than their counterparts in the private sector.²

Job stability compounded with a more compressed and less volatile wage distribution can be interpreted as a source of insurance against income risk. Indeed, whoever enters the public sector is exchanging a more volatile, but potentially higher, income for a less volatile one. Hence, by increasing its number of public employees, the government enhances the overall degree of insurance in the economy.³

The aim of this paper is to explore the welfare gains or losses due to a larger government. The novelty is to properly account for the aforementioned source of insurance. To do so, we introduce public employment in a standard incomplete markets model with overlapping generations (e.g. Huggett [1996]). In particular, the size of the government, defined by the number of agents employed in the public sector, not only affects the degree of insurance in the economy, but also its distribution of consumption. Hence, from an utilitarian perspective, whether a larger government increases or decreases welfare is an empirical question.

In a model economy calibrated to Brazil, we find that if changes in the public wage bill associated with changes in public employment are financed with consumption taxes,

¹High public job security is also present among other countries as noted in OECD [2008]: “A stronger protection against dismissals and other forms of termination of the employment is also normally a part of the special arrangements [of government employment]. This would traditionally guarantee employment for life with dismissal only possible for misconduct.”

²This pattern holds in several countries. See Gregory and Borland [1999] for a review.

³Notice that this source of insurance might not be available to everyone. If earnings dynamics in the public sector are too generous, there will be a larger number of candidates than public vacancies. Hence, a set of rules is necessary to match candidates and vacancies. In Brazil, for instance, most public servants are selected based on merit through a public exam. In particular, each exam is designed to test the knowledge necessary to perform a specific job.

the optimal size of public employment is nearly flat, ranging from 10 to 24 percent of the workforce. However, if the public employment is reduced from 22 to 10 percent, welfare losses due to a reduction in the degree of insurance are 4.5 percent, which are compensated by welfare gains due to level and inequality effects. In contrast, if changes in the wage bill are financed with lump-sum taxes, the optimal size of public employment is 4 percent of the workforce, which is associated with welfare gains of 17.7 percent due to a reduction in inequality.

The model has three main ingredients. First, we consider an overlapping generations model with heterogeneous agents. In particular, heterogeneity regards their income profiles that vary with age, a fixed level of human capital, and an uninsurable idiosyncratic risk (i.e. productivity).

Second, we consider a competitive economy with incomplete markets in the sense that borrowing-constrained agents can only save through a risk-free bond.

Third, there are two sectors: public and private. The private sector combines effective labor and capital to produce a single good. The public sector employs effective labor and capital to produce public goods, which have opposing effects on aggregate product. Since we consider a closed economy, private production is crowded out. In contrast, public goods enhance productivity in the private sector.

During their life-cycle, agents choose whether to work in the private sector or to apply for a public job. In line with the aforementioned evidence, we assume that public workers cannot be fired, but they may quit. Similarly, once in the public sector, risk becomes less volatile at the expense of a more compressed distribution of wages. Finally, we assume that income profiles also vary across sectors.

The government opens a given number of vacancies for each level of human capital it is willing to fill. Depending on the model's parameters, the public wage scheme might attract a larger number of candidates than open vacancies. If this is the case, in order to fill vacancies, the government only hires the most productive candidates. Notice that this selection mechanism emulates a public exam in which performance is positively

associated with productivity. Finally, as some agents with a high income profile in the private sector might not apply for a public job, the effects of a larger government on the overall distribution of income, wealth and consumption are ambiguous. In our benchmark calibration, for instance, only agents with intermediate levels of productivity are hired by the government.

The optimal size of public employment maximizes an ex-ante utilitarian welfare criterion. Following Conesa et al. [2009], we consider only the welfare of newborn agents. In particular, the overall welfare effect associated with a given policy is defined by how much lifetime consumption has to increase uniformly across newborn agents in the benchmark economy in order equalize welfare measures in a stationary equilibrium.

By adapting the methodology from Flodén [2001] to an environment with overlapping generations, we decompose the overall welfare effect of a change in public employment into three categories: (i) the level effect associated with changes in aggregate consumption; (ii) the inequality effect associated with changes in the distribution of consumption; and (iii) the uncertainty effect associated with changes in the degree of insurance in the economy.

Table 1 anticipates some of the results in this paper. It reports the welfare gains associated with different sizes of public employment when changes in the wage bill are financed with a single policy instrument. In our benchmark economy, public employment is calibrated at 22 percent of the workforce.

Instrument	public employ. (%)	total welfare effect (%)	level effect (%)	inequality effect (%)	uncertainty effect (%)
consumption taxes	10 to 24	0	1 to -0.1	3.4 to -0.5	-4.5 to 0.8
capital taxes	10 to 20	0.5	4.7 to 0.8	3 to 0.7	-7 to -1.2
lump-sum taxes	4	17.7	-1.2	18.3	0.7

Table 1: Results.

If the single instrument used to balance the government budget is a linear tax on consumption, the optimal size of public employment is nearly flat, ranging from 10 to 24 percent, which includes our benchmark calibration. Similar, if a linear tax on capital is consider, the optimal size ranges from 10 to 20 percent. Although total welfare gains are small in these ranges, losses due to the uncertainty effect is considerably high. If public

employment was reduced from 22 to 10 percent of the workforce, for example, losses due to uncertainty would be 4.5 and 7 percent if consumption and capital, respectively, taxes are considered. Hence, public employment is an important source of insurance in this economy.

Notice that capital taxes also amplify welfare gains due to the level effect. Intuitively, capital taxes are highly distortive in a stationary equilibrium, hindering capital accumulation.

If lump-sum taxes are considered, instead, the optimal size of public employment reduces to 4 percent of the workforce, which is associated with a total welfare effect of 17.7 percent. Notice that this welfare gain comes from the inequality effect. Intuitively, a large public sector benefits individuals with intermediate levels of productivity. Once the size of the government becomes smaller, the extra resources obtained from a reduction in the public wage bill is converted into lump-sum taxes, which particularly improves the welfare of those agents at the bottom of the consumption distribution.

We also study a policy experiment, in which public employment remains constant but the public wage distribution becomes even less volatile and more compressed. In this case, uncertainty does not affect much welfare, but the level and inequality effects generate losses. Hence, social welfare decreases.

Finally, we also claim that this model can explain, albeit imperfectly, some features of the distribution across age groups of workers in, admissions in and separations from the public sector.

The paper is structured as follows. Section 2 presents a brief review of the relevant literature. Section 3 presents the model. Section 4 presents the quantitative analysis, including the calibration procedure, results and sensitivity analysis. Section 5 concludes.

2 Related Literature

This paper relates to a vast literature studying different aspects of public policy and its welfare implications within an incomplete markets framework with heterogeneous agents and idiosyncratic risk.⁴ Flodén and Lindé [2001] and Alonso-Ortiz and Rogerson [2010], for example, study the optimal level of public insurance in an economy with distortive taxes. Public insurance, for instance, is achieved through lump-sum transfers.

Flodén and Lindé [2001], in particular, provide a strong motivation to account for public employment in this framework. They calibrate a model without public employment to both Sweden and the US economies. Given that wages are more persistent and volatile in the US than in Sweden, their model concludes that taxes and transfers (i.e. the degree of public insurance) should be higher in the US than in Sweden. However, these results would be biased if large transfer programs require a sizeable government to operate them. In particular, a sizeable government would further improve public insurance as public wages are less uncertain, which in turn would call for less generous transfers.⁵ Our paper properly accounts for this extra source of insurance associated with the size of government.

Other papers study the role of policy instruments, other than lump-sum transfers, to improve welfare. To the best of our knowledge, none of them consider public employment policies. Aiyagari and McGrattan [1998] and Flodén [2001], for instance, consider the role of public debt.⁶ Domeij and Heathcote [2004], Nishiyama and Smetters [2005], Conesa and Krueger [2006] and Conesa et al. [2009] study the effect of a variety of consumption, income and capital tax schedules. Berriel and Zilberman [2011] emphasize the role of targeted transfers to the poor. Imrohoroglu et al. [1995], Conesa and Krueger [1999], Huggett and Ventura [1999] and Storesletten et al. [1999] focus on the role of different

⁴See Heathcote et al. [2009] and Guvenen [2011] for recent reviews of this framework.

⁵Flodén and Lindé [2001] acknowledge but not address this possibility: “... although we look at wages before taxes and transfers, the relatively low degree of wage risk in Sweden may be a result of the big government sector. For example, a large fraction of the population work in the government sector and wage setting there seems to imply a significant amount of risk sharing.”

⁶To be precise, Flodén [2001] studies the interaction of lump-sum transfers and public debt.

social security arrangements. Finally, Hansen and Imrohoroglu [1992] explore the role of unemployment insurance.

In a different context, Rodrik [1998] and Rodrik [2000] explore a related idea to this paper. These articles argue that bigger governments might be an endogenous response to a higher level of external risk. As Rodrik [2000] points out:

“... relatively safe government jobs represent partial insurance against undiversifiable external risk faced by the domestic economy. By providing a larger number of “secure” jobs in the public sector, a government can counteract the income and consumption risk faced by the households in the economy.”

Also related is Jetter et al. [2011], who develop a model to study the effect of wage volatility on growth. The crucial assumption is that public wages are not volatile, but their counterparts in the private sector are. If volatility increases, both precautionary savings and the size of government increase for insurance reasons, affecting economy growth ambiguously.

Several papers study the implications of public wage and employment policies in macroeconomic workhorse models. Finn [1998] and Pappa [2009], for example, introduce public employment in standard real business cycle and new-Keynesian frameworks, respectively. Horner et al. [2007] and Quadrini and Trigari [2008] integrate public wage and employment policies into models with search and matching. However, we are not aware of any paper that introduces public employment in an incomplete markets model that follows in the tradition of Imrohoroglu [1989], Huggett [1993], Aiyagari [1994] and Huggett [1996]. This paper bridges this gap.

3 Model

We incorporate public employment in an overlapping generations framework with incomplete markets similar to Huggett [1996] and Imrohoroglu et al. [1999]. In particular, we consider a public sector, in which the government opens a given number of vacancies

every period. Agents can choose to apply for these jobs or to work in the private sector. Candidates who are not hired by the public sector work in the private sector. The aim is to study the welfare implications of public employment and wage policies.

3.1 Demographics, Preferences and Endowments

The economy is populated with overlapping generations whose decisions follow a well-defined life-cycle structure. At any point in time there is a measure one of agents indexed by age $t \in \{1, \dots, T\}$, who face an age-dependent probability of survival given by π_t . Once they reach the age of T , death is certain so $\pi_{T+1} = 0$. In order to make the distribution of population by age stationary, we assume an equal measure of agents is born at every period. Thus, at every period, agents at the age of t constitute a constant fraction $\mu_t \in (0, 1)$ of the population.

At $t = 1$, all agents have identical preferences over streams of consumption $\{c_t\}_{t=1}^T$, given by

$$\mathbb{E} \sum_{t=1}^T \beta^{t-1} \left(\prod_{i=1}^t \pi_i \right) u(c_t), \quad \text{with } u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \gamma > 0.$$

Notice we assume there is not altruism, so all bequests are accidental and distributed lump-sum to all agents alive.

Agents are not endowed with assets when they enter the labor market at $t = 1$ (i.e. when they are born). However, they are endowed with one unit of labor, which is supplied inelastically until the age of $t = T_r < T$, when they are forced to retire. Moreover, each agent experiences a productivity profile that determines the value of this unit of labor over time. In particular, this productivity profile depends on: (i) the experience at the labor market, which is equal to age t in our model; (ii) a fixed level of human capital $\theta \in \{\theta_1, \theta_2, \dots, \theta_m\}$ drawn by nature from a distribution, in which each θ has mass μ_θ , at the time the agent is born; and (iii) an uninsured idiosyncratic risk z that follows a finite state Markov chain with transition probabilities given by $\Pi(z', z) = \text{Prob}(z_{t+1} = z' | z_t = z)$, where $z, z' \in \{z_1, z_2, \dots, z_n\}$.⁷

⁷We rule out aggregate risk by assuming that this stochastic process is independent and identically

Let $s \in \{g, y\}$ be the sector an agent is working, where g stands for the public sector while y stands for the private sector.⁸ We assume the productivity profile, which may vary across sectors, is given by:

$$q_s(t, \theta, z) = \exp\{\gamma_1^s t + \gamma_2^s t^2 + \gamma_3^s(\theta) + \gamma_4^s(z)\}, \quad s \in \{g, y\}.$$

Notice that γ_1^s and γ_2^s are parameters whereas $\gamma_3^s(\cdot)$ and $\gamma_4^s(\cdot)$ are functions to be specified in the next section. Importantly, these objects may depend on the sector $s \in \{g, y\}$ the agent is working at. We assume that in the private sector, $\gamma_4^y(z) = z$, but as we discuss later, it is not clear how one's idiosyncratic productivity is affected by being employed in the public sector.

3.2 Private Production

There is a representative firm that produces consumption goods with a Cobb-Douglas function augmented with public goods,

$$Y = G^\xi K_y^\alpha H_y^{1-\alpha}, \quad \alpha, \xi \in (0, 1),$$

where K_y and H_y are aggregate capital and efficient labor units, respectively, employed at the private sector. Each period capital K_y depreciates at rate δ_y . Finally, we assume that public goods G , which are produced by the government, enhance productivity in the private sector.

3.3 Markets Arrangements

There are no insurance markets for the idiosyncratic risk z . In particular, markets are incomplete in the sense that agents can only accumulate wealth through risk-free bonds.

Moreover, agents are subject to a no-borrowing constraint.

distributed across agents.

⁸Since the public sector produces public goods G and the private sector produces consumption goods Y , we choose g and y , respectively, to denote these sectors throughout the paper.

We consider a closed economy with competitive markets. Hence, at every period, the interest rate r and the private wage rate w_y clear the markets for capital and efficient labor units, respectively.

Finally, accidental bequests are distributed lump-sum to all agents alive.

3.4 Public Sector

We suppose that the government taxes linearly labor income (τ_l), financial income (τ_a), consumption (τ_c) and bequests (τ_{beq}) in order to finance its consumption (C_g), investment in public capital (I_g), lump-sum transfers (Υ) and its payroll bill ($w_g H_g$), where w_g is the public wage rate set by the government. The government can also issue public debt D , at the equilibrium interest rate r , to finance its deficit.

The government also produces public goods G with efficient labor units H_g and capital K_g , which depreciates at a rate δ_g .⁹ In particular, we assume a Cobb-Douglas production function:

$$G = A_g K_g^\eta H_g^{1-\eta}, \quad \eta \in (0, 1),$$

where A_g is the total factor productivity in the production of public goods. Since we calibrate A_g to match the steady-state ratio G/Y we observe in the data, this formulation is general enough to accommodate a public sector in which only a fraction of public employment is used in productive activities.¹⁰

Notice that public sector production has opposing effects on aggregate product. Since we consider a closed economy, it crowds out private production. In contrast, it also enhances productivity in the private sector.

Finally, the government also manages a pay-as-you-go pension system. In particular, workers of both sectors contribute a fraction τ_{ss} of their labor income, while retired agents receive a flat benefit b . Since we calibrate the model economy to Brazil, where

⁹In a stationary equilibrium, the law of motion of public capital implies that $\delta_g = I_g/K_g$. Thus, given an investment decision I_g , K_g is determined endogenously.

¹⁰Indeed, if ω is the fraction of efficient labor units employed to produce public goods, $G = \tilde{A}_g K_g^\eta (\omega H_g)^{1-\eta} = A_g K_g^\eta H_g^{1-\eta}$, where $A_g = \tilde{A}_g \omega^{1-\eta}$.

pension schemes are in deficit, we include the pension system in the government budget constraint, which reads

$$\tau_a r(K_y + D) + \tau_c C_y + (\tau_l + \tau_{ss})(w_y H_y + w_g H_g) + \tau_{beq} beq = C_g + I_g + \Upsilon + rD + w_g H_g + B$$

in a stationary equilibrium. Notice that *beq* stands for accidental bequests and *B* stands for the aggregate level of pension benefits *b*.

We assume that tax instruments, public debt, pension benefits, and investment are exogenously set, in the sense that we calibrate them to capture how fiscal policy is conducted in Brazil. The consumption of the government is the policy variable used to balance its budget. It remains to discuss how employment is chosen and wages are set in the public sector.

3.4.1 Admission Policy

At every period, for each level of human capital $\theta \in \{\theta_1, \dots, \theta_m\}$, the government is willing to employ $\lambda(\theta)$ workers. Hence, it opens the number of vacancies necessary to accomplish this goal. Agents choose to either apply for a public job or work in the private sector. For simplicity, we assume an agent can only apply for vacancies assigned to her level of human capital. In our calibration, we proxy human capital θ by the level of schooling, which is observable by the government. In practice, depending on the complexity of the job, the government requires a minimum degree of schooling from candidates.

Depending on the model's parameters, public jobs may attract a larger number of candidates than open vacancies. If this is the case, in order to fill vacancies, the government only hires the most productive candidates.¹¹ Notice that this selection mechanism emulates a public exam in which performance is positively associated with productivity. Admissions to public jobs through public exams are widely spread across countries. In Brazil, for instance, most of the vacancies are filled with agents who perform well in a

¹¹Since labor is inelastically supplied, candidates work in the private sector if they are not hired by the government.

public exam designed to test the knowledge necessary to perform a specific job.

Although the age t also affects the productivity profile $q_s(t, \theta, z)$, $s \in \{g, y\}$, it is not clear how age t affects performance in a public exam. On one hand, older agents have more time to prepare themselves for the exam. On the other hand, performing well in a exam may require a specific skill that tends to depreciate with time, especially for those agents who spend some years working in the private sector. Hence, we assume that admission to public sector depends only on human capital θ and idiosyncratic risk z .

In a stationary equilibrium, the selection mechanism we explain above implies that for each level of θ , there is a threshold $\underline{z}(\theta)$, such that opened vacancies, necessary to keep $\lambda(\theta)$ workers in the public sector, are filled with type- θ agents who experience $z \geq \underline{z}(\theta)$. Importantly, not necessarily all type- θ agents with $z \geq \underline{z}(\theta)$ apply for a public job. Indeed, the private sector might be more attractive for some of them.

Finally, as we observe in practice, we assume public workers cannot be fired, but they may quit if the private sector becomes more attractive for them.

3.4.2 Wage Setting

Let w_y and w_g be the wage rates paid in the private and public, respectively, sectors. Recall that productivity is given by:

$$q_s(t, \theta, z) = \exp\{\gamma_1^s t + \gamma_2^s t^2 + \gamma_3^s(\theta) + \gamma_4^s(z)\}, \quad s \in \{g, y\}.$$

Since we assume that the private sector behaves competitively, the productivity profile $q_y(t, \theta, z)$ has a dual role. First, $q_y(t, \theta, z)$ is employed to produce consumption goods. Second, $w_y q_y(t, \theta, z)$ is the wage schedule in the private sector. Hence, by using data at the individual level on wages, experience and human capital, one can estimate γ_1^y , γ_2^y and $\gamma_3^y(\cdot)$ and, thus, calibrate the productivity profile in the private sector.

However, even in a competitive equilibrium, the government may choose to not remunerate productivity. In this case, $w_g q_g(t, \theta, z)$ might not be the wage schedule in

the public sector. Hence, we define a wage setting rule in the public sector denoted by $w_g \hat{q}_g(t, \theta, z)$, where

$$\hat{q}_g(t, \theta, z) = \exp\{\hat{\gamma}_1^g t + \hat{\gamma}_2^g t^2 + \hat{\gamma}_3^g(\theta) + \hat{\gamma}_4^g(z)\}.$$

In a similar fashion, we can use data on public workers to estimate $\hat{\gamma}_1^g$, $\hat{\gamma}_2^g$ and $\hat{\gamma}_3^g(\cdot)$,¹² and thus, calibrate the wage setting rule in the public sector.

We postpone to the next section the discussion on how we set $q_y(t, \theta, z)$, $q_g(t, \theta, z)$ and $\hat{q}_g(t, \theta, z)$ to solve numerically the model.

3.5 Recursive Equilibrium

In this paper, we focus on the properties of a stationary competitive equilibrium in which the measure of agents, defined over an appropriate family of subsets of the individual state space, remains invariant over time.

3.5.1 Agents' Problem

The agents make two types of decision during their lives. First, they choose how to allocate their disposable income between consumption and risk-free bonds. Second, they decide whether to work in the private or public sector. Once hired by the public sector, workers cannot be fired but they may quit. Finally, as we mentioned above, not all candidates may have the option to work in the public sector as their idiosyncratic productivity might not be high enough.¹³

In this context, there are five individual state variables: age t , a fixed level of human capital θ , a fixed idiosyncratic risk z , the previous sector s one works, and the amount of assets a accumulated. We assume that $s = y$ for those agents at the age of $t = 1$. Given

¹²Many empirical studies estimate these objects for both sectors and find substantial differences across them (e.g. Braga et al. [2009]). There are two possible complementary explanations for this discrepancy. First, the productivity profile varies across sectors. Second, productivity plays a minor role when setting public wages.

¹³Recall that for a given θ , government only hires those θ -type agents with $z \geq \underline{z}(\theta)$.

our assumptions on hiring and firing government employees, the agent problem prior to retirement, i.e. for $t < T_r$, is given by:

$$\begin{aligned}
V_t(a, z, s; \theta) &= \max_{c, s', a'} \left\{ u(c) + \beta \pi_t \sum_{z'} \Pi(z', z) V_{t+1}(a', z', s'; \theta) \right\}, \\
&\text{subject to} \\
(1 + \tau_c)c + a' &\leq [1 + (1 - \tau_a)r]a + (1 - \tau_l - \tau_{ss})w_{s'}\hat{q}_{s'}(t, \theta, z) + \Upsilon + (1 - \tau_{beq})beq, \\
c &\geq 0, \quad a' \geq 0, \\
s' &\in \begin{cases} \{y\} & \text{if } z \leq \underline{z}(\theta) \text{ and } s = y \\ \{g, y\} & \text{otherwise} \end{cases}, \\
V_{T_r}(a', z', s'; \theta) &= \tilde{V}_{T_r}(a'), \text{ for all } z', s', \theta,
\end{aligned}$$

where $\tilde{V}_{T_r}(a')$ is the value of retiring at the age of $t = T_r$. Notice we implicitly define $\hat{q}_y(t, \theta, z) = q_y(t, \theta, z)$, for all t, θ, z , so we can write a single problem for all agents.

After retiring, i.e. for $T_r \leq t < T$, the agent's problem is a cake-eating one:

$$\begin{aligned}
\tilde{V}_t(a) &= \max_{c, a'} \left\{ u(c) + \beta \pi_t \tilde{V}_{t+1}(a') \right\} \\
&\text{subject to} \\
(1 + \tau_c)c + a' &\leq [1 + (1 - \tau_a)r]a + b + \Upsilon + (1 - \tau_{beq})beq, \\
c &\geq 0, \quad a' \geq 0, \\
\tilde{V}_T(a') &= 0 \text{ for all } a'.
\end{aligned}$$

By solving the problems above, one obtains decision rules for consumption $c_t(a, z, s; \theta)$, savings $a'_t(a, z, s; \theta)$, and job sector $s'_t(a, z, s; \theta)$ along the life-cycle $t = 1, \dots, T$.

3.5.2 Definition and Policy Experiments

The definition of stationary competitive equilibrium is standard, except for the role the government has in hiring workers. In particular, (i) given prices and fiscal policies, the representative firm maximizes profits; (ii) given prices and fiscal policies, agents solve

their problems; (iii) the private wage rate w_y and the interest rate r clear the labor and capital, respectively, markets; (iv) for each age t and human capital θ , there is a stationary measure $\psi_{t,\theta}$ defined over an appropriate family of subsets of the individual state space;¹⁴ (v) the government produces public goods and chooses fiscal policy objects, which remain invariant over time, subject to a balanced budget constraint and the law of motion for public capital; finally, (vi) for each θ the government specifies the threshold $\underline{z}(\theta)$ such that it employs $\lambda(\theta)$ workers.

We are interest in welfare properties of the stationary equilibrium. In particular, we study the welfare implications of different levels of public employment, which is given by:

$$L_g = \sum_{t < T_r} \mu_t \sum_{\theta} \mu_{\theta} \int \mathbf{I}_{\{s'_t(a,z,s;\theta)=g\}} d\psi_{t,\theta}(z, a, s) = \sum_{\theta} \lambda(\theta)$$

in equilibrium.¹⁵ The policy experiment we study is to increase or decrease $\lambda(\theta)$ proportionally for all θ . In this case, public employment L_g increases or decreases, at the same time that the proportion of public workers across human capital levels remain the same.

We also study policy experiments, in which we consider different wage setting rules, $w_g \hat{q}_g(t, \theta, z)$, in the public sector.

3.5.3 Welfare Criterion

The optimal size of public employment maximizes an ex-ante utilitarian welfare criterion in a stationary equilibrium. Following Conesa et al. [2009], we consider only the welfare of newborn agents. Thus, social welfare reads

$$\sum_{\theta} \mu_{\theta} \int V_1(a, s, z; \theta) d\psi_{1,\theta}(z, a, s).$$

¹⁴The individual state space is the cartesian product of the spaces associated with the individual state variables, i.e., z, a, s .

¹⁵Notice that L_g is not equal to H_g , which is the aggregate level of efficient labor units employed at the public sector. In particular,

$$H_g = \sum_{t < T_r} \mu_t \sum_{\theta} \mu_{\theta} \int \mathbf{I}_{\{s'_t(a,z,s;\theta)=g\}} q_g(t, \theta, z) d\psi_{t,\theta}(z, a, s).$$

Throughout the paper, we report welfare effects in terms of consumption equivalence. In other words, the welfare effect associated with a given policy is defined by how much lifetime consumption has to increase uniformly across newborn agents in the benchmark economy in order to equalize social welfare measures across stationary equilibria.

Finally, by adapting the methodology from Flodén [2001] to this environment, we decompose the overall welfare effect of a change in public employment into three categories: (i) the level effect associated with changes in aggregate consumption; (ii) the inequality effect associated with changes in the distribution of consumption; and (iii) the uncertainty effect associated with changes in the degree of insurance in the economy.

4 Quantitative Analysis

This section assesses quantitatively the equilibrium effects of public employment and wage setting policies on welfare. The algorithm used to solve numerically for the stationary recursive equilibrium is standard. We use value function iterations to solve the household problem and a variant of the algorithm suggested by Imrohoroglu et al. [1999], augmented with an extra loop to pin down, for each θ , the value of $\underline{z}(\theta)$ that implies $\lambda(\theta)$ type- θ public employees.

4.1 Calibration

We calibrate the model to match some characteristics of the Brazilian economy in between 2000 and 2009.

4.1.1 Demography

We assume agents are born (i.e. enter the labor market) with 25 years old. They may live up to the age of 80, when death is certain. Each period corresponds to a five years interval, so that $T = 12$. The agents retire at the age of 65, that is $T_r = 9$. We calculate the age-dependent probability of survival, π_t , from mortality data provided by

the *Instituto Brasileiro de Geografia e Estatística* (IBGE) – the government department responsible for collecting data and processing official statistics.¹⁶

4.1.2 Productivity and Public Wage Setting

In order to specify the productivity process, one must proxy the level of human capital θ with an observable variable. In particular, we proxy θ by the degree of education an individual acquired before entering the job market. We suppose three levels of θ : (1) those with at most 10 years of schooling, which includes basic education and incomplete secondary education; (2) those who acquired between 11 and 14 years of schooling, which includes secondary education and incomplete college education; and (3) those with at least 15 years of schooling, which includes college education.

The distribution of θ is obtained from the 2009 *Pesquisa Nacional por Amostra de Domicílios* (PNAD) – an annual cross-sectional household data survey. In particular, we calculate the share of workers of each group of education, so that $\mu_{\theta_1} = 0.38$ (basic or no education), $\mu_{\theta_2} = 0.46$ (secondary education), and $\mu_{\theta_3} = 0.16$ (college education).

Recall that the productivity profile in the private sector is given by:

$$q_y(t, \theta, z) = \exp\{\gamma_1^y t + \gamma_2^y t^2 + \gamma_3^y(\theta) + z\}.$$

Under the assumption that markets behave competitively, by using data at the individual level on wages, experience and human capital, one can estimate γ_1^y , γ_2^y and $\gamma_3^y(\cdot)$. These estimations are obtained from Braga et al. [2009], who use the 2005 PNAD. In particular, $\gamma_1^y = 0.15$, $\gamma_2^y = -0.011$, $\gamma_3^y(\theta_1) = -0.54$, $\gamma_3^y(\theta_2) = 0$, and $\gamma_3^y(\theta_3) = 1.14$.

However, even in a competitive equilibrium, the government may not remunerate productivity. Hence, analogous estimations obtained from Braga et al. [2009] for public workers do not represent their productivity profile. Instead, we interpret them as the

¹⁶In particular, $\pi_t \in \{1, 0.990, 0.9870, 0.982, 0.975, 0.964, 0.948, 0.927, 0.895, 0.8440, 0.775, 0\}$.

wage setting rule in the public sector, given by:

$$\hat{q}_g(t, \theta, z) = \exp\{\hat{\gamma}_1^g t + \hat{\gamma}_2^g t^2 + \hat{\gamma}_3^g(\theta) + \hat{\gamma}_4^g(z)\}.$$

In particular, $\hat{\gamma}_1^g = 0.07$, $\hat{\gamma}_2^g = -0.006$, $\hat{\gamma}_3^g(\theta_1) = -0.60$, $\hat{\gamma}_3^g(\theta_2) = 0$, and $\hat{\gamma}_3^g(\theta_3) = 0.81$. It remains to specify $\hat{\gamma}_4^g(\cdot)$, which we turn later.

In the absence of a good strategy to estimate the productivity profile in the public sector, we suppose that productivity profiles are the same in both sectors but the government does not remunerate productivity. That is, $q_g(t, \theta, z) = q_y(t, \theta, z)$. We acknowledge this is an extreme assumption. Hence, we check sensitivity by reporting results when productivity profile varies across sectors and government remunerates productivity. That is, $q_g(t, \theta, z) = \hat{q}_g(t, \theta, z)$. In practice, reality should be in between these extremes scenarios.

4.1.3 Idiosyncratic Risk

The Markov process $\Pi(z', z)$ follows from an approximation of an AR(1) process in logs:

$$\ln z' = \rho \ln z + \varepsilon, \quad \text{where } \varepsilon \sim N(0, \sigma^2)$$

In Brazil, due to the lack of a household panel data survey, such as the Panel Study of Income Dynamics in the U.S., we cannot estimate ρ and σ properly. As an alternative strategy, we set $\rho = 0.82$ based on evidence for the U.S. economy,¹⁷ but adjust σ to match the Gini coefficient in Brazil. We consider the average of this coefficient between 2000 and 2009, calculated using multiple samples from PNAD, which is 0.56.

We use Rouwenhorst [1995]'s algorithm with 17 states to approximate this AR(1) process using a Markov chain. We assume that the initial distribution of the idiosyncratic risk is the invariant distribution associated with this Markov chain.

The Rouwenhorst [1995] method has a propriety that is useful to define $\hat{\gamma}_4^g(\cdot)$, i.e.

¹⁷The literature estimates this process to be very persistent. Flodén and Lindé [2001], for example, estimate $\rho = 0.91$, whereas French [2005] estimates $\rho = 0.98$ using annual data. Since a period in the model encompasses five years, we set $\rho = 0.96^5$.

the function that maps risk z into public wages. In particular, the transition matrix associated with the Markov chain does not depend on the variance of the AR(1) process. Hence, by reducing σ , the values of the states get more compressed, but the transition probabilities remain the same.

Many empirical studies have found that wages in the public sector are more compressed and less dispersed than their counterparts in the private sector. Hence, these empirical regularities can be captured by associating $\hat{\gamma}_4^g(z_i)$, $i = 1, \dots, n$, with the i -th state generated by the Rouwenhorst [1995]'s algorithm applied to an AR(1) process with the same persistence $\rho = 0.82$ but a smaller standard deviation than σ , say $\hat{\sigma}$.¹⁸ As the states get more compressed, whoever draws a low (high) z in the private sector might be paid more (less) in the public sector. Hence, the possibility to enter the public sector is a source of insurance in this economy.

In the absence of estimates for $\hat{\sigma}$, we set it equal to $\sigma/\sqrt{2}$. Since this parameter is of central importance, we perform some sensitivity analysis on it.

4.1.4 Preferences and Private Production

We follow the literature and set the coefficient of relative risk aversion γ at three. In addition, we set β to match the annual ratio of capital to output of three, which is obtained from national accounts provided by the IBGE.

The capital share α in Brazil is around 0.4 (e.g. Paes and Bugarin [2006]). The productivity of public goods ξ is set to 0.1. In the absence of a consensus on the magnitude of this coefficient, with estimates ranging from zero (e.g. Holtz-Eakin [1994]) to 0.2 (e.g. Lynde and Richmond [1993]), we perform sensitivity analysis on ξ . Finally, δ_y is set to match the annual ratio of investment to capital of 0.05, obtained from national accounts provided by the IBGE.

¹⁸Notice that in an extreme scenario in which $\hat{\sigma} = 0$, $\hat{\gamma}_4^g(\cdot)$ becomes constant.

4.1.5 Public Sector

The production function in the public sector is calibrated as follows. We set A_g and δ_g to match the 2001-9 average ratio of public goods production to output of 0.13 and annual ratio of public investment to public capital of 0.04, respectively, which are obtained from national accounts provided by the IBGE. In the absence of information on η , which is the parameter of the public production technology, we set it equal to its counterpart in the private sector α , which is 0.4.

We follow Pereira and Ferreira [2010] to calibrate some tax instruments. In particular, by using data on tax revenues and macroeconomic variables, we calculate the average consumption, labor income and capital tax rates, which are $\tau_c = 0.23$, $\tau_l = 0.21$ and $\tau_a = 0.14$, respectively. We follow the tax code to set the tax rate on bequests τ_{beq} at 0.04 and the contribution to the pension system τ_{ss} at 0.11, whereas the flat benefit b is set to match the 2001-9 average pension deficits as a percentage of GDP, obtained from the *Ministério da Previdência e Assistência Social* – the government branch responsible for managing the pension system.

The ratios of public investment I_g to GDP, lump-sum transfers Υ to GDP and debt D to GDP are set to 2.2, 8.4 and 47 percent, respectively. These figures are provided by the IBGE and the National Treasury. Finally, note that public consumption C_g is left free to balance the government budget.

Finally, we consider parameters related to public employment and wage policies. The public wage rate is set to match the ratio of the public wage bill to the private wage bill, i.e. $w_g H_g / w_p H_p = 0.3$, provided by the IBGE. Recall that $\lambda(\theta_1) + \lambda(\theta_2) + \lambda(\theta_3)$ is the share of public workers, calibrated at 22 percent. Hence, it remains to calibrate $\lambda(\theta_1)$ and $\lambda(\theta_2)$ to match the share of public workers with basic or no education (i.e. 18 percent) and secondary education (i.e. 45 percent). These figures are obtained from the 2009 PNAD.

4.1.6 Internally Calibrated Parameters: Summary

Table 2 summarizes the values assigned to internally calibrated parameters.

parameters	target variable	target data	target model
$\sigma = 0.09$	Gini	0.56	0.56
$\beta = 0.8$	annual K_y/Y	3	2.9
$\delta_y = 0.23$	annual I_y/K_y	0.05	0.05
$A_g = 0.75$	G/Y	0.13	0.13
$\delta_g = 0.18$	annual I_g/K_g	0.04	0.04
$w_g = 0.5$	$w_g H_g / w_y H_y$	0.30	0.31
$b = 0.29$	pension deficits/Y	0.014	0.014

Table 2: Internally calibrated parameters.

4.2 Results

This section reports the results. First, we argue that the model is able to replicate some dimensions of public employment dynamics. Second, we study the welfare implications of different public employment and wage policies. Finally, we perform some sensitivity analysis.

4.2.1 Public Employment Dynamics

The main objective of this paper is to study the welfare effects of public employment and wage policies accounting for their role in improving the insurance degree in the economy. Hence, it is desirable that the model replicates some features in the data associated with public employment dynamics. Due to data availability, we only consider the distribution across age groups of workers, admissions and separations in the public sector.¹⁹

In Figure 1, we compare the distribution of public workers across age groups in the model against the data, tabulated from the 2005 PNAD. Notice the share of workers increases up to a certain age, 35 in the model but 40 in the data, and then, declines.

¹⁹As each period in the model encompasses five years, we fit the data to age intervals. For the age of 25, for example, we group agents who have between 21 and 25 years of age; for the age of 30, we group the agents who have between 26 and 30 years of age; and so on.

However, the model predicts higher shares of both young and old workers. We conjecture that this discrepancy is due to: (1) the model does not allow retirement at different ages, so that the share of old agents are higher than in the data;²⁰ (2) in Brazil during the 2000s, agents have a more generous pension benefit if they stay longer in the public sector, so that the share of middle-aged workers are higher in the data; and (3) some public jobs require more previous training and experience (e.g. judges) than others, which might explain a smaller share of young workers.

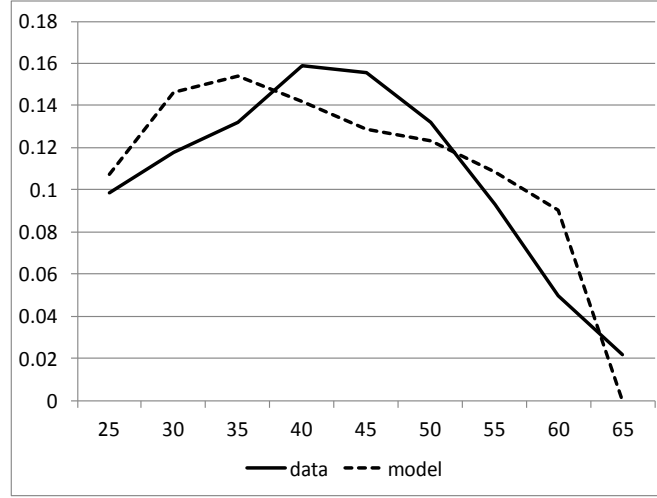


Figure 1: Distribution of Workers per Age

Data on admissions in and separations from the public sector were obtained from Pinheiro and Sugahara [2001], who reported the flows in and out of federal public servants registered by the *Sistema Integrado de Administração de Recursos Humanos* (SIAPE) – a database on federal public employees – for the years between 1995 and 1999.²¹

Figure 2 shows the distribution of admissions in the public sector across ages. Notice that the model replicates the share of admissions of old agents. However, it predicts more (less) admissions of young (middle-aged) agents. Again, we conjecture that this discrepancy is due to public job heterogeneity. Indeed, some jobs require more experience and training to be filled.

²⁰See Ferreira and dos Santos [forthcoming] for a model with endogenous choice of retirement.

²¹This dataset is not publicly available, so we could not generate these statistics for recent years.

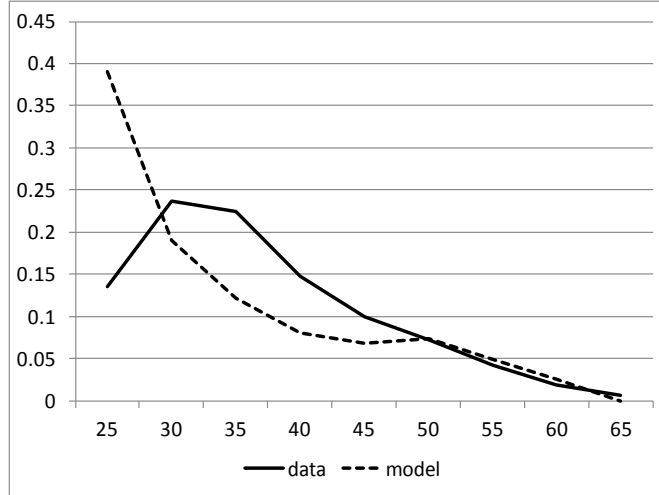


Figure 2: Distribution of Admissions per Age

Figure 3 shows the distribution of separations in the public sector across ages. Notice that, up to the age of 50, the model replicates the shape of the distribution of separations. However, it predicts less (more) separation for those with ages below (above) 50. Again, we conjecture that these discrepancies can be reconciled if we allow retirement at different ages and a more generous benefit if agents stay longer in the public sector.

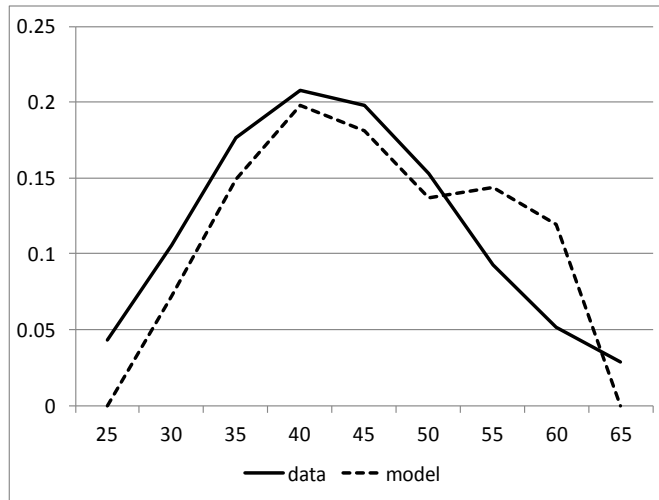


Figure 3: Distribution of Separations per Age

We conclude that, albeit imperfectly, this model can replicate important dimensions of the data associated with public employment.

Finally, Figure 4 plots the distribution of private and public workers across idiosyncratic risks z_i , $i = 1, \dots, 17$, for each level of education. In particular, the thresholds to

enter the public sector associated with basic or no education, secondary education and college education are $i = 11, 11$ and 7 , respectively.

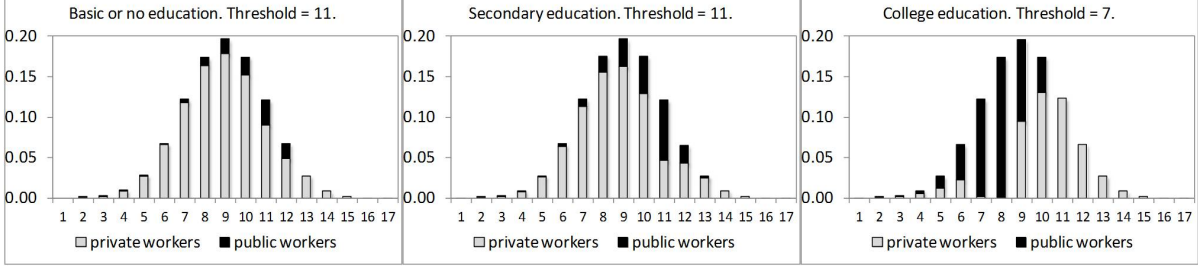


Figure 4: Distribution of Workers per Productivity

Notice that public employment is effective in increasing the welfare of intermediate types. Indeed, those with high shocks prefer to work in the private sector, whereas those with low shocks cannot enter the public sector. Moreover, a sizable number of individuals benefits from the fact that they cannot be fired.

4.2.2 Welfare Effects: Public Employment

This section shows our main results. In particular, we report the welfare implications of different sizes of public employment. Once the government changes the size of public employment, it affects the public wage bill and, thus, has to adjust its fiscal policy in order to balance its budget. We consider three types of policy adjustments: (1) consumption taxes τ_c ; (2) capital taxes τ_a ; and (3) lump-sum transfers Υ .²²

Results considering a lump-sum tax adjustment should be read with caution. As we argue above, lump-sum transfers capture the role of large welfare programs which require public workers to operate them. Hence, in practice, exchanging public employment for lump-sum transfers might not be feasible. In contrast, a simple change in the capital or consumption tax rate could be designed without an effective change in public employment.

Figure 5 plots the welfare gains (y-axis) against changes in the size of public employment ranging from 2 to 24 percent (x-axis). Recall that in our benchmark calibration,

²²We do not consider income taxes τ_l because labor is supplied inelastically. Hence, adjustments in τ_l are not distortive.

public employment is set to be 22 percent of the workforce.

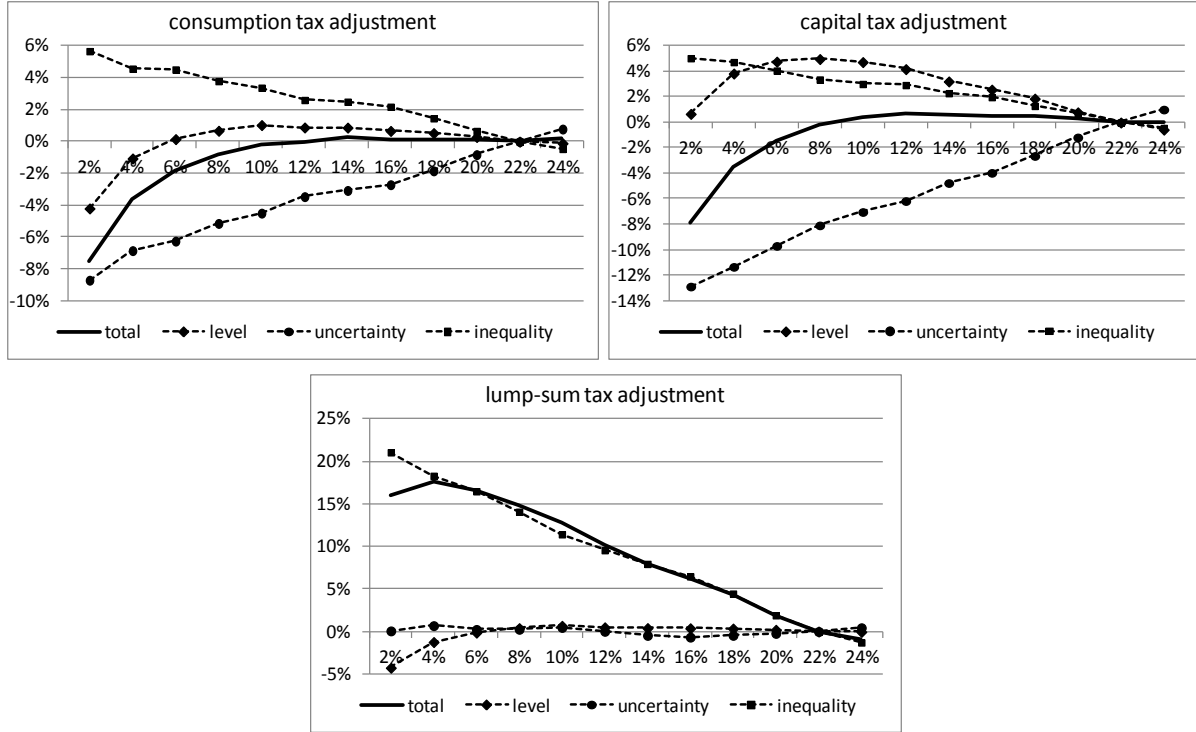


Figure 5: Welfare Implications (Public Employment)

First, consider the experiment with consumption tax adjustment (top-left plot). Although the social welfare remains flat for shares of public employment ranging from 10 to 24 percent, public employment plays an important role to increase the degree of insurance in the economy. Indeed, if the government reduced public employment to ten percent, the welfare loss due to uncertainty would be 4.5 percent. This loss is counteracted by an increase in welfare of one and 3.4 percent due to level and inequality, respectively, effects.

Second, results considering a capital tax adjustment (top-right plot) are qualitatively similar. Quantitatively, the optimal public employment ranges from 12 to 18 percent of the workforce, which represents a total welfare gain of roughly 0.5 percent. Moreover, welfare gains due the level effect and losses due to the uncertainty effect are amplified in comparison with the previous case. If the government reduced public employment

from 22 to ten percent, these figures would be 4.7 percent and 7 percent, respectively. Intuitively, capital taxes are highly distortive, hindering capital accumulation.

Finally, welfare gains can be considerably high if the government is allowed to exchange public employment for lump-sum transfers (bottom plot). In this case, the optimal level of public employment is four percent, which would represent a welfare gain of 17.7 percent. This gain is due to the inequality effect. Intuitively, as Figure 4 highlights, a large public sector benefits individuals with intermediate levels of productivity. Once the size of the government becomes smaller, the extra resources obtained from the reduction in the public wage bill are distributed lump-sum, which particularly improves the welfare of those agents at the bottom of the consumption distribution. Hence, consumption is distributed from intermediate to low types, increasing social welfare.

4.2.3 Welfare Effects: Public Wages as a Source of Insurance

Recall that in our benchmark calibration we set $\hat{\sigma}^2 = \iota\sigma^2$, with $\iota = 1/2$. By relying on a property of Rouwenhorst [1995] method, we argue that $\hat{\sigma}$ only governs how the public sector remunerates the idiosyncratic risk. In particular, a lower $\hat{\sigma}$ implies a less volatile and more compressed wage distribution. In this section, we study a policy experiment, in which the government varies ι from 0 to 0.6.²³

Figure 6 plots the welfare implications when the government adjusts consumption taxes to balance its budget.

Notice that if the government reduced $\hat{\sigma}$ to zero, the total welfare losses would be 5.6 percent. In particular, the uncertainty effect increases welfare a bit, but the level and inequality effects generate large welfare losses.

²³We cannot set ι higher than 0.6, otherwise there are less candidates than open vacancies in the public sector. Intuitively, a more volatile public wage schedule reduces the attractiveness of the public sector.

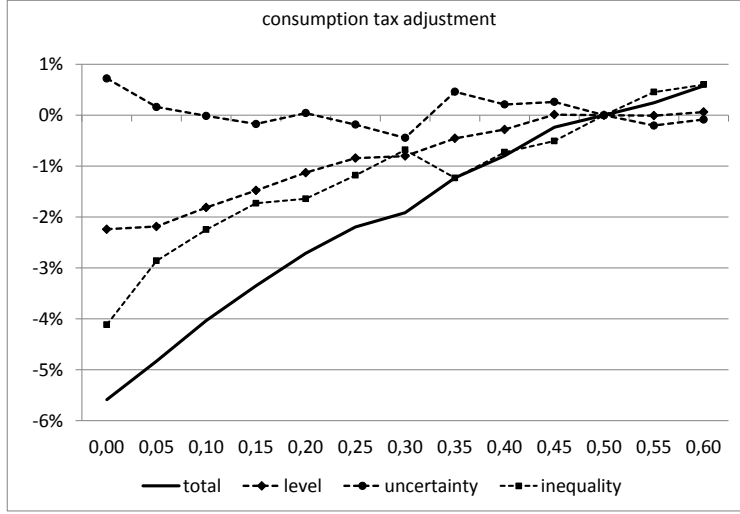


Figure 6: Welfare Implications (Public Wage Volatility)

4.3 Sensitivity Analysis

In this section, we check whether our results are sensitive to: (i) a different productivity profile in the public sector; (ii) different values of ξ , which captures the productivity of public goods. In both cases, we recalibrate the model to match the targets in Table 2.

4.3.1 Different Productivity Profiles ($q_g(t, \theta, z) = \hat{q}_g(t, \theta, z)$)

In the absence of a good strategy to estimate the productivity profile in the public sector, our benchmark results consider an extreme case in which productivity profiles are the same in both sectors but the government does not remunerate productivity. That is, $q_g(t, \theta, z) = q_y(t, \theta, z)$. Here, we suppose that the productivity profile varies across sectors and government remunerates productivity. That is, $q_g(t, \theta, z) = \hat{q}_g(t, \theta, z)$. In practice, reality should be in between these extremes scenarios.

In a stationary equilibrium, $q_g(t, \theta, z)$ only affects the aggregate level of efficient labor units employed at the public sector H_g . Indeed, since we calibrate the public total factor productivity A_g to match the ratio of public goods to product G/Y we observe in the data, any reduction in H_g is absorbed by an increase in A_g .²⁴ Hence, except for A_g and

²⁴In particular A_g increases from 0.75 to 0.96.

H_g , the stationary equilibrium has the same properties in both this and the benchmark cases.

Figure 7 shows the welfare implications under this alternative productivity profile.

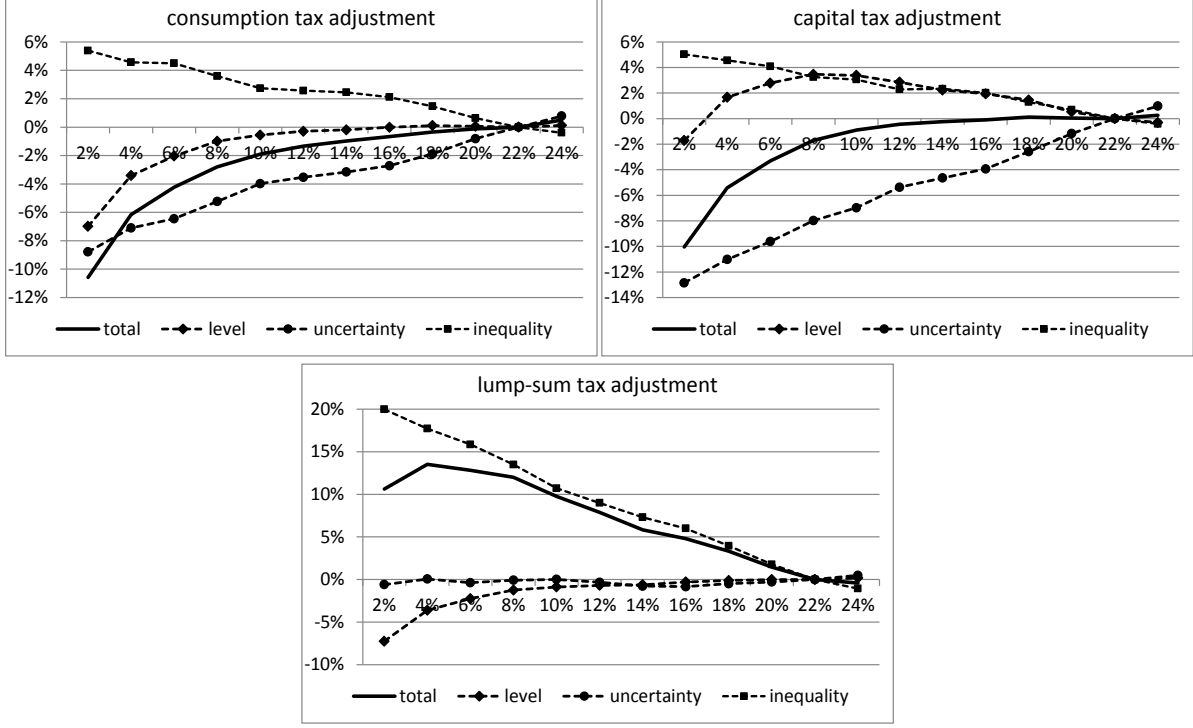


Figure 7: Welfare Implications (Alternative Productivity Profile)

The welfare implications due to both uncertainty and inequality effects are almost the same to the benchmark case. Intuitively, since agents are remunerated according to $w_s \hat{q}_g(t, \theta, z)$ when working in the public sector, the productive profile $q_g(t, \theta, z)$ does not enter directly in their optimization problems. However, they depend indirectly on $q_g(t, \theta, z)$, as it may affect prices r and w and thresholds $\underline{z}(\theta)$ through A_g and H_g . Hence, we conclude that general equilibrium and selection effects are not strong enough to modify the welfare implications due to uncertainty or inequality.

In contrast, welfare gains due to the level effect are smaller in this case. Intuitively, once the government reduces public employment, workers leave a relatively more produc-

tive public sector than in the previous case, which mitigates welfare gains. Nonetheless, the difference of level effects is not that large. For example, if the government reduced public employment to 10 percent of the workforce, the level effect would be near 1.5 percent higher in the benchmark case independent of the tax instrument used to balance its budget.

4.3.2 Productivity of Public Goods (ξ)

In this section, we analyze the role of ξ , which governs the productivity of public goods. In particular, we consider $\xi = 0.05$ and $\xi = 0.2$. Figure 8 plots total and level welfare effects for different values of ξ and tax adjustments.

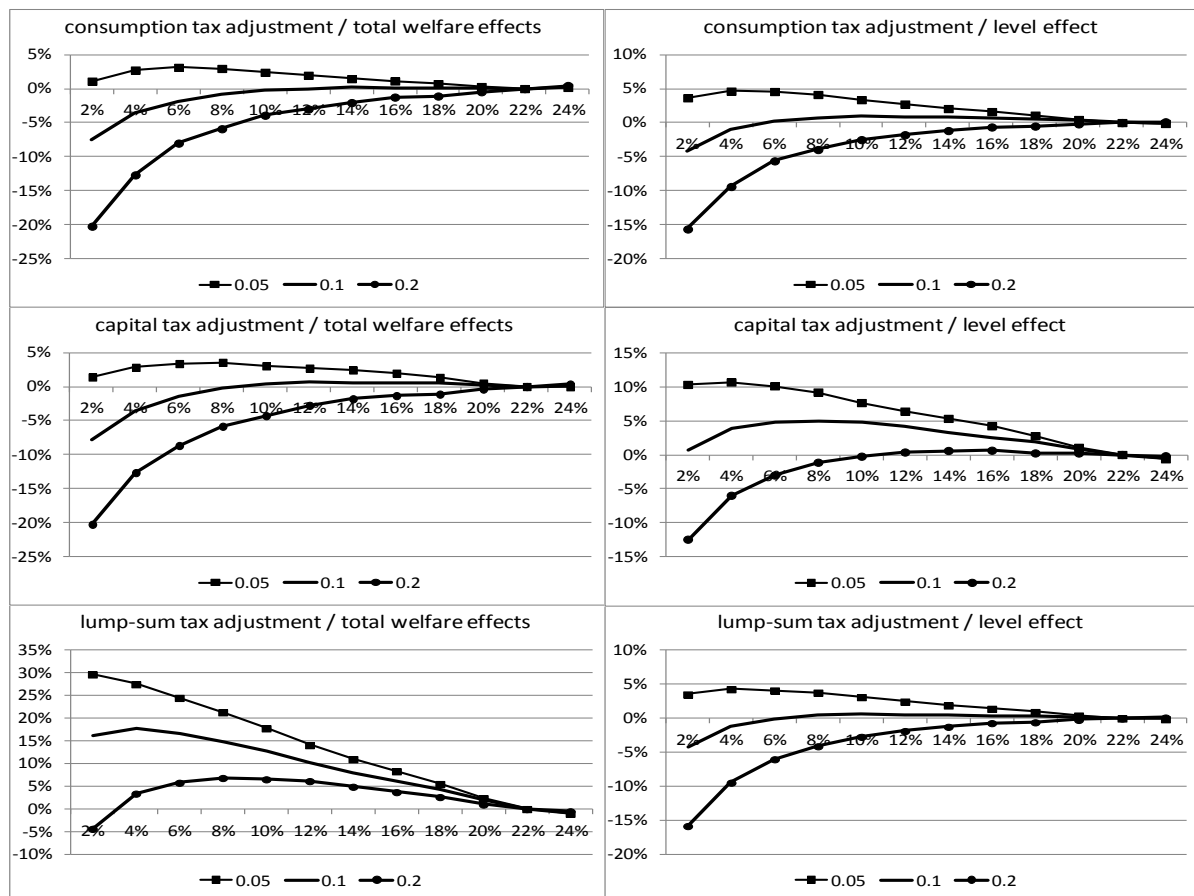


Figure 8: Welfare Implications (Alternative Valuer for ξ)

Notice that, as in the previous case, the level effect explains most of the changes in welfare gains due to different values of ξ . A similar intuition applies.

We conclude from these sensitivity analyses that, although a misspecification of the production technology associated with the public sector might bias social welfare, its part due to uncertainty and inequality effects is fairly robust to this misspecification.

5 Conclusion

In this paper, we show that public employment is an important source of social insurance. In our preferred experiment, although optimal public employment is nearly flat, ranging from 10 to 24 percent, if public employment was reduced from 22 to 10 percent of the workforce, losses due to a decrease in the degree of insurance would be 4.5 percent. Of course, this effect is counteracted by welfare gains due to inequality and level effects. Importantly, the welfare gains due to uncertainty are fairly robust to a misspecification of the production technology associated with the public sector. Finally, we also show that reducing the volatility of the public wage distribution may not improve the degree of insurance in the economy.

References

- S. R. Aiyagari. Uninsured idiosyncratic risk and aggregate saving. *Quarterly Journal of Economics*, 109(3):659–684, 1994.
- S. R. Aiyagari and E. R. McGrattan. The optimum quantity of debt. *Journal of Monetary Economics*, 42(3):447–469, 1998.
- J. Alonso-Ortiz and R. Rogerson. Taxes, transfers and employment in an incomplete marktes model. *Journal of Monetary Economics*, 57(8):949–958, 2010.
- T. C. Berriel and E. Zilberman. Targeting the poor: A macroeconomic analysis of cash transfer programs. Working Paper, 2011.
- B. G. Braga, S. Firpo, and G. Gonzaga. Escolaridade e diferencial de rendimentos entre o setor privado e o setor público no brasil. *Pesquisa e Planejamento Econômico*, 39(3):431–464, 2009.
- J. C. Conesa and D. Krueger. Social security reform with heterogeneous agents. *Review of Economic Dynamics*, 2(4):757–795, 1999.
- J. C. Conesa and D. Krueger. On the optimal progressivity of the income tax code. *Journal of Monetary Economics*, 53(7):1425–1450, 2006.
- J. C. Conesa, S. Kitao, and D. Krueger. Taxing capital? not a bad idea after all. *American Economic Review*, 99(1):25–48, 2009.
- D. Domeij and J. Heathcote. On the distributional effects of reducing capital taxes. *International Economic Review*, 45(2):523–554, 2004.
- P. C. G. Ferreira and M. R. dos Santos. The effect of social security, demography and technology on retirement. *Review of Economic Dynamics*, forthcoming.
- M. G. Finn. Cyclical effects of government’s employment and goods purchases. *International Economic Review*, 39(3):635–657, 1998.

- M. Flodén. The effectiveness of government debt and transfers as insurance. *Journal of Monetary Economics*, 48(1):81–108, 2001.
- M. Flodén and J. Lindé. Idiosyncratic risk in the u.s. and sweden: Is there a role for government insurance. *Review of Economic Dynamics*, 4(2):406–437, 2001.
- E. French. The effects of health, wealth, and wages on labour supply and retirement behaviour. *Review of Economic Studies*, 72(2):395–427, 2005.
- R. G. Gregory and J. Borland. Recent developments in public sector labor markets. In O. Ashenfelter and D. Card, editors, *Handbook of Labor Economics, Volume 3*, chapter 53, pages 3573–3630. Elsevier B.V., 1999.
- F. Guvenen. Macroeconomics with heterogeneity: A practical guide. *Federal Reserve Bank of Richmond Economic Quarterly*, 97(3):255–326, 2011.
- G. D. Hansen and A. Imrohoroglu. The role of unemployment insurance in an economy with liquidity constraints and moral hazard. *Journal of Political Economy*, 100(1):118–142, 1992.
- J. Heathcote, K. Storesletten, and G. L. Violante. Quantitative macroeconomics with heterogeneous households. *Annual Review of Economics*, 1:319–354, 2009.
- D. Holtz-Eakin. Public-sector capital and the productivity puzzle. *Review of Economics and Statistics*, 76(1):12–21, 1994.
- J. Horner, L. R. Ngai, and C. Olivetti. Public eenterprises and labor market performance. *International Economic Review*, 48(2):363–384, 2007.
- M. Huggett. The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of Economic Dynamics and Control*, 17(5-6):953–969, 1993.
- M. Huggett. Wealth distribution in life-cycle economies. *Journal of Monetary Economics*, 38(3):469–494, 1996.

- M. Huggett and G. Ventura. On the distributional effects of social security reform. *Review of Economic Dynamics*, 2(3):498–531, 1999.
- A. Imrohoroglu. The welfare cost of inflation under imperfect insurance. *Journal of Economic Dynamics and Control*, 16(1):79–91, 1989.
- A. Imrohoroglu, S. Imrohoroglu, and D. H. Jones. A life cycle analysis of social security. *Economic Theory*, 6(1):83–114, 1995.
- A. Imrohoroglu, S. Imrohoroglu, and D. H. Jones. Computing models of social security. In R. Marimon and A. Scott, editors, *Computational Methods for the Study of Dynamic Economies*, pages 221–237. Oxford University Press, 1999.
- M. Jetter, A. Nikolsko-Rzhevskyy, and W. T. Smith. The effects of wage volatility on growth. Working paper, 2011.
- C. Lynde and J. Richmond. Public capital and total factor productivity. *International Economic Review*, 34(2):401–414, 1993.
- S. Nishiyama and K. Smetters. Consumption taxes and economic efficiency with idiosyncratic wage shocks. *Journal of Political Economy*, 113(5):1088–1115, 2005.
- OECD. *The State of Public Service*. OECD, 2008.
- N. L. Paes and M. N. S. Bugarin. Parametros tributários da economia brasileira. *Estudos Econômicos*, 36(4):699–720, 2006.
- E. Pappa. The effects of fiscal shocks on employment and the real wage. *International Economic Review*, 50(1):217–244, 2009.
- R. A. C. Pereira and P. C. G. Ferreira. Avaliação dos impactos macro-econômicos e de bem-estar da reforma tributária no brasil. *Revista Brasileira de Economia*, 64(2):191–208, 2010.

- S. S. Pinheiro and T. Sugahara. Perfil dos funcionários públicos ativos nas Áreas federal, estadual e municipal – comparação de bases disponíveis: Rais, pnad e siapi. IPEA Texto para Discussão 837, 2001.
- V. Quadrini and A. Trigari. Public employment and the business cycle. *Scandinavian Journal of Economics*, 109(4):723–742, 2008.
- D. Rodrik. Why do more open economies have bigger governments? *Journal of Political Economy*, 106(5):997–1032, 1998.
- D. Rodrik. What drives public employment? *Review of Development Economics*, 4(3): 229–243, 2000.
- K. G. Rouwenhorst. Asset pricing implications of equilibrium business cycle models. In T. F. Cooley, editor, *Frontiers of Business Cycle Research*, pages 294–330. Princeton University Press, 1995.
- K. Storesletten, C. Telmer, and A. Yaron. The risk-sharing implications of alternative social security arrangements. *Carnegie-Rochester Conference Series on Public Polic*, 50(1):213–259, 1999.

Appendix

A Welfare Decomposition

The methodology used to decompose the welfare gains is based on Flodén [2001]. In particular, we adapt it to an environment with overlapping generations in which social welfare weights only newborn agents under the veil of ignorance. For further discussion on this methodology we refer the aforementioned article.

First, note that the expected lifetime utility of a newborn agent, i.e. with age $t = 1$, with human capital θ at state (a, z, s) is given by

$$V_1(a, z, s; \theta) = \mathbb{E} \left[\sum_{t=1}^T \beta^{t-1} \left(\prod_{i=1}^t \pi_i \right) \frac{c_t^{1-\gamma}}{1-\gamma} \middle| (a, z, s) \right].$$

The ex-ante utilitarian social welfare is given by the expected lifetime utility of a newborn agent under the veil of ignorance, which reads

$$W = \sum_{\theta} \mu_{\theta} \int V_1(a, z, s; \theta) d\psi_{1,\theta}(a, z, s).$$

Define economy A as the benchmark economy and economy B as the new stationary equilibrium after the policy change. We define total welfare gains ω by how much lifetime consumption has to increase uniformly across newborn agents in the benchmark economy in order equalize welfare measures in a stationary equilibrium.

Definition 1. *The total welfare gains ω of a given policy change is defined implicitly by*

$$\sum_{\theta} \mu_{\theta} \int \mathbb{E} \left[\sum_{t=1}^T \beta^{t-1} \left(\prod_{i=1}^t \pi_i \right) \frac{[(1+\omega)c_t^A]^{1-\gamma}}{1-\gamma} \middle| (a, z, s) \right] d\psi_{1,\theta}(a, z, s) = W^B.$$

Notice we use superscripts A and B to denote objects in their respective economies. The left hand side measures the social welfare under a hypothetical percentage change of ω in lifetime consumption, while the right hand side measures social welfare under the

new policy. Finally, it can be shown that $\omega = (W^B/W^A)^{1/(1-\gamma)} - 1$.

The total welfare effect can be decomposed into three categories: (i) the level effect associated with changes in aggregate consumption; (ii) the inequality effect associated with changes in the distribution of consumption; and (iii) the uncertainty effect associated with changes in the degree of uncertainty in the economy.

Consider the level effect. Define average consumption by

$$C = \sum_t \mu_t \sum_\theta \mu_\theta \int c_t(a, z, s; \theta) d\psi_{t,\theta}(a, z, s).$$

The level effect ω^{lev} is the percentage change in average consumption due to the new policy.

Definition 2. *The level effect ω^{lev} is given by*

$$\omega^{lev} = \frac{C^B}{C^A} - 1.$$

Consider the inequality and uncertainty effects. Let the certainty equivalent consumption bundle $\{\bar{c}(a, z, s; \theta)\}_{t=1}^T$ of a newborn agent at state (a, z, s) with human capital θ be defined implicitly by

$$V_1(a, z, s; \theta) = \sum_{t=1}^T \beta^{t-1} \left(\prod_{i=1}^t \pi_i \right) \frac{\bar{c}(a, z, s; \theta)^{1-\gamma}}{1-\gamma}.$$

Hence, the average certainty equivalent consumption of a newborn agent is given by

$$\bar{C} = \sum_\theta \mu_\theta \int \bar{c}(a, z, s; \theta) d\psi_{1,\theta}(a, z, s).$$

Notice that in a stationary equilibrium, $\bar{C}$ is also the average certainty equivalent consumption of all agents.

Let p^{unc} and p^{ine} be the cost associated with uncertainty and inequality, respectively.

In particular, p^{unc} is implicitly defined by

$$\sum_{t=1}^T \beta^{t-1} \left(\prod_{i=1}^t \pi_i \right) \frac{[(1 - p^{unc})C]^{1-\gamma}}{1 - \gamma} d\psi_{1,\theta}(a, z, s) = \sum_{t=1}^T \beta^{t-1} \left(\prod_{i=1}^t \pi_i \right) \frac{\bar{C}^{1-\gamma}}{1 - \gamma} d\psi_{1,\theta}(a, z, s).$$

In a stationary equilibrium, p^{unc} captures the cost of uncertainty in an equalitarian society, in which all agents consume the same amount of goods. It can be shown that $p^{unc} = \bar{C}/C - 1$.

Definition 3. *The uncertainty effect ω^{unc} is given by*

$$\omega^{unc} = \frac{1 - p^{unc,B}}{1 - p^{unc,A}} - 1 = \frac{\bar{C}^B}{\bar{C}^A} \frac{C^A}{C^B} - 1.$$

Similarly, p^{ine} is implicitly defined by

$$\sum_{t=1}^T \beta^{t-1} \left(\prod_{i=1}^t \pi_i \right) \frac{[(1 - p^{ine})\bar{C}]^{1-\gamma}}{1 - \gamma} d\psi_{1,\theta}(a, z, s) = W.$$

In a stationary equilibrium, p^{ine} captures the cost of inequality. It can be shown that $p^{ine} = W^{1/(1-\gamma)}/\bar{C} \times \text{constant} - 1$.

Definition 4. *The inequality effect ω^{ine} is given by*

$$\omega^{ine} = \frac{1 - p^{ine,B}}{1 - p^{ine,A}} - 1 = \frac{\bar{C}^A}{\bar{C}^B} \left(\frac{W^B}{W^A} \right)^{\frac{1}{1-\gamma}} - 1.$$

Finally, we can apply the previous definitions to prove the following proposition adapted from Flodén [2001].

Proposition 1. *Total welfare effect ω is decomposable into a level effect ω^{lev} , a inequality effect ω^{ine} , and a uncertainty effect ω^{unc} according to the following equation:*

$$(1 + \omega) = (1 + \omega^{lev})(1 + \omega^{ine})(1 + \omega^{unc}).$$