The Informativeness of Stock Prices, Misallocation and Aggregate Productivity*

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Abstract

Capital markets function as aggregators of private information and in an environment with imperfectly informed firms, guide investment and production decisions. We study the implications of poorly functioning capital markets for the misallocation of factors of production across heterogeneous firms. Our theoretical framework combines a noisy rational expectations model of asset markets with a standard model of production by heterogeneous firms. We use a model calibrated to cross-country stock market and firm-level data to investigate the extent to which differences in capital market conditions can explain TFP differences across countries.

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1 Introduction

A key role of financial markets is to aggregate dispersed information across investors. In an environment with imperfectly informed firms, the resulting market price of an equity claim on its profits may convey additional information to the firm regarding its prospects and influence real allocations in the economy. To the extent that financial markets are heterogeneous in their informational content, that is, some markets are less developed, or "noisier," than others, there may be a significant effect on the efficiency of resource allocations and through this channel, on aggregate performance of the economy.

In this paper, we propose a theory explicitly linking the informational content of financial markets to the efficiency of resource allocations, which in turn, impact macroeconomic variables such as aggregate productivity and output. Heterogeneous firms choose productive inputs observing only a noisy signal of their true productivity. Financial markets aggregate dispersed information across investors, but only imperfectly, and thus while not perfectly revealing of the firm's true state, convey an additional source of information to firms making their input decisions. Our framework then provides a tight link between the informativeness of stock prices, the degree of misallocation, and aggregate economic outcomes. We exploit this feature of the model to quantitatively evaluate the importance of information aggregation through financial markets in improving allocative efficiency and hence aggregate productivity and output. In particular, we are able to compute the losses to developing economies from having access to poorly functioning, and in particular less informative, financial markets, or in reverse, the potential gains from accessing markets with a level of informativeness observed in more developed countries such as the US. Preliminary results indicate that such gains would be substantial: [COMING SOON].

Our point of departure is a standard model of production by heterogenous firms and firm dynamics along the lines of Hopenhayn (1992). Imperfect information is modeled as the assumption that firms observe only noisy signals of their true productivity when choosing their factors of production. We characterize the competitive equilibrium in the presence of this type of friction, taking as given the information structure in the economy. The model shows that allocative efficiency and hence aggregate productivity and output are increasing in the precision of the firm's information and delivers a closed-form expression relating that precision, captured by the posterior variance of firms' beliefs over their own productivity, to these aggregates.

The information structure of the economy, and in particular the information conveyed to firms by financial markets, builds upon recent work under the noisy rational expectations paradigm by Albagli et al. (2011a) and Albagli et al. (2011b). Investors with access to private information about firm-level fundamentals trade claims to firm profits in a competitive stock market. Prices aggregate private information, but noise traders prevent the price from perfectly revealing all information. In other words, the price of its own stock provides the firm with an additional, albeit noisy, signal of fundamentals. Firms rely on this signal when making input decisions, with the extent of such reliance depending on its information content. Together then, these two building blocks capture the relationship we seek to explore: the informativeness of financial markets contributes to the efficiency of resource allocation and thereby to aggregate productivity and output.

We provide a quantitative evaluation of this question by calibrating the model to firm-level production and financial market data across a number of countries. Our production structure allows us to infer the degree of misallocation by examining dispersion in the marginal value of productive inputs. Our theory provides a tight link between the informativeness of stock prices and firm-level production variables. In particular, the strength of the relationship between firm performance, i.e., profits or earnings, and equity prices, and additionally the sensitivity of firm actions to those prices jointly pin down the informational content of financial markets. Intuitively, a higher correlation between asset prices and real performance and a stronger response of firms to movements in those prices reveal a greater degree of informativeness.

The calibrated model reveals significant dispersion in the performance of financial markets and in turn, a meaningful impact of these differences on allocative efficiency and so aggregate productivity and output. [COMING SOON]

Our paper relates to several existing branches of literature. Recent work such as Hsieh and Klenow (2009) and Restuccia and Rogerson (2008) have utilized a similar production structure to explore the impact of misallocation across countries. Our contribution here is to put a particular interpretation on the firm-level distortions implied by the data and investigate the influence of a particular factor we think important, namely the ability of financial markets to deliver useful information to the firm.

Greenwood and Jovanovic (1990) also investigate the effect of financial market development on resource allocation. Their focus in on the growth effects of reallocating investments to high-risk/high-reward projects, whereas we hone in on the implications for cross-sectional allocative efficiency of input usage across heterogeneous productive units...

Lastly, a number of papers have empirically investigated differences in the informational content of financial markets across countries. Recent examples include Durnev et al. (2003) and Morck et al. (2000)...

2 The Model

In this section, we develop our model of imperfect information in production and the role of financial markets in aggregating dispersed information across investors and delivering an additional source of information to firms. We turn first to the economy's production structure, outlining the impact of imperfect information on misallocation and aggregate productivity, but taking as giving the extent of information that is actually available to the firm. The goal here is make precise the effects informational frictions on the economy, remaining quite general how the level of information is determined. We next turn to the informational structure of the economy and and the mechanism through which financial markets aggregate dispersed information across investors and so deliver an additional piece of useful information to firms...

2.1 Imperfect Information in Production

A continuum of competitive firms produce a single consumption good and are heterogeneous over productivity A. The production technology for firm i and time t takes the form

$$Y_{it} = A_{it} N_{it}^{\alpha}, \alpha < 1$$

Production exhibits decreasing returns in labor, which is the only factor of production and which is supplied inelastically with aggregate endowment N. Firm-level productivity evolves stochastically according to the AR(1) process:

$$a_{it} = (1-\rho)\,\overline{a} + \rho a_{it-1} + \mu_{it}, \mu_{it} \sim \mathcal{N}\left(0, \sigma_{\mu}^{2}\right)$$

where we use lower-case to denote natural logs, a convention we follow throughout, so that, e.g., $a_{it} = \log A_{it}$. Labor market clearing implies total labor demand and supply be equated:

$$\int N_{it} di = N$$

Firms choose their labor input without knowledge of their true productivity, but rather having observed only a noisy signal with which to form expectations. Setting the price of output as numeraire and denoting with W the wage, the firm's profit maximization problem can be written as

$$\max_{N_{it}} \mathbb{E}_{it}\left[A_{it}\right] N_{it}^{\alpha} - W N_{it}$$

where $\mathbb{E}_{it}\left[A_{it}\right]$ denotes firm i's expectation of its time t productivity conditional on its information set when choosing its labor input. The first order condition for this problem is

$$\alpha N_{it}^{\alpha-1}\mathbb{E}_{it}\left[A_{it}\right]=W$$

i.e., firms choose labor to equate the expected marginal product of labor to the wage. Imposing labor market clearing to eliminate the wage gives firm employment and output as

$$\begin{aligned}
N_{it} &= \frac{\left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}} N}{\int \left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}} di} \\
Y_{it} &= A_{it}\left(\frac{\left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}} N}{\int \left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}} di}\right)^{\alpha}
\end{aligned}$$

To maintain tractability, we make two distributional assumptions, one on the structure of information for a single firm through time as well as one for the cross-section of productitivies across firms. The first is that conditional on information available at the time of input choices, the realizations of productivities are log-normal, that is, $a_{it}\sim\mathcal{N}\left(\widehat{a}_{it},\mathbb{V}\right)$, where $\widehat{a}_{it}=\mathbb{E}_{it}\left[a_{it}\right]$ denotes the conditional expectation of log productivity. The second is that the cross-sectional distribution of productivity and expected probabilities are jointly log-normal, i.e.,

$$X_{it}=\begin{bmatrix} a_{it} \\ \widehat{a}_{it}\end{bmatrix}\sim\mathcal{N}\left(\begin{bmatrix}\bar{a}\\ \bar{a}\end{bmatrix},\begin{bmatrix}\sigma_a^2 & r\sigma_a\sigma_{\widehat{a}}\\ r\sigma_a\sigma_{\widehat{a}} & \sigma_{\widehat{a}}^2\end{bmatrix}\right)$$

Note that

$$\begin{aligned}
\sigma_{\widehat{a}}^2 &= \sigma_a^2-\mathbb{V} \\
\mathbb{V} &= \sigma_a^2\left(1-r^2\right)
\end{aligned}$$

which implies

$$r\sigma_a\sigma_{\widehat{a}}=\sigma_a^2-\mathbb{V}$$

Using this structure, in conjunction with well-known properties of the log-normal distribution, we can write the log of firm-level employment and output as

$$\begin{aligned}
n_{it} &= \frac{1}{1-\alpha}\widehat{a}_{it}+n-\log\left(\int\exp\left(\frac{1}{1-\alpha}\widehat{a}_{it}\right)di\right) \\
y_{it} &= a_{it}+\frac{\alpha}{1-\alpha}\widehat{a}_{it}+\alpha n-\frac{\alpha}{1-\alpha}\bar{a}-\alpha\frac{1}{2}\left(\frac{1}{1-\alpha}\right)^2\sigma_{\widehat{a}}^2
\end{aligned}$$

Finally, we can write aggregate output succinctly as a function of model primitives as

$$y_t = \alpha n + \overline{a} + \frac{1}{2}\left(\frac{\sigma_a^2 - \alpha\mathbb{V}}{1-\alpha}\right)$$

where are taking as given the structure of information in the economy. It is straightforward to see that

$$\frac{dy_t}{d\mathbb{V}} = -\frac{1}{2}\left(\frac{\alpha}{1-\alpha}\right) < 0$$

i.e., output is decreasing in the posterior variance of a, which serves as a measure of informational frictions in the economy. Put another way, denoting with y_t^* the first-best allocation, which occurs where $\mathbb{V} = 0$,

$$y_t - y_t^* = -\frac{1}{2}\left(\frac{\alpha}{1-\alpha}\right)\mathbb{V}$$

captures the output loss caused by the presence of uncertainty regarding A.

Our framework is tractable enough to fully characterize the cross-sectional joint distribution of productivity, output, and employment as

$$\begin{bmatrix} a_{it} \\ y_{it} \\ n_{it} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \overline{a} \\ \mu_y \\ \mu_n \end{bmatrix}, \begin{bmatrix} \sigma_a^2 & \frac{\sigma_a^2-\alpha\mathbb{V}}{1-\alpha} & \frac{\sigma_a^2-\mathbb{V}}{1-\alpha} \\ \frac{\sigma_a^2-\alpha\mathbb{V}}{1-\alpha} & \frac{\sigma_a^2-\mathbb{V}}{(1-\alpha)^2}+\mathbb{V} & \frac{\sigma_a^2-\mathbb{V}}{(1-\alpha)^2} \\ \frac{\sigma_a^2-\mathbb{V}}{1-\alpha} & \frac{\sigma_a^2-\mathbb{V}}{(1-\alpha)^2} & \frac{\sigma_a^2-\mathbb{V}}{(1-\alpha)^2} \end{bmatrix}\right)$$

and thus

$$\begin{aligned} y_t &= \alpha n + \overline{a} + \frac{1}{2}\sigma_{ay} \\ TFP_t &= \overline{a} + \frac{1}{2}\sigma_{ay} \end{aligned}$$

that is, aggregate output and productivity are functions not only of the mean level of productivity, but depend additionally on the covariance between productivity and output at the firm-level.

Thus far, the model has delivered a sharp relationship between the extent of informational frictions, the degree of misallocation, and aggregate productivity and output, taking as given the set of information available to firms. We now make explicit the information structure in the economy, and in particular, the role of financial markets in serving as an aggregator of information across investors and delivering an informative signal to firms through equity prices.

2.2 Financial Markets

Our formulation of an imperfectly informative financial market is in the spirit of Albagli et al. (2011a) and Albagli et al. (2011b). For each firm i, there is a unit measure of outstanding stock or equity, which represents a claim on the firm's profits. There is a unit measure of risk-neutral investors. Each period, investors decide whether or not to purchase a single unit of equity in firm i at the current market price p_{it}. The market is also populated by noise traders who purchase a random quantity of the stock each period, $\Phi\left(z_{it}\right)$ where $z_{it} \sim \mathcal{N}\left(0, \sigma_z^2\right)$ and Φ denotes the standard normal CDF.

Information sets. It makes sense here to make explicit the information available to each set of informed agents in the economy at time t, i.e., investors and firms. Both observe the firm's productivity history perfectly, that is, a_{it-1}. Both observe the current stock price p_{it}. Each investor is endowed with a private signal about the firm's current period productivity a_{it} with precision (variance) σ_v^2, while each firm is endowed with a private signal about the same with precision σ_e^2.

Equilibrium. To characterize capital market equilibrium, we define the demand of informed traders for each asset as the total demand across the distribution of private signals, i.e.,

$$D\left(a_{it-1}, a_{it}, p_{it}\right) = \int d\left(a_{it-1}, s_{ijt}, p_{it}\right) dF\left(s_{ijt} | a_{it-1}, a_{it}\right)$$

where $d\left(a_{it-1}, s_{ijt}, p_{it}\right) \in [0, 1]$ is the demand of investor j observing private signal s_{ijt}. The expected payoff to purchasing the asset is given by

$$\int \left[\pi\left(a_{it-1}, a_{it}, p_{it}\right) + \beta \overline{p}\left(a_{it}\right)\right] dH\left(a_{it}, s_{ijt}, p_{it}\right)$$

where $H\left(a_{it}, s_{ijt}, p_{it}\right)$ denotes the investor's posterior beliefs over a_{it} and $\overline{p}\left(a_{it}\right)$ is the expected continuation value for a firm that realizes productivity a_{it}, defined by

$$\overline{p}\left(a_{it}\right) = \int_{\mu_{it+1}, z_{it+1}} p\left(a_{it}, a_{it+1}, z_{it+1}\right) d\Phi\left(\mu_{it+1}, z_{it+1}\right)$$

Because posterior beliefs are first-order stochastically increasing in s, and π and p_{it+1} are monotonically increasing in a_{it}, trading decisions are characterized by a threshold rule of the form

$$d\left(a_{it-1}, s_{ijt}, p_{it}\right)=\begin{cases}1 & \text{if } \int\left[\pi\left(a_{it-1}, a_{it}, p_{it}\right)+\beta \overline{p}\left(a_{it}\right)\right] dH\left(a_{it}, s_{ijt}, p_{it}\right)>p_{it} \\ \in[0,1] & \text{if } \int\left[\pi\left(a_{it-1}, a_{it}, p_{it}\right)+\beta \overline{p}\left(a_{it}\right)\right] dH\left(a_{it}, s_{ijt}, p_{it}\right)=p_{it} \\ 0 & \text{if } \int\left[\pi\left(a_{it-1}, a_{it}, p_{it}\right)+\beta \overline{p}\left(a_{it}\right)\right] dH\left(a_{it}, s_{ijt}, p_{it}\right)<p_{it}\end{cases}$$

where $p_{it}=p\left(a_{it-1}, a_{it}, z_{it}\right)$, i.e., there is a threshold signal $\widehat{s}_{it}$ such that investors observing signals above it decide to purchase and equity share, and those below it do not. Aggregating the demand decisions of all traders of asset i, market clearing implies

$$\Phi\left(\frac{\widehat{s}_{it}-a_{it}}{\sigma_v}\right)=\Phi\left(z_{it}\right)$$

which leads to a simple characterization of the threshold signal

$$\widehat{s}_{it}=a_{it}+\sigma_v z_{it}$$

This then represents the informational content of prices: the price reveals only a noisy signal of the firm's true productivity, where the level of informativeness is decreasing in the dispersion of private information and the size of the noise trader shock.

Substituting into the indifference condition for the marginal investor yields an implicit characterization of the price function

$$\begin{aligned} p\left(a_{it-1}, a_{it}, z_{it}\right) = & \int \pi\left(a_{it-1}, a_{it}, p_{it}\right) dH\left(a_{it} \mid a_{it-1}, a_{it}+\sigma_v z_{it}, p_{it}\right) \\ & +\beta \int \bar{p}\left(a_{it}\right) dH\left(a_{it} \mid a_{it-1}, a_{it}+\sigma_v z_{it}, p_{it}\right)\end{aligned}$$

3 Quantitative Analysis

3.1 Calibration

We plan to calibrate the model using firm-level production and financial data across a number of countries...

Production parameters. The primitives of the economy on the production side are $\alpha, \sigma_a^2, \mathbb{V}$. To pin down α, we note that the aggregate labor share in income is independent of the informational structure and is precisely equal to α, as in the full-information framework:

$$
\begin{aligned}
WN &= \alpha N^{\alpha}\left(\int\left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}}di\right)^{1-\alpha} \\
\frac{WN}{Y} &= \frac{\alpha N^{\alpha}\left(\int\left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}}\quad di\right)^{1-\alpha}\left(\int\left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}}\quad di\right)^{\alpha}}{N^{\alpha}\int A_{it}\left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{\alpha}{1-\alpha}}di}=\alpha\frac{\int\left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}}\quad di}{\int A_{it}\left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{\alpha}{1-\alpha}}di} \\
&= \alpha\frac{\int\left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{1}{1-\alpha}}\quad di}{\int A_{it}\left(\mathbb{E}_{it}\left[A_{it}\right]\right)^{\frac{\alpha}{1-\alpha}}di}=\alpha\frac{\int\exp\left(\frac{1}{1-\alpha}\hat{a}_{it}+\frac{1}{2}\frac{1}{1-\alpha}\mathbb{V}\right)\quad di}{\int\exp\left(a_{it}+\frac{\alpha}{1-\alpha}\hat{a}_{it}+\frac{1}{2}\frac{\alpha}{1-\alpha}\mathbb{V}\right)di} \\
&= \alpha\frac{\exp\left(\frac{\bar{a}}{1-\alpha}+\frac{1}{2}\left(\frac{1}{1-\alpha}\right)^{2}\left(\sigma_a^2-\mathbb{V}\right)+\frac{1}{2}\frac{1}{1-\alpha}\mathbb{V}\right)}{\exp\left(\frac{\bar{a}}{1-\alpha}+\frac{1}{2}\sigma_a^2+\frac{1}{2}\left(\frac{\alpha}{1-\alpha}\right)^{2}\left(\sigma_a^2-\mathbb{V}\right)+\frac{\alpha}{1-\alpha}\left(\sigma_a^2-\mathbb{V}\right)+\frac{1}{2}\frac{\alpha}{1-\alpha}\mathbb{V}\right)} \\
&= \alpha
\end{aligned}
$$

For simplicity, we assume a common labor share of 0.66 across countries and accordingly, set $\alpha = 0.66$. Later, we relax this assumption and explore how our results change with α.

Using this estimate for α, we use firm-level data on value-added and employment to estimate the stochastic properties of firm-level productivity. This entails inferring the two parameters of the AR(1) process described above, i.e., ρ and σ_e^2, for each country s. For notational convenience, we omit the country subscript s in our exposition.

Differences in the quality of information [PRELIMINARY]. We are currently working on using financial market data to infer the informational parameters of our model economy for the countries in or sample. However, before describing that procedure, we present a simple yet instructive analysis of cross-country production data through the lens of our model. This will show two important features of the data. First, we show that a measure of misallocation constructed using model-implied relationships varies systematically with the level of overall economic development, giving some promise of the quantitative significance of the mechanism. Second, substantial differences in the quality of information are needed to explain the observed levels of misallocation across countries.

Specifically, we use data on sales and employment from International Compustat and follow the procedure outlined above to measure α and σ_a^2. We then back out a measure of misallocation as

$$\tilde{\mathbb{V}} = \sigma_a^2 - (1-\alpha)\,\sigma_{an}$$

where $\tilde{\mathbb{V}}$ represents the posterior variance we would need to fully explain all misallocation in the data. We examine a cross section of countries in 1996 where we have at least 20 firms with the necessary data. We exclude several countries where the data suggest $\tilde{\mathbb{V}} > \sigma_a^2$ and a few outliers. The final sample consists of 84 countries.

We plot our preliminary results in Figure 1. The figure displays $\frac{\tilde{\mathbb{V}}}{\sigma_a^2}$, that is, the posterior variance needed to explain all misallocation as a proportion of the total variance, versus income per worker obtained from the Penn World Tables, along with the line of best fit. We normalize by σ_a^2 to roughly control for differences in the size of the underlying productivity shocks across countries. Although a fairly crude measure of the objects we are ultimately after, there is a large degree of dispersion across countries and a clear negative statistically significant relationship with income per worker. The correlation between the two is -0.44.

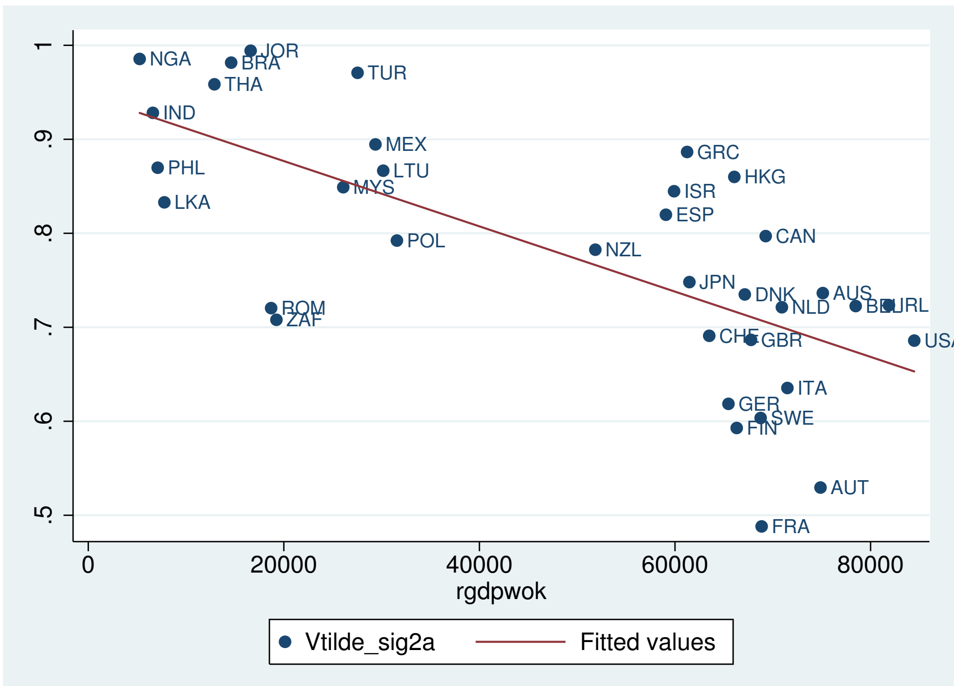

Figure 1: $\frac{\tilde{\mathbb{V}}}{\sigma_a^2}$ and Income per Worker

Informational parameters. A novel feature of our analysis is a calibration strategy which uses a combination of financial market and firm data to pin down the informational parameters for each country. The key parameters (again suppressing the country subscript s for brevity) are $\sigma_v^2, \sigma_z^2, \sigma_e^2$. Our strategy targets 3 moments - the variance of stock returns and their ability to forecast firm fundamentals (productivity a_{it} and actions (n_{it})). To see why these moments are informative, we first note that the variability of prices and their comovement with fundamentals are jointly determined by the quality of investors' private information (σ_v^2) and the size of the noise trader shock (σ_z^2). Though this dependence is in general non-monotonic, we focus on a region where higher levels of ($\sigma_v^2 \sigma_z^2$) lead to higher price volatility. Next, for a given level of 'noise' in prices, firm actions respond more to prices when the firm's own private information is less precise (σ_e^2 is high). [COMING SOON]

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