On the incidence of a financial transactions tax in a model with fire sales

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This version: February 5, 2013

Abstract

We study the impact of a financial transactions tax on a market where financial institutions trade with each other. There are two main results: First, if all banks can honor their short-term obligations, a financial transactions tax is entirely neutral. Second, in a model with correlated investment risk and short-term financing of banks, a financial transactions tax contributes to financial distress and undoes other policy measures that are used to stabilize financial markets. In the long run, however, this tax induced change of financial market outcomes gives banks an incentive to reduce their maturity mismatch.

Keywords: Financial transactions tax, financial stability, financial markets, cash-in-the-market-pricing, marking-to-market.

JEL-Codes: H22, G18, G21, G28

*I benefitted from conversations with Gadi Barlevy, Johannes Becker, Clemens Fuest, Peter Funk, Hans Gersbach, Hans-Peter Grüner, Martin Hellwig, Thomas Hemmelgarn, Alexander Ludwig, Thornton Matheson, Johannes Münster, Désirée Rückert and Philipp Weinschenk.

1 Introduction

The 2008/2009 financial crisis has generated a demand for new taxes to be raised in the financial sector with the objective to compensate tax payers for the costs of the financial crisis.[1] Among the conceivable tax instruments, one has since received particular interest of policy makers and the public at large, namely a financial transactions tax.[2]

This paper analyzes a financial transactions tax assuming that financial markets serve to facilitate trade between financial institutions, henceforth simply *banks*. Trade arises because banks differ in their liquidity needs: Banks that need liquidity in order to honor their current obligations sell their illiquid assets to banks that have more liquidity than they currently need.[3] We assume that financial markets do not operate in a frictionless way. Instead, there is cash-in-the-market-pricing, so that the price of an asset may fall below its "fundamental value". This situation arises if the demand on the financial market is restricted because many banks experience liquidity problems at the same time. Models with these properties have proven to be fruitful for an analysis of financial crises.[4]

Specifically, the contribution of the paper is as follows. First, we analyze how the introduction of a financial transactions tax affects financial market prices, and, more importantly, the number of distressed banks. Second, we study the long-run consequences of such a tax: How do banks respond to the tax induced change of financial market outcomes and what are the consequences for financial stability. Finally, we study the interaction of a financial transactions tax with other policy measures that have been taken with the intention to stabilize financial markets. Some of these policies (liquidity support by central banks and TARP or Bad Bank programs) were taken in order to stabilize financial markets in the short run. Others, such as the increased equity or liquidity requirements stipulated by Basel III, are taken with the intention to get "better" financial market outcomes in the long run. We relate a financial transactions tax to all of these policy measures.

A methodological contribution of the paper is to introduce a model of fire sales that

[1]For instance, *"At their September 2009 Pittsburgh Summit, G20 Leaders requested the International Monetary Fund to prepare a report on how the financial sector could make a fair and substantial contribution to meeting the costs associated with government interventions to repair it (IMF, 2010)"*.

[2]In September 2011, the European Commission proposed a EU-wide financial transactions tax (European Commission, 2011). In January 2013, 11 European countries, including France and Germany, decided to implement the commisions's proposal. Other European countries, including the UK and Sweden, do not plan to implement it. In the US, in the aftermath of the 2008/09 financial crises, various members of the democratic party have expressed their sympathy for a financial transactions tax. In the mid-eighties, the introduction of a financial transactions tax was seriously considered in the US, see Stiglitz (1989) and Summers and Summers (1989).

[3]We thereby follow a rich literature in finance. Examples include Bhattacharya and Gale (1987), or the work of Allen and Gale (1994, 2004a,b, 2005).

[4]See, for instance, Allen and Gale (2005), Cifuentes et al. (2005), Allen and Carletti (2008), Acharya and Yorulmazer (2008) or, for an overview, Shleifer and Vishny (2011).

is tractable in the sense that a comparative statics analysis can be undertaken analytically and be illustrated graphically. The previous literature has typically used numerical methods to obtain comparative statics results.

The formal analysis is based on a model that borrows essential features from the work of Allen and Gale (1994, 2004a,b). There are three periods, $T \in \{0, 1, 2\}$, and a large number of banks. In the initial period, each bank obtains funds from risk-averse debt-holders and risk-neutral providers of equity. A bank seeks to maximize its expected return on equity and has to respect a participation constraint for debt-holders. In the initial period it chooses a debt structure, that is, promises to debt-holders that are due in $T = 1$ and $T = 2$, respectively, and it decides how to invest its funds. It can either invest in projects that mature in $T = 1$ or in projects that mature in $T = 2$. Both investments are risky. It becomes known in $T = 1$ whether investments are successful or fail. There is both aggregate and idiosyncratic uncertainty: The performance of the investments of an individual bank, as well as the cross-section distribution of performances are ex ante unknown.

A long-term investment cannot be liquidated, but a claim on the cash flow that it generates, henceforth simply an *asset*, can be sold in $T = 1$ on a financial (spot) market. The motive for trade is that banks differ in their liquidity needs. After the state of the world has been revealed in $T = 1$, banks with a large number of successful short-run investments have more liquidity/cash than they need in order to honor their current obligations, and are therefore willing to buy other banks' assets at an appropriate price. Banks with only a small number of successful short-run investments, henceforth referred to as fire-sellers, do not have enough liquidity and therefore need to sell assets.

We model financial distress via a bankruptcy regime. A bank is bankrupt, if, at the given financial market price, the value of its liabilities exceeds the value of its assets. We assume that, in case of bankruptcy, a bank's debtors are bailed out by the government and the bank's holders of equity receive a payoff of zero, i.e., they are protected by limited liability. Both assumptions are descriptive of the experiences in the 2009/09 financial crises. Under these assumptions, there is a one-to-one relation between financial distress and the payments that the government has to make in order to bailout the debtors of distressed financial institutions. We therefore view financial distress as being undesirable from the perspective of tax-payers, and evaluate all policy interventions using *financial stability*, formalized via the number of distressed financial institutions, as our measure of welfare.

We obtain the following main results:

I. In the short-run, a financial transactions tax increases financial distress among banks with liquidity problems. However, the overall effect on financial stability is ambiguous. Banks that have enough liquidity but experience losses on their long term investments may benefit from the introduction of a financial transactions tax.

II. If all banks have sufficient liquidity, then a financial transactions is tax neutral. It neither affects equilibrium prices nor the number of distressed banks. It also has no long run consequences, i.e., banks do not adjust their investment decisions or their debt structures if a financial transactions tax is introduced. A policy implication is that, if, as a result of regulation, banks are forced to hold enough liquidity, a financial transactions tax does not threaten financial stability. The regulatory framework of Basel III contains such a liquidity requirement, the *net stable funding ratio*.

III. By contrast, if many banks have liquidity problems, possibly as a result of excessive short-term financing, and have correlated returns on their long-term investments, then a financial transactions tax has the following short run implications: Financial market prices fall and financial distress goes up. These changes need not be small. There may be multiple equilibria and the introduction of the tax may eliminate the "good" one that has only few bankruptcies and induce a jump into the "bad" one, with a drastic fall in prices and a drastic increase in the number of bankruptcies. We also study the impact of various policies (liquidity support by central banks, government purchases of assets), in this model and show that they would indeed have a stabilizing impact in the short run. This positive impact would be upset by the introduction of a financial transactions tax.

IV. Under the conditions in III., a financial transactions tax may have beneficial implications for financial stability in the long run. The tax induced change of financial market outcomes gives banks an incentive to reduce their maturity mismatch. However, these positive consequences could also be generated using a liquidity requirement. Moreover, this would not come at the cost of a destabilization in the short run.

All these observations lend themselves to the following main conclusion: As long as many financial institutions are on the brink of bankruptcy, the introduction of a financial transactions tax is likely to make things worse. If, by contrast, the financial system is by and large healthy, a financial transactions tax does not cause major problems. The formal analysis deliberately abstracts from issues that are emphasized by the proponents of a financial transactions tax, namely that it would discourage socially wasteful activities such as speculation or high-frequency trading. What the paper shows is that these potential benefits of a financial transactions tax have to be traded off against the costs in terms of financial stability – unless regulatory measures ensure that financial stability is no longer endangered. The paper also suggests that it may be wise to reform the regulatory framework for banks before a financial transactions tax ins introduced. If regulation ensures that banks do not have liquidity problems, then a financial transactions tax is neutral, i.e. it has no detrimental impact on the safety of the banking system.

Related Literature. The theoretical literature that looks at how financial institutions and financial markets are affected by taxation is surprisingly thin. Lockwood (2010) studies the optimal taxation of financial services in a dynamic Ramsey model. Philippon (2010) characterizes the optimal size of the financial sector relative to the "real" economy in an endogenous growth model. In his model, taxes can be used to improve upon the relative sizes of these sectors that would result under laissez-faire. There is also research on the treatment of financial services under a value added tax system, see Huizinga (2002), Auerbach and Gordon (2002), or Genser and Winker (1997). Some recent papers study Pigouvian taxation as a response to systemic risk-taking (Acharya e.a., 2010), maturity mismatch (Perotti and Suarez, 2011), or excessive leverage (Keen, 2011) . What all these articles have in common, however, is that they model the relation between a financial intermediary and agents who demand financial services, or they model the behavior of a representative bank. They do not model financial markets, i.e. markets where financial institutions trade financial products with each other.

This paper draws on the literature on fire sales on financial markets, and in particular on the models of Allen and Gale (1994, 2004a,b). As survey of the literature on fire sales is provided by Shleifer and Vishny (2011). Caballero and Simsek (2010) look at a setup similar to this paper. Their contribution is to model how uncertainty about the interconnectedness of financial institutions affects financial stability.

Overviews of the current practice in the taxation of the financial sector, and of the empirical and theoretical findings on financial transactions taxes are provided by Matheson (2011) and by Hemmelgarn and Nicodeme (2012). The view that financial transactions taxes are desirable as a way of discouraging speculation is, for instance, articulated in Keynes (1936),Tobin (1978), Stiglitz (1989) or Summers and Summers (1989). Recently, Brunnermeier et al. (2012) have provided a welfare analysis of speculative trade.

The remainder is organized as follows: Section 2 lays out a general model. In Section 3, we discuss equilibrium existence and comparative statics for this general framework. Section 4 contains the analysis of an economy without fire sales. Section 5 deals with the analysis of an economy with idiosyncratic liquidity risk and aggregate long-term investment risk. The last section concludes. All proofs are in an Appendix.

2 The Model

The Environment. There is a continuum of banks of measure 1. The set of banks is denoted by I, with typical element i. There are three dates, $T \in \{0, 1, 2\}$. Bank i is endowed with equity e_i and debt d_i. It promises its debtors payments of x_{i1} in period 1 and of x_{i2} in period 2. We can think of debtors either as depositors as in a model of liquidity insurance, or as lenders in the commercial paper or repo market.[5] Holders of equity receive the residual value of the bank's assets (i.e. what is left after the payments promised to depositors and other debtors are made) in case there is no failure. Hence, bank i has funds of $a_i := d_i + e_i$ to invest in $T = 0$. Bank i lends out or invests an amount y_{i1}^0 with a short term perspective. These investments are risky. They fail with a certain probability (possibly zero) in which case they return 0 in $T = 1$. If they do not fail, they return r_1 in $T = 1$. For ease of notation, we let $r_1 = 1$ in the following. In addition, bank i lends out an amount of $y_{i2}^0 = a_i - y_{i1}^0$ with a long run perspective. We think of these investments as risky loans to entrepreneurs, home-owners etc. A risky loan fails with a certain probability, in which case it returns 0. If it does not fail, it returns r_2 in $T = 2$. Whether or not a loan performs becomes known in $T = 1$.

Long-run investments can not be liquidated; that is, it is not possible to withdraw in $T = 1$ the funds invested in $T = 0$. However, it is possible to trade claims on the returns of these investments on a financial market in $T = 1$. We henceforth refer to such claims simply as assets. The financial market equilibrium is described in more detail below.

We assume that a bank's objective is to maximize the expected return on equity. We assume, for simplicity, that the holders of equity require a payment only in $T = 2$, and π_i^2 is the payment per unit of equity in $T = 2$. As of $T = 0$, π_i^2 is a random quantity. It depends both on idiosyncratic risk (such as the fraction of successful investments of bank i) and aggregate risk (such as the price on the financial market), which is realized in $T = 1$.

In $T = 1$, each bank i is hit by a shock. Formally, in $T = 1$, bank i is characterized by a vector $\sigma_i = (y_{i1}, y_{i2})$, where $y_{i1} \leq y_{i1}^0$ is the volume of performing short-run investments, and $y_{i2} \leq y_{i2}^0$ is the volume of performing long-run investments, i.e., of investments that will yield a return in $T = 2$. From the perspective of the initial period, σ_i is a random quantity; i.e., as of $T = 0$, bank i does not yet know how many of its long- and short-run investments will perform. We assume, for simplicity, that the banks' liabilities x_{i1} and x_{i2} are not random.[6]

In the following, we refer to the collection $\sigma = (\sigma_i)_{i \in I}$ as the state of the economy and denote the set of all states by S. There is a financial market where cash available in $T = 1$ can be traded against assets that yield a return in $T = 2$. Moreover, we consider the possibility that trades on the financial market are subject to an (ad valorem) transactions tax t, which drives a wedge between the revenue from selling an asset. We view the

[5] Consider a model with depositors who demand liquidity insurance, as in Diamond and Dybvig (1983), and suppose that depositors are promised $\tilde{x}_{i1}$ if they withdraw early and $\tilde{x}_{i2}$ if they withdraw late. Then $x_{i1} = s_i \tilde{x}_{i1}$ and $x_{i2} = (1 - s_i)\tilde{x}_{i2}$, where s_i is the fraction of early consumers among the depositors of bank i.

[6] In a model of liquidity insurance à la Diamond and Dybvig (1983), this is the case if s_i is a known parameter, so that, in the absence of a bank run, it is known a priori how many early consumers will show up in $T = 1$.

financial market prices as functions of the state of the economy σ and the tax rate t. The price $p(\sigma, t)$ that is received by a seller, and the price $q(\sigma, t)$ that is paid by a buyer are such that

$$q(\sigma, t) = (1 + t)p(\sigma, t) \ .$$

For most of the analysis we treat the state of the economy as fixed. To save on notation, we will suppress the dependence of the prices p and q on the state of the economy whenever this creates no confusion.

We assume that all economic agents have access to a storage technology in order to transfer resources available in $T = 1$ into resources available in $T = 2$. If instead of storing the resources available in $T = 1$, a bank buys assets on the financial market, it receives r_2/q per unit invested. Hence, there will be demand on the financial market only if $r_2 \geq q$.

Long-run equilibrium. We distinguish between a long-run and a short-run perspective. In the short-run, we treat e_i, d_i, x_{i1}, x_{i2}, y_{i1}^0 and y_{i2}^0 as predetermined and investigate how financial market outcomes are affected by the cross-section distribution of the banks' losses and by various policy interventions. The long-run perspective looks at how the banks' choices of x_{i1}, x_{i2}, y_{i1}^0 and y_{i2}^0 are affected if financial market outcomes change in response to a policy intervention. We now turn to the definition of a long-run equilibrium. We start with the profit-maximization problem of a generic bank i.

In this definition, we view a bank's capital structure, i.e., its endowment with debt d_i and and equity e_i as given. The bank's choices are its debt structure and its investments. We denote bank i's choice variable as $c_{0i} = (x_{i1}, x_{i2}, y_{i1}^0, y_{i2}^0)$, and write $h_{0i} = (d_i, e_i, c_i)$, for the collection of variables that are fixed after period 0. Bank i chooses c_{0i} with the understanding that in $T = 1$ it will be able to trade on a financial market and has expectations about the prices that it will face in the different states of the economy. Important assumptions are that a bank is protected by limited liability: If the maximal profit that a bank can reap when trading on the financial market in $T = 1$ is negative, then the bank will go out of business. Another important assumption is that in such an event, the bank's debt-holders will be bailed out by the government. We now turn to a formal statement of the bank's problem.

The bank's objective. Bank i's objective is given by

$$\Pi_i(h_{0i}, t) := E_\sigma[\pi_i^2(h_{0i}, \sigma_i, q(\sigma, t), p(\sigma, t))] \ ,$$

where the expectation is over the state of the economy, and $\pi_i^2(h_{0i}, \sigma_i, q(\sigma, t), p(\sigma, t))$ is the profit that bank i can realize on the financial market in $T = 1$ if its successful short-run and long-run investments, respectively, are given by $\sigma_i = (y_{i1}, y_{i2})$ and it faces prices of $q(\sigma, t)$ if it wants to buy assets and of $p(\sigma, t)$ if it wants to sell assets. Note that these prices depend both on the state of the economy σ and on the tax rate t.

Financial market profits. The expression $\pi_i^2(h_{0i}, \sigma_i, q(\sigma, t), p(\sigma, t))$ results from (i) profit-maximizing behavior on the financial market that opens in $T = 1$ and (ii) from limited liability. Formally, it equals

$$\max\{0, \pi_i^{2*}(h_{0i}, \sigma_i, q(\sigma, t), p(\sigma, t))\} ,$$

where $\pi^{2*}(h_{0i}, \sigma_i, q(\sigma, t), p(\sigma, t))$ follows from choosing a portfolio $z_i = (z_{i1}, z_{i2})$ in order to maximize the return on equity available in $T = 2$,

$$\frac{1}{e_i}(r_2 z_{i2} - x_{i2} + z_{i1} - x_{i1}) ,$$

subject to the constraints that the banks' budget constraint is satisfied,

$$z_{i2} - y_{i2} \leq \begin{cases} \frac{y_{i1} - z_{i1}}{q(\sigma,t)}, & \text{if} \quad z_{i2} \geq y_{i2} , \\ \frac{y_{i1} - z_{i1}}{p(\sigma,t)}, & \text{if} \quad z_{i2} \leq y_{i2} , \end{cases}$$

and that the bank is able to honor the promises to its debt-holders $z_{i1} - x_{i1} \geq 0$, and $r_2 z_{i2} - x_{i2} + z_{i1} - x_{i1} \geq 0$.

The bank's constraints. When choosing c_{0i} to maximize $\Pi_i(h_{0i}, t)$, bank i faces a budget constraint

$$y_{i1}^0 + y_{i2}^0 = a_i = e_i + d_i , \tag{1}$$

and, a participation constraint for debt-holders

$$U(x_{i1}, x_{i2}) \geq \bar{u}_i . \tag{2}$$

Hence, if bank i has promised payments of x_{i1} and x_{i2}, then the debtors' utility equals $U(x_{i1}, x_{i2})$, where U is taken to be a function that is increasing in both arguments and quasiconcave.[7] We do not impose any assumption on the utility level of $\bar{u}_i$ that debtors demand. In a model with perfect competition among banks such as, e.g., Allen and Gale (2004b), $\bar{u}_i$ would take a value that is so high that bank i's maximal profits are 0.

[7] In a Diamond and Dybvig (1983)-model of liquidity insurance, $U(x_{i1}, x_{i2})$ is the expected utility of a consumer who, as of $T = 0$, does not know whether he will have consumption needs in $T = 1$ or in $T = 2$. Consequently,

$$U(x_{i1}, x_{i2}) = s_i u\left(\frac{x_{i1}}{s_i}\right) + (1 - s_i)u\left(\frac{x_{i2}}{1 - s_i}\right) ,$$

where s_i is the probability of being an early consumer and u is an increasing and concave function that gives the instantaneous consumption utility. In such a model, one would also have to add the incentive compatibility constraint $\frac{x_{i1}}{s_i} \geq \frac{x_{i2}}{1-s_i}$, because otherwise late consumer would have an incentive to withdraw their money early. Here, we do not impose such an assumption because we are also interested in situations where banks have primarily short-run debt, so that x_{i1} is large relative to x_{i2}. This situation is descriptive of the excessive over-night financing that investment banks displayed in the run up of the 2008/09 financial crises.

The formulation of the participation constraint assumes that the promises to debtors are honored irrespectively of whether or not the bank goes out of business in $T = 1$. It therefore presupposes either the existence of a deposit insurance for debt-holders or that, in case of a bank failure, the government will intervene and bail out the bank's debt-holders. Moreover, it is assumed that these bailouts are complete, i.e. that there are no haircuts for debt-holders.[8] This full bailout assumption can be justified in a couple of ways. For commercial banks the assumption is descriptive because of deposit insurance. For investment banks, the experience in the 2008/09 financial crises suggests that a full bailout assumption is often a reasonable approximation.[9] Finally, we have assumed that debtors' are risk-averse. If we think of the government as having a superior ability to absorb expenditure shocks – e.g. because it can smooth tax distortions intertemporally by issuing government debt – then it maybe optimal to have a deposit insurance scheme that is backed by the government.

To sum up, bank i's problem is to choose c_{0i} in order to maximize $\Pi_i(h_{0i}, t)$ subject to the constraints in (1) and (2). We refer to this problem as *bank i' s long-run problem.* We denote by $C^*_{0i}(t)$ the set of solutions to this optimization problem.

Long-run equilibrium. A long-run equilibrium consists of a collection of state-dependent prices $\{p(\sigma, t), q(\sigma, t)\}_{\sigma \in S}$ and a collection of the bank choices $\{c_{i0}\}_{i \in I}$ such that

i) For every σ, the prices $p(\sigma, t)$ and $q(\sigma, t)$ are *financial market equilibrium prices.*

ii) For every i, c_{0i} solves bank i's long-run problem given the *financial market equilibrium prices.*

This definition contains the term *financial market equilibrium price*, which we have not yet defined. We now turn to financial market outcomes.

2.1 Financial Market Equilibrium

We now fix a state of the economy σ, and a history $\{h_{0i}\}_{i \in I}$ and turn to the financial market outcomes. The number of bankrupt banks is our measure of financial stability. We start with a clarification of the conditions under which a bank is bankrupt.

[8] Without this assumption, the debt-holders participation constraint would take a different form. The debt-holders would then receive the bank's liquidation value in case of a failure. This liquidation value would have to be split in some way between the holders of long-run debt and the holders of short-run debt. In addition, debt holders could receive some, possibly partial, bailouts in some states of the world, e.g. those with systemic events. All these considerations would have to enter the calculation of expected utility that enters the left-hand-side of the debtors' participation constraint.

[9] The failure of Lehman brothers is the prominent exception where debtors were not made whole. But given the drastic consequences that have been attributed to this political decision, it seems unlikely that this will happen again any time soon.

Bankruptcies. We assume that a bank goes bankrupt if, at the given price on the financial market, the value of its liabilities is larger than the value of its assets. Equivalently, the maximal profit that it can realize when staying in business, $\pi^{2*}(h_{0i}, \sigma_i, q(\sigma, t), p(\sigma, t))$, is negative. To clarify the conditions under which this is the case, the following terminology is useful: In $T = 1$, bank i has an endowment consisting of cash y_{i1} and assets y_{i2}. Since each asset generates r_2 units of resources available in $T = 2$, we can as well think of bank i as having an endowment of y_{i1} resources available in $T = 1$ and of $r_2 y_{i2}$ resources available in $T = 2$. It will prove convenient to also define the net endowments $\theta_{i1} = y_{i1} - x_{i1}$ and $\theta_{i2} = r_2 y_{i2} - x_{i2}$, respectively. We may distinguish the following types of banks:

i) *Safe and failed banks.* If $\theta_{i1} \geq 0$ and $\theta_{i2} \geq 0$, bank i is safe in that its ability to honor its promises does not depend on market conditions. If $\theta_{i1} \leq 0$ and $\theta_{i2} \leq 0$, with a least one inequality being strict, bank i fails irrespective of market conditions. In the following we denote the set of safe and failed banks by I^* and $I^\dagger$, respectively.

ii) *Fire-sellers.* Now suppose instead that $\theta_{i1} < 0$ and $\theta_{i2} > 0$. This bank has a liquidity problem in $T = 1$ and therefore needs to sell assets on the financial market. The relevant price is now p, and the bank survives provided that

$$\theta_{i1} + \frac{p}{r_2}\theta_{i2} \geq 0 \ .$$

Note that, for a bank that needs to sell assets, lower prices on the financial market make it more difficult to survive. We denote by

$$I^*_{sell}(p) = \left\{ i \in I \mid \theta_{i1} < 0, \theta_{i2} > 0, \frac{p}{r_2} \geq -\frac{\theta_{i1}}{\theta_{i2}} \right\}$$

the set of banks that have to sell assets on the financial market and manage to survive at a price of p. We denote by $I^\dagger_{sell}(p)$ the set of fire-selling banks that fail.

iii) *Fire-buyers.* Suppose that $\theta_{i1} > 0$ and $\theta_{i2} < 0$, so that bank i has more resources than it needs in $T = 1$, but that it does not have enough resources in $T = 2$. Therefore, it will have to use its excess liquidity in $T = 1$ to buy assets on the financial market and the relevant price is q. The bank will therefore survive provided that

$$\theta_{i1} + \frac{q}{r_2}\theta_{i2} \geq 0 \ .$$

Note, in particular, that, for a bank that needs to buy assets, lower prices on the financial market make it easier to survive.[10] In the following, we denote by

$$I^*_{buy}(q) = \left\{ i \in I \mid \theta_{i1} > 0, \theta_{i2} < 0, \frac{q}{r_2} \leq -\frac{\theta_{i1}}{\theta_{i2}} \right\}$$

[10]The bank could also use the storage technology, in which case it would be able to honor its promises provided that $\theta_{i1} + \theta_{i2} \geq 0$. However, since $\frac{q}{r_2} \leq 1$, it is more difficult to survive using the storage technology.

the set of banks that have to buy assets on the financial market and manage to survive at a price of q. Analogously, we denote by $I^{\dagger}_{buy}(q)$ the complementary set of banks that fail.

A bank that is bankrupt gets liquidated; that is, its assets are sold on the financial market.[11] The bank's debtors then receive the proceeds from these sales plus the cash that the bank has available in $T = 1$.[12] In case of bankruptcy, equity holders do not receive anything.

Behavior of non-bankrupt banks. Banks that do not go bankrupt can engage in trade on the financial market, i.e. they solve the problem to maximize their financial market profit which has been formally defined above. It is straightforward to verify that safe banks and fire-buyers buy as many assets as they can if $q < r_2$, and are indifferent between holding cash and assets if $q = r_2$. Their net demand on the financial market is therefore given by

$$z_{i2} - y_{i2} = \begin{cases} \frac{\theta_{i1}}{q}, & \text{if} \quad r_2 > q \ , \\ \in [-y_{i2}, \frac{\theta_{i1}}{q}], & \text{if} \quad r_2 = q \ . \end{cases} \tag{3}$$

A fire-selling bank sells as few assets as possible if $p < r_2$ and is indifferent between holding cash and assets if $p = r_2$. This yields a (negative) net demand of

$$z_{i2} - y_{i2} = \begin{cases} \frac{\theta_{i1}}{p}, & \text{if} \quad r_2 > p \ , \\ \in [-y_{i2}, \frac{\theta_{i1}}{p}], & \text{if} \quad r_2 = p \ . \end{cases} \tag{4}$$

Demand. The net demand for assets on the financial market stems from the safe banks and the banks that have to buy assets and manage to survive. Recall that all these banks have more liquidity than they need in $T = 1$, so that $\theta_{i1} > 0$. Given a price of $q \in (0, r_2)$, the demand on the financial market is given by

$$D(q) := \int_{\hat{I}^*(q)} (z_{i2} - y_{i2}) \, di = \int_{\hat{I}^*(q)} \frac{\theta_{i1}}{q} \, di$$

where $\hat{I}^*(q) := I^* \cup I^*_{buy}(q)$ is the set of banks that demand assets on the financial market. For $q = r_2$ the demand is bounded from above by $D(r_2)$ and from below by

$$- \int_{\hat{I}^*(q)} y_{i2} \, di \ .$$

Observe that the demand function D is downward-sloping for $q \in (0, r_2)$. If q goes up, this reduces the demand of each bank in $\hat{I}^*(q)$. In addition, the set $I^*_{buy}(q)$ and hence the set $\hat{I}^*(q)$ shrinks, since, for banks that have to buy, a higher price makes it more difficult

[11] In Section 5 below, we also consider an alternative scenario in which the assets of failed banks are acquired by the government.

[12] The higher this liquidation value, the lower is the cost of the government bailout.

to survive.

Supply. The net supply on the financial market has two sources: First, the assets of all banks that fail are sold. Second, some banks fire-sell to generate liquidity and thereby manage to survive. For these banks we have that $\theta_{i1} < 0$. Therefore, for $p \in (0, r_2)$ the supply on the financial market equals

$$\bar{S}(p) := \int_{\hat{I}^\dagger(p)} y_{i2}\, di - \int_{I^*_{sell}(p)} \frac{\theta_{i1}}{p}\, di\ ,$$

where $\hat{I}^\dagger(p) := I^\dagger \cup I^\dagger_{buy}(p(1+t)) \cup I^\dagger_{sell}(p)$. In this formula, we view the supply on the financial market as a function of the revenue p that a seller can realize. Since $q = p(1+t)$ we can as well interpret market supply as a function of q, and define

$$S(q) := \bar{S}\left(\frac{q}{1+t}\right)\ ,$$

i.e., S gives market supply as a function of the price q that buyers have to pay. Note that the slope of the supply function cannot be signed a priori.

For $p = r_2$, $\bar{S}(r_2)$ is only a lower bound on the supply on the financial market, since, at this price, the surviving fire-sellers would be willing to sell all their assets. The corresponding upper bound is

$$\int_{\hat{I}^\dagger(r_2) \cup I^*_{sell}(r_2)} y_{i2}\, di\ .$$

Financial market equilibrium prices. A price $q \in (0, r_2]$ is an equilibrium price if

$$D(q) = S\left(q\right), \quad \text{or if} \quad D(q) > S\left(q\right) \quad \text{and} \quad q = r_2\ .$$

Why is $q = r_2$ an equilibrium price even if $D(r_2) > S(r_2)$? At this price, banks on the demand side are willing to hold any number of assets smaller than $D(r_2)$. If $D(r_2) > S(r_2)$ it is therefore possible to arrange trade between buyers and sellers so that, at the given price of $q = r_2$, every non-bankrupt bank gets a profit-maximizing portfolio.

Proposition 1 *Suppose that D and S are continuous functions on $[0, r_2]$, and that the set $\hat{I}^*(0)$ has positive mass. Then, an equilibrium price exists.*

The Proposition asserts that a financial market equilibrium exists under very weak conditions. All that is needed, apart from continuity, is that demand exceeds supply at the minimal price of $q = 0$. Recall that the net demand of a bank that has excess liquidity in $T = 1$ equals $\frac{\theta_{i1}}{q}$. This grows without limits as q converges to 0. Hence, what is needed for existence is that there is a positive mass of banks that survive and have more cash than they need in $T = 1$.

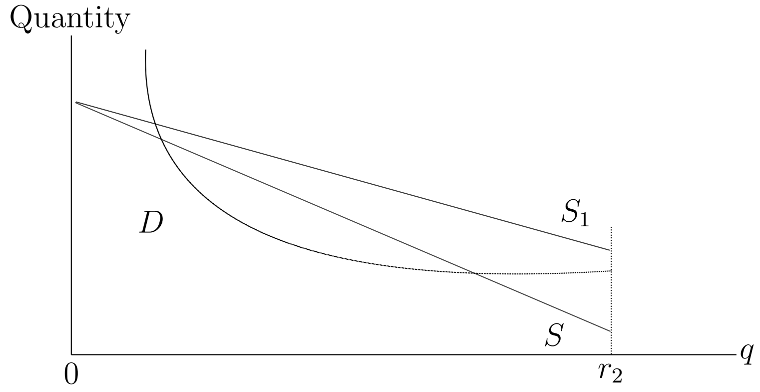

Figure 1: *Multiple equilibria.*

Obviously, an equilibrium exists if the demand and the supply curve intersect over the range of admissible financial market prices $q \in (0, r_2]$. An equilibrium exists even if they do not intersect. If there is no intersection, this means that, for all admissible q, demand exceeds supply in the sense that $D(q) > S(q)$. But this implies, in particular that $D(r_2) > S(r_2)$ so that r_2 is an equilibrium price.

There may exist multiple equilibria as illustrated in Figure 1. The reason is that a model with fire-sales may give rise to a downward-sloping supply curve: the fire-sellers have to sell a lot if the price is low, and they can afford to sell little if the price is high.[13]

The different equilibria cannot be Pareto-ranked. An equilibrium with a low price is preferred by those on the demand side. In fact, $q < r_2$ means that they can buy assets below their net present value and make a speculative profit. By contrast, for banks that have to fire-sell, equilibria with a high price are preferred. Higher prices make it more likely that they survive and that their shareholders receive a profit. If the government provides protection to the banks' debtors (via deposit insurance or bail-out guarantees), it will prefer the equilibrium with the lowest number of bankruptcies. However, it is not clear a priori that a higher financial market price is desirable from the government's perspective. This holds only if all banks at the brink of survival are fire-sellers. Fire-buyers need low asset prices in order to survive.

3 The impact of a financial transactions tax on financial market outcomes: The general case

In the following, we treat some state of the economy and the banks' decisions in the initial period as given. We then ask how financial market prices and the number of distressed

[13]In Section 5 we will look at a more specific version of this model, which shows that the situation in Figure 1 arises under a set of plausible assumptions.

banks respond to the introduction of a financial transactions tax. We proceed under a minimal set of assumptions. In subsequent sections, we will impose additional structure.

We can illustrate the impact of an increased tax t graphically. Consider the supply function $S : q \mapsto S(q)$ with $S(q) = \bar{S}\left(\frac{q}{1+t}\right)$, which gives the supply on the financial market as a function of the price q that a buyer has to pay on the financial market. Now, if we change the tax from a level of t to a level of $t_1 > t$ this yields a new supply function $S_1(q) = \bar{S}\left(\frac{q}{1+t_1}\right)$. Graphically, we can depict this as a shift of the supply curve to the right: A given supply is now associated with a higher price q for the buyers in order to compensate for the fact that the sellers receive a smaller fraction of the buyer's total spending. For the case of a downward-sloping supply curve, this is illustrated in Figure 1.[14]

Figures 1 and 2 show how an increase of the tax rate t can affect the set of equilibria. In Figure 1 there are initially three equilibria, namely the two intersections of the supply and the demand curve and the equilibrium at $q = r_2$. After a rightward shift of the supply curve, there is only one equilibrium left. Hence, the increase of t implies that we move from a situation with three equilibria to a situation with a unique equilibrium. In Figure 2, by contrast, the structure of the set of equilibria remains unaffected. Before and after the change of the tax rate, there are three equilibria.

This equilibrium multiplicity poses a challenge for an analysis of how financial market outcomes are affected by taxation. There are two approaches that one can take to address this problem. Both will be pursued in the following. In the first approach, one looks at *local changes* of *regular* equilibria,[15] i.e. one starts from a given equilibrium and asks how a small tax increase affects this given equilibrium, thereby disregarding the possibility of a jump into another equilibrium. In the second approach one looks at how the range of equilibrium prices responds to taxation – while maintaining the assumption that the structure of the equilibrium set is unaffected by taxation. In the subsequent Sections 4 and 5, we will introduce additional assumptions that make it possible to obtain a complete equilibrium characterization, and also a complete answer to the question of how the set of equilibria is affected by taxation.

Local changes of regular equilibria. It is not straightforward to determine how a given equilibrium price responds to taxation. This can be illustrated using Figure 2. If we focus on the "high-price equilibrium" or the "equilibrium in the middle", we conclude that the increase of t drives the financial market price for buyers, q, up. The "low-price equilibrium", by contrast, has a lower value of q after an increase of the tax rate. A typical approach is to view the "equilibrium in the middle" as irregular or unstable, and

[14]Since we have modeled the financial transactions tax as an ad valorem tax, the shift to the right is larger the larger is q.

[15]Regularity is a refinement that is explained below.

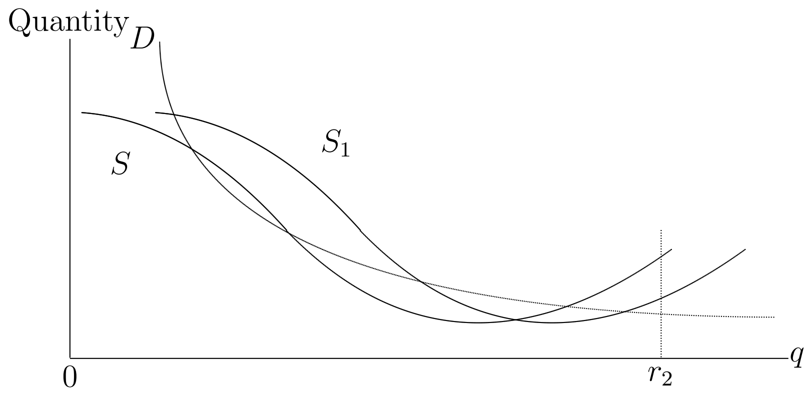

Figure 2: *Comparative static effects of an increase of the financial transactions tax: Regular versus irregular equilibria.*

the "low-price equilibrium" and the "high-price equilibrium" as regular, see Mas-Collel e.a. (1995). Formally, an equilibrium with price q^* is irregular if $D(q^*) = S(q^*)$ and $D'(q^*) > S'(q^*)$. The underlying idea is that excess demand should drive prices up, and that excess supply should drive prices down. Hence, starting from any non-equilibrium price, we will not approach the "equilibrium in the middle", but either the "low-price equilibrium" or the "high-price equilibrium". As the following Proposition shows, the focus on regular equilibria indeed makes it possible to offer one general conclusion:

Proposition 2 *Suppose that D and S are continuously differentiable and denote by t_0 an initial tax rate. Consider an equilibrium that is not irregular. There exists $\epsilon > 0$ so that moving to a higher tax rate $t_1 \in (t_0, t_0 + \epsilon)$ gives rise to a new equilibrium, which is in the neighborhood of the old equilibrium and such that:*

i) The price that sellers receive goes down: $p_1 < p_0$, where p_1 and p_0 are the equilibrium prices for sellers before and after the tax change, respectively.

ii) For fire-selling banks it is more difficult to survive in the new equilibrium: $p_1 < p_0$ implies that $I^*_{sell}(p_1) \subset I^*_{sell}(p_0)$.

According to the Proposition, if we are in a regular equilibrium, then, an increase of the tax rate will depress the revenue p that fire-selling banks realize per asset that they put on the financial market. As a consequence, it gets harder for them to generate the liquidity that is needed in order to survive, and (weakly) more fire-sellers will go bankrupt.

How does the range of equilibrium prices respond to taxation? We now turn to the question how the set of equilibrium prices responds to taxation. Proposition 2 can be used to answer this. A little reflection makes clear that both the minimal and the maximal element of the set of equilibrium prices are regular prices.[16] This yields the following Corollary to Proposition 2.

Corollary 1 *Suppose that D and S are continuously differentiable and denote by t_0 an initial tax rate. Let p_0^{min} and p_0^{max} by the minimal and the maximal element, respectively, in the set of equilibrium prices for sellers. There exists $\epsilon > 0$ so that moving to a higher tax rate $t_1 \in (t_0, t_0 + \epsilon)$ gives rise to a new set of equilibrium prices, with minimal element p_1^{min} and maximal element p_1^{max}, such that:*

i) The minimal price falls: $p_1^{min} \leq p_0^{min}$, with a strict inequality if $p_0^{min} \neq \frac{r_2}{1+t}$.

ii) The maximal price falls: $p_1^{max} \leq p_0^{max}$, with a strict inequality if $p_0^{max} \neq \frac{r_2}{1+t}$.

Implications for financial stability. Proposition 2 and Corollary 1 establish a general comparative statics result: An increase of the tax rate tends to decrease the price that sellers can realize and therefore tends to increase the number of distressed fire-sellers. However, this observation does not allow us to pin down the implications of a financial transactions tax for overall financial stability. This can be illustrated with the help of Figure 2. If we focus on the "low-price equilibrium" in Figure 2, then the implications of a small increase of the tax rate are the following: The equilibrium price q that buyers have to pay goes down. Hence, for fire-buying banks it is easier to survive in the new equilibrium since $q_1 < q_0$ implies that $I^{\dagger}_{buy}(q_0) \subset I^{\dagger}_{buy}(q_1)$. By contrast, if we focus on the "high-price-equilibrium" in Figure 2, then q goes up, so that, for all banks – i.e. for fire-selling as well as for fire-buying banks – it gets more difficult to survive.

To obtain more definite conclusions, we impose additional assumptions in what follows. In Section 5, we will look at a specific model with correlated long-run investment risk and idiosyncratic liquidity risk, which may be descriptive of the events following the collapse of Lehman brothers in September 2008. Before we turn to this case, we will, as a benchmark, investigate an economy in which fire sales do not arise.

[16]Consider a given irregular equilibrium. We first claim, that this equilibrium cannot be the equilibrium with the maximal level of q. This follows since (i) in any given irregular equilibrium, the supply curve lies above the demand curve for prices a little smaller than the equilibrium price and lies below the demand curve for prices a little larger than the equilibrium price, and (ii) for a price close to zero, the demand curve lies above the supply curve. Hence, there must be another intersection of the demand and the supply curve at a lower price. Similarly, we can show that to any irregular equilibrium with price q^*, we can find another equilibrium with a price that is higher: Either the supply and the demand curve intersect one more time at a price $q < r_2$, or they don't. If they do, we have another equilibrium with a higher price. If they do not, then at a price of $q = r_2$ the supply curve still lies below the demand curve, which implies that $q = r_2$ is another equilibrium price.

4 No fire sales

Assumption 1 *For all σ and all i, $\theta_{i1} \geq 0$.*

Under Assumption 1, all banks have enough cash in $T = 1$ to honor their current obligations. To make this assumption appear "natural" consider the following environment: The short-run investment is risk-free. Given our normalization $r_1 = 1$, investing funds with a short time horizon is equivalent to holding cash. Consequently, Assumption 1 is satisfied if all banks hold enough cash so that they can honor their obligations in $T = 1$.

Assumption 1 may be a consequence of regulation. For instance, the *net stable funding ratio* which is part of Basel III is a liquidity requirement for financial institutions. Alternatively, banks may voluntarily decide to hold enough liquidity. This may reduce the risk of bankruptcy or it may by driven by the hope to have cash reserves that make it possible to reap a profit if the financial market price q falls below its "fair value" of r_2.

Financial market outcomes. Under Assumption 1, we get clear-cut results on how the introduction of a financial transactions tax affects market prices and the number of distressed financial institutions.

Proposition 3 *Suppose that Assumption 1 holds and fix some arbitrary state σ of the economy. Suppose that q_0 and p_0 are equilibrium prices for buyers and sellers, respectively, at a tax rate of $t = 0$ and denote by $\hat{I}_0^\dagger$ the set of bankrupt banks in this equilibrium. Then, for any tax rate $t \neq 0$, there is an equilibrium so that:*

i) The equilibrium price for buyers is given by q_0.

ii) The equilibrium price for sellers is $\frac{q_0}{1+t}$.

iii) The set of bankrupt banks equals $\hat{I}_0^\dagger$.

The Proposition establishes a neutrality result. An increase of the tax rate does neither affect the price that buyers have to pay on the financial market nor the number of banks that go bankrupt. It only affects the revenue of a seller. Since the price that buyers pay is not affected by taxation, the price that sellers receive must go down if the tax rate goes up.

The intuition is as follows: Assumption 1 eliminates fire-sales. Therefore banks that face a risk of bankruptcy are those with $\theta_{i2} < 0$. These banks have to buy assets on the financial market. Otherwise they will not be able to honor their promises to debtors in $T = 2$. Hence, whether these banks survive depends only on the price that buyers face. As a consequence, the equilibrium condition for the buyers' price, $S(q) = D(q)$, becomes independent of the tax rate.

Long-run implications. In the following we will show that the short-run neutrality result in Proposition 3 extends to the long-run: If the bank's debtor's will be made whole in case of bankruptcy, then neither a bank's investment decisions nor its debt structure are affected if a financial transactions tax is introduced. As a first step, we show that a bank's expected profits are unaffected by taxation.

Proposition 4 *Under Assumption 1, a bank's expected profit is unaffected by the tax rate t. Formally, for any h_{0i}, and for any pair of tax rates t and t' with $t \neq t'$ we have that*

$$\Pi_i^2(h_{0i}, t) = \Pi_i^2(h_{0i}, t') \ .$$

The Proposition is a consequence of limited liability and of Proposition 3. In case of bankruptcy, limited liability implies that the holders of equity get a payoff of zero. Otherwise, profits depend on the price q which is unaffected by the tax rate. An increase of the tax rate therefore affects only the price p which, under Assumption 1, is irrelevant for the holders of equity. We now turn to the bank's initial choices, i.e. to bank i's long-run problem.

Proposition 5 *Under Assumption 1, a bank's optimal choices in the initial period are unaffected by the tax rate t, i.e. $C_{0i}^*(t) = C_{0i}^*(t')$, for any pair of tax rates t and t'.*

The Proposition follows immediately from two observations: (i) As shown in Proposition 4, the expected profit that is generated by any initial decision c_{0i} does not depend on the tax rate, and (ii) under a full bailout assumption, also the bank's constraints do not depend on the tax rate. Consequently, the set of optimal choices for bank i does not depend on the tax rate.

Propositions 3 and 5 identify conditions under which a financial transactions tax is inconsequential for the behavior of banks or for financial stability. There is also no effect on the government budget: The tax reduces the liquidation value for debtors in case a bank fails and generates some tax revenue for the government. If there is always a full bailout by the government, then the tax induced reduction of the liquidation value has to be compensated for by an increase of the payments to debtors.

5 Fire sales

We now investigate a model in which fire sales do occur. Specifically, we introduce a set of assumptions which are descriptive of the state of financial markets in the 2008/09 financial crises. Assumptions 2 - 5 below shift the focus to a financial system with the following features: Banks have primarily short-term debt and, moreover, their long-term performance is correlated. In addition, there is idiosyncratic liquidity risk.

Assumption 2 restricts attention to a symmetric environment. While banks may be heterogeneous ex post – that is, after the shock has hit in $T = 1$ – they are identical ex ante.

Assumption 2 *In $T = 0$, all banks are identical. This implies, in particular, that they all have the same structure of liabilities, i.e. there exist numbers X_1 and X_2 so that $x_{i1} = X_1$ and $x_{i2} = X_2$, for all i. Also, they choose the same short-run and long-run investments in $T = 0$. We denote the common investment levels by Y_1^0 and Y_2^0, respectively.*

Assumption 3 *There is aggregate risk with respect to the performance of long-run investments. All banks are equally affected by this aggregate risk. Formally, we assume that if bank i invests y_{i2}^0 into long-term projects, the volume of successful investments equals $y_{i2} = \sigma y_{i2}^0$, where $\sigma \in [0, 1]$ is an aggregate shock that affects all banks equally.*

All banks engage in the same long-run investment in $T = 2$, and they will all experience the same outcome, which can be good (high values σ) or bad (low values of σ). As an example, suppose that all banks have lent to a pool of home-owners. Within this pool, there is a certain rate of defaults. Since all banks have lent to the same pool, the default rate is the same for all banks. Put differently, their long-term lending risks are perfectly correlated. In a symmetric equilibrium where all banks choose the same investments – so that there is a number Y_2^0 such that $y_{i2}^0 = Y_2^0$, for all i – this implies, in particular, that they will experience the same performance of their long-run investments in $T = 1$. Hence, there exists a number $Y_2 = \sigma Y_2^0$ so that $y_{i2} = Y_2$, for all i.

Assumption 4 *We assume that $r_2 Y_2 > X_2$, or, equivalently, that $\theta_2 := r_2 Y_2 - X_2 \geq 0$, with probability 1. We also assume that $X_1 > \theta_2$ with probability 1.*

The assumption that $\theta_2 := Y_2 - X_2 > 0$ with probability 1 shifts the focus on fire-sales, and eliminates the possibility of fire-purchases from the model. Note that Assumption 4 is satisfied when banks have no long-term debt, i.e. if $X_2 = 0$, since Y_2 cannot become negative. Having primarily short-term debt and only little long term debt is typical for investment banks that obtain a lot of their financing on the repo or commercial paper market.

The assumption that $X_1 > \theta_2$ with probability 1 is made for ease of exposition. Economically, this assumption says that short-run liabilities are large relative to the net endowment θ_2 in period 2. As will become clear below, this assumption implies that even at a price of $p = r_2$, i.e. with the most favorable conditions for sellers on the financial market, a bank that experiences a complete failure of its short-run investments, so that $y_{i1} = 0$, will be unable to avoid bankruptcy. Put differently, the assumption ensures that for all $p \in [0, r_2]$, the set $I^{\dagger}_{sell}(p)$ has positive mass.[17]

Assumption 5 *There is idiosyncratic liquidity risk: Bank i's performing short-run investments are given by $y_{i1} = \beta_i Y_1^0$, where $\beta_i \in [0, 1]$. From the perspective of $T = 0$, β_i is a random variable that has a uniform distribution, and is stochastically independent from Y_2. Also, with an appeal to the law of large numbers, we assume that the cross-section distribution of β_i is a uniform distribution with probability 1, so that there is no aggregate liquidity risk. Finally, we assume that $Y_1^0 > X_1$.*

According to this assumption, the short-run investment gives a random return of β_i, where the random variable β_i captures an idiosyncratic investment risk. The expected return of bank i is $E[\beta_i]r_1 = \frac{1}{2}$. Due to the law of large numbers this is also the average return among all banks. The idiosyncratic liquidity risk is what generates a motive for trade on the financial market. Some banks lose a lot on their short-run investments and therefore have to sell some of their assets in order to honor their promises and to avoid bankruptcy. Other banks lose very little on their short-run investments and therefore have money left that they can invest on the financial market. The assumption that $Y_1^0 > X_1$ ensures that there are some banks with more money than they need in $T = 1$. Without this assumption there would be no demand on the financial market.

5.1 Financial Market Equilibrium

The given assumptions yield a financial market model that can be solved analytically. The following Proposition derives demand and supply on the financial market for a given state of the economy, or, equivalently, a given value of θ_2.

Proposition 6 *Under Assumptions 2 - 5 the demand and the supply functions are, respectively, given by*

$$D(q) = \frac{1}{q}\frac{(Y_1^0 - X_1)^2}{2Y_1^0} \quad \text{and} \quad S(q) = \frac{Y_2}{Y_1^0}X_1 - \frac{\theta_2}{2r_2Y_1^0}\left(Y_2 + \frac{X_2}{r_2}\right)\frac{q}{1+t} .$$

Since $\theta_2 > 0$ the supply curve is linear and downward sloping, as in Figure 1. The demand function D is downward sloping and convex, again as in Figure 1. This implies that the excess demand function $Z(q) := D(q) - S(q)$ is also globally convex so that there are

[17] Dropping this assumption would lead to a minor complication of the analysis. The supply function derived formally in Proposition 6 would look somewhat different, but it would still be a downward-sloping function.

at most two solutions to the equation $Z(q) = 0$. Generically, the set of equilibria will therefore have one of the following structures:

(a) There is only one equilibrium with $q = r_2$. This is the case if there is no $q \in [0, r_2]$ with $Z(q) = 0$.

(b) There is one regular equilibrium with $q < r_2$. This is the case if there is exactly one $q \in [0, r_2]$ with $Z(q) = 0$. This situation arises in Figure 1 with the supply function S_1.

(c) There is a both a regular and an irregular equilibrium with $q < r_2$. In addition, there is an equilibrium with $q = r_2$. This is the case if there are $q, q' \in [0, r_2]$ with $Z(q) = 0$ and $Z(q') = 0$, see Figure 2.

In case there are multiple equilibria as in (c) we may again dismiss the irregular equilibrium as implausible, in which case we are left with a "low-price equilibrium" with $q < r_2$ and an equilibrium with $q = r_2$. Moreover, under Assumptions 2 - 5, there is an unambiguous relation between the financial market price and the number of bankruptcies. The measure of bankrupt banks is given by

$$\int_{i \in I^{\dagger}_{sell}\left(\frac{q}{1+t}\right)} di = \int_0^{X_1 - \frac{q}{(1+t)r_2}\theta_2} \frac{1}{Y_1^0}\, ds = \frac{1}{Y_1^0}\left(X_1 - \frac{q}{(1+t)r_2}\theta_2\right) ,$$

which is a strictly decreasing function of q. Hence, from the viewpoint of financial stability – or from the perspective of a government that has to bail out debtors in case of bankruptcy – the "fair-price equilibrium" is also the "good equilibrium" and the "low-price equilibrium" is the "bad equilibrium."

Implications of an FTT. Proposition 6 makes it possible to clear-cut comparative statics results on the effects of a small increase of the tax t. If we focus on a "low-price equilibrium" and consider a small tax increase (graphically, a small rightward shift of the supply curve) this will lead to a decrease of the equilibrium value of q. If, by contrast, we focus on a "fair-price equilibrium", then we have $q = r_2$ before and after the tax change. These observations are summarized in the following Proposition, which we state without proof.

Proposition 7 *Suppose that Assumptions 2 - 5 hold and that there is an initial tax rate t_0. Consider an equilibrium that is not irregular. There exists $\epsilon > 0$ so that moving to a higher tax rate $t_1 \in (t_0, t_0 + \epsilon)$ gives rise to a new equilibrium which is in the neighborhood of the old equilibrium and has the following properties:*

i) The new equilibrium price p that sellers receive is lower.

ii) The new equilibrium price q that buyers pay is not higher.

iii) The new equilibrium has more bankrupt banks.

Proposition 7 deals with the consequences of a tax increase that leads to small price adjustments and which leaves the structure of the set of equilibria unchanged. An increase of the tax rate may however change the set of equilibria. Suppose we are initially in a situation with three equilibria, the bad "low-price equilibrium", the irrelevant "irregular equilibrium" and the good "fair-price equilibrium". An increase of the tax rate leaves the intercept of the supply curve unaffected but makes it flatter. Hence, if the tax rate is chosen sufficiently high, only the "low-price equilibrium" will be left, see the left part of Figure 3. Alternatively, the initial situation may be one in which the "fair-price equilibrium" is the only equilibrium because the supply curve and the demand curve do not intersect. Again, a big enough increase of the tax rate leads to a situation where the "low-price-equilibrium" is the only equilibrium, see the right part of Figure 3.

Proposition 8 *Suppose that Assumptions 2 - 5 hold and that*

$$r_2 > \frac{(Y_1^0 - X_1)^2}{Y_2 X_1} \,. \tag{5}$$

Suppose that there is an initial tax rate t_0 and that, at this tax rate, there exists a "fair-price equilibrium". There exists $\epsilon > 0$ so that moving to a higher tax rate $t_1 \geq t_0 + \epsilon$ gives rise to a new equilibrium set that contains only a "low-price equilibrium".

Proposition 8 shows that the introduction of a financial transactions tax can have drastic consequences for financial stability. Suppose that we are in an initial situation in which assets are fairly priced and the number of bankruptcies is modest. Then, an increase of the financial transactions tax may kill the "fair-price equilibrium" and so that only the "low-price equilibrium" is left. Consequently, there would be a large increase in the number of bankruptcies.

5.2 Interaction with other policy measures.

The experience of financial instability after the collapse of Lehman brothers in September 2008 has led to a number of policyinterventions. These included liquidity support by central banks (e.g. providing access to the Fed's discount window to investment banks in the US) and measures (such as the installation of bad banks in Europe or the toxic asset relief program in the US) with the intention to stop the vicious spiral of asset price falls, which trigger fire-sales, which trigger further reductions in asset prices etc. In addition, the excessive reliance of banks on short-term funding via the commercial paper or the repo

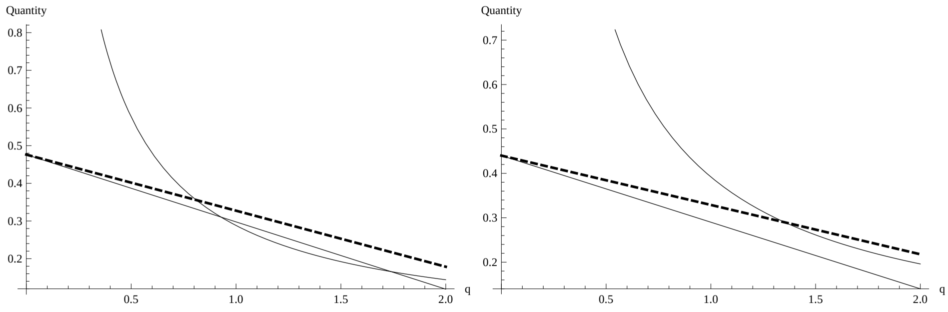

Figure 3: *Numerical examples: The parameters choices are $\theta_2 = X_1 = X_2 = Y_2 = 1$, $r_2 = 2$ and $Y_1^0 = 2.1$, in the figure on the left, and $Y_1^0 = 2.5$, in the figure on the right. The convex curves are the demand functions. The solid linear curves are the supply functions for a tax rate of $t = 0$. The dashed linear curves are the supply functions for a tax rate of $t = 0.2$ in the left figure and for tax rate of $t = 0.35$ in the right figure.*

market, has been identified as a major source of financial fragility. This problem could possibly be addressed via requirements that force banks to hold enough liquidity so that they can honor their short-term promises to debtors. In the following we will discuss these policy measures in the context of our model and study their interaction with a financial transactions tax. Our main finding is that these measure are all effective in the sense that they (i) make it more likely that there is a "fair-price equilibrium" and (ii) reduce the number of bankruptcies in a "low-price equilibrium". Hence, these measure have good implications from the perspective of financial stability. We will then show that a financial transactions tax has exactly the opposite effect: It makes the "fair-price equilibrium" less likely and increases the the number of bankruptcies in a "low-price equilibrium", i.e., it undoes the beneficial effects of other policy measures that serve to stabilize financial markets. We first look at policy interventions that serve to stabilize financial markets in the short-run. Subsequently, we turn to the long-run.

Liquidity Support. We formalize liquidity support by a central bank as a perfectly elastic supply of funds to non-bankrupt banks at an interest rate of 0. The restriction of lending to non-bankrupt banks ensures that the latter will indeed be able to pay back in $T = 2$ what they have borrowed in $T = 1$.

Assumption 6 *Suppose that all non-bankrupt banks can borrow in $T = 1$ as much as they want at an interest rate of* 0.

Proposition 9 *Under Assumptions 2-6, the "fair-price equilibrium" is the only equilibrium. An increase of the financial transactions tax*

i) does not affect the equilibrium price q that buyers have to pay.

ii) depresses the equilibrium price p that sellers receive and therefore increases the number of bankrupt banks.

The fact that banks can borrow as much as they want kills the "low-price equilibrium", which is the bad equilibrium from a financial stability viewpoint. The reason is that, at a price of $q < r_2$, buyers on the financial market can buy assets below their fair price and make a profit. This will trigger a huge demand for liquidity, which is then used to buy assets on the financial market. Consequently, at any price $q < r_2$, the demand on the financial market exceeds the supply, and this pushes prices up to the equilibrium level of $q = r_2$. Now, the beneficial effect of liquidity support is counteracted by the financial transactions tax. If the price for buyers is fixed at $q = r_2$, then $p = \frac{r_2}{1+t}$ implies that the price for sellers goes down if the tax rate t goes up. As a consequence, more fire-selling banks go bankrupt.

TARP/ Bad banks. An alternative way of restoring financial stability is to limit the amount of assets that are sold on the financial market. In 2008 the US government bought assets from distressed financial institutions via the the toxic asset relief program (*TARP*). An alternative measure is the installation of *bad banks*, i.e. of banks that hold rather than sell assets, and which are backed by a government guarantee. The main rationale for these policy measures was to limit the extent of adverse selection problems, by taking the worst, or most-difficult-to-value assets from the financial market. In the given model, we have abstracted from issues of adverse selection since we assumed that (i) it becomes commonly known in $T = 1$ which long-term investments fail and which ones perform, and (ii) that all performing long-term investments yield the same return in $T = 2$. We show in the following that, even in the given framework with no adverse selection problem, such programs can play a useful role simply because they limit the volume of fire-sales in a period of financial instability. We model the implications of a *TARP* or *Bad bank*-program via the following assumption:

Assumption 7 *The assets of bankrupt banks are not sold on the financial market.*

The financial market price still determines how many banks go bankrupt. The measure of bankrupt banks is given by the formula

$$\int_{i \in I^{\dagger}_{sell}\left(\frac{q}{1+t}\right)} di = \int_0^{X_1 - \frac{q}{(1+t)r_2}\theta_2} \frac{1}{Y_1^0}\, ds = \frac{1}{Y_1^0}\left(X_1 - \frac{q}{(1+t)r_2}\theta_2\right) .$$

However, the price that clears the financial market equates the demand for assets with the supply of assets by fire-selling banks who manage to survive. Without the government intervention, the supply would be larger since also the assets of bankrupt banks would be sold on the financial market. The equilibrium price q is therefore higher and the number of bankruptcies smaller than otherwise.

Proposition 10 *Under Assumptions 2-5 and 7, a financial market equilibrium has the following properties:*

i) The supply curve is increasing, implying that the equilibrium is unique.

ii) If the unique equilibrium is a "low-price equilibrium", i.e. an equilibrium with $q < r_2$, then the equilibrium price is higher and the number of bankruptcies is smaller than in a "low price equilibrium" that exists if the assets of bankrupt banks are sold on the financial market.

iii) An increase of the tax rate leads to a lower price p for sellers and an increased number of bankruptcies. The price q for buyers either goes up or remains constant.

Under Assumption 7 the supply function is increasing, rather than decreasing. The dominant force is that a higher price increases the number of fire-selling banks that can avoid bankruptcy which in turn increases the supply on the market. An implication of this is that there is a unique financial market equilibrium. To the extent that equilibrium multiplicity may be considered a problematic source of uncertainty for market participants (e.g. because it implies that it is difficult to determine the "true" value of an asset) it is a beneficial side effect of a *TARP* or *Bad-Bank* program that this multiplicity is eliminated.

With an increasing supply and a decreasing demand function, we get the "usual" comparative static effects of a tax increase on prices: the price p for buyers falls and the price q for seller rises, or remains constant. The fall of p, means, once more, that there is more financial distress.

Figure 4 illustrates the implications of a *TARP* or *Bad-Bank* program graphically, and also what happens if under such a program the tax rate is increased. In the left part, there is a unique "low-price equilibrium" with and without the program. The program gives rise to higher values of q and p, and hence fewer bankruptcies in case there is no taxation $t = 0$. If a tax is introduced the supply curve shifts to the right. This implies that q goes up, p goes down and the number of bankruptcies rises. In the right part, there are multiple equilibria without the program. With the program, only the "fair-price equilibrium" remains. Again, if a tax is introduced, the supply curve shifts to the right, implying that q stays put at a level of r_2, p goes down, and the number of bankruptcies goes up.

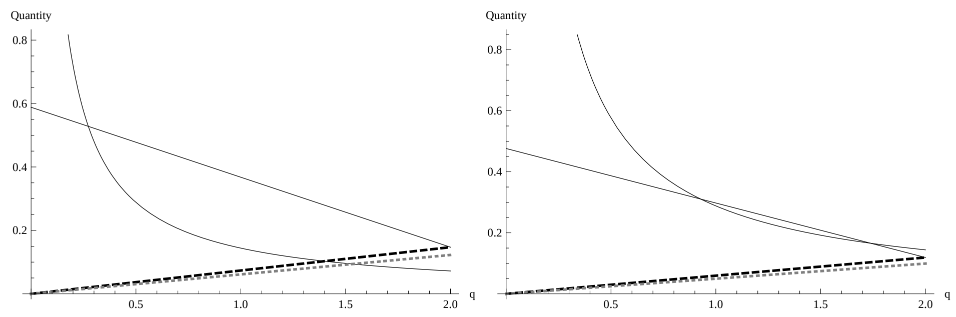

Figure 4: *Numerical examples: The parameters choices are* $\theta_2 = X_1 = X_2 = Y_2 = 1$, $r_2 = 2$ *and* $Y_1^0 = 1.7$, *in the figure on the left, and* $Y_1^0 = 2.1$ *in the figure right. The convex curves are the demand functions. The solid linear curves are the supply functions for a tax rate of* $t = 0$ *if there is no TARP-program. The dashed black curves are the supply functions with a TARP-program and a tax rate of* 0. *The dotted gray curves are the supply curves with a TARP-program and a tax rate of* $t = 0.2$.

5.3 Long-run implications

We now turn to a bank's choice of $c_{0i} = (y_{i1}^0, y_{i2}^0, x_{i1}, x_{i2})$ in the initial period. The following Proposition shows that the tax induced change of financial market outcomes gives banks an incentive to reduce their maturity mismatch.

Proposition 11 *Suppose there is a long-run equilibrium so that Assumptions 2 - 5 hold. Suppose that, for all* σ, $q(\sigma, t)$ *and* $p(\sigma, t)$ *are regular equilibrium prices and differentiable functions of the tax rate* t. *Then:*

i) The marginal return of long-run investments goes down and the marginal return of short-run investments goes up. Formally, for all c_{0i},

$$\frac{\partial^2 \Pi_i^2(c_{0i}, t)}{\partial t \, \partial y_{i1}^0} > 0, \quad \textit{and} \quad \frac{\partial^2 \Pi_i^2(c_{0i}, t)}{\partial t \, \partial y_{i2}^0} < 0 \, .$$

ii) The marginal cost of long-run debt goes up and the marginal cost of short-run debt goes down (weakly). Formally, for all c_{0i},

$$\frac{\partial^2 \Pi_i^2(h_{0i}, t)}{\partial t \, \partial x_{i1}} \leq 0, \quad \textit{and} \quad \frac{\partial^2 \Pi_i^2(h_{0i}, t)}{\partial t \, \partial x_{i2}} > 0 \, .$$

The Proposition shows that there is a beneficial effect of an increase of the financial transaction tax: Banks have incentive to reduce their maturity mismatch. Proposition 12 below implies that this leads to an increase of financial market prices and hence to fewer bankruptcies. The intuition is straightforward: The tax-induced increase of financial distress makes it more attractive to be a safe bank rather than a fire-seller. The chance of being safe bank is increased if maturity mismatch is reduced.

For reasons of tractability, the Proposition limits attention to local changes whose impact can be analyzed by looking at derivatives. We have seen previously, however, that tax increases can change the equilibrium structure so that, for some states σ, there is a jump from a "fair-price equilibrium" to a "low-price equilibrium". Allowing for such drastic shifts would reinforce the conclusions of Proposition 11. What is essential for the Proposition is the price response to a tax increase, namely that the price p falls in every state and that the price q either remains constant or falls. Now, the jump from a "fair-price equilibrium" to a "low-price equilibrium" has exactly this structure: both prices fall. Hence, Proposition 11 would extend if this possibility was acknowledged.

Propositions 7, 8 and 11 show that a financial transactions tax has two different, and opposing, implications for financial stability. In the short-run, for given investment decisions and debt structures, such a tax threatens financial stability. It depresses the prices on the financial markets and thereby makes life more difficult for a distressed financial institution. In the long run, however, banks have an incentive to adjust their investments and their debt structure so that maturity mismatch is reduced and distress becomes less likely. At first glance, this raises the question of how to strike a balance between these two effects. However, the beneficial corrective effects of the transactions tax could be generated in a different way, namely by a policy instrument that addresses maturity mismatch directly without having the destabilizing impact of a financial transactions tax. This is an implication of the following Proposition.

Proposition 12 *Suppose Assumptions 2-5 hold and that banks have only short-term debt, so that $X_1 > 0$ and $X_2 = 0$.*[18] *Suppose there is a "low-price-equilibrium" with an equilibrium price q^*.*

i) The equilibrium price q^* is an increasing function of Y_1^0.

ii) The equilibrium price q^* is a decreasing function of X_1.

iii) The equilibrium price q^* is a decreasing function of $Y_2 = \sigma Y_2^0$.

The Proposition says that a reduction of maturity mismatch is good for financial stability. At an aggregate level, this increase the liquidity that is available in $T = 1$. This implies that the demand on the financial market is higher, and this leads to a upward pressure on the financial market price, and reduces the number of distressed financial institutions.

As Propositions 11 reveals, a liquidity requirement has qualitatively the same long-run implications as a financial transactions tax. It forces banks to reduce their maturity mismatch. However, as Proposition 12 shows, this stabilizing long-run effect does not come at the cost of destabilizing short-run effect. With a liquidity requirement, for any given σ, the financial market outcome looks better. This positive effect would be weakened by a transactions tax that drives prices down and thereby increases financial distress.

[18] The Proposition trivially extends to situations where X_2 is not too large.

6 Concluding Remarks

This paper has looked at a stylized model of a financial market in order to trace out the implications of a financial transactions tax for financial stability. Key features of this model are that banks with liquidity needs may have to sell assets and that banks' assets are marked to the market. If many banks have to sell assets at the same time, this depresses the price, and, consequently, a large number of banks will be driven out of business. In such an environment, a financial transactions tax reduces the liquidity that is generated by the fire-sale of an asset because part of the revenue has to be shared with the government. Hence, distressed financial institutions have to sell more assets to meet their liquidity needs, and the number of distressed financial institutions rises. A financial transactions tax therefore has a destabilizing short-run effect.

The analysis has also shown that there may be a beneficial long-run effect. If banks understand the implications of a financial transactions tax, they have an incentive to reduce their maturity mismatch, i.e., to reduce the risk of ending up in the position of a fire-seller. It would be inappropriate, however, to advocate a financial transactions tax on this basis. A tax instrument, or some other regulatory tool, that addresses the discrepancy between banks' liquid assets and their short-term debt directly would have the same beneficial long-run consequences, without being destabilizing in periods of financial crisis.

The model in the paper was meant to be descriptive of the state of financial markets in the recent financial crises. It has deliberately abstracted from normative considerations. For instance, it was simply assumed that those who lend to a distressed bank will be bailed out by the government, without discussing the desirability of such bailouts. Also, it was noted that maturity mismatch lies at the heart of the crisis, without asking what an optimal degree of maturity mismatch would look like. These questions are of obvious importance for the design of optimal taxes and an optimal regulatory framework for the financial sector. They are, however, beyond the scope of this paper.

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Appendix

Proof of Proposition 1. We first show that $D(q) > S\left(\frac{q}{1+t}\right)$ as q approaches 0. Hence, at the minimal price of $q = p = 0$, demand exceeds supply on the financial market. Note that, for any q, $S\left(\frac{q}{1+t}\right)$ is bounded from above. This follows from the assumptions that in $T = 0$, each bank has a limited amount of funds that can be invested in the long asset, and that the number of performing assets in $T = 1$ cannot not exceed the investments made in $T = 0$. From

$$D(q) = \int_{\hat{I}^*(q)} \frac{\theta_{i1}}{q}\, di$$

and the assumption that the set $\hat{I}^*(0)$ has positive mass it follows that

$$\lim_{q \to 0} D(q) = \infty\ .$$

Now, since D and S are assumed to be continuous, there are only two possibilities: Either they do not intersect over the domain $(0, r_2]$. In this case it has to be true that $D(r_2) > S\left(\frac{r_2}{1+t}\right)$ so that r_2 is an equilibrium price. Or they do intersect, implying the existence of a price $q^* < r_2$ so that $D(q^*) = S\left(\frac{q^*}{1+t}\right)$, in which case q^* is an equilibrium price. $\square$

Proof of Proposition 2. For both types of equilibria, i.e. those with $D(q) = S(q)$ and $D' < S'$ and those with $D(r_2) > S(r_2)$, moving to a higher tax rate $t_1 \in (t_0, t + \epsilon)$ gives rise to a new equilibrium which is in the neighborhood of the old equilibrium. This can be illustrated graphically: The tax increase generates a "small" rightward shift of the supply curve. If the initial equilibrium is one with $D(q) = S(q)$ and $D' < S'$ then, because of continuous differentiability of D and S, the new equilibrium will inherit both of these properties. If, by contrast, the initial equilibrium is such that $q_0 = r_2$ and $D(r_2) > S(r_2)$, then, because of the continuity of D and S, after a small rightward shift of S it is still the case that $D(r_2) > S(r_2)$ implying that $q_1 = r_2$ is still an equilibrium price. With $p_0 = \frac{q_0}{1+t_0} = \frac{r_2}{1+t_0}$ and $p_1 = \frac{q_1}{1+t_1} = \frac{r_2}{1+t_1}$, the conclusion $p_0 < p_1$ then follows immediately for the latter type of equilibrium.

Now, consider an equilibrium with $D(q) = S(q)$ and $D'(q) < S'(q)$. We denote by $q^*(t)$ the equilibrium price for buyers as a function of the tax rate t. This price is implicitly defined by the equation $D(q^*(t)) = \bar{S}\left(\frac{q^*(t)}{1+t}\right)$. Analogously we define $p^*(t) := \frac{q^*(t)}{1+t}$. Straightforward calculations yield

$$p^{*\prime}(t) = \frac{q^*(t)}{(1+t)^2} \frac{D'(q^*(t))(1+t)}{\bar{S}'\left(\frac{q^*(t)}{1+t}\right) - D'(q^*(t))(1+t)} \tag{6}$$

For any q, and any given tax rate t, we have that $S(q) = \bar{S}\left(\frac{q}{1+t}\right)$. This implies that

$$S'(q)(1+t) = \bar{S}'\left(\frac{q}{1+t}\right) . \tag{7}$$

Using (7), we can rewrite equation (6) as

$$p^{*\prime}(t) = \frac{q^*(t)}{(1+t)^2} \frac{D'(q^*(t))}{S'(q^*(t)) - D'(q^*(t))} . \tag{8}$$

Since $D' < 0$, $p^{*\prime}(t) < 0$ if and only if $S' > D'$, i.e., if the initial equilibrium is regular.

To complete the proof we note that the claim in ii) is an immediate consequence of the definition of the set $I^*_{sell}(p)$.

$\square$

Proof of Proposition 3. *ad i)* Under Assumption 1, there are no failed banks and no fire-selling banks so that $I^\dagger = I^\dagger_{sell}(p) = I^*_{sell}(p) = \emptyset$. The set of bankrupt banks consists entirely of fire-buying banks that fail, i.e., of the set $I^\dagger_{buy}(q)$. Note that whether or not a bank fails depends only on the price q for buyers, but not on the price p for sellers. The supply on the financial market is therefore given by

$$S(q) = \int_{I^{\dagger}_{buy}(q)} y_{i2} \, di \; .$$

Observe that this supply function depends only on q. Consequently, the equilibrium condition – q is an equilibrium price if (i) $D(q) = S(q)$ and $q \leq r_2$, or (ii) $D(q) > S(q)$ and $q = r_2$ – is independent of the tax rate. Therefore, if q_0 fulfills the equilibrium condition for a tax rate of 0 then it fulfills the equilibrium condition for any tax rate.

ad ii) Immediate from part i) and the condition that $q = (1 + t)p$.

ad iii) Immediate from part i) and the observation in the proof of part i) that the set of bankrupt banks depends only on q. □

Proof of Proposition 4. Under Assumption 1, there are only three types of banks: safe ones, fire-buying banks that go bankrupt, and fire-buying banks that survive. Fix a state of the economy σ. Since banks are protected by limited liability, if $\theta_{i2} < 0$ and $\frac{q(\sigma,t)}{r_2} > -\frac{\theta_{i1}}{\theta_{i2}}$, a bank's profit is zero. Otherwise the bank makes a profit. Profit maximizing behavior of safe and fire-buying banks implies that the profit equals $\theta_{i2} + \frac{r_2}{q(\sigma,t)}\theta_{i1}$. We can therefore write

$$\pi_i^2(h_{0i}, \sigma_i, q(\sigma,t), p(\sigma,t)) = \mathbf{1}\left(\theta_{i2} + \tfrac{r_2}{q(\sigma,t)}\theta_{i1} \geq 0\right)\left(\theta_{i2} + \tfrac{r_2}{q(\sigma,t)}\theta_{i1}\right) \; .$$

Observe that the right hand side of this equation does not depend on $p(\sigma, t)$. Consequently, the ex ante expected profit

$$\Pi_i^2(h_{0i}, t) = E_\sigma\left[\mathbf{1}\left(\theta_{i2} + \frac{r_2}{q(\sigma,t)}\theta_{i1} \geq 0\right)\left(\theta_{i2} + \frac{r_2}{q(\sigma,t)}\theta_{i1}\right)\right] \; ,$$

also does not depend on $p(\sigma, t)$. It does depend on $q(\sigma, t)$. In Proposition 3, however, we have shown that for all σ, t and t', $q(\sigma, t) = q(\sigma, t')$. Consequently, we have for every σ, t and t' that

$$\mathbf{1}\left(\theta_{i2} + \tfrac{r_2}{q(\sigma,t)}\theta_{i1} \geq 0\right)\left(\theta_{i2} + \tfrac{r_2}{q(\sigma,t)}\theta_{i1}\right) = \mathbf{1}\left(\theta_{i2} + \tfrac{r_2}{q(\sigma,t')}\theta_{i1} \geq 0\right)\left(\theta_{i2} + \tfrac{r_2}{q(\sigma,t')}\theta_{i1}\right)$$

and therefore also that

$$\Pi_i^2(h_{0i}, t) = \Pi_i^2(h_{0i}, t') \; .$$

□

Proof of Proposition 5. By Proposition 4, under Assumption 1,

$$\Pi_i^2(h_{0i}, t) = \Pi_i^2(h_{0i}, t') \ ,$$

for any pair of tax rates t and t' and any vector h_{0i}. Hence, there exists a function $\bar{\Pi}_i^2 : h_{0i} \mapsto \bar{\Pi}_i^2(h_{0i})$ such that

$$\bar{\Pi}_i^2(h_{0i}) = \Pi_i^2(h_{0i}, t),$$

for any tax rate t. Bank i's long-run problem is therefore equivalent to the problem of choosing c_{0i} in order to maximize $\bar{\Pi}_i^2(h_{0i})$ subject to the constraints $U(x_{i1}, x_{i2}) \geq \bar{u}_i$ and $y_{i1}^0 + y_{i2}^0 = a_i$. Neither the objective function nor any of the constraints depends on the tax rate t. Hence, if c_{0i} is an optimal choice for some tax rate, then it is an optimal choice for any tax rate. □

Proof of Proposition 6. The assumption that $\theta_2 > 0$ implies that $I^\dagger = I^*_{buy}(q) = I^\dagger_{buy}(q) = \emptyset$, for all q. The demand for assets on the financial market is therefore given by

$$D(q) = \int_{i \in I^*} \frac{\theta_{i1}}{q} \, di$$

Under Assumptions 2 - 5, this can be written as

$$D(q) = \int_{X_1}^{Y_1^0} \frac{s - X_1}{q} \frac{1}{Y_1^0} \, ds = \frac{1}{q} \frac{(Y_1^0 - X_1)^2}{2Y_1^0} \ .$$

The supply on the financial market stems from banks in $I^\dagger_{sell}(p)$ who go bankrupt and whose assets are liquidated and from banks in $I^*_{sell}(p)$ that fire-sell and manage to survive. The supply of the former is given by

$$\bar{S}^\dagger(p) := \int_{i \in I^\dagger_{sell}(p)} Y_2 \, di = Y_2 \int_0^{X_1 - \frac{p}{r_2}\theta_2} \frac{1}{Y_1^0} \, ds = \frac{Y_2}{Y_1^0} \left(X_1 - \frac{p}{r_2} \theta_2 \right) \ .$$

The supply of the latter is given by

$$\bar{S}^*(p) := - \int_{i \in I^*_{sell}(p)} \frac{\theta_{i1}}{p} \, di = \int_{X_1 - \frac{p}{r_2}\theta_2}^{X_1} \frac{X_1 - s}{p} \frac{1}{Y_1^0} \, ds = \frac{1}{2} \frac{p}{Y_1^0} \left(\frac{\theta_2}{r_2} \right)^2$$

Summing these two expressions and exploiting that $\theta_2 = r_2 Y_2 - X_2$ yields

$$\bar{S}(p) = \frac{Y_2}{Y_1^0} X_1 - \frac{\theta_2}{2 r_2 Y_1^0} \left(Y_2 + \frac{X_2}{r_2} \right) p \ ,$$

and

$$S(q) = \frac{Y_2}{Y_1^0} X_1 - \frac{\theta_2}{2 r_2 Y_1^0} \left(Y_2 + \frac{X_2}{r_2} \right) \frac{q}{1+t} \ .$$

□

Proof of Proposition 8. The supply curve has an intercept of $\frac{Y_2}{Y_1^0}X_1$. The assumption $r_2 > \frac{(Y_1^0 - X_1)^2}{Y_2 X_1}$ ensures that a horizontal supply curve with this intercept (the limit case which arises as $t \to \infty$) intersects the demand curve at a price $q < r_2$. To see this, note that the price which equates $D(q)$ and $\frac{Y_2}{Y_1^0}X_1$ is given by

$$q = \frac{(Y_1^0 - X_1)^2}{Y_2 X_1} \, .$$

If (5) is violated, then there is no intersection of the supply and the demand curve at a price $q < r_2$, whatever the tax rate. Put differently, the "fair-price equilibrium" would be the only equilibrium, whatever the tax rate. By contrast, if (5) holds, then there is $\hat{t}$ so that for all $t \geq \hat{t}$, the "low-price equilibrium" is the only equilibrium. □

Proof of Proposition 9. Suppose there is an equilibrium with $q < r_2$. Then, if a bank lends q units of money from the central bank it can buy one asset and this asset will return r_2 in $T = 2$. In t_2 the bank has to repay q and therefore makes a profit of $r_2 - q > 0$. Hence, all banks will lend unlimited amounts from the central bank, and there will be an arbitrarily large demand on the financial market. Since the supply of assets on the financial market is bounded, this implies that $q < r_2$, and $D(q) > S(q)$. This contradicts the assumption that there is an equilibrium with $q < r_2$. There can also be no equilibrium with $q > r_2$, because in this case all non-bankrupt banks strictly prefer to hold cash over the purchase of assets on the financial market. Consequently, in any equilibrium, it has to be the case that $q = r_2$. This implies that, in equilibrium, $p = \frac{q}{1+t} = \frac{r_2}{1+t}$, so that an increase of t reduces p. The measure of bankrupt banks is given by

$$\int_{i \in I^{\dagger}_{sell}\left(\frac{q}{1+t}\right)} di = \int_0^{X_1 - \frac{q}{(1+t)r_2}\theta_2} \frac{1}{Y_1^0} \, ds = \frac{1}{Y_1^0}\left(X_1 - \frac{q}{(1+t)r_2}\theta_2\right) \, .$$

With $q = r_2$ this simplifies to $\frac{1}{Y_1^0}\left(X_1 - \frac{1}{(1+t)}\theta_2\right)$, an expression that is strictly increasing in t. □

Proof of Proposition 10. *ad i)* It follows from the argument in the proof of Proposition 6 that under Assumption 10, supply is given by a function S^*, with

$$S^*(q) = \frac{1}{2}\frac{q}{(1+t)Y_1^0}\left(\frac{\theta_2}{r_2}\right)^2 \, .$$

Obviously, this curve is increasing and hence has a unique intersection q' with the demand curve. If $q' \leq r_2$, then q' is the unique equilibrium price. Otherwise r_2 is the unique equilibrium price.

ad ii) Under Assumption 10, supply is given by S^*. Absent the government intervention supply is given by $S(q) = S^*(q) + S^{\dagger}(q)$, where $S^{\dagger}$ is defined formally in the proof of Proposition 6. Under Assumption 4, we have that $S^{\dagger} > 0$, for all $q < r_2$. Hence, for all

$q < r_2$, we have that $S(q) > S^*(q)$. Consequently, if q' is such that $S(q') = D(q')$, then $D(q') > S^*(q')$ and $q'' > q'$ for any q'' satisfying $D(q'') = S(q'')$.

ad iii) Consider a "low-price equilibrium." A small tax increase shifts the supply curve to the right and yields a new "low-price equilibrium". Since $S' > 0$ and $D' < 0$, the new equilibrium has a higher value of q and, by Proposition 2, a strictly lower value of p, and therefore a strictly larger number of bankrupt banks. A larger tax increase may eventually turn this "low-price equilibrium" into a "fair-price equilibrium". However, by the preceding argument, p falls and bankruptcies rise all along the way to the "fair-price equilibrium". Once we are in a "fair-price equilibrium", further tax increases leave the equilibrium value of q unaffected but depress the equilibrium value of p further, which leads to more bankruptcies. $\square$

Proof of Proposition 11. *Step 1.* We first show that, under Assumptions 2-5,

$$\begin{aligned}\Pi_i^2(h_{0i}, t) \quad = \quad & \frac{1}{2\ e_i\ y_{i1}^0} E_\sigma \left[\frac{r_2}{q(\sigma, t)} (y_{i1}^0 - x_{i1})^2 + 2(y_{i1}^0 - x_{i1})(r_2\ \sigma\ y_{i2}^0 - x_{i2}) \right. \\ & \left. + \frac{p(\sigma, t)}{r_2} (r_2\ \sigma\ y_{i2}^0 - x_{i2})^2 \right] ,\end{aligned} \tag{9}$$

To see this, note that, for any given pair of financial market prices p and q, we can write bank i's expected return on equity as

$$\pi_i^2(h_{0i}, \sigma_i, q, p) = \frac{1}{e_i} \left(V_i^{buy}(\sigma, q, h_{0i}) + V_i^{sell}(\sigma, p, h_{0i}) \right) , \tag{10}$$

where

$$V_i^{buy}(\sigma, q, h_{0i}) := \mathbf{1} \left(\theta_{i1} \geq 0, \theta_{i1} + \frac{q}{r_2} \theta_{i2} \geq 0 \right) \left(\theta_{i2} + \frac{r_2}{q} \theta_{i1} \right) \mid \sigma, q, h_{0i}$$

is the expected return on equity conditional on bank i having excess liquidity in $T = 1$ and therefore being a buyer on the financial market in $T = 1$, and

$$V_i^{sell}(\sigma, p, h_{0i}) := \mathbf{1} \left(\theta_{i1} \leq 0, \theta_{i1} + \frac{p}{r_2} \theta_{i2} \geq 0 \right) \left(\theta_{i2} + \frac{r_2}{p} \theta_{i1} \right) \mid \sigma, p, h_{0i}$$

is the expected return on equity conditional on bank i having to fire-sell on the financial market in $T = 1$. In these expressions $\mathbf{1}$ is the indicator function and expectations are taken with respect to the random variables θ_{i1} and θ_{i2}, respectively. Assumptions 4 and 5 imply that

$$\begin{aligned} V_i^{buy}(\sigma, q, h_{0i}) \quad &= \quad \textstyle\int_{x_{i1}}^{y_{i1}^0} \left(\theta_{i2} + \frac{r_2}{q}(s - x_{i1}) \right) \frac{1}{y_{i1}^0} ds \\ &= \quad \left(1 - \frac{x_{i1}}{y_{i1}^0} \right) \left(r_2\ \sigma\ y_{i2}^0 - x_{i2} + \frac{1}{2} \frac{r_2}{q(\sigma)} (y_{i1}^0 - x_{i1}) \right) . \end{aligned}$$

Assumptions 4 and 5 also imply that

$$\begin{aligned} V_i^{sell}(\sigma, q, h_{0i}) \quad &= \quad \textstyle\int_{x_{i1} - \frac{p}{r_2}\theta_{i2}}^{x_{i1}} \left(\theta_{i2} + \frac{r_2}{p}(s - x_{i1}) \right) \frac{1}{y_{i1}^0} ds \\ &= \quad \frac{1}{2} \frac{p(\sigma)}{r_2} \frac{(r_2\ \sigma\ y_{i2}^0 - x_{i2})^2}{y_{i1}^0} . \end{aligned}$$

Evaluating these expressions for $V_i^{sell}(\sigma, q, h_{0i})$ and $V_i^{sell}(\sigma, p, h_{0i})$ at the equilibrium prices $q(\sigma, t)$ and $p(\sigma, t)$, respectively, and substituting them into (10) yields

$$E[\pi_i^2 \mid \sigma, p(\sigma,t), q(\sigma,t), h_{0i}] = \\ \tfrac{1}{e_i}\left(\left(1-\tfrac{x_{i1}}{y_{i1}^0}\right)\left(r_2\ \sigma\ y_{i2}^0 - x_{i2} + \tfrac{1}{2}\tfrac{r_2}{q(\sigma,t)}(y_{i1}^0 - x_{i1})\right) + \tfrac{1}{2}\tfrac{p(\sigma,t)}{r_2}\tfrac{(r_2\ \sigma\ y_{i2}^0 - x_{i2})^2}{y_{i1}^0}\right) . \tag{11}$$

Our claim now follows from

$$\Pi_i^2(h_{0i}, t) = E_\sigma\left[\pi_i^2(h_{0i}, \sigma_i, q(\sigma,t), p(\sigma,t))\right]$$

and some straightforward algebraic manipulations.

Step 2. Fix some arbitrary vector h_{01}. Using equation (9) we can compute the following expressions

$$\begin{aligned}\frac{\partial^2 \Pi_i^2(h_{0i},t)}{\partial t\, \partial y_{i1}^0} &= \frac{-1}{2\ e_i\ (y_{i1}^0)^2} E_\sigma\left[\frac{r_2}{q(\sigma,t)^2}\frac{\partial q(\sigma,t)}{\partial t}((y_{i1}^0)^2 - (x_{i1})^2)\right. \\ &\qquad \left. + \frac{\partial p(\sigma,t)}{\partial t}(r_2\sigma y_{i2}^0 - x_{i2})^2\right] ,\end{aligned}$$

$$\frac{\partial^2 \Pi_i^2(h_{0i},t)}{\partial t\, \partial x_{i1}} = \frac{1}{e_i\ y_{i1}^0} E_\sigma\left[\frac{r_2}{q(\sigma,t)^2}\frac{\partial q(\sigma,t)}{\partial t}(y_{i1}^0 - x_{i1})\right] ,$$

$$\frac{\partial^2 \Pi_i^2(h_{0i},t)}{\partial t\, \partial y_{i2}^0} = \frac{1}{e_i\ y_{i1}^0} E_\sigma\left[\frac{1}{r_2}\frac{\partial p(\sigma,t)}{\partial t} r_2\sigma(r_2\sigma y_{i2}^0 - x_{i2})\right] ,$$

and

$$\frac{\partial^2 \Pi_i^2(h_{0i},t)}{\partial t\, \partial x_{i2}} = \frac{-1}{e_i\ y_{i1}^0} E_\sigma\left[\frac{1}{r_2}\frac{\partial p(\sigma,t)}{\partial t}(r_2\sigma y_{i2}^0 - x_{i2})\right] .$$

It follows from Proposition 7 that, for all σ,

$$\frac{\partial q(\sigma,t)}{\partial t} \leq 0 \quad \text{and} \quad \frac{\partial p(\sigma,t)}{\partial t} < 0.$$

Using these results and the assumption that, $y_{i1}^0 > x_{i1}$ and $\theta_{i2} := r_2\sigma y_{i2}^0 - x_{i2} > 0$, with probability 1, makes it possible to verify the following statements

$$\tfrac{\partial^2 \Pi_i^2(h_{0i},t)}{\partial t\, \partial y_{i1}^0} > 0, \quad \tfrac{\partial^2 \Pi_i^2(h_{0i},t)}{\partial t\, \partial x_{i1}} \leq 0, \quad \tfrac{\partial^2 \Pi_i^2(h_{0i},t)}{\partial t\, \partial y_{i2}^0} < 0 \quad \text{and} \quad \tfrac{\partial^2 \Pi_i^2(h_{0i},t)}{\partial t\, \partial x_{i2}} > 0 .$$

□

Proof of Proposition 12. Under Assumptions 2-5 and if $X_2 = 0$, demand and supply are given by

$$D(q) = \tfrac{1}{q}\tfrac{(Y_1^0 - X_1)^2}{2Y_1^0} \quad \text{and} \quad S(q) = \tfrac{Y_2}{Y_1^0}X_1 - \tfrac{1}{2Y_1^0}(Y_2)^2\tfrac{q}{1+t} .$$

The equation $D(q) = S(q)$ has two solutions. We focus on a "low-price-equilibrium" that is on the smallest q such that $D(q) = S(q)$, henceforth referred to as q^*. Straightforward computations yield

$$q^* = \frac{(1+t)X_1 - \sqrt{(1+t)^2(X_1)^2 - (1+t)(Y_1^0 - X_1)^2}}{Y_2} ,$$

an expression which increases in Y_1^0 and decreases in Y_2 and in X_1. □