

Does longevity improvement always raise the length of schooling through the longer-horizon mechanism? *

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Abstract

Hazan (2009) performs empirical analysis based on the conjecture that a necessary condition for higher life expectancy to cause longer schooling years is that it also increases lifetime labor supply, and reaches controversial conclusions. We aim to examine the theoretical validity of Hazan's (2009) conjecture, and more generally, to understand the relation between these two conditions in a standard life-cycle model. We find that the relation between the effects on optimal schooling years and expected lifetime labor supply differs systematically with respect to mortality reductions at different stages of the life cycle. Based on these systematic differences, we find that longer lifetime labor supply is not sufficient for increased schooling years for mortality reductions during the schooling years, and not necessary for increased schooling years for some mortality reductions during the working years. We provide explanations regarding why Ben-Porath's (1967) longer-horizon mechanism in the analysis of the timing of human capital investment is not always applicable to the question regarding the impact of mortality decline on human capital investment.

JEL Classification Numbers: I20; J10; J24

Keywords: mortality decline; schooling years; lifetime labor supply; longer-horizon mechanism

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1 Introduction

Hazan (2009) examines whether the increase in average years of schooling in USA from mid 1800s to mid 1900s was caused by mortality decline or not. Using a life-cycle model with perfectly rectangular survival function (with zero probability of death during lifetime, and then dying with certainty when reaching the maximum age), he suggests that a necessary condition for higher life expectancy to cause longer schooling years is that it also increases lifetime labor supply. His empirical analysis based on the data of USA (as well as several European countries) suggests that expected lifetime work hours actually decreased with a rise in life expectancy, and he concludes that the observed increase in schooling years cannot be explained by mortality decline. As mentioned in Cervellati and Sunde (2010), Hazan's (2009) conclusion, which challenges a conventional prediction of the human capital theory, has important policy implications regarding the benefit of health improvement and mortality decline.

Not surprisingly, Hazan's (2009) controversial conclusions generate heated responses.¹ In particular, Cervellati and Sunde (2010) argue that the rectangular survival function specification is not empirically relevant, since longevity increases from mid 1800s to mid 1900s were caused by mortality reductions at various ages (especially those in youth and working ages). They then show that in a life-cycle model with a general survival function, a mortality decline induces more schooling if and only if it increases the benefits of increased schooling relative to the costs, and that this condition is reduced to the condition on lifetime labor supply emphasized in Hazan (2009) when the survival function is rectangular. Moreover, they perform empirical analysis and show that while there is a pronounced decline in expected lifetime labor supply across the cohorts of men born at different decades from 1840 to 1930 (as in Hazan, 2009), there is no evidence of a decline in the benefits relative to the opportunity costs of schooling. Their empirical results challenge Hazan's (2009) conclusions that the increase in schooling cannot be caused by mortality decline.

This paper aims to contribute to this debate by examining a theoretic-

¹Some researchers (such as Hansen and Lønstrup, 2012; Strulik and Werner, 2012) respond by constructing theoretical models to rationalize the simultaneous increase in schooling years and decrease in expected lifetime labor supply when longevity improves. Other researchers (such as Cervellati and Sunde, 2010; 2012) question whether Hazan (2009) has used the appropriate condition or not when reaching his conclusion. Note that Cervellati and Sunde (2010) use a continuous-time framework, but they use a discrete-time one in their updated version (Cervellati and Sunde, 2012). Since this paper uses a continuous-time model, we refer mainly to their earlier paper.

cal issue: Is “mortality decline raising expected lifetime labor supply” (to be labeled as MDrELLS) a necessary or sufficient condition for “mortality decline raising schooling years” (to be labeled as MDrSY) in the standard life-cycle framework used in the literature. This investigation is important for at least three reasons. First, Hazan’s (2009) conclusions depend crucially on the theoretical conjecture that MDrELLS is *a necessary condition* for MDrSY, and his conclusions would be questionable if the theoretical foundation is not sound.² Second, while Hazan (2009) makes the above-mentioned conjecture, MDrELLS is interpreted, explicitly or implicitly, as *a sufficient condition* for MDrSY by a number of researchers. For example, Kalemli-Ozcan et al. (2000, p. 18) state that “(h)igher life expectancy raises the optimal quantity of schooling because investments in education will earn a return over a longer period of time.” Similarly, Bils and Klenow (2000, p. 1164) mention that “a higher life expectancy results in more schooling, since it affords a longer working period over which to reap the wage benefits of schooling.” Given these diverse interpretations,³ we aim to clarify the relationship between these two conditions and provide intuitive explanation. Third, Cervellati and Sunde (2010) argue that for a general non-rectangular survival function, the more appropriate condition is to compare the marginal benefit with marginal cost of extending schooling years, but not focusing on the lifetime labor supply. While we agree with Cervellati and Sunde (2010) that a non-rectangular survival function is more empirically relevant, we think that their theoretical argument is not fully developed, since they only point out that their condition (4) for a general survival function can be reduced to the lifetime labor supply condition emphasized by Hazan (2009) when the survival function is rectangular. It would have been more complete if they have done a step further by showing explicitly that MDrELLS is not a necessary condition for MDrSY for a general survival function. In this paper we address this question directly by providing comparative statics analysis of the optimal condition on the schooling year choice. In particular, we examine the derivative of optimal schooling years with respect to mortality changes at arbitrary ages, and obtain results that, to the best of our knowledge, have

²Hazan (2009) emphasizes that while satisfying a necessary condition (MDrELLS in this context) only provides supporting evidence for a hypothesis (MDrSY), the rejection of a necessary condition is enough to refute the hypothesis.

³We give one more example to illustrate the diverse interpretations about the relationship of these two conditions. While Hazan (2009) mentions several times that MDrELLS is a necessary condition for MDrSY, he also indicates that it is *a necessary and sufficient condition* in one instance, when he states that “as individuals live longer, they invest more in human capital if and only if their lifetime labor supply increases.” (Hazan, 2009, p. 1832)

not been emphasized in the literature.

Our novel results arise from the discovery of the systematic differences in the effects of mortality reductions at different ages,⁴ but not from any unconventional features. To highlight this point, we use a life-cycle model as similar as possible to those in the literature, especially in Hazan (2009) and Cervellati and Sunde (2010). After obtaining the first-order condition characterizing the optimal schooling years, we study the effect of mortality decline on optimal schooling years and expected lifetime labor supply. We obtain two main results. First, the effects on optimal schooling years are quite different in the three distinct phases of the life cycle: no effect in schooling and retirement phases, and a positive effect during the working phase. (We will provide the intuition in Section 3.) Second, we study the effect of mortality decline on expected lifetime labor supply, and find that it consists of two effects: a survival effect due to possible changes in survival probabilities at various ages, and a behavioral effect due to a change in optimal schooling years. We then combine the results to show that MDrELLS is neither necessary nor sufficient for MDrSY in the life-cycle model with a general non-rectangular survival function.

Our results raise doubts about Hazan’s (2009) conclusion that the increase in schooling years in the last century cannot be caused by mortality decline, because the theoretical foundation of his conclusion is invalid for a general survival function. More generally, we clarify the relationship between MDrELLS and MDrSY, and provide some explanations regarding why the misunderstandings about the relation between these two conditions arise in the literature.

This paper is organized as follows. Section 2 introduces the model and derives the first-order condition characterizing the schooling year choice. Section 3 examines the impact of mortality decline on optimal schooling years. Section 4 studies the impact on expected lifetime labor supply, and examines the relationship between MDrELLS and MDrSY. Section 5 provides the concluding remarks.

⁴Our interest in this question is partly stimulated by d’Albis et al. (2012) who show that mortality changes at young versus old ages may have different economic effects. Empirically, mortality decline concentrates mainly on children and young adults in the early stages of the demographic transition, but mainly on older people at the later stages. This pattern is well documented by demographers and health economists. For example, Wilmoth and Horiuchi (1999, pp. 484-5) mention that during the later stage of demographic transition, an “aging of mortality decline” has occurred, characterized by “successively larger reductions in mortality rates at older ages, and by smaller reductions at younger ages.” Eggleston and Fuchs (2012) discuss various behavioral and policy issues related to longevity improvement of high-income countries in recent decades, in which mortality decline occurs mainly late in life. They label it the “new demographic transition.”

2 The model

We consider a continuous-time life-cycle model in which an individual chooses the planned consumption path (from age 0 to T , the maximum age) and years of schooling (S) in the presence of lifetime uncertainty.⁵ The lifetime uncertainty is represented by a general survival function

$$l(x) = e^{-\int_0^x \mu(q) dq}, \quad (1)$$

where $l(x)$ is the probability of surviving up to at least age x , $l(0) = 1$, $l(T) = 0$, and $\mu(q) \geq 0$ (with $\lim_{q \rightarrow T} \mu(q) = \infty$) is the instantaneous mortality rate at age q .

We assume that schooling and labor supply choices at a particular time are indivisible, and that the progression from schooling to working phases is irreversible.⁶ These assumptions follow many researchers such as Kalemli-Ozcan et al. (2000), Hazan (2009), Heijdra and Romp (2009) and Cervellati and Sunde (2010). We also follow Kalemli-Ozcan et al. (2000), Heijdra and Romp (2009) and Cervellati and Sunde (2010) to assume that retirement age (R) is exogenously fixed.⁷ In this environment, an individual chooses a

⁵In most life-cycle models focusing on working and retirement phases, the age in the model is usually taken as adult age (such as actual age minus 20). When schooling decisions are included, Boucekine et al. (2003) interpret the model age as actual age minus 10. For high-income countries in recent years, it is probably better to interpret the model age as actual age minus 15, since it is likely to have mandatory age of working (usually 15 years old) and years of schooling (usually 9 years) in these countries.

⁶The assumption that the progression from schooling to work is irreversible, used here and by many researchers, is partly motivated by the theoretical results in Ben-Porath (1967).

⁷Empirical analysis on mortality decline, optimal schooling years and expected lifetime labor supply (such as in Hazan, 2009; Cervellati and Sunde, 2010) allow for different age-specific survival probabilities, labor force participation rates and working hours for different cohorts. However, for tractability reasons, many of these aspects are assumed to be fixed in the accompanying theoretical analysis. Since the key reason that some of the existing results may be misleading is that mortality reductions for different ages have not been carefully analyzed, we allow for age-specific mortality rates. On the other hand, we keep other aspects of the model simple. We find that MDrELLS is neither necessary nor sufficient for MDrSY for this model. Since these results provide counter-examples to the hypotheses in the literature, our simplifying assumptions are not misleading. For example, it can be shown that, using a disutility of labor specification (as in Bloom et al., 2007; d'Albis et al., 2012), the model with exogenous retirement age is a special case of the model with endogenous retirement age, when the slope of the disutility of labor function tend to infinity at the optimal retirement age. Had we used the endogenous retirement age model here, our counter-examples will still hold for this special case. In this paper, we choose the model with exogenous retirement age and other simplifying assumptions, which delivers sharp results but still serves our purposes.

consumption path and schooling years (between age 0 to age R) to maximize expected lifetime utility

$$\int_0^R e^{-\rho x} l(x) \left[\frac{c(x)^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} \right] dx + \int_R^T e^{-\rho x} l(x) \left[\frac{[(1 + \zeta) c(x)]^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} \right] dx, \quad (2)$$

subject to

$$a'(x) = \begin{cases} [r + \mu(x)] a(x) + h(S) w(x) - c(x) & \text{if } S < x \leq R \\ [r + \mu(x)] a(x) - c(x) & \text{if } x \leq S \text{ or } x > R \end{cases}, \quad (3)$$

and boundary conditions $a(0) = 0$, $a(T) \geq 0$, where ρ is the discount rate, σ is the intertemporal elasticity of substitution, ζ is a parameter capturing the utility from leisure during retirement, r is the constant real interest rate, $c(x)$ is consumption at age x , $a(x)$ is financial wealth at age x , $a'(x)$ is the derivative of $a(x)$ with respect to x , and $h(S) w(x)$ is the individual's labor income at age x , if he receives S years of schooling.⁸ We assume that $\rho \geq 0$, $r \geq 0$, $0 < \sigma < 1$, and $\zeta > 0$.

This life-cycle model with (1) to (3) also contains the following features. First, we assume that individuals have no bequest motive, and that annuities are perfect (as in Yaari, 1965; Blanchard, 1985). It is well known that in such an environment, individuals will find it optimal to purchase annuity contracts. As captured in (3), an individual aged x will surrender financial wealth $a(x)$ to the insurance company if death occurs, but will receive an extra amount equal to $\mu(x) a(x)$ if death does not occur. Second, we follow many researchers in the literature by assuming that foregone labor income is the only cost of schooling.⁹ Third, we follow Manuelli et al. (2012) to assume in (2) that the form of instantaneous utility function in the retirement phase is different from that during the schooling and working phases, so as to capture the (endogenous) drop in consumption level at retirement.

The individual's various choices are obtained as follows. (The detailed analysis is given in the Appendix.) First, conditional on a particular length of the schooling period, we obtain the optimal consumption path, defined as $c(S, x)$. It can be shown that the (conditional) optimal consumption path is

⁸Note that human capital is accumulated only through formal schooling in this model, following Bils and Klenow (2000), Kalemli-Ozcan et al. (2000), Hazan (2009) and Cervellati and Sunde (2010). On the other hand, human capital is also accumulated through on-the-job training in Manuelli et al. (2012).

⁹This assumption is made in Kalemli-Ozcan et al. (2000), Hazan (2009) and Cervellati and Sunde (2010). On the other hand, Bils and Klenow (2000) assume both forgone labor income and tuition cost. Our main results will also be applicable for a model with tuition cost, in a similar spirit as the reason mentioned in footnote 7.

characterized by

$$c(S, x) = \begin{cases} e^{\sigma(r-\rho)x} c(S, 0) & \text{if } x \leq R \\ (1 + \zeta)^{\sigma-1} e^{\sigma(r-\rho)x} c(S, 0) & \text{if } x > R \end{cases} . \quad (4)$$

Moreover, the initial consumption level, $c(S, 0)$, can be solved by substituting (4) into the intertemporal budget constraint at age 0, which is given by

$$\int_0^T e^{-rx} l(x) c(S, x) dx = \int_S^R e^{-rx} l(x) h(S) w(x) dx. \quad (5)$$

Second, conditional on the optimal consumption path in (4), the optimal (interior) length of the schooling period is given by¹⁰

$$h'(S^*) \left[\int_{S^*}^R e^{-rx} l(x) w(x) dx \right] = e^{-rS^*} l(S^*) h(S^*) w(S^*), \quad (6)$$

when the second-order condition

$$\frac{w'(S^*)}{w(S^*)} + \frac{2h'(S^*)}{h(S^*)} - \frac{h''(S^*)}{h'(S^*)} - \mu(S^*) - r > 0 \quad (7)$$

holds. The left-hand side of the first-order condition (6) is the marginal benefit of continuing to study (evaluated at the optimal choice S^*), which is measured by the increases in labor income throughout the working years from age S^* to age R , due to higher level of human capital. The right-hand side of (6) is the marginal cost (foregone labor income at age S^*) of postponing the entry into the labor market. It can be observed that (6) is similar to (3) in Cervellati and Sunde (2010).

3 Impact of mortality decline on optimal schooling years

Based on the first-order condition (6), we now consider the impact of a change in the mortality rate at an arbitrary age on optimal schooling years. These

¹⁰Note that the first-order condition (6) can also be derived by maximizing lifetime (human) wealth, given by the right-hand side of (5), with respect to schooling years. This result is based on the “separation theorem” for human capital investment, when capital markets are perfect. (See Acemoglu, 2009, Section 10.1.) Note also that (6) can be written as

$$h'(S^*) \left[\int_{S^*}^R e^{-r(x-S^*)} \frac{l(x)}{l(S^*)} w(x) dx \right] = h(S^*) w(S^*). \quad (6a)$$

In (6a), both marginal benefit and marginal cost of extending schooling years are expressed as the present discounted value back to age S^* , while they are expressed as the present discounted value back to age 0 in (6).

results will be useful when we examine, in the next section, the relationship between MDrSY and MDrELLS.

For the subsequent analysis, it turns out that a major channel of the effect of a mortality change at age x_0 is through a change in $l(x)$, survival probability up to age x , which may be different from x_0 . Since $\mu(x_0)$ is itself a function of age, the appropriate concept to use is the Volterra derivative of a functional (Volterra, 1959; Ryder and Heal, 1973; d’Albis et al., 2012). Using (1), it can be shown that¹¹

$$\frac{-\partial l(x)}{\partial \mu(x_0)} = \begin{cases} 0 & \text{if } x < x_0 \\ l(x) & \text{if } x \geq x_0 \end{cases}. \quad (8)$$

According to (8), a mortality decline at age x_0 only increases the survival probabilities at ages equal to or after x_0 , but has no effect on the survival probabilities before that age.¹²

Recognizing the dependence of $l(x)$ and S^* on $\mu(x_0)$ according to (1) and (6), we can differentiate (6) with respect to $\mu(x_0)$ to obtain

$$\frac{-\partial S^*}{\partial \mu(x_0)} = \frac{\frac{\int_{S^*}^R e^{-rx} \left(\frac{-\partial l(x)}{\partial \mu(x_0)} \right) w(x) dx}{\int_{S^*}^R e^{-rx} l(x) w(x) dx} - \frac{\left(\frac{-\partial l(S^*)}{\partial \mu(x_0)} \right)}{l(S^*)}}{\frac{w'(S^*)}{w(S^*)} + \frac{2h'(S^*)}{h(S^*)} - \frac{h''(S^*)}{h'(S^*)} - \mu(S^*) - r}. \quad (9)$$

According to the first-order condition (6), any exogenous mortality shock can potentially affect the optimal schooling choice S^* in two ways: (a) it affects

¹¹Note that we consider $\frac{-\partial l(x)}{\partial \mu(x_0)}$, which corresponds to the effect on survival probability to age x of a mortality decrease at age x_0 , instead of that of a mortality increase, $\frac{\partial l(x)}{\partial \mu(x_0)}$.

¹²The following results in a discrete-time framework may be useful to understand the intuition behind (8) of the continuous-time model. Let l_x be the survival probability to age x , where x is an integer from 0 to maximum lifetime T , with the initial value $l_0 = 1$ (normalization). Define $q_{i-1,i}$ ($i = 1, 2, \dots, T$) as the probability of death during period $i - 1$ to period i , with $q_{T-1,T} = 1$. It is easy to see that for $x = 1, 2, \dots, T$,

$$l_x = \prod_{i=1}^x (1 - q_{i-1,i}). \quad (1a)$$

Differentiating (1a) with respect to $q_{i-1,i}$, we have

$$\frac{-\partial l_x}{\partial q_{i-1,i}} = \begin{cases} 0 & \text{if } x + 1 \leq i \leq T \\ \frac{l_x}{1 - q_{i-1,i}} & \text{if } 1 \leq i \leq x \end{cases}. \quad (8a)$$

We now consider the thought experiment of increasing the number of “periods” to infinity when maximum lifetime T remains unchanged. In this case, the duration of each period decreases to zero and thus, $q_{i-1,i}$ tends to 0. Equation (8) of the continuous-time model can be interpreted as the limiting case of (8a) of the corresponding discrete-time model.

the expected stream of labor income during the working years through the survival probabilities $l(\cdot)$ from age S^* to age R , (b) it affects the expected value of foregone earning (at S^*) through the survival probability $l(S^*)$. Differentiating (6) with respect to $\mu(x_0)$ leads to various terms in the right-hand side of (9), with the first term in the numerator corresponding to the benefit of increasing schooling, and the second term corresponding to the cost. Note that the denominator of (9) is positive when the second-order condition (7) holds.

Using (8), we obtain

$$\frac{-\partial l(S^*)}{\partial \mu(x_0)} = \begin{cases} l(S^*) & \text{if } x_0 \leq S^* \\ 0 & \text{if } x_0 > S^* \end{cases}, \quad (10)$$

and

$$\int_{S^*}^R e^{-rx} \left(\frac{-\partial l(x)}{\partial \mu(x_0)} \right) w(x) dx = \begin{cases} \int_{S^*}^R e^{-rx} l(x) w(x) dx & \text{if } x_0 \leq S^* \\ \int_{x_0}^R e^{-rx} l(x) w(x) dx & \text{if } S^* < x_0 < R \\ 0 & \text{if } x_0 \geq R \end{cases}. \quad (11)$$

Combining (7) to (11), we have

$$\begin{aligned} \text{sign} \left[\frac{-\partial S^*}{\partial \mu(x_0)} \right] &= \text{sign} \left[\frac{\int_{S^*}^R e^{-rx} \left(\frac{-\partial l(x)}{\partial \mu(x_0)} \right) w(x) dx}{\int_{S^*}^R e^{-rx} l(x) w(x) dx} - \frac{\left(\frac{-\partial l(S^*)}{\partial \mu(x_0)} \right)}{l(S^*)} \right] \\ &= \begin{cases} 0 & \text{if } x_0 \leq S^* \\ +1 & \text{if } S^* < x_0 < R \\ 0 & \text{if } x_0 \geq R \end{cases}. \end{aligned} \quad (12)$$

We summarize the impact of mortality decline on the optimal schooling years in Proposition 1.

Proposition 1. *For the life-cycle model given by (1) to (3),*

- (a) $\frac{-\partial S^*}{\partial \mu(x_0)} = 0$ if $x_0 \leq S^*$,
- (b) $\frac{-\partial S^*}{\partial \mu(x_0)} > 0$ if $S^* < x_0 < R$, and
- (c) $\frac{-\partial S^*}{\partial \mu(x_0)} = 0$ if $x_0 \geq R$.

A mortality decline at age x_0 has a positive effect on optimal years of schooling during the working phase (when $S^* < x_0 < R$), but has no effect on S^* during the schooling or retirement phases (when $x_0 \leq S^*$ or $x_0 \geq R$). The intuitions of these results are set out in the following paragraphs.

When mortality decline occurs after retirement ($x_0 \geq R$), it has no effect on the survival probabilities before age R . Since both the marginal benefit and marginal cost schedules of increasing the schooling length are only related

to the variables from age S^* to age R , they are not affected by a change in $\mu(x_0)$ when $x_0 \geq R$. Thus, the optimal retirement age remains the same.

When mortality decline occurs during the schooling phase ($x_0 \leq S^*$), it affects the survival probabilities in the future, including the working years. As a result, both marginal benefit and marginal cost of increasing the schooling duration are affected, with marginal cost involves a term at a point in time (age S^*) and marginal benefit involves terms over a duration (from age S^* to age R). According to (9), a change in $\mu(x_0)$ affects the marginal cost through $l(S^*)$ and it affects marginal benefit through $\int_{S^*}^R e^{-rx} l(x) w(x) dx$. Moreover, the quantitative effects of these two factors are given by the percentage terms $\frac{\int_{S^*}^R e^{-rx} \left(\frac{-\partial l(x)}{\partial \mu(x_0)}\right) w(x) dx}{\int_{S^*}^R e^{-rx} l(x) w(x) dx}$ and $\frac{\left(\frac{-\partial l(S^*)}{\partial \mu(x_0)}\right)}{l(S^*)}$, and both terms equal to 1 when $x_0 \leq S^*$.¹³ Thus, both the marginal benefit and marginal cost curves shift upward by the same proportion at the optimal years of schooling S^* , resulting in an unchanged optimal schooling duration.

In contrast, a decline in $\mu(x_0)$ during the working phase (between ages S^* and R) has different effects on the marginal benefit and marginal cost of schooling decision. Specifically, it increases the marginal benefit (from age x_0 to age R) but does not affect the marginal cost. As a result, only the marginal benefit curve shifts up, and the optimal length of the schooling period increases.

The results of these three cases are summarized diagrammatically in Figure 1.

[Insert Figure 1 here.]

4 Is MDrELLS necessary or sufficient for MDrSY?

We now examine the relationship between MDrELLS and MDrSY in the life-cycle model given by (1) to (3). First, we consider the effect of mortality decline on expected lifetime labor supply (*ELLS*), which is defined as

$$ELLS = \int_{S^*}^R l(x) dx. \quad (13)$$

¹³It may be easier to see from (6a) in footnote 10 that the effects of a change in $\mu(x_0)$ in the schooling phase on $l(S^*)$ and $\int_{S^*}^R e^{-rx} l(x) w(x) dx$ cancel out exactly. When $x_0 \leq S^*$, a change in $\mu(x_0)$ affects both the numerator and denominator terms of the conditional survival probability $\frac{l(x)}{l(S^*)}$ in the left-hand side of (6a) by the same proportion, and thus, it does not affect $\frac{l(x)}{l(S^*)}$ for $x \in (S^*, R)$.

Note that the expected lifetime labor supply depends on the survival probabilities during the working years, as well as the optimal schooling years S^* , which is the lower limit of the integral.

Differentiating $ELLS$ with respect to $\mu(x_0)$ gives

$$\frac{-\partial ELLS}{\partial \mu(x_0)} = \int_{S^*}^R \left(\frac{-\partial l(x)}{\partial \mu(x_0)} \right) dx - l(S^*) \left(\frac{-\partial S^*}{\partial \mu(x_0)} \right). \quad (14)$$

Equation (14) can be interpreted as follows. A mortality decline at age x_0 can potentially affect the survival probabilities from age S^* to age R , which sums (integrates) to $ELLS$. This effect, which we label as the survival effect, corresponds to the first term on the right-hand side. A mortality decline at age x_0 may also affect the individual's optimal choice of schooling years. This effect, labelled as behavioral effect, corresponds to the second term.¹⁴

Using (8), we have

$$\int_{S^*}^R \left(\frac{-\partial l(x)}{\partial \mu(x_0)} \right) dx = \begin{cases} \int_{S^*}^R l(x) dx & \text{if } x_0 \leq S^* \\ \int_{x_0}^R l(x) dx & \text{if } S^* < x_0 < R \\ 0 & \text{if } x_0 \geq R \end{cases}. \quad (15)$$

Combining (12) to (15) gives

$$\frac{-\partial ELLS}{\partial \mu(x_0)} = \begin{cases} ELLS > 0 & \text{if } x_0 \leq S^* \\ \int_{x_0}^R l(x) dx - l(S^*) \left(\frac{-\partial S^*}{\partial \mu(x_0)} \right) & \text{if } S^* < x_0 < R \\ 0 & \text{if } x_0 \geq R \end{cases}. \quad (16)$$

A mortality decline during the schooling phase ($x_0 \leq S^*$) has no behavioral effect ($\frac{-\partial S^*}{\partial \mu(x_0)} = 0$), but leads to an increase in $ELLS$ because of the survival effect. On the other hand, the effect of a mortality decline during the working phase on $ELLS$ is ambiguous, because both of the survival and behavioral effects are positive. The overall effect is positive if the survival effect dominates. Finally, a mortality decline during the retirement phase ($x_0 \geq R$) has neither survival nor behavioral effect. These results are summarized in Proposition 2.

Proposition 2. *For the life-cycle model given by (1) to (3),*

$$(a) \quad \frac{-\partial ELLS}{\partial \mu(x_0)} > 0 \text{ if } x_0 \leq S^*,$$

¹⁴Examining various economic impacts of population aging, Bloom et al. (2010, especially Section III) discuss the decomposition of these impacts into accounting and behavioral effects. Their decomposition is similar in spirit to the survival and behavioral effects in (14). However, we think the terminology “survival effect” conveys the idea more clearly in our context, since the effect is related to changes in survival probabilities at different ages.

- (b) $\frac{\partial ELLS}{\partial \mu(x_0)}$ may be positive or negative if $S^* < x_0 < R$, and
(c) $\frac{\partial ELLS}{\partial \mu(x_0)} = 0$ if $x_0 \geq R$.

We now consider the implications of Propositions 1 and 2 on the relation between MDrSY and MDrELLS.

Based on Propositions 1(a) and 2(a), we conclude that MDrELLS is not a sufficient condition for MDrSY. As explained in Section 3, a mortality decline during the schooling phase shifts both marginal benefit and marginal cost up by the same proportion (at S^*), leading to no overall incentive to change the optimal choice of schooling years according to (6). On the other hand, it increases survival probabilities from age S^* to age R . The positive survival effect and zero behavioral effect lead to an overall positive effect on $ELLS$. Since MDrELLS is satisfied but MDrSY is not satisfied for a mortality decline during the schooling phase, MDrELLS is not a sufficient condition for MDrSY.

Next, we look at Propositions 1(b) and 2(b). A mortality decline at age x_0 during the working phase has a positive behavioral effect (since it increases the marginal benefit but not the marginal cost of extending schooling years). At the same time, it has a positive survival effect, since it increases the survival probabilities from age x_0 to age R .¹⁵ Because of these two effects, the overall effect is ambiguous in general. It can be concluded that MDrELLS is not a necessary condition for MDrSY for a mortality decline during the working phase, unless the survival effect *always* dominates the behavioral effect. To investigate this point further, we have performed several numerical analyses and found that the survival effect indeed does not always dominate the behavioral effect. We now present one example.

We choose an example with features commonly assumed in the literature and/or easily replicable. We follow Cervellati and Sunde (2010) to use Boucekine et al. (2002) specification for survival function

$$l^{BCL}(x) = \frac{\alpha - e^{\beta x}}{\alpha - 1}, \quad (17)$$

where $\alpha = 33.42$ and $\beta = 0.037$ (which corresponds to the estimates of the 1930 cohort of US men by according to Cervellati and Sunde's (2010) analysis). We then transform the survival function (17) to one based on

¹⁵Note that the magnitude of survival effect in the working phase, $\int_{x_0}^R l(x)dx$, which is decreasing in x_0 , is always less than that in the schooling phase, $\int_{S^*}^R l(x)dx$.

adult age (starting from actual age 15) according to¹⁶

$$l(x) = l_{adult}^{BCL}(x) = \frac{l^{BCL}(x+15)}{l^{BCL}(15)} = \frac{\alpha - e^{\beta(x+15)}}{\alpha - e^{\beta(15)}}. \quad (18)$$

We assume a constant wage path (for raw labor), $w(x) = 1$. For human capital function, it is assumed that $h(S) = e^{\gamma S}$, so that the rate of return to schooling (γ) is constant.¹⁷ Based on the above specifications, we can obtain $\frac{h'(S^*)}{h(S^*)} = \frac{h''(S^*)}{h'(S^*)} = \gamma$ and $\mu(S^*) = -\frac{l'(S^*)}{l(S^*)} = \frac{\beta e^{\beta(S^*+15)}}{\alpha - e^{\beta(S^*+15)}}$. We also assume $R = 50$ (correspond to 65 in actual age) and $\gamma = 6.8\%$. Finally, we choose r to solve (6) with $S^* = 4.28$ (which corresponds to 13.28 years of schooling for the 1930 cohort of US men according to Hazan (2009), assuming that a typical child starts schooling at 6 years old and has 9 years of schooling when reaching 15). It is found that $r = 6.033\%$. All the parameter values are quite reasonable.

The magnitude of the derivative $\frac{-\partial ELLS}{\partial \mu(x_0)}$ at different ages are presented in Figure 2. The value is negative from age 4.28 (S^*) to approximately age 43.6, but is positive (and very close to zero) afterwards up to age 50 (R). The behavioral effect dominates in early years of the working phase but the survival effect dominates in later years. In this example, MDrELLS is not satisfied during the earlier years of the working phase.

[Insert Figure 2 here.]

Partly guided by the numerical analysis, we are able to show analytically that the behavioral effect dominates the survival effect in early years of the working phase when the magnitude of the second-order condition (7) is relatively small.¹⁸ This result is given in the following Proposition. (The proof is presented in the Appendix.)

Proposition 3. *When a mortality decline occurs at age x_0 in early years of the working phase, the behavioral effect on expected lifetime labor supply*

¹⁶See footnote 5 regarding the use of actual age of 15 as the beginning of adult life. Note that $T \approx 80$ (in adult age) for the survival function (18).

¹⁷Hazan (2009) and Cervellati and Sunde (2010) assumed that $h(S) = e^{\psi(S)}$ with $\psi'(S) > 0$ and $\psi''(S) \leq 0$. On the other hand, Hall and Jones (1999) assume that $\psi(S)$ is a piecewise-linear function. They further assume that the rate of return to schooling, $\psi'(S)$, beyond the eighth year is 6.8% for OECD countries. The specification in this numerical example is consistent with these papers.

¹⁸Intuitively, a sufficiently small value of (7), which is equal to the denominator on the right-hand side of (9), guarantees that the magnitude of the behavioral effect is relatively large.

dominates the survival effect if¹⁹

$$\frac{w'(S^*)}{w(S^*)} + \frac{2h'(S^*)}{h(S^*)} - \frac{h''(S^*)}{h'(S^*)} - \mu(S^*) - r < \frac{1}{R - S^*}. \quad (19)$$

We conclude that for this life-cycle model with a general survival function, $\frac{-\partial ELLS}{\partial \mu(x_0)} > 0$ but $\frac{-\partial S^*}{\partial \mu(x_0)} = 0$ for a mortality decline during the schooling phase ($x_0 \leq S^*$). Thus, MDrELLS is *not a sufficient condition* for MDrSY. On the other hand, $\frac{-\partial S^*}{\partial \mu(x_0)} > 0$ but $\frac{-\partial ELLS}{\partial \mu(x_0)} < 0$ for some mortality reductions during the working phase ($S^* < x_0 < R$). As a result, MDrELLS is *not a necessary condition* for MDrSY.

5 Conclusion

According to conventional wisdom in human capital theory, mortality decline leads to an increase in human capital investment. Moreover, it has been widely perceived that this effect is related to the longer horizon over which the individual can receive the benefit of human capital investment. Hazan (2009) suggests that MDrELLS is a necessary condition for MDrSY, and it forms the theoretical foundation for his empirical analysis. On the other hand, the above-mentioned statements in Bils and Klenow (2000) and Kalemli-Ozcan et al. (2000) reflect that MDrELLS is interpreted as a sufficient condition for MDrSY.

In order to examine the theoretical validity of Hazan's (2009) conjecture, and more generally, to understand the relation between MDrELLS and MDrSY, we study a life-cycle model with lifetime uncertainty, endogenous schooling years and consumption. We use the Volterra derivative to study the effect of mortality decline at an arbitrary age on optimal schooling years (S^*) and expected lifetime labor supply ($ELLS$). We find that the relation between the effects of mortality decline on S^* and $ELLS$ differs systematically with respect to mortality reductions at different stages of the life cycle. Based on these systematic differences, we find that MDrELLS is not a sufficient condition for MDrSY for mortality reductions during the schooling phase, and not a necessary condition for MDrSY for some mortality reductions during the working phase.

Our results challenge Hazan's (2009) conclusion that increase in schooling years in USA in the last century was not caused by mortality decline.²⁰

¹⁹For the above numerical example, condition (19) is satisfied, with the values of the left- and right-hand sides equal to 0.0053 and 0.0219, respectively.

²⁰Note that our results challenge the conclusion of Hazan's (2009) empirical analysis, but

Together with Cervellati and Sunde’s (2010) empirical result that there is no decline in the benefits relative to the opportunity costs of schooling from mid 1800s to mid 1900s, Hazan’s (2009) conclusion is challenged from both theoretical and empirical perspectives. More generally, our results question existing hypotheses in the literature that MDrELLS is a sufficient condition of MDrSY (according to Bils and Klenow, 2000; Kalemli-Ozcan et al., 2000) or a necessary condition (according to Hazan, 2009).

We end this paper with a discussion about some misunderstandings related to the “longer horizon (of expected lifetime labor supply)” mechanism in the literature. In a seminal work, Ben-Porath (1967) examines investment of human capital in a life-cycle framework, and argues that “(t)he main reason why investment [in human capital] is undertaken most by the young is that they have a longer period over which they can receive returns on their investment” (Ben-Porath, 1967 p. 352). The longer-horizon argument is very intuitive in understanding at what ages an individual chooses to invest in human capital, since investing at younger (rather than older) ages implies a longer duration to reap the benefit of human capital investment. Perhaps because Ben-Porath’s (1967) argument is persuasive, later researchers apply this insight when examining an apparently similar question of the impact of mortality decline on human capital investment. In particular, when some researchers carry out formal analysis with a rectangular survival function, they find that longevity increase (as represented by an increase in maximum age) leads to longer schooling years, whether the retirement age is exogenous (as in Bils and Klenow, 2000) or endogenous (as in Hazan, 2009).²¹ In either case, the expected lifetime labor supply increases as well. These theoretical results for the model with a rectangular survival function, together with Ben-Porath’s (1967) insight, lead many researchers to believe that the same longer-horizon mechanism is important for both questions.

While the longer-horizon argument works well for the model with rectangular survival function, unfortunately rectangular survival function is not empirically relevant. The use of a general non-rectangular survival function is important to understand the effect of mortality decline on schooling years. In this paper, we consider a life-cycle model with a general survival function,

not its value. We think his empirical findings that schooling years increase dramatically but expected lifetime labor supply decreased when longevity improved in the last 150 years was valuable. We only disagree with the link between his empirical findings and conclusion.

²¹Kalemli-Ozcan et al. (2000) also obtain this result in a model with exponential survival function. However, it is well known (from, for example, Heijdra and Romp (2008), Figure 1) that exponential survival function deviates substantially from the observed human survival experience, especially for high-income countries in recent decades.

and find that the incentive to change the schooling years in response to a mortality decline, which is given in (6), is related to the effect on expected lifetime labor supply through both survival and behavioral effects in a rather complicated manner, according to (14). Furthermore, it is found that the relevance of the longer-horizon mechanism differs with respect to the stages of the life cycle.²² Based on Propositions 1 to 3, we conclude that the longer-horizon insight in the analysis of the timing of human capital investment (Ben-Porath, 1967) is not always applicable to the question regarding the impact of mortality decline on human capital investment.²³

6 Appendix

We derive the individual's consumption and schooling year choice in Section 6.1, and prove Proposition 3 in Section 6.2.

6.1 Consumption and schooling year choice

We solve the problem in two steps. The first step is to obtain the individual's optimal consumption path, conditional on schooling years. Define an indicator variable $E(x)$, which takes the value of 1 if $x \leq S$, and the value of 0 otherwise. Similarly, define an indicator variable $N(x)$, which takes the value of 1 if $S < x \leq R$, and the value of 0 otherwise. Therefore, (2) is equivalent to

$$\int_0^T e^{-\rho x} l(x) \left\{ [E(x) + N(x)] \left[\frac{c(x)^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} \right] + [1 - E(x) - N(x)] \left[\frac{[(1 + \theta) c(x)]^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} \right] \right\} dx,$$

and (3) can be written as

$$a'(x) = [r + \mu(x)] a(x) + N(x) h(S) w(x) - c(x).$$

²²Specifically, for a mortality decline during the schooling phase, there is no behavioral effect on S^* and the expected lifetime labor supply increases due to the survival effect. Thus, the longer-horizon argument *breaks down* in the schooling phase. For a mortality decline during the working phase, there is a positive behavioral effect on S^* and a positive survival effect. Moreover, it is shown that the survival effect does not always dominate the behavioral effect. As a result, the longer-horizon argument is *not required* for mortality reductions at some age $x_0 \in (S^*, R)$ in the working phase.

²³With hindsight, perhaps it is not difficult to see that these two questions are different, and therefore their mechanisms may not be the same. However, we admit that we also had that prior judgment before this project. Only after detailed analysis did we discover that the effect of mortality decline on schooling years differs systematically with respect to the age of mortality decline. This point is not apparent in advance.

Using standard techniques of dynamic optimization, it can be shown that the necessary first-order conditions for this optimization problem are

$$l(x) \left\{ [E(x) + N(x)] c(x)^{-\frac{1}{\sigma}} + [1 - E(x) - N(x)] (1 + \theta)^{1-\frac{1}{\sigma}} c(x)^{-\frac{1}{\sigma}} \right\} = \lambda(x),$$

and

$$\lambda(x) [r + \mu(x)] = \rho \lambda(x) - \lambda'(x),$$

where $\lambda(x)$ is the co-state variable associated with the state variable.

Solving these sets of equations, the optimal consumption path is characterized by:

(a)

$$c'(x) = \sigma(r - \rho)c(x) \text{ for } x \leq R. \quad (\text{A1})$$

(b) Because of

$$\lim_{x \rightarrow R^+} \lambda(x) = \lambda(R),$$

it can be shown that there is a discontinuous drop in consumption level at retirement, given by

$$\lim_{x \rightarrow R^+} c(x) = (1 + \theta)^{\sigma-1} c(R). \quad (\text{A2})$$

(c)

$$c'(x) = \sigma(r - \rho)c(x) \text{ for } x > R. \quad (\text{A3})$$

Combining (A1) to (A3) gives (4) in the main text. From now on, we write the dependence of the consumption path on S explicitly, as $c(S, x)$.

Differentiating (5) with respect to S gives

$$\begin{aligned} & \int_0^R e^{-rx} l(x) \frac{\partial c(S, x)}{\partial S} dx + \int_R^T e^{-rx} l(x) \frac{\partial c(S, x)}{\partial S} dx \\ &= h'(S) \int_S^R e^{-rx} l(x) w(x) dx - e^{-rS} l(S) h(S) w(S). \end{aligned} \quad (\text{A4})$$

Conditional on the optimal consumption path (4), the second step is to obtain the first-order condition for optimal schooling. Substitute (4) into (2) to express the objective function in terms of S only. Denote it by

$$V(S) = \int_0^R e^{-\rho x} l(x) \left[\frac{[c(S, x)]^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} \right] dx + \int_R^T e^{-\rho x} l(x) \left[\frac{[(1 + \theta) c(S, x)]^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} \right] dx.$$

The objective is to choose S to maximize $V(S)$. Differentiating it with respect to S and using (A4) gives

$$V'(S) = c(S, 0)^{-\frac{1}{\sigma}} \left[h'(S) \int_S^{\tilde{R}(S)} e^{-rx} l(x) w(x) dx - e^{-rS} l(S) h(S) w(S) \right].$$

Thus, the first-order condition for optimal schooling years is given by (6). When (7) holds, it can be shown that the second-order condition of the maximization problem is satisfied ($V''(S^*) < 0$). ■

6.2 Proof of Proposition 3

When $x_0 \in (S^*, R)$, the survival effect (SE) and the behavioral effect (BE), given by the first and second terms on the right-hand side of (14), become

$$SE(x_0) = \int_{S^*}^R \left(\frac{-\partial l(x)}{\partial \mu(x_0)} \right) dx = \int_{x_0}^R l(x) dx,$$

and

$$BE(x_0) = l(S^*) \left(\frac{-\partial S^*}{\partial \mu(x_0)} \right) = l(S^*) \frac{\frac{\int_{x_0}^R e^{-rx} l(x) w(x) dx}{\int_{S^*}^R e^{-rx} l(x) w(x) dx}}{\frac{w'(S^*)}{w(S^*)} + \frac{2h'(S^*)}{h(S^*)} - \frac{h''(S^*)}{h'(S^*)} - \mu(S^*) - r},$$

respectively. When condition (19) holds,

$$\begin{aligned} \lim_{x_0 \rightarrow S^*} BE(x_0) &= \frac{l(S^*)}{\frac{w'(S^*)}{w(S^*)} + \frac{2h'(S^*)}{h(S^*)} - \frac{h''(S^*)}{h'(S^*)} - \mu(S^*) - r} \\ &> (R - S^*) l(S^*) = \int_{S^*}^R l(S^*) dx \\ &\geq \int_{S^*}^R l(x) dx = \lim_{x_0 \rightarrow S^*} SE(x_0), \end{aligned}$$

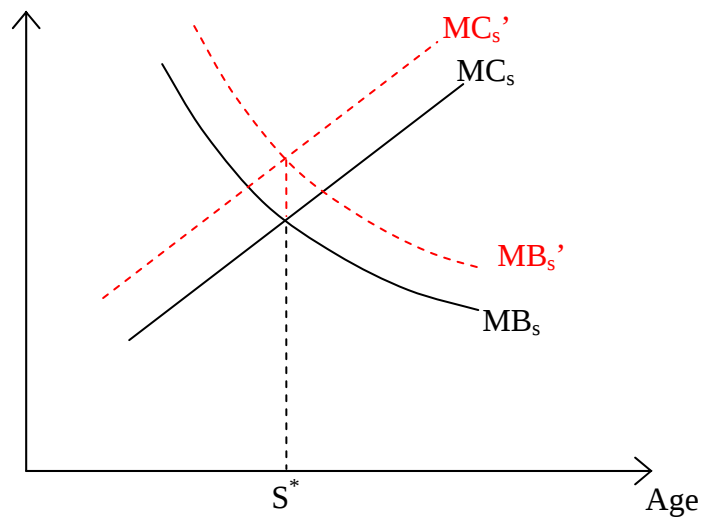
with the last inequality holding because $l(x)$ is weakly decreasing in x . We conclude that the behavioral effect dominates the survival effect when x_0 tends to S^* from above (during the working phase).

By continuity, there exists a $\delta > 0$ such that for all $x_0 \in (S^*, S^* + \delta)$, $BE(x_0) > SE(x_0)$ when (19) holds. ■

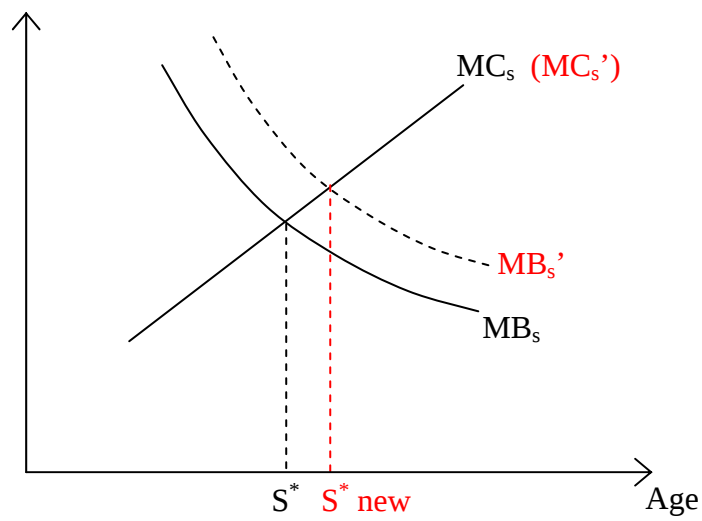
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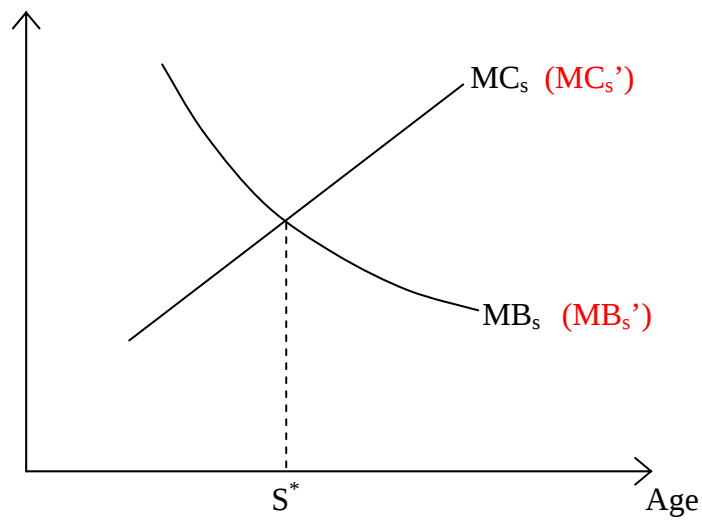
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(a) $x_0 \leq S^*$



(b) $S^* < x_0 < R$



(c) $x_0 \geq R$

Figure 1: Effects of Mortality Reductions on Optimal Schooling Years

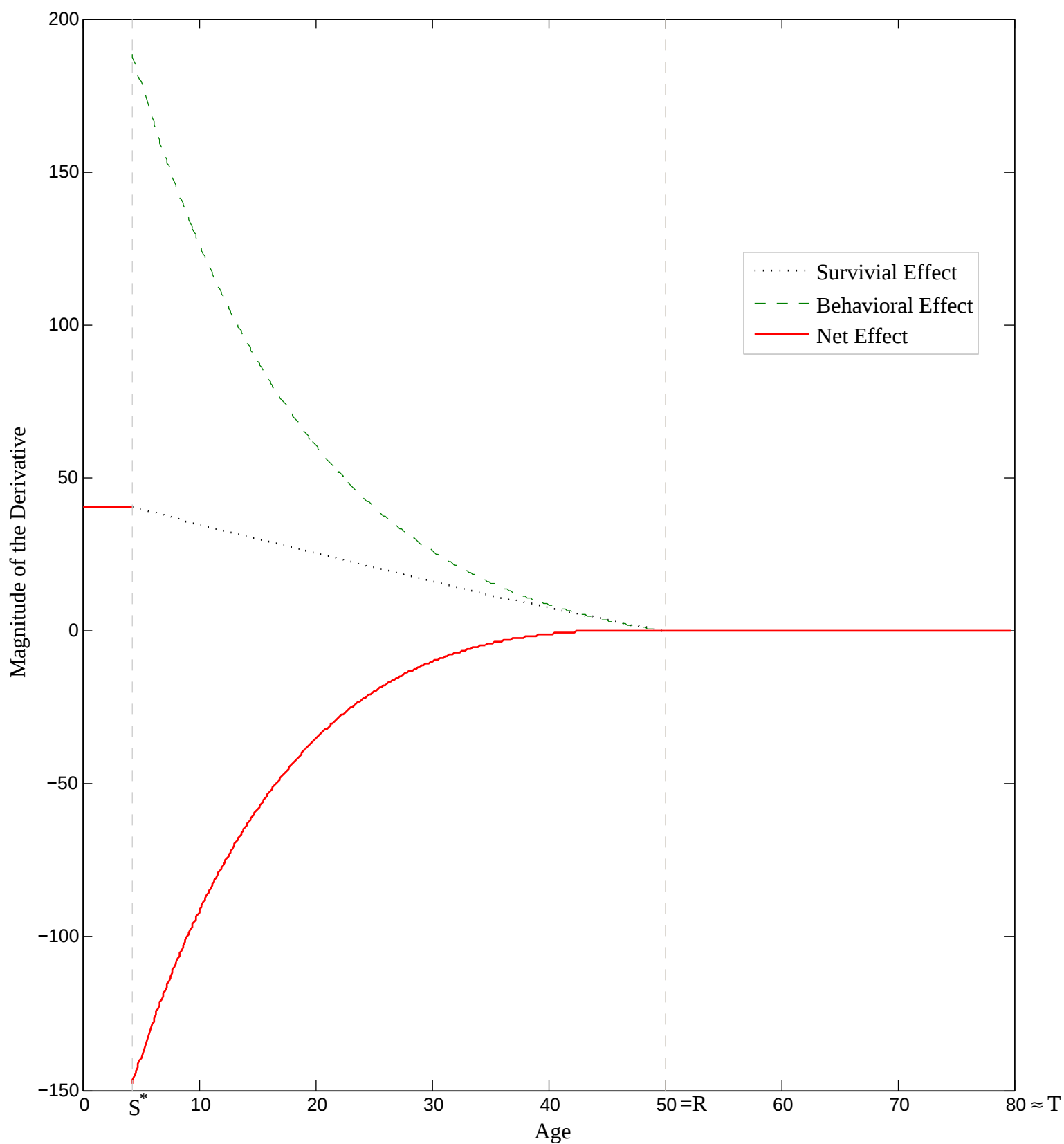


Figure 2: Survival and Behavioral Effects