

Liquidity and Welfare*

Yi Wen

(This version: October 1, 2012)

Abstract

This paper develops an analytically tractable Bewley model of money featuring capital and financial intermediation. It is shown that when money is a vital form of liquidity to meet uncertain consumption needs, the welfare costs of inflation can be extremely large. With log utility and parameter values that best match both the aggregate money demand curve suggested by Lucas (2000) and the variance of household consumption, agents in our model are willing to reduce consumption by 7% ~ 10% (or more) to avoid 10% annual inflation. In other words, raising the U.S. inflation target from 2% to 3% amounts to roughly a 0.5 percentage reduction in aggregate consumption. The astonishingly large welfare costs of inflation arise because inflation tightens liquidity constraints by destroying the buffer-stock value of money, thus raising the volatility of consumption at the household level. Such an inflation-induced increase in the idiosyncratic consumption-volatility at the micro level cannot be captured by representative-agent models or the Bailey triangle. Although the development of a credit and banking system can reduce the welfare costs of inflation by alleviating liquidity constraints, with realistic credit limits the cost of moderate inflation still remains several times larger than estimations based on the Bailey triangle. Our finding not only provides a justification for adopting a low inflation target by central banks, but also offers a plausible explanation for the robust positive relationship between inflation and social unrest in developing countries where money is the major form of household financial wealth.

Keywords: Liquidity Preference, Money Demand, Financial Intermediation, Velocity, Welfare Costs of Inflation.

JEL codes: D10, D31, D60, E31, E41, E43, E49, E51.

*This paper is a significantly revised version of Wen (2009, 2010). I thank Pengfei Wang for discussions on issues related to this project. My gratitude also goes to an anonymous referee, the associate editor, and the editor Bob King. I also thank Costas Azariadis, Jinghui Bai, Aleksander Berentsen, Gabriele Camera, Yongsung Chang, Mark Huggett, David Levine, Narayana Kocherlakota, Qing Liu, Rodolfo Manuelli, Steve Williamson, Tao Zhu, and seminar participants at Georgetown University, Hong Kong University of Science & Technology, Taiwan University, and the Federal Reserve Banks of San Francisco and St. Louis for comments; Judy Ahlers for editorial assistance; and Yu man Tam and Luke Shimek for research assistance. The views expressed do not reflect official positions of the Federal Reserve Bank of St. Louis and the usual disclaimer applies. Correspondence: Yi Wen, Research Department, Federal Reserve Bank of St. Louis, P.O. Box 422, St. Louis, MO, 63166, and School of Economics and Management, Tsinghua University, Beijing, China. Phone: 314-444-8559. Fax: 314-444-8731. Email: yi.wen@stls.frb.org.

1 Introduction

In developing countries, liquid money (cash and checking accounts) is the major form of household financial wealth and a vital tool of self-insurance to buffer idiosyncratic shocks because of the lack of a well-developed financial system. Based on recent data in China and India, more than 90% of the household financial wealth is held in the form of cash and checking accounts.¹ Even in developed countries, because of borrowing constraints and costs of participating in the financial markets, money remains one of the most important assets to provide liquidity to smooth consumption, especially for low-income households. Mulligan and Sala-i-Martin (2000) document that the majority of households in the United States do not hold financial assets other than checking accounts. In particular, based on the 1989 Survey of Consumer Finances (SCF), 59% of U.S. households do not hold any nonmonetary financial assets (interest-bearing assets), and 53% of those that hold checking accounts do not hold any interest-bearing assets. In addition, money demand is highly heterogeneous: The Gini coefficient of the distribution of money across households is greater than 0.85 in the United States. This degree of heterogeneity in money demand closely resembles the distribution of financial wealth instead of consumption (with a Gini coefficient less than 0.3). This suggests that the liquidity motive of money demand is at least as important (if not more so) as the transaction motive of money demand, even in developed countries such as the United States.²

When money is essential (as a store of value) for consumption smoothing and is unequally distributed across households, largely because of idiosyncratic needs for liquidity and the lack of sophisticated risk sharing, inflation can be far more costly than recognized by the existing literature. Historical evidence also suggests that moderate inflation (around 10% to 20% a year) may be significant enough to cause widespread social and political unrest in developing countries.³ Yet, the existing monetary literature suggests that the cost of inflation is small. For example, Lucas (2000) estimated that the welfare cost of increasing inflation from 4% to 14% is less than 1% of aggregate output. Such results are disturbing; if this is true, then the commonly accepted inflation

¹Townsend (1995) points out that currency and crop inventory are the major forms of liquid assets to provide self-insurance against idiosyncratic shocks for farmers in India and Thailand, and surprisingly, purchases and sales of real capital assets, including livestock and consumer durables, do not play a role in smoothing income fluctuations.

²Ragot (2009) reports that this stylized fact holds for other developed countries and argues that this is a problem for theories that directly link money demand to consumption, such as cash-in-advance (CIA), money-in-the-utility (MIU), or shopping-time models, but is consistent with incomplete-market models in which money is held as a form of financial asset that provides liquidity to smooth consumption.

³See Cartwright, Delorme, and Wood (1985) and Looney (1985) for empirical studies on the relationship between inflation and revolutions in recent world history. Using data from 54 developing countries, Cartwright, Delorme, and Wood (1985) find that inflation is the most significant economic variable to explain the probability and duration of social unrest and revolution, and is far more important than other economic variables, such as income inequality, GDP per capita, income growth, unemployment rate, and degree of urbanization. Their estimates show that a one-unit increase in the inflation rate raises the probability of revolution by 6 percentage points and increases the duration of revolution by 0.7 to 1.0 years.

target of 2% a year by most central banks in developed countries may be too conservative and not well justified, and this policy may have forgone too many social benefits of potentially higher employment through faster money growth.⁴

This paper argues that to properly assess the welfare costs of inflation, it is desirable to use a theoretical model that takes the liquidity function of money and the precautionary motives of money demand into account, so as to capture the "insurance value" of cash in addition to the opportunity cost of forgone interest as suggested by Bailey (1956). The loss of the insurance value of money under inflation may generate far larger welfare costs than implied by the Bailey triangle because inflation exposes cash-poor households to idiosyncratic shocks by destroying the family's buffer stock.

This paper constructs such a model by generalizing Bewley's (1980, 1983) precautionary money demand model to a tractable, dynamic stochastic general equilibrium (DSGE) framework where money coexists with other assets.⁵ The key feature distinguishing Bewley's model from the related literature, such as the heterogeneous-agent cash-in-advance (CIA) model of Lucas (1980) and the (S,s) inventory-theoretic model of Baumol (1952) and Tobin (1956), is that money is held solely as a store of value, completely symmetric to any other asset, and is not imposed from outside as the means of payments. Agents can choose whether to hold money depending on the costs and benefits. By freeing money from its role of medium of exchange, Bewley's approach allows us to focus on the function of money as a pure form of liquidity so the welfare implications of the liquidity-preference theory of money demand can be investigated in isolation. Beyond Bewley (1980, 1983), my generalized model is analytically tractable; hence, it greatly simplifies the computation of DSGE in environments with both idiosyncratic and aggregate shocks, capital accumulation, financial intermediation, and nontrivial distributions of cash balances, thus facilitating welfare and business-cycle analysis. Analytical tractability also makes the mechanisms of the model transparent.

The major finding of the paper, among other things, is that persistent money growth is very costly. When the model is calibrated to match not only the interest elasticity of aggregate money demand but also the extent of idiosyncratic risk faced by households in the data, the implied welfare cost of moderate inflation is astonishingly large, around 7% $\sim$ 10% of consumption under 10% annual inflation. Besides the large welfare cost of inflation, the generalized Bewley model is able to produce enough variability in velocity relative to output to match the data; in particular, it

⁴Small welfare costs of inflation are also obtained by many others, such as Cooley and Hansen (1989), Dotsey and Ireland (1996), Henriksen and Kydland (2010), and Khan, King, and Wolman (2003) in different models. Lagos and Wright (2005) obtain a significantly higher welfare cost of inflation in a search model of money—about 4% of aggregate consumption with a 10% inflation rate. Our welfare results are comparable to those obtained by Lagos and Wright (2005) in the order of magnitude, but with an entirely different mechanism and micro foundation.

⁵General-equilibrium analysis is important. Cooley and Hansen (1989) emphasize the general-equilibrium effect of inflation on output through substituting leisure for consumption in the face of positive inflation, which causes labor supply and output to decline. However, because these authors assume that money is held only for transaction purposes, the welfare cost of inflation is still small despite the general-equilibrium effects of inflation on output, about 0.4% $\sim$ 0.5% of GDP with 10% inflation.

can explain the negative correlation of velocity with real balances in the short run and its positive correlation with inflation in the long run. In addition, transitory lump-sum money injections can have positive real effects on aggregate activities despite flexible prices.⁶

Since holding money is both beneficial (providing liquidity) and costly (forgoing interest payment and bearing the inflation tax), agents opt to hold different amounts of cash depending on income levels and consumption needs (e.g., preference shocks). As a result, a key property of the model is an endogenously determined distribution of money holdings across households, with a strictly positive fraction of households being cash-constrained (i.e., with zero cash balances) in equilibrium. Hence, lump-sum money injections have an immediate positive impact on consumption for the liquidity-constrained agents, but not for agents with idle cash balances. Consequently, the aggregate price does not increase with the aggregate money supply one for one, so transitory monetary shocks are expansionary to aggregate output (even without open market operations), the velocity of money is countercyclical, and the aggregate price appears "sticky."

However, with anticipated inflation, permanent money growth reduces welfare significantly for several reasons: (i) Precautionary money demand induces agents to hold excessive amounts of cash to avoid liquidity constraints, raising the inflation tax on the population. (ii) Cash-poor agents suffer disproportionately more from the inflation tax because they are more likely to be subject to idiosyncratic risks without self-insurance; thus, for the same amount of reduction in real wealth, inflation reduces their expected utility more than it does for liquidity-abundant agents.⁷ (iii) The size of the liquidity-constrained population rises rapidly with inflation, leading to an increased portion of the population unable to smooth consumption against idiosyncratic shocks.⁸ This last factor can dramatically raise social welfare costs along the extensive margin.

The Bailey triangle is a poor measure of the welfare costs of inflation because it fails to capture the insurance function of money (as noted and emphasized by Imrohoroglu, 1992). At a higher inflation rate, not only does the opportunity cost of holding money increase (which is the Bailey triangle), but the crucial benefit of holding money also diminishes. In particular, when demand for money declines, the portion of the liquidity-constrained population rises; consequently, the welfare cost of inflation increases sharply due to the loss of self-insurance for an increasingly larger proportion of the population. This result is reminiscent of the analysis by Aiyagari (1994) in which he shows that the welfare cost of the loss of self-insurance in an incomplete-market economy is equivalent to a 14% reduction in consumption even though his calibrated model matches only

⁶To conserve space, this paper focuses only on the welfare costs of inflation. Interested readers are referred to Wen (2009a, 2010) for details of the other results.

⁷This asymmetric effect of inflation is related but different from the distributional effect emphasized by Erosa and Ventura (2002) in a heterogeneous-agent model where rich households rely more on credit transactions than low-income households.

⁸In this paper, the term "liquidity-constrained" agents is synonymous to households "with a binding liquidity constraint" or "zero cash balances."

one-third of the income and wealth inequalities in the data.⁹

This paper is also related to the work of Alvarez, Atkeson, and Edmond (2008). Both papers are based on an inventory-theoretic approach with heterogeneous money demand and can explain the short-run dynamic behavior of velocity and sticky aggregate prices under transitory monetary shocks. However, my approach differs from theirs in important aspects. Most notably, their model is based on the Baumol-Tobin inventory-theoretic framework where money is not only a store of value but also a means of payment (similar to CIA models). In their model, agents are exogenously and periodically segregated from the banking system and the CIA constraint always binds. For these reasons, the implications of the welfare costs of inflation in their model may also be very different from those in this paper. For example, Attanasio, Guiso, and Jappelli (2002) estimate the welfare costs of inflation based on a simple Baumol-Tobin model and find the cost to be less than 0.1% of consumption under 10% inflation. The main reason is that this segment of the literature has relied exclusively on Bailey's triangle (or the interest elasticity of money demand) to measure the welfare costs of inflation. Hence, despite having heterogeneous money holdings across households, such research is not able to obtain significantly larger estimates of the cost of inflation than those in representative-agent models.

Bewley's (1980) model has been studied in the recent literature, but the main body of this literature focuses on an endowment economy. For example, Imrohoroglu (1992), Imrohoroglu and Prescott (1991), and Akyol (2004) study the welfare costs of inflation in the Bewley model. To the best of my knowledge, Imrohoroglu (1992) is the first in the literature to recognize that the welfare costs of inflation in a Bewley economy are larger than those suggested by the Bailey triangle. However, like Bewley's (1980) work, this segment of the literature is based mainly on an endowment economy without capital and these models are not analytically tractable.¹⁰ Ragot (2009), on the other hand, uses a general-equilibrium version of Bewley's model with segregated markets (similar to Alvarez, Atkeson, and Edmond, 2008) to explain the joint distribution of money demand, consumption, and financial assets. He shows that standard models in which money serves only as a medium of exchange are inconsistent with the empirical distributions of these variables, whereas a general-equilibrium version of the Bewley model in which money is held as a store of value can better explain the empirical distributions. Nonetheless, Ragot (2009) also uses a numerical approach to solve the model and he does not study the welfare implications of inflation.

Our study complements the analysis of Telyukova (2011). Telyukova (2011) develops a liquidity-

⁹ Also see the more recent analyses of Krebs (2003a, 2007).

¹⁰ This literature tends to find higher welfare costs of inflation, but the absolute magnitude is still small. For example, Imrohoroglu (1992) shows the welfare cost of 10% inflation is slightly above 1% of consumption. The reason is that the distribution of liquidity demand in the model does not respond significantly to inflation if the support of idiosyncratic shocks is binary or does not have enough points (as typically assumed in this literature to reduce numerical computation burdens). Hence, an extensive margin would be missing and this margin is important for the welfare costs of inflation.

demand theory of money similar to ours to explain the "credit card debt puzzle": Households simultaneously revolve significant credit card debt and hold sizeable amounts of liquid deposits in low-interest checking accounts. In her model, money is assumed to be more liquid than credit cards in meeting uncertain consumption demand (driven by preference shocks). As a result, households opt to borrow at a high interest rate on their credit cards to smooth consumption against income shocks while simultaneously hold liquid cash to buffer unpredictable preference shocks—which by assumption cannot be buffered by credit cards. However, Telyukova’s model is a partial-equilibrium model and it is not analytically tractable. More importantly, she does not study the welfare implications of inflation in such an environment.

The rest of this paper is organized as follows: Section 2 presents the benchmark model on the household side and shows how to solve for individuals’ decision rules of money demand and consumption analytically. It reveals some of the basic properties of a monetary model based on liquidity preference. Section 3 extends the model to a production economy with capital and uses the model to evaluate the welfare costs of inflation. Section 4 introduces credit and banking into the general-equilibrium model and discusses some robustness issues of our welfare results. Section 5 concludes the paper.

2 The Benchmark Model

To highlight the liquidity value of money, the model features money as the only asset that can be adjusted quickly (costlessly) to buffer idiosyncratic shocks to consumption demand at any moment. Interest-bearing nonmonetary assets (such as capital) can be accumulated to support consumption but are not as useful (or liquid) as money in buffering idiosyncratic shocks. This setup captures the characteristics of incomplete markets, especially the lack of sophisticated risk-sharing arrangements in developing countries.

We make the model analytically tractable by introducing two important features: (i) We allow an endogenous labor supply with quasi-linear preferences (as in Lagos and Wright, 2005), and (ii) we replace idiosyncratic labor income shocks typically assumed in the incomplete-market literature (e.g., Imrohoroglu, 1989, 1992; Aiyagari, 1994; Huggett, 1993) by preference shocks. Even with quasi-linear preferences, the model is not analytically tractable if wage income is subject to idiosyncratic shocks. There are two ways to overcome this difficulty. One is to place idiosyncratic shocks on preferences (i.e., to the marginal utility of consumption as in Lucas, 1980), and the other is to place them on net wealth, which includes labor income. This paper takes the first approach. Both alternatives yield similar results for the welfare costs of inflation when the model is calibrated to match some key features of aggregate money demand and the idiosyncratic liquidity risk faced by households. This is reassuring because it suggests that the source of uninsurable idiosyncratic

shocks does not matter for our welfare results.¹¹

Time is discrete. There is a unit mass of continuum households in the interval $[0, 1]$. Each household is subject to an idiosyncratic preference shock, θ_t , which has the distribution $\mathbf{F}(\theta) \equiv \Pr[z \leq \theta]$ with support $[\theta_L, \theta_H]$. A household chooses sequences of consumption $\{c_t\}$, labor supply $\{n_t\}$, savings for interest-bearing assets $\{s_{t+1}\}$, and nominal money balance $\{m_{t+1}\}$ to maximize lifetime utilities, taking as given the paths of aggregate real wage $\{W_t\}$, real interest rate $\{r_t\}$, the aggregate price $\{P_t\}$, and the nominal lump-sum transfers $\{\tau_t\}$. The nominal rate of return to holding money is zero.¹²

Assume that in each period t the decisions for labor supply and holdings for interest-bearing assets must be made before observing the idiosyncratic shock θ_t in that period, and the decisions, once made, cannot be changed for the rest of the period (i.e., these markets are closed afterward for households until the beginning of the next period). Thus, if there is an urge to consume during period t after labor supply and capital investment decisions are made and the preference shock θ_t is realized, money is the only asset that can be adjusted to smooth consumption. Borrowing of liquidity (money) from other households is not allowed.¹³ These assumptions imply that households may find it optimal to carry money as self-insurance to cope with idiosyncratic uncertainty (as in Bewley, 1980), even though money is not required as a medium of exchange. As in the standard literature, any aggregate uncertainty, if it exists, is resolved at the beginning of each period before any decisions are made and is orthogonal to idiosyncratic uncertainty.

An alternative way of formulating the above information structure for decision-making is to divide each period into two subperiods, with labor supply and nonmonetary-asset investment determined in the first subperiod, the rest of the variables (consumption and money balances) determined in the second subperiod, and the idiosyncratic shocks θ_t realized only in the beginning of the second subperiod. Yet another alternative specification of the model is to have two islands, with labor supply and interest-rate bearing assets determined on island 1 and c_t and m_t determined on island 2 simultaneously by two spatially separated household members (e.g., a worker and a shopper), but only the shopper—who determines consumption and money balances in island 2—can observe θ_t in period t . Both members can observe aggregate shocks and the history of family decisions up to period t . At the end of each period the two members reunite and share everything perfectly (e.g., income, wealth, and information) and separate again in the beginning of the next

¹¹See an earlier version of this paper (Wen, 2010) for analyses based on the second approach.

¹²The zero lower bound on government bonds implies that the nominal return to money is zero. In addition, the nominal interest rate on checking accounts is essentially zero in many countries. However, since we may consider any perfectly liquid asset, including interest bearing checking accounts, as money, we can add a fixed interest rate on money. But doing so does not change our main results. This can be seen by simply redefining the time discount factor in our model as the product of β and a fixed deposit rate. However, see Section 4 in this paper for the analysis with a time varying and endogenously determined deposit rate.

¹³This assumption will be relaxed in Section 4.

period.¹⁴

2.1 Household Problem

We use lower-case letters to denote individual variables and upper-case letters to denote aggregate variables in this paper. Denote h^t as the history of an individual household up to period t , and h_{-1}^t as h^t excluding θ_t , namely, $h^t = h_{-1}^t \cup \theta_t$. Let H^t denote the history of the aggregate state up to period t . Then the problem of a household is to solve

$$\max_{c_t(h^t, H^t), m_{t+1}(h^t, H^t), n_t(h_{-1}^t, H^t)} E_0 \sum_{t=0}^{\infty} \beta^t \{ \theta_t \log c_t(h^t, H^t) - a n_t(h_{-1}^t, H^t) \} \quad (1)$$

subject to

$$c_t(h^t, H^t) + \frac{m_{t+1}(h^t, H^t)}{P_t(H^t)} \leq \frac{m_t(h^{t-1}, H^{t-1}) + \tau_t}{P_t(H^t)} + W_t(H^t) n_t(h_{-1}^t, H^t), \quad (2)$$

$m_{t+1}(h^t, H^t) \geq 0$, $n_t(h_{-1}^t, H^t) \in [0, \bar{n}]$, and $m_0 \geq 0$ given; where τ_t is an exogenous, uniform, lump-sum nominal transfer (to be specified later). To save notation, we drop the history indices $\{h^t, H^t\}$ by denoting $P_t = P_t(H^t)$, $W_t = W_t(H^t)$, $c_t = c_t(h^t, H^t)$, $m_{t+1} = m_{t+1}(h^t, H^t)$, and $n_t = n_t(h_{-1}^t, H^t)$, unless confusion may arise. Without loss of generality, assume $a = 1$ in the utility function.¹⁵

The household problem can be formulated recursively. Define

$$x_t \equiv \frac{m_t + \tau_t}{P_t} + W_t n_t \quad (3)$$

as total cash in hand, and denote $J_t(x_t, \theta_t)$ as the value function of the household based on the choice of c_t and m_{t+1} after the realization of θ_t . We then have

$$J_t(x_t, \theta_t) = \max_{c_t, m_{t+1}} \left\{ \theta_t \log c_t + \beta E_t V_{t+1} \left(\frac{m_{t+1}}{P_{t+1}} \right) \right\} \quad (4)$$

¹⁴Since the time period in the model can be short (e.g., t represents a month, a week, or a day), the assumption that labor supply and nonmonetary asset holdings (such as fixed capital) are predetermined and cannot be adjusted instantaneously in the second subperiod after the realization of θ_t is not as extreme as it appears. This information/timing structure amounts to creating a necessary friction for the existence of money as a liquid asset. In reality, especially in developing countries, it is costly to exchange labor and real assets (such as land and livestock) for consumption goods in spot markets (e.g., due to search frictions and other transaction costs). In developed countries, even government bonds are rarely held as a major form of liquid assets by low-income households and there are always costs involved in trading nonmonetary assets. As documented by Telyucova (2011) using household survey data, even credit cards are not as liquid as cash in meeting certain types of consumption demand.

¹⁵The model remains tractable if the utility function takes the more general form of $\frac{c_t^{1-\sigma} - 1}{1-\sigma} - n_t$. For simplicity we set $\sigma = 1$ in this paper. Setting $\sigma > 1$ can only enhance our conclusions.

subject to

$$c_t + \frac{m_{t+1}}{P_t} \leq x_t \quad (5)$$

$$m_{t+1} \geq 0, \quad (6)$$

where $V_t(\frac{m_t}{P_t})$ is the value function of the household based on the choice of n_t before observing θ_t .

That is,

$$V_t(\frac{m_t}{P_t}) = \max_{n_t} \left\{ -n_t + \int J_t(x_t, \theta_t) d\mathbf{F} \right\} \quad (7)$$

subject to (3) and $n_t \in [0, \bar{n}]$.

Since money is not required as a medium of exchange, choosing $m_{t+1} = 0$ for all t is always a Nash equilibrium. In what follows, we focus on monetary equilibria where money is accepted as a store of value and the aggregate price $P_t \in (0, \infty)$ is finite and bounded away from zero.

Proposition 1 *The decision rules for consumption, money demand, and cash in hand are given, respectively, by*

$$c_t = \min \left\{ 1, \frac{\theta_t}{\theta_t^*} \right\} x_t \quad (8)$$

$$\frac{m_{t+1}}{P_t} = \max \left\{ \frac{\theta_t^* - \theta_t}{\theta_t^*}, 0 \right\} x_t \quad (9)$$

$$x_t = W_t \theta_t^* R(\theta_t^*), \quad (10)$$

where the cutoff θ_t^* is independent of individual history h^t and is determined implicitly by the following Euler equation:

$$\frac{1}{W_t} = \left[\beta E_t \frac{P_t}{P_{t+1} W_{t+1}} \right] R(\theta_t^*), \quad (11)$$

where

$$R(\theta_t^*) \equiv \int \max \left\{ 1, \frac{\theta_t}{\theta_t^*} \right\} d\mathbf{F} > 1. \quad (12)$$

Proof. See Appendix A1. ■

Consumption is a concave function of cash in hand, with the marginal propensity to consume given by $\min \left\{ 1, \frac{\theta_t}{\theta_t^*} \right\}$, which is less than 1 in the case of a low urge to consume ($\theta_t < \theta_t^*$). Saving (money demand) is a buffer stock: Agents save in the low-return asset when consumption demand is low ($\frac{m_{t+1}}{P_t} > 0$ if $\theta_t < \theta_t^*$), anticipating that future consumption demand may be high ($\Pr[\theta > \theta^*] > 0$).

Equation (11) implicitly determines the optimal cutoff $\theta^*(H^t)$ as a function of the aggregate state only. The interpretation of equation (11) is straightforward. Treat $\frac{1}{W}$ as the marginal utility of consumption from wage income. The left-hand-side (LHS) of the equation is the opportunity cost of holding one more unit of real balances as inventories (as opposed to increasing consumption by one unit). The right-hand-side (RHS) is the expected gains by holding money, which take two possible values: The first term inside the integral of equation (12) reflects simply the discounted and inflation-adjusted next-period utility value of inventories (real balances) in the case of a low urge to consume (since 1 dollar is just 1 dollar if not consumed), which has probability $\int_{\theta \leq \theta^*} d\mathbf{F}(\theta)$. The second term is the marginal utility of consumption ($\left[\beta E_t \frac{P_t}{P_{t+1} W_{t+1}}\right] \frac{\theta_t}{\theta_t^*} = \frac{\theta_t}{x_t}$) in the case of a high urge to consume ($\theta > \theta^*$), which has probability $\int_{\theta > \theta^*} dF(\theta)$. The optimal cutoff θ_t^* (or cash in hand x_t) is chosen so that the marginal cost of holding money equals the expected marginal gains.

Hence, the rate of return to money is the inflation-adjusted real interest rate ($\beta \frac{P_t}{P_{t+1}}$) compounded by a liquidity premium $R(\theta^*)$. Notice that $R(\theta_t^*) > 1$, which implies that the option value of one dollar exceeds 1 because as inventories it provides liquidity in the case of a high consumption demand. This is why money has positive value in equilibrium despite the fact that its real rate of return is negative ($\beta \frac{P_t}{P_{t+1}} < 1$) or dominated by interest-bearing assets.

The optimal level of total cash reserve (x_t) is chosen such that the probability of running out of cash is strictly positive ($1 - \mathbf{F}(\theta_t^*) \in (0, 1)$) unless the real cost of holding money is zero (i.e., at the Friedman rule). Namely, the optimal cutoff θ_t^* and cash in hand x_t are chosen simultaneously (as they are two sides of the same coin) so that $0 < \Pr[\theta > \theta_t^*] < 1$. This inventory-theoretic formula of money demand is akin to that derived by Wen (2011a) in a optimal target inventory model based on the stockout-avoidance motive. Also note that aggregate shocks (if they exist) will affect the distribution of money holdings across households by affecting the cutoff $\theta^*(H^t)$.

Because θ_t^* is independent of h^t , the cutoff provides a sufficient statistics for the distribution of money demand in the economy. This property facilitates aggregation and makes the model analytically tractable. Consequently, numerical solution methods (such as the method of Krusell and Smith, 1998) are not needed to solve the model's general equilibrium and aggregate dynamics.

By equation (10), cash in hand is also independent of h^t whenever the cutoff is so. The intuition for x_t and θ_t^* being independent of individual history is that (i) they are determined before the realization of θ_t , and all households face the same distribution of idiosyncratic shocks when making labor supply decisions; and (ii) the quasi-linear preference structure implies that labor supply can be adjusted elastically to meet any target level of cash in hand ex ante. Hence, in the beginning of

each period agents opt to adjust labor income so that the target cash in hand x_t and the probability of a binding liquidity constraint $(1 - \mathbf{F}(\theta^*))$ maximize expected utility; as a result, x_t and θ_t^* are the same across all households regardless of their individual history and initial real balances. This result is reminiscent of the Lagos-Wright (2005) model where the distribution of cash balances is degenerate. However, here the distribution of cash holdings $(\frac{m}{P})$ is not degenerate even though x is degenerate.

2.2 Equilibrium Analysis

Aggregation. Given the sequences of $\{W_t, \tau_t\}$ and the initial distribution of m_0 , using capital letters to denote aggregate variables (i.e., $C_t \equiv \int c(h^t, H^t) d\mathbf{F}$), we can integrate individual decision rules by the law of large numbers. The resulting system of equations that determines the competitive equilibrium path of $\{C_t, M_{t+1}, N_t, X_t, \theta_t^*, P_t\}_{t=0}^\infty$ includes

$$\frac{1}{W_t} = \beta E_t \frac{1}{W_{t+1}} \frac{P_t}{P_{t+1}} R(\theta_t^*) \quad (13)$$

$$C_t = D(\theta_t^*) X_t \quad (14)$$

$$\frac{M_{t+1}}{P_t} = H(\theta_t^*) X_t \quad (15)$$

$$X_t = W_t \theta_t^* R(\theta_t^*) \quad (16)$$

$$N_t = \frac{1}{W_t} \left(X_t - \frac{M_t + \tau_t}{P_t} \right) \quad (17)$$

$$M_{t+1} = M_t + \tau_t, \quad (18)$$

where $D(\theta^*) \equiv \int \min \left\{ 1, \frac{\theta}{\theta^*} \right\} d\mathbf{F}$, $H(\theta^*) \equiv \int \max \left\{ 0, \frac{\theta^* - \theta_t}{\theta^*} \right\} d\mathbf{F}$, and these two functions satisfy $D(\theta^*) + H(\theta^*) = 1$. Equation (18) is the money market clearing equation. These six dynamic equations plus standard transversality conditions uniquely solve for the equilibrium path of $\{C_t, M_{t+1}, N_t, X_t, \theta_t^*, P_t\}_{t=0}^\infty$, given the initial distribution of money demand.¹⁶

¹⁶The value functions are given by

$$V\left(\frac{m_t}{P_t}\right) = V_0 + \frac{1}{W_t} \frac{m_t}{P_t}; \quad J(x_t, \theta_t) = J_0 + \begin{cases} \left[\beta E_t \frac{P_t}{P_{t+1} W_{t+1}} \right] x_t & \text{if } \theta_t \leq \theta_t^* \\ \theta_t \log x_t & \text{if } \theta_t > \theta_t^* \end{cases}.$$

Note that under preference shocks and with log utility function, the demand for real balances $(\frac{m}{P})$ is bounded from above for any positive value of P . So the value function V is also bounded. Also, if one is interested only in the dynamics near the steady state, the uniqueness of equilibrium can also be easily proven (checked) by the eigenvalue method.

The Quantity Theory. The aggregate relationship between consumption (equation 14) and money demand (equation 15) implies the "quantity" equation,

$$P_t C_t = M_{t+1} V_t, \quad (19)$$

where $V_t \equiv \frac{D(\theta_t^*)}{H(\theta_t^*)}$ measures the aggregate consumption velocity of money. A high velocity implies a low demand for real balances relative to consumption. Given the support of θ as $[\theta_L, \theta_H]$ and the mean of θ as $E\theta = \bar{\theta}$, by the definition for the functions D and H , it is easy to see that the domain of velocity is $\left[\frac{\bar{\theta}}{\theta_H - \bar{\theta}}, \infty\right)$, which is bounded below by zero but has no finite upper bound, in sharp contrast to CIA models where velocity is typically a constant of 1. A zero velocity means a liquidity trap (excessive money hoarding), and an infinite velocity means that either the value of money ($\frac{1}{P}$) is zero or nominal money demand (M) is zero. This property explains why the model can generate enough variability in velocity to match the data.

Steady-State Analysis. Assume that money supply follows a constant growth path with

$$\tau_t = \mu M_t, \quad (20)$$

where μ is the growth rate.¹⁷ A steady state is defined as the situation without aggregate uncertainty and with time-invariant distributions of individual variables. For simplicity, assume that the real wage is constant. Hence, in a steady state all real aggregate variables are constant over time, although the individual variables may be stochastic due to the iid shocks θ_t . Equation (13) implies that the steady-state cutoff θ^* is constant and determined by the relation

$$1 = \frac{\beta}{1 + \pi} R(\theta^*), \quad (21)$$

where $\pi \equiv \frac{P_t - P_{t-1}}{P_{t-1}}$ denotes the inflation rate. Hence, the cutoff θ^* is constant for a given level of inflation. The quantity relation (19) implies $\frac{P_t}{P_{t-1}} = \frac{M_{t+1}}{M_t} = 1 + \mu$ in the steady state, so the steady-state inflation rate is the same as the growth rate of money.

Since by equation (21) the return to liquidity R must increase with π , the cutoff θ^* must decrease with π (because $\frac{\partial R(\theta^*)}{\partial \theta^*} < 0$); therefore, $\frac{\partial \theta^*}{\partial \pi} < 0$. This means that when inflation rises, the required rate of return to liquidity must also increase accordingly to induce people to hold money. However, because the cost of holding money increases with π , agents opt to hold less money so that the probability of stockout ($1 - \mathbf{F}(\theta^*)$) rises, which reinforces a rise in the liquidity premium (i.e., $\frac{\partial^2 R}{\partial \theta^* \partial \pi} < 0$). Also, since the target wealth is given by $x(\theta^*) = W\theta^*R(\theta^*)$, we have $\frac{\partial x}{\partial \theta^*} =$

¹⁷See Wen (2010) for dynamic analysis under monetary shocks.

$W\mathbf{F}(\theta^*) > 0$, so the target wealth decreases with π . Therefore, a higher rate of inflation has two types of effects on welfare: The intensive margin and the extensive margin. On the intensive margin, $\frac{\partial x}{\partial \pi} < 0$, so higher inflation leads to lower consumption through a negative wealth effect for all agents. In addition, liquidity-constrained agents suffer disproportionately more because they (i) do not have self-insurance ($m_{t+1} = 0$) to buffer shocks and (ii) still face the same variance of idiosyncratic shocks (σ_θ^2) when having a lower wealth level. This second aspect of the intensive margin is emphasized by Imrohoroglu (1992) and Akyol (2004). On the extensive margin, $\frac{\partial \theta^*}{\partial \pi} < 0$; thus, a high inflation rate means that a larger portion of the population will become liquidity constrained and subject to idiosyncratic shocks without the buffer stock. This extensive margin will be shown to be an important force in affecting social welfare but it has not been fully appreciated by the existing literature.

Under the Friedman rule, $1 + \pi = \beta$, we must have $R = 1$ and $\theta^* = \theta_H$ according to equation (12). Consequently, $D(\theta^*) = \frac{\bar{\theta}}{\theta_H}$ and $H(\theta^*) = 1 - \frac{\bar{\theta}}{\theta_H}$. Hence, we have $x(H^t) = W\theta_H$ and

$$c_t = \min \left\{ 1, \frac{\theta_t}{\theta_H} \right\} x = \theta_t W. \quad (22)$$

That is, individual consumption is perfectly adjusted ("smoothed") based on preference shocks under the Friedman rule, suggesting perfect self-insurance or the first-best allocation. The probability of being liquidity constrained (running out of cash) is zero in this case because households opt to hold the maximum amount of money when the cost of doing so is zero: $\frac{m_{t+1}}{P_t} = (\theta_H - \theta_t)W > 0$ for all θ .

However, since θ^* is bounded below by θ_L , the liquidity premium is then bounded above by $R(\theta_L) = \frac{\bar{\theta}}{\theta_L}$. This means there must exist a maximum rate of inflation $\pi_{\max}$ such that equation (21) holds: $\frac{\bar{\theta}}{\theta_L} = \frac{1+\pi_{\max}}{\beta}$. At this maximum inflation rate,

$$\pi_{\max} = \beta \frac{\bar{\theta}}{\theta_L} - 1, \quad (23)$$

we have $D(\theta_L) = 1$ and $H(\theta_L) = 0$. That is, the optimal demand for real balances from all households goes to zero, $m(h^t, H^t) = 0$, if $\pi \geq \pi_{\max}$.¹⁸

When the cost of holding money is sufficiently high, agents opt not to use money as the store of value and the velocity becomes infinity: $V = \frac{D(\theta_L)}{H(\theta_L)} = \infty$. The velocity is a decreasing function of

¹⁸Obviously we need to assume that $\beta > \frac{\theta_L}{\bar{\theta}}$ to support a monetary equilibrium even at low inflation rates. If β is too small, there does not exist a high enough liquidity premium to induce people to hold money. As an example, suppose θ follows the Pareto distribution, $F(\theta) = 1 - \theta^{-\sigma}$ with support $(1, \infty)$, then $\theta_L = 1$ and $\bar{\theta} = \infty$, so the lower bound on β to support a monetary equilibrium is zero.

inflation because money demand drops faster than consumption as the inflation tax rises: $\frac{\partial V}{\partial \theta^*} \frac{\partial \theta^*}{\partial \pi} > 0$. This long-run implication is consistent with empirical data. For example, Chiu (2007) has found using cross-country data that countries with higher average inflation also tend to have significantly higher levels of velocity and argued that such an implication cannot be deduced from the Baumol-Tobin model with an exogenously segmented asset market.¹⁹

When money is no longer held as a store of value because of sufficiently high inflation ($\pi \geq \pi_{\max}$), it must be true that $c_t = \min \left\{ 1, \frac{\theta_t}{\theta_L} \right\} x = x = \bar{\theta}W$, so consumption is constant and completely unresponsive to preference shocks, suggesting the worst possible allocation with a significantly lower level of welfare than the case with the Friedman rule. Note that the aggregate (average) consumption under inflation $\pi \geq \pi_{\max}$ is identical to the aggregate (average) consumption under the Friedman rule by equation (22): $C = W \int \theta_t d\mathbf{F} = \bar{\theta}W$. This equivalence of aggregate allocations under drastically different inflation rates reveals the danger of measuring the welfare cost of inflation (or the welfare cost of the business cycle) based on representative-agent models. This point is quantified in the next section.

3 Welfare Costs of Inflation—General-Equilibrium Analysis

3.1 The Model with Capital

Households. Households accumulate illiquid capital asset through saving (s_t) and rent the capital stock to firms in a competitive rental market with the rental rate denoted by r_t . The illiquidity of the capital asset is captured by the assumption that saving decisions for fixed capital (s_{t+1}) in period t must be made in tandem with the decision of labor supply (n_t) in the first subperiod (i.e., before the preference shocks are realized). The household budget constraint becomes

$$c_t + \frac{m_{t+1}}{P_t} + s_{t+1} \leq (1 + r_t) s_t + \frac{m_t + \tau_t}{P_t} + W_t n_t \quad (24)$$

where $s_{t+1} = s_{t+1}(h_{-}^t, H^t)$ in accordance with the notations in equation (1). Compared with the previous benchmark model, there is now one more first-order condition for s_{t+1} on the household side²⁰:

$$\frac{1}{W_t} = \beta E_t \frac{1 + r_{t+1}}{W_{t+1}}, \quad (25)$$

which in the steady state becomes

$$1 = \beta (1 + r). \quad (26)$$

¹⁹Based on a long sample of the U.S. time-series data, Lucas (2000) also shows that the inverse of the velocity is negatively related to inflation.

²⁰See the proof for Proposition 2 in Appendix A2.

Compared with equation (21), we must have $(1 + r)(1 + \pi) = R(\theta^*)$ under the no-arbitrage condition, suggesting that the nominal rate of return to the illiquid capital asset must equal the liquidity premium of money in the steady state.

Firms. A representative firm produces final output according to the production technology, $Y_t = A_t K_t^\alpha N_t^{1-\alpha}$. The firm rents capital and hires labor from households. Perfect competition implies that factor prices equal their marginal products: $W_t = (1 - \alpha) \frac{Y_t}{N_t}$ and $r_t + \delta = \alpha \frac{Y_t}{K_t}$.

General Equilibrium. Define cash in hand as $x_t \equiv \frac{m_t + \tau_t}{P_t} + W_t n_t + (1 + r_t) s_t - s_{t+1}$. Wen (2009, 2010) shows that the household decision rules of consumption, real money balances, and cash in hand are identical in form to equations (8)-(12) except that the labor supply changes to $n_t = \frac{1}{W_t} \left[x_t - \frac{m_t + \tau_t}{P_t} + s_{t+1} - (1 + r_t) s_t \right]$ (also see Appendix A2 in this paper). That is, adding capital into the benchmark model does not change its basic properties, such as the fact that $\{x_t, \theta_t^*\}$ are independent of individual history h^t . In general equilibrium, the aggregate supply of capital equals the demand of capital, $\int s_{t+1}(h_-^t, H^t) d\mathbf{F} = K_{t+1}$, and the aggregate supply of labor equals the demand of labor, $\int n_t(h_-^t, H^t) d\mathbf{F} = N_t$.

Using upper-case letters to denote aggregate quantities, a general equilibrium is defined as the sequence $\{C_t, Y_t, N_t, K_{t+1}, M_{t+1}, P_t, W_t, r_t, \theta_t^*\}_{t=0}^\infty$, such that given prices $\{P_t, W_t, r_t\}$ and monetary policies, (i) all households maximize utilities subject to their resource and borrowing constraints (and initial capital and money holdings), (ii) firms maximize profits, (iii) all markets clear, (iv) the law of large numbers holds, and (v) the set of standard transversality conditions is satisfied.²¹ The equations needed to solve for the general equilibrium are stated in Proposition 2 below. Because the steady state is unique and the system is saddle stable, the distribution of money demand converges to a unique time-invariant distribution in the long run for any initial distributions of capital s_0 and money holdings m_0 .

Proposition 2 *The general equilibrium of the model can be characterized by the dynamic paths of ten aggregate variables, $\{C_t, K_{t+1}, M_{t+1}, N_t, X_t, Y_t, \theta_t^*, P_t, W_t, r_t\}$, which can be uniquely solved by the following ten aggregate equations:*

$$C_t = D(\theta_t^*) X_t \quad (27)$$

$$\frac{M_{t+1}}{P_t} = H(\theta_t^*) X_t \quad (28)$$

$$X_t = W_t \theta_t^* R(\theta_t^*) \quad (29)$$

²¹Such transversality conditions include $\lim_{t \rightarrow \infty} \beta^t E_t \frac{K_{t+1}}{W_t} = 0$ and $\lim_{t \rightarrow \infty} \beta^t E_t \frac{M_{t+1}}{P_t W_t} = 0$, where $\frac{1}{W}$ is the shadow value of capital and $\frac{1}{P}$ is the value of money.

$$X_t = \frac{M_t + \tau_t}{P_t} + (1 + r_t)K_t - K_{t+1} + W_t N_t \quad (30)$$

$$\frac{1}{W_t} = \beta E_t \frac{1}{W_{t+1}} \frac{P_t}{P_{t+1}} R(\theta_t^*) \quad (31)$$

$$\frac{1}{W_t} = \beta E_t \frac{1 + r_{t+1}}{W_{t+1}} \quad (32)$$

$$W_t = (1 - \alpha) \frac{Y_t}{N_t} \quad (33)$$

$$r_t + \delta = \alpha \frac{Y_t}{K_t} \quad (34)$$

$$C_t + K_{t+1} - (1 - \delta)K_t = Y_t \quad (35)$$

$$Y_t = A_t K_t^\alpha N_t^{1-\alpha}, \quad (36)$$

where $D(\theta^*) \equiv \int \min \left\{ 1, \frac{\theta}{\theta^*} \right\} d\mathbf{F}$, $H(\theta^*) \equiv \int \max \left\{ 0, \frac{\theta^* - \theta_t}{\theta^*} \right\} d\mathbf{F}$, and $R(\theta_t^*) \equiv \int \max \left\{ 1, \frac{\theta_t}{\theta_t^*} \right\} d\mathbf{F}$.

Proof. See Appendix A2. ■

The aggregate dynamics of the model (such as transitional dynamics or impulse responses to aggregate shocks) can be solved by standard methods popular in the representative-agent RBC literature, such as the log-linearization method of King, Plosser, and Rebelo (1988). In our steady-state analysis, we assume there is no aggregate uncertainty (e.g., $A_t = 1$ and money injection follows a constant growth rule).²²

3.2 Steady-State Allocations

In the steady state, the capital-to-output and consumption-to-output ratios are given by $\frac{K}{Y} = \frac{\beta\alpha}{1-\beta(1-\delta)}$ and $\frac{C}{Y} = 1 - \frac{\delta\beta\alpha}{1-\beta(1-\delta)}$, respectively, which are the same as in standard RBC models without money. Since $r + \delta = \alpha \frac{Y}{K}$ and $w = (1 - \alpha) \frac{Y}{N}$, the factor prices are given by $r = \frac{1}{\beta} - 1$ and $W = (1 - \alpha) \left(\frac{\beta\alpha}{1-\beta(1-\delta)} \right)^{\frac{\alpha}{1-\alpha}}$, respectively. Hence, the existence of money in this model does not alter the steady-state aggregate saving rate, the great ratios, and the real factor prices in the neoclassical growth model, in contrast to typical CIA models. However, the levels of aggregate income, consumption, employment, and capital stock are affected by money. These levels are given

²²See Wen (2009, 2010) for stochastic analyses under aggregate TFP and monetary shocks.

by

$$C = W\theta^*R(\theta^*)D(\theta^*), \quad Y = \frac{1 - \beta(1 - \delta)}{1 - \beta(1 - \delta) - \delta\beta\alpha}C, \quad K = \frac{\beta\alpha}{1 - \beta(1 - \delta)}Y, \quad N = \frac{1 - \alpha}{W}Y, \quad (37)$$

where the cutoff is determined by the inflation rate, $R(\theta^*) = \frac{1+\pi}{\beta}$. Thus, money affects aggregate allocations primarily through affecting aggregate consumption. Under the Friedman rule, we have $c = C = \bar{\theta}W$, so money ceases to be effective in influencing both individual consumption (c) and the aggregate allocations.

3.3 Measures of Welfare Costs

We measure the welfare costs of inflation as the state- and time-independent percentage increase (Δ) in household consumption that would make any individual household indifferent in terms of expected lifetime utilities between living in a high-inflation regime (π) and the Friedman-rule inflation regime ($\pi^o = \beta - 1$). That is, Δ is the compensation required for household consumption as the inflation rate increases above the Friedman rule. By the law of large numbers, the expected momentary utility of an individual i is the same as the aggregate (or average) utility of the population (with equal weights); namely, $\int \theta(i) \log c(i) d\mathbf{F}(\theta) - \int n(i) d\mathbf{F}(\theta) = \int \theta(i) \log c(i) di - N$. Thus, our welfare measure also corresponds to a social planner's measure with equal welfare weights.

Hence, the welfare cost of inflation Δ solves

$$\sum_{t=0}^{\infty} \beta^t \left\{ \int [\theta \log(1 + \Delta) c(\pi)] d\mathbf{F} - N(\pi) \right\} = \sum_{t=0}^{\infty} \beta^t \left\{ \int \theta \log c(\pi^o) d\mathbf{F} - N(\pi^o) \right\}. \quad (38)$$

By the household consumption policy, $c = \min\{1, \frac{\theta}{\theta^*}\}x$, and the property that $x = X$ is independent of individual history, the above equation implies

$$\Delta = \exp \left\{ \frac{1}{\bar{\theta}} \left[N(\pi) - N(\pi^o) + \bar{\theta} \log \frac{X(\pi^o)}{X(\pi)} + J_{\theta}(\pi^o) - J_{\theta}(\pi) \right] \right\} - 1, \quad (39)$$

where $J_{\theta}(\pi) \equiv \int_{\theta \leq \theta^*(\pi)} \left(\theta \log \frac{\theta}{\theta^*(\pi)} \right) d\mathbf{F}$ captures the effect of idiosyncratic risk (or the heterogeneity of consumption) on social welfare.

If instead we measure (incorrectly) the welfare costs of inflation by the utility of the average consumption of the society ($\log C = \log D(\theta^*)X$) in the steady state, then the equation to solve for the welfare cost ($\Delta^{\times}$) would be

$$\Delta^{\times} = \exp \left\{ \frac{1}{\bar{\theta}} \left[N(\pi) - N(\pi^o) + \bar{\theta} \log \frac{C(\pi^o)}{C(\pi)} \right] \right\} - 1. \quad (40)$$

As discussed previously, if $\pi \geq \pi_{\max}$ and $\pi^o = \beta - 1$, then we have identical aggregate allocations between the hyper-inflation regime and the Friedman-rule inflation regime with $N(\pi) = N(\pi^o)$ and $C(\pi) = C(\pi^o)$. Consequently, the incorrectly measured welfare cost of inflation is $\Delta^\times = 0$, whereas the correctly measured welfare cost of inflation is given by $\Delta = \exp \left\{ \frac{1}{\theta} [J_\theta(\pi^o) - J_\theta(\pi_{\max})] \right\} - 1 > 0$ because $J(\pi_{\max}) = \int_{\theta \leq \theta_L} \left(\theta \log \frac{\theta}{\theta_L} \right) d\mathbf{F} = 0$ and $J(\pi^o) = \int_{\theta \leq \theta_H} \left(\theta \log \frac{\theta}{\theta_H} \right) d\mathbf{F} > 0$. This result is striking; it suggests just how wrong representative-agent models can be when it comes to welfare implications.

3.4 Calibrations and Predictions

To facilitate calibration, we assume that the idiosyncratic shock θ follows the Pareto distribution,

$$F(\theta) = 1 - \theta^{-\sigma}, \quad (41)$$

with $\sigma > 1$ and the support $[1, \infty)$. The mean of this distribution is $\bar{\theta} = \frac{\sigma}{\sigma-1}$. Since θ is not bounded from above, at the Friedman rule money demand goes to infinity. In the following analysis, we assume that $\pi^o = \beta - 1 + \varepsilon$, where the positive number ε is arbitrarily close to 0 but not exactly equal to 0.²³ We refer to this inflation rate π^o as the "Friedman rule".

With the Pareto distribution function, we have $R(\theta^*) = 1 + \frac{1}{\sigma-1} \theta^{*- \sigma}$, $H(\theta^*) = R(\theta^*) - \frac{\sigma}{\sigma-1} \frac{1}{\theta^*}$, and $D(\theta^*) = 1 - H(\theta^*)$. The cutoff $\theta^*(\pi)$ can then be solved explicitly using the relation $R(\theta^*) = \frac{1+\pi}{\beta}$.

The time period t is a year.²⁴ Following the standard RBC literature, we set capital's income share $\alpha = 0.42$, the time discount factor $\beta = 0.95$, and the rate of capital depreciation $\delta = 0.1$. Under these parameter values, the implied capital-to-output ratio is $\frac{\beta\alpha}{1-\beta(1-\delta)} = 2.75$ and aggregate consumption-to-output ratio is 0.725. The remaining free parameters include σ in the Pareto distribution function and A in the quantity relationship:

$$\frac{M}{PY} = A \frac{H(\theta^*)}{D(\theta^*)} \left(\frac{1 - \beta(1 - \delta) - \delta\beta\alpha}{1 - \beta(1 - \delta)} \right), \quad (42)$$

where P denotes aggregate price level, Y aggregate output, M aggregate money supply, $\frac{PY}{M}$ is the empirical measure of the income velocity of money, and $\frac{H(\theta^*)}{D(\theta^*)} \left(\frac{1 - \beta(1 - \delta) - \delta\beta\alpha}{1 - \beta(1 - \delta)} \right)$ is the theoretical counterpart of income velocity implied by our model (where the ratio in the parenthesis is the

²³More specifically, we set $\varepsilon = 10^{-6}$.

²⁴We choose t to be a year because the aggregate money demand data used by Lucas (2000) are available only at the annual frequency.

consumption-to-income ratio). Since (i) the definition of money in the empirical data varies greatly (such as M1, M2, and M3), and (ii) the definition of the consumption-to-output ratio changes depending on whether government spending, net exports, and durable goods consumption are included in GDP, the measured velocity of money also varies accordingly. Hence, we introduce the scaling parameter A to reflect these variations in the measurement bias in the mean of velocity when calibrating our model to match the data.

These two free parameters $\{\sigma, A\}$ are calibrated by three independent methods, called Method 1, Method 2, and Method 3. Method 1 is our benchmark and the other two serve as robustness checks because Method 1 does not take into account the idiosyncratic risk. Under Method 1, we set $A = 1$ and use a least squares criterion to estimate the value of σ that enables our model to best match the empirical aggregate money demand curve ($\frac{M}{PY}$) suggested by Lucas (2000). This is also the calibration strategy of Lagos and Wright (2005). Under Method 2, we choose the values of $\{A, \sigma\}$ to jointly match the (i) the empirical money demand curve of Lucas (2000) and (ii) the probability of running out of cash (the likelihood of a binding liquidity constraint $\Pr[\theta > \theta^*]$) implied by the household survey data. Under Method 3, we choose the values of $\{A, \sigma\}$ to jointly match (i) the empirical money demand curve and (ii) the household consumption volatility implied by the inequality of household consumption. Appendix B provides details of these calibration procedures.²⁵

The calibrated parameter values are summarized in Table 1. Notice that the values of σ under various calibration methods imply that the variance of log consumption ($\log c_t$) in the model is in the range of $0.03 \sim 0.13$ (see the last column in Table 1). This range of household consumption volatility is consistent with the empirical estimates of Telyukova (2011, Table 9), who reports a range of $0.056 \sim 0.113$ for the variance of various types of household consumption. Under Method 1, the model-implied variance of household consumption is 0.055, which is on the lower bound of Telyukova's estimates. Hence, we use Method 1 as our benchmark calibration for $\{\sigma, A\}$ in this paper.²⁶

All the calibration methods amount to rationalizing the empirical money demand curve emphasized by Lucas (2000), in addition to various measures of consumption risks. Using historical data

²⁵ Because of the lack of long time-series panel data that can track the consumption expenditure and money demand of the same households for more than one year, we borrow information from cross-section data to infer consumption volatility. This is not entirely unreasonable. For example, if we survey households from the same villages with similar living standards and consumption needs, then cross-section variations may very well indicate over-time consumption risk of a typical household in the village.

²⁶ Telyukova's (2011) estimates are based on monthly data. However, she also reported similar estimates for the variance of household consumption based on quarterly data in an earlier 2009 version of her paper. Annual data are not available since the SCF data keep track of the same households for only one year. Following Telyukova, we compute in our model the variance of the logarithm of consumption, $\log c_t = \log [\min \{1, \frac{\theta_t}{\theta^*}\} x]$, based on simulated sample of θ_t with a sample size of 10^6 . Keeping the variance of preference shocks constant, the model would generate larger welfare costs of inflation if the time interval becomes shorter. Hence, using an annual model is conservative because calibrating our model at a quarterly or monthly interval would only enhance our results.

for GDP, money stock (M1), and the nominal interest rate, Lucas (2000) showed that the ratio of M1 to nominal GDP is downward sloping against the nominal interest rate. Lucas interpreted this downward relationship as a "money demand" curve and argued that it can be best rationalized by the Sidrauski (1967) model of money-in-utility (MIU). Lucas estimated that the empirical money demand curve can be best captured by an ad hoc power function of the form

$$\frac{M}{PY} = Ar^{-\eta}, \quad (43)$$

where A is a scale parameter, r the nominal interest rate, and η the interest elasticity of money demand. He showed that $\eta = 0.5$ gives the best fit. Because the money demand defined by Lucas is identical to the inverted velocity, a downward-sloping money demand curve is the same as an upward-sloping velocity curve (namely, velocity is positively related to the nominal interest rate or inflation). Similar to Lucas, the money demand curve implied by our model takes the form in equation (42). Figure 1 shows the fit of the theoretical model to the U.S. data under calibration Method 1.²⁷

The model's welfare implications under Method 1 are graphed in Figure 2. The top-left panel shows the correct measure of welfare cost (Δ). It is monotonically increasing with inflation. Hence, the Friedman rule is clearly optimal. The maximum welfare cost is reached at the maximum inflation rate $\pi_{\max} = 52.576\%$, where $\Delta = 14\%$ of consumption. Beyond this inflation rate money is no longer valued (held) by households, so the welfare cost of inflation remains constant at 14% for $\pi \geq \pi_{\max}$. When the inflation rate $\pi = 10\%$ (15% above the Friedman rule), the welfare cost is 9.6% of annual consumption. The cost would be even higher if we calibrate σ to match the variance of consumption in developing countries.²⁸

In contrast, the top-right panel of Figure 2 shows the incorrect measure of welfare cost ($\Delta^\times$) based on the average consumption of a household (or a representative agent). This measure is not monotonic; it equals 0 at two extreme points—the point of the Friedman rule and the point where $\pi = \pi_{\max}$. In the first case, individual consumption level ($c_t = \theta_t W$) is the first-best—because it is costless to hold money, so agents are perfectly insured against idiosyncratic risk. The average consumption in this case is $\bar{\theta}W$. In the latter case, individuals' consumption levels become homogeneous across households at a constant level ($c = \bar{\theta}W$) when money is no longer

²⁷The circles in Figure 1 show plots of annual time series of a short-term nominal interest rate (the commercial paper rate) against the ratio of M1 to nominal GDP, for the United States for the period 1900–1997, the data sample used by Lucas (2000). The data are downloaded from the online Historical Statistics of the United States–Millennium Edition. The solid line with crosses is the model's prediction.

²⁸Wen (2011) uses statistics based on medical spending, traffic incidents, and work-related injuries to argue that consumption risk in China is at least one order of magnitude larger than that in the U.S. For the sake of argument, suppose the variance of household consumption in China is twice of that in the U.S., then the implied welfare cost of 10% inflation would be 18 percent of consumption. Evidences show that China's 1989 Tian An Men Square anti-government movements was triggered by moderate inflation around 15% ~ 20%, and the movement was widely supported by low-income classes such as school teachers, workers, and farmers.

held as a store of value. Without money, inflation no longer has any adverse liquidity effects on consumption, so $\Delta^\times$ remains at zero for $\pi \geq \pi_{\max}$. The maximum cost of inflation by the incorrect welfare measure is $\Delta^\times = 0.78\%$ of consumption when $\pi = 2.7\%$. Under a 10% inflation rate (or 15% inflation above the Friedman rule), the incorrectly measured welfare cost is $\Delta^\times = 0.67\%$ of consumption. These values are similar in magnitudes to those obtained in the existing literature based on representative-agent models.

The bottom-left panel of Figure 2 shows the level of aggregate money demand ($\frac{M}{P}$), and the bottom-right panel shows the consumption velocity of money ($\frac{D(\theta^*)}{H(\theta^*)}$). The money demand function decreases monotonically with inflation, whereas the velocity increases monotonically with inflation. Near the Friedman rule, the demand for money is close to infinity and the velocity is close to zero. In contrast, the demand for real balances becomes zero for $\pi \geq \pi_{\max}$ and the velocity of money becomes infinity at $\pi_{\max}$.²⁹ The velocity of money is close to zero near the Friedman rule because households opt to hoard as much money as they can when its real rate of return equals the inverse of the time discount factor (i.e., the demand for money approaches infinity as the opportunity cost of holding money goes to zero). In this case, the aggregate price level is close to zero and the borrowing constraint ceases to bind for all households (or across all possible states). The velocity of money becomes infinity as $\pi \rightarrow \pi_{\max}$ because people want to divest their holdings of money as quickly as possible to avoid the inflation tax despite the need for a store of value to self-insure against idiosyncratic consumption-demand shocks. But since the cost of holding money is so high as $\frac{1}{P_t} \rightarrow \infty$ and the insurance value of money is destroyed, the demand for money becomes zero. This pertains to the "hot potato effect" of inflation found in hyper-inflation countries where people try to get rid of money as fast as they can to avoid the destruction of the liquidity value of money in hand (see Qian, Wang, and Wright, 2011).

These implications for money demand and velocity are quite different from standard CIA models, which imply a constant velocity and a strictly positive lower bound on money demand, because agents under the CIA constraint must hold money even with an infinite inflation rate $\pi = \infty$. In the real world, people often stop accepting domestic currency as a store of value or the means of payment when the inflation rate is too high (but before reaching infinity), consistent with our model's prediction.

Under Method 2, the values of (σ, A) are chosen to best match the empirical money demand curve suggested by Lucas (2000) and at the same time match the probability of running out of cash based on household surveys. Appendix B shows that it requires $\sigma \in (2.1, 3.1)$ and $A \in (0.54, 1.45)$ to generate a 10% to 20% probability of running out of cash in the model when the inflation rate

²⁹The graph shows the velocity only for $\pi < \pi_{\max}$.

is 4% a year (in addition to matching the Lucas curve in Figure 1). Based on these ranges of parameter values, the implied welfare costs of 10% inflation (or 15% above the Friedman rule) are 6.6% to 18%, whereas the implied variance of household consumption is in the range of $0.03 \sim 0.11$.

Under Method 3, we choose $\{\sigma, A\}$ to match the consumption risk implied by the cross-section distribution of household consumption in the data, in addition to fitting the Lucas curve. The idea is that the distribution of household consumption (i.e., the Gini coefficient) is positively correlated with the degree of consumption risk. Consumption risk in the model is determined by σ . The consumption Gini in the U.S. is about 0.3 and that in developing countries is about $0.4 - 0.43$ on average (see Appendix B). Again, based on the rule of thumb that overtime risk is about half of the cross-household dispersion, we choose $\{\sigma, A\}$ to generate consumption Gini in the range of $(0.15, 0.20)$ when the annual inflation rate is 4%. This yields $\sigma \in (2.0, 2.5)$ and $A \in (0.48, 0.86)$. Based on these ranges of parameter values, the implied welfare costs of 10% inflation are in the interval of $(11\%, 21\%)$, whereas the implied variance of household consumption is about $0.07 \sim 0.13$.

These welfare results are summarized in Table 2. The last column also reports the welfare costs of moving from the current 2% inflation target adopted by the U.S. Federal Reserve Bank to a new target of 3% per year under the three calibration methods.

In a heterogeneous-agent economy with incomplete markets, the larger the variance of idiosyncratic shocks (or smaller value of σ), the stronger the precautionary motive for holding money. This raises the inflation tax at a given inflation rate. More importantly, higher inflation shifts the mass of the distribution of money demand toward zero balances by reducing cash holdings across agents, resulting in a larger portion of the population without self-insurance against idiosyncratic shocks. This shift of the distribution of money demand in response to inflation is most critical in generating the large welfare cost.

To see the importance of this extensive margin, notice that $\frac{\partial R(\theta^*)}{\partial \pi} > 0$ and $\frac{\partial \theta^*}{\partial \pi} < 0$ by equation (21). So the probability of running out of cash $1 - \mathbf{F}(\theta^*)$ is positively affected by inflation. For example, under the Pareto distribution we have $1 - \mathbf{F}(\theta^*) = (\sigma - 1) \left(\frac{1 + \pi - \beta}{\beta} \right)$, which shows a positive linear relationship between the probability of a binding liquidity constraint (or the proportion of the liquidity-constrained population) and inflation. As inflation rises, the portion of the population holding zero balances increases rapidly. For example, given the parameter values under Method 1, when inflation increases from 0% to 10%, an additional 17% of the entire population is left without cash (thus without self-insurance), raising the total number of cashless agents to about 26% of the population. When holding money is too costly, the demand for real balances becomes so low that the probability of running out of cash is extremely high. The significantly reduced buffer stock or self-insurance amounts to large welfare costs.

Here we emphasize a point made by Lagos and Wright (2005). That is, we notice that our three different calibration methods (with quite different ranges of parameter values) can all match the empirical money demand curve in Figure 1 almost equally well, but the implied welfare costs are nonetheless quite different. In particular, the value of A is crucial for matching the Lucas money demand curve but plays no role in computing our welfare results (see equation (39)).³⁰ Hence, as noted by Lagos and Wright (2005), simply computing the area underneath the money demand curve as a measure of the welfare cost of inflation, as proposed by Bailey (1956) and favored by Lucas (2000), is not good enough. What is really needed is an explicit model of the micro foundations, especially the motives behind money demand, in order to properly estimate the welfare cost of inflation. Consistent with this spirit, here we offer a different model of money demand with micro foundations different from those of Lagos and Wright, and we obtain different welfare results (as expected) because we emphasize different functions of money in our models (medium of exchange versus store of value). One thing in common between our two approaches is that both models obtain substantially larger welfare costs of inflation than in the existing representative-agent monetary literature.

4 Welfare Implications of Credit and Banking

There may be at least two potential objections to the large welfare cost of inflation in the benchmark model. First, the model posits uninsurable idiosyncratic risk and that money is the only liquid asset to help self-insure against such risk. This setup rules out other types of insurance devices and, especially, does not take into account the role of credit and banking (such as consumer credit or credit cards) in mitigating the idiosyncratic risk through borrowing and lending. Second, it is a common belief in the existing literature that inflation benefits debtors by redistributing the burden of inflation toward creditors. For these reasons, the welfare costs of inflation may be overstated.³¹

To address these concerns, this section extends the general-equilibrium Bewley model to a setting with "narrow banking," where cash-rich agents can deposit their idle cash into a community bank, and cash-poor agents with a binding liquidity constraint can borrow from the bank by paying a nominal interest. The nominal interest rate of loans can be endogenously determined by the supply and demand of funds. This money-market interest rate can be significantly different from the rate of time preference and the rental rate of capital in the model.

The key friction in the benchmark model is the nonnegativity constraints ($m \geq 0$) on nominal

³⁰That is, A affects the estimated value of σ but does not directly enter the welfare function (39).

³¹Kehoe, Levine, and Woodford (1992) study the welfare effects of inflation in a Bewley model with aggregate uncertainty. They show that lump-sum nominal transfers can redistribute wealth from cash-rich agents to cash-poor agents, because the latter receive disproportionately more transfers than the former and thereby benefit from inflation. Consequently, inflation may improve social welfare. However, this positive effect on social welfare is quantitatively quite small and requires extreme parameter values in their model.

balances and the lack of risk sharing among households. With this setup, households opt to hold a zero-interest asset as store of value for precautionary reasons, and there is always an ex post inefficiency on money holdings since some agents end up holding idle balances while others end up liquidity constrained with zero balances. This creates a need for risk sharing, as suggested by Lucas (1980). However, without the necessary information- and record-keeping technologies, households cannot lend and borrow among themselves. In this section, we assume that a community bank (or credit union) emerges to resolve the risk-sharing problem by developing the required information technologies. The function of the bank is to accept nominal deposits from households and make nominal loans to bank members. Because of the lack of liquid assets to serve as collateral (other than the deposits), we assume that a household can borrow only up to a limit proportional to its average bank deposits in the past ($\int m_t d\mathbf{F}$) plus an additional fixed amount $\bar{b} \geq 0$.

For simplicity, we assume that (i) all deposits are withdrawn at the end of the same period (100-percent reserve banking) and (ii) all loans are one-period loans that charge the competitive nominal interest rate $1 + \tilde{i}$, which is determined by the demand and supply of funds in the community. The nominal interest rate on deposits is denoted by $1 + i^d$, with $i^d = \psi \tilde{i}$ and $\psi \in (0, 1)$. Any profits earned by the bank are distributed back to community members as lump-sum transfers.

Similar banking arrangements have been studied recently by Berentsen, Camera, and Waller (2006) and others. This literature shows that financial intermediation improves welfare.³² However, these authors study the issue in the Lagos-Wright (2005) framework, which focuses on the medium-of-exchange function of money and has no capital accumulation. In addition, in their model the welfare gains of financial intermediation come solely from the payment of interest on deposits and not from relaxing borrowers' liquidity constraints. In contrast, this paper focuses on welfare gains that derive mainly from risk sharing or relaxing borrowers' liquidity constraints.

The time line of events is as follows: In the beginning of each period, aggregate shocks (if any) are realized, each household then makes decisions on labor supply and capital investment, taking as given the initial wealth from last period. This is the first subperiod. In the second subperiod, idiosyncratic preference shocks are realized and each household chooses consumption, nominal balances, and the amount of new loans.

4.1 The Household Problem

As in the benchmark model, hours worked and nonmonetary asset investment in each period must be determined before the idiosyncratic preference shock is realized; the remaining decisions are all made after observing θ_t . Each household takes the bank's profit income (T_t) and government

³²However, Chiu and Meh (2008) show that banking may reduce welfare under moderate inflation rates if there exist transaction costs for using financial intermediation. For alternative approaches to money and banking, see Williamson (1986) and Andolfatto and Nosal (2003).

money transfers (τ_t) as given and chooses consumption, capital investment, labor supply, money demand, and credit borrowing (b_{t+1}) to maximize the objective function in equation (1) subject to

$$c_t + k_{t+1} + \frac{m_{t+1}}{P_t} + (1 + \tilde{i}_t) \frac{b_t}{P_t} \leq (1 + r_t)k_t + \left(1 + i_t^d\right) \frac{m_t}{P_t} + \frac{b_{t+1}}{P_t} + W_t n_t + \frac{T_t + \tau_t}{P_t} \quad (44)$$

$$m_{t+1} \geq 0 \quad (45)$$

$$b_{t+1} \geq 0 \quad (46)$$

$$b_{t+1} \leq \gamma \int m_t d\mathbf{F} + \bar{b}, \quad (47)$$

where $\tilde{i}$ denotes the nominal loan rate, i^d the nominal deposit rate, and r the real rental rate of capital. The nonnegativity constraints on nominal balances (m_{t+1}) and loans (b_{t+1}) capture the idea that households cannot borrow or lend outside the banking system. Equation (47) imposes a borrowing constraint on credit limits, where $\gamma \geq 0$.

Proposition 3 *Denoting total cash in hand by $x_t \equiv (1 + r_t)k_t - k_{t+1} + \frac{m_t}{P_t} + W_t n_t + \frac{T_t + \tau_t}{P_t} - (1 + \tilde{i}_t) \frac{b_t}{P_t}$ and $\underline{\theta}_t^* \leq \bar{\theta}_t^* \leq \tilde{\theta}_t^*$ as the cutoffs, the decision rules of cash in hand, consumption, money holdings, and loan demand are given, respectively, by*

$$x_t = \underline{\theta}_t^* W_t R_t, \quad (48)$$

$$c_t = \begin{cases} \frac{\theta}{\underline{\theta}^*} x_t & \text{if } \theta \leq \underline{\theta}_t^* \\ x_t & \text{if } \underline{\theta}_t^* < \theta \leq \bar{\theta}_t^* \\ \frac{\theta}{\bar{\theta}^*} x_t & \text{if } \bar{\theta}_t^* < \theta \leq \tilde{\theta}_t^* \\ x_t + \frac{\gamma M_t + \bar{b}}{P_t} & \text{if } \theta > \tilde{\theta}_t^* \end{cases} \quad (49)$$

$$\frac{m_{t+1}}{P_t} = \begin{cases} \left(1 - \frac{\theta}{\underline{\theta}^*}\right) x_t & \text{if } \theta \leq \underline{\theta}_t^* \\ 0 & \text{if } \theta > \underline{\theta}_t^* \end{cases} \quad (50)$$

$$\frac{b_{t+1}}{P_t} = \begin{cases} 0 & \text{if } \theta \leq \bar{\theta}_t^* \\ \left(\frac{\theta}{\bar{\theta}^*} - 1\right) x_t & \text{if } \bar{\theta}_t^* < \theta \leq \tilde{\theta}_t^* \\ \frac{\gamma M_t + \bar{b}}{P_t} & \text{if } \theta > \tilde{\theta}_t^* \end{cases} \quad (51)$$

where the cutoffs $\{\underline{\theta}_t^*, \bar{\theta}_t^*, \tilde{\theta}_t^*\}$ are determined jointly and uniquely by the following three equations:

$$\tilde{\theta}_t^* = \bar{\theta}_t^* \left[1 + \frac{\gamma M_t + \bar{b}}{P_t x_t}\right] \quad (52)$$

$$\frac{\bar{\theta}_t^*}{\underline{\theta}_t^*} = \frac{E_t \left[(1 + \tilde{i}_{t+1}) \frac{P_t}{P_{t+1} w_{t+1}} \right]}{E_t \left[(1 + i_{t+1}^d) \frac{P_t}{P_{t+1} w_{t+1}} \right]} \quad (53)$$

$$1 = \beta E_t \left[\left(1 + i_{t+1}^d \right) \frac{P_t W_t}{P_{t+1} W_{t+1}} \right] R(\underline{\theta}_t^*, \bar{\theta}_t^*, \tilde{\theta}_t^*), \quad (54)$$

where

$$R_t \equiv \int_{\theta < \underline{\theta}^*} d\mathbf{F}(\theta) + \int_{\underline{\theta}^* \leq \theta \leq \bar{\theta}^*} \frac{\theta}{\underline{\theta}^*} d\mathbf{F}(\theta) + \int_{\bar{\theta}^* < \theta < \tilde{\theta}^*} \frac{\bar{\theta}^*}{\underline{\theta}^*} d\mathbf{F}(\theta) + \int_{\theta > \tilde{\theta}^*} \frac{\bar{\theta}^*}{\underline{\theta}^*} \frac{\theta}{\tilde{\theta}^*} d\mathbf{F}(\theta) \quad (55)$$

measures the liquidity premium of money.

Proof. See Appendix A3. ■

The decision rules for illiquid capital assets and labor supply are similar to the previous models. There are four possible cases for money-credit demand: (i) If $m_{t+1} > 0$, then $b_{t+1} = 0$; namely, a household has no incentive to take a loan if it has idle cash in hand. (ii) If $b_{t+1} > 0$, then $m_{t+1} = 0$; namely, a household will take a loan only if it runs out of cash. (iii) It is possible that a household has no cash in hand but does not want to borrow money from the bank because the interest rate is too high; namely, $m_{t+1} = b_{t+1} = 0$. (iv) Finally, the optimal demand for credit may exceed the credit limit and in this case, $b_{t+1} = \gamma M_t + \bar{b}$ and $m_{t+1} = 0$. Which of these situations prevails in each period depends on the realized preference shock θ_t . So there exist three cutoff values with $\underline{\theta}^* \leq \bar{\theta}^* \leq \tilde{\theta}^*$ and these cutoffs divide the domain of θ into four regions.

Hence, the consumption function is easy to interpret. If the urge to consume is low ($\theta < \underline{\theta}^*$), then case (i) prevails and $c = \frac{\theta}{\underline{\theta}^*} x < x$. If the urge to consume is high ($\theta > \bar{\theta}^*$), then case (ii) prevails and $c = \frac{\theta}{\bar{\theta}^*} x > x$. In between (if $\underline{\theta}^* \leq \theta \leq \bar{\theta}^*$), consumption simply equals cash in hand, $c = x$, so case (iii) prevails. Finally, if the urge to consume is too high ($\theta > \tilde{\theta}^*$), then the household opts to hit its credit limit with $c = x + \frac{\gamma M_t + \bar{b}}{P}$ (case iv). The household in cases (ii) and (iv) is able to consume more than cash in hand because of the possibility of borrowing.

In the money market, the aggregate supply of credit is $\int m_t d\mathbf{F} = M_t$ and the aggregate demand is $\int b_t d\mathbf{F} = B(\tilde{i}_t)$. Note that credit demand can never exceed supply because the loan rate $\tilde{i}_t$ will always rise to clear the market. The nominal loan rate cannot be negative because people have the option not to deposit. Hence, the credit market-clearing conditions are characterized by the following complementarity conditions:

$$(M_t - B_t) \tilde{i}_t = 0; \quad M_t \geq B_t \text{ and } \tilde{i}_t \geq 0. \quad (56)$$

That is, the nominal loan rate is bounded below by zero. The market-clearing condition $(M - B) \tilde{i} = 0$ determines the nominal interest rate of money. When the supply of funds exceeds credit demand

($M_t > B_t$), the equilibrium interest rate is zero, $\tilde{i} = 0$. Otherwise, $\tilde{i}$ is determined by the equation $M = B(\tilde{i})$. Notice that the bank does not accumulate reserves because all reserves are distributed back to bank members by the end of each period. The bank's balance sheet is given by

$$\underbrace{M_t}_{\text{deposit}} + \underbrace{(1 + \tilde{i}_t)B_t}_{\text{loan payment}} \implies \underbrace{(1 + i_t^d)M_t}_{\text{withdraw+interest}} + \underbrace{B_t}_{\text{loan}} + \underbrace{T_t}_{\text{profit income}}, \quad (57)$$

where the LHS is total inflow of funds in period t and the RHS is total outflow of funds in period t . That is, at the beginning of period $t - 1$ (more precisely, the second subperiod of $t - 1$), the bank accepts deposit M_t and makes new loans B_t , and at the end of period $t - 1$ it receives loan payment $(1 + \tilde{i}_t)B_t$, faces withdrawal of M_t , and makes interest payments to depositors. Any profits are distributed back to households in lump sums at the end of period $t - 1$ in the amount $T_t = (\tilde{i}_t - i_t^d)B_t$, which becomes household income in the beginning of the next period.

4.2 Welfare Costs of Inflation with Financial Intermediation

Aggregating the household decision rules gives the following relationships linking aggregate consumption, aggregate money demand, and aggregate credit demand, respectively, to total cash in hand (x) in the steady state:

$$C = D(\underline{\theta}^*, \bar{\theta}^*, \tilde{\theta}^*)x \quad (58)$$

$$\frac{M'}{P} = H(\underline{\theta}^*, \bar{\theta}^*, \tilde{\theta}^*)x \quad (59)$$

$$\frac{B'}{P} = G(\underline{\theta}^*, \bar{\theta}^*, \tilde{\theta}^*)x, \quad (60)$$

where $D \equiv \int_{\underline{\theta} < \underline{\theta}^*} \frac{\theta}{\underline{\theta}^*} d\mathbf{F} + \int_{\underline{\theta}^* < \underline{\theta} < \bar{\theta}^*} d\mathbf{F} + \int_{\bar{\theta}^* < \underline{\theta} < \tilde{\theta}^*} \frac{\theta}{\tilde{\theta}^*} d\mathbf{F} + \int_{\underline{\theta} > \tilde{\theta}^*} \left[1 + \frac{\gamma M + \bar{b}}{Px}\right] d\mathbf{F}$, $H \equiv \int_{\underline{\theta} < \underline{\theta}^*} \left(1 - \frac{\theta}{\underline{\theta}^*}\right) d\mathbf{F}$, and $G \equiv \int_{\bar{\theta}^* < \underline{\theta} < \tilde{\theta}^*} \left(\frac{\theta}{\tilde{\theta}^*} - 1\right) d\mathbf{F} + \int_{\underline{\theta} > \tilde{\theta}^*} \frac{\gamma M + \bar{b}}{Px} d\mathbf{F}$. Notice that $D + H - G = 1$.

The model is closed by adding a representative firm as in the previous section. Hence, the general equilibrium of the model can be solved in the same way as in the previous section. The model has a unique steady state in which the following relationships hold: $\frac{K}{Y} = \frac{\alpha\beta}{1-\beta(1-\delta)}$, $\frac{C}{Y} = 1 - \frac{\delta\alpha\beta}{1-\beta(1-\delta)}$, $W = (1 - \alpha) \left(\frac{\beta\alpha}{1-\beta(1-\delta)}\right)^{\frac{\alpha}{1-\alpha}}$, and $1 + r = \frac{1}{\beta}$. In addition, we also have $R(\underline{\theta}^*, \bar{\theta}^*, \tilde{\theta}^*) = \frac{1+\pi}{\beta(1+i^d)}$, $X = \underline{\theta}^*WR$, and $N = (1 - \alpha) \frac{Y}{X} \underline{\theta}^*R$. The welfare cost function looks similar to equation (39), except the values of $\{X, N\}$ are different and the term $J_\theta(\pi)$ is now given by

$$J_\theta(\pi) = \int_{\underline{\theta} < \underline{\theta}^*} \theta \log \frac{\theta}{\underline{\theta}^*} d\mathbf{F} + \int_{\bar{\theta}^* < \underline{\theta} < \tilde{\theta}^*} \theta \log \frac{\theta}{\tilde{\theta}^*} d\mathbf{F} + \int_{\underline{\theta} > \tilde{\theta}^*} \theta \log \left[1 + \frac{\gamma M + \bar{b}}{Px}\right] d\mathbf{F}. \quad (61)$$

Proposition 4 *Suppose the deposit rate i^d is bounded above by the lending rate $\tilde{i}$; if the total supply of funds exceeds the total credit demand ($M_t > B_t$) at inflation rate π for some finite values of $\{\gamma, \bar{b}\}$, then the welfare cost function with banking is identical to that without banking. In other words, financial intermediation does not improve welfare whenever the money-market interest rate is at the zero lower bound.*

Proof. See Appendix A4. ■

Since bank lending has an upper limit when $\{\gamma, \bar{b}\}$ are finite, and since the optimal probability of running out of cash is an increasing function of inflation, under low inflation rates the demand for credit can be too low to exhaust the supply of funds. In this case, Proposition 4 states that there is no welfare gain from financial intermediation. In other words, at sufficiently low inflation rates, self-insurance can achieve identical allocations (distributions) to those with financial intermediation.

To understand this result, imagine first that the credit limit is infinitely small. Then by continuity the welfare cost function in the banking model is identical to that in the benchmark model for all inflation rates. Also, since the supply of funds exceeds the demand, the nominal lending rate is zero for all inflation rates. Second, imagine that the credit limit is unbounded (infinity). Then the welfare cost functions in the two models shall remain identical (cross each other) at the Friedman-rule inflation rate because at this point the supply of funds exceeds credit demand and the nominal lending rate is zero. Therefore, for any finite credit limits the welfare cost functions in the two models must overlap at low inflation rates toward the Friedman rule as long as $M_t > B_t$ or $\tilde{i} = 0$. In this overlapping interval, financial intermediation does not improve welfare.

We calibrate the credit limit in two ways. First, we set $\gamma = 1, \bar{b} = 0.05x$, and the deposit rate equals half of the lending rate: $i^d = \frac{1}{2}\tilde{i}$. Second, we set $\gamma = 0, i^d = 0$, and $\bar{b} = 0.5x$. Namely, in the first calibration we set the credit limit to the household average deposits plus an allowance worth 5% of total demand for cash in hand, and set the deposit rate equal to half of the lending rate.³³ In the second calibration, we set $\gamma = 0$ but the credit limit $\bar{b}$ to 50% of total demand for cash in hand and set the deposit rate to zero. The purpose of the second calibration is to study the separate effects of $\bar{b}$ on welfare.³⁴ The rest of the parameters are identical to Method 1 (see Table 1). The welfare cost functions with and without banking are graphed in Figure 3.

The dotted-dashed line in Figure 3 shows the welfare cost of inflation in the previous benchmark model (as a percent of annual consumption). It forms an upper envelope on the other two cost functions. The dotted line represents the welfare cost function with banking under the first calibra-

³³In the real world the deposit rates are in general significantly lower than the lending rates, especially in developing countries. For example, in China the average interest rate on demand deposits is at most 20% of the lending rate. Telyukova (2009) assumes that $i^d = 0.3\tilde{i}$ in the U.S. Setting a relatively higher deposit rate implies a lower welfare cost of inflation in our model, so it goes against our welfare results.

³⁴Since the average consumption $C = Dx \leq x$, the second calibration for credit limits is quite generous, as it allows a household to borrow more than 50% of its average consumption in the money market.

tion for the credit limit and deposit rate, $\{\gamma = 1, i^d = 0.5\tilde{i}, \bar{b} = 0.05x\}$. The dashed line represents the welfare cost function with banking under the second calibration, $\{\gamma = i^d = 0, \bar{b} = x/2\}$.

Clearly, under either forms of credit limit, financial intermediation does not improve welfare at sufficiently low inflation rates despite that the cost of borrowing is zero ($\tilde{i} = 0$), confirming Proposition 4. The intuition is as follows. First, with relatively low inflation, households opt to hold a sufficient amount of real balances to buffer consumption shocks since the cost of holding cash is relatively small. Second, the demand for credit is low because of borrowing limits. Hence, households are forced to self-insure against idiosyncratic shocks at relatively low inflation rates. Third, since households take the possibility of borrowing into account when determining the optimal amount of cash in hand, they opt to reduce the amount of cash in hand one for one with the increased credit limit if the borrowing cost is zero. Consequently, the consumption level is not affected and remains the same across all states of nature if $\tilde{i} = 0$ (see the proof in Appendix A4); so financial intermediation has no effects on welfare whenever $M_t > B_t$.

However, things change dramatically when the inflation rates are high enough. With sufficiently high inflation, the supply of funds shrinks and the demand for credit rises to a point such that the loan market clears ($M_t = B_t$). In this case, self-insurance is no longer sufficient and outside credit becomes beneficial even though it is now costly to borrow ($\tilde{i} > 0$). So for inflation rates larger than a critical value, the welfare cost of inflation with financial intermediation becomes lower than that in the benchmark model.

In particular, the two cost functions under banking (the dotted line and the dashed line in Figure 3) become essentially linear (instead of hump-shaped) when π is high enough. Under the first credit-limit calibration with $\{\gamma = 1, i^d = 0.5\tilde{i}, \bar{b} = 0.05x\}$, the cost function becomes essentially flat for $\pi > 21\%$ per year. This means two things: First, the welfare cost of inflation with banking can be significantly reduced at relatively high inflation rates. Second, since the cost increases only slowly with inflation for $\pi > 21\%$, the maximum tolerable rate of inflation for households to stop holding money is now much higher than in the benchmark model. This second feature of the banking model arises because the nominal deposit rate increases with inflation whenever $\tilde{i} > 0$, which can significantly reduce the inflation tax and the adverse liquidity effect of inflation on money holdings. However, the welfare cost of inflation at moderate inflation rates ($\pi \leq 20\%$) remains just as high as that in the no-banking model (see Table 3).

On the other hand, under the second credit-limit calibration with $\{\gamma = i^d = 0, \bar{b} = x/2\}$, even though deposits (checking accounts) do not pay interest, the welfare cost function starts to deviate from the benchmark model at a much lower inflation rate (around $\pi = 2\%$). Beyond this point, the cost function increases almost linearly and reaches the same maximum cost of 14% at $\pi_{\max} = 52.576\%$. At the moderate inflation rate ($\pi = 10\%$), the welfare cost is 7.5% of consumption, more

than 2 percentage points lower than in the other cases (see Table 3).

Therefore, the welfare gains of credit and banking depend on the form of credit limits and the inflation rate. There is little gain at low inflation rates because agents can self-insure against consumption risk when the cost of holding money is low, regardless of the form of credit limits. For very high inflation rates near $\pi_{\max}$, the insurance value of money is low and the cost of borrowing ($\tilde{i}$) is high, so redistributing idle cash balances through financial intermediation may not significantly improve welfare (such as under the second credit-limit calibration), unless the nominal interest rate on demand deposits is close to the lending rate or closely indexed to inflation (such as under the first credit-limit calibration).³⁵

Because inflation reduces the incentives for holding money, it thus increases the costs of borrowing in the credit market. Consequently, the nominal interest of loans in the money market can increase with inflation more than one for one in our model. It is precisely the higher interest costs of loans that may make debtors worse off (instead of better off) in the face of positive inflation, offsetting the redistributive effects noted by Kehoe, Levine, and Woodford (1992). Consequently, inflation can never be optimal in our model despite financial intermediation.

5 Conclusion

Developed countries, such as the United States and other OECD countries, usually adopt a low inflation target of about 2% per year. But why such a low target? Both historical and cross-country evidence suggest that moderate inflation (about 10% to 20% per year) may be significant enough to cause widespread social and political unrest. Yet, the existing monetary literature suggests that the cost of inflation is small.

This paper provides a tractable general-equilibrium Bewley model of money demand to rationalize the practice of a low inflation target in developed economies and the observed positive empirical relationship between inflation and social unrest in developing countries. The model shows that inflation can be very costly—about 7% to 10% of consumption under 10% inflation (or 15% above the Friedman rule). The cost may be substantially reduced if households can engage in borrowing and lending through an established credit and banking system. However, with realistic credit limits the welfare cost still remains several times larger than estimated in the existing literature.

The primary reason for the significantly higher welfare costs is that inflation destroys the buffer-stock value of money, thus leading to increased volatility in household consumption. Such an inflation-induced increase in the idiosyncratic consumption volatility at the household level cannot be captured by the Bailey triangle or representative-agent models.

³⁵Most types of interest-bearing checking accounts in the U.S. pay very low interest compared with the lending rate (such as the interest rate on credit cards), and the deposit interest rate is often sluggish to reflect inflation changes.

Two simplifying strategies allow our general-equilibrium Bewley model to be analytically tractable despite the existence of capital, financial intermediation, and possible aggregate uncertainty. First, the idiosyncratic shocks come from preferences (as in Lucas, 1980) rather than from labor income. Second, and more importantly, the utility function is linear in leisure. These simplifying strategies make the expected marginal utility of an individual's consumption and the target monetary wealth independent of idiosyncratic shocks and individual histories. With these properties, closed-form decision rules for individuals' consumption and money demand can be derived explicitly, and exact aggregation becomes possible under the law of large numbers. The aggregate variables form a system of nonlinear dynamic stochastic equations as in a representative-agent RBC model and can thus be solved by standard methods.

These simplifying strategies come at some costs. First, the assumption of preference shocks as the sole source of idiosyncratic risk rules out any positive correlation between the distributions of consumption and money demand. However, Wen (2010) shows that this correlation problem can be overcome by assuming wealth shocks—this alternative approach preserves closed-form solutions and generates similar welfare costs of inflation. Another cost is that the elasticity of labor supply is not a free parameter. This imposes some limits on the model's ability to study labor supply behavior and labor market dynamics within this framework. Nonetheless, the payoff of the simplifying assumptions is obvious: They not only make the generalized Bewley model analytically tractable regardless of aggregate uncertainty and capital accumulation, but they also reduce the computational costs for a heterogeneous-agent model to the level of solving a representative-agent RBC model. Because of these advantages, the model may prove useful in applied work and serve as an alternative to the Baumol-Tobin model and the Lagos-Wright (2005) model for monetary policy analysis.

A final remark is that we do not intend to study optimal monetary policy in this paper. There exist many frictions that can justify positive inflation and such frictions are absent from our model. As a future project, we can consider optimal monetary policies along the lines of Khan, King, and Wolman (2003).

Table 1. Calibrated Parameter Values

	α	β	δ	σ	A	Implied $\sigma_{\log c}^2$
Method 1	0.42	0.95	0.1	2.65	1	0.055
Method 2	0.42	0.95	0.1	2.1~3.1	0.54~1.45	0.03~0.11
Method 3	0.42	0.95	0.1	2.0~2.5	0.48~0.86	0.07~0.13

Note: α denotes capital share, β time discount factor, δ capital depreciation, σ shape parameter in Pareto distribution, A the scaling factor in equation (42), and $\sigma_{\log c}^2$ the variance of log household consumption.

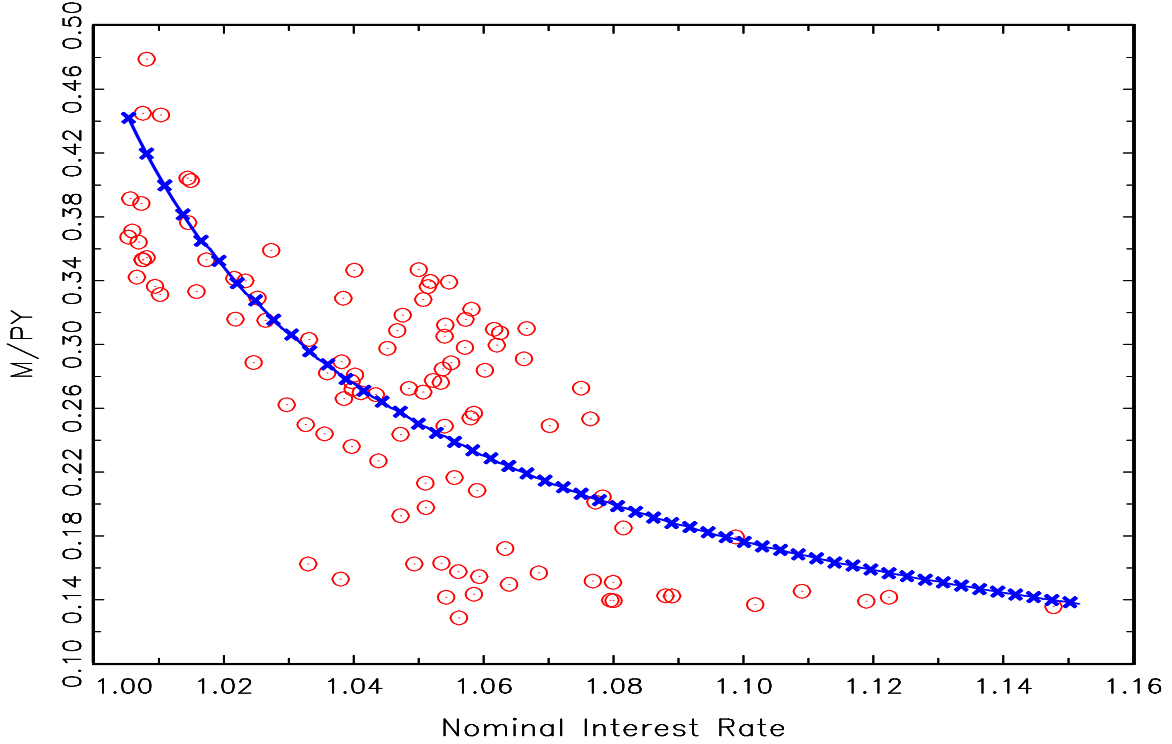
Table 2. Welfare Costs ($\Delta\%$ of Consumption)

	Inflation $\pi = 10\%$	Raising π from 2% to 3%
Calibration 1	9.6	0.5
Calibration 2	6.6 ~ 18	0.4 ~ 0.7
Calibration 3	11 ~ 21	0.6 ~ 0.8

Table 3. Welfare Costs with Banking ($\Delta\%$ of Consumption)

	Inflation $\pi = 10\%$	Raising π from 10% to 30%
Benchmark	9.6	3.5
Banking model 1	9.6	2.6
Banking model 2 ($i_t^d = 0$)	7.5	2.9

Note: Banking Model 1 features a credit limit of $\frac{M}{P} + \bar{b}$ and a deposit rate of $i_t^d = 0.5\tilde{i}_t$, Banking Model 2 features a credit limit of $\bar{b} = 0.5C$ and a zero deposit rate ($i_t^d = 0$).

Figure 1. Aggregate Money Demand Curve in the Model ($\times \times \times$) and Data (ooo).

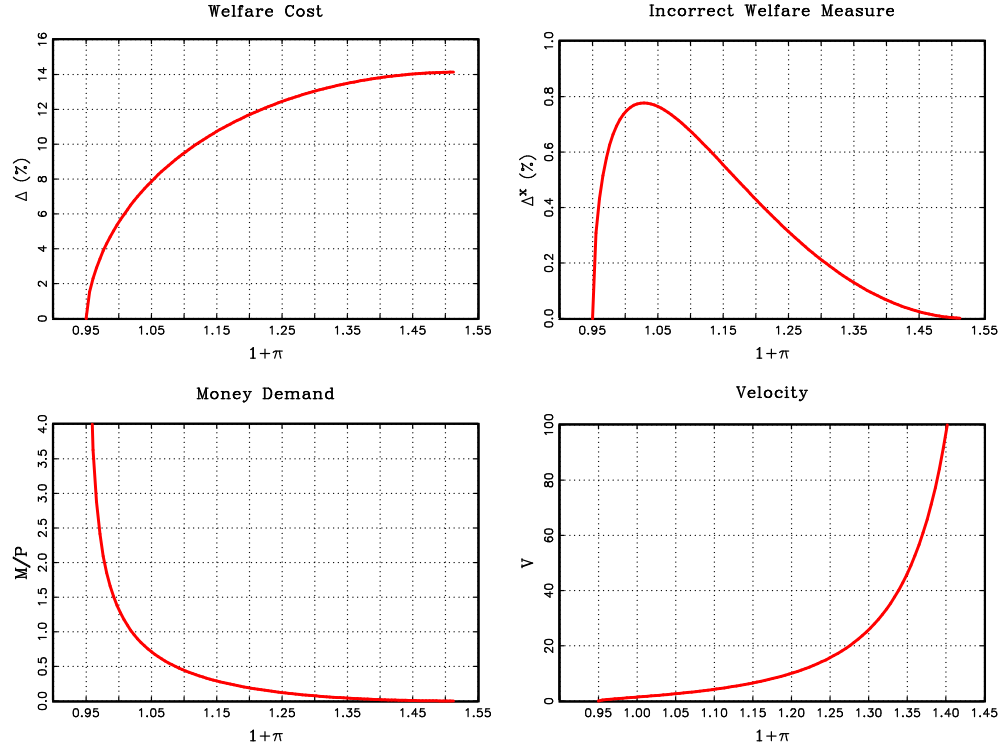


Figure 2. Welfare Costs, Money Demand, and Velocity under Calibration 1.

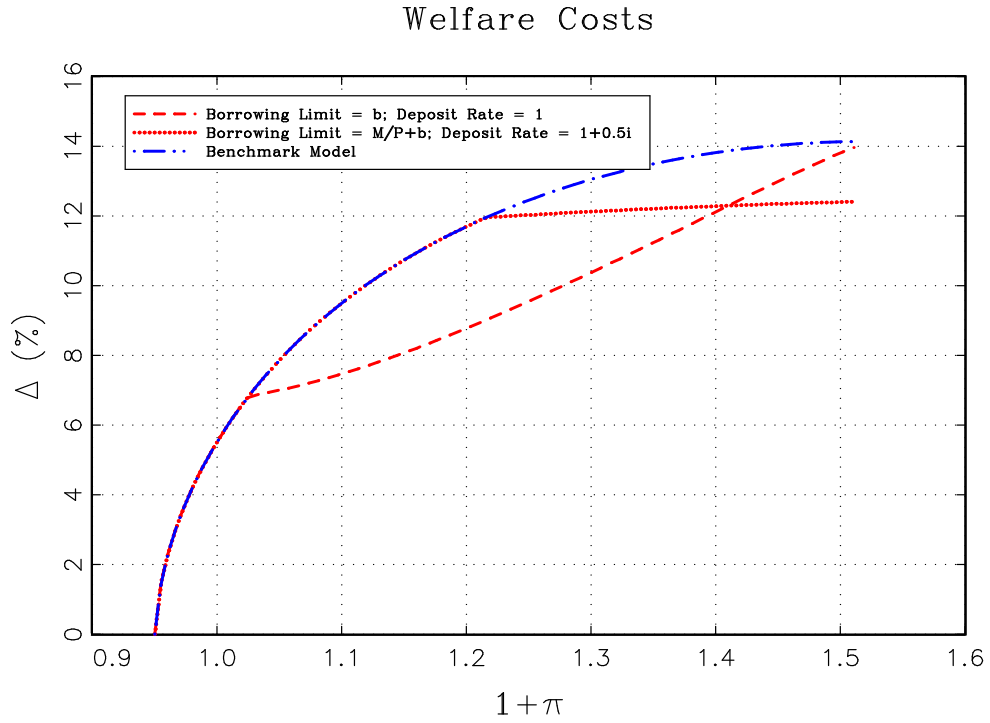


Figure 3. Welfare Implications of Banking.

Appendix

A1. Proof of Proposition 1

Proof. Denoting $\{\lambda_t, v_t\}$ as the Lagrangian multipliers for constraints (5) and (6), respectively, and assuming interior solution for n_t , the first-order conditions for $\{c_t, m_{t+1}, n_t\}$ are given, respectively, by

$$\frac{\theta_t}{c_t} = \lambda_t \quad (62)$$

$$\lambda_t = \beta E_t \left[\frac{\partial V_{t+1}}{\partial \tilde{m}_{t+1}} \frac{P_t}{P_{t+1}} \right] + P_t v_t \quad (63)$$

$$1 = \int \frac{\partial J_t}{\partial x_t} W_t d\mathbf{F}, \quad (64)$$

where $\tilde{m}_t \equiv \frac{m_t}{P_t}$ denotes real balances. The envelop theory implies

$$\frac{\partial J_t}{\partial x_t} = \lambda_t \quad (65)$$

$$\frac{\partial V_t}{\partial \tilde{m}_t} = \int \frac{\partial J_t}{\partial x_t} d\mathbf{F}. \quad (66)$$

Equation (64) reflects the assumption that decision for labor supply n_t must be made before the idiosyncratic preference shock θ_t (and hence the value of $J(x_t, \theta_t)$) are realized.

By the law of iterated expectations and the orthogonality assumption of aggregate and idiosyncratic shocks, equations (64) and (63) can be rewritten, respectively, as

$$\frac{1}{W_t} = \int \lambda_t d\mathbf{F} \quad (67)$$

$$\lambda_t = \beta E_t \frac{P_t}{P_{t+1} W_{t+1}} + v_t, \quad (68)$$

where $\frac{1}{W}$ pertains to the expected marginal utility of consumption in terms of labor income. The decision rules for consumption and money demand are characterized by a cutoff strategy, taking as given the aggregate environment. Denoting the cutoff by θ_t^* , we consider two possible cases below.

Case A. $\theta_t \leq \theta_t^*$. In this case, the urge to consume is low relative to a target. It is hence optimal to hold money as a store of value (to prevent possible liquidity constraints in the future). So $m_{t+1} \geq 0$, $v_t = 0$, and the shadow value of good (marginal utility of consumption) $\lambda_t = \beta E_t \frac{P_t}{W_{t+1} P_{t+1}}$. Thus,

$$c_t = \theta_t \left[\beta E_t \frac{P_t}{W_{t+1} P_{t+1}} \right]^{-1}.$$

Case B. $\theta_t(i) > \theta^*$. In this case, the urge to consume is high relative to a target. It is then optimal to spend all money in hand, so $v_t > 0$ and $m_{t+1} = 0$. By the resource constraint (5), we have $c_t = x_t$. Equation (62) then implies that the marginal utility of consumption is given by $\lambda_t = \frac{\theta_t}{x_t}$.

The above considerations imply

$$\lambda_t = \max \left\{ \beta E_t \frac{P_t}{W_{t+1} P_{t+1}}, \frac{\theta_t}{x_t} \right\}, \quad (69)$$

which determines the cutoff:

$$\beta E_t \frac{P_t}{W_{t+1} P_{t+1}} \equiv \frac{\theta_t^*}{x_t}. \quad (70)$$

Equation (67) then implies

$$\frac{1}{W_t} = \int \max \left\{ \left[\beta E_t \frac{P_t}{W_{t+1} P_{t+1}} \right], \frac{\theta_t}{x_t} \right\} d\mathbf{F}, \quad (71)$$

which implicitly determines the optimal cash in hand $x(H^t)$ as a function of aggregate state only (i.e., x_t is independent of individual history h^t). Using equation (70) to substitute cash in hand x_t , equation (71) implies the Euler equation (11) for money demand.

The above analyses and the first-order conditions imply the decision rules for consumption, money demand, cash in hand, and labor supply summarized in the equations in Proposition 1. Notice that labor supply, $n_t = \frac{1}{W_t} \left[x_t - \frac{m_t + \tau_t}{P_t} \right]$, may be negative if the existing real balances are too high. To ensure that we have an interior solution ($n > 0$), consider the worst possible case where money demand takes its maximum possible value, $\frac{m_t}{P_{t-1}} = \max \left\{ 0, \frac{\theta^* - \theta}{\theta^*} \right\} x = \frac{\theta^* - \theta_L}{\theta^*} x$. Suppose $M_{t+1} = M_t + \tau_t = (1 + \mu) M_t$ and $\frac{P_t}{P_{t-1}} = 1 + \pi = 1 + \mu$ in the steady state, then $\frac{\tau_t}{P_t} = \frac{\pi M_t}{P_t} = \frac{\pi}{1 + \pi} \frac{M_{t+1}}{P_t} = \frac{\pi}{1 + \pi} \int \frac{m_{t+1}}{P_t} d\mathbf{F} = \frac{\pi}{1 + \pi} H(\theta^*) x$ (by the decision rule), where $H(\theta^*) \equiv \int \max \left\{ 0, \frac{\theta^* - \theta}{\theta^*} \right\} d\mathbf{F}$. According to the definition of x in equation (3), $n > 0$ in the worst possible case (i.e., the minimum value of n is greater than 0) if $x_t - \frac{m_t + \tau_t}{P_t} = \left[1 - \frac{\theta^* - \theta_L}{\theta^*} - \frac{\pi}{1 + \pi} H(\theta^*) \right] x = \left[\frac{\theta_L}{\theta^*} - \frac{\pi}{1 + \pi} H(\theta^*) \right] x = \left[\frac{\theta_L}{\theta^*} - \frac{\pi}{\beta} \frac{H(\theta^*)}{R(\theta^*)} \right] x > 0$. This condition is clearly satisfied when $\pi = 0$. It is also satisfied in our model for $\pi \leq \pi_{\max}$ since the ratio $\frac{H(\theta^*)}{R(\theta^*)}$ is monotonically decreasing in π and approaches zero as $\pi \rightarrow \pi_{\max}$, whereas θ^* is decreasing in π but approaches θ_L , so the first term inside the bracket always exceeds the second term for any $\pi \in [\beta - 1, \pi_{\max}]$ under our parameter

calibrations for β . The intuition is simple: since consumption is bounded below by zero under log utility, under preference shocks cash holdings can never be too large to render hours worked negative. Thus, labor supply is always positive. It is also easy to see that n is bounded away from above by a sufficiently large constant $\bar{n}$ because cash holdings are bounded below by zero. ■

A2. Proof of Proposition 2

Proof. We provide only a briefly sketch of the proof here. Details can be found in Wen (2009, 2010). Define

$$\tilde{x}_t \equiv \frac{m_t + \tau_t}{P_t} + W_t n_t + (1 + r_t) s_t \quad (72)$$

as cash in hand, and denote $J_t(x_t, \theta_t)$ as the value function of the household based on the choice of c_t , m_{t+1} , and s_{t+1} . We then have

$$J_t(\tilde{x}_t, \theta_t) = \max_{c_t, m_{t+1}} \left\{ \theta_t \log c_t + \beta E_t V_{t+1} \left(\frac{m_{t+1}}{P_{t+1}}, s_{t+1} \right) \right\} \quad (73)$$

subject to

$$c_t + \frac{m_{t+1}}{P_t} + s_{t+1} \leq \tilde{x}_t \quad (74)$$

$$m_{t+1} \geq 0, \quad (75)$$

where $V_t(\frac{m_t}{P_t}, s_t)$ is the value function of the household based on the choices of n_t . That is,

$$V_t\left(\frac{m_t}{P_t}, s_t\right) = \max_{n_t} \left\{ -n_t + \int J_t(\tilde{x}_t, \theta_t) d\mathbf{F} \right\} \quad (76)$$

subject to (72) and $n_t \in [0, \bar{n}]$. Denoting $\{\lambda_t, v_t\}$ as the Lagrangian multipliers for constraints (74) and (75), respectively, and assuming interior solution for n_t , the first-order conditions for $\{c_t, m_{t+1}, s_{t+1}, n_t\}$ are given, respectively, by

$$\frac{\theta_t}{c_t} = \lambda_t \quad (77)$$

$$\lambda_t = \beta E_t \left[\frac{\partial V_{t+1}}{\partial \tilde{m}_{t+1}} \frac{P_t}{P_{t+1}} \right] + P_t v_t \quad (78)$$

$$\int \lambda_t d\mathbf{F} = \beta E_t \left[\frac{\partial V_{t+1}}{\partial s_{t+1}} \right] \quad (79)$$

$$1 = W_t \int \frac{\partial J_t}{\partial \tilde{x}_t} d\mathbf{F}, \quad (80)$$

where $\tilde{m}_t \equiv \frac{m_t}{P_t}$ denotes real balances and equation (79) is derived by differentiating both sides of equation (73) with respect to s_{t+1} . The envelop theory implies

$$\frac{\partial J_t}{\partial \tilde{x}_t} = \lambda_t \quad (81)$$

$$\frac{\partial V_t}{\partial \tilde{m}_t} = \int \frac{\partial J_t}{\partial \tilde{x}_t} d\mathbf{F} \quad (82)$$

$$\frac{\partial V_t}{\partial s_t} = (1 + r_t) \int \frac{\partial J_t}{\partial \tilde{x}_t} d\mathbf{F}. \quad (83)$$

Equation (79) reflects the assumption that decision for s_{t+1} is made before the idiosyncratic preference shock θ_t (and hence the value of λ_t) is realized. Equations (79)-(83) together imply equation (25). The first-order conditions for $\{c_t, \frac{m_{t+1}}{P_t}, n_t\}$ are identical to those in the benchmark model. After redefining cash in hand as $x_t = \tilde{x}_t - s_{t+1}$, the rest of the proof is similar to that in Appendix A1 for Proposition 1. ■

A3. Proof of Proposition 3

Proof. The proof is analogous to that in Appendix A1 or Appendix A2. The only important difference is to realize that we have three cutoffs here, $(\underline{\theta}^*, \bar{\theta}^*, \tilde{\theta}^*)$. Hence, we have four possible cases to consider as elaborated in the main text. Details can be found in Wen (2009, 2010). ■

A4. Proof of Proposition 4

Proof. It suffices to show that if $M_t > B_t$, then both the individual consumption level and the labor supply in the banking model are identical, respectively, to those in the benchmark model. First, notice that the real wage W in the banking model is identical to that in the benchmark model because the capital-to-output ratio is not affected by financial intermediation. Second, when $M_t > B_t$, equation (56) implies that $\tilde{i}_t = 0$. Hence, we also have $i_t^d = 0$. Equation (53) then implies $\bar{\theta} = \underline{\theta}$. Thus, equations (54) and (55) imply

$$\frac{1 + \pi}{\beta} = R(\tilde{\theta}^*) = \int_{\theta < \tilde{\theta}^*} d\mathbf{F}(\theta) + \int_{\theta > \tilde{\theta}^*} \frac{\theta}{\tilde{\theta}^*} d\mathbf{F}(\theta), \quad (84)$$

which is identical to equations (12) and (21) in the benchmark model. Hence, the liquidity premium function R is identical in the two models with $\tilde{\theta}^* = \theta^*$. In addition, equation (52) implies

$\left[1 + \frac{\gamma M_t + \bar{b}}{P_t x_t}\right] = \frac{\tilde{\theta}^*}{\tilde{\theta}}$, so the decision rules in equations (48) and (49) imply that household consumption in the banking model is given by

$$\begin{aligned} c_t &= \begin{cases} \theta WR & \text{if } \theta \leq \tilde{\theta}^* \\ \tilde{\theta}^* WR & \text{if } \theta > \tilde{\theta}^* \end{cases} \\ &= \min \left\{ 1, \frac{\theta}{\tilde{\theta}^*} \right\} \tilde{\theta}^* WR, \end{aligned} \quad (85)$$

which is identical to equation (8) in the benchmark model after substituting out x by equation (10). As a result, the aggregate consumption level C and aggregate output Y are also identical across the two models, respectively. Finally, labor supply in the banking model is given by $N = (1 - \alpha) \frac{Y}{W}$, which is identical to that in the benchmark model because the aggregate output level is. Therefore, the welfare cost function in the banking model is identical to that in the benchmark model whenever the credit market for loanable funds does not clear (or the lending rate is at the zero lower bound) because of excess liquidity on the supply side and borrowing constraints on the credit demand side. ■

Appendix B. Calibration Methods (Not for Publication)

Method 1. We choose the value of σ to minimize the distance between the model and the data using the following least square function,

$$\Gamma = \sum_{i=1}^T \left((x_i^m - x_i^d)^2 + (y_i^m - y_i^d)^2 \right) + 2 \left(\max \{x_i^m\} - \max \{x_i^d\} \right) + 2 \left(\max \{y_i^m\} - \max \{y_i^d\} \right), \quad (86)$$

where T denotes sample size, x_i^d denotes the i th observation of the nominal interest rate in the Lucas data, x_i^m its model counterpart, y_i^d the i th observation of the money demand in the Lucas data, and y_i^m its model counterpart. The last two terms in the above function serve to put more weight on the two end points of the demand curve.

Method 2. In addition to minimizing the moment condition in (86), we add an additional constraint that the model-implied likelihood of running out of cash is consistent with the household survey data. This alternative calibration method is to choose the parameters $\{\sigma, A\}$ so that the probability of running out of cash $1 - \mathbf{F}(\theta^*)$ in our model equals the proportion of liquidity-constrained population in the United States. According to the Survey of Consumer Finances (SCF), the portion of households having zero balances in checking accounts is 19.3% of the population based on surveys in the years between 1989 and 2007 (with standard deviation of 1.3%), the portion of

households having less than \$10 in checking accounts is 20% (with standard deviation of 1.4%), and that having less than \$20 is 20.6% (with standard deviation 1.3%).³⁶ Households with little balances in their checking accounts also tend to have very little balances in other types of accounts, such as saving accounts and money-market accounts. Hence, if we define a household with zero balances in checking accounts as those facing a binding liquidity constraint in our model, we can choose $\{\sigma, A\}$ such that (i) $1 - \mathbf{F}(\theta^*) = 0.2$ when the inflation rate is 4% per year and (ii) the value of Γ in equation (86) is minimized.³⁷ One problem with this approach is that the fraction of the population with zero balances at any moment may overstate the likelihood of running out of cash for a typical individual. These two statistics are identical only under the assumption of iid idiosyncratic shocks. However, based on the rule of thumb that over-time risk is roughly half of the cross-household dispersion in income and consumption, we can set the probability of running out of cash in the model to the interval (0.1, 0.2). To generate a 10% $\sim$ 20% probability of running out of cash in the model when the inflation rate is 4% a year (in addition to minimize Γ), it requires $\sigma \in (2.1, 3.1)$ and $A \in (0.54, 1.45)$, respectively.

Method 3. Similar to Method 2, we require the model to match the consumption risk or unexpected shocks to consumption demand. One measure of consumption risk is household expenditure volatility. It is arguable that aside from income uncertainty, a perhaps more important source of consumption volatility (especially in developing countries) is expenditure uncertainty, such as unexpected spending for housing, education, and health care, or unpredictable expenditures related to accidents, property damages, and volatile fluctuations in consumption goods prices. Such expenditure uncertainty is especially large and highly uninsurable in developing countries than developed countries because of the lack of insurance markets. An ideal proxy of spending risk would be the frequency of illness and the associated costs or accessibility of medical services, but such data are either unavailable or highly inadequate. Wen (2011b) reports that the risk related to car accidents in China is 24 to 35 times higher than in the U.S. Also, the risk of work-related injuries in China is two orders of magnitude higher than in the U.S. For example, the average annual incidence rate of fatal injuries in the U.S. mining industry is 0.026% (or 2.6 individuals per 1000 persons for the period 2005-09). The comparable incidence rate in China (for the period 1981-94) is about 15%. Alternatively, if the accident rate is measured by the number of fatal injuries (death) per millions of tons of coal output, the value is 0.02% in the U.S. and 4% in China.

³⁶ On the other hand, the portion of households with balances greater than \$3,000 in checking accounts is larger than 32% with standard deviation of 2.2%. See Wen (2010) for more details.

³⁷ A large body of empirical literature suggests that 19% of the U.S. population is liquidity constrained. For example, Hall and Mishkin (1982) use the Panel Study of Income Dynamics and find that 20% of American families are liquidity constrained. Mariger (1986) uses a life-cycle model to estimate this fraction to be 19.4%. Hubbard and Judd (1986) simulate a model with a constraint on net worth and find that about 19.0% of United States consumers are liquidity constrained. Jappelli (1990) uses information on individuals whose request for credit has been rejected by financial intermediaries and estimates through a Tobit model that 19.0% of families are liquidity constrained. Therefore, the emerging consensus points to a fraction of approximately 20% of the population to be liquidity constrained.

Since there do not exist long enough time-series panel data for household expenditures (SCF do not keep track of the same households for more than one year), an alternative way to gauge expenditure uncertainty is the Gini coefficient for households with similar income and living standards. The following table shows the Gini coefficients for consumption expenditure and health care expenditure across households in villages of developing countries. Based on the rule of thumb that over-time risk is roughly half of the cross-household dispersion in consumption expenditure,³⁸ we can infer from the table the approximate consumption spending risk faced by households in these countries.

Expenditure Inequality for Developing Countries							
Consumption Gini	Burkina Faso	Guate- mala	Kazakh- stan	Kyrgyz- stan	Para- guay	South Africa	Thailand
	0.43	0.39	0.37	0.45	0.47	0.54	0.39
Health care Gini	Burkina Faso	Guate- mala	Kazakh- stan	South Africa	Para- guay	Zambia	Thailand
	0.43	0.42	NA	0.67	0.18	0.32	0.38

The average consumption Gini across these developing countries is 0.43 and the average health-care Gini is 0.4; these values are both significantly larger than the consumption Gini (0.3) in the United States. So we calibrate the model to generate a consumption Gini of 0.15 for the U.S. and 0.2 for developing countries. This calibration yields the range of parameter values of σ in the last row in Table 1 under Method 3. The implied consumption volatility under these calibrated values of σ turn out consistent with the empirical estimates provided by Telyukova (2011, Table 9) for U.S. households. Hence, we believe that our calibration provides a reasonable benchmark value on the consumption risk in developing countries.

³⁸See Wen (2010, 2011b) for references.

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