

Liquidity Trap and Excessive Leverage

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February 17, 2013

Abstract

We investigate the role of debt market policies in mitigating liquidity traps driven by household leverage. When borrowers engage in deleveraging, the interest rate needs to fall to induce lenders to pick up the decline in aggregate demand. However, if the fall in the interest rate is limited by the zero lower bound, aggregate demand is insufficient and the economy enters a liquidity trap. In such an environment, households' borrowing and saving decisions are associated with aggregate demand externalities. The competitive equilibrium allocation is constrained inefficient. Welfare can be improved by ex-ante restrictions on leverage to mitigate prospective deleveraging. Ex-post policies to write down debt also generate positive demand externalities.

JEL Classification: E32, E44

Keywords: Leverage, liquidity trap, zero lower bound, aggregate demand externality, constrained efficiency.

Preliminary and incomplete. Please do not circulate.

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1 Introduction

Household leverage has been proposed as a key contributing factor to the recent recession and the slow recovery in the US. Several authors have documented the dramatic increase of leverage in the household sector before 2006 as well as the subsequent deleveraging episode. Using county-level data, Mian and Sufi (2010) have argued that household deleveraging is responsible for much of the job losses between 2007 and 2009. This view has recently been formalized in a number of theoretical models, e.g., Hall (2011), Eggertsson and Krugman (2012), and Guerrieri and Lorenzoni (2012). These models have emphasized that interest rates need to fall when borrowers engage in deleveraging to induce lenders to make up for the lost aggregate demand. However, *the zero lower bound (ZLB)* on nominal rates may prevent the real interest rate from declining sufficiently to clear the goods market, plunging the economy into a liquidity trap.

An important question concerns the optimal policy response to liquidity traps associated with household deleveraging. The US government and the Federal Reserve have responded to the recent recession by utilizing fiscal stimulus and unconventional monetary policies. These policies are (at least in part) supported by a growing theoretical literature that analyzes the policy implications of the liquidity trap. Curiously, even though the problems are thought to have originated in the debt market, theoretical analyses of the liquidity trap have typically not focused on policies for the debt market.¹ Could it be welfare improving to intervene during the earlier leveraging phase? Could it be welfare improving to write down some debt during a deleveraging episode?

To address these questions, we present a stylized model of a recession driven by household deleveraging and the ZLB constraint. The distinguishing feature of our model is that households *endogenously accumulate leverage*, even though they *anticipate* the upcoming deleveraging episode. Our first result is that, when households have a sufficiently strong rationale to incur leverage, there is a demand-driven recession even though it is fully anticipated. We do not pretend that most households in the US expected the deleveraging episode. However,

¹Several papers capture the liquidity trap in a representative household framework which leaves no room for debt market policies (see Eggertsson and Woodford (2003), Werning (2012), and Correia et al. (2012)). An exception is Eggertsson and Krugman (2011), which features debt but does not focus on debt market policies.

some households and, more importantly, regulators might have taken such a possibility into account, especially in 2006 and 2007.

Our main result is that it is desirable to use *preventive policies* to slow down the accumulation of household leverage in such instances. In the run-up to deleveraging episodes, households borrow too much from a social point of view. A simple debt market policy that restricts household leverage (coupled with appropriate ex-ante transfers) could make all households better off. Put differently, the competitive equilibrium is constrained inefficient and features excessive leverage. This result obtains whenever the deleveraging episode is severe enough to trigger a liquidity trap (assuming that the ZLB constraint cannot be fully circumvented ex post, say with unconventional monetary or fiscal policies).

The mechanism behind the inefficiency is an *aggregate demand externality* that applies whenever output is at least in part influenced by demand. In these situations, the decisions of economic agents that affect aggregate demand also affect aggregate output (and other agents' income). Individual agents naturally do not take into account these general equilibrium effects, which may lead to inefficiencies (see Farhi and Werning, 2012). In our economy, the liquidity trap ensures that output is demand-determined and below its efficient level. Greater ex-ante leverage leads to a greater reduction in aggregate demand and a deeper recession. Borrowers who choose their leverage (and lenders who finance them) do not take the negative demand externalities into account, leading to excessive leverage.

Our analysis also reveals that, during deleveraging episodes, *ex-post interventions* in debt markets might be associated with significant welfare gains. We identify a special case of household preferences (that feature strong substitutability between consumption and leisure) for which an *ex-post debt writedown* leads to a *Pareto improvement*. More generally, although a debt writedown typically reduces lenders' welfare, the negative effects are substantially mitigated by positive demand externalities. When the economy is in a liquidity trap, lenders' consumption is effectively bounded from above (as higher consumption would require a lower interest rate). In contrast, borrowers' consumption is not bounded and is responsive to their net wealth. Consequently, a debt writedown that transfers wealth from lenders to borrowers considerably increases aggregate demand and output, and in some cases, generates a Pareto improvement.

1.1 Related literature

Our paper is related to a long economic literature studying the zero lower bound on nominal interest rates and liquidity traps, starting with Hicks (1937) and Krugman (1998) in simple IS/LM-style frameworks to investigations in a New Keynesian framework by e.g. Eggertsson and Woodford (2003, 2004). A growing recent literature has investigated the optimal fiscal and monetary policy response to liquidity traps (see e.g. Eggertsson, 2009; Correia et al., 2011; Werning, 2012). Our contribution to this literature is that we focus on ex-ante policies.

A related strand of literature has shown that a shock to borrowing limits leads to declines in interest rates because it forces borrowers to delever, reducing their effective demand for credit (see e.g. Korinek et al., 2010; Guerrieri and Lorenzoni, 2012). If the nominal interest rate hits the zero-lower bound, this pushes the economy into a liquidity trap (see e.g. Eggertsson and Krugman, 2012). Whereas this literature focuses on the positive implications of episodes of deleveraging, we study the normative implications for optimal ex-ante regulation of borrowing and ex-post policies to mitigate liquidity traps.

The aggregate demand externality we identify in our paper is similar to the macro externality described in Farhi and Werning (2012ab). The broad idea is that, when output is influenced by aggregate demand, decentralized allocations are inefficient because agents do not internalize the impact of their actions on aggregate demand. In Farhi and Werning (2012ab), output responds to aggregate demand because prices are sticky and countries are in a currency union (and thus, under the same monetary policy).² They emphasize the inefficiencies in cross-country insurance arrangements. In our model, output is demand-determined because of the liquidity trap, and we emphasize the inefficiencies in household leverage in a closed economy setting.

Our results on excessive borrowing and risk-taking also resemble the recent literature on pecuniary externalities, including Caballero and Krishnamurthy (2003), Lorenzoni (2008), Bianchi and Mendoza (2010), Jeanne and Korinek (2010ab) and Korinek (2011). In those papers, agents do not internalize the impact of individual decisions on aggregate prices, and a planner can improve on outcomes by moving asset prices in a way that relaxes financial con-

²In related work, Schmitt-Grohe and Uribe (2012abc) identify a distinct externality that is driven by the downward rigidity of nominal wages.

straints. The aggregate demand externality of this paper works through a completely different channel – individual agents do not internalize that their private deleveraging reduces aggregate demand, and the interest rate cannot decline sufficiently to induce borrowers to make up for the lost demand and clear markets, creating an inefficient labor wedge. A planner internalizes that reducing leverage ex-ante supports aggregate demand during episodes of deleveraging and reduces the labor wedge.

The rest of this paper is structured as follows. The ensuing section introduces the key aspects of our environment. Section 3 characterizes an equilibrium that features an anticipated demand-driven recession. Section 4 establishes the constrained inefficiency of this equilibrium and presents our main results. Section 5 characterizes the constrained efficient allocations and Section 6 concludes. Most proofs are in the appendix.

2 Environment and equilibrium

This section describes the basic environment and defines the equilibrium. The environment has three key features. First, there is an economic motive for borrowing and leverage. Second, there is a possibility of a future tightening of borrowing constraints that would lead to deleveraging. Importantly, this possibility is anticipated by households. Third, there is a zero lower bound constraint on the nominal interest rate and inflation is sticky, which could trigger a demand driven recession. We analyze to what extent the anticipated recession restrains household leveraging and whether there is room for policy interventions in the debt market.

Financial markets and borrowing constraints

The economy is set in infinite discrete time $t \in \{0, 1, \dots\}$. The baseline model has no uncertainty. Let r_{t+1} denote the interest rate between dates t and $t+1$, and d_{t+1}^h denote households' outstanding debt (or savings if negative) for the next period. Households are subject to borrowing constraints at each date t . In particular, households' total outstanding debt at the next date $t+1$ cannot exceed a threshold: $d_{t+1}^h \leq \phi_t$. Here, $\phi_t \geq 0$ denotes an exogenous debt limit in the same spirit as Aiyagari (1994), or more recently, Guerrieri and Lorenzoni (2012) and

Eggertsson and Krugman (2012). The debt limit plays an important role in our analysis, so a brief discussion is in order.

We view the limit, ϕ_t , as determined by the borrowing capacity provided by households' durable goods, e.g., their houses or cars (which we don't explicitly model). Empirically, most household borrowing in the US (roughly 90% as calculated by Rampini and Viswanathan, 2012) is collateralized by durable goods.³ Moreover, a significant fraction of this borrowing is associated with households' non-durables consumption. For instance, Kermani (2012) illustrates that, out of the 5.7 trillion dollar increase in households mortgage borrowing between 2000 and 2006, only 2.4 trillion dollars were spent on residential investment. The remaining 3.3 trillion dollars were cashed out from home equity (see also Mian and Sufi, 2010).

We allow the limit, ϕ_t , to be time varying. A change in the value of durable goods would naturally generate such variation. Perhaps more importantly, a change in loan to value ratios for durable goods would also generate this variation. A recent theoretical literature, e.g., Geanakoplos (2010) and Simsek (2013), emphasizes that an increase in uncertainty can naturally reduce loan to value ratios, which would translate into a reduction of ϕ_t in our setting.

Our focus is on understanding the effect of an anticipated tightening of borrowing constraints, which we capture with the following assumption:

Assumption (A1). The debt limits satisfy $\phi_0 = \infty$ and $\phi_t \equiv \phi$ for each $t \geq 1$, where $\phi \in [0, \infty)$.

Date 0 can be thought of as an episode of high prices for durable goods and relatively high loan to value ratios. Date 1 (and onwards) can be thought of as an anticipated future state in which durable goods prices and loan to value ratios will be lower. To keep the analysis simple, we abstract away from uncertainty and assume that the borrowing constraints are tightened at date 1 for sure. The qualitative results we emphasize would be even stronger if the borrowing constraints are tightened with some probability less than 1.

³Theory also suggests that households' borrowing capacity is likely to be determined by their durable goods. Households' wage income is difficult to pledge to potential lenders either because of information frictions (as emphasized by Townsend, 1979) or because labor supply cannot be legally enforced (as emphasized by Hart and Moore, 1994). In contrast, durable goods are relatively easier to pledge because their value is less subject to information or incentive frictions.

The demand side and households' leverage decisions

There are two types of households, borrowers and lenders, denoted by $h \in \{b, l\}$. The types have the same measure, and they are identical except two respects:

Assumption (A2). The borrowers have a (weakly) smaller discount factor, $\beta^b \leq \beta^l$, and a (weakly) greater initial debt level, $d_0^b = -d_0^l \geq 0$.

This assumption enables us to capture two distinct and relevant scenarios. If $\beta^b < \beta^l$ and d_0^b is low, then leverage increases endogenously at date 0 and we analyze whether the leveraging process is efficient. If $\beta^b = \beta^l$ and d_0^b is high, the economy starts with some initial (previously accumulated) leverage and we analyze whether the deleveraging process is efficient (that is, whether the economy can manage a “smooth landing” to lower leverage). As we will see, the efficiency analysis is very similar across the two scenarios.

Households also supply labor endogenously. For the baseline model, we assume their state utility function takes the particular form, $u(\tilde{c}_t^h - v(n_t^h))$. In particular, we assume consumption and leisure are perfect substitutes and we define $c_t = \tilde{c}_t^h - v(n_t^h)$ as *net consumption*. This assumption enables us to abstract away from labor supply effects (which is not our focus). It also makes our welfare results particularly stark. That said, this assumption is not necessary for our main result and will be dropped in Section 5. Households' optimization problem can be written as:

$$\begin{aligned} \max_{\{\tilde{c}_t^h, d_{t+1}^h, n_t^h\}_t} \quad & U_0^h = \sum_{t=0}^{\infty} (\beta^h)^t u(c_t^h) \\ \text{s.t. } \quad & c_t^h = e_t - d_t^h + \frac{d_{t+1}^h}{1+r_{t+1}} \text{ for all } t, \\ \text{where } \quad & e_t = w_t n_t^h + T_t - v(n_t) \text{ and } d_{t+1}^h \leq \phi_t \text{ for each } t \geq 1. \end{aligned} \tag{1}$$

Here, T_t denotes lump-sum subsidies that are (symmetrically) distributed by the planner, and e_t denotes households' *net income*, that is, their income net of labor costs.

The supply side

There is a competitive final goods sector that can convert one unit of labor to one unit of the consumption good. Thus, the final good firms' optimization problem is given by:

$$\max_{n_t} y_t (1 - \tau_t) - w_t n_t, \text{ where } y_t = n_t. \quad (2)$$

Here, τ_t denotes a linear tax on output, or equivalently the labor wedge, chosen by the planner. We assume that the planner chooses the wedge to maximize households' net income, e_t , subject to equilibrium conditions. The planner will typically select $\tau_t = 0$ since there is no reason to distort output. However, when the zero lower bound (ZLB) constraint binds, the planner will be unable to set $\tau_t = 0$ as this would lead to excess demand in the goods market. As a second best, she will be induced to set a positive wedge, $\tau_t > 0$, which will restore equilibrium in the goods market by generating a recession.

In practice, the output adjustment in the short run is of course not mediated through changes in tax policies. This is a simple modeling device to capture a demand-driven recession caused by the ZLB constraint. A similar recession is also obtained in New-Keynesian models, e.g., Eggertsson and Woodford (2003), that introduce sticky prices or wages (or both). We chose not to adopt the New-Keynesian framework for two reasons.

First, abstracting away from price setting simplifies the model considerably and enables us to focus on the debt market and to characterize the efficiency properties of equilibrium. The downside is that we lose the amplification mechanisms related to the endogenous response of inflation (see, for instance, Eggertsson and Krugman (2011) or Werning (2012)).

Second, and more importantly, we view the demand-driven recession induced by the ZLB constraint as a phenomenon that is not necessarily tied to the New-Keynesian framework. Note that the ZLB constraint effectively restricts demand even in an economy in which *all relative prices are flexible* (and inflation is sticky for reasons that we will discuss below). Hence, when the ZLB constraint binds, output cannot be supply determined regardless of whether one adopts the New-Keynesian framework. In fact, an earlier literature, e.g., Solow and Stiglitz (1968) and Barro and Grossman (1971), modeled a shortage of demand as a disequilibrium phenomenon with rationing (see Hall, 2011, for a recent treatment). The common denominator across

different approaches is that, when there is a demand shortage driven by the ZLB constraint, output is influenced by demand. Our modeling choice with the labor wedge enables us to capture this common aspect in an otherwise standard equilibrium model.

The zero lower bound and the monetary policy

Let P_t denote the nominal price of the consumption good at date t . Households can also invest in government bonds, which are in zero net supply and which provide a nominal interest rate, i_{t+1} . From no-arbitrage, real and nominal interest rates are related according to the Fisher equation:

$$r_{t+1} = i_{t+1} \frac{P_t}{P_{t+1}} \text{ for each } t. \quad (3)$$

Nominal interest rates are set by the planner subject to *the ZLB constraint*:

$$i_{t+1} \geq 0. \quad (4)$$

This constraint captures an arbitrage restriction introduced by the availability of cash. In particular, if the constraint was violated, then households would hoard cash instead of holding bonds (and the bond market would not clear).

Apart from the ZLB constraint, the planner follows a standard Taylor rule of the form:

$$\begin{aligned} i_{t+1}(\tilde{\pi}_t) &= \max(0, r_{t+1}^n + \psi_\pi \tilde{\pi}_t) \text{ for each } t, \\ \text{where } 1 + r_{t+1}^n &= \min_{h \in \{b, l\}} \frac{u'(c_t^h)}{\beta^h u'(c_{t+1}^h)} \text{ and } \psi_\pi > 1. \end{aligned} \quad (5)$$

Here, r_{t+1}^n denotes the natural rate of interest defined as the minimum of the marginal rate of substitution across all households (since, in equilibrium, at least one type of household will be unconstrained). The expression, $\tilde{\pi}_t$, denotes the current inflation level. The inflation target is set to zero, and the monetary policy responds by a factor greater than 1 to deviations of inflation from target. These assumptions ensure that there will be zero inflation in equilibrium that is, $P_t = P_0$ for each t .

By specifying the monetary policy in this fashion, we are implicitly assuming that the plan-

ner cannot make policy commitments. It is well known that committing to a higher inflation target in the future could mitigate the ZLB problem (see, for example, Krugman, 1997 or Eggertsson and Woodford, 2003). It is also known that committing to create a future output boom could be helpful (see Werning, 2011). However, these policies might be difficult to implement in practice since they are ex-post inefficient. We assume that this type of commitment is not possible (or is imperfect) and focus on alternative debt market policies. We first define and analyze the Laissez-faire equilibrium.

Definition 1 (Equilibrium). *Given assumptions (A1) and (A2), the competitive equilibrium is a path of allocations, $\{(c_t^h, d_{t+1}^h, n_t^h), [y_t(\nu)]_\nu\}_t$, and prices, $\{P_t, [p_t(\nu)]_\nu, w_t, r_{t+1}, i_{t+1}\}_t$ such that households' allocation solves (1), the final good firms solve (2), the Fisher equation (3) is satisfied, the monetary policy follows the Taylor rule in (5), and the planner sets the labor wedge, τ_t , at each date to maximize the net income, e_t .*

3 An anticipated demand-driven recession

This section characterizes the equilibrium and establishes the possibility of a recession that is fully anticipated by households. The next section analyzes the efficiency properties of this equilibrium.

The labor demand by the final goods sector implies: $w_t = 1 - \tau_t$. Households' labor supply is determined by: $v'(n_t^h) = w_t$. Hence, the goods market equilibrium condition is given by:

$$1 - \tau_t = v'(n_t^h) \text{ for each } h. \quad (6)$$

In particular, both types supply the same level of labor, $n_t^h \equiv n_t$, that is determined by the labor wedge, τ_t . Whenever possible, the planner sets the labor wedge to zero, $\tau_t = 0$. By doing so, it provides all households a maximum level of net income:

$$e^* \equiv \max_n n - v(n). \quad (7)$$

The planner might be unable to set the efficient level of output in (7) because of the ZLB constraint. When this is the case, she is constrained to set a higher wedge, $\tau_t > 0$, and

implement a lower level of income, $e_t < e^*$, that will be endogenously determined.

We next turn to the debt market. As we have noted, the monetary policy rule without commitment implies inflation is zero, that is, $P_t = P_0$ for each t . In view of the Fisher equation (3), the ZLB constraint on the nominal rate also restricts the real interest rate to be positive $r_t \geq 0$. Put differently, the ZLB constraint effectively sets a lower bound on the real interest rate, which will be central to the analysis of the debt market.

We consider equilibria in which the borrowing constraints bind at all future dates, that is, $d_{t+1} = \phi$ for each $t \geq 1$. First consider dates $t \geq 2$ at which the economy is in a steady-state. At these dates, the interest rate is constant and given by $1/\beta^l - 1 > 0$. Hence, the ZLB constraint does not bind and households' net income is maximized, $e_t = e^*$. Their consumption is given by:

$$c_t^b = e^* - \phi \left(1 - \beta^l\right) \text{ and } c_t^l = e^* + \phi \left(1 - \beta^l\right).$$

Next consider date $t = 1$. At this date, borrowers' consumption is given by $c_1^b = e_1 - d_1 + \frac{\phi}{1+r_2}$. In particular, if borrowers' outstanding debt level is large, $d_1 > \phi$, they reduce their consumption relative to the steady state to meet the debt limit. The slack consumption demand is absorbed by lenders: $c_1^l = e_1 + d_1 - \frac{\phi}{1+r_2}$. The increase in lenders' consumption is also required to be consistent with their Euler equation:

$$\frac{u' \left(e_1 + d_1 - \frac{\phi}{1+r_2} \right)}{\beta^l u' \left(e^* + \phi \left(1 - \beta^l\right) \right)} = 1 + r_2.$$

In particular, the increase in lenders' consumption is mediated through a decrease in the interest rate, r_2 .

The key observation is that the ZLB constraint, $r_2 \geq 0$, effectively sets an upper bound on lenders' consumption. The upper bound, $\bar{c}_1^l$, is obtained as the solution to:

$$u' \left(\bar{c}_1^l \right) = \beta^l u' \left(e^* + \phi \left(1 - \beta^l\right) \right). \quad (8)$$

It follows that there are two possibilities. If the initial level of debt is not too large, i.e., $d_1 \leq \bar{c}_1^l + \phi - e^*$, then the required deleveraging is relatively small and the ZLB constraint does not bind. The output is at its efficient level, $e_1 = e^*$, lenders' consumption is below the upper

bound, $c_1^l < \bar{c}_1^l$, and the interest rate is positive $r_2 > 0$.

The second and the more interesting possibility obtains when the initial level of debt is relatively large:

$$d_1 > \bar{d}_1 = \bar{c}_1^l + \phi - e^*. \quad (9)$$

In this case, required deleveraging is large and the ZLB constraint binds. The interest rate is at its lower bound, $r_2 = 0$, and lenders' consumption is at its upper bound, $c_1^l = e_1 + d_1 - \phi = \bar{c}_1^l$. Rewriting this expression, output is below its efficient level and given by:

$$e_1 = \bar{c}_1^l + \phi - d_1 < e^*. \quad (10)$$

Intuitively, when the outstanding debt level is large (or the debt limit is low), borrowers are forced into a severe deleveraging episode which lowers their consumption demand. In view of the ZLB constraint, lenders' consumption demand is also bounded from above. Consequently, the deleveraging episode leads to a shortage of aggregate demand, which in turn triggers a demand-driven recession.

A natural question is whether the economy enters this recession even though it is anticipated by households. Our first result identifies two natural scenarios in which this is the case.

Proposition 1. *Consider the following two scenarios:*

(i) **Leveraging scenario:** *There is no initial debt, $d_0 = 0$, but borrowers have a sufficiently low discount factor: $\beta^b \leq \bar{\beta}^b < \beta^l$, where $\bar{\beta}^b > 0$ is a threshold characterized in the appendix.*

(ii) **Deleveraging scenario:** *Households have the same discount factor, $\beta^l = \beta^b$, but the initial level of debt is relatively large: $d_0 \in (\bar{d}_0, \tilde{d}_0)$, where $\bar{d}_0, \tilde{d}_0 > 0$ are two thresholds characterized in the appendix.*

In both scenarios, date 1 leverage satisfies $d_1 > \bar{d}_1$ [where $\bar{d}_1$ is characterized in (9)]. At date 1, the real interest rate is zero, net income is determined by Eq. (10), and there is a demand-driven recession [that is, $e_1 < e^$]. At date 0, the real interest rate is positive, net income is at its efficient level, and there is no recession [that is, $e_0 = e^*$].*

The first scenario concerns situations in which there is a rationale to accumulate leverage (e.g., because households have different discount factors). The role of this scenario is to

understand to what extent the impending recession restrains the endogenous accumulation of leverage. The second scenario concerns situations in which the economy starts with a high level of (previously accumulated) leverage so there is a need to delever. The role of this scenario is to understand whether the economy can manage a smooth landing, that is, a transition to low leverage without triggering a severe recession.

The common denominator across the two scenarios is that borrowers have a rationale to have outstanding debt at date 1. Proposition 1 establishes that, when this rationale is sufficiently strong, households endogenously choose a sufficiently high level of d_1 and the economy enters an *anticipated* recession.⁴ Moreover, the economy enjoys a quiet period at date 0 with a positive interest rate and the efficient level of output. A natural question is whether households' leverage choices in these quiet times is efficient despite the fact that they generate a recession in the future. We next turn to this question.

4 Excessive leverage and the inefficiency of equilibrium

A growing literature analyzes various policies that could improve welfare in environments with the ZLB constraint. As we have noted, we rule out monetary policies that require commitment. We also rule out fiscal and tax policies which are studied elsewhere (see Werning, 2012, and Correia et al., 2012). Instead, our focus is on policies related to the debt market. Given that the recession in our model is caused by leverage, the debt market policies are the natural candidates for effective government intervention. As we will see, they are also relatively easy to implement in the sense that they do not require commitment.

We first illustrate that the equilibrium in our model can be Pareto improved *even ex post*, that is, starting date 1. Although this result is somewhat special, it illustrates the source of the inefficiency in our setting. We then show our main result that the equilibrium is Pareto inefficient also *ex ante*, and that it can be improved by using simple debt market policies.

⁴Naturally, a combination of the two scenarios could also generate the recession at date 1.

Ex-post inefficiency and debt writedowns

The equilibrium in our baseline setting can be Pareto improved by a simple ex-post debt writedown. To see this, suppose borrowers forgive some of lenders' outstanding debt so that leverage is reduced from d_1 to the threshold, $\bar{d}_1$, given by Eq. (9). By our earlier analysis, the recession is avoided and the net income increases to its efficient level, e^* . Borrowers' welfare increases both because their outstanding debt is lower and because their net income is higher. Importantly, lenders' net consumption remains the same at the upper bound, $\bar{c}_1^l$, in (8). We thus obtain the following result.

Proposition 2 (Ex-post inefficiency). *Consider the equilibrium characterized in Proposition 1 starting at date 1 (in either scenario). Given the state preferences $u(c_t^h - v(n_t^h))$, reducing all borrowers' outstanding debt to $\bar{d}_1$ in Eq. (9) strictly increases borrowers' welfare without affecting lenders' welfare, generating a Pareto improvement.*

The intuition for the result comes from the endogenous determination of output in Eq. (10). When the ZLB constraint binds, output is below its efficient level and it is influenced by aggregate demand. A reduction in leverage, d_1 , increases consumption demand and brings output closer to its efficient level. Each lender, left to her own choice, would not write down debt because she would not take into account the beneficial effects on demand and output. We refer to this effect as an *aggregate demand externality*. In our setting, these externalities are sufficiently strong that, if the lenders could coordinate to write down debt, all households would be better off.

The demand externalities are strong in part because of the special form of lenders' state-preferences, $u(c_1^l - v(n_1))$. This form ensures that, when the ZLB constraint binds, lenders' net consumption, $c_1^l - v(n_1)$, is a purely forward looking variable that is independent of the current state of the economy [cf. Eq. (8)]. Consequently, writing down debt *endogenously* leads to sufficiently strong general equilibrium effects to leave lenders' utility unchanged.

While Proposition 2 is special, its logic applies more generally and suggests that ex-post debt writedowns during a liquidity trap could be associated with large welfare gains (if not a Pareto improvement). To see this, consider a version of the model with separable preferences, $u(c_1^l) - v(n_1)$, as in Eggertsson and Krugman (2012). In this case, the same logic would imply

that lenders' consumption, c_1^l , is a purely forward looking variable that is independent of the current state of the economy. Hence, a debt writedown would not affect lenders' consumption (and it would actually *raise their consumption* in versions of the model that feature deflation or the Fisher effect). Unlike Proposition 2, lenders' welfare in this case, $u(c_1^l) - v(n_1)$, might not increase because their labor costs, $v(n_1)$, also increase when the economy is stimulated. Nonetheless, the point remains that there are strong and positive externalities associated with writing down debt in a liquidity trap.

Ex-ante inefficiency and debt limits

We next turn to the main result of our paper which concerns households' leverage choices at date 0. Suppose a debt writedown at date 1 is not possible perhaps because of legal restrictions or concerns with moral hazard. An alternative way to reduce debt is to prevent it from accumulating in the first place. To capture this possibility, suppose households' date 0 leverage choices are subject to an additional constraint, $d_1^h \leq \phi_0^{pl}$, where ϕ_0^{pl} is an *endogenous* debt limit set by the planner. To trace the Pareto frontier, we also allow the planner to transfer wealth at date 0 between borrowers and lenders. This is equivalent to setting an initial debt level, d_0^{pl} , that is potentially different d_0 .

Proposition 3 (Ex-ante inefficiency). *Consider the equilibrium characterized in Proposition 1 (in either scenario). Then, the equilibrium is constrained Pareto inefficient. There exists an endogenous debt limit, ϕ_0^{pl} , and initial wealth transfer to borrowers, $d_0^{pl} < d_0$, that strictly increase borrowers' utility while leaving lenders indifferent.*

We provide a sketch proof for this result which is completed in the appendix. Suppose the planner sets the debt limit, ϕ_0^{pl} , equal to the threshold, $\bar{d}_1$, characterized in Eq. (9). By Proposition 2, this debt limit (weakly) increases all households' welfare starting at date 1. However, it also changes households' date 0 consumption: Lenders' net consumption is higher (since they save less) and borrowers' net consumption is lower (since they borrow less). An appropriately chosen initial wealth transfer, $d_0^{pl} < d_0$, ensures that households' date 0 net consumption is also unchanged. It follows that the combination of the debt limit and the initial wealth transfer generates a Pareto improvement.

Intuitively, households' date 0 leverage choices are also associated with aggregate demand externalities. Borrowers that choose their leverage (or equivalently, lenders that finance them) do not take into account the adverse effects on demand and output at date 1, which leads to excessive leverage.

5 Constrained efficient allocations

Propositions 2 and 3 established the constrained inefficiency of the Laissez-faire equilibrium allocations. However, they have not characterized the constrained efficient allocations, that is, those allocations that cannot be Pareto improved further. We next turn to this question. We also drop the specific form of the state preferences, $u(c - v(n))$, and consider general state preferences, $U(c, n)$, with the usual regularity properties: $U_c > 0, U_{cc} < 0$ and $U_n < 0$ (and Inada conditions).

The planner is subject to a number of constraints that correspond to the equilibrium requirements. First, we assume as before that the planner cannot commit to policies that are ex-post inefficient. Second, we also assume that the planner is subject to the same borrowing constraints as the competitive equilibrium at the constrained dates $t \geq 1$. In our model, both conditions are satisfied by requiring the planner to respect the (efficient) competitive equilibrium allocations starting date 2. In particular, we allow the planner to choose an initial debt level, $d_2 \in [-\phi, \phi]$ that is consistent with the constraint at date 1. We then require the planner to implement the corresponding competitive equilibrium allocations starting date 2, which are characterized by the following conditions:

For each $t \geq 2$:

$$\begin{aligned} y_t &\equiv y \text{ where } -U_n(y, y)/U_c(y, y) = 1, & (\text{Ex-post efficiency in the goods market}) \\ c_t^b &= e + d_2(1 - \beta^l) \text{ and } c_t^l = e - d_2(1 - \beta^l) & (\text{Constrained efficiency in the debt market}) \end{aligned} \tag{11}$$

Third, and most importantly, we also require the planner to be subject to a ZLB constraint at dates 0 and 1. As we have seen, the ZLB constraint in our framework effectively sets a lower bound on the real interest rate. Therefore, we write the planner's ZLB constraint as a lower bound on households' marginal rate of substitution between periods:

$$\beta^h U_c(c_{t+1}^h, n_{t+1}^h) \leq U_c(c_t^h, n_t^h) \text{ for each } t \in \{0, 1\} \text{ and } h. \quad (12)$$

Finally, the planner is also subject to resource constraints at dates 0 and 1. We allow the planner to set a potentially different labor wedge across the two types (unlike the analysis in the previous sections). This is a simplifying assumption which we could relax by imposing slightly more structure on preferences, $U(c, n)$. With this assumption, the planner's resource constraints are simply given by:

$$\sum_{h \in \{b, l\}} c_t^h \leq \sum_{h \in \{b, l\}} n_t^h \text{ for each } t \in \{0, 1\}. \quad (13)$$

We define $\tau_t^h = 1 + \frac{U_n(c_t, n_t)}{U_c(c_t, n_t)}$ as the implicit labor wedge set by the planner at date $t \in \{0, 1\}$ for type h households.

The planner's problem can then be written as:

$$\begin{aligned} & \max_{(c_t^h, n_t^h)_{h, t \in \{0, 1\}, d_2}} \sum_{t=0}^{\infty} (\beta^b)^t U(c_t^b, n_t^b) \\ & \text{subject to } \sum_{t=0}^{\infty} (\beta^l)^t U(c_t^l, n_t^l) \geq U^l \text{ and Eqs. (11) -- (13)}. \end{aligned} \quad (14)$$

Here, changing lenders' required level of utility, U^l , enables us to trace the constrained Pareto frontier. Our next result characterizes the properties of constrained efficient allocations for which the ZLB constraint binds at date 1.

Proposition 4. *Consider a solution to the planning problem in (14) (with a corresponding U^l) for which the ZLB constraint strictly binds only at date 1 and only for lenders.*

(i) *Households' date 0 and 1 consumption allocations satisfy:*

$$\frac{U_c(c_0^l, n_0^l)}{\beta^l U_c(c_1^l, n_1^l)} < \frac{U_c(c_0^b, n_0^b)}{\beta^b U_c(c_1^b, n_1^b)}. \quad (15)$$

(ii) *At date 0, the labor wedge is zero, $\tau_0^h = 0$ for each h . At date 1, the borrowers' labor wedge is zero, $\tau_1^b = 0$. However, lenders' labor wedge is weakly positive, $\tau_1^l \geq 0$, as long as preferences satisfy the regularity condition: $U_{cn}(c_t^l, n_t^l) \leq -U_{cc}(c_t^l, n_t^l)$ [and strictly positive if*

$$U_{cn}(c_t^l, n_t^l) < -U_{cc}(c_t^l, n_t^l)/.$$

The first part establishes that the optimal allocation strictly “distorts” households’ relative marginal rates of substitution relative to the usual Euler equation. Moreover, the distortion takes the form that borrowers consume relatively more at date 1 and relatively less at date 0. This is equivalent to imposing a debt limit at date 0 consistent with Proposition 3 in the previous section.

The second part of Proposition 4 characterizes the sign of the labor wedge at dates 0 and 1, which depends on whether the ZLB constraint is binding. Given that the constraint is not binding at date 0, there is no labor wedge at date 0. Similarly, there is no labor wedge for borrowers at date 1. However, under a regularity condition that labor and leisure are not too substitutable (which is satisfied for typical preferences),⁵ there is a positive labor wedge for lenders at date 1. Hence, constrained efficient allocations are *qualitatively* similar to the Laissez-faire equilibrium allocation characterized in Proposition 1, in the sense that quiet times at date 0 are followed by a recession at date 1. However, note from part (i) that the recession induced by the planner is milder relative to the Laissez-faire allocation (because it is associated with greater consumption demand by borrowers). Put differently, it is typically optimal for the planner to mitigate the recession but not to completely eliminate it.⁶

We provide two complementary intuitions for the first part of Proposition 4, which is the main result of this section. In a Laissez-faire equilibrium, households do not take into account the ZLB constraint (because this is a macroeconomic constraint). Consequently, their consumption allocations between dates 0 and 1 satisfy the usual Euler equation. In contrast, the planner takes into account that lenders’ consumption at date 1 is effectively bounded from above in view of the ZLB constraint. The planner also recognizes that borrowers’ consumption at date 1 is not subject to the same constraint. Consequently, the planner induces lenders to consume relatively less at date 1 and borrowers to consume relatively more, leading to the distortion in Eq. (15).

⁵For example, this condition is satisfied as strict inequality for the usual separable case, $u(c) - v(n)$.

⁶The difference between Propositions 3 and 4 stem from the fact that the former focuses on substitutable preferences, $u(c - v(n))$. With these preferences, the regularity condition is satisfied as equality and there is no labor wedge: $\tau_1^l = 0$.

A second intuition is provided by the aggregate demand externalities emphasized in the previous section. Given that lenders' consumption is bounded by the ZLB constraint, borrowers are in a unique position to increase aggregate consumption demand at date 1. Given the demand-driven recession (captured here by the positive labor wedge), an increase in aggregate demand would bring output closer to efficiency. Borrowers that allocate their consumption across dates 0 and 1 (and lenders that finance them) do not take the demand externalities into account. Consequently, borrowers consume too little at date 1 relative to the planner's allocation as captured in Eq. (15).

It is also useful to illustrate the relationship of our result to the inefficiency identified in Farhi and Werning (2012). They consider an open economy setting in which demand affects output because countries with separate labor markets are part of the same currency union (and thus, they share the same monetary policy). Their main result shows that the optimal allocation in each country is characterized by a distorted Euler equation for the representative household in each country. In a Laissez-faire equilibrium, the representative household chooses too little (resp. too much) consumption for states in which there is a positive (resp. negative) labor wedge. This result and its intuition can be directly mapped to our Eq. (15) by considering the behavior of *borrowers*, whose consumption is unconstrained at both dates 0 and 1. In contrast, the behavior of lenders in our setting is inefficient in *the opposite direction* of the representative household in Farhi and Werning (2012). In particular, since lenders' consumption is constrained at date 1, their Laissez-faire consumption demand is too high at the recession date 1 and too low at the quiet date 0.

6 Conclusion

[To be written]

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Appendix

Proof of Proposition 1.

At date 0, households' Euler equations are satisfied and can be written as:

$$1 + r_1 = \frac{u'(c_0^l)}{\beta^l u'(c_1^l)} = \frac{u'(c_0^b)}{\beta^b u'(c_1^b)}.$$

We conjecture an equilibrium in which a recession is triggered at date 1 but not date 0, that is: $e_1 < 0$ and $e_0 = e^*$. In this case, the previous equations can be written as:

$$1 + r_1 = \frac{u'\left(e^* + d_0 - \frac{d_1}{1+r_1}\right)}{\beta^l u'(\bar{c}_1^l)} = \frac{u'\left(e^* - \left(d_0 - \frac{d_1}{1+r_1}\right)\right)}{\beta^b u'(\bar{c}_1^l + 2(\phi - d_1))}. \quad (16)$$

Note that borrowers' date 1 consumption is determined by combining the expressions $c_1^b = e_1 - d_1 + \phi$ and $e_1 = \bar{c}_1^l + \phi - d_1$. Eqs. (16) represent two equations in two unknowns, $(1 + r_1, d_1)$. Under regularity conditions, there is a unique solution.

For the conjectured allocation to be an equilibrium, we also need the solution, d_1 , to satisfy:

$$e_1 = \bar{c}_1^l + \phi - d_1 < e^*.$$

We next show that this is the case for the two scenarios identified in the proposition.

First consider the leveraging scenario, i.e., set $d_0 = 0$. Note that d_1 is decreasing in borrowers' discount factor, β^b . Suppose, for a second, that the outstanding debt is at its threshold level:

$$\bar{d}_1 = \phi + \bar{c}_1^l - e^* > 0.$$

From the lenders' Euler equation, this would imply an interest rate, $\bar{r}_1 > 0$, determined by:

$$1 + \bar{r}_1 = \frac{u'\left(e^* - \frac{\phi + \bar{c}_1^l - e^*}{1 + \bar{r}_1}\right)}{\beta^l u'(\bar{c}_1^l)}.$$

Consider a discount factor for borrowers:

$$\bar{\beta}^b = \frac{1}{1 + \bar{r}_1} \frac{u' \left(e^* + \frac{\bar{c}_1^l - e^* + \phi}{1 + \bar{r}_1} \right)}{u' (2e^* - \bar{c}_1^l)}$$

such that the pair, $(\bar{d}_1, 1 + \bar{r}_1)$, would also satisfy borrowers' Euler equation. Note that $\bar{\beta}^b \in (0, \beta^l)$. It follows that for any $\beta^b < \bar{\beta}^b$, the outstanding debt level is above the threshold, $d_1 > \bar{d}_1$, and the economy enters the recession.

Second consider the deleveraging scenario, i.e., set $\beta^l = \beta^b$. In this case, note that d_1 is increasing in the outstanding debt level, d_0 . As in the previous case, suppose the outstanding debt is at its threshold level, $d_1 = \bar{d}_1$. Equating borrowers' and lenders' Euler equations then uniquely determine the wealth transfer at date 0 as:

$$d_0 - \frac{\bar{d}_1}{1 + r_1} = \bar{c}_1^l - e^* > 0.$$

Combining this with lenders' Euler equation pins down the interest rate as $1 + \bar{r}_1 = 1/\beta^l > 0$. Hence, the debt level will be at its threshold at date 1 if it is at the following level at date 0:

$$\bar{d}_0 = \bar{c}_1^l - e^* + \beta^l \bar{d}_1.$$

If $d_0 > \bar{d}_0$, then there is a demand-driven recession at date 1 but not at date 0.

In the second scenario, there is another threshold, $\tilde{d}_0$, above which there would also be a recession at both dates 0 and 1. To find this threshold, suppose instead the interest rate at date 1 is zero, that is: $\bar{r}_1 = 0$. Solving the Euler equations in this case gives a unique $\tilde{d}_0 > \bar{d}_0$ along with $\tilde{d}_1 > \bar{d}_1$. Hence, when $d_0 > \tilde{d}_0$, there is a recession at both dates. This completes the proof of the proposition.

Proof of Proposition 4.

Let $\lambda^b = 1$ and λ^l denote the Lagrange multiplier on lenders' minimum-utility constraint. With this notation, λ^h corresponds to the "Pareto weight" corresponding to type h agents. In addition, let $\left\{ \left(\gamma_t^{h,Z} \right)_h \geq 0 \text{ and } \gamma_t^R \geq 0 \right\}_{t \in \{0,1\}}$ denote the Lagrange multipliers corresponding

to the ZLB and the resource constraints.

The FOC with respect to c_t^h is:

$$\lambda^h \left(\beta^h \right)^t U_c \left(c_t^h, n_t^h \right) - \gamma_t^R + \gamma_t^{h,Z} U_{cc} \left(c_t^h, n_t^h \right) - \gamma_{t-1}^{h,Z} \beta^h U_{cc} \left(c_t^h, n_t^h \right) = 0 \text{ and} \quad (17)$$

for each $t \in \{0, 1\}$ and h . Given the assumptions that $\gamma_0^{h,Z} = 0$ for each h , $\gamma_1^{b,Z} = 0$, and $\gamma_1^{l,Z} > 0$, it follows that:

$$\begin{aligned} \lambda^h U_c \left(c_0^h, n_0^h \right) &= \gamma_0^R \text{ for each } h, \\ \lambda^b U_c \left(c_1^b, n_1^b \right) &= \gamma_1^R \text{ and } \lambda^l U_c \left(c_1^l, n_1^l \right) > \gamma_1^R. \end{aligned}$$

Combining these inequalities establishes the inequality in (15), proving the first part.

Next consider the FOC with respect to n_t^h :

$$\lambda^h \left(\beta^h \right)^t U_n \left(c_t^h, n_t^h \right) + \gamma_t^R + \gamma_t^{h,Z} U_{cn} \left(c_t^h, n_t^h \right) - \gamma_{t-1}^{h,Z} \beta^h U_{cn} \left(c_t^h, n_t^h \right) = 0. \quad (18)$$

Since $\gamma_0^{h,Z} = 0$ for each h , Eqs. (17) and (18) for a given h at $t = 0$ can be summed to obtain:

$$\lambda^h U_c \left(c_0^h, n_0^h \right) \tau_0^h = 0 \text{ for each } h.$$

This establishes $\tau_0^h = 0$ for each h . Similarly, since $\gamma_1^{l,Z} > 0$ and $\gamma_1^{b,Z} = 0$, the equations for a given h at $t = 1$ can be summed to obtain:

$$\begin{aligned} \lambda^l \beta^l U_c \left(c_1^l, n_1^l \right) \tau_1^l + \gamma_1^{l,Z} \left(U_{cc} \left(c_1^l, n_1^l \right) + U_{cn} \left(c_1^l, n_1^l \right) \right) &= 0, \\ \lambda^b \beta^b U_c \left(c_1^b, n_1^b \right) \tau_1^b &= 0. \end{aligned}$$

This establishes $\tau_1^b = 0$. Given the regularity condition $U_{cc} + U_{cn} \leq 0$, it also implies $\tau_1^l \geq 0$.

The inequality is strict if the regularity condition is strictly satisfied, proving the second part.