Time and space aggregation of the labor market flows

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PRELIMINARY AND INCOMPLETE

Abstract

I present new empirical evidence on the relationship between the job flows, worker flows, and the time horizon at which these flows are measured. In particular, I show that worker flows grow linearly with the horizon at which they are measured; job flows grow approximately with the square root of the horizon. Further, I show that these patterns hold for all firm size categories separately, and that the magnitude of the job and worker flows decreases with employer's size. To interpret these patterns, I explore simple models of a representative firm which faces employment adjustment costs. I show that such a model can replicate some of the observed patterns. I discuss the implications of presented facts for interpreting the differences in the characteristics of the labor markets in Europe and the U.S.

1 Introduction

I present new empirical evidence on the relationship between the job flows, worker flows, and the time horizon at which these flows are measured. I document the following facts:

1. Worker flows, namely the separation and hiring rates, grow linearly with the horizon at which they are measured.

2. Job flows, namely the job creation and destruction rates, grow less than linearly with the horizon at which they are measured. At horizons longer than 3 months, they grow approximately with the square root of the horizon. At horizons shorter than 3 months, the growth is close to linear.

3. The magnitude of the job and worker flows decreases with the employer size, and more so for the job than worker flows. As a consequence, the ratio of the worker to job flows increases with employer size.

4. Facts 1 and 2 hold for the each size category separately.

To establish these facts, I use the Austrian Social Security Dataset (ASSD). This is a panel dataset which covers the universe of the private sector employees in Austria for the period 1972-2007. The dataset contains the establishment code of the employed workers, which allows me to construct the time series for the establishment-level employment, hiring and separations.

I now discuss the implications of these facts. The key determinant of the Fact 1 and 2 is the nature of the productivity shocks faced by a firm, and the form of the adjustment costs. To illustrate this point, I analyze a simple model of a representative firm with no adjustment costs. I show that a model where shocks to firm's productivity growth are independent and drawn from a normal distribution with zero mean, delivers the result that the job flows rise with the square root of the horizon. This is consistent with the data at long but not at the short horizons. Adjustment costs on the labor size can partly remedy this drawback. These observations imly that Fact 1 and 2 can be used as an alternative way of estimating the productivity processes at the level of individual firms.

The literature has noticed a big difference in magnitudes of the job and worker flows, and several papers attempted to explain this difference, for example Kiyotaki and Lagos (2007) and Pries and Rogerson (2005). The leading explanation is that the labor market involves two types of reallocation - one from less to more productive firm, and one from bad to good worker-firm matches. The first type of reallocation primarily generates the job flows, while the second type of reallocation is linked to the worker flows (on top of the job flows). This literature also examines welfare costs of policies which interfere with one or the other reallocation mechanisms. Since the difference in magnitudes of these flows is the key for both model calibration and the policy evaluation, it is important at which frequency these flows are measured: the facts 1 and 2 imply that the ratio of the worker to job flows grows at $k/\sqrt{k} = \sqrt{k}$, where k is the horizon at which they are measured - one finds striking differences in magnitudes when looking at the annual data, but very small differences at the weekly frequency.

The cross-country empirical studies have shown that the labor markets in Europe and the U.S. are characterized by similar magnitudes of the job flows but very different magnitudes of the worker flows. Several papers attribute these differences to tighter labor market polices in Europe. However, Fact 3 suggests that aggregation over the size classes is part of the story. The average job creation rate is a weighted average of the job creation rates for different size categories, with weights given by the share of employees in each category. Thus, even if it is the case that the job creation rate in Europe is lower than in the U.S. for each size category, the weighted average can be the same. At the same time, since the worker– job flow ratio is higher for larger firms, the difference between Europe and the U.S. in this ratio can be partly due to aggregation over size classes.

Finally, understanding the nature of differences between the worker and job flows could provide a guidance for the calibration of the search models. The canonical search model considers one-to-one matches as the unit of analysis, and thus by construction has job flows being equal to the worker flows. A researcher then has to choose which of the two flows to target in the calibration. Since such a model can never be consistent with both flows simultaneously, it would be good to understand implications of choosing one over the other.

The rest of the paper is organized as follows. The next section shows the pronounced patterns in the data. To make sense of the presented evidence, I analyze a simple model of representative firm and discuss which predictions of the model are supported by the data.

2 Empirical evidence

2.1 Data description and construction of the main variables

I use Austrian Social Security Database (ASSD) for the empirical analysis. The ASSD records the labor market spells of individuals that contribute to the determination of eligibility and amount of the social security, sickness, health and accident benefits. These spells are then translated into the labor market status. For the analysis in this paper, the only relevant status is employment.

For each employment spell, I observe the begin and end date of the spell, the identification code of the establishments and annual earnings associated with this employer.

I use the establishment identification code to merge workers into establishments. For each establishment I construct the time series of employment, hires and separations using the begin and end date of workers' employment spell. I construct these time series at different frequencies, starting from monthly.

For each month, I define a reference day to be Wednesday of the third week of that month. I make this particular choice because I want to avoid measurement problems associated with the patterns for the hires and separations, see Appendix A. In particular, most hires occur in the first week of the month, and on Mondays. Similarly, most separations happen in the last week of the month, and on Fridays to Sundays. Thus, choosing Wednesday of the third month should take care of the hiring and separating regularities.

I say that a worker is employed in establishment e in period t if she works in the establishment e on the reference day of that period. This means that her employment spell started on or before and ended on or after the reference day. I consider a worker to be a hire in period t if her employment spell started between two reference days, within the period $(t-1, t]$. Separations are defined in the similar way: a worker is counted as a separation in period t if the end date of her spell falls into the interval $[t-1, t)$. If the flow of hires and separations are defined this way, it holds that

$$L_{e,t+1} = L_{e,t} + H_{e,t} - S_{e,t}$$

where $L_{e,t}$ is employment (stock), $H_{e,t}$ is hires (flow), $S_{e,t}$ is separations (flow) in establishment e at time t.

I follow Davis, Haltiwanger, and Schuh (1998) and define the job creation (JC) and destruction (JD) in establishment e at time t as

$$JC_{e,t} = \max\left[L_{e,t} - L_{e,t-1}, 0\right], \quad JD_{e,t} = \max\left[-\left(L_{e,t} - L_{e,t-1}\right), 0\right].$$

I measure the employer size by its average rather than the current size,

$$Z_{e,t} = \frac{1}{2}\left(L_{e,t} + L_{e,t-1}\right),$$

and use this measure to define the job creation, destruction, hiring and separation rates,

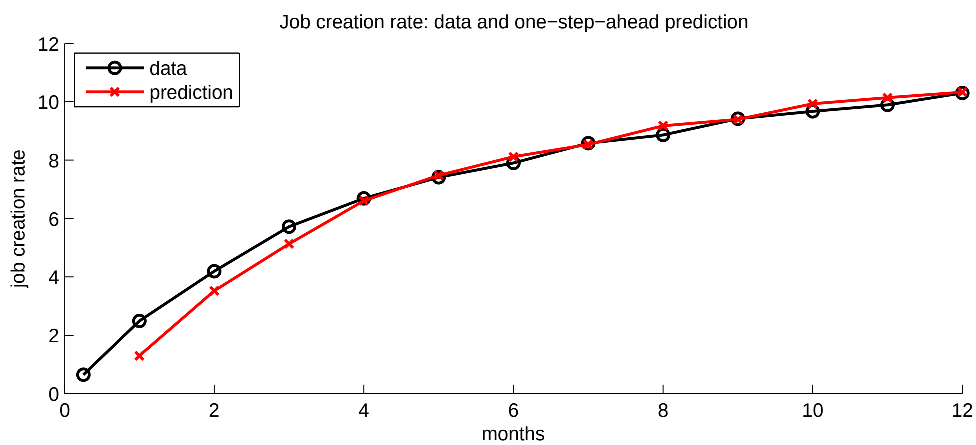
$$jc_{e,t} = \frac{JC_{e,t}}{Z_{e,t}}, jd_{e,t} = \frac{JD_{e,t}}{Z_{e,t}}, h_{e,t} = \frac{H_{e,t}}{Z_{e,t}}, s_{e,t} = \frac{S_{e,t}}{Z_{e,t}}.$$

When examine the job and worker flows for different size categories, I classify them based on $Z_{e,t}$. As discussed in Davis, Haltiwanger, and Schuh (1998), this measure is better than begin- or end-of-period employment because it addresses the problem of the regression bias. Such a bias arises due to the interaction of the fact that the establishments are reclassified each period based on their size, and the possibility of fluctuations in size due to the transitory shocks.

2.2 Empirical evidence

Figures 1, 2, 3 and Table 1 show the key patterns in the data.

Figure 1: Job creation rate - data and one-step-ahead linear prediction



Notes: The figure shows the average job creation rate in the data (black line) together with the one-step-ahead prediction using the equation (1).

The Figures 1 and 2 show the job creation rate and hiring rate measured at different horizons. The black lines depict the data, the red lines show the one-step-ahead predictions. In particular, to calculate the prediction for the job creation rate measured over $k+1$ periods, denote it by $\hat{jc}(k+1)$, I used equation (1)

$$\hat{jc}(k+1) = jc(k)\sqrt{\frac{k+1}{k}} \tag{1}$$

Figure 2: Hiring rate - data and one-step-ahead linear prediction

Notes: The figure shows the average hiring rate in the data (black line) together with the one-step-ahead prediction using the equation (2).

where jc is the measured job creation rate. I use a linear prediction for the hiring rate, equation (2)

$$\hat{h}(k+1) = h(k)\frac{k+1}{k}. \tag{2}$$

The figures illustrate the Facts 1 and 2. The patterns for the job destruction and separation rate are almost identical to the presented figures, thus I omit them.

The same pattern holds for different size categories, see Figure 3. Each line corresponds to one size category, and they are ranked: the largest job creation rate corresponds to smallest size category, the smallest job creation rate to the largest size category, see also Table 1.

Table 1 shows the quarterly labor market flows for different size categories. All flows - job creation, destruction, hiring and separation rates - are decreasing with the employers' size. However, the job flows decrease faster with the size, and thus the ratio of the worker and job flows increases with the size, as shown in column 5 of the Table 1. These observations are true at all frequencies.

2.3 Comparison to the US data

A comparison of these results to the U.S. is not straightforward because a similar dataset does not exist for the U.S. Nevertheless, using different datasets which measure job and worker flows at different frequencies exist, and these are presented in Table 2. The pattern is very similar to the shown in the previous section.

Table 1: Establishment size and employment shares, quarterly

employer size	jc	jd	h	s	$(h+s)/(jc+jd)$	emp. share
$1-5$	11.0	10.7	17.0	16.7	1.5	13.1
$6-20$	7.4	7.1	15.1	14.8	2.0	17.4
$21-50$	5.6	5.4	13.6	13.4	2.4	12.7
$51-100$	4.4	4.2	12.4	12.2	2.8	9.8
$101-501$	3.0	3.0	9.9	9.7	3.3	22.9
$501-1000$	1.9	1.8	7.9	7.8	4.1	8.1
>1000	1.4	1.2	6.4	6.1	4.8	16.1
all	4.9	4.7	11.7	11.5	2.4	–

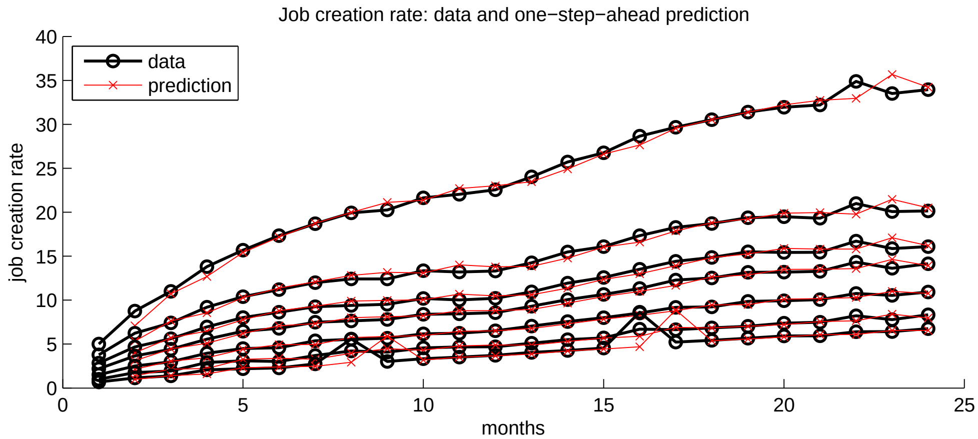
Notes: The table shows quarterly job creation, job destruction, hiring and separation rates, and employment shares in establishments of in different size categories. The reported employment shares are similar at all frequencies. The rates depend on the horizon, see Figures 1, 2 .

Table 2: Labor market flows at different frequencies for the U.S.

	annual	quarterly	monthly	weekly
job creation	9.1	3.4	1.5	–
job destruction	10.3	3.1	1.5	–
hiring rate	35-60	9.5	3.2	–
separation rate	35-60	9.2	3.1	–

Notes: The sources are Pries and Rogerson (2005) for the annual worker flows, and Davis, Faberman, and Haltiwanger (2006) for the rest.

Figure 3: Job creation rate for different size classes - data and one-step-ahead linear prediction



Notes: The figure shows the average job creation rate in the data (black line) together with the one-step-ahead prediction using the equation (1). The different lines correspond to different size classes, and they are sorted. That is, the largest job creation rate corresponds to smallest size category, the smallest job creation rate to the largest size category.

The relationship between the magnitude of the flows and the employer size is not new, and has already been shown in Chapter 4 of Davis, Haltiwanger, and Schuh (1998). The authors use data for the manufacturing sector in the U.S. over for the period 1973-1988 to compute the average job flow rates and show that they are decreasing in the size of the employer.

3 Model

To interpret this evidence and understand better its implications for the labor market dynamics, I start with simple models of the firm employment. I use these models to examine their implications for the magnitude of the job and worker flows at different horizons. I set up the models in discrete time.

3.1 Frictionless representative firm

Consider a representative firm which uses labor as the only input for the production. It takes wage rate w as given and decides how much labor L to hire to maximize its profit, $\pi = AL^{\alpha} - wL$. The first order condition implies that the optimal level of employment is given by

$$L^* = \left(\frac{\alpha A}{w}\right)^{1/(1-\alpha)}. \tag{3}$$

I will consider different types of the productivity processes to analyze its implications for the flows.

Assume first that the increments of the $\log A(t)$ iid and normally distributed with mean 0 and variance σ^2,

$$\log A(t+1) - \log A(t) \sim N\left(0, \sigma^2\right).$$

Equation (3) then implies that the increments of $\log L^*(t)$ are also iid and normally distributed with mean zero and standard deviation $\sigma_L \equiv \sigma/(1-\alpha)$,

$$\log L^*(t+1) - \log L^*(t) \sim N\left(0, \sigma_L^2\right).$$

What does this model imply for the job flows? The job creation rate[1] measured over the period of length k is given by

$$jc_t(k) = E\left[\max\left\{\log L(t+k) - \log L(t), 0\right\} | L(t)\right].$$

To simplify the notation, define a random variable $X^k = \log L(t+k) - \log L(t)$. Normality and independence of increments implies that given the employment at time t, X^k is normally distributed, $X^k \sim N(0, k\sigma_L^2)$. The job creation rate then is

$$\begin{aligned} jc_t(k) &= \Pr\left[X^k > 0\right] E\left[X^k | X^k > 0\right] \\ &= \frac{1}{2}\sqrt{k\sigma_L^2}\frac{\phi(0)}{1-\Phi(0)} \\ &= \sigma_L\sqrt{\frac{k}{2\pi}} \end{aligned}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are pdf and cdf of the standard normal distribution. To derive this result I utilized the formula for the mean of the truncated normal distribution.

This simple model implies that the job creation rate grows with the square root of the horizon, $\sqrt{k}$, which is consistent with the presented empirical evidence.

The worker flows in this model are not so straightforward to define because workers do not have identity which would enable us to determine who joined the firm as a new worker and who has left the firm. To proceed further, I will assume that within 1 period, a firm

[1] To simplify the calculation, I will define job creation rate through logarithms.

can only hire or only fire workers. This is not necessarily true in the data, but it is not completely off either - recall that most hires occur on Mondays while separations on Fridays to Sundays. With this definition at hand, I can define the hires between time t and $t+k$, $H_t(k)$, to be

$$\begin{aligned} H_t(k) &= \max\{0, L(t+1) - L(t)\} + \max\{0, L(t+2) - L(t+1)\} + \dots \\ &\quad + \max\{0, L(t+k) - L(t+k-1)\}. \end{aligned}$$

The expected hiring rate between t and $t+k$ is

$$h_t(k) = E\left[\frac{H_t(k)}{L(t)}\right] = k\frac{1}{L(t)}E[\max\{0, L(t+1) - L(t)\}],$$

where I used the fact that $E[\max\{0, L(t+1) - L(t)\}] = \max\{0, L(t+j) - L(t+j-1)\}$ for all j. Thus, the hiring rate is growing linearly with the horizon, the pattern observed in the data.

Now suppose that the increments of the $\log A(t)$ are iid but drawn from the normal distribution with mean μ and variance σ^2 . Then the distribution of X^k is $X^k \sim N(k\mu_L, k\sigma_L^2)$, where $\mu_L \equiv \mu/(1-\alpha)$ and $\sigma_L \equiv \sigma/(1-\alpha)$. The derivation is the same as before. The job creation rate is given by

$$\begin{aligned} jc_t(k) &= jc_t(k) = \Pr\left[X^k > 0\right] E\left[X^k | X^k > 0\right] \\ &= (1 - \Phi(a))\left(\mu_L k + \sqrt{k\sigma_L^2}\frac{\phi(a)}{1 - \Phi(a)}\right), \end{aligned}$$

where $a \equiv (0 - \mu_L k)/\sqrt{\sigma_L^2 k}$. The simplification yields,

$$jc_t(k) = k\mu_L \Phi\left(\frac{\mu_L}{\sigma_L}\sqrt{k}\right) + \sqrt{k}\sigma_L \phi\left(\frac{\mu_L}{\sigma_L}\sqrt{k}\right).$$

The job creation rate at the horizon k is a combination of two terms - one grows linearly with k, one grows with $\sqrt{k}$. For low k, the second term dominates and job creation grows approximately with $\sqrt{k}$. For high k, it is the linear terms which drives the job creation rate, which is inconsistent with the data.

What did we learn from this simple model? This model takes us quite far in explaining the relationship between the job and worker flows when measured at different horizons. The worker flows grow linearly, while the job flows grow with the square root of the horizon. The second prediction is consistent with the data at long, but not at the low horizons. In the data, it looks like the increments in the log-employment are correlated at short horizons, and thus in the next step, I explore the adjustment costs model to see if such a model would perform better.

The magnitude of the job flows depends on σ, the standard deviation of the process for the log-productivity. This could be an explanation for the differences in the magnitude of the job creation across size categories. If smaller employers face productivity shocks with larger variance, they would experience larger job creation rates. Another possible explanation for this phenomena, which I am currently exploring, is the importance of an integer constraint on employment. The smallest possible increase in employment is 1 worker, which constitutes a large percentage increase for a small employer, but a small percentage increase for a large employer. This is true even if the productivity processes for small and large employers are identical. The integer constraint works similarly as an adjustment cost model - a small firm cannot always react to instantaneous productivity changes, and needs to wait until there is a justification for increasing the employment by at least 1 worker. Therefore, I now turn to the analysis of an adjustment costs model.

3.2 Adjustment costs model

The frictionless model with independent increments in log-productivity got us quite far in explaining the relationship between the job flows and the horizon. However, the job flows at very high frequencies grow faster than with $\sqrt{k}$ which suggests that the employment changes at high frequency might be correlated, which can be achieved through adjustment costs.

A representative firm solves the dynamic problem

$$V\left(A, L_{-1}\right)=\max_{L \geq 0}\left\{A L^{\alpha}-w L-C\left(L, L_{-1}\right)+E_{A' \mid A}\left[V\left(A', L\right)\right]\right\}$$

where $C\left(L, L_{-1}\right)$ is the adjustment costs. The firm enters the period with employment stock from the previous period, observes the current productivity shock A and decides whether to adjust its employment. If it does, it has to pay the adjustment costs $C\left(L, L_{-1}\right)$. I consider two specific forms of the adjustment costs, proportional and fixed costs of adjustment. In case of proportional adjustment costs, function $C(\cdot, \cdot)$ is of a form

$$C\left(L, L_{-1}\right)=\tau \max \left\{L_{-1}-L, 0\right\}+\mu \max \left\{L-L_{-1}, 0\right\},$$

where τ and μ can be interpreted as firing and hiring taxes. In the model of fixed costs of adjustment, every time a firm wants to adjustment its employment, it has to pay costs proportional to the its employment,

$$C\left(L, L_{-1}\right) = \lambda L_{-1} \mathcal{I}_{L \neq L_{-1}}$$

where $\mathcal{I}_x$ is an indicator function for the event x. As before, the process for A is such that $\log A\left(t+1\right) - \log A\left(t\right) \sim N\left(0, \sigma^2\right)$.

Define variable $z_{-1} = L_{-1}/L^*$ and $z = L/L^*$, where L^* is the optimal employment from the frictionless model in the previous section. The variables z_{-1} and z express how far the employment is from the desired frictionless target. The solution of the model with the proportional adjustment costs is of a form

$$z = \begin{cases} \bar{z} & \text{if } z_{-1} > \bar{z} \\ z_{-1} & \text{if } z_{-1} \in \left[\underline{z}, \bar{z}\right] \\ \underline{z} & \text{if } z_{-1} < \underline{z} \end{cases}$$

for some thresholds $\underline{z}, \bar{z}$. Thus, there is an inaction region $\left[\underline{z}, \bar{z}\right]$ within which the firm does not change its employment. Outside the inaction region, the firm adjusts its employment to the $\underline{z}$ or $\bar{z}$. In the case of the fixed costs of adjustment, the solution is characterized by 2 thresholds $\underline{z}, \bar{z}$ and a target level z^* such that the firm does not adjust its employment as long as $z_{-1} \in \left[\underline{z}, \bar{z}\right]$, and it adjusts its employment to reach z^* if z_{-1} is outside the inaction region,

$$z = \begin{cases} z^* & \text{if } z_{-1} > \bar{z} \\ z_{-1} & \text{if } z_{-1} \in \left[\underline{z}, \bar{z}\right] \\ z^* & \text{if } z_{-1} < \underline{z} \end{cases}$$

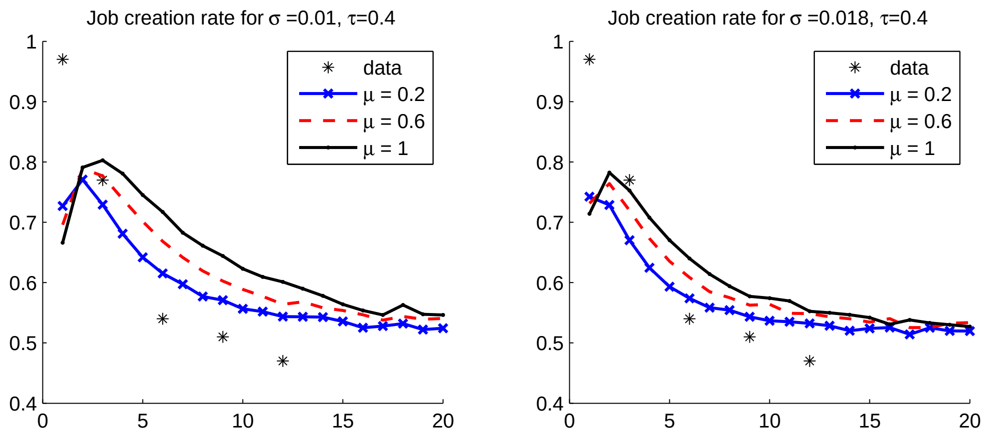
I cannot derive the predictions of this model for the job and worker flows at different horizons analytically. Therefore, I solve the model numerically, simulate time series for the firm-level employment and use it to calculate the job flows at different frequencies. I depict the predictions for the job flows in the following form. I define the θ_k as

$$\theta_k \equiv \lim_{s \to 0} \frac{\log\left(\frac{jc(k+s)}{jc(k)}\right)}{\log\left(\frac{k+s}{k}\right)}.$$

The θ_k measures the rate at which the job creation grows with the horizon. The value $\theta_k = 0.5$ means that the job creation grows with the $\sqrt{k}$, while the value $\theta_k = 1$ indicate a linear relationship.

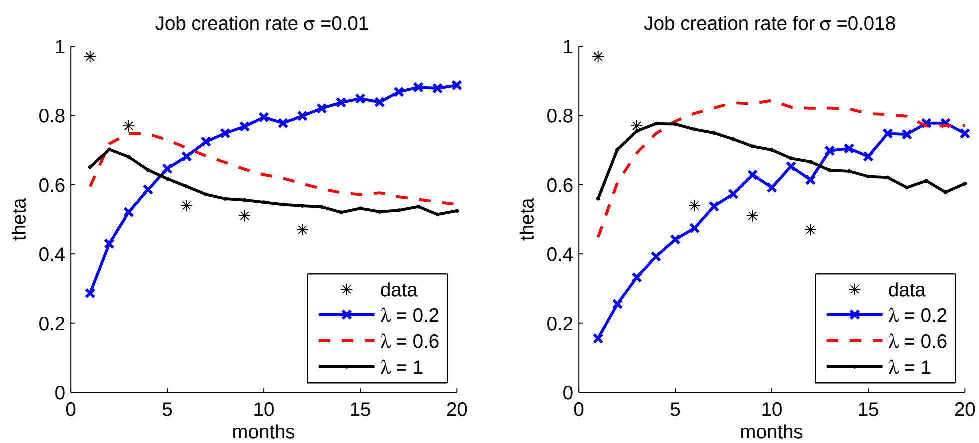
For the numerical calculation, I take 1 day as 1 period and set the discount factor $\beta = 0.9998$. I choose $\alpha = 0.7$ and normalize the wage, $w = 1$. I choose σ to match the magnitude of the monthly job creation rate at 2.5%. I explore the effect of other parameters, i.e. τ and μ in the proportional adjustment model, and λ in the fixed costs model - on the time pattern of the job flows. Figures 4 and 5 show the evolution of θ_k as a function of k for different parameter values.

Figure 4: Proportional adjustment costs model



Notes: The figure shows the implications of the proportional adjustment costs model for the growth of the job creation rate as a function of the horizon, θ_k. The different lines correspond to different parameter values.

Figure 5: Fixed adjustment costs model



Notes: The figure shows the implications of the fixed adjustment costs model for the growth of the job creation rate as a function of the horizon, θ_k. The different lines correspond to different parameter values.

Both models share some common features. First, they predict that the value of θ_k is close to 0.5 at longer horizons, which is consistent with the data. For low values of k, the value of θ_k is greater than 0.5, but the values observed in the data are higher than what either of the models can generate. Generally, the θ_k as a function of k is hump-shaped. This is particularly the case in the fixed adjustment costs model where, if the costs are large enough, the θ_k can even be increasing over the depicted period. This observation can discipline how high the costs can be since in both models, an increase in the costs leads to a more pronounced hump-shaped pattern.

What are the predictions for the worker flows? I assume that a firm cannot hire and fire people on the same day. Thus, I define hires as the positive employment change and separations as a negative employment change within one period. Both models imply that the hiring and separation rates increase linearly with the horizon, but the level of the hiring rate for the chosen parameter values is way too high: 17% in the proportional adjustment costs, and 20% in the fixed costs model at monthly frequency, as compared to 4% in the data.

This analysis has shown that an adjustment costs model can to a large extent explain the relationship between the job creation rate and the horizon at which they are measured.

4 Conclusion

I present new empirical evidence on the relationship between the job flows, worker flows, and the time horizon at which these flows are measured. Some of these patterns can be explained by an adjustment costs model.

The presented model does not explain the pattern for employers of the different size, which is something I am currently working on.

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A Timing of hires and separations

Figures 6 and 7 illustrate the timing of the hiring and separation process. The hires mostly happen on Mondays and during the first week of a month. Separations typically occur at the end of week (Fridays and Sundays), and the last week of the month.

Figure 6: Timing of hires and separations - days of the week

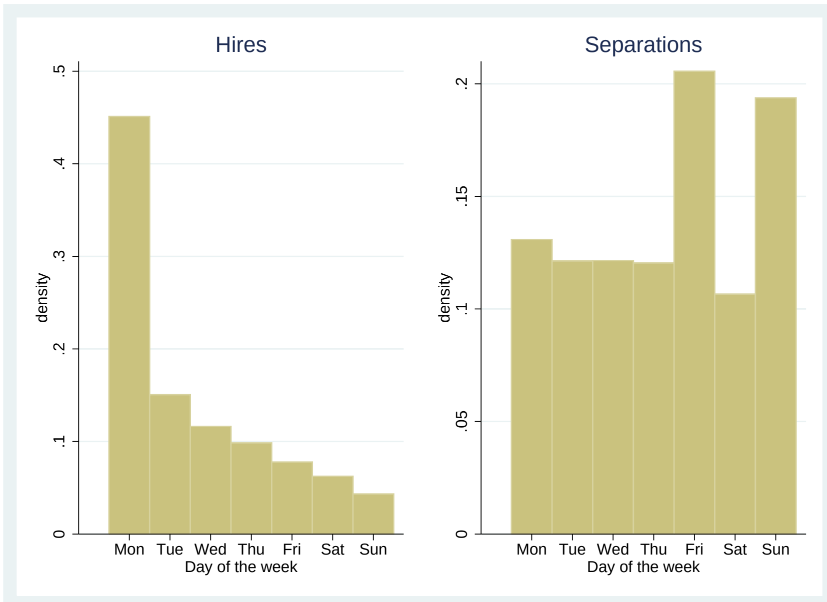

Figure 7: Timing hires and separations - week of the month