Sectoral Shocks, Reallocation and Unemployment in a Model of Competitive Labor Markets.

Maximiliano Dvorkin
Yale University

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Abstract

TO BE COMPLETED

1 Introduction

> *A leading question -perhaps* the *leading question- in macroeconomics since the publication in 1982 of David Lilien's paper, "Sectoral Shifts and Cyclical Unemployment," is whether sectoral, rather than aggregate, shocks are the key factor responsible for fluctuations in the unemployment rate.*
>
> *Janet Yellen (1989)*

A common characteristic of economic recessions is an increase in the reallocation of labor across sectors. If workers' skills are specific to some activities and not easily transferable, this reallocation can be protracted, leading to higher unemployment and longer duration of unemployment spells. A classic debate in macroeconomics, started by the works of Lilien (1982) and Abraham and Katz (1986), is whether reallocation and unemployment are the result of sectoral or aggregate shocks. The recent economic recession sparked a renewed interest for this topic as academics and policymakers want to disentangle whether the high rates of unemployment and long term duration are the result of an insufficient aggregate demand or because job seekers are not qualified for the type of jobs that are available.

While there is a large literature in macro and labor economics centered around the sectoral versus aggregate shocks debate in the labor market, a fully structural approach to this question is rare. In this work I contribute to fill this gap by developing a structural model with segmented labor markets in the spirit of Lucas and Prescott (1974) where both aggregate and sectoral shocks influence labor demand and wages in each sector. Mobility and reallocation frictions prevent rapid adjustments to these shocks generating unemployment.

This paper makes three contributions. First, on the technical side, I adapt and extend numerical techniques to solve a Lucas and Prescott (1974) type of model with a sufficiently large number of islands and aggregate uncertainty. The main difficulty in these models is that the whole distribution of wages and economic conditions over islands is part of the state variables in the worker's decision problem, and with aggregate uncertainty this distribution is not time invariant. Second, I analyze the cyclical properties of island models. Despite the original paper was published almost 40 years ago, little is known about the business cycle properties of this model. Last, I use the model to quantify the importance of sectoral shocks in generating cyclical fluctuations in the labor market and in particular, whether the dynamics of the labor market in the Great Recession are a result of a adverse sectoral shocks or a weak aggregate demand.

One advantage of having a structural model is to conduct controlled experiments. The main experiment I will conduct here is whether the evolution of unemployment and sectoral employment during the great recession can be rationalized by some specific shocks: an aggregate shock, a shock to an specific sector. One possible candidate is a shock to construction. This is clear after observing the high contribution of construction to total employment destruction. However, this sector do not have strong links to the rest of the economy. It accounts for less than 2% of total intermediate use. Rather its production is devoted almost entirely to final demand. Reinhart and Rogoff (2009) document that recessions that are accompanied by a financial crisis, like the Great Recession, tend to last longer and trigger stronger movements in employment and unemployment. This points into the direction of looking at a shock to the financial sector. On the other hand, the output of the financial sector is extensively used by other sectors, representing roughly 20% of total intermediate and 20% final demand. The route taken here to analyze the severity of a financial crisis is different from a large part of the literature since I focus on input-output linkages as the main amplification and propagation mechanism.

1.1 Related Literature

In this work I follow a large literature started by Lucas and Prescott (1974) on unemployment under competitive labor markets better known as island models. More recently, Alvarez and Shimer (2009, 2011) introduce differential costs to switching islands, allowing workers to remain attached to their current island, yet not working.[1] In this paper I allow for a similar mechanism. This is consistent with empirical definitions of islands more frequently used in the literature given that not all unemployed switch to a different activity.

My contribution to the literature of island models and the largest departure from it is to introduce aggregate uncertainty. Although the first island model was published almost 40 years ago, little is known about the business cycle properties of this model. Veracierto (2008) is one exception. By assuming complete markets and fully random search, he is able to write down the problem of the social planner. This reduces to a stochastic nonlinear control problem which he solves using a "linear-quadratic" approximation. Judd (1996) discusses the limitations and potential problems that may arise in using linear-quadratic approximations. Here I use perturbation methods which are not subject to those problems. In addition, in Veracierto (2008), like in all traditional island models, labor market flows and duration are not defined. The reason is that, in equilibrium, a positive measure of workers are indifferent between being employed or not. This implies that we can interchange the employment status of many agents without altering equilibrium. However, by doing that, key labor market variables like UE, EU and duration are not uniquely pinned down as they depend of individual labor histories. I show that my model can uniquely pin down labor market histories for workers from micro-foundations.

In my model I have a finite number of islands which I associate to economic sectors. Islands are connected not only by workers flows across them, but also by input-output linkages. It has long been recognized that input-output linkages can propagate sectoral shocks to the rest of the economy, producing aggregate effects ((Horvath, 2000; Acemoglu et al., 2012)). In a recent article, Foerster et al. (2011) argue that for proper identification of the shocks, it is necessary to take into account these linkages, otherwise sectoral shocks will mistakenly be identified as aggregate shocks. Foerster et al. (2011) find that sectoral shocks can explain close to 50% of the fluctuations in the industrial production index. The Industrial Production Index includes durable and non-durable manufacturing, construction, mining and logging, which is a small part of agriculture. These industries account for less than 15% of total employment and less than 20% of Value Added. At the sectoral level, economic accounts with data on sectoral value added are only available yearly.[2]. Here I use labor

[1]See also Hamilton (1988); Gouge and King (1997) for earlier definitions of a similar type of unemployment

[2]The BEA has recently developed economic accounts for sectors at quarterly frequency but the series start in 2007

market data, which is available at the monthly frequency, covering the whole economy and at the sectoral level. This allows me to identify the role of shocks originating in sectors not included in the Industrial Production Index and therefore is an extension of Foerster et al. (2011).

Carrillo-Tudela and Visschers (2013) have a model of mobility and reallocation and my paper has a similar flavor to theirs. However, there are important differences. First they focus on occupational mobility. Second, they specify a model of the labor market with frictions to move across islands and search and matching within island. While they do have an aggregate shock, there are no sectoral shocks and even their occupational shock is purely idiosyncratic and drawn from a time invariant distribution.[3] The only shock correlated across workers is the aggregate one. This prevents them to address the discussion of whether sectoral (or occupational) shocks are a main driving force triggering reallocation and unemployment. In addition, they assume constant returns to scale both in the production function and the matching function together with a free entry condition, enabling them to use the block recursive equilibrium specified in Menzio and Shi (2011). In this way the model becomes very tractable as the distribution of workers over islands is not part of the state variables in the decision problem of the workers and firms. While this specification is certainly convenient, it departs substantially from traditional island models where congestion and a downward sloping labor demand are an essential part of the model and reallocation. To put simply, in their model, islands with "too many" workers do not suffer from lower wages or lower matching probability. Whether this matters or not is a quantitative question. However there are reasons to believe that this feature may be relevant in the real world as part of the policy and academic debate is whether "too many" construction workers are depressing conditions in that sector.

In this paper I follow a large literature on dynamic discrete choice models. Recent applications to segmented labor markets include Lee and Wolpin (2006), Kline (2008), Artuc et al. (2010), yet these papers focus on mobility across labor markets without unemplevment. Mangum (2010) is the first application (that I am aware of) that specifies a discrete choice model with unemployment and search and matching frictions as in Diamond-Mortensen-Pissarides and uses it to study labor reallocation across states. I deviate from his work in two important ways. First, I remain close to traditional island models and assume no search and matching frictions within a labor market. Second, Mangum (2010) solves his model using the Oblivious Equilibrium concept, which deviates from rational expectations. In the model I develop here, agents form rational expectations about future conditions which includes the distribution of wages.

[3] To put simply, there are no islands in their model. At least no in the Lucas and Prescott (1974) sense. Alternatively you can think of each worker being part of her own individual island, with no connection to the rest except through the economy-wide aggregate shock.

Two recent papers, developed simultaneously and independently, are related to the one I present here. The first is Pilossoph (2012). She develops a two island dynamic discrete choice model of unemployment with frictions across and within islands similar to Mangum (2010). She calibrates the model to construction and the rest of the economy and use it to answer whether sectoral shocks in construction can explain much of the dynamics of unemployment in the great recession. My work departs from her in a number of ways. First I do not include matching frictions. Second I am able to solve and calibrate a model with 14 sectors and input-output linkages. This is important in order to identify which sector contribute more to fluctuations or if it is indeed the result of a common aggregate force. Finally, in her work wages are fully fixed at their steady state value and do not change due to any type of shock, while in my work wages fluctuate changing the incentives to work or relocate.

The second is the paper by Wiczer (2013), where he analyzes whether occupation specific human capital affect the duration of unemployment in a model with search frictions both across and within labor markets. His model is closely related to Carrillo-Tudela and Visschers (2013) with the exception that search over occupations is semi-directed, in stead of fully random, and he has occupational shocks affecting all workers in that occupation. None-theless, the distribution of workers across occupations is not part of the state variables of the agents since constant returns to scale both in production and matching functions together with free entry in terms of vacancy posting kill any congestion effects, a feature that is an essential part of traditional island models and that is missing here as well. While this reduce the computational burden as the distribution of agents over islands is not part of state variables of workers, it is subject to the same critique I mentioned before. I depart from his work by abstracting from matching frictions within labor markets and having congestion effects and a downward sloping demand within markets (which requires agents in my model to keep track of the distribution of workers over markets). In addition, the focus of my paper is on quantifying the importance of sectoral shocks vis a vis aggregate shocks.[4]

2 An Island Model with Aggregate Shocks.

In this section I describe the main assumptions and structure of the model. Time is discrete and there is a *finite* number of islands in the economy which I will interpret as a labor market

[4] Wiczer (2013) also specifies a dynamic discrete choice model, but departs from the standard setup by using a taste shock with an exponential distribution in stead of the typical Type I Extreme Value used in this literature. While he has good reasons for this departure, it is worth noting that exponential shocks are bounded from below by zero, a fact that seems to be overlooked in the proof of his main proposition and which cast doubts on the validity of the results.

and will use these terms interchangeably.[5]

It is well known that input-output linkages can propagate sectoral shocks to the rest of the economy and have aggregate effects (Horvath, 2000; Acemoglu et al., 2012). For the goal of this paper, it is therefore necessary to incorporate input-output relations to the model. As has become standard in the literature, I will assume a Cobb-Douglas production function with intermediate inputs. I will denote y_i the output from island i that is used in the production of the final good (final demand or value added), and x_i to total production. φ_{ij} is the amount of good produced in island i that is used as intermediate in island j.

I assume that the representative firm in island i operates a technology that transforms labor and intermediate inputs into units of an island-specific good:

$$x_i = e^{\lambda_i z + \theta_i} L_i^{\alpha_i} \prod_{j=1}^{J} \varphi_{ji}^{(1-\alpha_i)\phi_{ij}}; \qquad \text{where } \sum_{j=1}^{J} \phi_{ij} = 1 \tag{1}$$

where α_i is the share of labor (value added) and $\phi_{ij} \geq 0$ is the share of good j in intermediate expenditures. L_i is the amount of labor hired by the firm, z is an aggregate productivity shock and θ_i is an island specific productivity shock. Both z and θ_i are distributed AR(1) independent of each other. λ_i is a parameter that captures the sensitivity of the island to the aggregate shock. All firms in an island share the same productivity.[6]

Firms and households are price takers in the labor market. Production has constant returns to scale and there are no fixed costs nor any entry or exit barrier, therefore the total number of firms is not pinned-down and it suffices to analyze the "representative firm". The optimal conditional demands for labor and intermediates are:

$$L_i = \alpha_i A_i w_i^{\alpha_i} \prod_{j=1}^{J} p_j^{(1-\alpha_i)\phi_{ij}} \frac{x_i}{w_i} \tag{2}$$

$$\varphi_{ij} = (1-\alpha_i)\phi_{ij} A_i w_i^{\alpha_i} \prod_{j=1}^{J} p_j^{(1-\alpha_i)\phi_{ij}} \frac{x_i}{p_j} \tag{3}$$

where $A_i = \left[e^{\lambda_i z + \theta_i} \alpha_i^{\alpha_i} (1-\alpha_i)^{(1-\alpha_i)} \prod_{j=1}^{J} \phi_{ij}^{(1-\alpha_i)\phi_{ij}}\right]^{-1}$, w_i is the real wage paid in the is-

[5] In the empirical application of the model I will assume that an island is an industry. This assumption is in line with a large literature on island models. None-the-less, it is clear that other dimensions like occupation and geography may be relevant. See the discussion in Section XX.

[6] I specify both types of shocks as affecting productivity as this has been the traditional approach to island models. It is clear that affect the demand for products, either aggregate or sectoral, would also have a similar impact. Therefore, these shocks should not be interpreted literally as supply or productivity shocks, but rather as capturing, in reduced form, external forces affecting behavior in the model.

land and p_j is the price of island j good relative to the price of the final good. The minimum cost to produce an unit of good i is

$$c_i = A_i w_i^{\alpha_i} \prod_{j=1}^{J} p_j^{(1-\alpha_i)\phi_{ij}}$$

There is a final good producer that aggregates island intermediate goods using a CES production function:

$$y = \kappa \left(\sum_i \psi_i y_i^{\frac{\chi-1}{\chi}} \right)^{\frac{\chi}{\chi-1}}$$

where $\chi > 0$ is the elasticity of substitution over island goods, κ is a constant scale parameter which will be calibrated to make average wages approximately equal to one in equilibrium and ψ_i is the weight of good i in the final demand. The demand for good i is downward sloping and so is labor demand. The price of the final good, which is the good consumed by the household, is normalized to 1.

There is a unit measure of households or workers. Workers start the period attached to a labor market and can work in that market if they choose to. Transitioning to a different labor market is costly and partially irreversible. Therefore, the worker's problem is dynamic. These costs prevent perfect arbitrage implying that wages and lifetime utility will not be fully equalized across islands. For simplicity I will assume no savings. Markets are incomplete and households cannot insure against shocks.

Workers can be in one of two employment states: working in island i or searching for a job. If working they obtain labor income which they consume, and if unemployed they get some consumption in terms of home production or a consumption equivalent value of leisure. In addition to aggregate and island specific shocks, households face idiosyncratic shocks that affect their return to work *vis a vis* their value of leisure or home production and their preferences for an island.

Workers are heterogeneous in 3 dimensions. The first is the island they start the period in. This is the standard dimension of heterogeneity in islands models. The second is a worker-island specific productivity τ, which represents the number of efficiency units or the quality of labor services that the worker has in island i if working. Kambourov and Manovskii (2009) have a similar type of worker heterogeneity. They assume that τ is an experience premium. Here τ will have a similar interpretation. However, I will assume that τ follows a regular Markov chain, which is iid across workers, persistent but not perfect.

In this case fluctuations in τ represent changes in workers' return to market work in the island.[7] The last dimension of heterogeneity is a purely transitory preference shock to the work and search decision, which is iid across households and time. This shock captures, in reduced form, short lived variations in the disutility of working and home production opportunities.

While in principle the first two sources of heterogeneity would be sufficient for the model to reproduce many dynamic patterns observed in the real world, as will become evident later, the introduction of a purely transitory shock is convenient from a computational point of view.

The timing is as follows, workers start the period in a particular island, which is the result of past location decisions. Aggregate and island specific shocks for the period are realized. Idiosyncratic shocks to the return to work τ and the disutility of working are realized. Households observe these idiosyncratic shocks and the aggregate state of the economy, which I will denote Υ, and decide whether to work in the current island or search for a job. If searching, households cannot work and face one period of unemployment. There is a random component to search. If households are searching for a job they observe the economic conditions in all islands. As will be discussed next, the mobility decision is affected by economic conditions, the costs of switching and idiosyncratic preferences for the different islands.

In setting up the household problem, I will deviate from the standard Lucas and Prescott (1974) model by specifying the problem as a dynamic discrete choice optimization with random utility.[8] The problem of the worker that starts the period in island i is:

$$U(i, \epsilon, \tau, \Upsilon) = \max \left\{ U_W(i, \tau, \Upsilon) + \sigma_1\, \epsilon_W ;\, U_S(i, \tau, \Upsilon) + \sigma_1\, \epsilon_S \right\} \tag{4}$$

where $U_W(i, \tau, \Upsilon)$ and $U_S(i, \tau, \Upsilon)$ are the value of working and searching for workers of island i with type τ. $\Upsilon = \{z, \{x(i, \tau)\}, \{\theta_i\}\}$ is the distribution of productivities and households across islands. $\{\epsilon_W; \epsilon_S\}$ are transitory preference shocks reflecting the (dis)utility of working and home production opportunities. Each period the worker draws a new set of ϵ, which is i.i.d. across workers and time.

It is clear from the problem that workers decide the best alternative for them, that is, whether to work at their current island or to enter unemployment and search. In making this decision, workers take into account the rewards from working, the expected future

[7] In the context of a single homogeneous labor market, this dimension of worker heterogeneity is related to Bils et al. (2011) and Krusell et al. (2012).

[8] The literature on dynamic discrete choice is vast, with innumerable applications in labor, IO and trade. For recent applications to segmented labor markets see Kline (2008), Artuc et al. (2010) and Mangum (2010).

conditions and the idiosyncratic preferences. Workers have different reservation wages, depending on the realization of aggregate and idiosyncratic shocks.

I can establish some connection between some aspects of the model presented here and the standard Mortensen-Pissarides labor model. It is clear that the price setting mechanism and the functioning of markets is fundamentally different. The model I present here follows the competitive markets approach of Lucas and Prescott (1974). However, I can draw an analogy between the preference shock ϵ and the job destruction and matching shocks in the Mortensen-Pissarides model. When ϵ_W is sufficiently large compared to ϵ_S, the worker will work, which can be interpreted as no destruction if previously employed, or matched if previously unemployed, and vice-versa when ϵ_W is sufficiently small compared to ϵ_S. In this model ϵ will interact with the wage and other economic conditions the worker faces, something that is ruled out in the standard Mortensen-Pissarides model.

$U_W(i, \tau, \Upsilon)$ is a common value of working for all individuals in the island that have the same productivity τ.

$$U_W(i, \tau, \Upsilon) = u_i(w(i, \Upsilon)\, \tau) + \beta\, E_{\epsilon', \tau', \Upsilon'} \left[U(i, \epsilon', \tau', \Upsilon') | \tau, \Upsilon \right] \tag{5}$$

where $w(i, \Upsilon)$ is the market wage the worker gets for each of her efficiency units τ in the current island, $u_i(.)$ is the period utility function, and β is the discount factor. I will assume that $u_i(.)$ is increasing and concave. As islands differ in the amount of risk workers are exposed to, workers will demand a premium to locate in riskier islands.[9]

A worker that chooses to work in the current island will start next period in that island. The second part of equation (5) reflects the continuation value. As next periods' conditions are not know beforehand, expectations are taken with respect to the evolution of the aggregate state Υ', next period realization of the worker-island productivity τ' and the preference shocks ϵ'.

The value of searching is,

$$U_S(i, \tau, \Upsilon) = u_i(b) + \beta\, E_{\tilde{\epsilon}} \left[\max_j \left\{ E_{\epsilon', \tau', \Upsilon'} \left[U(j, \epsilon', \tau', \Upsilon') | \tau, \Upsilon \right] + \sigma_2 \tilde{\epsilon}_j \right\} \right] \tag{6}$$

where b is the consumption equivalent value of leisure or home production the household gets when unemployed. $\tilde{\epsilon}$ is a vector of preference shocks over islands that are iid across households and time which affects the decision of searchers. Here I assume that this shock is observed after the worker has decided to search. Once this preference shocks are observed,

[9] In addition, as discussed in Dixit and Rob (1994), the combination of incomplete insurance markets, general equilibrium and mobility costs generates a pecuniary externality and equilibrium will not be efficient.

workers will choose in which market they want to participate in next period.

The maximum over islands in the second part of equation (6) includes all islands, including the worker's current island. This allows the model to be closer to the data since, as discussed later, in the real world not all unemployed empirical definitions of islands. [10] In Hamilton (1988); Gouge and King (1997); Alvarez and Shimer (2011) a similar mechanism is in place and not all unemployed switch islands.

From the point of view of a single worker, search over islands is directed once the $\tilde{\epsilon}$ is realized. However, *ex-ante*, search can be interpreted as semi-directed, as $\tilde{\epsilon}$ introduces a random component to search. $\tilde{\epsilon}$ introduces an additional source of heterogeneity across workers which will lead to different mobility and unemployment patterns.

In a similar spirit as in Kambourov and Manovskii (2009) and Alvarez and Shimer (2009), I assume that a worker that moves to a different island looses her island specific skills τ. They interpret this as loosing her experience in that island. The fact that, in the model presented here, τ may decrease even if a worker does not change islands makes the interpretation of τ as experience less straightforward. In stead, τ is better interpreted as island specific skills or abilities, which evolve stochastically and are not perfectly persistent. In this sense, a worker may loose (part of) her skills even without changing islands, implying that her idiosyncratic "match" with the island became less rewarding. A worker that switches to a new island will get a new draw of τ from some distribution, independently of her past history. Carrillo-Tudela and Visschers (2013) have a similar evolution of skills.

Some comments are in order. First I am assuming no recall in the sense that workers loose their skills or cannot remember the island they used to be good at in the past. Second, I am assuming ex-post heterogeneity in skills as workers do not know beforehand their productivity level in other islands before they move there and work. Last, the randomness in the search process and the acquisition of skills upon switching islands is reminiscent of a McCall (1970) model, and depending on economic conditions both at the individual and aggregate levels workers decide to work or continue searching. Although these assumptions greatly simplify the computation of the model at the expense of not capturing some effects that may be present in the labor market, there will be a high degree of selection, with high skills being less inclined to search.

In order to simplify the problem, I will assume that each element in both types of pref-

[10] In the data, a large fraction of unemployed individuals return to work in the same industry or occupation of last employment. Carrillo-Tudela and Visschers (2013) document that close to 50% of unemployed workers switch occupation upon reentering employment. I document close to 30% for industry switches. In addition, Fujita and Moscarini (2012) document that a large fraction of the unemployed, both temporarily *and* permanently laid-off, return to work with their previous employer.

erence shocks $\{\epsilon; \tilde{\epsilon}\}$ is distributed standardized Type I Extreme Value. This is a widely used and studied distribution in the literature of dynamic discrete choice models (Rust, 1987, 1994).[11] I define $V(i,\tau,\Upsilon) = E_{\tilde{\epsilon}}\left[U(i,\bar{\epsilon},\Upsilon\right]$, which is the expected utility of a worker in island i and skills τ that has observed the aggregate and island productivity shocks and his τ but has not observed her own realization of the preference shock ϵ. $V(i,\tau,\Upsilon)$ can also be interpreted as the utility of the "representative agent" of island i and type τ.

Therefore, the *ex-ante* problem for the worker can be written as:

$$V(i,\tau,\Upsilon) \quad = \quad E_{\epsilon} \max \left\{ \begin{array}{l} u_i(w(i,\Upsilon)\,\tau) + \beta\, E_{\tau',\Upsilon'}\left[V(i,\tau',\Upsilon')|\tau,\Upsilon\right] + \sigma_1\,\epsilon_W; \\ u_i(b) + \beta\, E_{\tilde{\epsilon}}\left[\max_j \left\{E_{\tau',\Upsilon'}\left[V(j,\tau',\Upsilon')|\tau,\Upsilon\right] + \sigma_2\tilde{\epsilon}_j\right\}\right] + \sigma_1\,\epsilon_S \end{array} \right\}$$

It is clear that given $V(i,\tau,\Upsilon)$, and the realization for the preference shocks, we can recover the original problem for the individual worker. By properties of the Extreme Value distribution, we can simplify the previous expression:

$$V(i,\tau,\Upsilon) = \sigma_1\,\log\left(e^{(u_i(w(i,\Upsilon)\,\tau)+\beta\,E[V(i,\tau',\Upsilon')|\tau,\Upsilon])/\sigma_1} + e^{\left[u_i(b)+\beta\,\sigma_2\,\log\left(\sum_j e^{E[V(j,\tau',\Upsilon')|\tau,\Upsilon]/\sigma_2}\right)\right]/\sigma_1}\right) \tag{7}$$

where past values of τ affect the conditional expectation over V only if the worker stays in the same island. Equation (7) is a more tractable expression. In sum, the Extreme Value assumption allows us to collapse the ϵ heterogeneity across consumers and obtain a "representative agent". The utility of the representative agent is defined recursively in equation (7).

An additional advantage of the Extreme Value distribution are simple expressions for the proportions of workers that will choose each alternative. Let $N(i,\tau,\Upsilon)$ be the measure of workers in island i and skills τ that decide to stay in the island and work, and $M(i,j,\tau,\Upsilon)$ be the mass of unemployed workers of skills τ that move from island i to island j.

$$N(i,\tau,\Upsilon) \quad = \quad \frac{e^{(u_i(w(i,\Upsilon)\,\tau)+\beta\,E[V(i,\tau',\Upsilon')|\tau,\Upsilon])/\sigma_1}}{e^{V(i,\tau,\Upsilon)/\sigma_1}} \tag{8}$$

$$M(i,j,\tau,\Upsilon) \quad = \quad \frac{e^{\left[u_i(b)+\beta\,\sigma_2\,\log\left(\sum_k e^{E[V(k,\tau',\Upsilon')|\tau,\Upsilon]/\sigma_2}\right)\right]/\sigma_1}}{e^{V(i,\tau,\Upsilon)/\sigma_1}}\,\frac{e^{E[V(j,\tau',\Upsilon')|\tau,\Upsilon]/\sigma_2}}{\sum_k e^{E[V(k,\tau',\Upsilon')|\tau,\Upsilon]/\sigma_2}} \tag{9}$$

Note that $M(i,i,\tau,\Upsilon)$ is the measure of unemployed workers in island i that search for

[11] The main advantage of the Extreme Value distribution, which lead to its popularity, is the closed form solution for the expectation of the maximum of a set of random variables and also for the probability of a particular choice being maximal.

work and find island i to be their best alternative.[12] Total Labor supply in the island is $L^s(i,\Upsilon) = \sum_\tau \tau\, x(i,\tau)\, N(i,\tau,\Upsilon)$.[13]

Definition 1. *Recursive Competitive Equilibrium: A stationary equilibrium in this economy is a set of Value Functions $V(i,\tau,\Upsilon)$, proportion of workers $N(i,\tau,\Upsilon)$, proportion of searchers $M(i,j,\tau,\Upsilon)$, labor demand and supply $L^d(i,\Upsilon), L^s(i,\Upsilon)$, prices for intermediate (island) goods $p(i,\Upsilon)$, island and aggregate production $y(i,\Upsilon), y$ and wages $w(i,\Upsilon)$ for all islands $i = \{1 \dots J\}$ together with an operator over the distribution of workers over islands, $G(\Upsilon)$ such that*

- *Given wages, $V(i,\tau,\Upsilon)$ is the maximum lifetime utility of the representative agent of skills τ in island i, and $N(i,\tau,\Upsilon)$, $M(i,j,\tau,\Upsilon)$ and $L^d(i,\Upsilon)$ are defined as before.*
- *Given wages, firms maximize profits.*
- *All labor and product markets clear: $L^d(i,\Upsilon) = L^s(i,\Upsilon)$, $x_i = y_i + \sum_{j=1}^J \varphi_{ij}$, for all i.*
- *$x' = G(\Upsilon)\, x$, where G is consistent with $N(i,\tau,\Upsilon)$ and $M(i,j,\tau,\Upsilon)$.*

In addition, in a competitive equilibrium it must be that $p_i = c_i$, therefore,

$$\log(p_i) = \log(A_i) + \alpha_i \log(w_i) + (1-\alpha_i) \sum_{j=1}^J \phi_{ij} \log(p_j)$$

which, given parameters, defines a system of linear equations between the vector of (log) wages w and the vector of (log) prices p for the island good.

Using equations (2) and (3) together with market clearing and rearranging we get:

$$y_i = \frac{L_i}{A_i \alpha_i} \left(\frac{w_i}{\prod_{j=1}^J p_j^{\phi_{ij}}} \right)^{(1-\alpha_i)} - \sum_{j=1}^J (1-\alpha_j)\phi_{ji}\, \frac{w_j}{p_j} \frac{L_j}{\alpha_j} \tag{10}$$

In addition, the final demand for good i is $y_i = y\kappa^{\chi-1} \left(\frac{p_i}{p}\right)^{-\chi}$.

It is important to note that with a finite number of islands, both z *and* θ will have aggregate effects in the economy. The equilibrium is not stationary and the distribution x is not time invariant. These shocks affect the incentives to move as workers trade-off current wages in their island versus the costs and rewards to moving. In addition, the aggregate shock z is complementary to the island productivity θ. The more productive islands will

[12] There is a close link between $M(i,i,\tau,\Upsilon)$ and the definition of rest unemployment or skill unemployment used by Alvarez and Shimer (2011) and Alvarez and Shimer (2009).

[13] Equations (7), (8) and (9) are derived in the Appendix.

see fewer workers leaving and more arriving in expansions and the opposite in recessions, making x' change in time with movements in z, θ and x.

It is convenient to define here the solution to a version of the model with no aggregate uncertainty. A stationary equilibrium in this model requires that z and θ are equal to their expected value of zero at all times. In this case, the distribution of households over islands is time-invariant and wages are constant. None-the-less, workers transit along the distribution by effect of the idiosyncratic productivity and preference shocks, generating gross flows. Gross job creation and destruction will typically be larger than unemployment, which is invariant in the stationary equilibrium. Similarly, gross mobility across islands is positive but, on net, will cancel.[14] The characteristics of the stationary version in this model are different from that of Lucas and Prescott (1974) model. This is a direct consequence of the finite number of islands.

3 Numerical Solution

The model does not have a closed form solution and in order to solve it we need to use numerical techniques. The main limitation with incorporating aggregate shocks to island models has always been technical. We understand well how to write down the model, what the state variables are and the conditions that define equilibrium, yet we do not know how to solve it. Therefore, the purpose of this section is to discuss the algorithm and techniques that allow me to solve the model for a sufficiently large number of islands.

The solution to the recursive competitive equilibrium is a set of *functions* $\{x'(i,\tau,\Upsilon), V(i,\tau,\Upsilon), w(i,\Upsilon), p(i,\Upsilon), N(i,\tau,\Upsilon), M(i,j,\tau,\Upsilon)\}_{\forall\, i,j,\tau}$ that solves the following system for any value of $\Upsilon = \{z, \{x(i,\tau)\}, \{\theta_i\}\}$,

[14] Coen-Pirani (2010) is also able to generate gross flows that are different from net flows in an island model by introducing a binary taste shock.

$$
\begin{aligned}
0 &= V(i,\tau,\Upsilon) - \sigma_1 \log\left(e^{\left(u_i(w(i,\Upsilon)\,\tau) + \beta\, E\left[V(i,\tau',\Upsilon')|\tau,\Upsilon\right]\right)/\sigma_1} + e^{\left[u_i(b) + \beta\,\sigma_2\, \log\left(\sum_j e^{E\left[V(j,\tau',\Upsilon')|\tau,\Upsilon\right]/\sigma_2}\right)\right]/\sigma_1}\right) \\
0 &= y(\Upsilon)\,\kappa^{\chi-1} p(i,\Upsilon)^{-\chi} - \frac{\sum_\tau \tau\, x(i,\tau)\, N(i,\tau,\Upsilon)}{\left[e^{\lambda_i z+\theta_i}\alpha_i^{\alpha_i}(1-\alpha_i)^{(1-\alpha_i)}\prod_{j=1}^J \phi_{ij}^{(1-\alpha_i)\phi_{ij}}\right]^{-1}\alpha_i}\left(\frac{w_i}{\prod_{j=1}^J p(j,\Upsilon)^{\phi_{ij}}}\right)^{(1-\alpha_i)} + \\
&\quad + \sum_{j=1}^J (1-\alpha_j)\phi_{ji}\,\frac{w_j}{p(j,\Upsilon)}\,\frac{\sum_\tau \tau\, x(j,\tau)\, N(j,\tau,\Upsilon)}{\alpha_j} \\
0 &= x'(i,\tau,\Upsilon) - \left[\sum_{\hat\tau} \pi(\tau|\hat\tau)\, x(i,\hat\tau)\left(N(i,\hat\tau,\Upsilon) + M(i,i,\hat\tau,\Upsilon)\right) + \pi(\tau)\sum_{j\neq i}\sum_{\hat\tau} x(j,\hat\tau)\, M(j,i,\hat\tau,\Upsilon)\right] \\
0 &= y(\Upsilon)^{1-1/\chi} - \kappa\sum_i\left[\frac{\sum_\tau \tau\, x(i,\tau)\, N(i,\tau,\Upsilon)}{\left[e^{\lambda_i z+\theta_i}\alpha_i^{\alpha_i}(1-\alpha_i)^{(1-\alpha_i)}\prod_{j=1}^J \phi_{ij}^{(1-\alpha_i)\phi_{ij}}\right]^{-1}\alpha_i}\left(\frac{w_i}{\prod_{j=1}^J p(j,\Upsilon)^{\phi_{ij}}}\right)^{(1-\alpha_i)} - \right. \\
&\quad \left. - \sum_{j=1}^J (1-\alpha_j)\phi_{ji}\,\frac{w_j}{p(j,\Upsilon)}\,\frac{\sum_\tau \tau\, x(j,\tau)\, N(j,\tau,\Upsilon)}{\alpha_j}\right]^{1-1/\chi} \\
0 &= \log(p(i,\Upsilon)) + \log\left(e^{\lambda_i z+\theta_i}\alpha_i^{\alpha_i}(1-\alpha_i)^{(1-\alpha_i)}\prod_{j=1}^J \phi_{ij}^{(1-\alpha_i)\phi_{ij}}\right) - \alpha_i\log(w_i) - (1-\alpha_i)\sum_{j=1}^J \phi_{ij}\log(p(j,\Upsilon)) \\
0 &= z' - \rho_z\, z - \sigma_z\,\varepsilon_z \qquad (11) \\
0 &= \theta_i' - \rho_{\theta_i}\,\theta_i - \sigma_{\theta_i}\,\varepsilon_{\theta_i}
\end{aligned}
$$

where $\pi(\tau|\hat\tau)$ is the conditional transition probability between $\hat\tau$ and τ, and $\pi(\tau)$ is the probability of getting a draw τ upon arriving to a new island. The third equation highlights the fact that workers that switch islands get a new draw of τ . Also $N(i,\tau,\Upsilon)$ and $M(i,j,\tau,\Upsilon)$ are defined in (8) and (9) :[15]

The high dimensionality of the state space and the fact that Υ contains all continuous variables makes problem (11) difficult to solve using standard methods. Even for a small number of islands the curse of dimensionality has a strong bite. Here I overcome the curse of dimensionality using perturbation methods. These methods are used extensively in economics, mainly in models with a representative agent, and can easily accommodate a very large number of state variables.[16] In the model presented here there are several representative agents, one per island, and their relative importance (measure) changes in time as the

[15] It is possible to reduce the system (11) further by substituting $y(\Upsilon)$.

[16] For a complete exposition on perturbation theory in economics see Judd (1998).

distribution of workers changes. The idea of using perturbation in problems with a distribution of agents and aggregate shocks goes back to Campbell (1998) where he used it in analyzing entry and exit of firms over the business cycle. More recently Reiter (2009) and Mertens and Judd (2012) used perturbation in a Bewley-Hugget-Aiyagari model with aggregate shocks. In addition, I will be perturbing the Value Function which is the key element in the extensive margin decision on whether to work and where. Perturbation of the Value function is discussed in Judd (1998) but has seldom been applied in economic problems. A recent exception is Caldara et al. (2011) where they use it to solve a problem with recursive preferences. Kline (2008) uses perturbation methods to solve a model with 3 islands but no unemployment.

In order to use perturbation methods in this problem the policy functions must be differentiable as the technique relies in Taylor's and the Implicit Function theorems. Note that in the original Lucas and Prescott (1974) model, all households in the island have identical preferences with islands being perfect substitutes. In this case, both at the level of the household and the island, the solution does not deliver smooth differentiable functions.[17] In the model I develop here, the single household still has preferences of perfect substitutes over islands, but the introduction of the idiosyncratic preference shock generates a smooth labor supply for the island and the value function of the representative agent. While in principle this can be achieved with any shock, the use of the Extreme Value distribution substantially simplifies the problem by delivering almost closed-form expressions.

To apply perturbation theory, let me re-write the process for z in the following way:[18]

$$\begin{aligned} z_t &= \rho_z \, z_{t-1} + \lambda \, \sigma_z \, \varepsilon_{z,t} \\ \theta_{i,t} &= \rho_{\theta_i} \, \theta_{i,t-1} + \lambda \, \sigma_{\theta_i} \, \varepsilon_{\theta_i,t}; \qquad \forall i \end{aligned}$$

λ is the perturbation parameter. When $\lambda = 1$ aggregate shocks $\{z, \theta\}$ are active and when $\lambda = 0$ aggregate shocks are shut-down and the model is stationary. To understand the method, I make the dependence of the solution on the perturbation parameter explicit: $\{x'(i,\tau,\Upsilon;\lambda), V(i,\tau,,\Upsilon;\lambda), w(i,\Upsilon;\lambda), p(i,\Upsilon;\lambda), N(i,\tau,\Upsilon;\lambda), M(i,j,\tau,\Upsilon;\lambda)\}_{\forall\, i,j,\tau}$.

To keep the exposition simple, in what follows I will only perform a second order perturbation. I will approximate the solution around the point in which $\lambda = 0$, i.e. the stationary model. Higher orders can easily be achieved but the notation gets cumbersome. Let me define $\Upsilon^{ss} = \left\{0, \{x^{ss}(i,\tau)\}_{\forall i,\tau}, 0, \ldots, 0\right\}$. In this way, the second order approximation to

[17] The reader can see the shape of the solution in the traditional island model in Alvarez and Veracierto (2000), where it is clear that the Value Function and the labor supply present sharp kinks and areas where the derivative is zero, violating the conditions for the Taylor and Implicit Function Theorems.

[18] Here I adapt the notation used in Caldara et al. (2011) to the setup developed here.

the Value Function in (11) is:

$$\begin{aligned}
V(i,\tau,\Upsilon;\lambda) \;\approx\;& V^{ss}(i,\tau) + V^{ss}_{z}(i,\tau)\,z_t + \sum_{j,\tilde{\tau}} V^{ss}_{x(j,\tilde{\tau})}(i,\tau)\,\hat{x}_t(j,\tilde{\tau}) + \sum_{j} V^{ss}_{\theta_j}(i,\tau)\,\theta_{j,t} + V^{ss}_{\lambda}(i,\tau)\,\lambda + \\
& \frac{1}{2}\left[V^{ss}_{z,z}(i,\tau)\,(z_t)^2 + \sum_{j,\tilde{\tau}} V^{ss}_{z,x(j,\tilde{\tau})}(i,\tau)\,z_t\,\hat{x}_t(j,\tilde{\tau}) + \sum_{j} V^{ss}_{z,\theta_j}(i,\tau)\,z\,\theta_{j,t} + V^{ss}_{z,\lambda}(i,\tau)\,z\,\lambda\right] + \\
& \frac{1}{2}\sum_{k,\bar{\tau}}\Bigg[V^{ss}_{z,x(k,\bar{\tau})}(i,\tau)\,z_t\hat{x}_t(k,\bar{\tau}) + \sum_{j,\tilde{\tau}} V^{ss}_{x(k,\bar{\tau}),x(j,\tilde{\tau})}(i,\tau)\,\hat{x}_t(k,\bar{\tau})\,\hat{x}_t(j,\tilde{\tau}) + \\
& \qquad \sum_{j} V^{ss}_{x(k,\bar{\tau}),\theta_j}(i,\tau)\,\hat{x}_t(k,\bar{\tau})\,\theta_{j,t} + V^{ss}_{x(k,\bar{\tau}),\lambda}(i,\tau)\,\hat{x}_t(k,\bar{\tau})\,\lambda\Bigg] + \\
& \frac{1}{2}\sum_{k}\left[V^{ss}_{z,\theta_k}(i,\tau)\,z_t\,\theta_{k,t} + \sum_{j,\tilde{\tau}} V^{ss}_{\theta_k,x(j,\tilde{\tau})}(i,\tau)\,\theta_{k,t}\,\hat{x}_t(j,\tilde{\tau}) + \sum_{j} V^{ss}_{\theta_k,\theta_j}(i,\tau)\,\theta_{k,t}\,\theta_{j,t} + V^{ss}_{\theta_j,\lambda}(i,\tau)\,\theta_{j,t}\,\lambda\right] + \\
& \frac{1}{2}\left[V^{ss}_{z,\lambda}(i,\tau)\,z_t\,\lambda + \sum_{j,\tilde{\tau}} V^{ss}_{\lambda,x(j,\tilde{\tau})}(i,\tau)\,\lambda\,\hat{x}_t(j,\tilde{\tau}) + \sum_{j} V^{ss}_{\lambda,\theta_j}(i,\tau)\,\lambda\,\theta_{j,t} + V^{ss}_{\lambda,\lambda}(i,\tau)\,\lambda^2\right]
\end{aligned}$$

where $V^{ss}(i,\tau) = V(i,\tau,\Upsilon^{ss};0)$, $V^{ss}_{\gamma}(i,\tau) = \frac{\partial V(i,\tau,\Upsilon^{ss};0)}{\partial\gamma}$, $V^{ss}_{\gamma,\delta} = \frac{\partial^2 V(i,\tau,\Upsilon^{ss};0)}{\partial\gamma\partial\delta}$ and $\gamma,\delta \in \{z,\{x(j,\tilde{\tau})\},\{\theta_j\}\}$. And $\hat{x}_t(k,\bar{\tau}) = x_t(k,\bar{\tau}) - x^{ss}(k,\bar{\tau})$. The previous expression can be simplified using Young's theorem and making $\lambda = 1$. The law of motion for $\{x'(i,\tau,\Upsilon;\lambda), w(i,\Upsilon;\lambda), p(i,\Upsilon;\lambda), N(i,\tau,\Upsilon;\lambda), M(i,j,\tau,\Upsilon;\lambda)\}_{\forall i,j,\tau}$ can be derived in a similar way.

$V^{ss}(i,\tau)$, $V^{ss}_{\gamma}(i,\tau)$, $V^{ss}_{\gamma,\delta}(i,\tau)$ are the coefficients in the approximate solution which are not known and must be solved for. To do this we perturb the equilibrium conditions in (11). The system in (11) can be re-written more compactly as:

$$F(\Upsilon;\lambda) = H(\{V(i,\tau,\Upsilon), E_{\Upsilon}[V(i,\tau',\Upsilon')], x(i,\tau), x'(i,\tau,\Upsilon), w(i,\Upsilon), p(i,\Upsilon), \theta_i\}_{\forall i,\tau}, z, y;\lambda) = \mathbf{0}$$

where F is a function which links implicitly the solution of the model with the state variables.[19]

Note that

$$F(\Upsilon^{ss};0) = H(\{V(i,\tau,\Upsilon^{ss}), V(i,\tau',\Upsilon^{ss}), x(i,\tau), x(i,\tau,\Upsilon^{ss}), w(i,\Upsilon^{ss}), p(i,\Upsilon^{ss}), 0\}_{\forall i,\tau}, 0, y^{ss};0) = \mathbf{0}$$

is just the solution of the stationary version of the model where aggregate shocks z,θ are shut-down. This defines a non-linear system in the variables

[19] I omit variables $N^{ss}(i,\tau), M^{ss}(i,j,\tau)$ since they are a function of the rest.

$\{x^{ss}(i,\tau), V^{ss}(i,\tau), w^{ss}(i,), p^{ss}(i), N^{ss}(i,\tau), M^{ss}(i,j,\tau)\}_{\forall i,j,\tau}$. While it is possible to solve for the stationary values with any standard non-linear solver, sometimes derivative based methods may fail in finding the general equilibrium with a large number of markets. If that is the case, I found useful to use the following algorithm which describes a *tatonnement* process and turns out to be very robust and fast:

1. Guess a vector of wages w. Solve the linear system of log prices and log wages.
2. Solve the problem of the representative worker for the island i and productivity τ. To do this use Value Function Iteration to solve equation (7), guessing an initial vector of V and iterate until convergence. Get N and M.
3. Guess a distribution of workers over islands at the beginning of the period (x).
4. Use N, M and x to update the distribution of agents over islands for the beginning of the next period x'. Compare x' with x and if they are not approximately equal, use x' as a new guess and go back to step 3.
5. Compute labor supply and demand and check if markets clear. Use the firms' condition to get the wages, w^{+1} using labor supply. Compare w^{+1} with the guess w and if they are not approximately equal, go back to step 1 using bisection or similar method.

The derivatives of F with respect to each of the elements in the state space

$$F_{\gamma}(\Upsilon^{ss};0) = \mathbf{0}; \qquad \text{for } \gamma = \{z, \{x(j,\tilde{\tau})\}, \{\theta_j\}\} \tag{12}$$

define a quadratic system of equations in the unknowns $\{x^{ss}_{\gamma}(i,\tau), V^{ss}_{\gamma}(i,\tau), w^{ss}_{\gamma}(i), N^{ss}_{\gamma}(i,\tau), M^{ss}_{\gamma}(i,j,\tau)\}_{\forall i,j,\tau}$. We need to use the solution that satisfies Blanchard and Kahn (1980) conditions, provided it exists.

For the elements of the second order, we need to take second order derivatives on F and equalize to zero,

$$F_{\gamma,\delta}(\Upsilon^{ss};0) = \mathbf{0}; \qquad \text{for } \gamma, \delta = \{z, \{x(j,\tilde{\tau})\}, \{\theta_j\}\} \tag{13}$$

which defines a linear system in $\left\{x^{ss}_{\gamma,\delta}(i,\tau), V^{ss}_{\gamma,\delta}(i,\tau), w^{ss}_{\gamma,\delta}(i), N^{ss}_{\gamma,\delta}(i,\tau), M^{ss}_{\gamma,\delta}(i,j,\tau)\right\}_{\forall i,j,\tau}$.

3.1 Order of approximation

The goodness of the approximation depends on the shape of the unknown functions. While all numerical methods provide only an approximation to the true solution, a finite order perturbation may deliver poor approximations away from the stationary world.[20] None-the-less, it has been shown that in many economic applications perturbation methods perform reasonably well and the approximation error is similar to that of global approximations (Aruoba et al. (2006) and Caldara et al. (2011).)

At a computational level, one important question is whether an approximation of order one is sufficient. There are several reasons to use perturbation of a higher order. On the one hand, in the model $x'(i, \tau, \Upsilon), N(i, \tau, \Upsilon), M(i, j, \tau, \Upsilon)$ are guaranteed to be strictly between zero and one. However, in the approximate solution this may not hold and some large shocks may lead to implausible values for these variables.[21] This is just an indication that it is necessary to increase the order of the approximation.

But besides a more accurate approximation, there are other important reasons to use higher order perturbations. As is well known, a linear approximation to the solution displays Certainty Equivalence and risk plays no role in the agents decisions. In the context of simple representative agent economies, a first order perturbation provides a reasonably good approximation to the dynamics of the endogenous variables.[22] However, in this model a first order perturbation may provide a poor approximation, even with standard shocks processes. The reason is that workers' decision on where to locate (and the value of working versus not) is done at the extensive margin and differences in the risk of the different choices affect this decision. Since flows are are proportional to stocks, the dynamics of the key variables in the model will in turn be affected by the characteristics of the shock process, an element that is completely missed with a first-order/certainty equivalent solution. In other words, in this model, in contrast with much of the literature of perturbation in representative agents economies, risk can be of first order importance for accurately describing the dynamics of the labor market.

In addition, as will be discussed in depth later, in this model there may be non-linear effects of shocks, in particular, asymmetric impact and persistence. These effects stem from the *interaction* between the shocks and the distribution of agents. These non-linear are missed

[20] Note that for differentiable functions, Taylor's theorem states that the approximation converges to the true function globally as the order of the perturbation tends to infinity, making the approximation truly global.

[21] It is well known that perturbation techniques cannot handle inequality constraints. Therefore, a constraint like $M(i, j, \tau, \Upsilon) \geq 0$ cannot be enforced.

[22] In simple representative agent models, with no stochastic volatility or strong non-linearities, a linear approximation to the policy function generally misses the level of the endogenous variables but captures approximately well the changes. Therefore, only welfare calculations can be severely affected (Kim and Kim, 2003).

with a linear approximation.

4 A Simple Example

In this section I present a simple model to illustrate how to use perturbation methods to approximate the solution to the model. Let the number of islands be $J = 3$ and the worker-island productivity be approximated by a 3 state Markov process. In this case, there are 13 state variables in Υ: the aggregate shock z, the three island productivity shock θ and the distribution of agent-types over islands x which has 9 elements. The system (11) has a total of 26 equations: 9 Bellman equations, 3 wage equations, 9 law of motion for the distribution, one for output, one for the evolution of the aggregate shock and 3 for the evolution of island shocks.

I will compute a second order approximation to the solution. There are many methods to achieve this, all of them equivalent. For ease of exposition I use the method developed by Gomme and Klein (2011). Let's define the vector of variables as $\Lambda_t = \{z_t, \{\theta_{i,t}\}, \{x_{i,\tau,t}\}, \{w_{i,t}\}, \{V_{i,\tau,t}\}, y_t\}$, and $\Lambda_{t+1} = \{z_{t+1}, \{\theta_{i,t+1}\}, \{x_{i,\tau,t+1}\}, \{w_{i,t+1}\}, \{EV_{i,\tau,t}\}, y_{t+1}\}$.

Given some values for the parameters, the first step is to compute the stationary steady state in which $\Lambda_t = \Lambda_{t+1} = \Lambda$. As I mentioned before, this requires us to solve a system of 26 non-linear equations in 26 variables. Next, we find the Jacobian of the system with respect to Λ_{t+1} and Λ_t. This can be represented by a matrix of dimensions 26 times 52. We evaluate the Jacobian at the stationary steady state.

As an example, the derivative of the first equation, the Value Function, in (11) with respect to z, evaluated at the steady state is

$$\begin{aligned} V_z^{ss}(i,\tau) \quad - \quad & N^{ss}(i,\tau)\left(u'(w^{ss}(i)\tau)\,\tau\, w_z^{ss}(i) + \beta\sum_{\tau'} pr(\tau'|\tau)\left(\rho\, V_z^{ss}(i,\tau') + \sum_{j,s} V_{x(j,s)}^{ss}(i,\tau')\, x_z^{ss}(j,s)\right)\right) - \\ & \beta\, M^{ss}(i,i,\tau)\sum_{\tau'} pr(\tau'|\tau)\left(\rho\, V_z^{ss}(i,\tau') + \sum_{j,s} V_{x(j,s)}^{ss}(i,\tau')\, x_z^{ss}(j,s)\right) - \\ & \beta\sum_{j\neq i} M^{ss}(i,j,\tau)\sum_{\tau'} pr(\tau')\left(\rho\, V_z^{ss}(j,\tau') + \sum_{k,s} V_{x(k,s)}^{ss}(j,\tau')\, x_z^{ss}(k,s)\right) = 0 \end{aligned}$$

If we were only interested in the linear approximation of the model, we can stop here as we have all the necessary elements. In a linear approximation, for each of the endogenous variables $\{\{x_{i,\tau,t+1}\},\{w_{i,t}\},\{V_{i,\tau,t}\},y_t\}$, the solution is a linear equation on the 13 state variables and a constant.

The third step is to compute the Hessian of each equation in the model with respect to Λ_{t+1} and Λ_t. For each equation we have a symmetric matrix of size 56. The method requires to stack these matrices vertically. Therefore, we get a matrix of size 1456 times 56 with all the second order derivatives for all equations. With a perturbation of second order, the solution is a polynomial of second order for each of the endogenous variables with 13 linear terms, 91 second order terms and a constant.[23]

I will use some simple parametrization just to get a flavor of how the model performs.[24] I assume islands are symmetric with the exception of their sensitivity to the aggregate shock. Because of this difference, some islands will be more exposed to aggregate risk than others.

4.1 Impulse-Responses

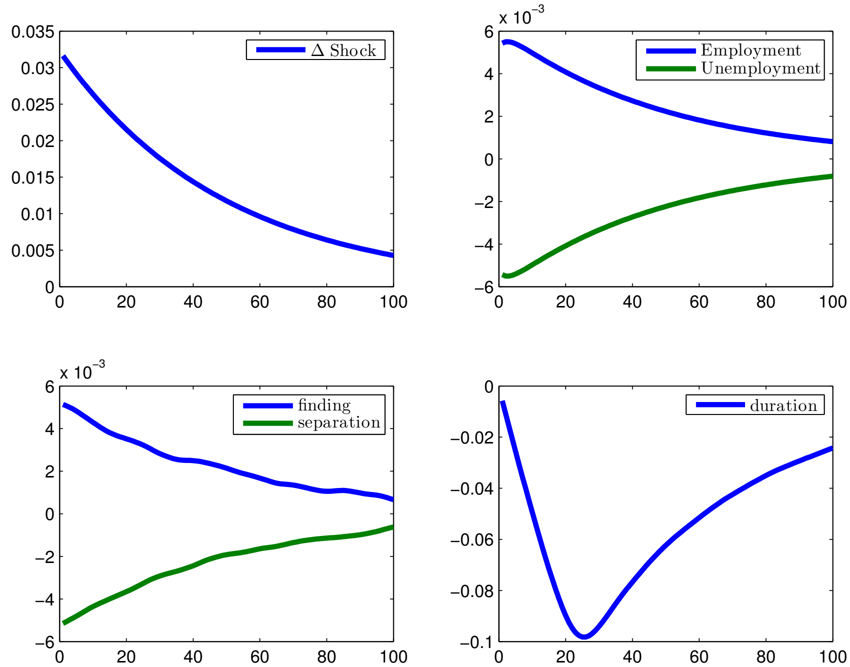
To get a flavor of how the model performs, I will compute the response of the main labor market variables to an aggregate shocks and a sector specific shock. As the model is nonlinear, the IRFs depend on the previous history of shocks. At the same time, the ergodic mean of the variables is not their stationary value, therefore computing IRFs in deviations from the steady state is uninformative. I follow Fernandez-Villaverde et al. (2011) in constructing of the IRFs as absolute deviations from the ergodic mean. To construct the IRFs of labor market flows and duration of unemployment spells, I simulate labor histories for 50000 workers in an economy hit by a single shock and differentiate from an economy with no shock but with the same individual specific shocks. As this introduce some small sample noise, I HP filter these IRFs.

Figure 1 shows the response to an aggregate shock of one standard deviation that affects all sectors. The shocks is persistent and its evolution is displayed in panel (a). As the shock is positive, wages in the economy are higher than average, making work more attractive. Employment increases and unemployment falls. The job finding and separation probabilities also have the right sign, with an increased finding probability and lower separations in a boom. Panel (d) shows that a positive shock lowers the average duration of unemployment spells. Figure 2 show the response to a sector specific shock. We can see that the effect of this shock is very similar, although as it primarily affects a smaller part of the economy, its

[23]Using Young's Theorem and $\lambda = 1$, the number of therms in the second order is 91 instead of 169.

[24]Here the objective is not to match moments in the data, therefore I will not discuss the value of these parameters. In the next section I will perform a rigorous exercise and a careful empirical strategy will be discussed.

Figure 1: Response to an aggregate shock
(deviation from ergodic mean)



effects are of smaller magnitude.

4.2 Amplification and Asymmetry

TO BE COMPLETED

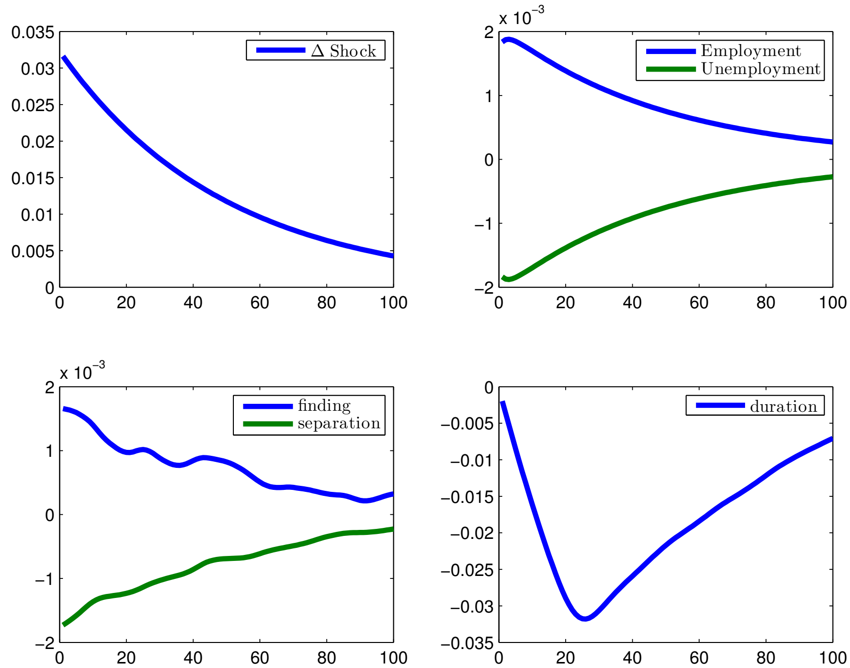
5 Numerical Analysis and Evaluation of the Model

As I mentioned in the introduction, there are not many examples of island models with business cycle dynamics.[25] Therefore, a necessary step is to evaluate whether the model I presented in the previous section is able to capture the relevant features of the labor market.

One difficulty in linking the model with the real world is defining what constitutes an island. In the model, workers can switch employers and firms can replace workers at no cost within the island. However, changing labor markets is costly for workers both in terms of unemployment and a possible loss of skills. This implies that switching costs should be a

[25] Veracierto (2008) is the only exception.

Figure 2: Response to an aggregate shock



guiding criteria. In the model two more features define an island. First, an island produces a differentiated good. Second islands are subject to idiosyncratic shocks and have different sensitivities to the aggregate shock.

In the literature of worker mobility and displacement, several dimensions have been considered in defining an island: industry, occupation and geography. Most likely all of them are relevant and an island should be defined in terms of the intersection of all three. Since that specification would be computationally very demanding and given that the focus of this work is on the effects of sectoral shocks, in the present work I will assume that an island is defined in terms of an industry or sector, with workers having industry specific human capital.[26]

In terms of how many sectors to include, there is a trade-off between disaggregation, data availability and computational burden. Labor market data for narrowly defined sectors is harder to come by at the monthly frequency. While it is possible to construct it using microdata, small sectors will contain few observations and thus the exercise will be more influenced by measurement error. Therefore, I calibrate the model to 14 aggregate sectors for the US economy using the NAICS classification.[27] As I discuss in detail later, I use a 5 point

[26]The works of Jacobson et al. (1993); Neal (1995); Lee and Wolpin (2006); Sullivan (2010), among others, find evidence of industry specific human capital.

[27]I include all sectors except mining, accounting for 99.5% of total employment. The characteristics of mining are very different from other sectors, which makes merging it to any other industry questionable. As this

grid for the worker-island productivity τ, totaling 85 state variables. This implies that, at the second order, each policy function contains 85 linear terms, 3655 quadratic (cross) terms and a constant. As there are 184 variables in the model, the total number of coefficients is over 680,000. [28] While 14 industries may seem too coarse, it is worth noting that in the data workers that switch more narrowly defined industries, 80% of the time also switch these aggregate groups. In other words, industry switches are almost always across these large industry groups.

5.1 Data

Data comes from different sources. I use microdata from the monthly CPS from 1979 to 2010. The advantage of the CPS is that it is the source of official labor market statistics, it has a large sample size with a monthly frequency and it has a short panel dimension. The limitation is that attrition and classification errors can be a source of bias (see the discussion in (Elsby et al., 2012). From the CPS I compute moments on employment, unemployment, EU and UE flows, at the aggregate and at industry level.

In addition, from the CPS I compute gross and net industry mobility. As discussed in Moscarini and Thomsson (2007), gross mobility is prone to large biases due to coding errors. In 1994 the CPS changed to a "dependent coding" system, substantially reducing the error.[29] Therefore, I use data from 1995 onwards to compute mobility moments.

The CPS does not have an homogeneous industry classification over time. The classification evolves to better take into account an increasing number of new goods and services. Clear examples are mobile communications and Internet services which became widely available in the late 1990s. In 2003 the classification underwent a major change, moving from the Standard Industrial Classification (SIC) to the North American Industrial Classification System (NAICS). None-the-less, even within the SIC system there were changes in 1983 and 1992. Here I use the current NAICS definition and adjust years 1979 to 2002 in order to construct an homogeneous industry series.[30]

sector comprises around 0.5% of employment, including mining as a separate sector can potentially affect the estimation and the model's performance. In this way, energy and oil shocks will be part of the aggregate shocks in the economy.

[28] In terms of computational burden, I use a second order perturbation for the reasons discussed previously, and most of the parameters will be estimated structurally, which require solving the model repeatedly.

[29] In a dependent coding system, the interviewer will ask the household if she still works for the same employer she reported last period. If that is the case, answers for industry and occupation will be taken from the previous month.

[30] While for many industries the change in classification from SIC to NAICS does not lead to substantial differences, for others there is a sharp break in 2003. While the break is noticeable in the levels of employment and unemployment, the unemployment rate is only mildly affected. None-the-less, I correct for these abrupt changes by computing differences in the levels and replacing the change in 2003 by the centered average 6

Time series for industrial employment and wages are taken directly from the BLS. The BLS reports aggregate information from the Current Employment Survey (CES) and are reported using NAICS classification over the whole period.[31] I detrend using the Hodrick-Prescott filter with smoothing parameter equal to 1600 at the quarterly frequency and I seasonally adjust all CPS series using X13-ARIMA-SEATS. Wages are deflated by the CPI-U.

5.2 Parametrization

The model has many parameters. I only calibrate exogenously the discount factor, the elasticity of substitution and one parameter governing the law of motion for τ to values usually assumed in the literature. However, I will calibrate the rest of the parameters "endogenously", to minimize the distance between some targeted moments generated by model and their data counterpart.[32]

I take the time period to be one month. This implies that searching takes at least one month of unemployment and that duration will be multiples of one month. I calibrate the discount factor to an annual rate of 5% as in Shimer (2005). I set the elasticity of substitution over sector goods (χ) to 2.2, which is the median value estimated by Broda and Weinstein (2006) and is consistent with the estimates by Alvarez and Shimer (2011).[33] These are the only parameters I impose exogenously.

Different sectors will have different volatilities. As I discussed before, risk plays a role in this model, therefore I define period utility as $u_i(c) = \log(c) + \mu_i$. Logarithm utility implies a coefficient of relative risk aversion equal to one and is a common functional form in the literature. Since flows are proportional to stocks, it is important for the model to approximate the distribution of workers over industries. μ_i is an island specific "amenity" or island (dis)utility which I introduce to affect the total measure (level) of workers in the island. In the real world, employment shares for different industries are not constant, and the model cannot capture this. Therefore, the moments I match are the average employment shares for each sector from 2000 to 2010.[34]

months back and ahead and integrating back the original series.

[31] The CES does not measure agricultural employment, therefore I use the CPS for this variable.

[32] I follow the algorithm proposed by Guvenen (2011) in searching for the optimal parameters. This algorithm combines a local minimization stage with a global one. In the first, given a set of starting values, a derivative-free numerical minimization algorithm finds the best parameters. The global stage involves getting a sufficiently large number of starting conditions drawn quasi-randomly from the feasible set of parameters.

[33] Broda and Weinstein (2006) estimate this elasticity for a larger number of varieties. Is would be expected that the fewer options, the harder it is to substitute among them and the elasticity would be lower.

[34] An alternative to the use of μ_i is to calibrate θ_i to have different means for the different industries. I prefer the specification with amenities since the effect of aggregate and sectoral shocks can easily be compared across industries.

The scale parameter κ is targeted to deliver an economy-wide average earnings of approximately one, which is a normalization. Productivity shocks are not directly observable and their properties must be inferred from the dynamics of observable variables. In addition, as argued by Foerster et al. (2011), disentangling aggregate and sectoral shocks importance is difficult due to inter-sectoral linkages. Here I estimate the properties of the aggregate and industry shock processes and the sensitivity parameters λ to match the volatility and correlations of aggregate and sectoral employment and wages.

As is usual in the literature of heterogeneous agents, the idiosyncratic island-worker productivity τ will be a discrete approximation to an AR1 process. I use Rouwenhorst (1995) method to discretize this process using a 5 state Markov Chain.[35] Krusell et al. (2012) calibrate this process to have an autoregressive parameter of 0.9931 and standard deviation of 0.1017 (see their Table 3) in accordance to previous literature. In the model presented here there will be a strong selection effect, and workers with a low skill type will be inclined to search and change islands. As they sample from a different distribution, the effective persistence in skills will be different, leading to a distribution for τ with larger mass in the right tail. I assume the same value for the autoregressive parameter, but the standard deviation will be estimated to target a value for residual earnings inequality that is close to the data (average from 2003 to 2010).[36]

There are 3 more parameters to calibrate. The value of leisure b and σ_1, σ_2. As has been discussed in Artuc et al. (2010), identification may be an issue as is possible to obtain approximately the same level for unemployment by changing b and σ_1 in the same direction. However, as they point out, changes in the economic conditions (wages) will induce a larger or lower unemployment and mobility response which depend on σ_1 σ_2 for a given b. In other words, we need both the level of unemployment and its cyclical properties to pin-down the b and σ_1. Therefore, I will target a level for the unemployment rate of 6% and a standard deviation of log unemployment 8 times larger than the standard deviation of GDP.

Given that the costs to switch industries are imposed by the structure of the model, σ_2 will target the level of gross intersectoral mobility. In the model, all switches involve a spell of unemployment. The CPS is not well suited for computing moments for industry mobility through unemployment. As the CPS is a short panel, there will be a important fraction of households for which we do not observe any prior industry if they are sampled while unemployed. In addition, some households that do enter unemployment may not exit it while in the sample. Therefore, I use the Survey if Income and Program Participation (SIPP) to

[35] See the discussion in Kopecky and Suen (2010) on the advantages of this procedure.

[36] To compute earnings inequality, I regress log-earnings using the CPS (Annual Social and Economic Supplement) against age, age squared, dummies for sex, race, education attainment, full-time worker status, industry and occupation for a sample of workers age 25 to 60. Inequality is computed as the standard deviation of residual log-earnings, which is 0.61 (average from 2003-2010).

Table 1: Parameters and targeted moments

Description	parameter	Value / Target
Unit of time		one month
Number of islands		14 (NAICS sectors)
Discount factor	β	5% annual
Elasticity Substitution CES	χ	2.2 (Broda and Weinstein, 2006)
Period Utility	$u(c)$	$\log(c)$
Scale param. in prod function	κ	$E(w) \approx 1$ (across industry and time)
Sector's amenity	μ_i	Sector labor share ($\approx$ 2000-2010)
Sector sensitivity to aggregate shock	λ_i	Match sectoral Employment, standard
Sector shock autocorrelation	ρ_{θ_i}	deviation, autocorrelation
Sector shock variance	$\sigma^2_{\theta_i}$	and correlation w/ GDP
Aggregate shock autocorrelation	ρ_z	Match standard deviation
Aggregate shock variance	σ^2_z	and autocorrelation of GDP
Autocorrelation for τ	ρ_τ	0.9931 (Krusell et al., 2012)
Standard deviation for τ	σ_τ	log residual earnings std = 0.61.
Home production / leisure	b	Match unemployment level
Std. Work Preference	σ_1	and standard deviation
Std. Sector Preference	σ_2	level of sectoral mobility
		through unemployment of 30%

compute the level of intersectoral gross mobility that involves at least one month of unemployment. The SIPP is survey composed of a series of panels. In each panel, households are surveyed between 2.5 to 4 years. I find that for those households that exit unemployment, roughly 29.5% do so with a major industry change.[37]

In the model I do not directly observe *realized* switches. I need to construct this variable by simulation of individual employment histories. This type of simulation is computationally very demanding, severely slowing down the estimation procedure. As an approximation, I compute model switches and target a value for them that is somewhat larger than the fraction of realized moves we observe in the data. Ex-post, I simulate individual employment histories and check whether realized moves in the model are in line with those observed in the data.

[37] Carrillo-Tudela and Visschers (2013) use the SIPP to compute occupational mobility through unemployment. They find that roughly 50% of households switch occupations through unemployment.

A summary of the previous description is displayed in Table 1. There are 63 parameters to calibrate and I target 63 moments in the data. While the previous discussion linked parameters to moments, it is clear that all parameters have an effect on all moments.

5.3 Business cycle dynamics in Island Economies

Here I show how the model performs in terms on approximating the moments we observe in the data. Besides the fit in terms of the variables that where targets in the calibration, I compare in terms of EU and UE flows, duration and intersectoral mobility.

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6 Sources of Business Cycle: Aggregate vs. Sectoral Shocks

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7 Conclusions

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Appendix A: Value Function, Labor Supply and Mobility.

Here I derive equations (7), (8) and (9) describing the lifetime expected utility the representative agent of the island, the proportion of agents that decide to work and the proportion of workers that are unemployed and potentially switch to a different island.

As I discussed in the text, the assumption for each of the realizations of the preference shocks is that they are drawn independently from a Type I Extreme Value distribution. The density and distribution functions for a Type I Extreme Value random variable are:

$$\begin{aligned} f(x) &= e^{-(x+\gamma)-e^{-(x+\gamma)}} \\ F(x) &= e^{-e^{-(x+\gamma)}} \end{aligned}$$

The domain for x is the real line. The constant $\gamma \approx 0.5772$ (Euler's constant) is a correction term to get a mean of zero.[38] I assume a total of J islands, with $2 \leq J < \infty$.

The following is the problem for an agent which has observed the realization of the the aggregate shock, the island productivity shock and his realization of τ, but has not yet observed the realization of his preference shocks on the (dis)utility of working and the value of home production. I call this the problem of the representative agent for the island.

$$V(i,\tau,\Upsilon) \quad = \quad E_{\epsilon} \max \left\{ \begin{array}{l} u_i(w(i,\Upsilon)\,\tau) + \beta\, E_{\tau',\Upsilon'}\left[V(i,\tau',\Upsilon')|\tau,\Upsilon\right] + \sigma_1\,\epsilon_W; \\ u_i(b) + \beta\, E_{\tilde{\epsilon}}\left[\max_j \left\{E_{\tau',\Upsilon'}\left[V(j,\tau',\Upsilon')|\tau,\Upsilon\right] + \sigma_2 \tilde{\epsilon}_j\right\}\right] + \sigma_1\,\epsilon_S \end{array} \right\}$$

To simplify notation let me write

$$V_k = E_{\tau',\Upsilon'}\left[V(k,\tau',\Upsilon')|\tau,\Upsilon\right]$$

Now, the expected maximum for an unemployed worker that is searching over islands and has not yet observed his island preference shocks is:

$$\begin{aligned} E_{\tilde{\epsilon}}\left[\max_k \left\{V_k + \sigma_2\,\tilde{\tilde{\epsilon}}_k\right\}\right] \quad &= \quad \sum_{k=1}^{J} \int_{-\infty}^{\infty} (V_k + \sigma_2\,\tilde{\epsilon}_k) f(\tilde{\epsilon}_k) \left[\prod_{j\neq k} F\left(\frac{V_k - V_j}{\sigma_2} + \tilde{\epsilon}_k\right)\right] d\tilde{\epsilon}_k \\ &= \quad \sum_{k=1}^{J} \int_{-\infty}^{\infty} (V_k + \sigma_2\,\tilde{\epsilon}_k) e^{-(\tilde{\epsilon}_k+\gamma) - e^{-(\tilde{\epsilon}_k+\gamma)}} \left[\prod_{j\neq k} e^{-e^{-\left(\frac{V_k - V_j}{\sigma_2} + \tilde{\epsilon}_k + \gamma\right)}}\right] d\tilde{\epsilon}_k \\ &= \quad \sum_{k=1}^{J} \int_{-\infty}^{\infty} (V_k + \sigma_2\,\tilde{\epsilon}_k)\, e^{-(\tilde{\epsilon}_k+\gamma)}\, e^{-e^{-(\tilde{\epsilon}_k+\gamma)} \sum_{j=1}^{J} e^{-\left(V_k - V_j\right)/\sigma_2}} d\tilde{\epsilon}_k \end{aligned}$$

let's call $\sum_{j=1}^{J} e^{-(V_k - V_j)/\sigma_2} = e^{\lambda}$, then

$$= \sum_{k=1}^{J} e^{-\lambda} \left[V_k + \sigma_2\,(\lambda - \gamma) + \sigma_2 \int_{-\infty}^{\infty} (\tilde{\epsilon}_k + \gamma - \lambda)\, e^{-(\tilde{\epsilon}_k+\gamma-\lambda) - e^{-(\tilde{\epsilon}_k+\gamma-\lambda)}} d\tilde{\epsilon}_k\right]$$

[38] The effective variance of the preference shocks will be adjusted by the parameter σ in the agent's problem.

$$= \sum_{k=1}^{J} e^{-\lambda} \left[V_k + \sigma_2 \, \lambda \right]$$

$$= \sigma_2 \left[\log \left(\sum_{j=1}^{J} e^{V_j/\sigma_2} \right) \right] \left[\frac{\sum_{k=1}^{J} e^{V_k/\sigma_2}}{\sum_{j=1}^{J} e^{V_j/\sigma_2}} \right]$$

$$= \sigma_2 \left[\log \left(\sum_{j=1}^{J} e^{V_j/\sigma_2} \right) \right]$$

Which is the expression in Equation (7) once we substitute back for what V_j stands for.

To compute the proportions of agents that will chose to work or switch islands, we just need to know that an agent in island i will move to island k at the end of the period if:

$$V_k + \sigma_2 \, \tilde{\epsilon}_k > V_j + \sigma_2 \, \tilde{\epsilon}_j \qquad \forall j$$

By the law of large numbers, the proportion of people that will switch from i to k is

$$\begin{aligned} Pr\left(V_k + \sigma_2 \, \tilde{\epsilon}_k > V_j + \sigma_2 \, \tilde{\epsilon}_j ; \, \forall j\right) &= Pr\left(\tilde{\epsilon}_j < V_k/\sigma_2 \, + \tilde{\epsilon}_k - V_j/\sigma_2 \, ; \, \forall j\right) \\ &= \int_{-\infty}^{\infty} f(\tilde{\epsilon}_k) \left[\prod_{j \neq k} F\left(\frac{V_k - V_j}{\sigma_2} + \tilde{\epsilon}_k \right) \right] d\tilde{\epsilon}_k \\ &= e^{-\lambda} \int_{-\infty}^{\infty} e^{-(\tilde{\epsilon}_k + \gamma - \lambda)} \, e^{-e^{-(\tilde{\epsilon}_k + \gamma - \lambda)}} d\tilde{\epsilon}_k \\ &= \frac{1}{e^{\lambda}} \\ &= \frac{e^{V_k/\sigma_2}}{\sum_{j=1}^{J} e^{V_j/\sigma_2}} \end{aligned}$$

Which after proper substitution of V_j leads to equations (8) and (9).

Appendix B: Labor demand

Here I derive the equations related to labor demand and prices faced by the island-good producers. In the island, production is constant returns to scale and uses labor only.

$$y_{i,t} = e^{\lambda_i \, z_t + \theta_{i,t}} \, L_{i,t} \tag{14}$$

The firm is competitive and maximizes profits. As there are no dynamic links between periods, the problem reduces to solve the following static maximization:

$$\max_{L_{i,t}} p_{i,t}\, e^{\lambda_i\, z_t + \theta_{i,t}}\, L_{i,t} - w_{i,t}\, L_{i,t}$$

which leads to

$$L_{i,t} = \begin{cases} 0 & if\, p_{i,t}\, e^{\lambda_i\, z_t + \theta_{i,t}} < w_{i,t} \\ [0, \infty) & if\, p_{i,t}\, e^{\lambda_i\, z_t + \theta_{i,t}} = w_{i,t} \\ \infty & if\, p_{i,t}\, e^{\lambda_i\, z_t + \theta_{i,t}} > w_{i,t} \end{cases}$$

In equilibrium, only the second case will hold and firms will make zero profits.

The final good producers have the following production function

$$y_t = \kappa \left(\sum_i y_{i,t}^{\frac{\chi-1}{\chi}} \right)^{\frac{\chi}{\chi-1}}$$

Their problem also reduces to a static profit maximization:

$$\max_{\{y_{i,t}\}} p_t\, \kappa \left(\sum_i y_{i,t}^{\frac{\chi-1}{\chi}} \right)^{\frac{\chi}{\chi-1}} - \sum_i p_{i,t}\, y_{i,t}$$

The first order condition is

$$y_{i,t} = y_t\, \kappa^{\chi-1}\, p_t^{\chi}\, p_{i,t}^{-\chi}$$

Replacing $y_{i,t} = e^{\lambda_i\, z_t + \theta_{i,t}} L_{i,t}$ and $p_{i,t}\, e^{\lambda_i\, z_t + \theta_{i,t}} = w_{i,t}$ and re-arranging we have:

$$L_{i,t} = \kappa^{\chi-1}\, y_t\, \frac{\left(e^{\lambda_i\, z_t + \theta_{i,t}}\right)^{\chi-1}}{(w_{i,t}/p_t)^{\chi}}$$

And taking p_t to be the numeraire, we have the expressions used in section 2

References

Abraham, K. and Katz, L. (1986). Cyclical unemployment: Sectoral shifts or aggregate disturbances? *Journal of Political Economy*, 94(3):507–522.

Acemoglu, D., Carvalho, V., Ozdaglar, A., and Tahbaz-Salehi, A. (2012). The network origins of aggregate fluctuations. *Econometrica*, 80(5):1977–2016.

Alvarez, F. and Shimer, R. (2009). Unemployment and human capital. *University of Chicago, mimeo*.

Alvarez, F. and Shimer, R. (2011). Search and rest unemployment. *Econometrica*, 79(1):75–122.

Alvarez, F. and Veracierto, M. (2000). Labor-market policies in an equilibrium search model. *NBER Macroeconomics Annual 1999*, page 265.

Artuc, E., Chaudhuri, S., and McLaren, J. (2010). Trade shocks and labor adjustment: A structural empirical approach. *American Economic Review*, 100(3):1008–45.

Aruoba, S., Fernandez-Villaverde, J., and Rubio-Ramirez, J. (2006). Comparing solution methods for dynamic equilibrium economies. *Journal of Economic Dynamics and Control*, 30(12):2477–2508.

Bils, M., Chang, Y., and Kim, S. (2011). Worker heterogeneity and endogenous separations in a matching model of unemployment fluctuations. *American Economic Journal: Macroeconomics*, 3(1):128–154.

Blanchard, O., Diamond, P., Hall, R., and Yellen, J. (1989). The beveridge curve. *Brookings papers on economic activity*, 1989(1):1–76.

Blanchard, O. and Kahn, C. (1980). The solution of linear difference models under rational expectations. *Econometrica: Journal of the Econometric Society*, pages 1305–1311.

Broda, C. and Weinstein, D. (2006). Globalization and the gains from variety. *The Quarterly Journal of Economics*, 121(2):541–585.

Caldara, D., Fernandez-Villaverde, J., Rubio-Ramirez, J., and Yao, W. (2011). Computing dsge models with recursive preferences and stochastic volatility. *Review of Economic Dynamics*.

Campbell, J. (1998). Entry, exit, embodied technology, and business cycles. *Review of Economic Dynamics*, 1(2):371–408.

Carrillo-Tudela, C. and Visschers, L. (2013). Unemployment and endogenous reallocation over the business cycle. *Working Paper*.

Coen-Pirani, D. (2010). Understanding gross worker flows across us states. *Journal of Monetary Economics*, 57(7):769–784.

Dixit, A. and Rob, R. (1994). Switching costs and sectoral adjustments in general equilibrium with uninsured risk. *Journal of Economic Theory*, 62(1):48–69.

Elsby, M., Hobijn, B., and Sahin, A. (2012). On the importance of the participation margin for labor market fluctuations. *Working Paper*.

Fernandez-Villaverde, J., Guerron-Quintana, P., Rubio-Ramirez, J., and Uribe, M. (2011). Risk matters: The real effects of volatility shocks. *American Economic Review*, 101(6).

Foerster, A., Sarte, P., and Watson, M. (2011). Sectoral versus aggregate shocks: A structural factor analysis of industrial production. *Journal of Political Economy*, 119(1):1–38.

Fujita, S. and Moscarini, G. (2012). Recall and unemployment. *Working Paper*.

Gomme, P. and Klein, P. (2011). Second-order approximation of dynamic models without the use of tensors. *Journal of Economic Dynamics and Control*, 35(4):604–615.

Gouge, R. and King, I. (1997). A competitive theory of employment dynamics. *The Review of Economic Studies*, 64(1):1–122.

Guvenen, F. (2011). Macroeconomics with heterogeneity: A practical guide. *Working Paper, National Bureau of Economic Research*.

Hamilton, J. D. (1988). A neoclassical model of unemployment and the business cycle. *The Journal of Political Economy*, pages 593–617.

Horvath, M. (2000). Sectoral shocks and aggregate fluctuations. *Journal of Monetary Economics*, 45(1):69–106.

Jacobson, L., LaLonde, R., and Sullivan, D. (1993). Earnings losses of displaced workers. *The American Economic Review*, pages 685–709.

Judd, K. (1998). *Numerical methods in economics*. MIT press.

Judd, K. L. (1996). Approximation, perturbation, and projection methods in economic analysis. *Handbook of computational economics*, 1:509–585.

Kambourov, G. and Manovskii, I. (2009). Occupational mobility and wage inequality. *Review of Economic Studies*, 76(2):731–759.

Kim, J. and Kim, S. (2003). Spurious welfare reversals in international business cycle models. *journal of International Economics*, 60(2):471–500.

Kline, P. (2008). Understanding sectoral labor market dynamics: An equilibrium analysis of the oil and gas field services industry. *Department of Economics, UC Berkeley, mimeo*.

Kopecky, K. and Suen, R. (2010). Finite state markov-chain approximations to highly persistent processes. *Review of Economic Dynamics*, 13(3):701–714.

Krusell, P., Mukoyama, T., Rogerson, R., and Şahin, A. (2012). Is labor supply important for business cycles? *National Bureau of Economic Research*.

Lee, D. and Wolpin, K. (2006). Intersectoral labor mobility and the growth of the service sector. *Econometrica*, 74(1):1–46.

Lilien, D. (1982). Sectoral shifts and cyclical unemployment. *The Journal of Political Economy*, pages 777–793.

Lucas, R. and Prescott, E. (1974). Equilibrium search and unemployment. *Journal of Economic Theory*, 7(2):188–209.

Mangum, K. (2010). A dynamic model of local labor markets. *unpublished, Department of Economics, Duke University*.

McCall, J. J. (1970). Economics of information and job search. *The Quarterly Journal of Economics*, pages 113–126.

Menzio, G. and Shi, S. (2011). Efficient search on the job and the business cycle. *Journal of Political Economy*, 119(3):468–510.

Mertens, T. and Judd, K. (2012). Equilibrium existence and approximation for incomplete market models with substantial heterogeneity. *Available at SSRN*.

Moscarini, G. and Thomsson, K. (2007). Occupational and job mobility in the us. *The Scandinavian Journal of Economics*, 109(4):807–836.

Neal, D. (1995). Industry-specific human capital: Evidence from displaced workers. *Journal of labor Economics*, pages 653–677.

Pilossoph, L. (2012). A multisector equilibrium search model of labor reallocation. *Working Paper*.

Reinhart, C. M. and Rogoff, K. S. (2009). *This time is different: Eight centuries of financial folly*. Princeton University Press.

Reiter, M. (2009). Solving heterogeneous-agent models by projection and perturbation. *Journal of Economic Dynamics and Control*, 33(3):649–665.

Rouwenhorst, G. K. (1995). Asset pricing implications of equilibrium business cycle models. In Cooley, T., editor, *Frontiers of Business Cycle Research*, pages 294–330. Princeton University Press.

Rust, J. (1987). Optimal replacement of gmc bus engines: An empirical model of harold zurcher. *Econometrica: Journal of the Econometric Society*, pages 999–1033.

Rust, J. (1994). Structural estimation of markov decision processes. *Handbook of econometrics*, 4:3081–3143.

Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. *American economic review*, pages 25–49.

Sullivan, P. (2010). Empirical evidence on occupation and industry specific human capital. *Labour Economics*, 17(3):567–580.

Veracierto, M. (2008). On the cyclical behavior of employment, unemployment and labor force participation. *Journal of Monetary Economics*, 55(6):1143–1157.

Wiczer, D. (2013). Long-term unemployment: Attached and mismatched. *Working Paper*.