Investment-specific Technology Shocks and Recursive Preferences[1]

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Håkon Tretvoll,[2]

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Abstract

Investment-specific technology (IST) shocks have been shown to play a significant role in explaining the business cycle dynamics of the US economy. Recently a debate has arisen over the extent to which these shocks aid our understanding of international business cycles as well. This paper presents an international business cycle model where agents have recursive preferences, and shows that this specification of preferences plays a crucial role for the impact of IST shocks on the joint dynamics of international prices and quantities. The most striking result is that the model generates real exchange rate fluctuations that exhibit both a high volatility relative to output and are uncorrelated with relative consumptions across countries.

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[1]All errors are mine.

[2]BI Norwegian Business School, N-0442 Oslo, Norway. Email: hakon.tretvoll@bi.no

1 Introduction

Investment-specific technological change has been shown to play an important role in accounting for the business cycle in the United States. First introduced in a neoclassical framework by Greenwood et al. (1988), the importance of shocks to the investment technology has been established by Greenwood et al. (1997) and Fisher (2006). More recently Justiniano and Primiceri (2008) and Justiniano et al. (2010) establish that investment-specific technology (IST) shocks account for about 50 percent of the variance of output while productivity (TFP) shocks account for about 25 percent.

Raffo (2010) argues that IST shocks also help explain the joint dynamics of international prices and quantities. He focuses on two features of international business cycles that have proven difficult to account for in the standard international real business cycle framework developed by Backus et al. (1992, 1994). First, international relative prices tend to appreciate when domestic consumption and output increase more than those abroad. Second, both relative prices and trade flows are volatile over the business cycle. Raffo develops a model with both TFP and IST shocks and incorporates two key features from Greenwood et al. (1988): variable capacity utilization and preferences with no intertemporal substitution of labor (GHH preferences). His main finding is that the model can match the two features of the data, and he therefore argues that IST shocks are a key component of international business cycle models.

Mandelman et al. (2011) take a different approach to analyzing whether IST shocks can explain key features of international data. Whereas Raffo calibrates the exogenous shock processes so that IST shocks account for most of the variability of output (as in Justiniano et al. (2010)), Mandelman et al. instead estimate processes for IST shocks from data on investment and consumption deflators. Crucially, their estimates imply a much lower volatility for the IST shock process than that used in Raffo's calibration. They show that in a very similar model where the exogenous processes for TFP and IST shocks are estimated rather than calibrated, the ability of the IST shocks to account for puzzling features of international business cycles is greatly diminished.

In this paper we model agents with recursive preferences in an environment similar to that of Mandelman et al. The model features TFP and IST shocks that are parameterized based on Mandelman et al.'s estimates, but the change in the preference specification nevertheless delivers real exchange rate fluctuations that are very different. The real exchange rate is the relative price of consumption across countries. In the standard model, where agents have power utility over a composite of consumption and leisure, only the short-run differences in these quantities across countries affect the current currency value. The low volatility of these differences therefore implies a low standard deviation of the real exchange rate, which also tends to be perfectly correlated with relative consumption across countries. As pointed out by Backus and Smith (1993) this is not the case in the data.

In contrast, in the model with recursive preferences, agents' attitudes towards risk and substitution over time are modeled separately. This makes preferences non-separable over future states and implies that agents want to smooth future utility as well as consumption and leisure. They are therefore sensitive to the long-run prospects of the economy, and the exchange rate adjusts to reflect both current and future differences in allocations. The dynamics in quantities and prices are driven by TFP and IST shocks with unit roots as estimated by Mandelman et al. These shocks therefore have a long-run impact on the economy and the long-run effects impact recursive agents' marginal utilities for consumption today.

The two countries specialize in the production of intermediate goods. These goods are traded internationally, and used as inputs in the production of final goods that are allocated to consumption and investment. There is home bias in the production of final goods, so that the domestic intermediate good is the main input in each country. Persistent shocks lead to persistent fluctuations in the relative price of the intermediate inputs. Home bias implies that these relative price fluctuations affect the cost of consumption differently across countries, and therefore lead to large movements in the real exchange rate. Sharing of the long-run risks related the permanent shocks also break the perfect correlation between the real exchange rate and relative consumptions. The model can therefore account for both the volatility of real exchange rates and their disconnect from current quantities (the Backus-Smith puzzle).

Since Tallarini (2000) it is well known that the alternative preference specification applied in this paper enables a closed production economy to jointly match key moments of both quantities and asset prices. In the model presented here it is also the case that agents' sensitivity to news about future consumption growth shows up as additional components of their stochastic discount factors. With permanent TFP and IST shocks the model is able to produce highly volatile stochastic discount factors due to these "long-run" components. As in Tallarini (2000) the model therefore implies a high market price of risk when risk aversion is high.

1.1 Related literature

- Greenwood et al. (1988, 2000)
- Raffo (2010)

- Mandelman et al. (2011)
- Justiniano and Primiceri (2008); Justiniano et al. (2010)
- Fisher (2006)
- Cummins and Violante (2002)
- Heathcote and Perri (2002)
- Tretvoll (2013)
- Colacito et al. (2012)

1.2 Organization

This paper is organized as follows. Section 2 describes the model and the equilibrium conditions for that economy. Section 3 discusses a recursive formulation of the social planner's problem which allows us to solve for equilibrium allocations. In section 4 we parameterize the model using standard values from the international macroeconomics literature and discuss our main results. Section 6 concludes.

2 The model

The model presented here builds on the standard IRBC framework developed by Backus et al. (1992, 1994) and in particular to the two-country, two-goods model described in Heathcote and Perri (2002). Following Raffo (2010) and Mandelman et al. (2011) we include both investment-specific technology shocks and TFP shocks in the model. As those authors we also include endogenous capital utilization as in Greenwood et al. (1988) and a quadratic adjustment cost in the capital stock. Our main innovation is to model agents with recursive preferences in this framework.

The two countries are referred to as "home" h, and "foreign" f. In each country an intermediate good is produced which is traded internationally, and a final good is produced using the two intermediate goods as inputs. For convenience we will occasionally refer to the final good produced in country $i \in \{h, f\}$ as "final good i", and the intermediate good produced in country i as "intermediate good i". International financial asset markets are complete.

Time is discrete. Each period there is a realization of a random event s_t, and $s^t = (s_0, s_1, \dots, s_t)$ denotes the history of events up to an including time t. The probability at time 0 of a particular history of events is $\pi(s^t)$, and s_0 is given. In general allocations in period t are functions of the history s^t and of initial values for capital stocks $K_{i,0}$ and asset holdings $A_{i,0}$ for $i \in \{h, f\}$, but for notational convenience we will suppress the dependence on these initial values.

2.1 Households

In each country the representative, infinitely lived, agent maximizes a recursive utility function over an aggregate of the final consumption good and leisure. We will refer to the representative consumer in country i as "agent i". As in Epstein and Zin (1989) and Weil (1990), preferences are modeled using the formulation from Kreps and Porteus (1978). The utility of agent $i \in \{h, f\}$ is defined by[1]

$$U_{i,t} = \left[(1-\beta)\mathcal{C}(C_{i,t}, n_{i,t})^{\rho} + \beta\left(\mathbb{E}_t(U_{i,t+1}^{\alpha})\right)^{\rho/\alpha}\right]^{1/\rho} \tag{1}$$

The function $\mathcal{C}(C_{i,t}, n_{i,t})$ determines the contribution to utility from consumption and labor in the current period. Two different functional forms are considered for this function. The first is the standard Cobb-Douglas form as in Heathcote and Perri (2002), and the second is the GHH specification from Greenwood et al. (1988):

$$\mathcal{C}(C_{i,t}, n_{i,t}) = C_{i,t}^{\tau}(1-n_{i,t})^{1-\tau} \tag{2}$$

$$\mathcal{C}(C_{i,t}, n_{i,t}) = C_{i,t} - \psi n_{i,t}^{\nu} \tag{3}$$

The parameter τ in the first specification is the share of consumption in the consumption-leisure bundle. As discussed in section 4, the parameters ν and ψ in the GHH specification will be set to imply the same steady-state value for the household's labor supply and the same labor supply elasticity as in the Cobb-Douglas case.

The parameter β is the discount factor, the intertemporal elasticity of substitution (IES) over the consumption-leisure aggregate is $1/(1-\rho)$, and risk aversion towards static gambles over the aggregate is measured by $1-\alpha$.

[1]The special cases where $\rho = 0$ or $\alpha = 0$ are easily handled by taking the appropriate limits. For brevity these limiting expressions are not presented here.

The household maximizes (1) subject to a budget constraint, a capital evolution equation, and given initial values of the capital stock $K_{i,0}$ and asset holdings $A_{i,0}$. The budget constraint in country h is given by:

$$C_{h,t}(s^t) + I_{h,t}(s^t) + \sum_{s_{t+1}|s_t} q_t(s_{t+1}|s^t)A_{h,t}(s_{t+1}|s^t) = \\ W_{h,t}(s^t)n_{h,t}(s^t) + r_{h,t}(s^t)u_{h,t}(s^t)K_{h,t}(s^{t-1}) + A_{h,t}(s^t) \quad (4)$$

Here $C_{h,t}(s^t) \geq 0$ is units of consumption of the final good in country h, $I_{h,t}(s^t) \geq 0$ is investment, $n_{h,t}(s^t) \in [0,1]$ is the fraction of time spent working, and $K_{h,t}(s^{t-1}) \geq 0$ is the capital stock in country h at the beginning of period t. $u_{h,t}(s^t)$ is the endogenous utilization rate of capital which is discussed further below. $W_{h,t}(s^t)$ is the wage in country h, and $r_{h,t}(s^t)$ is the rental rate of capital. The prices of both factor inputs are measured in units of the final good in country h. The household has access to a complete set of state contingent securities that deliver one unit of the final good from country h. $A_{h,t}(s^t)$ is the number of such claims owned by the representative agent in country h, and $q_t(s_{t+1}|s^t)$ is the price of one such claim in time $t+1$ after history s^t measured in units of final good h. Households issue state contingent securities subject to state-by-state debt limits that ensure that it is always feasible for the household to repay its state contingent debt.[2]

The budget constraint in country f is similar, but is written in terms of the final good in that country. So the prices of the factor inputs $W_{f,t}(s^t)$ and $R_{f,t}(s^t)$ are measured in terms of the final good in f. The budget constraint is thus

$$C_{f,t}(s^t) + I_{f,t}(s^t) + \frac{1}{e_t(s^t)} \sum_{s_{t+1}|s_t} q_t(s_{t+1}|s^t)A_{f,t}(s_{t+1}|s^t) = \\ W_{f,t}(s^t)n_{f,t}(s^t) + r_{f,t}(s^t)u_{f,t}(s^t)K_{f,t}(s^{t-1}) + \frac{A_{f,t}(s^t)}{e_t(s^t)} \quad (5)$$

where $e_t(s_t)$ is the real exchange rate defined as the relative price of the final good in country f relative to the final good in country h.

The capital evolution equation in country $i \in \{h, f\}$ is

$$K_{i,t+1}(s^t) = \left[1 - \delta(u_{i,t}(s^t))\right] K_{i,t}(s^{t-1}) + \\ Z^I_{i,t}(s^t)\left\{ I_{i,t}(s^t) - \frac{\phi}{2} I_{i,t-1}(s^{t-1}) \frac{Z^I_{i,t-1}(s^{t-1})}{Z^I_{i,t}(s^t)} \left[\frac{I_{i,t}(s^t)Z^I_{i,t}(s^t)}{I_{i,t-1}(s^{t-1})Z^I_{i,t-1}(s^{t-1})} - \Delta_I \right]^2 \right\} \quad (6)$$

[2]These limits are often referred to as "natural" debt limits, and in the equilibrium of this model, they are never binding.

The depreciation rate of the stock of capital, $\delta(u_{i,t}(s^t))$, is a function of the rate of utilization $u_{i,t}(s^t)$. Following Greenwood et al. (1988) this function is given by

$$\delta(u_{i,t}(s^t)) = \bar{\delta} + \frac{b}{1+\varepsilon} u_{i,t}(s^t)^{1+\varepsilon} \tag{7}$$

where $b \geq 0$ and $\varepsilon \geq 0$. The parameter ε is the elasticity of marginal depreciation with respect to the utilization rate, and b and $\bar{\delta}$ pin down the rates of utilization and depreciation in the steady state.

Finally, $Z^I_{i,t}(s^t)$ is the investment-specific technology shock, Δ_I is the long-run gross rate of growth of investment along the balanced growth path, and the elasticity of the capital adjustment cost to changes in investment is determined by ϕ.

2.2 Firms

In each country there are producers of intermediate and final goods. In this section we describe these firms' problems and the processes for productivity and investment-specific technology shocks in the two countries.

2.2.1 Final goods production

A representative firm in the final goods sector produces using a CES production function where the inputs are the intermediate goods produced in both countries. In each country there is a bias in the final goods production towards the intermediate good produced in that country. The demand in country i for intermediate good j is denoted by $X_{ij,t}$ for $i, j \in \{h, f\}$. The production functions of the final goods in the two countries are given by

$$\begin{aligned} G_h\left(X_{hh,t}(s^t), X_{hf,t}(s^t)\right) &= \left[\eta (X_{hh,t}(s^t))^{\frac{\sigma-1}{\sigma}} + (1-\eta)(X_{hf,t}(s^t))^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\ G_f\left(X_{fh,t}(s^t), X_{ff,t}(s^t)\right) &= \left[(1-\eta)(X_{fh,t}(s^t))^{\frac{\sigma-1}{\sigma}} + \eta (X_{ff,t}(s^t))^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \end{aligned} \tag{8}$$

The parameter σ is the elasticity of substitution between the two intermediate goods[3], and home bias in final goods production is given by each country putting weight $\eta > 1/2$ on the intermediate good produced in that country.

[3]For $\sigma = 1$ the final goods production function is Cobb-Douglas.

The representative final goods producers take the prices of the intermediate inputs as given and solve the following problem:

$$\max_{X_{ih,t}(s^t),X_{if,t}(s^t)\geq 0} G_i\left(X_{ih,t}(s^t),X_{if,t}(s^t)\right) - q_{ih,t}(s^t)X_{ih,t}(s^t) - q_{if,t}(s^t)X_{if,t}(s^t) \tag{9}$$

where $q_{ij,t}(s^t)$ is the price in country i of intermediate good j in units of final good i.

2.2.2 Intermediate goods production

In each country, a representative firm in the intermediate goods sector produces using capital and labor with a standard constant returns to scale production function. These firms take prices of goods and factor inputs as given and maximize profits by solving

$$\begin{aligned}\max_{n_{i,t}(s^t),K_{i,t}(s^{t-1})\geq 0} & q_{ii,t}(s^t)Z^P_{i,t}(s^t)(u_{i,t}(s^t)K_{i,t}(s^{t-1}))^{\theta}n_{i,t}(s^t)^{1-\theta} \\ & - W_{i,t}(s^t)n_{i,t}(s^t) - r_{i,t}(s^t)u_{i,t}(s^t)K_{i,t}(s^{t-1})\end{aligned} \tag{10}$$

where $Z^P_{i,t}(s^t)$ is the productivity shock in country i, and θ is the share of capital in production which is the same across countries. Gross domestic product in country i in units of final good i is denoted $Y_{i,t}(s^t)$ and is given by

$$Y_{i,t}(s^t) = q_{ii,t}(s^t)Z^P_{i,t}(s^t)(u_{i,t}(s^t)K_{i,t}(s^{t-1}))^{\theta}n_{i,t}(s^t)^{1-\theta} \tag{11}$$

2.2.3 Processes for TFP and IST shocks

Following Mandelman et al. (2011) we let both the log-TFP shocks, $\log Z^P_{i,t}$, and the log-IST shocks, $\log Z^I_{i,t}$, be cointegrated of order $C(1,1)$ across countries. This is consistent with estimates of these processes in Rabanal et al. (2011) and Mandelman et al. (2011) which the parameterization of the processes is based on. The law of motion for the log-first differences of shock $F \in \{P, I\}$ in the home and the foreign country is specified by the following VECM:

$$\begin{pmatrix} \Delta \log Z^F_{h,t} \\ \Delta \log Z^F_{f,t} \end{pmatrix} = \begin{pmatrix} \phi^F_{0h} \\ \phi^F_{0f} \end{pmatrix} + \left[\log Z^F_{h,t-1} - \kappa^F \log Z^F_{f,t-1}\right] \begin{pmatrix} \phi^F_{1h} \\ \phi^F_{1f} \end{pmatrix} + \begin{pmatrix} \varepsilon^F_{h,t} \\ \varepsilon^F_{f,t} \end{pmatrix} \tag{12}$$

where $(1, -\kappa^F)$ is the cointegrating vector, $(\varepsilon^F_{h,t}, \varepsilon^F_{f,t})^T \sim N(0, \Sigma^F)$, and Δ is the first-difference operator.

2.3 International trade and prices

International trade in the intermediate inputs is costless, and the law of one price holds. As the prices in the respective countries are denoted in terms of the local final goods, the following equation holds for the price of intermediate good $i \in \{h, f\}$:

$$q_{hi,t}(s^t) = e_t(s^t) q_{fi,t}(s^t) \tag{13}$$

Therefore the real exchange rate can be expressed in terms of the prices of either of the intermediate goods in the two countries.

$$e_t(s^t) = \frac{q_{hi,t}(s^t)}{q_{fi,t}(s^t)}, \qquad i = h, f \tag{14}$$

In addition, the terms of trade is defined as the price of imports in country h relative to exports from country h (i.e. the price of intermediate good f relative to intermediate good h):

$$p_t(s^t) = \frac{q_{hf,t}(s^t)}{q_{hh,t}(s^t)} \tag{15}$$

Other variables of interest related to international trade are the import ratio and net exports. For country i we denote the import ratio $ir_{i,t}(s^t)$ and, following Heathcote and Perri (2002), define it as the ratio of imports to non-traded domestic intermediate good production:

$$ir_{i,t}(s^t) = \frac{X_{ij,t}(s^t)}{X_{ii,t}(s^t)} \qquad j \neq i \tag{16}$$

Net exports as a fraction of GDP in country i is denoted $nx_{i,t}(s^t)$ and is given by

$$nx_{i,t}(s^t) = \frac{q_{ii,t}(s^t) X_{ji,t}(s^t) - q_{ij,t}(s^t) X_{ij,t}(s^t)}{Y_{i,t}(s^t)} \qquad j \neq i \tag{17}$$

2.4 Market clearing

The model is closed with market clearing conditions in the markets for the final and intermediate goods in each country, as well as in the international financial asset market. The state-contingent claims are in zero net supply, so after every history s^t we have for every possible state s_{t+1} in the subsequent period that

$$A_{h,t}(s_{t+1}|s^t) + A_{f,t}(s_{t+1}|s^t) = 0 \tag{18}$$

In the markets for the final and intermediate goods the market clearing conditions are given by

$$G_i\left(X_{ih,t}(s^t), X_{if,t}(s^t)\right) = C_{i,t}(s^t) + I_{i,t}(s^t) \qquad i \in \{h, f\} \tag{19}$$

$$Z^P_{i,t}(s^t)(u_{i,t}(s^t)K_{i,t}(s^{t-1}))^{\theta} n_{i,t}(s^t)^{1-\theta} = X_{hi,t}(s^t) + X_{fi,t}(s^t) \qquad i \in \{h, f\} \tag{20}$$

2.5 Definition of equilibrium

Given the law of motion for productivity and investment-specific shocks defined by equation (12), an equilibrium in this economy is a set of allocations for households in both countries, $C_{i,t}(s^t)$, $n_{i,t}(s^t)$, $I_{i,t}(s^t)$,$u_{i,t}(s^t)$, $K_{i,t}(s^t)$, and $A_{i,t}(s^t)$ for $i \in \{h, f\}$; allocations for intermediate good producers in both countries $X_{ij,t}(s^t)$ for $i, j \in \{h, f\}$; allocations for final good producers in both countries $G_i(X_{ih,t}(s^t), X_{if,t}(s^t))$ for $i \in \{h, f\}$; intermediate goods prices in both countries $q_{ij,t}(s^t)$ for $i, j \in \{h, f\}$; factor prices in both countries $W_{i,t}(s^t)$ and $r_{i,t}(s^t)$, and the prices of state-contingent assets $q_t(s_{t+1}|s^t)$ such that: (*i*) given prices, the households' allocations solve the households' problem; (*ii*) given prices, the intermediate goods producers' allocations solve the intermediate goods producers' problem; (*iii*) given prices, the final goods producers' allocations solve the final goods producers' problem; (*iv*) markets clear.

2.6 Equilibrium conditions

The equilibrium conditions include the first-order conditions of the households, and the intermediate and final goods producing firms in both countries. In addition, there are the laws of motion for capital and the productivity and investment-specific shocks, the production functions, and the market clearing conditions.

2.6.1 Households

The households' first-order conditions for consumption and labor supply are

$$(1-\beta)U_{i,t}(s^t)^{1-\rho}\mathcal{C}_{i,t}(s^t)^{\rho-1}\frac{\partial \mathcal{C}_{i,t}(s^t)}{\partial C_{i,t}(s^t)} = \lambda_{i,t}(s^t) \tag{21}$$

$$\frac{\partial \mathcal{C}_{i,t}(s^t)/\partial n_{i,t}(s^t)}{\partial \mathcal{C}_{i,t}(s^t)/\partial C_{i,t}(s^t)} = W_{i,t}(s^t) \tag{22}$$

where $\lambda_{i,t}(z^t)$ is the lagrange multiplier on the household's budget constraint, and for convenience we use the notation $\mathcal{C}_{i,t}(s^t) = \mathcal{C}(C_{i,t}(s^t), n_{i,t}(s^t))$.

The first-order condition with respect to the choice of capital utilization is

$$r_{i,t}(s^t) = q^K_{i,t}(s^t)\delta^{'}(u_{i,t}(s^t)) \tag{23}$$

where $q^K_{i,t}(s^t)$ is the price of a unit of capital in terms of the final consumption good:

$$q^K_{i,t}(s^t) = \left\{ Z^I_{i,t}(s^t)\left(1 - \phi\left[\frac{I_{i,t}(s^t)Z^I_{i,t}(s^t)}{I_{i,t-1}(s^{t-1})Z^I_{i,t-1}(s^{t-1})} - \Delta_I\right]\right)\right\}^{-1} \tag{24}$$

With the preferences specified in equation (1), the households' stochastic discount factors $m_{i,t+1}(s_{t+1}|s^t)$ are given by

$$m_{i,t+1}(s_{t+1}|s^t) = \\ \beta\left(\frac{\mathcal{C}_{i,t+1}(s^{t+1})}{\mathcal{C}_{i,t}(s^t)}\right)^{\rho-1}\frac{\partial \mathcal{C}_{i,t+1}(s^{t+1})/\partial C_{i,t+1}(s^{t+1})}{\partial \mathcal{C}_{i,t}(s^t)/\partial C_{i,t}(s^t)}\left(\frac{U_{i,t+1}(z_{t+1}|z^t)}{\zeta_t(U_{i,t+1})}\right)^{\alpha-\rho} \tag{25}$$

where $\zeta_t(U_{i,t+1}) = \left(\mathbb{E}_t U^{\alpha}_{i,t+1}\right)^{1/\alpha}$ is the certainty equivalent of future utility, and $\mathbb{E}_t$ is the conditional expectation operator conditioning on information available at time t.

The first-order condition with respect to capital delivers the standard equation for pricing returns

$$1 = \mathbb{E}_t\left(m_{i,t+1}(s_{t+1})R^K_{i,t+1}(s_{t+1})|s^t\right) \tag{26}$$

where $R^K_{i,t+1}(s_{t+1}|s^t)$ is the return on capital investment given by

$$R^K_{i,t+1}(s_{t+1}|s^t) = \frac{1}{q^K_{i,t}(s^t)}\Bigg(q_{ii,t+1}(s^{t+1})Z^P_{i,t+1}(s^{t+1})\theta\left(\frac{n_{i,t+1}(s^{t+1})}{u_{i,t+1}(s^{t+1})K_{i,t+1}(s^{t+1})}\right)^{1-\theta}u_{i,t+1}(s^{t+1}) \\ + q^K_{i,t+1}(s^{t+1})\left[1 - \delta(u_{i,t+1}(s^{t+1}))\right]\Bigg) \tag{27}$$

The remaining first-order condition for the households with respect to the choice of state-contingent assets, deliver the following expressions for the asset prices:

$$\begin{aligned} q_t(s_{t+1}|s^t) &= \pi(s_{t+1}|s^t)m_{h,t+1}(s_{t+1}|s^t) \\ &= \pi(s_{t+1}|s^t)m_{f,t+1}(s_{t+1}|s^t)\frac{e_t(s^t)}{e_{t+1}(s_{t+1}|s^t)} \end{aligned} \tag{28}$$

This delivers the relationship between the stochastic discount factors across countries and real exchange rate dynamics discussed in Backus, Foresi, and Telmer (2001).

2.6.2 Firms

The final goods firms' first-order conditions for demand of intermediate inputs from both countries determine the prices of the intermediate goods in each country in terms of the final good in that country.

$$\frac{\partial G_i\left(X_{ih,t}(s^t), X_{if,t}(s^t)\right)}{\partial X_{ij,t}(s^t)} = q_{ij,t}(s^t) \tag{29}$$

The intermediate goods firms' first-order conditions imply that labor and capital are paid their marginal product:

$$W_{i,t}(s^t) = (1-\theta)q_{ii,t}(s^t)Z^P_{i,t}(s^t)\left(\frac{u_{i,t}(s^t)K_{i,t}(s^{t-1})}{n_{i,t}(s^t)}\right)^{\theta} \tag{30}$$

$$r_{i,t}(s^t) = \theta q_{ii,t}(s^t)Z^P_{i,t}(s^t)\left(\frac{n_{i,t}(s^t)}{u_{i,t}(s^t)K_{i,t}(s^{t-1})}\right)^{1-\theta} \tag{31}$$

2.7 Terms of trade and real exchange rate

The expressions for intermediate goods prices in equation (29) and the definition of the terms of trade in equation (15) gives the following equilibrium expressions for the terms of trade in terms of intermediate goods allocations in either country h or f:

$$p_t(s^t) = \frac{1-\eta}{\eta}\left(\frac{X_{hh,t}(s^t)}{X_{hf,t}(s^t)}\right)^{\frac{1}{\sigma}} = \frac{\eta}{1-\eta}\left(\frac{X_{fh,t}(s^t)}{X_{ff,t}(s^t)}\right)^{\frac{1}{\sigma}} \tag{32}$$

The equations for intermediate goods prices, the definition of the real exchange rate in equation (14), and equation (32) for the terms of trade imply the following expression which shows the equilibrium relationship between the real exchange rate and the terms of trade:

$$e_t(s^t) = \left(\frac{(1-\eta)^\sigma + \eta^\sigma (p_t(s^t))^{1-\sigma}}{\eta^\sigma + (1-\eta)^\sigma (p_t(s^t))^{1-\sigma}}\right)^{\frac{1}{1-\sigma}} \tag{33}$$

2.7.1 Remaining conditions

The remaining conditions necessary to solve for equilibrium are the laws of motion of capital (6); the law of motion of productivity (12); the relationships between intermediate goods prices across countries (13); and the market clearing conditions (18), (19), and (20).

3 The planner's problem

Since international financial markets are complete, we can solve the social planner's problem for the competitive equilibrium allocations in this economy, and use the allocations to solve for prices. When households have recursive preferences as specified in equation (1) as opposed to standard additive preferences, an additional state variable is necessary to obtain a recursive value function for the planner. The formulation used here is that the planner maximizes the utility of agent h subject to a constraint on the utility of agent f. The additional state variable is then the promised utility to agent f, $\bar{U}_{f,t}$.[4]

In the recursive formulation of the problem allocations at time t are only a function of the vector of state variables at time t rather than the entire history. The exogenous state variables associated with the productivity and investment specific shocks we label

$$\tilde{Z}_t = (Z^P_{h,t}, Z^P_{f,t}, Z^I_{h,t}, Z^I_{f,t}) \tag{34}$$

This vector endogenous and exogenous statevariables s_t, is then given by

$$s_t = (K_{h,t}, K_{f,t}, \bar{U}_{f,t}, \tilde{Z}_t) \tag{35}$$

[4]For an alternative formulation of a recursive planner's problem see Colacito and Croce (2013, 2011) who follow Anderson (2005).

In each period the planner chooses allocations and a promised utility to agent f for each possible realization of the exogenous state variables next period. The planner's problem is then:

Given s_t, choose $C_{i,t} \geq 0$, $n_{i,t} \in [0,1]$, $K_{i,t+1} \geq 0$, $I_{i,t} \geq 0$, $u_{i,t} \geq 0$, $X_{ij,t} \geq 0$, for $i, j \in \{h, f\}$, and the function $\bar{U}_{f,t+1}(\tilde{Z}_{t+1})$ for all possible realizations of $\tilde{Z}_{t+1}$, in order to solve

$$Q(s_t) = \max \quad \left[(1-\beta)\mathcal{C}(C_{h,t}, n_{h,t})^{\rho} + \beta\left(\mathbb{E}(Q(s_{t+1})^{\alpha}|s_t)\right)^{\rho/\alpha}\right]^{1/\rho}$$

subject to

$$\left[(1-\beta)\mathcal{C}(C_{f,t}, n_{f,t})^{\rho} + \beta\left(\mathbb{E}(\bar{U}_{f,t+1}(\tilde{Z}_{t+1})^{\alpha}|s_t)\right)^{\rho/\alpha}\right]^{1/\rho} \geq \bar{U}_{f,t} \tag{36}$$

+ the laws of motion for capital in equation (6), the laws of motion for productivity and investment-specific shocks in equation (12), and the market clearing conditions for final and intermediate goods in equations (19) and (20). For notational convenience the dependence of all the choice variables on the state s_t is captured by writing variable $X(s_t)$ as X_t in the formulation above. Note that since preferences exhibit non-satiation we can assume that constraint (36) holds with equality at the optimum. The utility obtained by agent f is then given by $U_f(s_t) = \bar{U}_{f,t}$, $\forall t$.

3.1 Solving the planner's problem

`TODO: rewrite/update the rest of this section.`

In the solution to the planner's problem there are two conditions that are of particular interest. First, the ratio of the first-order conditions with respect to the consumptions of the two agents implies the following condition for the planner's allocations of consumption across the two countries:

$$\lambda^*(s_t)\left(\frac{C_f(s_t)}{C_h(s_t)}\right)^{\tau\rho-1}\left(\frac{1-n_f(s_t)}{1-n_h(s_t)}\right)^{(1-\tau)\rho} = \frac{\lambda_{Cf}(s_t)}{\lambda_{Ch}(s_t)} \tag{37}$$

The term λ^* is the ratio of Pareto weights that determines how the planner allocates goods across countries, and λ_{Ci} for $i \in \{h, f\}$ are the lagrange multipliers on the resource constraints for the final goods in (19).

We let $\lambda_U(s_t)$ be the lagrange multiplier on the planner's promise-keeping constraint (36), and $\lambda^*(s_t)$ is defined as

$$\lambda^*(s_t) = \lambda_U(s_t) \left(\frac{U_f(s_t)}{U_h(s_t)} \right)^{1-\rho} \tag{38}$$

The Pareto weight on agent h is then given by $\mu^*(s_t)$ where

$$\mu^*(s_t) = \frac{1}{1 + \lambda^*(s_t)} \qquad \text{or} \qquad \lambda^*(s_t) = \frac{1 - \mu^*(s_t)}{\mu^*(s_t)} \tag{39}$$

and the Pareto weight on agent f is $1 - \mu^*(s_t)$. Equation (38) implies that there is a one-to-one mapping between the continuation utility state variable $\bar{U}_{f,t}$ and the Pareto weight ratio $\lambda^*(s_t)$. We therefore consider the optimal allocations functions of λ^* and discuss the dynamics of the model in terms of the dynamics of this ratio.

The second condition of particular interest governs the dynamics of the Pareto weight ratio. For a given realization of the productivity shocks next period the first-order condition with respect to $\bar{U}_f(s_{t+1})$, the envelope condition with respect to $\bar{U}_{f,t}$, and the definition of $\lambda^*(s_t)$ imply

$$\frac{\lambda^*(s_{t+1})}{\lambda^*(s_t)} = \left(\frac{U_h(s_{t+1})}{\zeta_t(U_{h,t+1})} \right)^{\rho-\alpha} \left(\frac{U_f(s_{t+1})}{\zeta_t(U_{f,t+1})} \right)^{\alpha-\rho} \tag{40}$$

3.1.1 Allocations from the planner's problem

`TODO: rewrite/update the rest of this section.`

In the case where $\alpha = \rho$, the model reduces to the standard model where agents have additive expected utility preferences. Equation (40) shows that the Pareto weight ratio is constant in that case. It is then common to set the Pareto weight on each country to 1/2 so that $\lambda^*(s_t) = 1 \; \forall t$. Therefore it is not crucial to emphasize how allocations depend on this ratio, and the dependence can be left implicit.

In contrast when $\alpha < \rho$ and agents have recursive preferences,[5] the dynamics of λ^* reflect a key feature of the efficient allocations: Shocks not only affect the contemporaneous allocations of consumption and leisure, but also agents' continuation utilities.

[5] When $\alpha < \rho$ agents prefer early resolution and low persistence of uncertainty. We restrict our attention to this case which is considered most relevant in the literature.

As discussed below, the response in the continuation utilities will also be reflected in prices in the decentralized equilibrium, and will therefore imply that the response in the real exchange rate is partially disconnected from the responses in consumption and leisure across countries.

The change in the Pareto weight ratio after a shock at time $t+1$ is given by equation (40). As the equation shows, the change is related to the continuation utility of each agent relative to that agent's certainty equivalent. The terms $U_i(s_{t+1})/\zeta_t(U_{i,t+1})$ can be thought of as innovations in the agents' future utilities, and changes in λ^* reflect differences in these innovations. For shocks to imply dynamics that are very different from the case with additive preferences they therefore have to have different long-run implications for the two countries. Such shocks imply differences in continuation utilities which are reflected in the dynamics of the Pareto weight ratio.

Consider for example a positive productivity shock in country h. This leads to an increase in the production of intermediate good h. Since both agents derive utility from final goods that are produced with intermediate good h as an input, the planner can increase the utility of both agents by increasing allocations of intermediate good h. Both innovations in future utility are therefore greater than 1. However, due to home bias the planner increases the consumption of agent h more. The production of final good h uses intermediate good h more intensively and therefore increases more than the production of final good f. The innovation in the utility of agent h is therefore larger than the innovation in the utility of agent f. Agent h benefits more when productivity increases for the good that he is biased towards.

Since $\alpha < \rho$, equation (40) shows that a positive productivity shock in country h which leads to a larger innovation in the future utility of agent h, also leads to an increase in the Pareto weight ratio. The planner increases the Pareto weight on agent f to partially compensate him for the productivity shock that is relatively less favorable for him. The changes in the Pareto weight ratio thus reflect the agents' desire to smooth future utility.

3.2 Allocations and prices

`TODO: rewrite/update the rest of this section.`

In the decentralized equilibrium the dynamics of the allocations reflected in the variation of the Pareto weight ratio correspond with trade in state-contingent securities. The agents partially insure each other against relatively bad productivity shocks. The price of these securities is determined by agents' stochastic discount factors in equation (25). It is common in the literature on recursive preferences to decompose the stochastic discount factors into a "short-run component" (SRC) and a "long-run component" (LRC) (see for example Kaltenbrunner and Lochstoer (2010)), as follows:

$$m_i(s_{t+1}, s_t) = \underbrace{\beta \left(\frac{C_i(s_{t+1})}{C_i(s_t)}\right)^{\tau\rho-1} \left(\frac{1-n_i(s_{t+1})}{1-n_i(s_t)}\right)^{(1-\tau)\rho}}_{SRC_i(s_{t+1},s_t)} \underbrace{\left(\frac{U_i(s_{t+1})}{\zeta_t(U_{i,t+1})}\right)^{\alpha-\rho}}_{LRC_i(s_{t+1},s_t)} \tag{41}$$

The dynamics in the Pareto weight ratio therefore correspond to heterogeneous responses to productivity shocks in the long-run components of the stochastic discount factors.

This will also be reflected in movements in the real exchange rate. One way to see this is through equation (28). Taking logs of that equation we obtain:

$$\Delta \log e(s_{t+1}) = \log m_f(s_{t+1}, s_t) - \log m_h(s_{t+1}, s_t) \tag{42}$$

Continuing the example from above, a positive productivity shock in country h leads to a larger innovation in future utility in country h. With $\alpha < \rho$ the resulting drop in m_h is larger than the drop in m_f, and the real exchange rate depreciates.

Another way to see the connection between the dynamics of the Pareto weight ratio in the planner's problem and movements in the real exchange rate in the decentralized equilibrium, is through equation (37). The ratio of lagrange multipliers on the resource constraints for final goods in the planner's problem, $\lambda_{Cf}/\lambda_{Ch}$ correspond to the real exchange rate in the decentralized equilibrium. Thus equation (37) can be written:

$$\lambda^*(s_t) \left(\frac{C_f(s_t)}{C_h(s_t)}\right)^{\tau\rho-1} \left(\frac{1-n_f(s_t)}{1-n_h(s_t)}\right)^{(1-\tau)\rho} = e(s_t) \tag{43}$$

Dynamics in the Pareto weight ratio therefore imply that the model delivers real exchange rate movements that are partially disconnected from relative movements in consumption and leisure. To evaluate whether this implies that the model can explain the high variability of the real exchange rate in the data, we solve the model numerically.

3.3 Balanced growth and restricting the cointegrating vector

`TODO: rewrite/update the rest of this section.`

Since $\log Z_{h,t}(z^t)$ and $\log Z_{f,t}(z^t)$ are both integrated processes we need to normalize the equilibrium conditions to obtain a stationary system. The country i variables that have a trend are rescaled by dividing through by the level of productivity in country i, $Z_{i,t}(z^t)$.[6] The full set of normalized equations are presented in appendix (A).

This model satisfies the restrictions on the production function and preferences from King, Plosser, and Rebelo (1988) that are necessary for the existence of a balanced growth path. Along with a productivity process with a unit root these conditions are also sufficient in a closed economy, but as discussed in Rabanal et al. (2011), a two-country model requires an additional restriction on the cointegrating vector to ensure balanced growth. The restriction is that the ratio of productivities $Z_h(s_t)/Z_f(s_t)$ must be stationary.

To see the need for this additional restriction consider for example the normalized version of equation (37):

$$\lambda^*(s_t)\tilde{z}(s_t)^{1-\tau\rho}\left(\frac{c_f(s_t)}{c_h(s_t)}\right)^{\tau\rho-1}\left(\frac{1-n_f(s_t)}{1-n_h(s_t)}\right)^{(1-\tau)\rho} = e(s_t) \tag{44}$$

where $c_i(s_t)$ are rescaled consumptions, and $\tilde{z}(s_t) = Z_h(s_t)/Z_f(s_t)$. Since the real exchange rate and labor supply in each country are stationary, if the ratio $\tilde{z}(s_t)$ was non-stationary then either the ratio of consumptions or the ratio of Pareto weights would be non-stationary. In either case balanced growth would not exist. A sufficient condition for stationarity of $\tilde{z}(s_t)$ is that the cointegrating vector in the law of motion for productivity (12) is $(1, -1)$. That is, we require that $\kappa = 1$.

3.4 Solution method

`TODO: rewrite/update the rest of this section.`

Heathcote and Perri (2002) and Rabanal et al. (2011) respectively linearize and log-linearize their models and compute moments by averaging across simulations. Here we follow a similar procedure, but to adequately capture the dynamics in this model a higher-order approximation is necessary. The reason is that the parameter α does not appear in a first-order approximation around a deterministic steady state. As shown

[6] There are two exceptions to this: (i) the utilities of the households $U_{i,t}(z^t)$ are divided by $Z_{i,t}(z^t)^\tau$ because there is a trend in consumption, but not in leisure, and (ii) the capital stocks $K_{i,t}(z^{t-1})$ are divided by $Z_{i,t-1}(z^{t-1})$ so that the normalized capital stock is determined at the start of period t.

in equation (40) the difference between α and ρ is key in determining the dynamics of the Pareto weights, so a good approximation of the effect of α on the dynamics of the model is necessary. In a second-order approximation α only shows up in the constant terms. Hence, we use a third-order approximation to capture the impact of α.[7,8]

4 Parameterization and results

4.1 Productivity and investment-specific shocks

As shown by Mandelman et al. (2011), whether IRBC models with investment-specific technology shocks can address the standard puzzles in international macroeconomics hinges on the parameterization of the process for these shocks. They argue that Raffo (2010)'s success in addressing the puzzles relies on a calibration where the IST shocks are much more volatile than suggested by the data. Here we therefore follow closely the parameterization used in Mandelman et al. (2011) and use estimated values as the basis for our parameterization.

The parameters of the VECM for the productivity shocks are based on the estimated values from Rabanal et al. (2011) They use data for the United States and an aggregate for the rest of the world consisting of the Euro area plus the United Kingdom, Canada, Japan and Australia. They use data on real GDP, hours, employment and real capital stocks. The key result from their estimates are that they cannot reject the hypothesis that the TFP series are cointegrated with a cointegrating vector equal to $(1, -1)$. Hence, $\kappa^P = 1$ and the restriction required for balanced growth is satisfied. A second important result is that the coefficients for the speed of adjustment are significant, but small. This implies that the TFP processes converge slowly over time and that differences in productivity across countries are persistent. Consistent with their estimates we set $\phi^P_{1h} = -0.007$ and $\phi^P_{1f} = 0.007$.

We impose a symmetric productivity process across the two countries and set $\phi^P_{0h} = \phi^P_{0f} = 0.0036$.[9] In Rabanal et al. these estimates differ, but due to cointegration the

[7] See van Binsbergen et al. (2008) for a discussion of the application of perturbation methods to models with recursive preferences. See Colacito and Croce (2011) for a discussion of the need for a third-order approximation to capture the dynamics in an endowment economy with risk-sensitive agents.

[8] The coefficients of the third-order approximation are easily computed using Dynare++, which can also compute approximations of a higher order. It is therefore straightforward to check that our results are not substantially affected by using an approximation of a higher order.

[9] Symmetry in the productivity process is imposed since we will start the economy from a symmetric steady state in our simulations below.

TFP processes nevertheless grow at the same rate along the balanced growth path. Their estimates imply an annualized long-run growth rate of TFP of 1.44 percent which is also the implication from our choice of parameters. Finally we set the standard deviation of the innovations to the productivity processes to 0.0067 which is the estimate from Rabanal et al. for the United States.

The parameters of the VECM for the investment-specific technology shocks are taken from Mandelman et al. (2011) They construct series for the IST shocks using data for the United States and the same aggregate for the rest of the world as Rabanal et al. In both cases the series are constructed using data on investment and consumption deflators. For the U.S. the IST shock is defined as

$$Z^I_{h,t}(s^t) = \frac{\text{PCE}_t^{\text{U.S.}}}{\text{PI}_t^{\text{U.S.}}}$$

where $\text{PCE}_t^{\text{U.S.}}$ is the personal consumption expenditures deflator, and $\text{PI}_t^{\text{U.S.}}$ is the investment deflator. For the rest of the world the IST shock is defined as

$$Z^I_{f,t}(s^t) = \sum_i w^i_t \frac{\text{PCE}^i_t}{\text{PI}^i_t}$$

where i identifies the country, and w^i_t is the trade weight of country i at time t. The weights are currency weights used in the Broad Index of the Foreign Exchange Value of the dollar calculated by the U.S. Federal Reserve. The particular deflators used differs depending on the data available for each country. See Mandelman et al. (2011) for the details.

The estimates by Mandelman et al. support the assumption IST series is also cointegrated with a cointegrating vector equal to $(1, -1)$. Hence, we also set $\kappa^I = 1$. They normalize the shocks so that the constant terms $\phi^I_{0h} = \phi^I_{0h} = 0$, and table 1 shows the estimates for the other parameters of the process.

Note that for both the TFP shocks and the IST shocks the correlations between the innovations in the VECMs are set to 0 as this restriction was not rejected by the data by either Rabanal et al. (2011) or Mandelman et al. (2011).

4.2 Recursive preference parameters

The key innovation in this paper is to model agents with recursive preferences in an IRBC model with both TFP and IST shocks. As shown in Tretvoll (2013), an IRBC model where agents have recursive preferences implies much greater variability in real

Exogenous processes

VECM TFP shocks		VECM IST shocks	
$\phi^P_{0h} = \phi^P_{0f}$	$= 0.0036$	$\phi^I_{0h} = \phi^I_{0f}$	$= 0$
ϕ^P_{1h}	$= -0.007$	ϕ^I_{1h}	$= -0.035$
ϕ^P_{1f}	$= 0.007$	ϕ^I_{1f}	$= -0.017$
σ^P_h	$= 0.0067$	σ^I_h	$= 0.0051$
σ^P_f	$= 0.0067$	σ^I_f	$= 0.0052$

Table 1: Parameters for the exogenous processes for TFP and IST shocks.

exchange rates than in the standard case with additive preferences. This is the case when the only shocks are TFP shocks with a standard deviation estimated from the data.

With both TFP and IST shocks in the model the debate over the model's ability to address key puzzles in international macroeconomics hinges on the calibration of the volatility of the IST shocks. Raffo (2010) calibrates the IST and TFP shocks to ensure that the IST shocks explain about 50 percent of the variance of output, and TFP shocks explain 25 percent. He argues that this is consistent with evidence from Fisher (2006) and the findings of Justiniano and Primiceri (2008). In the calibration the variance of the IST shocks is almost three times the variance of the TFP shocks, and Mandelman et al. (2011) argue that this is inconsistent with their estimates. They proceed to argue that a model with IST shocks with their estimated parameters of the shock process cannot successfully address the puzzles.

Here we use Mandelman et al. (2011) estimates for the IST shock process, and investigate whether modelling agents with recursive preferences in this framework alter the conclusion drawn by Mandelman et al.. Recursive preferences allow us to disentangle agents' intertemporal elasticity of substitution from their risk aversion. To analyze the impact of doing so, we first fix the intertemporal elasticity of substitution $1/(1-\rho)$ to be 0.5 which a standard value in the literature. We then consider different values for risk aversion $1-\alpha$. First, we set $1-\alpha = 2$ which gives the standard case with additive preferences. Second, we set $1-\alpha = 50$ in the baseline parameterization and later investigate how the results change for an intermediate range of values of this key parameter.

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TODO: Discuss Swanson (2009) and the interpretation of
risk aversion with labor in the utility function.
How does this affect the GHH case?
```

4.3 Other parameters

To set other parameters in the model we closely follow Mandelman et al. (2011). The discount factor β is set to 0.99. When preferences are specified with a Cobb-Douglas aggregate in equation (2), the consumption share in the consumption-leisure composite τ, is set to 0.34. When using the GHH specification, the parameters ν and ψ are set to 8.01 and 1163.4 respectively to obtain the same steady-state value for households' labor supply and the same labor supply elasticity as in the Cobb-Douglas case. This is the procedure followed by Raffo (2010) as well.

Standard values are used for the technology parameters. The capital share in production θ, is set to 0.36. The capital utilization rate is normalized to be 1 in the steady-state, and the elasticity of marginal depreciation ε, is fixed at 1. The steady-state depreciation rate $\delta(1)$ is set to 0.025. Together these restrictions pin down the values of b and $de\bar{l}ta$.

The trade elasticity is set to $\sigma = 0.62$ as in Mandelman et al. (2011). This is a key parameter in international macroeconomics, and using a low value is common. Corsetti et al. (2008) and Raffo (2010) set it to 0.5, Heathcote and Perri (2002) set it to 0.9, while Mandelman et al. and Rabanal et al. (2011) consider the values 0.62 and 0.85. We will use the lowest of these values in our baseline parameterization, but also discuss our results in the case where $\sigma = 0.85$.[10]

A choice of σ imposes a corresponding choice of the home bias parameter η, since together these parameters determine the import ratio in the deterministic steady state. The parameters for the productivity process are based on estimates by Rabanal et al. who use data for the period $1973Q1 - 2006Q4$. We therefore choose η to match the import ratio in the United States of about 12% in that period. That is, η solves the equation

$$0.12 = \left(\frac{1-\eta}{\eta}\right)^{\sigma} \tag{45}$$

4.4 Deterministic vs. stochastic steady state

It is important to note that since we solve the model with a third-order approximation, certainty equivalence does not hold. Therefore the deterministic steady state does not correspond to the mean of the ergodic distribution of the variables in the model. To

[10]Note that since the model features complete financial markets it does not suffer from the potential of multiple equilibria as discussed in Bodenstein (2010) despite a low value for σ.

Benchmark parameters

Preferences	Discount factor	$\beta = 0.99$
	Cobb-Douglas consumption share	$\tau = 0.34$
	GHH preference parameters	$\nu = 8.01$, $\psi = 1163.4$
	Intertemporal elasticity of substitution	$1/(1-\rho) = 0.5$
	Risk aversion	$1-\alpha = 2$ or $1-\alpha = 50$
Technology	E.o.s. between intermediate goods	$\sigma = 0.62$
	Import ratio	$0.12 \quad (\Rightarrow \eta = 0.96)$
	Capital share	$\theta = 0.36$
	Depreciation rate (steady-state)	$\delta(1) = 0.025$

Table 2: Benchmark parameters: In the additive preferences case we set $1-\alpha = 2$. In the recursive preferences case we set $1-\alpha = 50$.

obtain an approximation to this mean, we start our economy at the deterministic steady state and "simulate" the economy under a sequence of productivity shocks set equal to zero. The economy converges to a point which we refer to as the stochastic steady state, where agents take into account the uncertainty in the economy. They therefore work more and invest more to maintain a higher capital stock, which lowers the return on capital. In our simulations and impulse response functions we use the stochastic steady state as our starting point.[11]

4.5 Results for the benchmark parameters

As the model is non-stationary we follow the approach of Rabanal et al. and compute HP-filtered moments by stochastic simulation. The model is simulated for 125 periods starting from the stochastic steady state, and the HP-filter is applied to the series for output, consumption, investment, employment and the real exchange rate. Second moments are computed from the filtered series, and the averages over 100 simulations are reported in table 3. The first and second rows of each panel in table 3 report the results for the economy with additive and recursive preferences respectively. With additive preferences the model discussed here is a complete markets version of the economy in Mandelman et al., and the results are similar to the results they report for that case.

[11] An alternative approach is to start simulations at the deterministic steady state, but discard a large number of periods at the start of each simulation as a burn-in. See for example van Binsbergen et al. (2008). We find that using this approach does not affect our results.

The most striking result concerns real exchange rate fluctuations. Extending the standard model to allow for recursive preferences generates a real exchange rate that is both highly volatile and uncorrelated with relative consumption across countries. This is the case even though the model features complete financial asset markets.

As in Tallarini (2000) the model also generates a high market price of risk when preferences are recursive and risk aversion is high. The market price of risk in the model is about 26%, which matches the 25% that Tallarini finds in the data.[12] Table 4 reports the market price of risk for the model with additive and recursive preferences. In addition it reports the standard deviation of the stochastic discount factor, and the standard deviations of the short-run and long-run components of the stochastic discount factor.

The results show that the model with additive preferences does not come close to matching the market price of risk. With additive preferences $\alpha = \rho$ and the stochastic discount factor consists of the short-run component only. Since the variability of this component is low, so is the market price of risk. With recursive preferences on the other hand, the variability of the stochastic discount factor is high, and this is mainly driven by the variability of the long-run component. Since there is a unit root in the process for productivity, technology shocks have permanent effects. They therefore produce large innovations in future utility and hence, a high volatility of the stochastic discount factors.

4.5.1 Real exchange rate movements

4.5.2 Cross-country correlations

4.6 A note on survival

`TODO: check/update this section`

An issue that arises in this economy due to the dynamics in the Pareto weights is whether a non-degenerate stationary distribution of these weights exists. Anderson (2005) finds that in a one-good endowment economy where agents have risk-sensitive preferences, dynamics in the Pareto weights only arises when agents have different degrees of risk-sensitivity. In that case a non-degenerate distribution of Pareto weights

[12] Tallarini (2000) calculates the market price of risk from U.S. data from $1948:2-1993:4$. The data used are the value-weighted NYSE portfolio and the 3-month Treasury bill.

Results — additive and recursive preferences w/GHH aggregate

	sd(y)	rsd(c)	rsd(ι)	rsd(n)	rsd(e)	$\rho(e)$
Data	1.58	0.76	4.55	0.75	3.06	0.82
Additive	0.77	0.91	2.25	0.16	2.45	0.41
Recursive	0.68	0.87	2.33	0.20	4.72	0.53

	Correlations				
	(y, n)	(y, c)	(y, ι)	(y, nx)	$(e, c_h/c_f)$
Data	0.87	0.84	0.91	-0.49	-0.04
Additive	-0.23	0.92	0.81	-0.48	1.00
Recursive	-0.18	0.89	0.79	-0.23	-0.02

	Correlations			
	(y_h, y_f)	(c_h, c_f)	(ι_h, ι_f)	(n_h, n_f)
Data	0.44	0.36	0.28	0.40
Additive	0.49	0.33	0.04	0.14
Recursive	0.77	0.88	0.20	-0.17

Table 3: Second moments from HP-filtered simulated series: 'sd' denotes standard deviation; 'rsd' denotes standard deviation relative to output; (y, c, ι, n, e, nx) are respectively output, consumption, investment, labor, the real exchange rate, and net exports; 'ρ' denotes first-autocorrelation. Moments are calculated as the average over 100 simulations of 125 periods. Additive: $1 - \alpha = 2$. Recursive: $1 - \alpha = 50$. This table was produced with the GHH specification for the aggregate of consumption and labor.

Parameterization	MPR	sd(m)	sd(SRC)	sd(LRC)
Additive	0.58	0.57	0.57	0.00
Recursive	25.8	25.3	1.07	25.1

Table 4: 'MPR' denotes the market price of risk, 100 $sd(m)/\mathbb{E}(m)$; 'm' denotes the stochastic discount factor; 'SRC' denotes the short-run component and 'LRC' denotes the long-run component; 'sd' denotes the standard deviation. Each term is calculated as the average over 100 simulations of 125 periods.

does not exist. Colacito and Croce (2011) obtain dynamics in the Pareto weights in a two-good endowment economy where agents have risk-sensitive preferences over an aggregate consumption good. The consumption good is produced from two intermediate goods with home bias in the aggregation. Colacito and Croce prove there is a non-degenerate distribution of Pareto weights in their economy.

In the economy presented here, the dynamics in Pareto weights are partly due to the mechanism discussed by Colacito and Croce. We cannot apply their proof however, since our specification of preferences is more general and since our economy includes production and endogenous labor supply. Production introduces a separate channel that generates dynamics in the Pareto weights. Even in a production economy with a single good and no heterogeneity in preference parameters the Pareto weights will change over time. The reason is that different realizations of productivity shocks will induce different optimal labor supply decisions across agents. Hence, heterogeneity in agents' utilities from consumption can arise despite equal preference parameters.

While we cannot prove analytically that a non-degenerate distribution of Pareto weights exists for this economy, we can approximate the stationary distribution through simulation. Figure **??** plots the Pareto weight on agent h in our economy in a simulation over 1 million quarters. Panel (a) of the figure shows the simulated path and panel (b) shows the distribution. The figure shows that during the simulation the Pareto weight remains far from the boundaries of 0 and 1. As with simulating any Markov chain to approximate a distribution, we only observe the states the chain enters during the simulation. That is, we can say that we have not detected any problems concerning survival, but we cannot prove that no such problems exist. However, this exercise suggests that the survival of the agents in the short simulations considered above is not a primary concern.

5 Discussion

In this section we discuss how the results are affected by varying key parameters.

5.1 Varying risk aversion

5.2 Varying the elasticity of substitution and home bias

6 Conclusion

A Normalized system of equations

Following Mandelman et al. (2011) we account for the unit roots in both $Z^P_{i,t}$ and $Z^I_{i,t}$ by detrending the variables by $Z_{i,t} = (Z^P_{i,t})^{1/(1-\theta)}(Z^I_{i,t})^{\theta/(1-\theta)}$. The detrended variables are:

$$V_{i,t} = \frac{U_{i,t}}{Z_{i,t}}, \;\; c_{i,t} = \frac{C_{i,t}}{Z_{i,t}}, \;\; x_{i,t} = \frac{X_{i,t}}{Z_{i,t}}, \;\; y_{i,t} = \frac{Y_{i,t}}{Z_{i,t}}, \;\; x_{ij,t} = \frac{X_{ij,t}}{Z_{i,t}}, \;\; \iota_{i,t} = \frac{I_{i,t}}{Z_{i,t}}$$
$$k_{i,t} = \frac{K_{i,t}}{Z_{i,t-1}Z^I_{i,t-1}}, \;\; \tilde{z}_t = \frac{Z_{f,t}}{Z_{h,t}}, \;\; \tilde{z}^F_t = \frac{Z^F_{f,t}}{Z^F_{h,t}}$$

where $i \in \{h, f\}$ and $F \in \{P, I\}$.

A.1 Equations to solve for equilibrium

Cross-country consumption allocations:

$$\lambda^*_t \tilde{z}^{\rho-1}_t \left(\frac{c_{f,t} - \frac{\psi}{\tilde{g}_{f,t}} n^\nu_{f,t}}{c_{h,t} - \frac{\psi}{\tilde{g}_{h,t}} n^\nu_{h,t}} \right)^{\rho-1} = e_t$$

Intermediate goods allocations:

$$
\begin{aligned}
e_t = \frac{q_{hh,t}}{q_{fh,t}} &= \frac{\eta}{1-\eta}\left(\frac{x_{fh,t}}{x_{hh,t}}\right)^{\frac{1}{\sigma}}\left(\frac{G_h}{G_f}\right)^{\frac{1}{\sigma}} \\
e_t = \frac{q_{hf,t}}{q_{ff,t}} &= \frac{1-\eta}{\eta}\left(\frac{x_{ff,t}}{x_{hf,t}}\right)^{\frac{1}{\sigma}}\left(\frac{G_h}{G_f}\right)^{\frac{1}{\sigma}} \\
\frac{g^P_{h,t}}{\tilde{g}_{h,t}}(u_{h,t}k_{h,t})^{\theta}n_{h,t}^{1-\theta} &= x_{hh,t} + x_{fh,t}\tilde{z}_t \\
\frac{g^P_{f,t}}{\tilde{g}_{f,t}}(u_{f,t}k_{f,t})^{\theta}n_{f,t}^{1-\theta} &= \frac{x_{hf,t}}{\tilde{z}_t} + x_{ff,t}
\end{aligned}
$$

Final goods allocations:

$$
\begin{aligned}
G_h(x_{hh,t}, x_{hf,t}) &= c_{h,t} + \iota_{h,t} \\
G_f(x_{fh,t}, x_{ff,t}) &= c_{f,t} + \iota_{f,t}
\end{aligned}
$$

Labor allocations:

$$
\begin{aligned}
\psi\nu n_{h,t}^{\nu-1} &= \eta\left(\frac{G_{h,t}}{x_{hh,t}}\right)^{\frac{1}{\sigma}} g^P_{h,t}(1-\theta)\left(\frac{u_{h,t}k_{h,t}}{n_{h,t}}\right)^{\theta} \\
\psi\nu n_{f,t}^{\nu-1} &= \eta\left(\frac{G_{f,t}}{x_{ff,t}}\right)^{\frac{1}{\sigma}} g^P_{f,t}(1-\theta)\left(\frac{u_{f,t}k_{f,t}}{n_{f,t}}\right)^{\theta}
\end{aligned}
$$

Stochastic discount factors: In units of final goods in the respective countries

$$
\begin{aligned}
m_{h,t+1} &= \beta\left(\frac{c_{h,t+1}\tilde{g}_{h,t+1} - \psi n_{h,t+1}^{\nu}}{c_{h,t} - \frac{\psi}{\tilde{g}_{h,t}}n_{h,t}^{\nu}}\right)^{\rho-1}\left(\frac{\tilde{g}_{h,t+1}V_{h,t+1}}{\zeta_t(\tilde{g}_{h,t+1}V_{h,t+1})}\right)^{\alpha-\rho} \\
m_{f,t+1} &= \beta\left(\frac{c_{f,t+1}\tilde{g}_{f,t+1} - \psi n_{f,t+1}^{\nu}}{c_{f,t} - \frac{\psi}{\tilde{g}_{f,t}}n_{f,t}^{\nu}}\right)^{\rho-1}\left(\frac{\tilde{g}_{f,t+1}V_{f,t+1}}{\zeta_t(\tilde{g}_{f,t+1}V_{f,t+1})}\right)^{\alpha-\rho}
\end{aligned}
$$

Returns to capital: In units of final goods in the respective countries

$$
\begin{aligned}
r_{h,t+1} &= (1-\phi[\cdot_{h,t}])\left(q_{hh,t+1}g^P_{h,t+1}\theta(u_{h,t+1}k_{h,t+1})^{\theta-1}u_{h,t+1}n_{h,t+1}^{1-\theta} + \frac{1-\delta(u_{h,t+1})}{g^I_{h,t+1}(1-\phi[\cdot_{h,t+1}])}\right) \\
r_{f,t+1} &= (1-\phi[\cdot_{f,t}])\left(q_{ff,t+1}g^P_{f,t+1}\theta(u_{f,t+1}k_{f,t+1})^{\theta-1}u_{f,t+1}n_{f,t+1}^{1-\theta} + \frac{1-\delta(u_{f,t+1})}{g^I_{f,t+1}(1-\phi[\cdot_{f,t+1}])}\right)
\end{aligned}
$$

Pricing:

$$1 = \mathbb{E}_t m_{h,t+1} r_{h,t+1}$$
$$1 = \mathbb{E}_t m_{f,t+1} r_{f,t+1}$$

Capital evolution:

$$k_{h,t+1} = (1-\delta(u_{h,t}))\frac{k_{h,t}}{\tilde{g}_{h,t} g^I_{h,t}} + \iota_{h,t} - \frac{\phi}{2}\frac{\iota_{h,t-1}}{\tilde{g}_{h,t} g^I_{h,t}}\left[\frac{\iota_{h,t}}{\iota_{h,t-1}}\tilde{g}_{h,t} g^I_{h,t} - \Delta_I\right]^2$$
$$k_{f,t+1} = (1-\delta(u_{f,t}))\frac{k_{f,t}}{\tilde{g}_{f,t} g^I_{f,t}} + \iota_{f,t} - \frac{\phi}{2}\frac{\iota_{f,t-1}}{\tilde{g}_{f,t} g^I_{f,t}}\left[\frac{\iota_{f,t}}{\iota_{f,t-1}}\tilde{g}_{f,t} g^I_{f,t} - \Delta_I\right]^2$$

Capital utilization:

$$q_{hh,t}\; g^P_{h,t}\theta(u_{h,t}k_{h,t})^{\theta-1} n^{1-\theta}_{h,t} = \frac{\delta'(u_{h,t})}{g^I_{h,t}(1-\phi[\cdot_{h,t}])}$$
$$q_{ff,t}\; g^P_{f,t}\theta(u_{f,t}k_{f,t})^{\theta-1} n^{1-\theta}_{f,t} = \frac{\delta'(u_{f,t})}{g^I_{f,t}(1-\phi[\cdot_{f,t}])}$$

Pareto weight evolution:

$$\frac{\lambda^*_{t+1}(s_{t+1})}{\lambda^*_t(s_t)} = \left(\frac{\tilde{g}_{h,t+1}V_{h,t+1}(s_{t+1})}{\zeta_t(\tilde{g}_{h,t+1}V_{h,t+1})}\right)^{\rho-\alpha}\left(\frac{\tilde{g}_{f,t+1}\bar{V}_{f,t+1}}{\zeta_t(\tilde{g}_{f,t+1}\bar{V}_{f,t+1})}\right)^{\alpha-\rho}$$

This gives one equation for each possible realization of the state next period.

Exogenous variables[13]

$$\log g^F_{h,t+1} = \phi^F_{0h} - \phi^F_{1h}\log \tilde{z}^F_t + \varepsilon^F_{h,t+1}$$
$$\log g^F_{f,t+1} = \phi^F_{0f} - \phi^F_{1f}\log \tilde{z}^F_t + \varepsilon^F_{f,t+1}$$
$$\log \tilde{z}^F_{t+1} = \log \tilde{z}^F_t + \log g^F_{f,t+1} - \log g^F_{h,t+1}$$
$$\tilde{z}_t = (\tilde{z}^P_t)^{1/(1-\theta)}(\tilde{z}^I_t)^{\theta/(1-\theta)}$$
$$\tilde{g}_{h,t} = (g^P_{h,t})^{1/(1-\theta)}(g^I_{h,t})^{\theta/(1-\theta)}$$
$$\tilde{g}_{f,t} = (g^P_{f,t})^{1/(1-\theta)}(g^I_{f,t})^{\theta/(1-\theta)}$$

[13] The laws of motion for the productivity state variables after imposing that the cointegrating vector is $(1,-1)$ and no constant term in the cointegrating relationship.

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