

Math Matters: Student Ability, College Majors, and Wage Inequality^{*}

Michelle Rendall[†] and Andrew Rendall[‡]
The University of Zurich

February 16, 2013

ABSTRACT

This paper assess increasing wage inequality by showing that the top deciles of college earners are enjoying significant relative wage growth, which is underpinned by the link between *ex ante* math ability, math-heavy college majors and highly quantitative occupations. This mechanism is further strengthened by the strong and accelerating shift away from math-heavy college majors and occupations. We develop a general equilibrium model with multiple education choices whose outcomes depends on *ex ante* abilities, coupled with preferences, that lead to occupational opportunities that mimic the facts presented. This research shows that a large portion of wage inequality is determined by initial math/quantitative abilities. Furthermore, these results imply that policy measures aimed at increasing college enrollment to decrease wage inequality do not address the underlying process and, in some cases, may exacerbate wage inequality.

JEL classification: E20, E24, E25, I20, J24, J31

Keywords: wage inequality, college majors, occupations, mathematics abilities

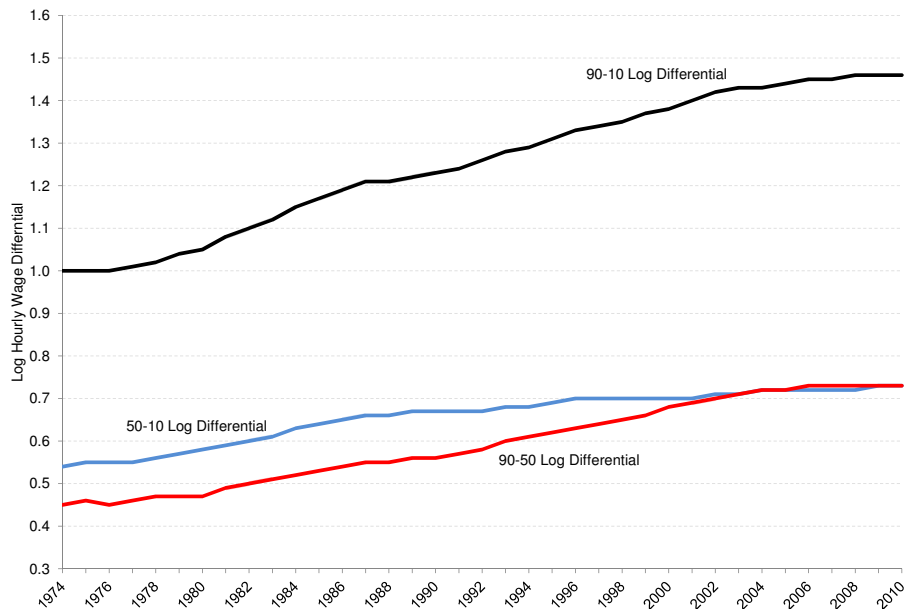
^{*}We would like to thank seminar and conference participants at University of Zurich for valuable comments. Michelle Rendall gratefully acknowledges financial support from the European Research Council (ERC Advanced Grant IPCDP-229883).

[†]University of Zurich, Department of Economics, Muehlebachstrasse 86 CH-8008 Zurich.
Corresponding Email: michelle.rendall@econ.uzh.ch.

[‡]University of Zurich, Graduate School of Business, Plattenstrasse 14, 8032 Zurich, Switzerland.
Email: andrew.rendall@uzh.ch. We are solely responsible for errors and omissions.

1 Introduction

Higher education, broadly defined as college or university education beyond the secondary level, is typically seen as a mechanism driving both social and economic mobility. The completion of higher education is correlated with many positive *ex post* outcomes, including higher wages, better quality of life and longer life expectancy (see [Baum et al., 2010](#)). These correlations have prompted significant government intervention to promote higher education, with the goal of extending these benefits to a larger section of the population ([Baum et al., 2010](#)). However, this paper shows that this conclusion is erroneous due to the heterogeneity both intra- and inter-education groups.



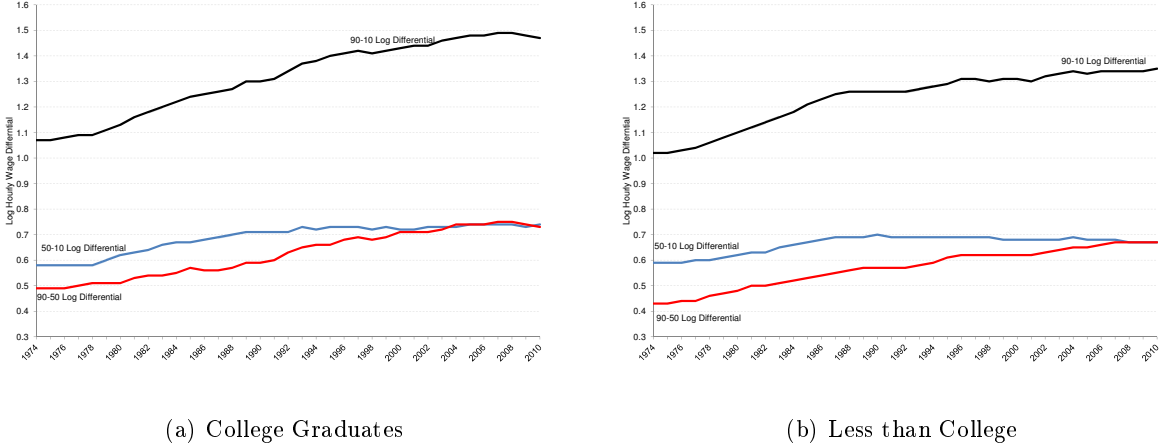
Source: Current Population Survey

Figure 1: US Wage Inequality

The US has seen a large rise in wage inequality. Figure 1 plots the 90th-to-10th, 90th-to-50th and 50th-to-10th percentile of hourly log wages for full-time, full-year¹ males aged 25 to 54 from

¹At least 40 weeks and 35 hours per week.

the Current Population Survey (CPS).² Inequality has risen faster at the upper half of the wage distribution. This points to the well established fact that there is increasing wage inequality amongst college graduates. Figure 2 plots the same percentiles by education group. Here college graduates are all individuals with at least a university degree (i.e., associate, bachelors or graduate degree). While the 90th-to-10th percentile wage differential increased by 33 percentage points for individuals



Source: Current Population Survey

Figure 2: *US Wage Inequality by Education*

with no-college degree, it increased by 40 percentage points within the group of college graduates. Moreover, the rise in the 50th-to-10th percentile wage inequality was twice as large for college graduates, with 16 percentage points (versus 8 percentage points), while the 90th-to-50th percentile wage inequality increased by the same 24 percentage points. While the rise was larger in the bottom half, the inequality was larger initially for college graduates in the top half of the distribution and equal for the bottom half. The seminal research by [Kambourov and Manovskii \(2009\)](#) explains a large part of within group wage inequality by focusing on occupational mobility and the cost of switching occupations. They find that occupational mobility accounts for a significant portion of wage inequality. Their approach differs from this paper by concentrating on the given conditions at the time of occupational choice (i.e., educational choices are not modeled). Thus, we expand on their research by modeling the initial conditions that proceed the labor force choices that are

²CPS data is obtained from the IPUMS-CPS project ([King et al., 2010](#)).

central to their results. We believe this to be important, since we show below that education choices matter for occupation decisions later in life. We further show that separating the population strictly by educational attainment alone, some very significant points are missed. That is, the following five facts result from a decomposition of the within and between group heterogeneity previously discussed.

1. The top earners without any college education earn substantially more than the bottom half of college graduates (see Figure 3).
2. The main group driving wage inequality are the top college graduates. Specifically, the top 20 percent of college earners have dramatically increased their wages compared to all other education-wage groups (see Figure 4).
3. *Ex ante* mathematics abilities are highly correlated with higher *ex post* outcomes, whereas verbal skills are not (see Figure 5).
4. Mathematics requirements for college majors are highly correlated with occupation-specific mathematics skills, but not verbal skills (see Figure 6).
5. When associating majors with their required mathematical ability, there has been a dramatic shift toward the tails (i.e., an increase in the bottom quintiles and the top quintiles). This shift is depicted in Figures 7 and 8.

Thus, by concentrating on aggregates of educational attainment alone, policy makers may design policies that are inconsistent with a thorough analysis across the wage distribution conditional on educational attainment. By basing education policy on blunt and broad measures of the benefits of college education, there will be a misallocation of both human and physical capital. Therefore, the aim of this paper is to show the importance of combining the five points above in an analysis aimed at correcting wage inequality issues related to education. To our knowledge, there is no prior research that has explored these five facts together. The recent paper by [Altonji et al. \(2012\)](#) is the first to establish point three above, while the other four facts are new contributions to the literature. [Altonji et al. \(2012\)](#) are “motivated by the large discrepancies in labor market outcomes across college

majors” noting that, “there is much less work on why individuals choose between different types of education,” despite the, “returns across college majors rival(ing) the college wage premium.” They note that the 2009 American Community Survey (ACS) show a 56.1-percent gap between male electrical engineering and male general education majors, compared to a college wage premium of 57.7-percent. Due to the empirical focus of their research, the primary results are detailed estimated returns to college majors with associated math and verbal SAT scores. The authors note that this area of research is relatively untouched³ and is important for understanding the structural mechanisms underpinning the *ex post* outcomes of higher education. A crucial difference between our research and [Altonji et al. \(2012\)](#) is, by using the information on mathematic skill requirement within occupations from the Dictionary of Occupational Titles, we show that mathematics-focused majors are highly correlated with *ex post* wage outcomes through the occupational choices available to these majors.

Addressing similar wage discrepancies, [Silos and Smith \(2012\)](#) look at the trade-off between acquiring specific and targeted human capital. They concentrate on individuals’ choices between education paths leading to specific occupations versus those that have broader applicability, and thus more occupational choice. This research fits extremely well within the broader educational transition described above. The authors show that policies directed at occupation-specific human capital accumulation lead to lower income growth and lower inequality. In this research, we emphasize the importance of the skill types accumulated, with particular attention given to mathematics scores as either a specific necessary ability or as a strong indicator of associated abilities. Specifically, those who have majored in math-intensive areas may initially sort into high-wage occupations, with little prospect of switching to alternative high-wage occupations.

The aim here is to explicitly model the education decision based on the five facts outline above, and presented in detail in section 2, with math abilities being the main driver of wage inequality. Specifically, we hypothesize that wage inequality is driven to a large extent by individuals’ initial abilities, which limits an individual’s educational choices. Therefore, that part of the within group wage inequality is explained by individuals’ education choices, which are largely dependent on

³A comprehensive review of the existing empirical studies on the returns to college major can be found in Table 2 of [Altonji et al. \(2012\)](#).

individuals' innate mathematics abilities. A secondary concern of this research is highlighting the effects of currently enacted policy goals directed toward increasing college attendance rates. Lastly, this research suggests that individuals maximize their expected wages based on their initial abilities, with the learning of mathematics skill in school being the most important initial determinant.

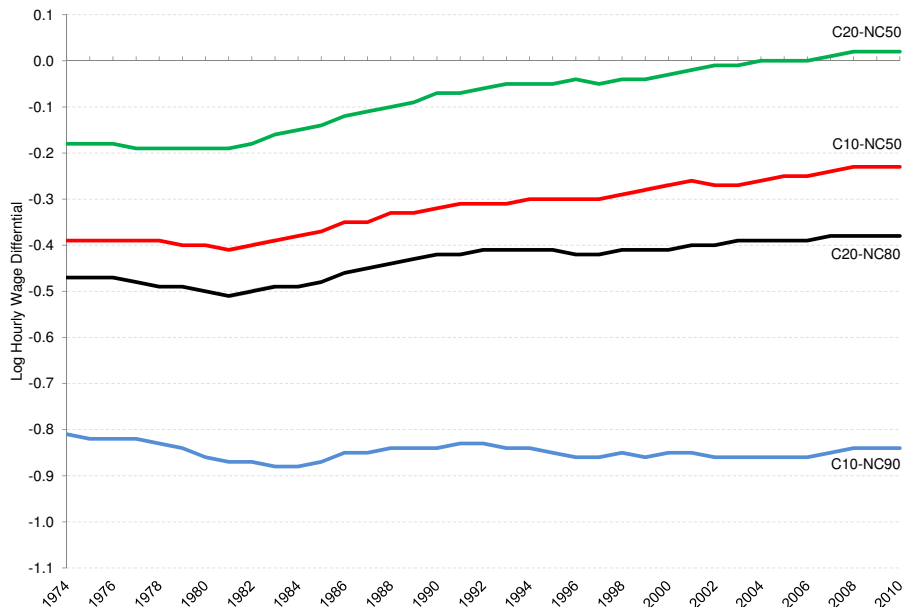
The model in this paper is loosely based on [Hendricks and Schoellman \(2012\)](#). The authors look at the discrete education choices of individuals (i.e., high school, some college, and college), focusing on *ex ante* abilities as measured by IQ scores. Their results show that one-third of the college wage premium and one-fourth of its growth is driven by ability ("ability premium"). While looking at ability as a driver of wage outcomes, [Hendricks and Schoellman \(2012\)](#) define broad education categories that mask/dilute the subset mainly driving wage inequality: the top earning college graduates, who exhibit strong mathematical abilities.

As wage inequality across different education groups, mathematics requirements of college majors and quantitative occupation requirements are the key features within this study, Section 2 provides a summary of the data facts related to wage inequality, college majors and occupations over time and across cohorts. The general equilibrium model is outlined in Section 3, and Section 4 provides analytical results. Section 5 concludes.

2 Data

This research relies on four main points summarized below and supported in the following subsections:

1. The top earning college graduates (i.e., top two deciles) are primarily driving wage inequality;
2. Wages are highly correlated with *ex ante* mathematical ability as measured by SAT scores;
3. Highly quantitative occupations are highly correlated with *ex ante* mathematical ability as measured by SAT scores; and
4. There has been a significant and accelerating shift away from college majors requiring high mathematical abilities.



Source: Current Population Survey

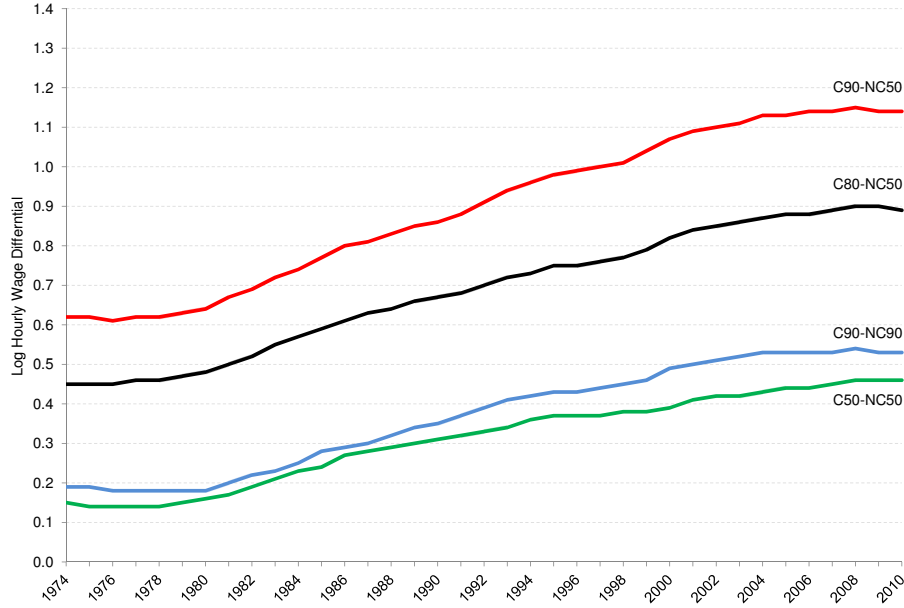
Figure 3: Bottom Wage College Graduate Performance

These four points together present a coherent story of *ex ante* mathematical ability dictating initial college major options, from which occupations and, ultimately, wages are determined. I.e., those with higher mathematics abilities pursue math-heavy majors and occupations. These particular occupations also enjoy the highest wages. Furthermore, the math intensive majors that lead to higher wage occupations are increasingly shunned by each subsequent generation. This shift away from math-heavy majors is further exacerbating wage inequality.

2.1 Who is Driving Wage Inequality?

To illustrate which education group subsets are driving wage inequality, we use data from the 2009 American Community Survey (ACS)⁴ from which the residual of a Mincer wage regression is

⁴The 2009 American Community Survey (ACS) is the first year in which college major is included. Note that the trends observed in the ACS 2009 are identical in the ACS 2010.



Source: Current Population Survey

Figure 4: Top Wage College Graduate Performance

derived from log hourly wages of full-time, full-year males. The regression controls for age, age-squared, race, marital status, and state of residence (using ACS/Census weights). The unexplained residual for various education-wage groups are compared in Figures 3 and 4. These cross-education-wage group comparisons highlight the importance of high-earning college graduates in driving wage inequality, especially since the mid-1980s (see Figure 2).

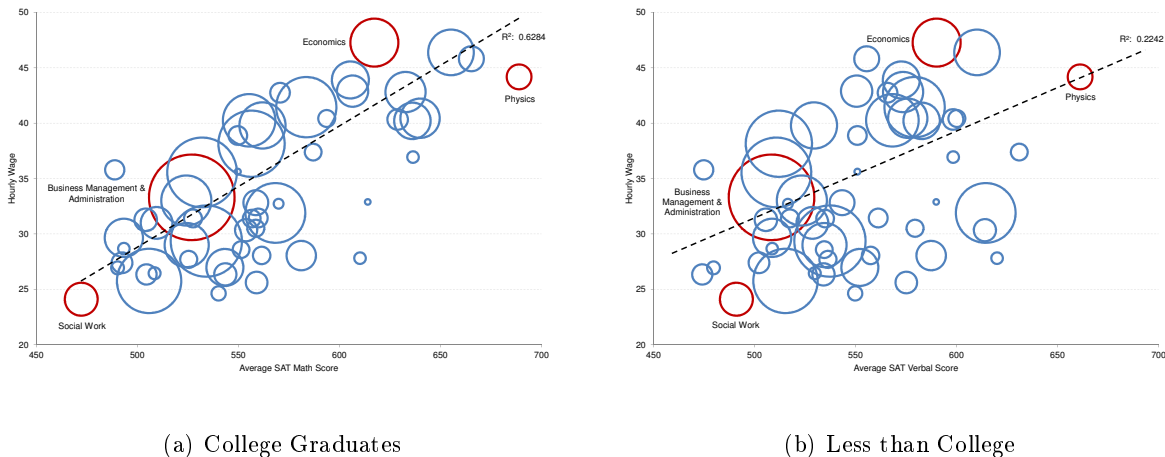
Figure 3 compares the residual wages of the bottom 10th and 20th percent of college graduates with the middle and upper non-college wage groups (50, 80 and 90th percentiles). It is important to note that the bottom earning college graduates have significantly lower wages than the middle and upper non-college wage groups. While there is a trend toward convergence for the lower college (20th percentile) and middle non-college (50th percentile) groups, the other group comparisons show a flat to very mild convergence, with continuing substantial wage inequality favoring non-college graduates. The final comparison within Figure 3 shows that the bottom college-wage decile has lost

ground against the top non-college-wage decile.

In contrast to Figure 3, Figure 4 compares the residual wages of the middle and top college and non-college wage groups. The 90th percentile college-wage group has outpaced the median non-college-wage group by more than 50 percent since the mid-1970s. This is a remarkable performance considering the top college-wage groups have also increased their wage premium against the top non-college-wage groups by approximately 35 percent. The implications of this figure are summarized in two points: (1) the top college-wage groups are outpacing all other groups, but (2) are sprinting ahead of the median non-college earners. Thus, a large part of wage inequality growth is driven by the top college-wage groups, while the bottom college-graduates are behind compared to a large share of non-college graduates.

2.2 Math and Wages

Figure 5 links hourly wage for full-time, full-year males aged 30 to 59 with math and verbal SAT scores. The ACS 2009 data contains individual-level data containing the college major information that is then associated with the SAT scores from the Baccalaureate and Beyond Longitudinal Study 2009 similar to Altonji et al. (2012). The strong correlation of math abilities and earnings is in sharp



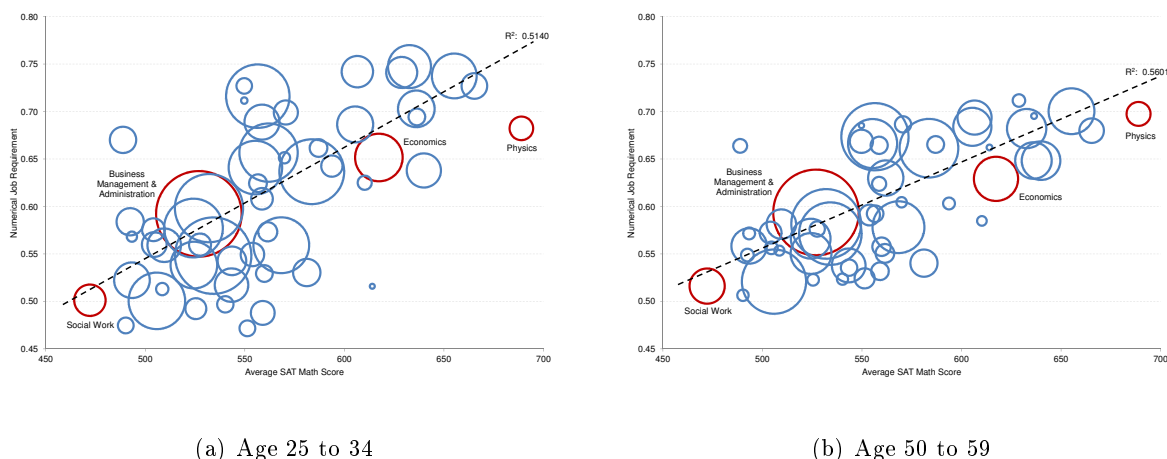
Source: Baccalaureate and Beyond Longitudinal Study 2009 and American Community Survey 2009

Figure 5: *SAT Scores and Wages*

contrast to that of verbal skills, with correlation coefficients of 0.63 and 0.22, respectively.

2.3 Math and Occupations

Figures 6 link college majors with the skill requirements of occupations for cohorts aged 25-34 and 50-59. Using the Dictionary of Occupational Titles (DOT), numerical job requirements associated with occupations are applied to the ACS 2009 data. The ACS data contains individual-level observations with college major and occupation information. As in the previous section, SAT scores are merged with the ACS data using the Baccalaureate and Beyond Longitudinal Study 2009. There is a strong link across cohorts between numerical requirements of occupations and the math SAT scores of those filling the positions. Hence, those with high SAT scores are generally entering occupations requiring high quantitative skills, and this is a consistent link across generations. However,



Source: Dictionary of Occupational Titles 1991, Baccalaureate and Beyond Longitudinal Study 2009 and American Community Survey 2009

Figure 6: *SAT Scores and Numerical Occupation Requirements*

SAT Verbal scores are not highly correlated with verbal requirements in occupation, that is the R-squared for SAT Verbal and verbal abilities required at work are 0.10 for individuals aged 25 to 34 and 0.20 for individuals aged 50 to 59. Lastly, correlations are also consistent for the distribution of occupation requirements (second moments). Table 1 computes correlations by percentiles of the wage distribution for a given major. The correlations are consistent across age and majors over the entire distribution. In addition, the table also highlights the importance of numerical skills, whereas verbal ability is not highly correlated.

Table 1: Correlation with SAT Math

Age 25 to 34

	Mean	Bottom			Top		
		10%	25%	50%	50%	25%	10%
General	0.64	0.56	0.61	0.61	0.60	0.47	0.38
Numerical	0.72	0.65	0.69	0.70	0.71	0.62	0.56
Verbal	0.52	0.40	0.47	0.50	0.51	0.40	0.30

NB: All values are significant at a 99% confidence level.

Age 35 to 49

	Mean	Bottom			Top		
		10%	25%	50%	50%	25%	10%
General	0.60	0.33	0.57	0.65	0.54	0.44	0.38
Numerical	0.74	0.57	0.68	0.73	0.73	0.65	0.55
Verbal	0.47	0.03	0.31	0.48	0.45	0.36	0.32

NB: All values are significant at a 99% confidence level.

Age 50 to 59

	Mean	Bottom			Top		
		10%	25%	50%	50%	25%	10%
General	0.53	0.26	0.41	0.54	0.51	0.44	0.40
Numerical	0.75	0.49	0.63	0.72	0.74	0.69	0.55
Verbal	0.35	0.01	0.13	0.30	0.38	0.34	0.33

NB: All values are significant at a 99% confidence level.

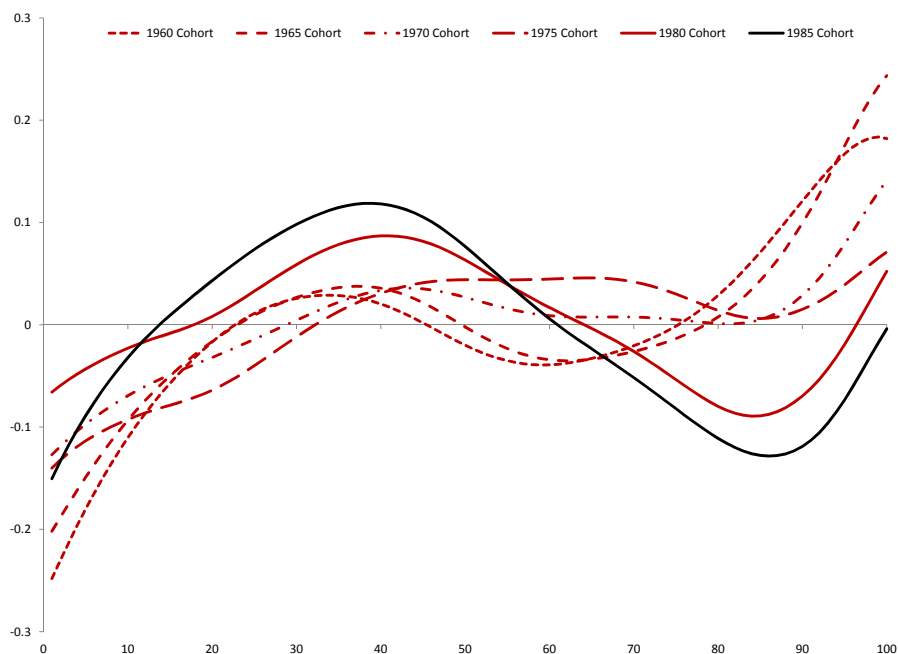
Source: Dictionary of Occupational Titles 1991, Baccalaureate and Beyond Longitudinal Study 2009 and American Community Survey 2009

2.4 Cohorts and Shifting Majors

To advance the picture illustrated in the previous figures, Figures 7 and 8 illustrate the change in degrees obtained and occupations over time by ranking them by their mathematical/quantitative difficulty. The line with the shortest dashes compares the 1955 cohort, with each subsequent cohort represented by longer dashed lines culminating in the latest cohort (1985) represented by the solid black line. That is, it compares individuals that were age 25 by 1980 with later cohorts. To do this, we plot smoothed changes in the share of college graduates between the 1955 cohort and subsequent cohorts at each percentile of the SAT mathematics scores associated with the college degrees obtained and occupations, respectively. The figures use data for full-time, full-year males. This is done to align this data with the wage inequality focus of this research and the previous figures. The degree and occupation data over time are paired with SAT math scores from the 2009

Baccalaureate & Beyond (B&B) and college majors from the ACS 2009.⁵ This method used is similar to Autor et al. (2003).

Figure 7 shows a clear compositional transition toward majors requiring math abilities in the bottom three quintiles, except for the bottom decile, which falls off. This might be accounted for by the dropping of low-level college majors from the curriculum offerings, such as home economics [citation missing]. The trend toward majors requiring lower math abilities is offset by a significant decrease in those completing majors requiring higher math abilities. This trend is very strong except for those majors near the absolute top of the distribution, which have stayed at the same levels seen in the 1955 cohort.



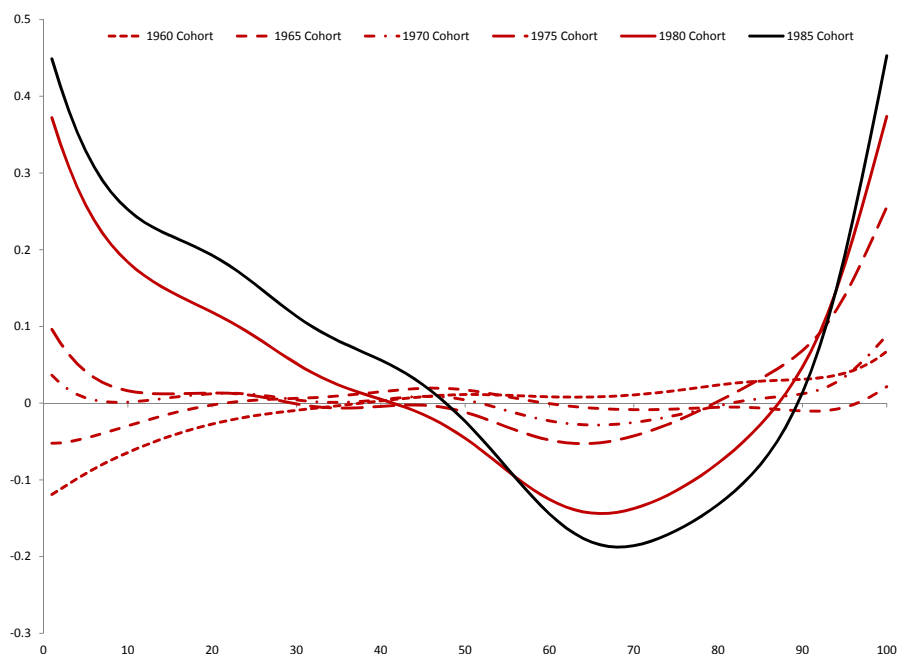
Source: Current Population Survey

Figure 7: Cohort Changes by College Major

Figure 8 extends the above relationship with quantitative requirements to inter-temporal changes in occupation outcomes. This figure is setup in a similar manner as Figure 7, with the latest

⁵The results from the ACS 2010 are similar.

cohort (1985) represented by a solid black line. For this figure average SAT Mathematics scores by occupation for the 1955 cohort are computed and changes over time relative to this cohort are then plotted. That is, since college majors do not correlate one-to-one with occupations, this figure controls for a potential change in occupational choices by different cohorts. Practically, this figure depicts the strong shift away from occupations between the 50 and 90th percentiles, as ranked by their math requirements. This is offset by massive growth in occupations in the top decile and bottom half. Given the previously presented data, this figure provides the basis from which increasing wage inequality can be explained. Note that the transition away from the 50 and 90th percentile occupations accelerated starting in the 1980 cohort.



Source: Current Population Survey

Figure 8: Cohort Changes by Occupation

3 Model [INCOMPLETE]

The model is loosely based on [Hendricks and Schoellman \(2012\)](#). In addition, analyzing the schooling decision in a general equilibrium set-up, the model is also modified to incorporate the choice of college majors. Individuals choose whether to go to college or not at age 0 (i.e., age 18 in the data). If agents decide to pursue college-level education, they choose which major to specialize in. However, different majors require different abilities (i.e., some are easier to complete than others). We rank majors by difficulty, that is schooling choice s^0 is no college education, s^1 is the easiest major and s^n the most difficult to complete. Individuals know their own innate ability θ and the expected wages by schooling choice $E(w|s)$. Actual wages, drawn after graduating, depend on an individual's ability, schooling choice, and the aggregate supply of college graduates with a given skill set (in the context of wage inequality and schooling choice, the general equilibrium effects cannot be ignored). Let κ be the cost of attending college which is increasing in the complexity of a major, $\kappa(s^j|\theta) < \kappa(s^{j+1}|\theta)$. Moreover, assume that higher ability individuals face a lower cost of completing more difficult majors, $\kappa(s|\theta') < \kappa(s|\theta)$, where $\theta' > \theta$. This assumption will lead to high ability individuals selecting more difficult majors, and very low ability individuals forgoing a college-level education. While there is positive sorting by ability into more complex schooling types in the data (see [Altonji et al., 2012](#)), the sorting is not perfect. Therefore, we introduce a random component to κ which is iid across individuals, which summarizes the individual-specific cost(s), $\lambda(s, \theta) = \kappa(s|\theta) + \epsilon_i$. The random component could be interpreted either as differences in the tolerance for studying or preferences across majors in school. I.e., a high ability individual with a negative ϵ_i will choose a less “complex” major potentially because he prefers the subject matter of that major. We will be agnostic about the exact nature of this component, and calibrate it to match the data on ability and major choice in the data.

While in school individuals forego working and, after graduating, they will supply labor inelastically to the labor market at wage w_t^i . Individuals maximize expected lifetime utility,

$$E \left(\sum_{t=1}^T \log(c) \right) - \lambda(s, \theta), \quad (1)$$

subject to a life-time budget constraint,

$$\sum_{t=1}^T \frac{c}{(1+r)^t} = \sum_{t=1}^T \frac{w_t^i}{(1+r)^t}, \quad (2)$$

where $w_1^i = 0$ if the individual decides to attend college. Clearly, individuals will pick the major that provides the highest expected lifetime utility given the disutility of schooling $\lambda(s, \theta) = \kappa(s|\theta) + \epsilon_i$.

In modeling individuals' efficiency units of labor we follow [Violante et al. \(2008\)](#), where an individual's efficiency units of labor, $e^i = \theta^\zeta$, depend on their innate ability θ . The individual's wage follows, $w_t^i = w(s)_t e^i$, the product of the schooling-specific price in the labor market multiplied by their labor units. The schooling-specific prices are determined in general equilibrium, where a representative firm operates a CES production function to produce the final consumption good in the economy. The firm hires labor of all education types, which are imperfect substitutes,

$$Y = \{\eta_t L_c^\rho + (1 - \eta_t) L_0^\rho\}^{1/\rho}, \quad (3)$$

where L_0 , is non-college labor, $\frac{1}{1-\rho}$ is the elasticity of substitution between non-college and college labor, L_c . College labor is itself an imperfect substitute between majors,

$$Y = \left\{ \sum_{j=1}^n \gamma_j L_j^\nu \right\}^{1/\nu}. \quad (4)$$

An increase in the supply of a certain college major will results in a fall of their relative wages, assuming $0 < \nu < 1$, i.e., they are imperfect substitutes;

$$\frac{w(s^j)}{w(s^k)} = \frac{\gamma_j}{\gamma_k} \left(\frac{L_j}{L_k} \right)^{\nu-1}. \quad (5)$$

A rise in η_t , is usually regarded as skill-biased technical change in the literature and will result in a relative wage increase for college educated individuals, assuming the college supply-side effects do not outweigh the increase;

$$\frac{w(c)}{w(s^0)} = \frac{\eta_t}{1 - \eta_t} \left(\frac{L_c}{L_0} \right)^{\rho-1}. \quad (6)$$

That is, with skill-biased technical change, more individuals will find college attractive. However, if the new college entrants have, on-average, lower ability levels, the wage inequality within college majors will widen, given the assumption that lower ability individuals find it more difficult to major in more complex fields. As a consequence, the larger the absolute number of college entrants, the larger the post-education wage inequality will be. This follows intuitive supply-demand dynamics.

4 Estimation [TO BE ADDED]

5 Results [TO BE ADDED]

6 Conclusion [TO BE ADDED]

References

- Altonji, Joseph G., Erica Blom, and Costas Meghir**, “Heterogeneity in Human Capital Investments: High School Curriculum, College Major, and Careers,” Working Paper 17985, National Bureau of Economic Research April 2012.
- Autor, David H., Frank Levy, and Richard J. Murnane**, “The Skill Content Of Recent Technological Change: An Empirical Exploration,” *The Quarterly Journal of Economics*, November 2003, *118* (4), 1279–1333.
- Baum, Sandy, Jennifer Ma, and Kathleen Payea**, *Education Pays 2010*, CollegeBoard Advocacy & Policy Center, 2010.
- Hendricks, Lutz and Todd Schoellman**, “Student Abilities During the Expansion of U.S. Education, 1950-2000,” MPRA Paper 12798, University Library of Munich, Germany January 2012.
- Kambourov, Gueorgui and Iourii Manovskii**, “Occupational Mobility and Wage Inequality,” *Review of Economic Studies*, 04 2009, *76* (2), 731–759.
- King, Miriam, Steven Ruggles, J. Trent Alexander, Sarah Flood, Katie Genadek, Matthew B. Schroeder, Brandon Trampe, and Rebecca Vick**, “Integrated Public Use Microdata Series, Current Population Survey: Version 3.0. [Machine-readable database],” 2010. Minneapolis: University of Minnesota.
- Silos, Pedro and Eric Smith**, “Human capital portfolios,” Working Paper 2012-03, Federal Reserve Bank of Atlanta 2012.
- Violante, Giovanni L., Costas Meghir, and Giovanni Gallipoli**, “Equilibrium Effects of Education Policies: a Quantitative Evaluation,” 2008 Meeting Papers 868, Society for Economic Dynamics 2008.