Sand in the Wheels of Capitalism

On the Political Economy of Capital Market Frictions*

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Abstract

We present a positive theory of capital market frictions that raise the cost of capital for new firms and lower the cost of capital for incumbent firms. Capital market frictions arise from a political conflict across voters who differ in two dimensions: (i) a fraction of voters owns capital, the rest receives only labor income; and (ii) voters have different vintages of human capital. We identify young workers as the decisive voter group, with preferences in between capitalists who favor a free capital market, and old workers, who favor restricted capital mobility. We show that capital market frictions do not naturally arise in a static framework, or even in a dynamic framework if capital market frictions are reversible. But if capital market frictions can be made to persist over time, we show that young workers favor capital market frictions as a way to smooth income, especially if wealth is concentrated and if technological obsolescence is high.

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1 Introduction

> Political economists say that capital sets towards the most profitable trades, and that it rapidly leaves the less profitable non-paying trades. But in ordinary countries this is a slow process. (Bagehot, 1873)

In a free market, capital moves naturally towards its most profitable use, leaving less productive activities. In practice, capital markets are regulated by politicians and the allocation of capital across industries reflects political as well as economic choices. When in 2010 the French car company Renault planned to shut down production facilities in Flins (France), president Nicolas Sarkozy personally intervened and summoned Renault's top executives to his office. Sarkozy held considerable leverage over Renault as the French state was a major shareholder, creditor, and strategic partner of the company.[1] Indeed, after the meeting, Renault agreed to update the facilities instead of shutting them down and pledged to maintain employment.

The example of Renault illustrates three wider issues that interest us in this paper. First, politicians can and do intervene in individual companies and whole industries; thus influencing the allocation of capital in the economy. Second, political intervention occurs through many distinct channels: states regularly own company stakes, bail them out, or extend cheap credit to favored industry. Politicians exert further influence by setting the rules and regulations that govern the capital market; think of bankruptcy law or takeover law (cf. Pagano and Volpin, 2001). Finally, the example of Renault illustrates the wider issue that politicians often intervene in companies or industries to maintain employment, or protect other incumbent interests (cf. Hellwig, 2006).

[1] The state held 15% of Renault's stock, bailed the company out to the tune of €3 billion after the financial crisis, and helped to promote electric vehicles–where Renault had a big interest. On top of that, French car sales were still being propped up by a cash-for-clunkers scheme (cf. 'Renault Agrees to Sarkozy's Demands,' Wall Street Journal, 17 Jan 2010.)

Wurgler (2000) provides further evidence that suggests the state's important role in the capital market: Wurgler (2000) shows that growing industries invest more and declining industries divest more in countries where the extent of state ownership in the economy is less, i.e., capital is allocated more efficiently in those countries.[2] In a neoclassical economic framework–where the financial sector is functional to the needs of industry and trade–deviations from the free market allocation are attributed to frictions in the capital market. An important open question is then how capital market frictions arise and persist over time, as they suggest inefficiency; furthermore, what can explain their documented variation across countries? We address these questions qualitatively and provide a comprehensive theory of capital market frictions rooted in political economy.

This paper argues that democracies may choose to resist free capital markets, and throw *sand in the wheels of capitalism*, depending on demographical context, the concentration of wealth, and the rate of technological progress. In effect, democracies favor income stability over economic efficiency when the population is older, when the wealth distribution is uneven, and when the rate of technological growth is high. We show our results using a political economy model where agents, who differ in age and wealth, vote over capital market frictions that lower the cost of capital for incumbent firms, while raising the cost of capital for new firms.[3] New firms are more productive than incumbent firms so that there is an economic rationale for allocating more capital in new firms than in incumbent firms. But the balance of power between different economic constituencies may be such that a free capital market is opposed.

Our starting insight is that technological progress affects workers more

[2]Wurgler (2000) also finds that across countries, (i) more developed financial markets, and (ii) a higher degree of minority investor protection, are associated with a more efficient allocation of capital.

[3]Specifically, we adopt a production economy model with vintage technology, two overlapping generations, and specific human capital; we extend this model with a simple majority vote.

than capitalists. While labor and capital are complementary factors of production, labor is less mobile across technologies than capital. If human capital risk cannot be fully insured–e.g. because of moral hazard reasons–then workers are exposed to the risks that are specific to their human capital. The result of this human capital specificity is a political conflict between citizens with different vintages of sunk human capital: old agents who work in incumbent firms resist the reallocation of capital to new firms, as this leads to a reduction in their wages. Hence, old workers favor capital market frictions as a a way to maintain higher wages. Old capitalists on the other hand favor a free capital market. The young, finally, are the most interesting group.

The pivotal constituency in elections is formed by the young workers. Young workers do not gain from capital market frictions while they are still young, but they would like to limit future capital reallocation, anticipating that they will be less productive when they are old. A consumption smoothing motive then leads young workers' preferences to be partially aligned with the preferences of the old workers. Rapid technological change implies that the productivity gap between young and old workers is bigger, and therefore that the motive to obstruct free capital markets is stronger. We consider two scenarios: society chooses a *persistent policy* that is chosen once; or society maintains a *social contract* that needs to be politically supported in every vote. We show that the alliance against capital reallocation can be sustained under a persistent policy; multiple outcomes can be sustained as a social contract, since the lack of commitment to a reallocation friction may discourage the young to slow down capital reallocation today.

Our analysis suggests that capital market frictions which are hard to reverse can be understood as political instruments to reduce income variation. There are clear examples such frictions in capital markets. Bankruptcy law, for instance, defines specific conditions to the assignment of assets from declining sectors. While in some countries–such as the United Kingdom–bankruptcy law is designed to protect financial interests, in others–such as

France and Italy–it explicitly instructs the liquidator to reassign capital in a manner which protects employment.

The rest of the paper is organized as follows: section 2 reviews the literature; section 3 gives the model; section 4 solves for the economic equilibria, taking capital market frictions as given; section 5 endogenizes the capital market frictions, by adding a voting stage; finally, section 6 concludes.

2 Related Literature

Our paper contributes to the literature on the political economy of finance, which stresses the political determinants of financial market regulations and corporate governance–see Pagano and Volpin (2001) for an early review. Several papers in this literature have emphasized that the economic interests of capital investors can be subordinated due to political considerations. In **?**, labor forms an alliance with inside shareholders to set up a stakeholder society with low investor protection and high employment protection. In Perotti and von Thadden (2006), a majority limits the ability of shareholders to allocate capital in order to limit risk for other stakeholders. Here, we emphasize an orthogonal channel: young and old workers form an alliance in favor of capital market frictions so that incumbent firms retain more of the scarce capital. Other related papers include Krusell and Ríos-Rull (1996) and Saint-Paul (2002), who study the political support for technological innovation, and labor market flexibility. The capital market plays no role in these papers, while it is central in ours. We offer an alternative channel to advance stakeholder interests: capital market frictions.

Hassler et al. (2003) study the political support for a distortionary welfare state. The welfare state distorts private incentives to invest in education, which in turn gives rise to a constituency that supports the welfare state. Hassler et al. (2003) provide an example of how repeated majority voting in an OLG model can generate persistence in support of an inefficient welfare policy, we provide another. A key difference in our model is that all constituencies vote, whereas the young are disenfranchised in Hassler et al. (2003) Another difference is that our current framework does not allow for dynamic feedback of political choices through incentives as in Hassler et al. (2003); the composition of constituencies is fixed in our paper.

We study different specifications of the political model, both in terms of possible voting strategies (open-loop, subgame perfect, and Markov perfect) and in terms of the persistence of the chosen policy (reversible and irreversible). This approach is similar to Azariadis and Galasso (2002), who study the political support for intergenerational transfers from young to old generations (such as pay-as-you go pension systems) under different specifications of the political model. An important difference to our approach is that the young are a majority in Azariadis and Galasso (2002), while the outcome of our voting game is more complex: based on age and wealth differences, we identify four distinct voter classes, none of which forms a majority. Young workers are decisive because their preferences are less extreme than the preferences of other voter classes, not because of their number.

As usual in political economy models, one may wonder why economically suboptimal outcomes cannot be resolved by bargaining. Should it not be possible to compensate the old workers for allowing capital reallocation? Such efficient bargaining is prohibited by the hold-up problem that is associated with giving up political power (cf. Acemoglu, 2003). If old workers vote in favor of a free capital market, other groups have no incentive to compensate them; likewise, if old workers receive compensation upfront, they will not vote in favor of a free capital market. This hold-up problem is a core issue in political economy and a source of much inefficiency. While capital market frictions cannot be bargained away, capital market participation can be broadened; thus, young and old workers' preferences can be changed (cf. Biais and Perotti, 2002).

3 Model

We consider an infinite horizon production economy with vintage technology. In each period there are 2 overlapping generations–the young and the old–with age-specific human capital: the young generation works in firms that use the latest vintage technology; the old generation works in firms that use an older vintage technology. All firms use capital and labor to produce a perishable consumption good. While labor is technology-specific, a fixed supply of capital can be used by all firms. We ignore capital growth in order to focus on the question how capital is allocated among different sectors. Time is denoted by subscripts $t = 0, 1, 2, ...,$; firm and agent characteristics are denoted by superscripts.

3.1 Firms

Each period, a unit mass of firms is set up. Firms exist for two periods: in the first period, firms employ the young generation and we refer to them as young firms (or y-firms); in the second period, firms employ the same generation as before–which is now the old generation–and we refer to them as old firms (or o-firms). Firms use a vintage technology, capital, and labor to produce a common consumption good that cannot be stored or saved.

Production is given by an age-specific productivity factor, θ^j, and a general production function, F:

$$\theta^j F(K^j, L^j), \;\; j = y, o$$

where K^j and L^j denotes capital and labor used in j-firms, $\theta^o < \theta^y$, and the price of output is normalized to 1.[4] The production function F satisfies the common conditions, i.e., (i) production is increasing in both factors, at a decreasing rate; (ii) capital and labor are complementary factors of

[4]We assume that old firms are less productive than young firms because they are stuck with an older technology.

production; and (iii) the Inada conditions are satisfied. Formally, F satisfies (i) $F_K, F_L > 0$ and $F_{KK}, F_{LL} < 0$, (ii) $F_{LK} = F_{KL} > 0$, and (iii) $\lim_{K\to 0} F_K = \lim_{L\to 0} F_L = \infty$, and $\lim_{K\to\infty} F_K = \lim_{L\to\infty} F_L = 0$.

Firms maximize profits in competitive factor and output markets. The labor market is segmented due to human capital specificity: old firms hire old workers and pay wages w_t^o; young firms hire young workers and pay wages w_t^y. The capital market has two important features: the cost of capital is r_t for capital that the firm retains from the previous period; firms incur an additional cost f_t if they wish to add to the capital they have employed last period. The additional cost f_t represents a pure deadweight loss; it drives a wedge between the interest rate that capitalists receive and the cost of capital that firms pay. We refer to f_t as the capital market friction, or the reallocation cost.

As there are two costs of capital in the economy—one for retained capital and one for newly obtained capital—a firm's total cost of capital depends on its capital stock at the start of each period. We use a hat notation to denote the initial capital stock of firms. For example, $\hat{K}_t^j$ denotes the initial capital stock of j-firms in period t. Firm profits for j-firms in period t are then given by

$$\theta^j F(K_t^j, L_t^j) - r_t K_t^j - f_t \max(0, K_t^j - \hat{K}_t^j) - w_t^j L_t^j \tag{3.1}$$

This is a standard expression for firm profits, except for the third term. Firms pay a marginal cost of capital $r_t + f_t$, if they want to attract capital beyond the initial stock of $\hat{K}_t{}^j$.

Young firms have no capital ($\hat{K}_t^y = 0$) and must attract all capital at a unit cost of $r_t + f_t$. When young firms turn into old firms—and the young generation turns into the old generation—they retain last period's capital ($\hat{K}_t^o = K_{t-1}^y$). As old firms are less productive than young firms, there is an economic rationale to move capital from old to young firms, i.e., to reallocate capital. Finally, old firms become obsolete when the old generation dies. Then the capital that they previously employed comes available to use elsewhere.

3.2 Agents

Each period, a generation of agents of unit mass is born. Agents live for two periods: in the first period, agents work in young firms; in the second period, agents work in old firms. Agents are endowed with vintage human capital work in firms of the same vintage. Furthermore, each generation consists of workers and capitalists. Old capitalists own the fixed capital stock, $\bar{K}$, and all the firms–in other words old capitalists own all the debt and equity in the economy. Young capitalists earn only wage income, but will inherit the capital stock and all the firms from their parents. The fraction of capitalists in each generation, which is constant, is given by η.

All agents inelastically supply their labor and earn wages; we normalize each agent's labor supply to $\bar{L} = 1$ per period. On top of their wage income, old capitalists receive interest payments and firm profits, as they own all the debt and equity in the economy. Old capitalists are identical and hold diversified portfolios; it follows that each capitalist receives wage income, w_t^o, and capital income,

$$s_t := \frac{r_t \bar{K} + \Pi_t}{\eta}$$

where Π_t denotes aggregate firm profits.[5] When capitalists die, their children inherit the capital stock; thus, a fraction η of the young turns into capitalists. The fraction of capitalists, $\eta \in (0, 1]$, is a measure of inequality: higher η means more capitalists and less wealth per capitalist.

The population can now be split into four groups, based on their lifetime income profile: young workers (yw), old workers (ow), young capitalists (yc), and old capitalists (oc). As income cannot be stored or saved, agents consume

[5] We assume that capitalists receive wages as well as capital income. This assumption is not crucial for our results: as we will show later, capitalists do not favor capital market frictions even if they benefit from a wage increase.

their income in every period. Lifetime utility of young workers and your capitalists is then given by

$$U_t^{yw} := u(w_t^y) + \delta u(w_{t+1}^o) \tag{3.2}$$

$$U_t^{yc} := u\left(w_t^y\right) + \delta u\left(w_{t+1}^o + s_{t+1}\right) \tag{3.3}$$

where $\delta \epsilon (0,1]$ is a time discount factor; u is a standard felicity function with $u' > 0$ and $u'' < 0$; and $s_t := \frac{1}{\eta}\left(r_t \bar{K} + \Pi_t\right)$. Remaining lifetime utility of the old generation at time t is then

$$U_t^{ow} = u(w_t^o) \tag{3.4}$$

for the old workers, and

$$U_t^{oc} = u(w_t^o + s_t) \tag{3.5}$$

for the old capitalists. Note that agents do not optimize over economic choices, as they don't save and they supply labor inelastically. Instead, agents optimize over political choices, by choosing a capital market friction in each period (cf. section 5).

3.3 Markets

Firms and agents interact in competitive factor and product markets. The product market is competitive and we normalize the price of the unique consumption good to 1. Each segment of the labor market is competitive and wages, w_t^j, adjust until the markets for young and old workers clear. There is one market for capital. The capital market is competitive in the sense that the interest rate, r_t, adjusts until the market clears; but, transactions on this market are subject to the capital market friction, f_t. Timing in each period is as follows,

1. an *initial allocation* of capital is retained from the previous period;

2. agents *vote* over the capital market friction f_t;

3. *economic activity* results in a new allocation of capital; and

4. agents get their *payoff*, i.e., their wage and capital income.

The political conflict has two dimensions: there is a class conflict between capitalists and workers, and there is an age conflict between the young generation and the old generation. Old workers stand to lose most from free capital mobility: because of the complementarity of capital and labor, their wage drops if capital is reallocated from old firms to young firms. Old capitalists also see their labor income drop, but their capital income increases. Preferences of the young generation depend on the nature of the capital market frictions, in particular, whether they are expected to persist over time. Young workers may vote in favor of a positive capital market friction, if they expect the friction to prevail until they are old. We analyze policy preferences and the resulting political equilibria in section 5. First we characterize the set of economic equilibria for a given sequence of capital market frictions.

4 Economic Equilibrium

For a given sequence of capital market frictions, $\{f_t\}_{t=0}^{\infty}$, with $0 \leq f_t < \infty$, an *economic equilibrium* is given by a sequence of factor prices and capital allocations $E = \{r_t, w_t^y, w_t^o, K_t^y, K_t^o\}_{t=0}^{\infty}$ such that in every period (i) firms maximize profits, and (ii) markets clear. We prove the existence and uniqueness of an economic equilibrium for an arbitrary sequence of capital market frictions in the following.

4.1 Existence and Characterization

Each period, firms take prices as given and maximize profits.[6] Young firms have no capital, and incur the capital market friction on each unit of capital they choose to employ, cf. profit function (3.1). They solve

$$\max_{K_t^y, L_t^y} \theta^y F(K_t^y, L_t^y) - (r_t + f_t) K_t^y - w_t^y L_t^y$$

which leads to standard first-order conditions

$$\theta^y F_K(K_t^y, L_t^y) = r_t + f_t \tag{4.1}$$

$$\theta^y F_L(K_t^y, L_t^y) = w_t^y \tag{4.2}$$

and corresponding capital and labor demand, K_t^y and L_t^y.

Old firms start the period with the capital they have employed in the previous period, $\hat{K}_t^o = K_{t-1}^y$. They solve

$$\max_{K_t^o, L_t^o} \theta^o F(K_t^o, L_t^o) - r_t K_t^o - f_t \max(0, K_t^o - \hat{K}_t^o) - w_t^o L_t^o$$

which leads to standard first-order condition

$$\theta^o F_L(K_t^o, L_t^o) = w_t^o \tag{4.3}$$

and corresponding labor demand L_t^o. As for their capital demand, the profit function of old firms is not differentiable at $\hat{K}_t^o$; thus, capital demand depends on whether old firms choose to adjust their capital. If old firms acquire extra capital in equilibrium, then K_t^o must satisfy

$$\theta^o F_K(K_t^o, L_t^o) = r_t + f_t \tag{4.4}$$

[6] Firms maximize profits that accrue to their owners, i.e. the old capitalists; hence, firms maximize current profits.

which is consistent if old firms indeed scale up, i.e., if

$$\theta^o F_K(K_t^o, L_t^o) < \theta^o F_K(\hat{K}_t^o, L_t^o)$$

or

$$r_t < \theta^o F_K(\hat{K}_t^o, L_t^o) - f_t$$

Old firms increase their capital if the interest rate is sufficiently small; likewise, old firms decrease their capital if the interest rate is sufficiently big, or

$$r_t > \theta^o F_K(\hat{K}_t^o, L_t^o).$$

For intermediate values of the interest rate, old firms keep the capital they used in the previous period, $\hat{K}_t^o = K_{t-1}^y$.

In each period, young and old agents inelastically supply their labor, normalized to 1. In equilibrium, both segments of the labor market must clear and we obtain equilibrium wage rates from (4.2) and (4.3):

$$\begin{aligned} w_t^y &= \theta^y F_L(K_t^y, 1) && (4.5) \\ w_t^o &= \theta^o F_L(K_t^o, 1). && (4.6) \end{aligned}$$

Capital and labor are complementary factors of production, so that wages increase along with an increase in capital for young and old firms.

Capital demand of young firms follows from (4.1) and, by letting $g(K) := F_K(K, 1)$, we can write

$$K_t^y\left(r_t; f_t\right) = K_t^y\left(r_t\right) = g^{-1}\left(\frac{r_t + f_t}{\theta^y}\right) \tag{4.7}$$

Our earlier discussion shows that capital demand of old firms is

$$K_t^o(r_t; f_t) = K_t^o(r_t) = \begin{cases} g^{-1}\left(\frac{r_t + ft}{\theta^o}\right) & \text{if} \quad r_t < \underline{r}_t \\ g^{-1}\left(\frac{r_t}{\theta^o}\right) & \text{if} \quad r_t > \bar{r}_t \\ \hat{K}_t^o & \text{if} \quad \underline{r}_t \le r_t \le \bar{r}_t \end{cases} \tag{4.8}$$

with $\underline{r}_t \equiv \theta^o g\left(\hat{K}_t^o\right) - f_t$, and $\bar{r}_t \equiv \theta^o g\left(\hat{K}_t^o\right)$. Total capital demand then is

$$\varphi_t(r_t; f_t) = \varphi_t(r_t) = K_t^y(r_t) + K_t^o(r_t)$$

and the capital market clears if $\varphi_t(r_t) = \bar{K}$. The following lemma shows that a market clearing interest rate exists and is unique in each period.

Lemma 4.1. *For $0 \le f_t < \infty$, there exists a unique market clearing interest rate $r_t^* = r_t^*(f_t)$.*

Proof. For any $0 \le f_t < \infty$, φ_t is continuous, strictly decreasing, and piecewise differentiable in r_t (with kinks at $\underline{r}_t$ and $\bar{r}_t$). Furthermore, by the Inada conditions we have

$$\lim_{r_t \to \infty} \varphi_t(r_t) = 0 \quad \text{and} \quad \lim_{r_t \to -f_t} \varphi_t(r_t) = \infty$$

Hence by the continuity of φ_t, there is $r_t^* > -f_t$ such that

$$\varphi_t(r_t^*) = \bar{K}$$

Uniqueness of r_t^* follows from the strict monotonicity of φ_t. □

From the market clearing interest rate, r_t^*, period t sector wages and capital demands readily follow.[7] Hence, we have proven the following proposition

[7]If the interest rate is negative, we set it to 0, which implies that some capital remains unused. We show later, however, that no voter group wishes to set f_t so high as to induce a negative interest rate.

Proposition 4.1. *For any sequence of capital market frictions $\{f_t\}_{t=0}^{\infty}$ with $0 \leq f_t < \infty$, there exists a unique economic equilibrium E.*

Proof. See above. $\square$

The previous argument and in particular capital demand of old firms, given by (4.8), shows that the economic equilibrium in each period is characterized by the initial capital of the old firms, $\hat{K}_t^o$, and the prevailing capital market friction, f_t. Depending on the interest rate, old firms then adjust their capital, downward, upward, or they keep the capital they employed last period. We characterize these cases in turn.

First, old firms adjust their capital upward if and only if the interest rate is sufficiently small, or $r_t^* < \underline{r}_t$. The capital market clearing condition then reads

$$g^{-1}\left(\frac{r_t^* + f_t}{\theta^y}\right) + g^{-1}\left(\frac{r_t^* + f_t}{\theta^o}\right) = \bar{K} \tag{4.9}$$

which implicitly gives the equilibrium interest rate, r_t^*. In this case, condition (4.9) implies that $r_t^* + f_t$ does not depend on f_t; hence, old firms' equilibrium capital is independent of f_t. We define this value of old firms' capital as $\underline{K}^o$, so

$$\underline{K}^o := g^{-1}\left(\frac{r_t^* + f_t}{\theta^o}\right) \tag{4.10}$$

with r_t^* given by (4.9). Consistency in this case requires that $\hat{K}_t^o < \underline{K}^o$.

Second, old firms adjust their capital downward if and only if the interest rate is sufficiently large, or $r_t^* > \bar{r}_t$. Market clearing then reads

$$g^{-1}\left(\frac{r_t^* + f_t}{\theta^y}\right) + g^{-1}\left(\frac{r_t^*}{\theta^o}\right) = \bar{K} \tag{4.11}$$

which gives the equilibrium interest rate. In this case, old firms' equilibrium capital is given by

$$\overline{K}^o := g^{-1}\left(\frac{r_t^*}{\theta^o}\right) \tag{4.12}$$

which is an increasing function of the capital market friction, f_t, cf. capital market clearing condition (4.11) . Consistency in this case requires that $\hat{K}_t^o > \underline{K}^o$.

Finally, old firms keep their initial capital unchanged if and only if the interest rate falls in an intermediate range, or $\underline{r}_t \leq r_t^* \leq \bar{r}_t$. The capital market clearing condition then reads

$$g^{-1}\left(\frac{r_t^* + f_t}{\theta^y}\right) + \hat{K}_t^o = \bar{K} \tag{4.13}$$

which gives the equilibrium interest rate. In this case, old firms' capital is simply given by $\hat{K}_t^o$ and consistency requires that

$$\underline{K}^o \leq \hat{K}_t^o \leq \overline{K}^o(f_t)$$

We see that old firms' behavior in period t is fully characterized by their initial capital, $\hat{K}_t^o$, and the prevailing capital market friction, f_t.

Young firms' behavior is easily described: as young firms have no initial capital, they must adjust capital upward. It follows that equilibrium capital of young firms is given by

$$K^y = g^{-1}\left(\frac{r_t^* + f_t}{\theta^y}\right)$$

where r_t^* depends on the equilibrium behavior of old firms, i.e., on $\hat{K}_t^o$ and f_t. This concludes the characterization of the economic equilibrium in period t. We have shown that for arbitrary sequences of capital market frictions, $\{f_t\}_{t=0}^{\infty}$ with $0 \leq f_t < \infty$, capital market activity in any given period necessarily has little structure. As we are mostly interested in stable political outcomes that yield constant sequences of f_t , we focus on steady state equilibria in the following.

4.2 Steady States

A *steady state economic equilibrium* is an economic equilibrium such that $(K_t^y, K_t^o) = (K_{t+1}^y, K_{t+1}^o)$ for all t. In a steady state, capital is reallocated from old firms to young firms as the following lemma shows.

Lemma 4.2. *In a steady state economic equilibrium, old firms do not scale up.*

Proof. Consider a steady state equilibrium and suppose that old firms scale up, i.e., $K^o > \hat{K}^o = K^y$–where we've suppressed subscripts because these are steady state values. As old firms scale up in every period, we know that in any given period t, market clearing is given by condition (4.9), or

$$g^{-1}\left(\frac{r_t^* + f_t}{\theta^y}\right) + g^{-1}\left(\frac{r_t^* + f_t}{\theta^o}\right) = \bar{K}$$

It follows from $\theta^y > \theta^o$ that $K_t^o < K_t^y$, a contradiction. □

The intuition for lemma 4.2 is that old firms are less productive than young firms, and so in a steady state–where old firms adjust their capital by the same amount in every period–the economic rationale for reallocating capital from old to young firms prevails.

We can now characterize all steady states that exist for constant sequences of capital market frictions.

Proposition 4.2. *Given a constant sequence of capital market frictions* $\{f\}_{t=0}^{\infty}$*, with* $0 \leq f < \infty$*, there exist bounds* $0 < \bar{f} < \bar{\bar{f}} < \infty$ *given by* $\bar{f} = (\theta^y - \theta^o)\, g\left(\frac{1}{2}\bar{K}\right)$*, and* $\bar{\bar{f}} = \theta^y g\left(\frac{1}{2}\bar{K}\right)$ *, such that*

1. for $f \in [0, \bar{f})$, the unique steady state equilibrium is given by

$$K^o = g^{-1}\left(\frac{r^*}{\theta^o}\right) \quad , \quad K^y = g^{-1}\left(\frac{r^* + f}{\theta^y}\right) \tag{4.14}$$

where r^* is given by market clearing condition (4.11), or

$$g^{-1}\left(\frac{r^*+f}{\theta^y}\right)+g^{-1}\left(\frac{r^*}{\theta^o}\right)=\overline{K} \tag{4.15}$$

2. for $f \in \left[\bar{f}, \bar{\bar{f}}\right)$, the unique steady state equilibrium is given by

$$K^y = K^o = \frac{1}{2}\bar{K} \tag{4.16}$$

where r^* is given by market clearing condition (4.13), or

$$g^{-1}\left(\frac{r^*+f}{\theta^y}\right)+\hat{K}^o=\bar{K} \tag{4.17}$$

Proof. We proof the proposition by construction. Let $\{f\}_{t=0}^{\infty}$ be a constant sequence of capital market frictions, with $0 \leq f < \infty$. By lemma 4.2, $K^o \leq \hat{K}^o = K^y$ in any steady state.

For 1: consider potential steady states with $K^o < K^y$. As old firms must scale down, the capital allocations are given by (4.14) and the market clearing interest rate, r^*, is by (4.15), cf. section 4.1. For consistency, we need to check that indeed $K^o < K^y$. The inequality holds as long as

$$g^{-1}\left(\frac{r^*}{\theta^o}\right)<g^{-1}\left(\frac{r^*+f}{\theta^y}\right)$$

Clearly, $K^o < K^y$ for $f = 0$, and–through (4.15)–K^o is increasing in f, while K^y is decreasing in f. Furthermore, we note that

$$\lim_{f\to\infty} g^{-1}\left(\frac{r^*}{\theta^o}\right)=\bar{K}\ ,\ \lim_{f\to\infty} K^y = 0$$

so that, by the continuity of g^{-1}, and by (4.15), there exists $\bar{f} \in (0,\infty)$ such that

$$g^{-1}\left(\frac{r^*}{\theta^o}\right) = g^{-1}\left(\frac{r^* + \bar{f}}{\theta^y}\right) = \frac{1}{2}\bar{K}$$

Solving explicitly for $\bar{f}$ gives

$$\bar{f} = (\theta^y - \theta^o)\, g\left(\frac{1}{2}\bar{K}\right) \tag{4.18}$$

For 2: consider potential steady states with $K^o = K^y$. As the capital market must clear, capital allocations are necessarily given by $K^o = K^y = \frac{1}{2}\bar{K}$, and the market clearing interest rate, r^*, is given by (4.17). For consistency, we need to check that young firms wish to employ $\frac{1}{2}\bar{K}$, and that old firms indeed do not wish to adjust their capital. Young firms wish to employ $\frac{1}{2}\bar{K}$ if

$$g^{-1}\left(\frac{r^* + f}{\theta^y}\right) = \frac{1}{2}\bar{K}$$

from where $(r^* + f) = \theta^y g\left(\frac{1}{2}\bar{K}\right)$. As the interest rate is restricted to be nonnegative, we obtain the upperbound $\bar{\bar{f}} = \theta^y g\left(\frac{1}{2}\bar{K}\right)$. Old firms face a cost of capital r^* when they consider to adjust capital downward. It follows from part 1 that old firms do not wish to scale down as long as $f \geq \bar{f}$, which concludes the proof. □

Proposition 4.2 gives all steady state equilibria that exist for constant sequences of capital market frictions $\{f\}_{t=0}^{\infty}$. We denote these steady states by E_f.[8] Capital market frictions distort the allocation of capital in the economy, so that production output stays below its potential maximum. This economic distortion, however, does not constitute a Pareto inefficiency per se. As usual in political economy models, there may be groups of voters that gain from the economic distortion.

[8]From the proof of proposition 4.2, we see that a steady state with $K^y = K^o = \frac{1}{2}\bar{K}$ also exists for non-constant sequences $\{f_t\}_{t=0}^{\infty}$ as long as $\bar{f} \leq f_t < \bar{\bar{f}}$. We do not consider these in the following.

5 Political Equilibrium

Will democracies support a free capital market? To answer this question, we let capital market frictions be politically chosen in this section: preceding economic interaction, agents vote on a capital market friction in each period. Specifically, there is a majority vote in which all voters vote for their preferred capital market friction.[9]

Formally, a *vote* (or action) at time t for a member of voter class i is a capital market friction $a_t^i \in [0, \infty)$. At time t, the publicly known history of the voting game consists of the sequence of past capital market frictions, $h_t = (f_0, .., f_{t-1}) \epsilon H_t$ with $H_t = [0, \infty)^t$. A voting strategy for voter class i at time t is then given by a mapping $v_t^i : H_t \to [0, \infty)$.

A *political economic equilibrium* is a sequence of factor prices and allocations $E = \{r_t, w_t^y, w_t^o, K_t^y, K_t^o\}_{t=0}^{\infty}$, supplemented with a voting strategy profile $v = \{v_t^{yw}, v_t^{yc}, v_t^{ow}, v_t^{oc}\}_{t=0}^{\infty}$, such that (i) v is an equilibrium of the voting game; and (ii) E is an economic equilibrium given the sequence of reallocation costs $\{f_t\}_{t=0}^{\infty}$ in the outcome of v. We say that an economic equilibrium, E, is *politically supported* if there exists a voting strategy profile, v, such that $\{E, v\}$ is a political economic equilibrium.

In the following, we focus on steady state equilibria that are politically supported. To obtain closed-form solutions, we assume that production is Cobb-Douglas,

$$F(K, L) = K^{\alpha} L^{\beta} \tag{5.1}$$

with $0 < \alpha + \beta < 1$. We derive all steady state equilibrium values in appendix A.

The following lemma summarizes the comparative statics of factor prices and sector capital with respect to f_t; the lemma is formulated for deviations from a steady state in a given period to have the necessary generality to

[9] We assume a pure majority rule, i.e., democracy is direct, voters vote sincerely, and there is an open agenda (cf. Persson and Tabellini, 2000).

analyze potential out-of-steady-state deviations.

Lemma 5.1. *For any given t, let $\bar{f}_t$ be defined by*

$$\hat{K}_t^o = \overline{K}^o(\bar{f}_t). \tag{5.2}$$

where the function $\overline{K}^o$ has been defined by (4.12). Then (i) K_t^y and w_t^y are strictly decreasing in $f_t \in [0, \bar{f}_t]$ and constant for $f_t > \bar{f}_t$; (ii) K_t^o and w_t^o are strictly increasing in $f_t \in [0, \bar{f}_t]$ and constant for $f_t > \bar{f}_t$; (iii) $w_t^o + s_t$ is decreasing in f_t.

Proof. See Appendix B. □

The equilibrium cut-off value $\bar{f}_t$ is the boundary value at which old firms retain their initial capital, $\hat{K}_t^o$. For lower values of the friction, old firms scale down.[10] Note that once f_t is fixed for a given period, the equilibrium interest rate r_t^* and all other time t equilibrium values readily follow. Hence we can write agents' lifetime utility as a function of the capital market frictions prevailing today and tomorrow, cf. (3.2) - (3.5):[11]

$$U_t^i = U_t^i(f_t, f_{t+1}) \quad \text{for} \quad i \epsilon \{ow, oc, yw, yc\}$$

5.1 Policy Preferences

Voters' policy preferences depend on whether they expect today's chosen policy to affect tomorrow's policy. As a benchmark, we derive the equilibrium that results when intertemporal linkeages are absent, i.e., when voters play *open-loop* strategies that do not depend on history.

[10] We consider deviations out-of-steady-state, which means that $\hat{K}_t^o \geq \frac{1}{2}\bar{K}$ (cf. proposition 4.2). It follows that $\bar{f}_t \geq \bar{f}$, where $\bar{f}$ is the boundary value of the capital market friction for which no reallocation of capital takes place in steady state.

[11] The old generation's utility depends only on f_t and is constant in f_{t+1}.

Proposition 5.1. *If voters play open-loop strategies, then the unique political economic equilibrium is given by* E_0 *and the voting strategy profile* $v = \{v_t^{yw} = 0, v_t^{yc} = 0, v_t^{ow} = \bar{f}_t, v_t^{oc} = 0\}_{t=0}^{\infty}$.

Proof. Lemma 5.1 shows that U_t^{yw}, U_t^{yc}, and U_t^{oc} are monotonically decreasing in f_t. Likewise, U_t^{yw} is monotonically increasing in f_t. The voting strategy profile, v, then follows from sincere voting and a majority of agents votes for $f_t = 0$ in every period. The corresponding political economic equilibrium is E_0. □

Proposition 5.1 is the first key result of our paper: a free capital market is supported in political economic equilibrium if voters (i) vote in every period, and (ii) play strategies that do not depend on history. A majority consisting of old capitalists and all the young favor a free capital market, as wages paid by young firms and capitalists' income are maximized if the capital market is free. Unsurprisingly, capitalists achieve maximum lifetime utility in this steady state: their wages in young age are maximized, and so is their capital income in old age. Note that workers on the other hand achieve maximum lifetime utility if there is no capital market friction when they are young and if the maximum friction prevails when they are old.

We turn to the question in which steady state the utility, U_t^i, of the different voters is maximized. Because in steady state the capital market friction is time independent, as are the voters' utilities, we may drop subscripts and write $U^i(f) = U_t^i(f, f)$. It follows from the above discussion that $U^{yc}(f)$ and $U^{oc}(f)$ are maximized for $f = 0$. Old worker utility, $U^{ow}(f)$, is maximized if no capital reallocation takes place, i.e., for $f \geq \bar{f}$. Young workers have more interesting preferences as we will see in the following.

Consider young worker utility $U^{yw}(f) = u\left(w^y(f)\right) + \delta u\left(w^o(f)\right)$. Steady state wages follow from (4.5) and (4.6) and the capital allocations we derived in Appendix A. We have

$$w^y(f) = \theta^y \beta \left(\frac{\alpha \theta^y}{r(f) + f} \right)^{\frac{\alpha}{1-\alpha}} \ ; \ w^o(f) = \theta^y \beta \left(\frac{\alpha \theta^o}{r(f)} \right)^{\frac{\alpha}{1-\alpha}}$$

for $f \in [0, \bar{f}]$, and

$$w^y(f) = \theta^y \beta \left(\frac{1}{2} \bar{K} \right)^{\frac{\alpha}{1-\alpha}} \quad ; \quad w^o(f) = \theta^y \beta \left(\frac{1}{2} \bar{K} \right)^{\frac{\alpha}{1-\alpha}}$$

for $f > \bar{f}$. Young workers face a real trade off: higher wages when young against higher wages when old. The following lemma gives the preferred steady state policy of the young worker.

Lemma 5.2. *Young workers preferred steady state capital market friction, f^{yw}, is given by*

$$\frac{u'(w^y)}{\delta u'(w^o)} = \frac{r(f^{yw})}{r(f^{yw}) + f^{yw}} \tag{5.3}$$

If condition (5.3) admits no interior solution then $f^{yw} = 0$ if $\frac{u'(w^y)}{\delta u'(w^o)} > \frac{r(f)}{r(f)+f}$ on $(0, \bar{f})$; and, $f^{yw} = \bar{f}$ if $\frac{u'(w^y)}{\delta u'(w^o)} < \frac{r(f)}{r(f)+f}$ on $(0, \bar{f})$.

Proof. See appendix B. □

Lemma 5.2 shows that young workers may prefer a steady state with a positive capital market friction. A positive friction allows the young worker to smooth consumption over their lifetime; accordingly, equation (5.3) can be interpreted as an optimal smoothing condition. For example, if utility is linear, there is no consumption smoothing motive and (5.3) admits no interior solution. If utility is concave, and voters are risk-averse, then f^{yw} will be given by condition (5.3) and depend on the discount rate, δ, and technological factors, θ^y and θ^o. The assumption that agents cannot save is crucial, as lemma 5.2 reveals that the motive to install a capital market friction is to smooth consumption. But if we reinterpret the drop in wages as an increase in unemployment, then our findings hold even if households can store their income.[12] The capital market friction then becomes an unemployment insurance that young workers wish to install.

We have seen that capital market frictions cannot arise in political equilibrium when voters play open-loop strategies (cf. proposition 5.1). We have also seen that workers achieve maximum lifetime utility in a different steady state (cf. lemma 5.2). Young workers are the pivotal voter class, but they do not choose $f_t = f^{yw}$, as there is no guarantee that next period's young workers will do the same and set $f_{t+1} = f^{yw}$. Open-loop strategies leave no scope for cooperation between young workers in different periods. But once we allow for richer voting strategies, cooperation between subsequent generations of young workers can be sustained in a subgame perfect Nash equilibrium (SPNE) of the repeated voting game.

Proposition 5.2. *If voters play history dependent strategies, then there exists $f^* \in [0, \bar{f}]$ such that for every $f \in [0, f^*]$, the steady state equilibrium E_f can be politically supported.*

Proof. Consider the set

$$D := \{f \epsilon [0, \infty) | U^{yw}(f, f) \geq U^{yw}(0, 0)\}$$

D is nonempty, as $0 \in D$; D is closed, as U^{yw} is continuous; and D is convex, as U^{yw} is single-peaked. It follows that D is a closed interval which contains f^{yw}, the preferred lifetime policy of the young workers (cf. lemma 5.2). Now, let $f^* := \min\{\bar{f}, \sup D\}$ and choose an arbitrary $f \in [0, f^*]$. Then E_f can be politically supported. Consider the voting strategy profile

[12] Unemployment is easily introduced in our model by making wages downward rigid.

$v^* = \{v_t^{yw}, v_t^{yc}, v_t^{ow}, v_t^{oc}\}_{t=0}^{\infty}$ such that

$$\begin{aligned} v_t^{yw} &= \begin{cases} f \quad \text{if} \quad c_{t-s} = f \quad \text{for} \quad s = 1, ...t \\ 0 \qquad\qquad\qquad \text{otherwise} \end{cases} \\ v_t^{yc} &= v_t^{oc} = 0 \\ v_t^{ow} &= \bar{f}_t \end{aligned}$$

With this strategy profile the voting game equilibrium is f in every period provided the play started with $f_0 = f$. The best deviation of the yw class at time t is to set $f_t = 0$ given that $v_{t+1}^{yw} = 0$. This deviation is not profitable since $U_t^{yw}(0,0) \leq U_t^{yw}(f,f)$. It remains to check that $v_{t+1}^{yw} = 0$ is incentive compatible. It is because $U_{t+1}^{yw}(0,0) > U_{t+1}^{yw}(f_{t+1},0)$ for any $f_{t+1} > 0$. We have shown that v^* is an SPNE of the repeated voting game. We conclude that $\{E_f, v^*\}$ is a political economic equilibrium. □

Proposition 5.2 shows that each steady state in which young workers get a higher utility than their open loop utility can be politically supported. Proposition 5.2 is the second key result of our paper: capital market frictions can arise in political economic equilibrium if voters (i) vote in every period, and (ii) play strategies that depend on history. Young workers are pivotal, as their vote determines the outcome of the voting game in each period. Young workers achieve maximum lifetime utility in this steady state if they choose $v_t^{yw} = f^{yw}$.[13]

Political equilibria with history dependent voting strategies can be interpreted as arising from a *social contract* which allows for cooperation between current and future young generations: young workers vote for a positive capital market friction as long as they expect future generations to do the same. But there is no guarantee that cooperation is achieved in the repeated voting game, as there are multiple equilibria.

Following the literature on dynamic political economy, we reduce the mul-

[13] As there are multiple equilibria, there is no guarantee that they will.

tiplicity of equilibria by restricting the solution concept by studying policies that can be sustained without commitment (cf. Hassler et al., 2003; Klein et al., 2008). In particular, we look for Markovian equilibria of the voting game, i.e., subgame-perfect equilibria in which the the policy variable is a time-invariant function of the state variable. In the present context, the state variable is old firms' initial capital stock, $k_t := \hat{K}_t^o = K_{t-1}^y$. We restrict the choice of the decisive voter class to a Markovian policy function,

$$f_t = \mu(k_t) \tag{5.4}$$

which is assumed to be time invariant and differentiable.

The transition function, T, gives next period's state variable as a function of this period's state variable and capital market friction. The transition function

$$T : [0, \bar{K}] \times [0, \infty) \to [0, \bar{K}]$$

given by $T(k_t, f_t) = K_t^y$, follows from firm behavior evaluated for a Cobb-Douglas production function (cf. section 4). Remember that firm behavior does not depend on the future prevailing capital market friction, f_{t+1}, which implies that the transition function takes a simple form as it does not depend on the policy function μ.

Young workers now solve

$$\begin{aligned} \mu(k_t) &= \arg\max_{f_t} U_t^{yw}(f_t, f_{t+1}; k_t) \\ \text{subject to } f_{t+1} &= \mu(k_{t+1}), \quad f_t \in [0, \infty] \\ k_{t+1} &= T(k_t, f_t) \end{aligned}$$

Intuitively, the policy function, μ, must yield a capital market friction, f_t, such that the utility of young workers is maximized, taking into account the effect the choice of has on future policy through the state variable k_{t+1}.

First, we consider trivial Markovian policy functions of the form

$$\mu(k_t) = C$$

where C is a constant. Then the optimization problem reduces simply to

$$\max_{f_t} U_t^{yw}(f_t, C)$$

which is uniquely solved by $f_t = 0$. It follows that

$$\mu(k_t) = 0$$

is an admissible Markovian policy function. If young workers adopt the trivial Markovian voting strategy $v_t^{yw} = 0$ for every t, then $f_t = 0$ is the outcome of the voting game for every t. Thus we have shown that E_0, the steady state without frictions, is politically supported when voters play Markovian voting strategies. A priori, there could be other steady states that are politically supported. But we are able to rule this out in the following.

Proposition 5.3. *If voters play Markovian strategies, then E_0 is the unique steady state that is politically supported. The corresponding voting strategy profile is $v = \{v_t^{yw} = 0, v_t^{yc} = 0, v_t^{ow} = \bar{c}_t, v_t^{oc} = 0\}_{t=0}^{\infty}$.*

Proof. Consider a steady state E_f with $f > 0$. In steady state we have

$$K^y(f) \geq \bar{K}^o(f)$$

which implies that the transition function, T, reduces to

$$h(f) = T(k, f) = \left(\frac{\alpha \theta^y}{r^*(f) + f} \right)^{\frac{1}{1-\alpha}}$$

where $r^*(f)$ is given by

$$\left(\frac{\alpha\theta^y}{r^*(f)+f}\right)^{\frac{1}{1-\alpha}} + \left(\frac{\alpha\theta^o}{r^*(f)}\right)^{\frac{1}{1-\alpha}} = \bar{K}$$

Note that h is a strictly decreasing function of f on $[0, \bar{f}]$. Since in steady state, we must also have

$$f = \mu(h(f))$$

we see that $\mu(k) = h^{-1}(k)$. But this cannot be a solution to the young worker's optimization problem as

$$\arg\max_{f_t} U_t^{yw}(f_t, f_t)$$

is independent of k, cf. lemma 5.2. We conclude that there are no steady states, besides E_0, that can be politically supported using Markovian policy functions. □

Proposition 5.3 shows that only the free capital market steady state can be political supported if the policy function is Markovian. The impossibility to politically support other steady states results from the fact that the preferred policy of the pivotal young worker class is independent of the state; thus, the form imposed by $f_t = \mu(k_t)$ leads to trivial solutions.

5.2 Policy Persistence

What is the equilibrium if voters choose persistent capital market frictions? To answer this question, we pursue an assumption of *policy persistence* in the following. Such a restriction of the political model amounts to assuming that current voters can commit future voters to their choice (as in Azariadis and Galasso (2002), or Saint-Paul (2002)). Specifically, voters at time t set a persistent policy that lasts throughout their lifetime, i.e., $f_t = f_{t+1}$.[14]

All agents vote sincerely, and so their votes follow from preferences derived in section 5.1. Recall that utility of old capitalists is maximized for $f_t = 0$, and utility of young capitalists is maximized for $f_t = 0$ and $f_{t+1} = 0$; hence, all capitalists vote for a zero friction,

$$v_t^{oc} = v_t^{yc} = 0$$

It follows that if capitalists are a majority, the outcome of the voting game is $f_t = f_{t+1} = 0$; and consequently, E_0 is the unique steady state that can be politically supported under policy persistence. If capitalists are a minority on the other hand, then worker preferences are decisive for the political equilibrium. We have seen before that utility of the old worker class is maximized if old firms can retain all their initial capital, $\hat{K}_t^o$. It follows that old workers vote for a high capital market friction, $v_t^{ow} \geq \bar{f}_t$ (cf. lemma 5.1). As for the young workers, they face a tradeoff: a higher capital market friction leads to a decrease in their young-age wages and an increase in their old-age wages. We show that young workers vote *as if* choosing among steady state utility levels:[15]

Lemma 5.3. *Consider a vote at time t under policy persistence. Then young workers choose $a_t^{yw} = f^{yw}$, where f^{yw} is given by lemma 5.2.*

Proof. See Appendix B. □

We have the following proposition.

[14] This assumption means that the next generation of voters is disenfranchised. As we will see, however, the disenfranchised median voter–a young worker–is better off in the policy persistence equilibrium than in the equilibrium without commitment (cf. proposition 5.1).

[15] We show in Appendix B that the economy reaches a new steady state right after the vote at time t.

Proposition 5.4. *Under policy persistence, the voting strategy profile is $v = \{v_t^{yw} = f^{yw}, v_t^{yc} = 0, v_t^{ow} = \bar{f}_t, v_t^{oc} = 0\}_{t=0}^{\infty}$ and the unique political equilibrium is given by (i) $\{E_0, v\}$ if capitalists are a majority, or (ii) $\{E_{f^{yw}}, v\}$ if capitalists are a minority.*

Proof. Strategy profile v follows from sincere voting and voters policy preferences discussed above. For (i): if capitalists are a majority, $f_t = 0$ is the outcome of the voting game in every period that there is a vote, and E_0 is the corresponding steady state. For (ii): as all preferences are single peaked, we can apply a median voter theorem. The median voter is a young worker, so $f_t = f^{yw}$ is the outcome of the voting game in every period that there is a vote. The corresponding economic equilibrium is $E_{f^{yw}}$. $\square$

When capitalists are a minority, the more plausible case on which we focus, then the median voter is a young worker. Young workers are pivotal, as their preferences lie in between the preferences of capitalists and the preferences of old workers. Proposition 5.4 is the third key result of our paper: under policy persistence, the preferred steady state of the young worker is the unique steady state that is politically supported. Workers achieve maximum lifetime utility in this political equilibrium.

As an illustration of proposition 5.4, we solve explicitly for the political economic equilibrium under persistent policy voting with CRRA utility. Let the felicity function u be given by

$$u_g(w) := \begin{cases} \frac{w^{1-g}}{1-g} & \text{for} \quad g > 0, g \neq 1 \\ \ln w & \text{for} \qquad g = 1 \end{cases}$$

The following lemma gives the preferred capital market friction of the young worker,

Lemma 5.4. *If voters choose persistent policies, the unique outcome f^{yw} of* *the voting game in every period is given by*

$$f^{yw} = \begin{cases} 0 & \textit{if} \quad \delta A^{\frac{-g}{1+g(\gamma-1)}} \leq 1 \\ \left(\delta A^{\frac{-g}{1+g(\gamma-1)}} - 1\right) r & \textit{if} \quad 1 < \delta A^{\frac{-g}{1+g(\gamma-1)}} < \frac{\theta^y}{\theta^o} \\ \bar{c} & \textit{if} \quad \delta A^{\frac{-g}{1+g(\gamma-1)}} \geq \frac{\theta^y}{\theta^o} \end{cases}$$

with $\gamma := \frac{1}{1-\alpha}$ *and* $A := \left(\frac{\theta^o}{\theta^y}\right)^\gamma$.

Proof. See appendix □

Implicit derivation of f^{yw} gives comparative statics

Lemma 5.5. *f^{yw} is increasing in δ; decreasing in θ^o; and increasing in θ^y.*

Proof. Omitted □

The intuition for lemma 5.5 is that the consumption smoothing motive increases if young workers value future consumption more and if the rate of technological obsolescence is higher, so that the wage gap between young and old age is bigger.

6 Conclusion

In this paper we use an OLG model to study a political conflict between economic agents who differ in age (young vs. old) and wealth (workers vs. capitalists). The political conflict centers on the capital market: capitalists, be they young or old, wish to see a free capital market where capital is allocated to its most productive use in each period. Old workers, on the other hand, wish to obstruct the capital market, as reallocation of capital to new firms lowers their wages. Young workers, finally, have two opposing considerations: while they favor a free capital market when they are young, which gives them higher wages; they also anticipate being old, which tends to align their preferences with the preferences of old workers. This political conflict exists as long as (i) capital and labor are complementary factors of production, (ii) human capital is less mobile than physical capital, and (iii) human capital risk cannot be fully insured.

We assume that the political conflict is settled democratically, i.e., by a majority vote that precedes economic interaction in each period. The main contribution of our paper is to identify the young worker as the decisive interest group in society–under the plausible assumption that capitalists are a minority.[16] Young workers are decisive because their preferences are less polarized than preferences of other groups. While young workers are hurt by capital market frictions in the short term, they may still favor them to smooth consumption over their lifetime, i.e., to decrease the wage gap between young and old age. Young workers prefer a higher friction (i) if technology grows at a faster pace, (ii) if they place more weight on the future, and (iii) if they are more risk averse. This result holds as long as young workers expect the capital market friction to persist.

We rule out lump sum transfers as a means to reach a free capital market, i.e., we rule out vote buying. As Acemoglu (2003) has pointed, such political bargains are unlikely to take place as there is an essential hold-up problem: if workers vote for a free capital market, the capitalists have no incentive to compensate them ex post; likewise, the workers have no incentive to vote for a free capital market after they've received the compensation.

The voting process is critical: the political equilibrium depends on the ability of a current majority to establish a persistent policy. When policies may be overturned in each period, the model features multiple equilibria. By contrast, the equilibrium prediction is the unique outcome favored by the young worker if the outcome of the vote is irreversible. Our model explains why a free capital market is opposed in democracies as a result of the wealth and age distribution in a country's population.

[16]A special case is when capitalists form a majority; in this case, our model predicts that democracies will choose a free capital market. Broad capital market participation is found in some democracies, in particular those with funded pension schemes.

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A Cobb-Douglas Production

We derive the steady state economic equilibria for a Cobb-Douglas production economy, where F is given by

$$F(K,L) = K^\alpha L^\beta \tag{A.1}$$

with $0 < \alpha + \beta < 1$. Steady-state capital allocations for $f \in [0, \bar{f}]$ are

$$K^y = \left(\frac{\alpha\theta^y}{r^* + f}\right)^{\frac{1}{1-\alpha}}; \quad K^o = \left(\frac{\alpha\theta^o}{r^*}\right)^{\frac{1}{1-\alpha}} \tag{A.2}$$

with r^* given by capital market clearing condition

$$\left(\frac{\alpha\theta^y}{r^* + f}\right)^{\frac{1}{1-\alpha}} + \left(\frac{\alpha\theta^o}{r^*}\right)^{\frac{1}{1-\alpha}} = \bar{K}$$

The boundary value $\bar{f}$ follows from (**??**) and is given by

$$\bar{f} = \alpha\frac{\theta^y - \theta^o}{\left(\frac{1}{2}\bar{K}\right)^{1-\alpha}}$$

For $f \in \left[\bar{f}, \bar{\bar{f}}\right)$ we have steady state capital allocations

$$K^y = \left(\frac{\alpha\theta^y}{r^* + f}\right)^{\frac{1}{1-\alpha}};, \quad K^o = \frac{1}{2}\bar{K}$$

where the capital market clearing condition

$$\left(\frac{\alpha\theta^y}{r^* + f}\right)^{\frac{1}{1-\alpha}} = \frac{1}{2}\bar{K}$$

allows us to obtain the equilibrium interest rate explicitly,

$$r^* = \frac{\alpha\theta^y}{\left(\frac{1}{2}\bar{K}\right)^{1-\alpha}} - f$$

from where the upperbound follows, by setting $r^* = 0$,

$$\bar{\bar{f}} = \frac{\alpha\theta^y}{\left(\frac{1}{2}\bar{K}\right)^{1-\alpha}}$$

B Proofs of Lemmas

B.1 *Proof of Lemma 5.1:*

Consider the equilibrium interest rate in period t. We show that it must be decreasing in the capital market friction. First, for $f_t < \bar{f}_t$, we have $\hat{K}_t^o \geq \overline{K}^o$ so that old firms scale down and the interest rate is given by

$$\left(\frac{\alpha\theta^y}{r_t + f_t}\right)^{\frac{1}{1-\alpha}} + \left(\frac{\alpha\theta^o}{r_t}\right)^{\frac{1}{1-\alpha}} = \bar{K}$$

Total differentation yields

$$-\frac{1}{1-\alpha}\frac{K_t^y}{r_t + f_t}\left(\frac{dr_t}{df_t} + 1\right) - \frac{1}{1-\alpha}\frac{K_t^o}{r_t}\frac{dr_t}{df_t} = 0 \tag{B.1}$$

which rewrites as

$$\frac{dr_t}{df_t} = -\frac{K_t^y}{K_t^y + K_t^o\left(\frac{r_t+f_t}{r_t}\right)}$$

so that $\frac{dr_t}{df_t}\epsilon(-1,0)$. Next, for $f_t \geq \bar{f}_t$, we have $K_t^o = \hat{K}_t^o$ and the interest rate is given by

$$\left(\frac{\alpha\theta^y}{r_t + f_t}\right)^{\frac{1}{1-\alpha}} = \bar{K} - \hat{K}_t^o$$

$$r_t = \frac{\alpha\theta^y}{\left(\bar{K} - \hat{K}_t^o\right)^{1-\alpha}} - f_t$$

and we see that $\frac{dr_t}{df_t} = -1$.

Coming to part (i) of the lemma, capital in the y-sector is given by

$$K_t^y = \left(\frac{\alpha\theta^y}{r_t + f_t}\right)^{\frac{1}{1-\alpha}}$$

Taking the derivative with respect to f_t gives

$$\frac{dK_t^y}{df_t} = \frac{-1}{1-\alpha}\frac{K_t^y}{r_t + f_t}\left(\frac{dr_t}{df_t} + 1\right)$$

so that $\frac{dK_t^y}{df_t} < 0$ for $f_t \in [0, \bar{f}_t)$ and $\frac{dK_t^y}{df_t} = 0$ for $f \geq \bar{f}_t$. Next, wages in the y-sector are

$$w_t^y = \theta^y \beta \left(K_t^y\right)^\alpha$$

Taking the derivative with respect to f_t yields

$$\frac{dw_t^y}{df_t} = -\frac{\beta}{1-\alpha}K_t^y\left(\frac{dr_t}{df_t} + 1\right) \tag{B.2}$$

so that $\frac{dw_t^y}{df_t} < 0$ for $f_t \in [0, \bar{f}_t]$; $\frac{dw_t^y}{df_t} = 0$ for $f_t > \bar{f}_t$.

As for part (ii) of the lemma, capital in the o-sector is given by

$$K_t^o = \left(\frac{\alpha\theta^o}{r_t}\right)^{\frac{1}{1-\alpha}}$$

for $f_t < \bar{f}_t$; and

$$K_t^o = \hat{K}^o$$

for $f_t \geq \bar{f}_t$. Hence $\frac{dK_t^o}{df_t} > 0$ for $f_t \in [0, \bar{f}_t)$ and $\frac{dK_t^o}{df_t} = 0$ for $f_t \geq \bar{f}_t$. Wages in the o-sector are

$$w_t^o = \theta^o \beta \left(K_t^o\right)^\alpha$$

Taking the derivative with respect to f_t gives

$$\frac{dw_t^o}{df_t} = -\frac{\beta}{1-\alpha} K_t^o \frac{dr_t}{df_t} \tag{B.3}$$

so that $\frac{dw^o}{df_t} > 0$ for $f_t \epsilon [0, \bar{f}_t)$ and $\frac{dw_t^o}{df_t} = 0$ for $f_t \geq \bar{f}_t$.

Finally, for part (iii) of the lemma, first consider profits: let π_t^y and π_t^o denote profits in the young and old sector respectively. Then

$$\pi_t^y = \theta^y \left(K_t^y\right)^\alpha - (r_t + f_t) K_t^y - w_t^y$$

and

$$\pi_t^o = (\theta^o) \left(K_t^o\right)^\alpha - r_t K_t^o - w_t^o$$

We take the derivative of π_t^y with respect to f_t and obtain

$$\frac{d\pi_t^y}{df_t} = \frac{\alpha+\beta-1}{1-\alpha}\left(\frac{dr_t}{df_t} + 1\right) K_t^y$$

so that $\frac{d\pi_t^y}{df_t} < 0$ for $f_t \in [0, \bar{f}_t)$ and $\frac{d\pi_t^y}{df_t} = 0$ for $f \geq \bar{f}_t$. Similarly, for old firms' profits we get

$$\frac{d\pi_t^o}{df_t} = \frac{\alpha+\beta-1}{1-\alpha}\frac{dr_t}{df_t} K_t^o$$

so that $\frac{d\pi_t^o}{df_t} > 0$ for $f_t \in [0, \bar{f}_t)$ and $\frac{d\pi_t^o}{df_t} > 0$ for $f_t \geq \bar{f}_t$. Turning to total profits, $\Pi_t = \pi_t^y + \pi_t^o$, we have

$$\frac{d\Pi_t}{df_t} = \frac{\alpha + \beta - 1}{1 - \alpha} \left[\left(\frac{dr_t}{df_t} + 1 \right) K_t^y + \frac{dr_t}{df_t} K_t^o \right] \tag{B.4}$$

and we see that Π_t is decreasing in f_t iff

$$\frac{dr_t}{df_t} \bar{K} + K_t^y \geq 0$$

Recall that for $f_t \in [0, \bar{f}_t)$ we have

$$\frac{dr_t}{df_t} = -\frac{K_t^y}{K_t^y + K_t^o \left(\frac{r_t + f_t}{r_t} \right)}$$

so that $-\frac{dr_t}{df_t} \bar{K} \leq K_t^y$ and total profits are nonincreasing in f_t. For $f_t \geq \bar{f}_t$, we have $\frac{dr_t}{df_t} = -1$ so that total profits are increasing in f_t. This result is due to the fact that r_t declines in f_t while the allocation of capital does not change; hence, the cost of capital goes down for old firms. While profits can increase in f_t, capital income cannot. Recall that capital income is given by

$$s_t = \frac{r_t \bar{K} + \Pi_t}{\eta}$$

For $f_t \in [0, \bar{f}_t)$, we have $\frac{d\Pi_t}{df_t} \leq 0$ so that $\frac{ds_t}{df_t} \leq 0$. For $f_t \geq \bar{f}_t$ we have

$$\frac{ds_t}{df_t} = \frac{1}{\eta} \left(-\bar{K} + \frac{1 - \alpha - \beta}{1 - \alpha} K_t^o \right)$$

so that $\frac{ds_t}{df_t} < 0$. Now, old capitalists income is given by

$$w_t^o + s_t$$

Let $f_t \in [0, \bar{f}_t)$, then we have

$$\begin{aligned}
\frac{d(w_t^o + s_t)}{df_t} &= -\frac{\beta}{1-\alpha} K_t^o \frac{dr_t}{df_t} + \frac{1}{\eta}\left(\frac{\beta}{1-\alpha}\frac{dr_t}{df_t}\bar{K} + \frac{\alpha+\beta-1}{1-\alpha}K_t^y\right) \\
&< -\frac{\beta}{1-\alpha} K_t^o \frac{dr_t}{df_t} + \frac{\beta}{1-\alpha}\frac{dr_t}{df_t}\bar{K} \\
&< 0
\end{aligned}$$

Next, for $f_t \geq \bar{f}_t$, $\frac{dw_t^o}{df_t} = 0$ and $\frac{ds_t}{df_t} < 0$ which shows part (iii). $\square$

B.2 *Proof of Lemma 5.2:*

Consider $U^{yw}(f)$ on $(0, \bar{f})$. Taking the derivative of U^{yw} with respect to f, we get

$$\frac{dU^{yw}}{df} = u'(w^y)\frac{dw^y}{df} + \delta u'\left(w^o\right)\frac{dw^o}{df} \tag{B.5}$$

Substituting $\frac{dw^y}{df}$ and $\frac{dw^o}{df}$ in (B.5) yields

$$\frac{dU^{yw}}{df} = \frac{-\beta\alpha}{1-\alpha}\left[u'(w^y)\theta^y\frac{(K^y)^\alpha}{r+f}\left(\frac{dr}{df}+1\right) + \delta u'\left(w^o\right)\theta^o\frac{(K^o)^\alpha}{r}\frac{dr}{df}\right]$$

where K^y and K^o are the steady state capital allocations given by (A.2) and r follows from $K^y + K^o = \bar{K}$. Implicit differentiation of the capital market clearing condition with respect to f shows that

$$\frac{dr}{df} + 1 = -\left(\frac{\theta^o}{\theta^y}\right)^{\frac{1}{1-\alpha}}\left(\frac{r+f}{r}\right)^{\frac{1}{1-\alpha}}\frac{dr}{df}$$

which we use to obtain

$$\frac{dU^{yw}}{df} = \frac{-\beta}{1-\alpha}\frac{dr}{df}\frac{1}{r}K^o[\delta u'\left(w^o\right)r - u'\left(w^y\right)(r+f)]$$

Since r and K^o are nonnegative and $\frac{dr}{df} < 0$, the first part of this expression is positive for all f. Hence $\frac{dU^{yw}}{df} \geq 0$ if and only if

$$\delta u'(w^o)\, r - u'(w^y)(r+f) \geq 0 \tag{B.6}$$

Note that $u'(w^y)(r+f)$ strictly increases in f while $\delta u'(w^o)\, r$ strictly decreases in f. It follows that if for some f condition (B.6) is satisfied with equality then it is the unique utility maximizing steady state policy. Otherwise, either $\delta u'(w^o)\, r - u'(w^y)(r+f) > 0$ for all $f \epsilon (0, \bar{f})$ so that

$$Arg \max_f U^{yw}(f) = [\bar{f}, \infty)$$

or $\delta u'(w^o)\, r - u'(w^y)(r+f) < 0$ for all $f \epsilon (0, \bar{f})$ so that

$$Arg \max_f U^{yw}(f) = \{0\}$$

Finally we note that young worker preferences are single peaked, as young worker's utility is decreased by moving the policy further away from their preferred policy. □

B.3 ***Proof of Lemma 5.3:***

To prove lemma 5.3,we first prove two auxiliary lemmas that give the economic equilibrium in periods t and $t+1$. Since we consider out-of steady-state dynamics we assume that the equilibrium at time $t-1$ is given by steady state allocations for some arbitrary $f \in [0, \bar{f}]$. The first lemma describes the economic equilibrium after the friction is adjusted downwards:

Lemma. *Let the economic equilibrium at time $t-1$ be given by steady state values for some $f \leq \bar{f}$. Consider a downward change in policy $f_t = f_{t+1} \leq f$, then*

$$K_t^y = \left(\frac{\alpha \theta^y}{r_t^* + f_t} \right)^{\frac{1}{1-\alpha}}$$

and

$$K_t^o = \left(\frac{\alpha\theta^o}{r_t^*}\right)^{\frac{1}{1-\alpha}}$$

with r_t^* given by $K_t^y + K_t^o = \bar{K}$, furthermore $K_{t+1}^y = K_t^y$ and $K_{t+1}^o = K_t^o$.

Proof. Since $f_t \leq f$, we also have $\overline{K}^o(f_t) \leq \overline{K}^o(f)$, where $\bar{K}^o$ is the equilibrium cut-off value function defined by (4.12). It follows that

$$\hat{K}_t^o \geq \overline{K}^o(f_t)$$

so that old firms shed capital and K_t^y and K_t^o are as posed. Note that because $f_t \leq f$, we have $K_t^o < K_{t-1}^o$ and, hence, also $K_t^y > K_{t-1}^y$ by market clearing . We must verify that the same allocation obtains in period $t+1$. Moving forward one period we have

$$\hat{K}_{t+1}^o = K_t^y > \bar{K}^o(f_{t+1})$$

so that again old firms shed capital. It follows that the same interest rate obtains in both periods, and allocations are as posed. □

Lemma B.3 shows that a downward change in policy results in steady state values that correspond to a lower capital market friction. Intuitively, we can say that changing the policy to a lower capital market friction moves the economy to a new steady state corresponding to the lower value of the friction. The same need not be true for a change in policy to a higher capital market friction as the next lemma shows.

Lemma. *Let the economic equilibrium at time $t-1$ be given by steady state values for some $f \leq \bar{f}$. Consider an upward change in policy $f_t = f_{t+1} > f$, then*

(i) if $f_t \leq \bar{f}$ we have

$$K_t^y = \left(\frac{\alpha\theta^y}{r_t^* + f_t}\right)^{\frac{1}{1-\alpha}} \quad ; \quad K_t^o = \left(\frac{\alpha\theta^o}{r_t^*}\right)^{\frac{1}{1-\alpha}}$$

with r_t^* given by $K_t^y + K_t^o = \bar{K}$, furthermore $K_{t+1}^y = K_t^y$ and $K_{t+1}^o = K_t^o$;

(ii) if $\bar{f} < f_t \leq \bar{f}_t$ we have

$$K_t^y = \left(\frac{\alpha\theta^y}{r_t^* + f_t}\right)^{\frac{1}{1-\alpha}} \quad ; \quad K_t^o = \left(\frac{\alpha\theta^o}{r_t^*}\right)^{\frac{1}{1-\alpha}}$$

with r_t^* given by $K_t^y + K_t^o = \bar{K}$, furthermore $K_{t+1}^y = K_t^o$ and $K_{t+1}^o = K_t^y$; and

(iii) if $f_t > \bar{f}_t$ we have

$$K_t^y = \bar{K} - \hat{K}_t^o \quad ; \quad K_t^o = \hat{K}_t^o$$

furthermore $K_{t+1}^o = K_t^y$ and $K_{t+1}^y = K_t^o$.

Proof. For (i): suppose $f < f_t \leq \bar{f}$. Then also $f_t \leq \bar{f}_t$, where $\bar{f}_t$ is given by (5.2). Hence we have

$$K_{t-1}^y = \hat{K}_t^o \geq \bar{K}^o(f_t)$$

by the monotonicity of $\bar{K}^o$. It follows that old firms scale down and K_t^y and K_t^o are as posed. Consider period $t+1$. Since $f_t = f_{t+1} \leq \bar{f}$, we have

$$K_t^y(f_t) = \hat{K}_{t+1}^o \geq \bar{K}^o(f_{t+1})$$

by the definition of $\bar{f}$, given by (4.18), and the monotonicity in of K^y and $\bar{K}^o$ in f_t. Hence K_{t+1}^y and K_{t+1}^o are as posed.

For (ii): suppose $\bar{f} < f_t \leq \bar{f}_t$. Then we have also have

$$\hat{K}_t^o \geq \bar{K}^o(f_t)$$

so that old firms scale down and allocations are as posed. Moving forward one period it follows from $\bar{f} < f_t = f_{t+1}$ that

$$K_t^y(f_t) < \bar{K}^o(f_{t+1})$$

Hence old firms do not adjust capital and $K^o_{t+1} = K^y_t$. By market clearing then $K^y_{t+1} = K^o_t$.

For (iii): suppose $\bar{f}_t < f_t$, then $\hat{K}^o_t < \bar{K}^o(f_t)$ so that old firms do not adjsut capital. It follows that $K^o_t = \hat{K}^o_t$ and, by market clearing, $K^y_t = \bar{K} - \hat{K}^o$. In period $t+1$, since $\bar{f} < f_t = f_{t+1}$ we have $\hat{K}^o_{t+1} < \bar{K}^o(f_{t+1})$ and so $K^o_{t+1} = K^y_t$ and $K^y_{t+1} = K^o_t$. □

Lemma B.3 shows that a small upward change in policy $(f_t \leq \bar{f})$ results in steady state allocations that correspond to a higher friction. Now, consider lifetime utility U^{yw}_t of the young worker at time t. Lemma B.3 also implies that young workers will not vote for a higher friction than $\bar{f}$. To see this note that K^y_t and K^o_{t+1} are strictly decreasing in f_t for $\bar{f} < f_t \leq \bar{f}_t$; they are constant in f_t for $f_t > \bar{f}_t$. Hence young workers strictly prefer $f_t = \bar{f}$ over any $f_t > \bar{f}$. With the auxiliary lemmas, we can now proof lemma 5.3.

Let f^{yw}_t denote the preferred policy of the young workers. We have argued that $f^{yw}_t \epsilon [0, f]$ and we have shown that, if this policy is set, the economy attains steady state values corresponding to the friction f^{yw}_t. It follows that $f^{yw}_t = f^{yw}$, where f^{yw} is given by lemma 5.2. By sincere voting we have $v^{yw}_t = f^{yw}$, which concludes the proof. □

B.4 *Proof of Lemma 5.4:*

Proof. Taking the derivative of U^{yw} with respect to f we have shown (cf. lemma 5.2) that $\frac{dU^{yw}}{df} \geq 0$ if and only if

$$u'(w^y)(r+f) - \delta u'(w^o) r \leq 0 \tag{B.7}$$

With CRRA utility this rewrites as

$$\beta^{-g} \alpha^{-g(\gamma-1)} (\theta^y)^{-g\gamma} \left[(r+f)^{1+g(\gamma-1)} - \delta A^{-g} r^{1+g(\gamma-1)} \right] \leq 0$$

so that lifetime utility of the young is increasing in the reallocation cost iff

$$\left(\frac{r+f}{r}\right) \leq \delta A^{\frac{-g}{1+g(\gamma-1)}} \tag{B.8}$$

The utility maximizing policy now follows from this condition. First note that $\left(\frac{r+f}{r}\right)$ is increasing in f; and that we have $\left(\frac{r+\bar{f}}{r}\right) = \frac{\theta^y}{\theta^o}$, so that

$$1 \leq \frac{r+f}{r} \leq \frac{\theta^y}{\theta^o}$$

for $f\epsilon[0,\bar{f}]$. We see that if

$$\delta A^{\frac{-g}{1+g(\gamma-1)}} \leq 1$$

then U^{yw} is decreasing in f so that $f^{yw} = 0$. Likewise if

$$\delta A^{\frac{-g}{1+g(\gamma-1)}} \geq \frac{\theta^y}{\theta^o}$$

then U^{yw} is increasing in f so that $f^{yw} = \bar{f}$ Finally if

$$1 < \delta A^{\frac{-g}{1+g(\gamma-1)}} < \frac{\theta^y}{\theta^o}$$

then $\frac{dU^{yw}}{df}$ switches sign on $[0,\bar{f}]$ and f^{yw} is given by

$$f^{yw} = \left(\delta A^{\frac{-g}{1+g(\gamma-1)}} - 1\right) r\left(f^{yw}\right)$$

$\square$