

Dealers' Insurance, Market Structure And Liquidity

Francesca Carapella^{*1} and Cyril Monnet^{†2}

¹Federal Reserve Board of Governors

²University of Bern and Study Center Gerzensee

February 15, 2013

Abstract

Dealers intermediate transactions between buyers and sellers in many markets. In particular this is true in financial markets where many contracts are over-the-counter (OTC), especially derivatives: they are negotiated bilaterally and subject their users to the risk of counterparty default. Nearly all OTC derivatives today are negotiated between a dealer and end users¹. Beginning in the late 1990s, some derivatives negotiated OTC, began to be cleared and settled through central counterparties (CCPs): by interposing itself as the counterparty of record for all transactions, the CCP protects trading participants from both settlement risk and replacement cost losses arising from a counterparty default. Since then, OTC-cleared derivatives volume has grown steadily in those and several other clearinghouses; however, many market participants still prefer traditional OTC derivatives (with bilateral credit risk management).

Then a question naturally arises: why in certain markets and for certain transactions CCPs are not used? This is the first question that this paper answers.

^{*}Francesca.Carapella@frb.gov

[†]cyril.monnet@gmail.com

¹Or between two dealers, but for the purpose of this paper this does not change much, since dealers are the principals in the transactions they arrange.

In the wake of the credit crisis of 2007 and 2008, policy makers developed an elevated interest in CCP clearing and settlement solutions for OTC derivatives, culminated with the Dodd-Frank Act passed on July 21, 2010. As part of the new regulatory framework for OTC derivatives, the Dodd-Frank Act mandates that many OTC derivatives transactions be cleared and settled through regulated CCPs. In essence, clearing via CCPs reallocates the risk of loss arising from non-performance in derivatives transactions. This reallocation, however, may affect the behavior of market participants and the structure of the market.

This is the second question that this paper answers: it studies the effect that the provision of counterparty risk insurance to dealers has on market liquidity and market structure. We interpret our insurance mechanism as the introduction of a central counterparty². Contrary to most papers in the literature, we analyze how insurance will impact the trading behavior of incumbents dealers as well as the effect on dealers' entry. We find that more counterparty risk shifts trading toward more efficient dealers, who are the incumbents: so these dealers would reject the introduction of an insurance as it allows more (and relatively inefficient) dealers to enter the market, thus lowering their market power. As a result, insurance implies a decrease in the bid-ask spread. This feature of the equilibrium shows a trade off for end users: they benefit from the reduction of the bid-ask spread, but they lose from facing on average less efficient dealers. Therefore, whether the overall welfare generated by the industry increases with the insurance or not, depends on which of the two effects dominates.

1 Introduction

In many markets transactions take place through intermediaries (dealers) that are in charge of channeling the supply to the demand. There are several ways they can do so. Dealers hold a stock of the object that they then try to sell as soon as possible. This works fine when the object is rather standardized and the demand is frequent. However, this does not work very well when the object is more tailored to the requirements of the buyer. In this case, dealers take orders and try to fulfill these orders from sellers they know. However, this exposes them to counterparty risk: once an order has

²Or of an efficient interdealer market.

been placed, and the dealer purchased the object from a seller, the dealer is exposed to the risk that the buyer bails out from the trade. If the buyer fails to honor his obligation, the dealer would end-up with an object that he has to sell and few people want. The dealer can contact other dealers to sell this object, but, if the object is highly customized, demand is likely to be low. Hence, even in the presence of an inter-dealer market, a dealer can end-up in the costly situation of having to pay the seller while keeping the object idle. What is the value of insuring dealers against such counterparty risk then? Intuitively, dealers would seem to benefit from such an insurance, but it does not seem to exist. If anything, dealers try to insure themselves by requiring a downpayment of some sorts to the buyer, in the hope of somehow controlling the default probability of the buyer. In this paper we argue that the *absence* of a dealer's insurance protecting against counterparty risk *can be optimal* for some agents. The reason is simple: providing insurance makes the business less risky and more profitable to dealers. This implies that more dealers can enter the market, even those who are not very efficient dealers. This indirectly hurts incumbent dealers and, somewhat surprisingly, sellers. Under some conditions and for certain distributions of dealers across their efficiency levels, this hurts even buyers.

This paper analyzes how providing insurance against counterparty risk impacts liquidity and the structure of the market: the measure of active dealers, buyers and sellers, the share of the market that each dealer services, and finally the equilibrium bid-ask spread. Such a comprehensive characterization of the equilibrium allows an analysis of who gains and who loses from the introduction of an insurance arrangement, and why.

We use a model first proposed by Spulber ? and further developed by Rust and Hall ?. There, buyers and sellers of a real asset have to trade through dealers. Dealers vary in their efficiency to intermediate trades. However search frictions prevent buyers and sellers to always trade with the most efficient dealers. Instead, dealers post (and commit to) bid and ask prices to attract buyers and sellers. Buyers and sellers sample dealers randomly and decide whether to trade at the posted price with the dealers they have sampled, or whether they should carry on searching. The search friction implies that the equilibrium bid-ask spread will be positive, and even less efficient dealers will be active.

Within this framework, dealers face an inventory problem: if they post low bid and ask prices, they will end up with too much demand for the asset relative to what they can cover. Alternatively, if they post high bid and ask

prices, they will end up with too many assets in inventory. So in the model dealers face a resource constraint period by period that requires they always have sufficient assets to face buyers' demand. Our innovation to Spulber ? is to introduce uncertainty in a simplified version of his model.³ We assume that dealers face uncertain demand by buyers. Buyers place orders at the beginning of every period but are subject to exogenous default probability before they have the chance to settle their orders. When dealers can predict how many buyers will default, they will just acquire less assets, a strategy that is similar to airline's overbooking policy. However, when dealers cannot predict the severity of defaults, they may find themselves with too many assets in inventory, which is costly.

We show that a small amount of default risk has the following impacts on the markets: each dealer will increase their bid-ask spread. Interestingly, for relatively efficient dealers, both the bid and the ask prices increase. Finally, less dealers will find it profitable to operate when there is uninsurable default risk.

We then introduce insurance. Providing dealers with insurance against buyers' default, results in a reduction of the bid-ask spread (under some conditions): with a smaller measure of buyers who may default at settlement dealers prefer to charge a lower *mark up* per transaction on a larger volume of transactions. More buyers and sellers are served, and the measure of dealers⁴ active on the market thus increases with the provision of insurance. This, however, introduces a trade off for the most efficient dealers: the most efficient dealers are active and make positive profits even without insurance against buyers' default, but the least efficient dealers are not. They are able to enter the market only when some degree of insurance is feasible and provided, thus *stealing* some market share from the most efficient dealers. The effect of such increased competition in the dealers' market, on the profits of the most efficient dealer is negative. Despite the provision of insurance increases the volume of transactions they execute by shrinking their bid-ask spread, the most efficient dealers are harmed by the presence of more active dealers who serve part of the demand and supply that they were serving in the absence of insurance. In fact, if the most efficient dealers could choose the degree of insurance provided, they would not choose full insurance.

³We also worked on a version of his model but we found the results did not qualitatively change. This simplified version allows us to characterize results more sharply.

⁴Indexed by their efficiency level, that is the cost of executing a transaction.

Buyers and sellers face a trade off: the introduction of insurance causes the bid-ask spread to shrink, which benefits them as they have to pay less (are paid more⁵) to buy (sell) the object. The introduction of insurance against buyers' default has also an indirect effect on buyers' and sellers' welfare by affecting the probability that they will be matched with any given dealer: more dealers who are relatively inefficient become active with insurance. Because of the search friction, this implies that the probability of meeting any given active dealer decreases: so buyers and sellers will meet efficient dealers less often. Because efficient dealers charge lower ask prices this hurts them.

This paper thus makes two contributions: first it offers a perspective that can explain the opposition of some dealers to the recent political pressure⁶ for clearing all derivatives traded OTC on central counterparties. Second, it offers an explanation for the coexistence of bilaterally cleared OTC derivatives with derivatives that are traded OTC but are centrally cleared.

2 A Model of Dealers and Risk

We base our analysis on a modified version of Spulber ? and Rust and Hall ? equilibrium search model. The presentation of the model follows closely the one in Rust and Hall ?. There are three types of agents, buyers, sellers and dealers. Buyers and sellers cannot trade directly an asset and all trades must be intermediated by dealers.

There is a continuum $[0, 1]$ of heterogeneous and infinitely-lived buyers, sellers, and dealers. A seller of type v can sell at most one unit of the asset at an opportunity cost v . A buyer of type v can hold at most one unit of the asset and is willing to pay at most v to hold it.

Dealers face no counterparty risk in Rust and Hall ?, as dealers' clients exit the market after they settle their claim. In this paper, we introduce counterparty risk for dealers by assuming that buyers first place *orders* with dealers, but then exit the market with probability λ , *before* they have the chance to settle their orders. A buyer who exits the market is replaced

⁵For sellers the effect of insurance on welfare is less straightforward: as it will become clear from the model, very efficient dealers actually decrease their bid price as insurance is introduced, and if risk is small. This has a direct negative effect on sellers' welfare because sellers who are matched with very efficient dealers get paid less for the object they sell.

⁶Dodd-Frank Act for example, ?. European financial markets legislation has also been moving in the same direction.

with a new buyer which v is drawn from the uniform $[0, 1]$. We do not consider strategic default and λ is exogenous. This is akin to the risk that a counterparty goes bust for reasons that are independent of his trading activities. Contrary to buyers, sellers always settle their orders.⁷

In and by itself, this type of counterparty risk is not interesting: There is nothing a dealer can do to insure against it. So we also assume that dealers face idiosyncratic risk: Nature does not allocate buyers perfectly across dealers who can be in two states, 1 or -1 . In state 1, a dealer has a measure $\lambda - \varepsilon$ of his buyers exiting the market, while in state -1 a measure $\lambda + \varepsilon$ of his buyers exit. This default shock is independent of whether the buyers placed an order at the bid-ask spread posted by the dealer. Dealers cannot observe state s before it actually occurs: They only observe the actual measure of buyers exiting the market once that is realized.

This shock is i.i.d. and each state occurs with probability $1/2$, so that there is no aggregate uncertainty. Notice also that on average buyers exit the market before settlement with probability λ .

At time $t = 0$, the initial distribution of types of buyers and sellers is $v \sim U[0, 1]$. Since new agent's type is drawn randomly over the same distribution, in all subsequent periods $t = 1, 2, 3, \dots$ the distribution of types will also be $U[0, 1]$. Therefore $U[0, 1]$ is the unique invariant distribution of types in each subsequent period $t = 1, 2, 3, \dots$.

There is a continuum of dealers indexed by their trading cost k , the marginal cost of taking a seller's order before the seller actually pays for the good.⁸ Trading costs are uniformly distributed over the interval $[\underline{k}, 1]$, where $\underline{k}$ is the marginal cost of the most efficient dealer.

In equilibrium, only dealers who can make a profit will operate a trading post and there will be a cost $\bar{k} \leq 1$ such that no dealer with a cost greater than $\bar{k}$ operate a post. A dealer of type $k \in [\underline{k}, \bar{k}]$ chooses a pair of bid-ask prices $(b(k), a(k))$ that maximizes his expected discounted profits. A dealer is willing to buy the asset at price $b(k)$ from a seller and he is willing to sell the asset at the ask price $a(k)$. We consider a stationary equilibrium so that

⁷This asymmetry between buyers and seller is not substantial. Analogous results would arise if sellers exited the market before settlement.

⁸This introduces an asymmetry regarding the cost of dealing with a buyer or a seller. However, this can be justified in real contracts as the cost of dealing with the handling of the underlying good. The result would not be substantially modified if we introduced a handling cost of the buyer as well, k^b as long as $k^b < k$. Here we set $k^b = 0$. For financial contracts, this is the cost of designing the contracts.

$b(k)$ and $a(k)$ will be constant through time.

Buyers and sellers engage in search. Each period, if he decides to search, a trader gets a price quote from a random dealer. Since dealers post stationary bid and ask prices depending on their types, traders face a distribution $F(a)$ of ask prices and $G(b)$ of bid prices. These distributions are equilibrium objects. Traders discount the future at rate β .

Timing, also shown in the Figure below, is as follows: At time 0, dealers $k \in [\underline{k}, \bar{k}]$ choose a bid and ask quote. $\forall t \geq 0$, buyers and sellers decide whether they want to search or not. If so, they contact a dealer at random, and they either accept the quoted price or keep searching. If they agree, they place an order to buy/sell a unit of the asset. Then each buyer exits with probability λ . If dealers are in state $s \in \{-1, 1\}$ a measure $\lambda - s\varepsilon$ of their buyers exits. Finally, settlement occurs: Each operating dealer receives assets from sellers who placed an order and delivers one asset to each of the $(1 - \lambda + s\varepsilon)$ buyers. Dealers dispose of the surplus of assets⁹.

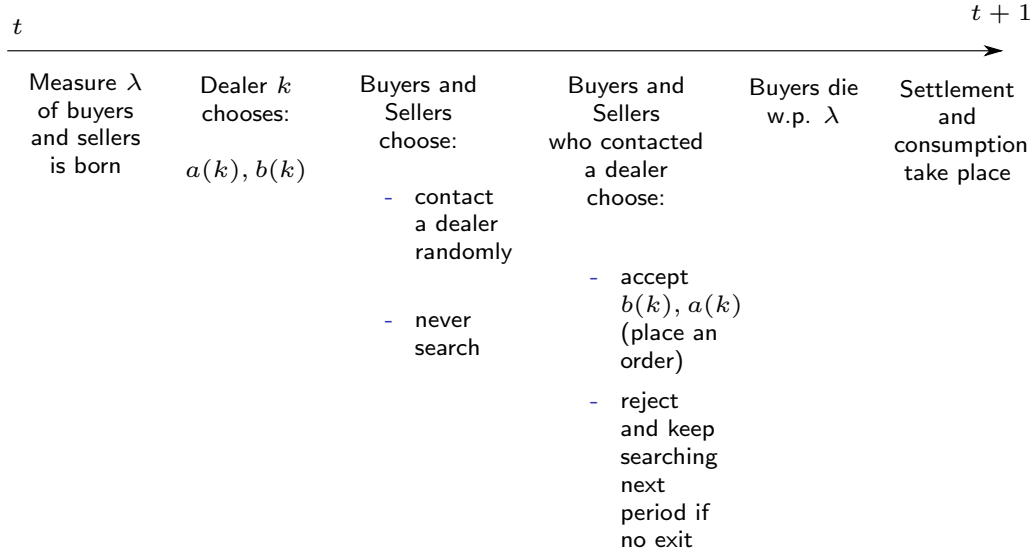


Figure 1: Timing

The main difference from Spulber ? is that buyers do not give up on future options by trading in a given period. In ?, buyers exit the market after

⁹The asset fully depreciates in the hand of the dealers.

they trade. Here, trading today does not exclude traders from future trades. Hence, their trading decision is simpler. However, dealers will not “compete” and so they will behave as monopolists. A common feature between ? and our set-up is that each active dealer has a higher probability of intermediating funds whenever they are fewer dealers. This is key to our result.

3 No settlement fails

In this section we assume that there is no settlement failure so that $\lambda = 0$. The decision of consumers is simply to accept the selected ask price a whenever $v \geq a$ and reject otherwise. Their payoff is

$$V_c(v) = \int_{\underline{a}}^v (v - a) dF(a) + \beta V_c(v)$$

where $\underline{a}$ is the lowest ask price.

The decision of producers is to accept the selected bid price b whenever $v \leq b$ and reject otherwise. Their payoff is

$$V_p(v) = \int_v^{\bar{b}} (b - v) dG(b) + \beta V_p(v)$$

Dealers that post an ask-price a face the following demand

$$D(a) = \frac{1}{N} \int_a^1 dv = \frac{1}{N}(1 - a)$$

where N is the measure of active dealers. Only those consumers with a value greater than the posted price will accept the offer. Similarly, dealers that post a bid-price b face the following demand

$$S(b) = \frac{1}{N} \int_0^b dv = \frac{1}{N}b$$

A dealer of type k maximizes his profit by choosing a and b , subject to the resource constraint, or

$$\Pi(k) = \max_{a,b} \frac{1}{N} \{aD(a) - (b + k)S(b)\}$$

subject to $D(a) \leq S(b)$. The resource constraint will bind, so that $b = 1 - a$ and a dealer chooses a to maximize

$$\Pi(k) = \frac{1}{N}(1 - a)(2a - 1 - k)$$

with solution

$$a(k) = \frac{3 + k}{4}$$

so that

$$b(k) = \frac{1 - k}{4}$$

Notice that, because of the linearity of the bid and ask prices and because the distribution of dealer cost is uniform, then the distribution of bid and ask prices are also uniform on $[a(0), a(\bar{k})]$ and $[b(\bar{k}), b(0)]$.

In equilibrium, all dealers with intermediation cost k such that $\Pi(k) \geq 0$ will be active. Therefore, all dealers with $k \leq \bar{k}$ where $\Pi(\bar{k}) = 0$ will be active so that the measure of active dealers N is $N = \bar{k}$. It is easy to see that $\bar{k} = 1$ such that $a(\bar{k}) = 1$ and $b(\bar{k}) = 0$. Therefore the least efficient dealer is indifferent between operating and not as he will anyway not face any demand at the price he would set. Any dealer $k < \bar{k} = 1$ makes a strictly positive profit equal to

$$\Pi(k) = \frac{(1 - k)^2}{8N} = \frac{(1 - k)^2}{8\bar{k}}.$$

And therefore:

$$\begin{aligned} \bar{a} &= a(\bar{k}) = \frac{3 + \bar{k}}{4} = 1 \\ \underline{a} &= a(0) = \frac{3}{4} \\ \bar{b} &= b(0) = \frac{1}{4} \\ \underline{b} &= b(\bar{k}) = \frac{1 - \bar{k}}{4} = 0 \end{aligned}$$

Clearly, each dealer charges its monopoly price, as there is no competition (contrary to Spulber's ? model): The bid/ask prices posted by *other* dealers do not influence the decision of agents to accept or reject the price they obtain as they can anyway search again next period, independently of their

decision today. So contrary to Spulber's ? model, agents do not forfeit the option of getting a better deal tomorrow if they accept the proposed deal today. Since dealers charge the monopoly price, even inefficient dealers can make a profit, which implies that they have the incentive to enter the market: Hence we should expect that the equilibrium number of active dealers is too high relative to what a planner would choose. This is what we analyze next.

To define the optimal number of dealers, we now define the surplus of dealers, buyers and producers as a function of $\bar{k}$. Total economy-wide profits, or surplus of dealers, are:

$$\begin{aligned} S_d(\bar{k}) &= \int_0^{\bar{k}} \Pi(k) dk = \int_0^{\bar{k}} \frac{(1-k)^2}{8\bar{k}} dk \\ &= \frac{3 - (3 - \bar{k})\bar{k}}{24} \end{aligned}$$

which is always decreasing in $\bar{k} \leq 1$.

The surplus of consumers is:

$$\begin{aligned} S_c(\bar{k}) &= \int_{a(0)}^1 \left[\int_{a(0)}^{a(\bar{k}) \vee v} \frac{(v-a)}{a(\bar{k}) - a(0)} da \right] dv \\ &= \frac{1}{a(\bar{k}) - a(0)} \left[\int_{a(0)}^{a(\bar{k})} \int_{a(0)}^v (v-a) dadv + \int_{a(\bar{k})}^1 \int_{a(0)}^{a(\bar{k})} (v-a) dadv \right] \\ &= \frac{1}{6} (3 + a(0)^2 + a(0)(-3 + a(\bar{k})) + (-3 + a(\bar{k}))a(\bar{k})) \\ &= \frac{1}{6} [3 + a(0)^2 - (3 - a(\bar{k})) (a(0) + a(\bar{k}))] \\ &= \frac{(3 - (3 - \bar{k})\bar{k})}{96} = \frac{S_d(\bar{k})}{4} \end{aligned}$$

Hence, $S_c(\bar{k})$ is always decreasing in $\bar{k}$.

Finally, the surplus of producers is

$$\begin{aligned}
S_p(\bar{k}) &= \int_0^{b(0)} \left[\int_{b(\bar{k}) \wedge v}^{b(0)} \frac{(b-v)}{b(0) - b(\bar{k})} db \right] dv \\
&= \frac{1}{b(0) - b(\bar{k})} \left[\int_{b(\bar{k})}^{b(0)} \int_v^{b(0)} (b-v) db dv + \int_0^{b(\bar{k})} \int_{b(\bar{k})}^{b(0)} (b-v) db dv \right] \\
&= \frac{1}{6} (b(0)^2 + b(\bar{k})^2 + b(0)b(\bar{k})) \\
&= \frac{(3 - (3 - \bar{k})\bar{k})}{96} = \frac{S_d(\bar{k})}{4}
\end{aligned}$$

Hence $S_p(\bar{k})$ is always decreasing in $\bar{k}$. Therefore, as expected, neither dealers, nor consumers or producers benefit from the free entry of dealers. Given intermediation is needed, the best solution is to have only the most efficient dealers $k = 0$ intermediate all trades. Notice that this is true because the most efficient dealer charges the same price, independent of the presence of other dealers. This is not true in a model like Spulber ? : there, even the most efficient dealers may wish to lower their price when other dealers are operating. In the next section we introduce settlement fails.

4 Version with settlement fails

In this section, we introduce a risk of settlement fails for dealers. Settlement fails when the consumer fails to collect his purchase and pay. We assume that this happens on average with probability λ so that a measure λ of consumers will fail to settle. However, dealers are also subject to an idiosyncratic risk $\varepsilon \in (0, \lambda)$: With probability $1/2$, a dealer experiences a fraction $\lambda + \varepsilon$ of its consumers failing to settle and with probability $1/2$, only a fraction $\lambda - \varepsilon$ will fail to settle.¹⁰ The cost of settlement fail for dealers is that they still have to honour their obligations toward sellers. The cost of settlement fails for buyers is that they cannot consume the good. The shock is i.i.d across dealers and across time.

The consumers' and producers' problems are as above, so that $D(a) =$

¹⁰We can extend this to a symmetrically distributed ε around $[-\bar{\varepsilon}, \bar{\varepsilon}]$, where $\bar{\varepsilon} < \lambda$ and $E(\varepsilon) = 0$. Then everything below holds with $\varepsilon = \bar{\varepsilon}$.

$\frac{1}{N}(1-a)$ and $S(b) = \frac{1}{N}b$. Dealers' profits and maximization problem are:

$$\begin{aligned}\Pi(k; \lambda, \varepsilon) &= \max_{\{a, b\}} E_s \{a(1 - \lambda + s\varepsilon) D(a) - (b + k) S(b)\} \\ &\text{s.t. } (1 - \lambda + s\varepsilon) D(a) \leq S(b) \quad \forall s \in \{-1, 1\}\end{aligned}$$

As in our version of Spulber ?, the constraint binds when $s = 1$. Therefore

$$S(b) = (1 - \lambda + \varepsilon) D(a) \equiv \lambda_\varepsilon D(a).$$

Notice that dealers expect to have to deliver $(1 - \lambda)D(a)$ securities. However, dealers have to purchase more securities than they expect will be necessary, as they have to satisfy their promise to deliver it in all possible states. Hence, the possibility of settlement fails implies that dealers over-buy the security. Together with $D(a) = \frac{1}{N}(1 - a)$ and $S(b) = \frac{1}{N}b$ we have:

$$\Pi(k; \lambda, \varepsilon) = \max_{\{a\}} \{a(1 - \lambda) - [\lambda_\varepsilon(1 - a) + k] \lambda_\varepsilon\} \frac{1}{N} (1 - a)$$

Taking the number of operating dealers as given, Figure 2 shows the profit of a dealer with $\varepsilon = 0$ and as ε is increased to a positive value. As the figure shows, the direct effect of increasing risk is to decrease the dealer's profit. The reason is intuitive: Given an ask-price, the dealer faces a number of order a . The dealer expects only $(1 - \lambda)a$ buyers to collect the asset and pay for it. However, he needs to buy sufficient assets to cover the demand in the high state and this amount increases with ε . This extra purchase yields to a direct decrease in the dealer's profit. To account for this, dealers adjust their ask price upward. In this way they face less orders, which lowers the effective demand in the high state.

The first order condition gives us

$$a(k) = 1 - \frac{1 - \lambda - k\lambda_\varepsilon}{2(1 - \lambda + \lambda_\varepsilon^2)} = \frac{1 - \lambda + 2\lambda_\varepsilon^2 + k\lambda_\varepsilon}{2(1 - \lambda + \lambda_\varepsilon^2)} \quad (1)$$

$$\begin{aligned}b(k) &= \lambda_\varepsilon(1 - a(k)) \\ &= \lambda_\varepsilon \frac{1 - \lambda - k\lambda_\varepsilon}{2(1 - \lambda + \lambda_\varepsilon^2)}\end{aligned} \quad (2)$$

It is worth emphasizing the effect of risk (ε) on the bid-ask spread. Since the ask price is increasing with risk, dealers do not need to serve as many consumers as before, so that they should decrease their bid price to purchase

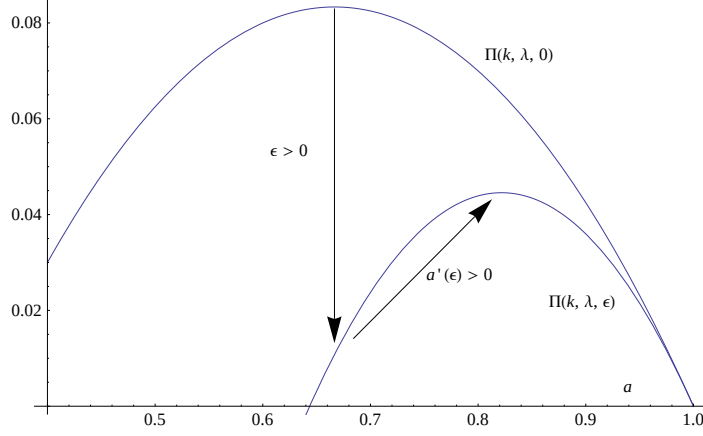


Figure 2: Dealer's profits as a function of ε

a lower quantity of the asset. However, notice the factor λ_ε which multiplies $1 - a(k)$: With a higher risk of settlement fails, dealers have to over-buy the security and this pushes the bid price up. The overall effect on the bid price is therefore uncertain depending on which effects dominates. It turns out that if λ and ε are sufficiently small, then the bid price will increase in settlement failure risk for some k . Indeed, we have

$$\frac{\partial b(k)}{\partial \lambda_\varepsilon} = \frac{(1 - \lambda)}{2(1 - \lambda + \lambda_\varepsilon^2)^2} (1 - \lambda - 2k\lambda_\varepsilon - \lambda_\varepsilon^2)$$

so that the bid price of dealer k increases with settlement risk whenever

$$k < \frac{1 - \lambda - \lambda_\varepsilon^2}{2\lambda_\varepsilon} \equiv \kappa(\varepsilon)$$

as the figures below show.

Notice that $\kappa(\varepsilon) = 0$ whenever $\varepsilon = \sqrt{1 - \lambda}(1 - \sqrt{1 - \lambda})$. In general we have the following result

Lemma 1 *For all $\varepsilon \leq \bar{\varepsilon} \equiv \sqrt{1 - \lambda}(1 - \sqrt{1 - \lambda})$, $b(k)$ is increasing in ε whenever $k < \kappa(\varepsilon)$ and decreasing otherwise. For all $\varepsilon > \bar{\varepsilon}$ the bid price is always decreasing in ε for all $k \leq \bar{k}$.*

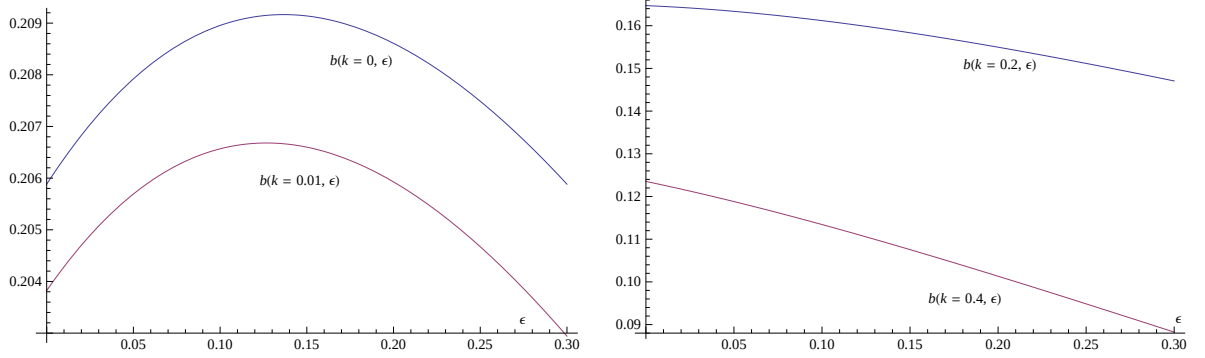


Figure 3: Bid prices as a function of ε for different dealers

We can now characterize the demand and supply for each dealer:

$$D(a) = \frac{1}{N}(1-a) = \frac{1}{2N} \frac{1-\lambda-k\lambda_\varepsilon}{(1-\lambda+\lambda_\varepsilon^2)}$$

$$S(b) = \frac{1}{N}\lambda_\varepsilon(1-a) = \frac{1}{2N}\lambda_\varepsilon \frac{1-\lambda-k\lambda_\varepsilon}{(1-\lambda+\lambda_\varepsilon^2)}$$

Replacing for $a(k)$ and $b(k)$ as well as $N = \bar{k}$ in the profit we obtain the profit for a dealer of type k :

$$\Pi(k; \lambda, \varepsilon) = \frac{\lambda_\varepsilon(1-\lambda-k\lambda_\varepsilon)^2}{4(1-\lambda)(1-\lambda+\lambda_\varepsilon^2)}$$

so that $\Pi(0; \lambda, \varepsilon)$ is increasing in ε whenever ε is small enough.¹¹ The marginal active dealer $\bar{k}$ is such that $\Pi(\bar{k}; \lambda, \varepsilon) = 0$, which gives us

$$\bar{k} = \frac{1-\lambda}{\lambda_\varepsilon} < 1. \quad (3)$$

It is then easy to compute that $a(\bar{k}) = 1$. In the sequel, we show our main result.

Proposition 2 *The consumers' expected surplus is decreasing with the risk of settlement failure as measured by ε . The producers' surplus is increasing in ε if ε is small enough, and it is decreasing otherwise. The total dealers'*

¹¹The sign of $\partial\Pi(0)/\partial\varepsilon$ is the sign of $1-\lambda-\lambda_\varepsilon^2$. Hence, for all ε such that $\varepsilon < \sqrt{1-\lambda}(1-\sqrt{1-\lambda})$ the profit of the most efficient dealer will be increasing.

surplus is decreasing in ε . However, the most efficient dealers always benefit from an increase in settlement failure risk. The overall welfare as measured by the equally weighted sum of all expected surplus is decreasing in ε .

The surplus of dealers is simply

$$\begin{aligned} S_d(\varepsilon) &= \int_0^{\bar{k}} \Pi(k; \lambda, \varepsilon) dk \\ &= \frac{1}{12} \frac{(1 - \lambda)^2}{(1 - \lambda + \lambda_\varepsilon^2)} \end{aligned}$$

which is always decreasing in ε .

Lemma 3 *The dealers' surplus is decreasing in the risk of settlement fails.*

The surplus of consumers now has to take into account that consumers may not obtain the good if they fail to settle. Therefore, their surplus is scaled down by the probability of being hit by a settlement fail, λ :

$$\begin{aligned} S_c(\bar{k}) &= (1 - \lambda) \int_{a(0)}^1 \left[\int_{a(0)}^{a(\bar{k}) \vee v} \frac{(v - a)}{a(\bar{k}) - a(0)} da \right] dv \\ &= \frac{1}{6} (1 - \lambda) (1 - a(0))^2 \end{aligned}$$

Recall that $a(0) = 1 - \frac{1 - \lambda}{2(1 - \lambda + \lambda_\varepsilon^2)}$. Hence, the consumers' surplus is strictly decreasing with ε .¹²

Lemma 4 *The consumers' surplus is decreasing with the risk of settlement fails.*

Finally, we compute the surplus of producers, as

$$\begin{aligned} S_p(\bar{k}) &= \int_0^{b(0)} \left[\int_{b(\bar{k}) \wedge v}^{b(0)} \frac{(b - v)}{b(0) - b(\bar{k})} db \right] dv \\ &= \frac{b(0)^2}{6} \end{aligned}$$

¹²This can be simplified to:

$$S_c(\bar{k}) = \frac{1}{6} \frac{(1 - \lambda)^3}{4(1 - \lambda + \lambda_\varepsilon^2)^2}$$

Recall that $b(0) = \lambda_\varepsilon \frac{1-\lambda}{2(1-\lambda+\lambda_\varepsilon^2)}$ and from lemma 1, $\frac{\partial b(0)}{\partial \lambda_\varepsilon} > 0$ for λ_ε small enough and negative otherwise. Therefore, the surplus of producers is increasing when there is little risk of settlement fails, while it is decreasing otherwise.

Lemma 5 *The producers' surplus is increasing with the risk of settlement fails whenever ε is small and it is decreasing otherwise.*

We now want to analyze whether the surplus for this economy is increasing with settlement fails. Hence, we compute

$$S_d(\bar{k}) + S_p(\bar{k}) + S_c(\bar{k})$$

It is more convenient to operate a change of variable to compute the surplus of dealers. In the Appendix we show that

$$S_d(\bar{k}) = \frac{2(1-\lambda+\lambda_\varepsilon^2)^2}{3(1-\lambda)}(1-a(0))^3$$

Therefore,

$$\begin{aligned} S_d(\bar{k}) + S_p(\bar{k}) + S_c(\bar{k}) &= \frac{2(1-\lambda+\lambda_\varepsilon^2)^2}{3(1-\lambda)}(1-a(0))^3 \\ &\quad + \frac{b(0)^2}{6} \\ &\quad + \frac{1}{6}(1-\lambda)(1-a(0))^2 \end{aligned}$$

and using $b(k) = \lambda_\varepsilon(1-a(k))$ we obtain

$$\begin{aligned} S_d(\bar{k}) + S_p(\bar{k}) + S_c(\bar{k}) &= \frac{2(1-\lambda+\lambda_\varepsilon^2)^2}{3(1-\lambda)}(1-a(0))^3 \\ &\quad + \frac{\lambda_\varepsilon^2(1-a(0))^2}{6} + \frac{1}{6}(1-\lambda)(1-a(0))^2 \end{aligned}$$

or simplifying,

$$\begin{aligned} S \equiv S_d(\bar{k}) + S_p(\bar{k}) + S_c(\bar{k}) &= (1-a(0))^2 \left[\frac{2(1-\lambda+\lambda_\varepsilon^2)^2}{3(1-\lambda)}(1-a(0)) + \frac{1-\lambda+\lambda_\varepsilon^2}{6} \right] \\ &= \frac{(1-\lambda+\lambda_\varepsilon^2)}{6}(1-a(0))^2 \left[\frac{4(1-\lambda+\lambda_\varepsilon^2)}{(1-\lambda)}(1-a(0)) + 1 \right] \\ &= \frac{(1-\lambda)^2}{8(1-\lambda+\lambda_\varepsilon^2)} \end{aligned}$$

which is strictly decreasing in ε .

5 Risk aversion

We now consider the case where traders are risk averse in the following sense: The surplus from trade of a buyer is $x = v - a(k)$ whenever he accepts the bid price $a(k)$. Similarly the surplus from trade of a seller is $x = b(k) - v$. We assume that traders value the surplus from trade according to a CRRA utility function,

$$u(x) = \frac{(x + c)^{1-\sigma} - c^{1-\sigma}}{(1-\sigma)},$$

where $\sigma > 1$ and $c > 0$ is small. We need $c > 0$ so that traders prefer to trade than to exit the market without searching.¹³ This specification implies that their decision to accept a bid or an ask price is the same as in the previous section. Therefore, the optimal bid and ask prices set by dealers (1)-(2) are unchanged. As a consequence, the least efficient dealer in operation is still $\bar{k}$ defined by (3). Also, the effect of settlement risk on the bid-ask prices is unchanged: Increased settlement risk makes entry less profitable so that the least efficient dealers exit the market. As a consequence, the distribution of ask-prices becomes more concentrated. While they face higher ask price, buyers face a lower dispersion of ask price. Since they are risk averse, they may prefer that dealer face a little more risk. Obviously, buyers face a trade-off as on one hand they face a higher average ask-price, but on the other hand, the distribution of ask price is more compressed.

It is tedious to compute the overall buyers' welfare with $c > 0$ and we do so in the Appendix where we show that with $c > 0$,

$$U_c = \frac{(1-\lambda)}{(1-\sigma)} \left\{ \frac{(1-a(0)+c)^{3-\sigma} - c^{3-\sigma}}{(1-a(0))(2-\sigma)(3-\sigma)} - \frac{c^{1-\sigma}}{2}(1-a(0)) - \frac{c^{2-\sigma}}{2-\sigma} \right\}$$

Hence, we obtain

$$\frac{\partial U_c}{\partial \varepsilon} = \frac{(1-\lambda)}{(1-\sigma)} \left\{ -\frac{(1-a(0)+c)^{2-\sigma}}{(1-a(0))(2-\sigma)} + \frac{(1-a(0)+c)^{3-\sigma} - c^{3-\sigma}}{(1-a(0))^2(2-\sigma)(3-\sigma)} + \frac{c^{1-\sigma}}{2} \right\} \frac{\partial a(0)}{\partial \varepsilon}$$

Computation with different values for σ reveals that the payoff of consumers is always decreasing with an increasing in settlement risk. Therefore, concavity of the buyer's payoff function is not enough to generate the desirability of settlement risk. We turn next to different distribution of the dealers' cost.

¹³This is the case if $\sigma > 1$ as $x^{1-\sigma}/(1-\sigma) < 0$ for all $x \geq 0$, and this affects the decision of traders to accept or reject an offer.

5.1 Distribution function for dealers transaction cost

In this section of the paper we assume that dealers are distributed according to a beta probability distribution $f(k; \alpha, \beta) = \frac{\alpha k^{\alpha-1} (1-k)^{\beta-1}}{\mathcal{B}(\alpha, \beta)}$ with support $[0, 1]$. Let $\beta = 1$ so that $\mathcal{B}(\alpha, \beta) = 1$. Then the cdf associated with it is

$$F(k) = \int_0^k \alpha s^{\alpha-1} ds = k^\alpha$$

Now, because only $\bar{k} = \frac{1-\lambda}{\lambda_\varepsilon} < 1$ are active, then¹⁴

$$F_{\bar{k}}(k) = \frac{k^\alpha}{\bar{k}^\alpha}$$

and the probability distribution function is then simply $f_{\bar{k}}(k) = \alpha \frac{k^{\alpha-1}}{\bar{k}^\alpha}$.

Notice that ask prices are an affine transformation of the dealer's cost of the form $a(k) = a(0) + \xi k$ where $a(\bar{k}) = 1$ and $\xi = \frac{\lambda_\varepsilon}{2(1-\lambda+\lambda_\varepsilon^2)}$, then the cdf of $a(k)$ is derived from $F_{\bar{k}}(k)$:

$$\begin{aligned} F_a(\hat{a}) &= \frac{\left(\frac{\hat{a}-a(0)}{\xi}\right)^\alpha}{\bar{k}^\alpha} \\ f_a(a) &= \frac{1}{\xi} f_{\bar{k}}\left(\frac{a-a(0)}{\xi}\right) \end{aligned}$$

Similarly for the bid price

$$b(k) = b(0) - \lambda_\varepsilon \xi k$$

And

$$\begin{aligned} F_b(\hat{b}) &= 1 - \frac{\left(\frac{b(0)-\hat{b}}{\lambda_\varepsilon \xi}\right)^\alpha}{\bar{k}^\alpha} \\ f_b(b) &= \frac{1}{\lambda_\varepsilon \xi} f_{\bar{k}}\left(\frac{b(0)-b}{\lambda_\varepsilon \xi}\right) \end{aligned}$$

¹⁴Or, similarly, from $F(k) = k^\alpha$ we have that the truncated distribution $F_{\bar{k}}(k) = \Pr(s \leq k \mid s \leq \bar{k}) = \frac{\Pr(s \leq k \cap s \leq \bar{k})}{\Pr(s \leq \bar{k})} = \frac{F(k)}{F(\bar{k})}$.

5.2 Consumers' surplus

Then consumers' surplus (with linear preferences), using integration by parts, is:

$$\begin{aligned} S_c &= \int_{a(0)}^1 \left[\int_{a(0)}^v (v - a) f_a(a) da \right] dv \\ &= \frac{(1 - a(0))^{\alpha+2}}{\xi^\alpha \bar{k}^\alpha (\alpha + 1) (\alpha + 2)} \end{aligned}$$

Using $a(k) = 1 - \frac{1-\lambda-k\lambda_\varepsilon}{2(1-\lambda+\lambda_\varepsilon^2)}$, $\xi = \frac{\lambda_\varepsilon}{2(1-\lambda+\lambda_\varepsilon^2)}$ and $\bar{k} = \frac{1-\lambda}{\lambda_\varepsilon}$ we then have:

$$S_c = \frac{\left(\frac{1-\lambda}{2(1-\lambda+\lambda_\varepsilon^2)} \right)^2}{(\alpha + 1) (\alpha + 2)}$$

which is decreasing in ε . Also notice that the smaller α is the faster S_c decreases in ε .

5.3 Producers' surplus

Similarly for producers' surplus, using integration by parts:

$$\begin{aligned} S_p &= \int_0^{b(0)} \left[\int_v^{b(0)} (b - v) f_b(b) db \right] dv \\ &= \frac{b(0)^{\alpha+2}}{(\alpha + 1) (\alpha + 2) (\lambda_\varepsilon \xi \bar{k})^\alpha} \end{aligned}$$

Using $b(k) = \lambda_\varepsilon \frac{1-\lambda-k\lambda_\varepsilon}{2(1-\lambda+\lambda_\varepsilon^2)}$ we then have:

$$S_p = \lambda_\varepsilon^2 S_c$$

which is increasing¹⁵ in ε if and only if $\varepsilon \in [0, \bar{\varepsilon}]$ (where $\bar{\varepsilon} = -(1 - \lambda) + \sqrt{1 - \lambda}$ as defined above). Also notice that the smaller α is the faster S_p increases in ε .

5.4 Dealers' surplus

For dealers let us rewrite the expected demand and supply faced in their decision problem:

$$D(a) = \int_a^{\bar{r}_c} \tilde{h}(r) dr$$

where $\tilde{h}(r)$ is the conditional probability density of consumers' reservation prices among the fraction $1 - \underline{v}_c$ who chose to participate in the dealers' market. Therefore, $\tilde{h}(r)$ is derived as follows: the reservation price of a consumer with valuation v , denoted $r_c(v)$, is simply that specific consumer's valuation:

$$r_c(v) = v$$

Now, $v \sim U[\underline{v}_c, 1]$ therefore

$$\begin{aligned} \Pr(r_c(v) \leq r) &= \Pr(v \leq r) \\ &= \frac{r - \underline{v}_c}{1 - \underline{v}_c} \end{aligned}$$

and the probability density function associated with it is simply $h(r) = \frac{1}{1 - \underline{v}_c}$. Then the per dealer k density of consumers is $(1 - \underline{v}_c) f_k(k) h(r)$. So

¹⁵Where

$$\begin{aligned} \frac{\partial S_p}{\partial \varepsilon} &= \frac{(1 - \lambda)^2}{4(\alpha + 1)(\alpha + 2)} \frac{\partial \left(\frac{\lambda_\varepsilon}{(1 - \lambda + \lambda_\varepsilon^2)} \right)^2}{\partial \varepsilon} \\ &= \frac{(1 - \lambda)^2}{4(\alpha + 1)(\alpha + 2)} \frac{2\lambda_\varepsilon}{(1 - \lambda + \lambda_\varepsilon^2)} \left(\frac{1 - \lambda + \lambda_\varepsilon^2 - 2\lambda_\varepsilon^2}{(1 - \lambda + \lambda_\varepsilon^2)^2} \right) \\ &= \frac{\lambda_\varepsilon (1 - \lambda)^2}{2(\alpha + 1)(\alpha + 2)} \frac{(1 - \lambda - \lambda_\varepsilon^2)}{(1 - \lambda + \lambda_\varepsilon^2)^3} \end{aligned}$$

which is always strictly positive if and only if ε is such that $1 - \lambda - \lambda_\varepsilon^2 > 0$.

that the mass of consumers who place an order when the ask price they face is a (i.e. demand faced by a dealer who posts ask price a if his type is k -because here the mass of consumers that contact him is a function of k) is simply

$$\begin{aligned} D(a(k)) &= \int_{a(k)}^{\bar{r}_c} (1 - \underline{v}_c) f_{\bar{k}}(k) h(r) dr \\ &= (1 - a(k)) f_{\bar{k}}(k) \end{aligned}$$

And similarly for the supply:

$$S(b(k)) = b(k) f_{\bar{k}}(k)$$

For dealers, we also need to take into account the constraint of meeting demand period by period, so that substituting the expected demand and supply per dealer k into the objective function of a dealer we have, as before, that expected profits of dealer k with the optimal choice of a , are

$$\begin{aligned} \pi(k; \lambda, \varepsilon) &= f_{\bar{k}}(k) \{a(k)(1 - \lambda) - [\lambda_\varepsilon(1 - a(k)) + k] \lambda_\varepsilon\} (1 - a(k)) \\ &= \alpha \frac{k^{\alpha-1}}{\bar{k}^\alpha} \frac{(1 - \lambda - k \lambda_\varepsilon)^2}{4(1 - \lambda + \lambda_\varepsilon^2)} \end{aligned}$$

Then aggregate dealers' surplus is given by the total discounted profits of all dealers participating in the dealer market are:

$$\begin{aligned} S_d(\varepsilon) &= \int_0^{\bar{k}} \Pi(k; \lambda, \varepsilon) dk \\ &= \frac{1}{4(\alpha + 1)(1 - \lambda + \lambda_\varepsilon^2)} \left\{ (1 - \lambda - \lambda_\varepsilon \bar{k}) [(\alpha + 1)(1 - \lambda) + [2\lambda_\varepsilon - (\alpha + 1)\lambda_\varepsilon] \bar{k}] + \frac{2\lambda_\varepsilon^2 \bar{k}^2}{(\alpha + 2)} \right\} \end{aligned}$$

And using $\bar{k} = \frac{1-\lambda}{\lambda_\varepsilon}$ we then have:

$$S_d = \frac{(1 - \lambda)^2}{2(\alpha + 1)(\alpha + 2)(1 - \lambda + \lambda_\varepsilon^2)}$$

Also notice that the smaller α is the faster S_d decreases in ε .

Overall we have the following result:

Claim 6 S_d decreases in ε . The smaller α is the larger is the decrease in S_d . S_p increases in ε , for $\varepsilon \in [0, \bar{\varepsilon}]$, and decreases in ε , for $\varepsilon \in [\bar{\varepsilon}, \lambda]$. The smaller α is the larger is the increase (decrease) in S_d . S_c is decreasing in ε . The smaller α is the faster S_c decreases in ε .

5.5 Total welfare

Summing up consumers', producers' and dealers' welfare we have:

$$\begin{aligned} W &= S_c + S_p + S_d \\ &= \frac{(1-\lambda)^2 (3 + 3\lambda_\varepsilon^2 - 2\lambda)}{4(\alpha+1)(\alpha+2)(1-\lambda+\lambda_\varepsilon^2)^2} \end{aligned}$$

And:

$$\frac{\partial W}{\partial \varepsilon} = \frac{(1-\lambda)^2 \lambda_\varepsilon}{2(\alpha+1)(\alpha+2)} \frac{(\lambda - 3(1+\lambda_\varepsilon^2))}{(1-\lambda+\lambda_\varepsilon^2)^3}$$

which is always negative since

$$\lambda - 3(1+\lambda_\varepsilon^2) < 0$$

Claim 7 *Total welfare is always decreasing in ε regardless of the value of α .*

5.6 Different parameters for beta distribution

In this section of the paper we assume that dealers are distributed according to a beta probability distribution $f(k; \alpha, \beta) = \frac{\beta k^{\alpha-1} (1-k)^{\beta-1}}{B(\alpha, \beta)}$ with support $[0, 1]$. Let $\alpha = 1$ so that $f(k; \alpha, \beta) = \beta (1-k)^{\beta-1}$ and the cdf associated with it is

$$F(k) = 1 - (1-k)^\beta$$

Now, because only $\bar{k} = \frac{1-\lambda}{\lambda_\varepsilon} < 1$ are active, then¹⁶

$$F_{\bar{k}}(k) = \frac{1 - (1-k)^\beta}{1 - (1-\bar{k})^\beta}$$

and the probability distribution function is then simply $f_{\bar{k}}(k) = \beta \frac{(1-k)^{\beta-1}}{1 - (1-\bar{k})^\beta}$.

¹⁶Or, similarly, from $F(k) = k^\alpha$ we have that the truncated distribution $F_{\bar{k}}(k) = \Pr(s \leq k \mid s \leq \bar{k}) = \frac{\Pr(s \leq k \cap s \leq \bar{k})}{\Pr(s \leq \bar{k})} = \frac{F(k)}{F(\bar{k})}$.

Notice that ask prices are an affine transformation of the dealer's cost of the form $a(k) = a(0) + \xi k$ where $a(\bar{k}) = 1$ and $\xi = \frac{\lambda_\varepsilon}{2(1-\lambda+\lambda_\varepsilon^2)}$, then the cdf of $a(k)$ is derived from $F_{\bar{k}}(k)$:

$$\begin{aligned} F_a(\hat{a}) &= \frac{1 - \left(1 - \frac{\hat{a} - a(0)}{\xi}\right)^\beta}{1 - (1 - \bar{k})^\beta} \\ f_a(a) &= \frac{1}{\xi} f_{\bar{k}}\left(\frac{a - a(0)}{\xi}\right) \end{aligned}$$

Similarly for the bid price

$$b(k) = b(0) - \lambda_\varepsilon \xi k$$

And

$$\begin{aligned} F_b(\hat{b}) &= \frac{\left(1 - \frac{b(0) - \hat{b}}{\lambda_\varepsilon \xi}\right)^\beta - (1 - \bar{k})^\beta}{1 - (1 - \bar{k})^\beta} \\ f_b(b) &= \frac{1}{\lambda_\varepsilon \xi} f_{\bar{k}}\left(\frac{b(0) - b}{\lambda_\varepsilon \xi}\right) \end{aligned}$$

5.6.1 Consumers' surplus

Then consumers' surplus (with linear preferences), using integration by parts, is:

$$\begin{aligned} S_c &= \int_{a(0)}^1 \left[\int_{a(0)}^v (v - a) f_a(a) da \right] dv \\ &= \frac{1 - a(0)}{[1 - (1 - \bar{k})^\beta]} \left(\frac{1 - a(0)}{2} - \frac{\xi}{\beta + 1} \right) - \frac{\frac{\xi}{\beta + 1} \frac{\xi}{\beta + 2}}{[1 - (1 - \bar{k})^\beta]} \left[\left(1 - \frac{1 - a(0)}{\xi}\right)^{\beta + 2} - 1 \right] \end{aligned}$$

And using $a(k) = 1 - \frac{1 - \lambda - k \lambda_\varepsilon}{2(1 - \lambda + \lambda_\varepsilon^2)}$, $\xi = \frac{\lambda_\varepsilon}{2(1 - \lambda + \lambda_\varepsilon^2)}$ and $\bar{k} = \frac{1 - \lambda}{\lambda_\varepsilon}$ we then have that for $\lambda = 0.3, \beta = 0.2$ consumers' surplus as a function of ε is increasing for small values of ε , as figure 4 shows.

Let ε^* denote the threshold such that $\forall \varepsilon \leq \varepsilon^*$ we have that $\frac{\partial S_c}{\partial \varepsilon} > 0$ and $\forall \varepsilon > \varepsilon^*$ we have that $\frac{\partial S_c}{\partial \varepsilon} < 0$. Then as $\beta > 0$ decreases we have that ε^* increases.

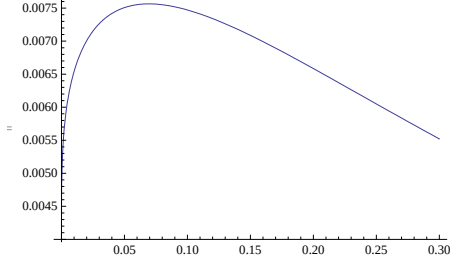


Figure 4: Consumers' surplus as a function of ε : $\lambda = 0.3, \beta = 0.2$

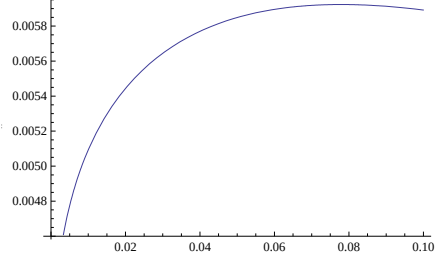


Figure 5: Consumers' surplus as a function of ε : $\lambda = 0.1, \beta = 0.2$

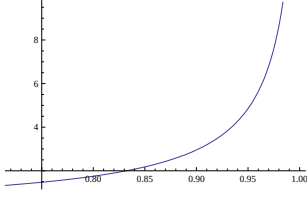


Figure 6: $f_a(a)$: $\beta = 0.2, \lambda = 0.3, \varepsilon = 0.02$

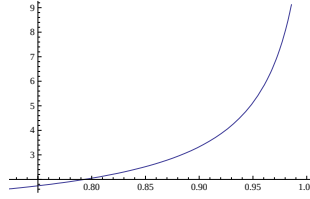


Figure 7: $f_a(a)$: $\beta = 0.2, \lambda = 0.3, \varepsilon = 0.05$

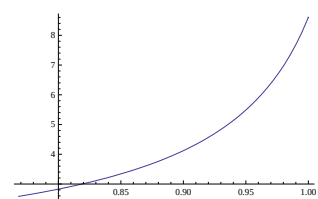


Figure 8: $f_a(a)$: $\beta = 0.2, \lambda = 0.3, \varepsilon = 0.2$

Also, for the same value of β , the smaller λ is the larger ε^* is. Figure 5 shows S_c as a function of ε for $\lambda = 0.1, \beta = 0.2$.

And substituting out $a(k) = 1 - \frac{1-\lambda-k\lambda_\varepsilon}{2(1-\lambda+\lambda_\varepsilon^2)}$, $\xi = \frac{\lambda_\varepsilon}{2(1-\lambda+\lambda_\varepsilon^2)}$ and $\bar{k} = \frac{1-\lambda}{\lambda_\varepsilon}$ we then have that:

$$S_c = \frac{(\beta + 2)(1 - \lambda)(\beta + 1)(1 - \lambda) - 2\lambda_\varepsilon^2 \left(1 - \frac{1-\lambda}{\lambda_\varepsilon}\right)^{\beta+2} + 2\lambda_\varepsilon(\lambda_\varepsilon - (\beta + 2)(1 - \lambda))}{8(\beta + 1)(1 - \lambda + \lambda_\varepsilon^2)^2 \left[1 - \left(1 - \frac{1-\lambda}{\lambda_\varepsilon}\right)^\beta\right](\beta + 2)}$$

The whole positive effect of ε comes from $\left[1 - \left(1 - \frac{1-\lambda}{\lambda_\varepsilon}\right)^\beta\right]$ at the denominator which is coming from $\bar{k}$ through the distribution of ask prices. Figure 6 shows the pdf of the ask price, $f_a(a)$, for $\beta = 0.2, \lambda = 0.3, \varepsilon = 0.02$. Notice that when ε increases, the mass on every surviving dealer increases. Figure 7 shows the pdf of the ask price, $f_a(a)$, for $\beta = 0.2, \lambda = 0.3, \varepsilon = 0.05$. Figure 8 shows the pdf of the ask price, $f_a(a)$, for $\beta = 0.2, \lambda = 0.3, \varepsilon = 0.2$.

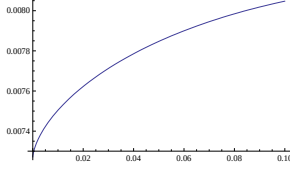


Figure 9: $S_p(\varepsilon)$: $\beta = 0.7, \lambda = 0.1$

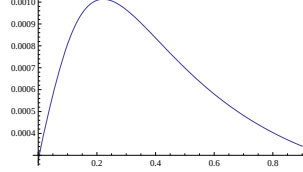


Figure 10: $S_p(\varepsilon)$: $\beta = 0.7, \lambda = 0.9$

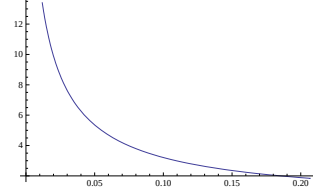


Figure 11: $f_b(b)$: $\beta = 0.2, \lambda = 0.3, \varepsilon = 0.02$

5.6.2 Producers' surplus

Similarly for producers' surplus, using integration by parts:

$$\begin{aligned}
 S_p &= \int_0^{b(0)} \left[\int_v^{b(0)} (b-v) f_b(b) db \right] dv \\
 &= \frac{1}{\left[1 - \left(1 - \bar{k}\right)^\beta\right]} \left\{ \frac{b(0)^2}{2} - \frac{\lambda_\varepsilon \xi}{\beta+1} b(0) + \frac{\lambda_\varepsilon \xi}{\beta+1} \frac{\lambda_\varepsilon \xi}{\beta+2} \left[1 - \left(1 - \frac{b(0)}{\lambda_\varepsilon \xi}\right)^{\beta+2} \right] \right\}
 \end{aligned}$$

Using $b(k) = \lambda_\varepsilon \frac{1-\lambda-k\lambda_\varepsilon}{2(1-\lambda+\lambda_\varepsilon^2)}$, $\xi = \frac{\lambda_\varepsilon}{2(1-\lambda+\lambda_\varepsilon^2)}$ and $\bar{k} = \frac{1-\lambda}{\lambda_\varepsilon}$, we then have:

$$S_p = \frac{\lambda_\varepsilon^2}{\left[1 - \left(1 - \frac{1-\lambda}{\lambda_\varepsilon}\right)^\beta\right]} \frac{(\beta+1)(1-\lambda)^2 + \frac{2\lambda_\varepsilon^2}{(\beta+2)} \left(1 - \left(1 - \frac{1-\lambda}{\lambda_\varepsilon}\right)^{\beta+2}\right) - 2\lambda_\varepsilon(1-\lambda)}{4(\beta+1)(1-\lambda+\lambda_\varepsilon^2)^2}$$

Interestingly, also the producers' surplus is decreasing in ε for large values of β : for example for $\beta = 2, \lambda = 0.3$ it is decreasing, but for $\beta = 1, \lambda = 0.3$ it is hump shaped with a threshold ε^* such that $\forall \varepsilon \leq \varepsilon^*$ we have that $\frac{\partial S_p}{\partial \varepsilon} > 0$ and $\forall \varepsilon > \varepsilon^*$ we have that $\frac{\partial S_p}{\partial \varepsilon} < 0$. As in the consumers' surplus case, as $\beta > 0$ decreases we have that ε^* increases. Figure 9 shows producers' surplus, S_p , as a function of ε when $\beta = 0.7, \lambda = 0.1$. Also notice that despite how large λ may be, as long as β is small enough then S_p will always be hump shaped as a function of ε . Figure 10 shows producers' surplus S_p as a function of ε when $\beta = 0.7, \lambda = 0.9$. In order to gain insight on what is going on with the distribution of bid prices, Figure 11 shows the pdf of the bid price, $f_b(b)$, for $\beta = 0.2, \lambda = 0.3, \varepsilon = 0.02$.

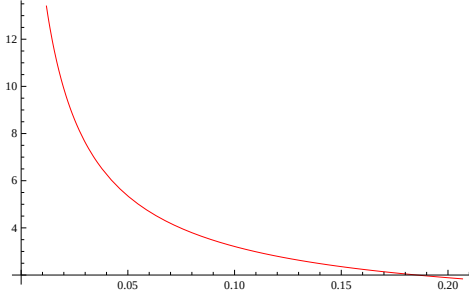


Figure 12: $f_b(b)$: $\beta = 0.2, \lambda = 0.3, \varepsilon = 0.02$

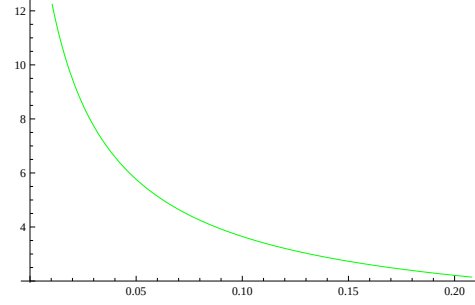


Figure 13: $f_b(b)$: $\beta = 0.2, \lambda = 0.3, \varepsilon = 0.05$

Therefore there is a lot of mass on inefficient dealers so that when they exit all that mass gets thrown onto more efficient dealers: recall that more efficient dealers are the ones who charge the highest (lowest) bid (ask) price because they are the only ones who can afford to do so. Therefore the above picture means that few dealers (the efficient ones) charge the highest bid prices, whereas many dealers (the inefficient ones) charge the lowest bid prices.

Notice that when ε increases, the mass on bid prices offered by very efficient dealers increases. Figure 12 (red) shows the pdf of $f_b(b)$ for $\beta = 0.2, \lambda = 0.3, \varepsilon = 0.02$ and figure 13 (green) for $\beta = 0.2, \lambda = 0.3, \varepsilon = 0.05$.

Notice that $\underline{b} = 0$ is unchanged because it is the bid price quoted by the marginal operating dealer (which is making zero profits); however $\bar{b}$ increases with ε because it is the bid price quoted by the most efficient dealer whose demand and supply change as ε increases because there are less dealers who are active (since $\bar{k}$ decreases). Therefore the most efficient dealer is more likely to get a random call by a buyer and a seller ($f_{\bar{k}}(k=0)$ increases) and he is efficient enough that it is profitable for him to increase the bid price and serve a larger share of the market.

5.6.3 Dealers' surplus

If we take into account that expected demand and supply are $D(a) = (1 - a(k)) f_{\bar{k}}(k)$ and $S(b(k)) = b(k) f_{\bar{k}}(k)$ then expected profits are $\pi(k; \lambda, \varepsilon) = \beta \frac{(1-k)^{\beta-1} (1-\lambda-k\lambda\varepsilon)^2}{1-(1-\bar{k})^\beta 4(1-\lambda+\lambda\varepsilon^2)}$. Either way we know that the calculation of aggregate dealers' surplus is the same regardless of which interpretation we give (match-

ing or probability). Therefore aggregate dealers' surplus is:

$$\begin{aligned}
S_d(\varepsilon) &= \int_0^{\bar{k}} \Pi(k; \lambda, \varepsilon) dk \\
&= \frac{(1-\lambda)}{(1-(1-\bar{k})^\beta)^4(1-\lambda+\lambda_\varepsilon^2)} \left\{ (1-\lambda) - \frac{2\lambda_\varepsilon}{\beta+1} + \left(2\lambda_\varepsilon \bar{k} - (1-\lambda) + \frac{2\lambda_\varepsilon}{\beta+1} (1-\bar{k}) \right) (1-\bar{k})^\beta \right\} + \\
&\quad + \frac{\lambda_\varepsilon^2}{(1-(1-\bar{k})^\beta)^4(1-\lambda+\lambda_\varepsilon^2)} \left\{ \frac{2}{(\beta+1)(\beta+2)} - (1-\bar{k})^\beta \left[\bar{k}^2 + \frac{2((1-\bar{k})^2 + (\beta+2)(1-\bar{k})\bar{k})}{(\beta+1)(\beta+2)} \right] \right\}
\end{aligned}$$

and using $\bar{k} = \frac{1-\lambda}{\lambda_\varepsilon}$, we then have:

$$\begin{aligned}
S_d &= \frac{(1-\lambda)}{(1-(1-\bar{k})^\beta)^4(1-\lambda+\lambda_\varepsilon^2)} \left\{ (1-\lambda) - \frac{2\lambda_\varepsilon}{\beta+1} + \left((1-\lambda) + \frac{2\varepsilon}{\beta+1} \right) \left(\frac{\varepsilon}{\lambda_\varepsilon} \right)^\beta \right\} + \\
&\quad + \frac{\lambda_\varepsilon^2}{(1-(1-\bar{k})^\beta)^4(1-\lambda+\lambda_\varepsilon^2)} \left\{ \frac{2}{(\beta+1)(\beta+2)} - \left(\frac{\varepsilon}{\lambda_\varepsilon} \right)^\beta \left(\frac{1-\lambda}{\lambda_\varepsilon} \right)^2 - \left(\frac{\varepsilon}{\lambda_\varepsilon} \right)^\beta \frac{2\varepsilon(\varepsilon+(\beta+2)(1-\lambda))}{\lambda_\varepsilon^2(\beta+1)(\beta+2)} \right\}
\end{aligned}$$

Dealers' surplus for a given λ is inverse U shaped in ε : in general the smaller λ the larger the value of β^* , where $\beta^* = \{\beta > 0 : \frac{\partial S_d}{\partial \varepsilon} > 0, \forall \beta < \beta^*\}$. For a given λ , as we increase β the peak of the inverse U shaped function is reached at a value $\hat{\varepsilon} < 0$; analogously for β small the peak of the inverse U shaped function is reached at a value $\hat{\varepsilon} > \lambda$, therefore in these two polar cases we have that dealers' surplus is either decreasing or increasing in any feasible value of $\varepsilon \in [0, \lambda]$, as we can see from figures 14 and 15, that show S_d as a function of ε for $\lambda = 0.3, \beta = 2$ and $\lambda = 0.1, \beta = 0.2$ respectively.

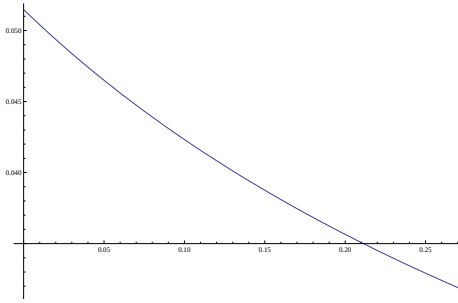


Figure 14: $S_d(\varepsilon)$: $\lambda = 0.3, \beta = 2$

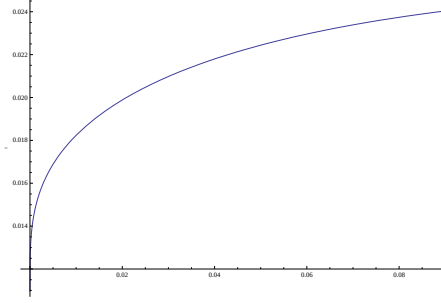


Figure 15: $S_d(\varepsilon)$: $\lambda = 0.1, \beta = 0.2$

But for intermediate values the inverse U shape is clear, as figure 18 shows for $\lambda = 0.3, \beta = 0.2$.

Then a natural question is: what happens to the distribution of active dealers when β changes. Figure 16 shows the mass of active dealers $f_{\bar{k}}(k)$ for $\lambda = 0.1, \beta = 0.2$ and figure 17 for $\lambda = 0.1, \beta = 5$.

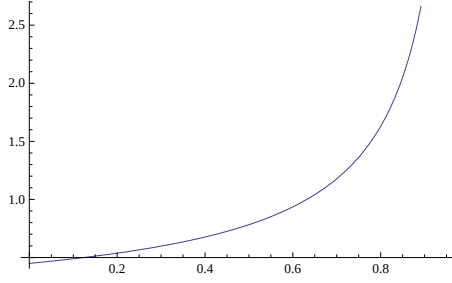


Figure 16: $f_{\bar{k}}(k)$: $\lambda = 0.1, \beta = 0.2$

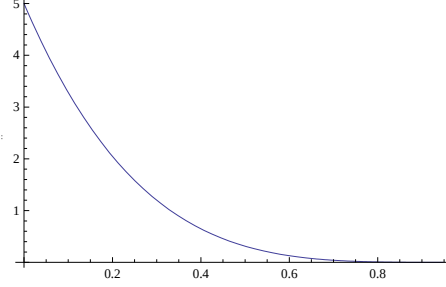


Figure 17: $f_{\bar{k}}(k)$: $\lambda = 0.1, \beta = 5$

For any $\beta < 1$ we have that the pdf has a lot more mass on inefficient dealers, and the opposite is true for $\beta > 1$. Therefore when ε increases and inefficient dealers exist, the pdf moves so that the mass increases for all active k but more so for the relatively inefficient ones. Figure 19 shows $f_{\bar{k}}(k)$ for $\lambda = 0.3, \beta = 0.2, \varepsilon_1 = 0.1, \varepsilon_2 = 0.2, \varepsilon_3 = 0.3$:

It is easy to see that the measure of active dealers shrinks as ε increases and that even though it is not easily visible from the graph, we can calculate the absolute increases in the mass for each dealer who has survive the increase in ε and the mass increases more for relatively more inefficient dealers. However the percentage increase in the mass is actually the same (the increase relative to the mass before change in ε).

6 Social planner's solution

We are interested in studying a benchmark to which we can compare our market equilibrium. In this economy the first best solution (maximizing total surplus from trade subject to resource constraint only -i.e. unit upper bound on asset holding and production, so if a consumer gets an asset it must be that a producer is producing an asset) is such that no dealers operate ($\bar{k} = \underline{k} = 0$) and that the top half of the consumers' distribution is served ($\int_{\underline{v}_c}^1 dv = \frac{1}{2}, \underline{v}_c = \frac{1}{2}$) using production by the bottom half of the producers' distribution ($\int_{0_c}^{\bar{v}_p} dv = \frac{1}{2}, \bar{v}_p = \frac{1}{2}$).

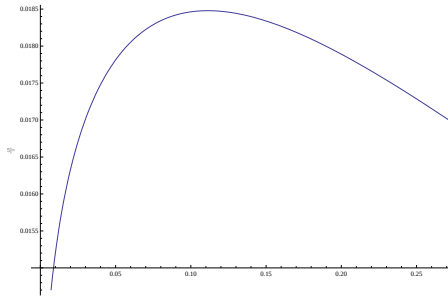


Figure 18: $S_d(\varepsilon)$ for $\lambda = 0.3, \beta = 0.2$

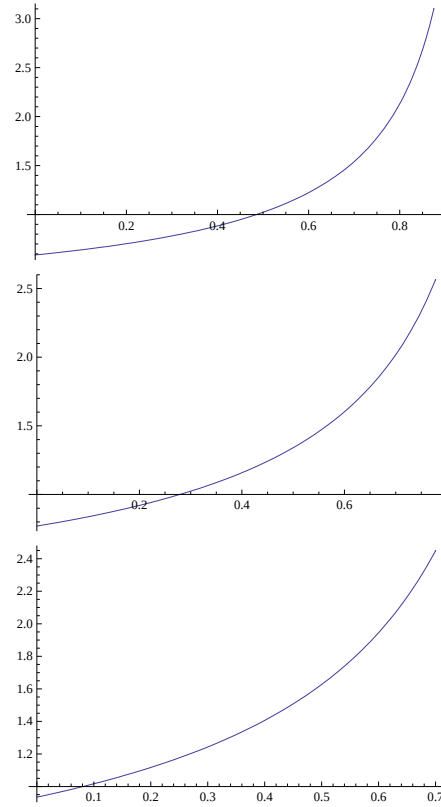


Figure 19: $f_k^-(k)$ for $\lambda = 0.3, \beta = 0.2, \varepsilon = 0.1, 0.2, 0.3$

Now consider a social planner who is subject to two frictions. First, matches between buyers and dealers, sellers and dealers occur according to the random matching present in the market economy: each buyer (seller) meets a dealer with probability $\frac{1}{\bar{k}}$, since dealers are uniformly distributed between $[0, \bar{k}]$. Second, the planner cannot offer contingent contracts (taxes or transfers) to buyers and sellers: that is to say the contract cannot be contingent on the valuation v of a buyer (seller). Then given these restrictions to the planner's problem, if he could choose $\bar{k}$ then he would choose $\bar{k} = 0$ so that all trades are intermediated by the most efficient dealer who is the only one to be matched with buyers and sellers. In this case we find that the top quarter of the consumers' distribution is served ($\int_{\underline{v}_c}^1 dv = \frac{1}{4}, \underline{v}_c = \frac{3}{4}$) using production by the bottom quarter of the producers' distribution¹⁷ ($\int_0^{\bar{v}_p} dv = \frac{1}{4}, \bar{v}_p = \frac{1}{4}$).

If we consider now that the planner cannot choose $\bar{k}$, then he maximizes the surplus from trade in every match for every k . Let the expected *revenue* from operating a post k given that the probability that buyers and sellers get to post k is $\frac{1}{\bar{k}}$, is:

$$R_k = \frac{(1 - \lambda)}{\bar{k}} \int_{v_k^b}^1 \tau_k^b dv - \frac{1}{\bar{k}} \int_0^{v_k^s} (\tau_k^s + k) dv$$

Therefore he solves:

$$\begin{aligned} \max \quad & R_k \\ \text{s.t.} \quad & \tau_k^b(v) \leq v \\ & \tau_k^s(v) \geq v \\ & \tau_k^b(v) = \tau_k^b(v') \quad \forall v, v' \\ & \tau_k^s(v) = \tau_k^s(v') \quad \forall v, v' \end{aligned}$$

where the first two constraints are feasibility constraints on the taxes/transfers that the planner can charge/pay to agents. The third and fourth constraints restrict taxes/transfers to be uncontingent on the valuation of each agent. This problem is the benchmark for our market equilibrium without settlement fails: in this case we find that for every k , less than the top quarter

¹⁷We develop these benchmarks social planner problems in an online appendix. They are however only descriptive of the environment and the mechanisms at work relative to efficiency. No result present in the paper depends on these benchmarks social planner problems.

of the consumers' distribution is served ($v_c = \frac{3-k}{4}$) using production by less than the bottom quarter of the producers' distribution¹⁸ ($\bar{v}_p = \frac{1-k}{4}$).

Let us now consider an additional constraint: $\lambda_\varepsilon \int_{v_k^b}^1 dv \leq \int_0^{1v_k^s} dv$ which is a feasibility constraint on the delivery of assets when the planner also is subject to settlement fail constraint that each dealer is subject to. This benchmark social planner now maximizes revenue for each active post,

$$\begin{aligned} \max \quad & \int_0^{\bar{k}} R_k \\ \text{s.t.} \quad & \tau_k^b(v) \leq v \\ & \tau_k^s(v) \geq v \\ & \tau_k^b(v) = \tau_k^b(v') \quad \forall v, v' \\ & \tau_k^s(v) = \tau_k^s(v') \quad \forall v, v' \\ & \lambda_\varepsilon \int_{v_k^b}^1 dv \leq \int_0^{v_k^s} dv \end{aligned} \tag{4}$$

where $\bar{k} = \frac{1-\lambda}{\lambda_\varepsilon}$ is given. From the constraints we have that $\tau_k^b(v) = v_k^b$ and $\tau_k^s(v) = v_k^s$. Then the last constraints is simply $\lambda_\varepsilon (1 - v_k^b) \leq v_k^s$ and the revenue/surplus from trade from dealer k is $R_k = (1 - \lambda) \frac{(1-v_k^b)}{\bar{k}} v_k^b - \frac{v_k^s}{\bar{k}} (v_k^s + k)$. Therefore the objective function (4) is

$$\int_0^{\bar{k}} \frac{1}{\bar{k}} \left\{ (1 - \lambda) (1 - v_k^b) v_k^b - \lambda_\varepsilon^2 (1 - v_k^b)^2 - k \lambda_\varepsilon (1 - v_k^b) \right\} dk$$

Notice that the objective function is concave in v_k^b so that the first order condition is a necessary and sufficient condition for a solution, v_k^{b*} , is:

$$\begin{aligned} (1 - \lambda) (1 - 2v_k^b) + 2\lambda_\varepsilon^2 (1 - v_k^b) + k\lambda_\varepsilon &= 0 \\ (1 - \lambda + 2\lambda_\varepsilon^2) - 2v_k^b (1 - \lambda + \lambda_\varepsilon^2) + k\lambda_\varepsilon &= 0 \end{aligned}$$

so that

$$v_k^{b*} = \frac{1 - \lambda + 2\lambda_\varepsilon^2 + k\lambda_\varepsilon}{2(1 - \lambda + \lambda_\varepsilon^2)}$$

¹⁸We develop these benchmarks social planner problems in an online appendix. They are however only descriptive of the environment and the mechanisms at work relative to efficiency. No result present in the paper depends on these benchmarks social planner problems.

x Now substituting it back into the objective function we obtain a function of ε :

$$\int_0^{\bar{k}} R_k dk = \frac{(1-\lambda)^2}{12(1-\lambda+\lambda_\varepsilon^2)}$$

which is always decreasing in ε . Not surprisingly, this corresponds to the expected revenue of dealers in the market.

7 Conclusion

A Appendix

A.1 Change of variables

$$S_d(\bar{k}) = \int_0^{\bar{k}} \{a(k)(1-\lambda) - [\lambda_\varepsilon(1-a(k)) + k]\lambda_\varepsilon\} \frac{1}{k} (1-a(k)) dk$$

Since

$$a(k) = \frac{(1-\lambda) + 2\lambda_\varepsilon^2 + \lambda_\varepsilon k}{2(1-\lambda+\lambda_\varepsilon^2)}$$

we have

$$k(a) = \frac{2(1-\lambda+\lambda_\varepsilon^2)a - (1-\lambda+2\lambda_\varepsilon^2)}{\lambda_\varepsilon}$$

Hence

$$dk = \frac{2(1-\lambda+\lambda_\varepsilon^2)}{\lambda_\varepsilon} da$$

so that the surplus of dealers is simply

$$S_d(\bar{k}) = \int_{a(0)}^{a(\bar{k})} \{a(1-\lambda) - [\lambda_\varepsilon(1-a) + k(a)]\lambda_\varepsilon\} \frac{1}{k} (1-a) \frac{2(1-\lambda+\lambda_\varepsilon^2)}{\lambda_\varepsilon} da$$

Hence,

$$\begin{aligned} S_d(\bar{k}) &= \int_{a(0)}^{a(\bar{k})} \left\{ a(1-\lambda) - \left[\lambda_\varepsilon(1-a) + \frac{2(1-\lambda+\lambda_\varepsilon^2)a - (1-\lambda+2\lambda_\varepsilon^2)}{\lambda_\varepsilon} \right] \lambda_\varepsilon \right\} \frac{1}{k} (1-a) \frac{2(1-\lambda+\lambda_\varepsilon^2)}{\lambda_\varepsilon} da \\ &= \int_{a(0)}^{a(\bar{k})} \{a(1-\lambda) - 2(1-\lambda)a - \lambda_\varepsilon^2 a + (1-\lambda) + \lambda_\varepsilon^2\} \frac{1}{k} (1-a) \frac{2(1-\lambda+\lambda_\varepsilon^2)}{\lambda_\varepsilon} da \\ &= \int_{a(0)}^{a(\bar{k})} \{(1-\lambda+\lambda_\varepsilon^2)(1-a)\} \frac{1}{k} (1-a) \frac{2(1-\lambda+\lambda_\varepsilon^2)}{\lambda_\varepsilon} da \end{aligned}$$

Using $\bar{k} = \frac{1-\lambda}{\lambda_\varepsilon}$ we obtain

$$\begin{aligned} S_d(\bar{k}) &= \frac{2(1-\lambda+\lambda_\varepsilon^2)^2}{(1-\lambda)} \int_{a(0)}^{a(\bar{k})} (1-a)^2 da \\ &= \frac{2(1-\lambda+\lambda_\varepsilon^2)^2}{(1-\lambda)} \left[\frac{1}{3}(1-a(0))^3 - \frac{1}{3}(1-a(\bar{k}))^3 \right] \end{aligned}$$

and since $a(\bar{k}) = 1$ we obtain

$$\begin{aligned} S_d(\bar{k}) &= \frac{2(1-\lambda+\lambda_\varepsilon^2)^2}{3(1-\lambda)} (1-a(0))^3 \\ &= \frac{2(1-\lambda+\lambda_\varepsilon^2)^2}{3(1-\lambda)} \frac{(1-\lambda)^3}{8(1-\lambda+\lambda_\varepsilon^2)^3} \\ &= \frac{1}{12} \frac{(1-\lambda)^2}{(1-\lambda+\lambda_\varepsilon^2)} \end{aligned}$$

A.2 Dealer's distribution

$$\begin{aligned} S_c &= \int_{a(0)}^1 \left[\int_{a(0)}^v (v-a) f_a(a) da \right] dv \\ &= \int_{a(0)}^1 \left[\int_{a(0)}^v (v-a) \frac{1}{\xi} g_{\bar{k}} \left(\frac{a-a(0)}{\xi} \right) da \right] dv \end{aligned}$$

and since $a(k) = a(0) + \xi k$ we have $da = \xi dk$, hence changing variable, we obtain

$$\begin{aligned} S_c &= \int_{a(0)}^1 \left[\int_0^{\frac{v-a(0)}{\xi}} \{v-a(0)-\xi k\} g_{\bar{k}}(k) dk \right] dv \\ &= \int_{a(0)}^1 \left[\int_0^{\frac{v-a(0)}{\xi}} \{v-a(0)-\xi k\} \frac{\alpha k^{\alpha-1}}{\bar{k}^\alpha} dk \right] dv \end{aligned}$$

and

$$\begin{aligned}
& \int_0^{\frac{v-a(0)}{\xi}} \{v - a(0) - \xi k\} \frac{\alpha k^{\alpha-1}}{\bar{k}^\alpha} dk = \\
& \int_0^{\frac{v-a(0)}{\xi}} [v - a(0)] \frac{\alpha k^{\alpha-1}}{\bar{k}^\alpha} dk - \int_0^{\frac{v-a(0)}{\xi}} \xi \frac{\alpha k^\alpha}{\bar{k}^\alpha} dk = \\
& \frac{v - a(0)}{\bar{k}^\alpha} [k^\alpha]_0^{\frac{v-a(0)}{\xi}} - \frac{\xi}{\bar{k}^\alpha} \frac{\alpha}{(\alpha+1)} [k^{\alpha+1}]_0^{\frac{v-a(0)}{\xi}} = \\
& \frac{1}{(\xi \bar{k})^\alpha} [v - a(0)]^{\alpha+1} - \frac{1}{(\xi \bar{k})^\alpha} \frac{\alpha}{(\alpha+1)} [v - a(0)]^{\alpha+1} = \\
& \frac{1}{(\alpha+1)} \frac{1}{(\xi \bar{k})^\alpha} [v - a(0)]^{\alpha+1}
\end{aligned}$$

and as $\xi \bar{k} = 1 - a(0)$ we obtain

$$\int_0^{\frac{v-a(0)}{\xi}} \{v - a(0) - \xi k\} \frac{\alpha k^{\alpha-1}}{\bar{k}^\alpha} dk = \frac{1}{(\alpha+1)} \frac{[v - a(0)]^{\alpha+1}}{[1 - a(0)]^\alpha}$$

which is strictly decreasing in ε .

The surplus of producers is

$$\begin{aligned}
S_p &= \int_0^{b(0)} \left[\int_v^{b(0)} (b - v) f_b(b) db \right] dv \\
&= \int_0^{b(0)} \left[\int_v^{b(0)} (b - v) \frac{1}{\lambda_\varepsilon \xi} g_{\bar{k}} \left(\frac{b(0) - b}{\lambda_\varepsilon \xi} \right) db \right] dv
\end{aligned}$$

and since $b = b(0) - \xi \lambda_\varepsilon k$ we obtain $db = -\xi \lambda_\varepsilon dk$. Hence, again we can change variable to find,

$$\begin{aligned}
S_p &= \int_0^{b(0)} \left[\int_{\frac{b(0)-v}{\lambda_\varepsilon \xi}}^0 -(b(0) - \lambda_\varepsilon \xi k - v) g_{\bar{k}}(k) dk \right] dv \\
&= \int_0^{b(0)} \left[\int_0^{\frac{b(0)-v}{\lambda_\varepsilon \xi}} (b(0) - \lambda_\varepsilon \xi k - v) \frac{\alpha k^{\alpha-1}}{\bar{k}^\alpha} dk \right] dv
\end{aligned}$$

and so we obtain

$$\begin{aligned}
& \int_0^{\frac{b(0)-v}{\lambda_\varepsilon \xi}} (b(0) - \lambda_\varepsilon \xi k - v) \frac{\alpha k^{\alpha-1}}{\bar{k}^\alpha} dk = \\
& \int_0^{\frac{b(0)-v}{\lambda_\varepsilon \xi}} (b(0) - v) \frac{\alpha k^{\alpha-1}}{\bar{k}^\alpha} dk - \int_0^{\frac{b(0)-v}{\lambda_\varepsilon \xi}} \lambda_\varepsilon \xi \frac{\alpha k^\alpha}{\bar{k}^\alpha} dk = \\
& \frac{(b(0) - v)}{\bar{k}^\alpha} \left[\frac{b(0) - v}{\lambda_\varepsilon \xi} \right]^\alpha - \frac{\alpha}{1 + \alpha} \frac{\lambda_\varepsilon \xi}{\bar{k}^\alpha} \left[\frac{b(0) - v}{\lambda_\varepsilon \xi} \right]^{\alpha+1} = \\
& \left[\frac{1}{\lambda_\varepsilon \xi} \right]^\alpha \frac{(b(0) - v)^{\alpha+1}}{\bar{k}^\alpha} \frac{1}{1 + \alpha} =
\end{aligned}$$

and since $\lambda_\varepsilon \xi \bar{k} = b(0)$ we obtain

$$S_p = \int_0^{b(0)} \left[\frac{1}{1 + \alpha} \frac{(b(0) - v)^{\alpha+1}}{b(0)^\alpha} \right] dv$$

which is strictly increasing with ε whenever ε is small enough (as $\frac{\partial b(0)}{\partial \varepsilon} > 0$ in that case).

A.3 Different specification for dealers' distribution

In this section of the paper we assume that dealers are distributed according to a beta probability distribution $f(k; \alpha, \beta) = \frac{\alpha k^{\alpha-1} (1-k)^{\beta-1}}{\mathcal{B}(\alpha, \beta)}$ with support $[0, 1]$. Let $\beta = 1$ so that $\mathcal{B}(\alpha, \beta) = 1$. Then the cdf associated with it is

$$\begin{aligned}
F(k) &= \frac{\alpha \mathcal{B}(k; \alpha, \beta)}{\alpha \mathcal{B}(\alpha, \beta)} \\
&= \frac{\int_0^k f(s; \alpha, \beta) ds}{\int_0^1 f(s; \alpha, \beta) ds} \\
&= \int_0^k f(s; \alpha, \beta) ds \\
&= \int_0^k \alpha s^{\alpha-1} ds = k^\alpha
\end{aligned}$$

Now, because only $\bar{k} = \frac{1-\lambda}{\lambda_\varepsilon} < 1$ are active, then¹⁹

$$\begin{aligned} F_{\bar{k}}^-(k) &= \frac{\int_0^k f(s; \alpha, \beta) ds}{\int_0^{\bar{k}} f(s; \alpha, \beta) ds} \\ &= \frac{k^\alpha}{\bar{k}^\alpha} \end{aligned}$$

and the probability distribution function is then simply $f_{\bar{k}}^-(k) = \alpha \frac{k^{\alpha-1}}{\bar{k}^\alpha}$.

Notice that ask prices are an affine transformation of the dealer's cost of the form $a(k) = a(0) + \xi k$ where $a(\bar{k}) = 1$ and $\xi = \frac{\lambda_\varepsilon}{2(1-\lambda+\lambda_\varepsilon^2)}$, then the cdf of $a(k)$ is derived from $F_{\bar{k}}^-(k)$:

$$\begin{aligned} F_a(\hat{a}) &= \Pr(a \leq \hat{a}) \\ &= \Pr(a(0) + \xi k \leq \hat{a}) \\ &= F_{\bar{k}}^-\left(k \leq \frac{\hat{a} - a(0)}{\xi}\right) \\ &= \frac{\left(\frac{\hat{a} - a(0)}{\xi}\right)^\alpha}{\bar{k}^\alpha} \\ f_a(a) &= \frac{\alpha \left(\frac{a - a(0)}{\xi}\right)^{\alpha-1}}{\xi \bar{k}^\alpha} \\ &= \frac{1}{\xi} f_{\bar{k}}^-\left(\frac{a - a(0)}{\xi}\right) \end{aligned}$$

Similarly for the bid price

$$\begin{aligned} b(k) &= \lambda_\varepsilon (1 - a(k)) \\ &= \lambda_\varepsilon (1 - a(0) - \xi k) \\ &= b(0) - \lambda_\varepsilon \xi k \end{aligned}$$

¹⁹Or, similarly, from $F(k) = k^\alpha$ we have that the truncated distribution $F_{\bar{k}}^-(k) = \Pr(s \leq k \mid s \leq \bar{k}) = \frac{\Pr(s \leq k \cap s \leq \bar{k})}{\Pr(s \leq \bar{k})} = \frac{F(k)}{F(\bar{k})}$.

And

$$\begin{aligned}
F_b(\hat{b}) &= \Pr\left(b(0) - \lambda_\varepsilon \xi k \leq \hat{b}\right) \\
&= \Pr\left(k \geq \frac{b(0) - \hat{b}}{\lambda_\varepsilon \xi}\right) \\
&= 1 - F_{\bar{k}}\left(\frac{b(0) - \hat{b}}{\lambda_\varepsilon \xi}\right) \\
&= 1 - \frac{\left(\frac{b(0) - \hat{b}}{\lambda_\varepsilon \xi}\right)^\alpha}{\bar{k}^\alpha} \\
f_b(b) &= \alpha \frac{\left(\frac{b(0) - b}{\lambda_\varepsilon \xi}\right)^{\alpha-1}}{\lambda_\varepsilon \xi \bar{k}^\alpha} \\
&= \frac{1}{\lambda_\varepsilon \xi} f_{\bar{k}}\left(\frac{b(0) - b}{\lambda_\varepsilon \xi}\right)
\end{aligned}$$

A.4 Consumers' surplus

Then consumers' surplus (with linear preferences), using integration by parts, is:

$$\begin{aligned}
S_c &= \int_{a(0)}^1 \left[\int_{a(0)}^v (v-a) f_a(a) da \right] dv \\
&= \int_{a(0)}^1 \left[\int_{a(0)}^v (v-a) \frac{\alpha \left(\frac{a-a(0)}{\xi} \right)^{\alpha-1}}{\xi \bar{k}^\alpha} da \right] dv \\
&= \frac{\alpha}{\xi \alpha \bar{k}^\alpha} \int_{a(0)}^1 \left[\int_{a(0)}^v v (a-a(0))^{\alpha-1} - a (a-a(0))^{\alpha-1} da \right] dv \\
&= \frac{\alpha}{\xi \alpha \bar{k}^\alpha} \int_{a(0)}^1 \left\{ \left[\frac{v}{\alpha} (a-a(0))^\alpha \right]_{a(0)}^v - \left(\left[\frac{a}{\alpha} (a-a(0))^\alpha \right]_{a(0)}^v - \int_{a(0)}^v \frac{(a-a(0))^\alpha}{\alpha} da \right) \right\} dv \\
&= \frac{\alpha}{\xi \alpha \bar{k}^\alpha} \int_{a(0)}^1 \left\{ \frac{v}{\alpha} (v-a(0))^\alpha - \frac{v}{\alpha} (v-a(0))^\alpha + \frac{1}{\alpha} \left[\frac{(a-a(0))^{\alpha+1}}{\alpha+1} \right]_{a(0)}^v \right\} dv \\
&= \frac{\alpha}{\xi \alpha \bar{k}^\alpha} \int_{a(0)}^1 \left\{ \frac{v}{\alpha} (v-a(0))^\alpha - \frac{v}{\alpha} (v-a(0))^\alpha + \frac{(v-a(0))^{\alpha+1}}{\alpha(\alpha+1)} \right\} dv \\
&= \frac{\alpha}{\xi \alpha \bar{k}^\alpha \alpha (\alpha+1)} \left[\frac{(v-a(0))^{\alpha+2}}{\alpha+2} \right]_{a(0)}^1 \\
&= \frac{(1-a(0))^{\alpha+2}}{\xi \alpha \bar{k}^\alpha (\alpha+1)(\alpha+2)}
\end{aligned}$$

Using $a(k) = 1 - \frac{1-\lambda-k\lambda_\varepsilon}{2(1-\lambda+\lambda_\varepsilon^2)}$, $\xi = \frac{\lambda_\varepsilon}{2(1-\lambda+\lambda_\varepsilon^2)}$ and $\bar{k} = \frac{1-\lambda}{\lambda_\varepsilon}$ we then have:

$$\begin{aligned}
S_c &= \frac{\left(\frac{1-\lambda}{2(1-\lambda+\lambda_\varepsilon^2)} \right)^{\alpha+2}}{\left(\frac{\lambda_\varepsilon}{2(1-\lambda+\lambda_\varepsilon^2)} \right)^\alpha \left(\frac{1-\lambda}{\lambda_\varepsilon} \right)^\alpha (\alpha+1)(\alpha+2)} \\
&= \frac{\left(\frac{1-\lambda}{2(1-\lambda+\lambda_\varepsilon^2)} \right)^2}{(\alpha+1)(\alpha+2)}
\end{aligned}$$

which is decreasing in ε . Also notice that the smaller α is the faster S_c decreases in ε .

A.5 Producers' surplus

Similarly for producers' surplus, using integration by parts:

$$\begin{aligned}
S_p &= \int_0^{b(0)} \left[\int_v^{b(0)} (b-v) f_b(b) db \right] dv \\
&= \int_0^{b(0)} \left[\frac{\alpha}{\lambda_\varepsilon \xi \bar{k}^\alpha} \int_v^{b(0)} (b-v) \left(\frac{b(0)-b}{\lambda_\varepsilon \xi} \right)^{\alpha-1} db \right] dv \\
&= \frac{\alpha}{(\lambda_\varepsilon \xi \bar{k})^\alpha} \int_0^{b(0)} \left[\int_v^{b(0)} b(b(0)-b)^{\alpha-1} db - \int_v^{b(0)} v(b(0)-b)^{\alpha-1} db \right] dv \\
&= \frac{\alpha}{(\lambda_\varepsilon \xi \bar{k})^\alpha} \int_0^{b(0)} \left\{ \left[-b \frac{(b(0)-b)^\alpha}{\alpha} \right]_v^{b(0)} - \int_v^{b(0)} \frac{-(b(0)-b)^\alpha}{\alpha} db - v \left[-\frac{(b(0)-b)^\alpha}{\alpha} \right]_v^{b(0)} \right\} dv \\
&= \frac{\alpha}{(\lambda_\varepsilon \xi \bar{k})^\alpha} \int_0^{b(0)} \left\{ \left[v \frac{(b(0)-v)^\alpha}{\alpha} \right] + \left[-\frac{(b(0)-v)^{\alpha+1}}{\alpha(\alpha+1)} \right]_v^{b(0)} + v \left[-\frac{(b(0)-v)^\alpha}{\alpha} \right] \right\} dv \\
&= \frac{\alpha}{(\lambda_\varepsilon \xi \bar{k})^\alpha} \int_0^{b(0)} \left\{ \frac{(b(0)-v)^{\alpha+1}}{\alpha(\alpha+1)} \right\} dv \\
&= \frac{1}{(\alpha+1)(\lambda_\varepsilon \xi \bar{k})^\alpha} \left[-\frac{(b(0)-v)^{\alpha+2}}{\alpha+2} \right]_0^{b(0)} \\
&= \frac{b(0)^{\alpha+2}}{(\alpha+1)(\alpha+2)(\lambda_\varepsilon \xi \bar{k})^\alpha}
\end{aligned}$$

Using $b(k) = \lambda_\varepsilon \frac{1-\lambda-k\lambda_\varepsilon}{2(1-\lambda+\lambda_\varepsilon^2)}$ we then have:

$$\begin{aligned}
S_p &= \frac{\left(\lambda_\varepsilon \frac{1-\lambda}{2(1-\lambda+\lambda_\varepsilon^2)} \right)^{\alpha+2}}{(\alpha+1)(\alpha+2) \left(\lambda_\varepsilon \frac{\lambda_\varepsilon}{2(1-\lambda+\lambda_\varepsilon^2)} \frac{1-\lambda}{\lambda_\varepsilon} \right)^\alpha} \\
&= \frac{\left(\frac{\lambda_\varepsilon(1-\lambda)}{2(1-\lambda+\lambda_\varepsilon^2)} \right)^2}{(\alpha+1)(\alpha+2)} \\
&= \lambda_\varepsilon^2 S_c
\end{aligned}$$

which is increasing²⁰ in ε if and only if $\varepsilon \in [0, \bar{\varepsilon}]$ (where $\bar{\varepsilon} = -(1 - \lambda) + \sqrt{1 - \lambda}$ as defined above). Also notice that the smaller α is the faster S_p increases in ε .

A.6 Dealers' surplus

For dealers let us rewrite the expected demand and supply faced in their decision problem:

$$D(a) = \int_a^{\bar{r}_c} \tilde{h}(r) dr$$

where $\tilde{h}(r)$ is the conditional probability density of consumers' reservation prices among the fraction $1 - \underline{v}_c$ who chose to participate in the dealers' market. Therefore, $\tilde{h}(r)$ is derived as follows: the reservation price of a consumer with valuation v , denoted $r_c(v)$, is simply that specific consumer's valuation:

$$r_c(v) = v$$

Now, $v \sim U[\underline{v}_c, 1]$ therefore

$$\begin{aligned} \Pr(r_c(v) \leq r) &= \Pr(v \leq r) \\ &= \frac{r - \underline{v}_c}{1 - \underline{v}_c} \end{aligned}$$

and the probability density function associated with it is simply $h(r) = \frac{1}{1 - \underline{v}_c}$. Then the per dealer k density of consumers is $(1 - \underline{v}_c) f_k^-(k) h(r)$. So

²⁰Where

$$\begin{aligned} \frac{\partial S_p}{\partial \varepsilon} &= \frac{(1 - \lambda)^2}{4(\alpha + 1)(\alpha + 2)} \frac{\partial \left(\frac{\lambda_\varepsilon}{(1 - \lambda + \lambda_\varepsilon^2)} \right)^2}{\partial \varepsilon} \\ &= \frac{(1 - \lambda)^2}{4(\alpha + 1)(\alpha + 2)} \frac{2\lambda_\varepsilon}{(1 - \lambda + \lambda_\varepsilon^2)} \left(\frac{1 - \lambda + \lambda_\varepsilon^2 - 2\lambda_\varepsilon^2}{(1 - \lambda + \lambda_\varepsilon^2)^2} \right) \\ &= \frac{\lambda_\varepsilon (1 - \lambda)^2}{2(\alpha + 1)(\alpha + 2)} \frac{(1 - \lambda - \lambda_\varepsilon^2)}{(1 - \lambda + \lambda_\varepsilon^2)^3} \end{aligned}$$

which is always strictly positive if and only if ε is such that $1 - \lambda - \lambda_\varepsilon^2 > 0$.

that the mass of consumers who place an order when the ask price they face is a (i.e. demand faced by a dealer who posts ask price a if his type is k -because here the mass of consumers that contact him is a function of k) is simply

$$\begin{aligned} D(a(k)) &= \int_{a(k)}^{\bar{r}_c} (1 - \underline{v}_c) f_{\bar{k}}(k) h(r) dr \\ &= \int_{a(k)}^1 (1 - \underline{v}_c) f_{\bar{k}}(k) \frac{1}{1 - \underline{v}_c} dr \\ &= (1 - a(k)) f_{\bar{k}}(k) \end{aligned}$$

And similarly for the supply:

$$S(b(k)) = b(k) f_{\bar{k}}(k)$$

For dealers, we also need to take into account the constraint of meeting demand period by period, so that substituting the expected demand and supply per dealer k into the objective function of a dealer we have, as before, that expected profits of dealer k with the optimal choice of a , are

$$\begin{aligned} \pi(k; \lambda, \varepsilon) &= f_{\bar{k}}(k) \{a(k)(1 - \lambda) - [\lambda_\varepsilon(1 - a(k)) + k] \lambda_\varepsilon\} (1 - a(k)) \\ &= \alpha \frac{k^{\alpha-1}}{\bar{k}^\alpha} \{a(k)(1 - \lambda) - [\lambda_\varepsilon(1 - a(k)) + k] \lambda_\varepsilon\} (1 - a(k)) \\ &= \alpha \frac{k^{\alpha-1}}{\bar{k}^\alpha} \left[\left(1 - \frac{1 - \lambda - k\lambda_\varepsilon}{2(1 - \lambda + \lambda_\varepsilon^2)}\right) (1 - \lambda) - \lambda_\varepsilon^2 \left(\frac{1 - \lambda - k\lambda_\varepsilon}{2(1 - \lambda + \lambda_\varepsilon^2)}\right) - \lambda_\varepsilon k \right] \left(\frac{1 - \lambda - k\lambda_\varepsilon}{2(1 - \lambda + \lambda_\varepsilon^2)}\right) \\ &= \alpha \frac{k^{\alpha-1}}{\bar{k}^\alpha} \left[\left(\frac{1 - \lambda + 2\lambda_\varepsilon^2 + k\lambda_\varepsilon}{2(1 - \lambda + \lambda_\varepsilon^2)}\right) (1 - \lambda) - \lambda_\varepsilon^2 \left(\frac{1 - \lambda - k\lambda_\varepsilon}{2(1 - \lambda + \lambda_\varepsilon^2)}\right) - \lambda_\varepsilon k \right] \left(\frac{1 - \lambda - k\lambda_\varepsilon}{2(1 - \lambda + \lambda_\varepsilon^2)}\right) \\ &= \alpha \frac{k^{\alpha-1}}{\bar{k}^\alpha} \left[\left(\frac{1 - \lambda + 2\lambda_\varepsilon^2 - \lambda_\varepsilon^2}{2(1 - \lambda + \lambda_\varepsilon^2)}\right) (1 - \lambda) + k\lambda_\varepsilon \left(\frac{(1 - \lambda + \lambda_\varepsilon^2)}{2(1 - \lambda + \lambda_\varepsilon^2)} - 1\right) \right] \left(\frac{1 - \lambda - k\lambda_\varepsilon}{2(1 - \lambda + \lambda_\varepsilon^2)}\right) \\ &= \alpha \frac{k^{\alpha-1}}{\bar{k}^\alpha} \left[\frac{(1 - \lambda)(1 - \lambda + \lambda_\varepsilon^2)}{2(1 - \lambda + \lambda_\varepsilon^2)} - k\lambda_\varepsilon \frac{1 - \lambda + \lambda_\varepsilon^2}{2(1 - \lambda + \lambda_\varepsilon^2)} \right] \left(\frac{1 - \lambda - k\lambda_\varepsilon}{2(1 - \lambda + \lambda_\varepsilon^2)}\right) \\ &= \alpha \frac{k^{\alpha-1}}{\bar{k}^\alpha} \left[\frac{1 - \lambda - k\lambda_\varepsilon}{2} \right] \left(\frac{1 - \lambda - k\lambda_\varepsilon}{2(1 - \lambda + \lambda_\varepsilon^2)}\right) \\ &= \alpha \frac{k^{\alpha-1}}{\bar{k}^\alpha} \frac{(1 - \lambda - k\lambda_\varepsilon)^2}{4(1 - \lambda + \lambda_\varepsilon^2)} \end{aligned}$$

Then aggregate dealers' surplus is given by the total discounted profits of all dealers participating in the dealer market are:

$$\begin{aligned}
S_d(\varepsilon) &= \int_0^{\bar{k}} \Pi(k; \lambda, \varepsilon) dk \\
&= \frac{\alpha}{4\bar{k}^\alpha (1 - \lambda + \lambda_\varepsilon^2)} \int_0^{\bar{k}} (1 - \lambda - \lambda_\varepsilon k)^2 k^{\alpha-1} dk \\
&= \frac{\alpha}{4\bar{k}^\alpha (1 - \lambda + \lambda_\varepsilon^2)} \int_0^{\bar{k}} (1 - \lambda - \lambda_\varepsilon k)^2 k^{\alpha-1} dk \\
&= \frac{\alpha}{4\bar{k}^\alpha (1 - \lambda + \lambda_\varepsilon^2)} \left\{ \left[(1 - \lambda - \lambda_\varepsilon k)^2 \frac{k^\alpha}{\alpha} \right]_0^{\bar{k}} - \int_0^{\bar{k}} -2\lambda_\varepsilon (1 - \lambda - \lambda_\varepsilon k) \frac{k^\alpha}{\alpha} dk \right\} \\
&= \frac{\alpha}{4\bar{k}^\alpha (1 - \lambda + \lambda_\varepsilon^2)} \left\{ (1 - \lambda - \lambda_\varepsilon \bar{k})^2 \frac{\bar{k}^\alpha}{\alpha} - \frac{2}{\alpha} \int_0^{\bar{k}} -\lambda_\varepsilon (1 - \lambda - \lambda_\varepsilon k) k^\alpha dk \right\} \\
&= \frac{\alpha}{4\bar{k}^\alpha (1 - \lambda + \lambda_\varepsilon^2)} \left\{ (1 - \lambda - \lambda_\varepsilon \bar{k})^2 \frac{\bar{k}^\alpha}{\alpha} + \frac{2\lambda_\varepsilon}{\alpha} \left\{ \left[(1 - \lambda - \lambda_\varepsilon k) \frac{k^{\alpha+1}}{\alpha+1} \right]_0^{\bar{k}} - \int_0^{\bar{k}} (-\lambda_\varepsilon) \frac{k^{\alpha+1}}{\alpha+1} dk \right\} \right\} \\
&= \frac{\alpha}{4\bar{k}^\alpha (1 - \lambda + \lambda_\varepsilon^2)} \left\{ \frac{(1 - \lambda - \lambda_\varepsilon \bar{k})^2 \bar{k}^\alpha}{\alpha} + \frac{2\lambda_\varepsilon (1 - \lambda - \lambda_\varepsilon \bar{k}) \bar{k}^{\alpha+1}}{\alpha(\alpha+1)} + \frac{2\lambda_\varepsilon^2}{\alpha(\alpha+1)} \int_0^{\bar{k}} k^{\alpha+1} dk \right\} \\
&= \frac{\alpha}{4\bar{k}^\alpha (1 - \lambda + \lambda_\varepsilon^2)} \left\{ \frac{(1 - \lambda - \lambda_\varepsilon \bar{k}) \bar{k}^\alpha}{\alpha} \left((1 - \lambda - \lambda_\varepsilon \bar{k}) + \frac{2\bar{k}\lambda_\varepsilon}{(\alpha+1)} \right) + \frac{2\lambda_\varepsilon^2}{\alpha(\alpha+1)} \left[\frac{k^{\alpha+2}}{\alpha+2} \right]_0^{\bar{k}} \right\} \\
&= \frac{\alpha}{4\bar{k}^\alpha (1 - \lambda + \lambda_\varepsilon^2)} \left\{ \frac{(1 - \lambda - \lambda_\varepsilon \bar{k}) \bar{k}^\alpha}{\alpha(\alpha+1)} ((\alpha+1)(1 - \lambda) + [2\lambda_\varepsilon - (\alpha+1)\lambda_\varepsilon] \bar{k}) + \frac{2\lambda_\varepsilon^2 \bar{k}^{\alpha+2}}{\alpha(\alpha+1)(\alpha+2)} \right\} \\
&= \frac{1}{4(\alpha+1)(1 - \lambda + \lambda_\varepsilon^2)} \left\{ (1 - \lambda - \lambda_\varepsilon \bar{k}) [(\alpha+1)(1 - \lambda) + [2\lambda_\varepsilon - (\alpha+1)\lambda_\varepsilon] \bar{k}] + \frac{2\lambda_\varepsilon^2 \bar{k}^2}{(\alpha+2)} \right\}
\end{aligned}$$

And using $\bar{k} = \frac{1-\lambda}{\lambda_\varepsilon}$ we then have:

$$\begin{aligned}
S_d &= \frac{1}{4(\alpha+1)(1 - \lambda + \lambda_\varepsilon^2)} \left\{ (1 - \lambda - 1 + \lambda) [(\alpha+1)(1 - \lambda) + [2\lambda_\varepsilon - (\alpha+1)\lambda_\varepsilon] \bar{k}] + \frac{2\lambda_\varepsilon^2 \left(\frac{1-\lambda}{\lambda_\varepsilon}\right)^2}{(\alpha+2)} \right\} \\
&= \frac{(1 - \lambda)^2}{2(\alpha+1)(\alpha+2)(1 - \lambda + \lambda_\varepsilon^2)}
\end{aligned}$$

A.7 Total welfare

Summing up consumers', producers' and dealers' welfare we have:

$$\begin{aligned}
W &= S_c + S_p + S_d \\
&= \frac{\left(\frac{1-\lambda}{2(1-\lambda+\lambda_\varepsilon^2)}\right)^2}{(\alpha+1)(\alpha+2)} + \frac{\left(\frac{\lambda_\varepsilon(1-\lambda)}{2(1-\lambda+\lambda_\varepsilon^2)}\right)^2}{(\alpha+1)(\alpha+2)} + \frac{(1-\lambda)^2}{2(\alpha+1)(\alpha+2)(1-\lambda+\lambda_\varepsilon^2)} \\
&= \frac{(1+\lambda_\varepsilon^2)(1-\lambda)^2}{4(\alpha+1)(\alpha+2)(1-\lambda+\lambda_\varepsilon^2)^2} + \frac{(1-\lambda)^2}{2(\alpha+1)(\alpha+2)(1-\lambda+\lambda_\varepsilon^2)} \\
&= \frac{(1-\lambda)^2}{2(\alpha+1)(\alpha+2)(1-\lambda+\lambda_\varepsilon^2)} \left(\frac{(1+\lambda_\varepsilon^2)}{2(1-\lambda+\lambda_\varepsilon^2)} + 1 \right) \\
&= \frac{(1-\lambda)^2(3+3\lambda_\varepsilon^2-2\lambda)}{4(\alpha+1)(\alpha+2)(1-\lambda+\lambda_\varepsilon^2)^2}
\end{aligned}$$

And:

$$\begin{aligned}
\frac{\partial W}{\partial \varepsilon} &= \frac{(1-\lambda)^2}{4(\alpha+1)(\alpha+2)} \left[\frac{6\lambda_\varepsilon(1-\lambda+\lambda_\varepsilon^2)^2 - (3+3\lambda_\varepsilon^2-2\lambda)(1-\lambda+\lambda_\varepsilon^2)4\lambda_\varepsilon}{(1-\lambda+\lambda_\varepsilon^2)^4} \right] \\
&= \frac{(1-\lambda)^2\lambda_\varepsilon}{2(\alpha+1)(\alpha+2)} \left[\frac{3(1-\lambda+\lambda_\varepsilon^2) - 2(3+3\lambda_\varepsilon^2-2\lambda)}{(1-\lambda+\lambda_\varepsilon^2)^3} \right] \\
&= \frac{(1-\lambda)^2\lambda_\varepsilon}{2(\alpha+1)(\alpha+2)} \frac{(\lambda-3(1+\lambda_\varepsilon^2))}{(1-\lambda+\lambda_\varepsilon^2)^3}
\end{aligned}$$

which is always negative since

$$\lambda - 3(1+\lambda_\varepsilon^2) < 0$$

A.8 Different parameters for beta distribution

In this section of the paper we assume that dealers are distributed according to a beta probability distribution $f(k; \alpha, \beta) = \frac{\beta k^{\alpha-1}(1-k)^{\beta-1}}{B(\alpha, \beta)}$ with support

$[0, 1]$. Let $\alpha = 1$ so that

$$\begin{aligned}
\mathcal{B}(\alpha, \beta) &= \int_0^1 \beta k^{\alpha-1} (1-k)^{\beta-1} ds \\
&= \int_0^1 \beta (1-s)^{\beta-1} ds \\
&= \beta \left[-\frac{(1-s)^\beta}{\beta} \right]_0^1 \\
&= 1
\end{aligned}$$

Then $f(k; \alpha, \beta) = \beta (1-k)^{\beta-1}$ and the cdf associated with it is

$$\begin{aligned}
F(k) &= \frac{\mathcal{B}(k; \alpha, \beta)}{\mathcal{B}(\alpha, \beta)} \\
&= \frac{\int_0^k \beta (1-s)^{\beta-1} ds}{\int_0^1 \beta (1-s)^{\beta-1} ds} \\
&= \frac{\beta \left[-\frac{1}{\beta} (1-s)^\beta \right]_0^k}{\beta \left[-\frac{1}{\beta} (1-s)^\beta \right]_0^1} \\
&= \frac{\beta \left[-\frac{1}{\beta} (1-k)^\beta + \frac{1}{\beta} \right]}{\beta \frac{1}{\beta}} \\
&= 1 - (1-k)^\beta
\end{aligned}$$

Now, because only $\bar{k} = \frac{1-\lambda}{\lambda_\varepsilon} < 1$ are active, then²¹

$$\begin{aligned}
F_{\bar{k}}(k) &= \frac{\int_0^k \beta (1-s)^{\beta-1} ds}{\int_0^{\bar{k}} \beta (1-s)^{\beta-1} ds} \\
&= \frac{1 - (1-k)^\beta}{1 - (1-\bar{k})^\beta}
\end{aligned}$$

and the probability distribution function is then simply $f_{\bar{k}}(k) = \beta \frac{(1-k)^{\beta-1}}{1-(1-\bar{k})^\beta}$.

²¹Or, similarly, from $F(k) = k^\alpha$ we have that the truncated distribution $F_{\bar{k}}(k) = \Pr(s \leq k \mid s \leq \bar{k}) = \frac{\Pr(s \leq k \cap s \leq \bar{k})}{\Pr(s \leq \bar{k})} = \frac{F(k)}{F(\bar{k})}$.

Notice that ask prices are an affine transformation of the dealer's cost of the form $a(k) = a(0) + \xi k$ where $a(\bar{k}) = 1$ and $\xi = \frac{\lambda_\varepsilon}{2(1-\lambda+\lambda_\varepsilon^2)}$, then the cdf of $a(k)$ is derived from $F_{\bar{k}}(k)$:

$$\begin{aligned}
F_a(\hat{a}) &= \Pr(a \leq \hat{a}) \\
&= \Pr(a(0) + \xi k \leq \hat{a}) \\
&= F_{\bar{k}}\left(k \leq \frac{\hat{a} - a(0)}{\xi}\right) \\
&= \frac{1 - \left(1 - \frac{\hat{a} - a(0)}{\xi}\right)^\beta}{1 - (1 - \bar{k})^\beta} \\
f_a(a) &= \frac{-\beta \left(1 - \frac{a - a(0)}{\xi}\right)^{\beta-1} \left(-\frac{1}{\xi}\right)}{1 - (1 - \bar{k})^\beta} \\
&= \frac{\beta \left(1 - \frac{a - a(0)}{\xi}\right)^{\beta-1}}{\xi [1 - (1 - \bar{k})^\beta]} \\
&= \frac{1}{\xi} f_{\bar{k}}\left(\frac{a - a(0)}{\xi}\right)
\end{aligned}$$

Similarly for the bid price

$$\begin{aligned}
b(k) &= \lambda_\varepsilon (1 - a(k)) \\
&= \lambda_\varepsilon (1 - a(0) - \xi k) \\
&= b(0) - \lambda_\varepsilon \xi k
\end{aligned}$$

And

$$\begin{aligned}
F_b(\hat{b}) &= \Pr\left(b(0) - \lambda_\varepsilon \xi k \leq \hat{b}\right) \\
&= \Pr\left(k \geq \frac{b(0) - \hat{b}}{\lambda_\varepsilon \xi}\right) \\
&= 1 - F_{\bar{k}}\left(\frac{b(0) - \hat{b}}{\lambda_\varepsilon \xi}\right) \\
&= 1 - \frac{1 - \left(1 - \frac{b(0) - \hat{b}}{\lambda_\varepsilon \xi}\right)^\beta}{1 - (1 - \bar{k})^\beta} \\
&= \frac{\left(1 - \frac{b(0) - \hat{b}}{\lambda_\varepsilon \xi}\right)^\beta - (1 - \bar{k})^\beta}{1 - (1 - \bar{k})^\beta} \\
f_b(b) &= \beta \frac{\left(1 - \frac{b(0) - b}{\lambda_\varepsilon \xi}\right)^{\beta-1} \frac{1}{\lambda_\varepsilon \xi}}{1 - (1 - \bar{k})^\beta} \\
&= \frac{1}{\lambda_\varepsilon \xi} \beta \frac{\left(1 - \frac{b(0) - b}{\lambda_\varepsilon \xi}\right)^{\beta-1}}{1 - (1 - \bar{k})^\beta} \\
&= \frac{1}{\lambda_\varepsilon \xi} f_{\bar{k}}\left(\frac{b(0) - b}{\lambda_\varepsilon \xi}\right)
\end{aligned}$$

A.8.1 Consumers' surplus

Then consumers' surplus (with linear preferences), using integration by parts, is:

$$\begin{aligned}
S_c &= \int_{a(0)}^1 \left[\int_{a(0)}^v (v-a) f_a(a) da \right] dv \\
&= \int_{a(0)}^1 \left[\int_{a(0)}^v (v-a) \frac{\beta \left(1 - \frac{a-a(0)}{\xi}\right)^{\beta-1}}{\xi [1 - (1-\bar{k})^\beta]} da \right] dv \\
&= \frac{\beta}{\xi [1 - (1-\bar{k})^\beta]} \int_{a(0)}^1 \left[\int_{a(0)}^v v \left(1 - \frac{a-a(0)}{\xi}\right)^{\beta-1} da - \int_{a(0)}^v a \left(1 - \frac{a-a(0)}{\xi}\right)^{\beta-1} da \right] dv \\
&= \frac{\beta}{\xi [1 - (1-\bar{k})^\beta]} \int_{a(0)}^1 v \left[\frac{-\xi}{\beta} \left(1 - \frac{a-a(0)}{\xi}\right)^\beta \right]_{a(0)}^v dv - \\
&\quad - \frac{\beta}{\xi [1 - (1-\bar{k})^\beta]} \int_{a(0)}^1 \left\{ \left[a \frac{-\xi}{\beta} \left(1 - \frac{a-a(0)}{\xi}\right)^\beta \right]_{a(0)}^v - \int_{a(0)}^v \frac{-\xi}{\beta} \left(1 - \frac{a-a(0)}{\xi}\right)^\beta da \right\} dv \\
&= -\frac{1}{[1 - (1-\bar{k})^\beta]} \left\{ \int_{a(0)}^1 v \left(1 - \frac{v-a(0)}{\xi}\right)^\beta dv - \frac{(1-a(0)^2)}{2} \right\} + \\
&\quad + \frac{1}{[1 - (1-\bar{k})^\beta]} \int_{a(0)}^1 \left\{ v \left(1 - \frac{v-a(0)}{\xi}\right)^\beta + \frac{\xi}{\beta+1} \left(1 - \frac{v-a(0)}{\xi}\right)^{\beta+1} \right\} dv - \frac{1-a(0)}{[1 - (1-\bar{k})^\beta]} \left(a(0) + \frac{\xi}{\beta+1}\right) \\
&= \frac{(1-a(0)^2)}{2[1 - (1-\bar{k})^\beta]} + \frac{1}{[1 - (1-\bar{k})^\beta]} \int_{a(0)}^1 \left\{ \frac{\xi}{\beta+1} \left(1 - \frac{v-a(0)}{\xi}\right)^{\beta+1} \right\} dv - \frac{1-a(0)}{[1 - (1-\bar{k})^\beta]} \left(a(0) + \frac{\xi}{\beta+1}\right) \\
&= \frac{1-a(0)}{[1 - (1-\bar{k})^\beta]} \left(\frac{1+a(0)}{2} - a(0) - \frac{\xi}{\beta+1} \right) + \frac{\xi}{[1 - (1-\bar{k})^\beta]} \int_{a(0)}^1 \left(1 - \frac{v-a(0)}{\xi}\right)^{\beta+1} dv \\
&= \frac{1-a(0)}{[1 - (1-\bar{k})^\beta]} \left(\frac{1-a(0)}{2} - \frac{\xi}{\beta+1} \right) + \frac{\xi}{[1 - (1-\bar{k})^\beta]} \left[\frac{-\xi}{\beta+2} \left(1 - \frac{v-a(0)}{\xi}\right)^{\beta+2} \right]_{a(0)}^1 \\
&= \frac{1-a(0)}{[1 - (1-\bar{k})^\beta]} \left(\frac{1-a(0)}{2} - \frac{\xi}{\beta+1} \right) - \frac{\frac{\xi}{\beta+1} \frac{\xi}{\beta+2}}{[1 - (1-\bar{k})^\beta]} \left[\left(1 - \frac{1-a(0)}{\xi}\right)^{\beta+2} - 1 \right]
\end{aligned}$$

Let ε^* denote the threshold such that $\forall \varepsilon \leq \varepsilon^*$ we have that $\frac{\partial S_c}{\partial \varepsilon} > 0$ and $\forall \varepsilon > \varepsilon^*$ we have that $\frac{\partial S_c}{\partial \varepsilon} < 0$. Then as $\beta > 0$ decreases we have that ε^*

increases.

And substituting out $a(k) = 1 - \frac{1-\lambda-k\lambda_\varepsilon}{2(1-\lambda+\lambda_\varepsilon^2)}$, $\xi = \frac{\lambda_\varepsilon}{2(1-\lambda+\lambda_\varepsilon^2)}$ and $\bar{k} = \frac{1-\lambda}{\lambda_\varepsilon}$ we then have that:

$$\begin{aligned}
S_c &= \frac{(\beta+2)(1-\lambda)[(\beta+1)(1-\lambda) - 2\lambda_\varepsilon] - 2\lambda_\varepsilon^2 \left[\left(1 - \frac{1-\lambda}{\lambda_\varepsilon}\right)^{\beta+2} - 1 \right]}{8(\beta+1)(1-\lambda+\lambda_\varepsilon^2)^2 \left[1 - \left(1 - \frac{1-\lambda}{\lambda_\varepsilon}\right)^\beta \right] (\beta+2)} \\
&= \frac{(\beta+2)(1-\lambda)(\beta+1)(1-\lambda) - 2\lambda_\varepsilon^2 \left(1 - \frac{1-\lambda}{\lambda_\varepsilon}\right)^{\beta+2} + 2\lambda_\varepsilon(\lambda_\varepsilon - (\beta+2)(1-\lambda))}{8(\beta+1)(1-\lambda+\lambda_\varepsilon^2)^2 \left[1 - \left(1 - \frac{1-\lambda}{\lambda_\varepsilon}\right)^\beta \right] (\beta+2)}
\end{aligned}$$

A.8.2 Producers' surplus

Similarly for producers' surplus, using integration by parts:

$$\begin{aligned}
S_p &= \int_0^{b(0)} \left[\int_v^{b(0)} (b-v) f_b(b) db \right] dv \\
&= \int_0^{b(0)} \left[\int_v^{b(0)} (b-v) \frac{1}{\lambda_\varepsilon \xi} \beta \frac{\left(1 - \frac{b(0)-b}{\lambda_\varepsilon \xi}\right)^{\beta-1}}{1 - \left(1 - \bar{k}\right)^\beta} db \right] dv \\
&= \frac{\beta}{\lambda_\varepsilon \xi \left[1 - \left(1 - \bar{k}\right)^\beta \right]} \int_0^{b(0)} \left[\int_v^{b(0)} b \left(1 - \frac{b(0)-b}{\lambda_\varepsilon \xi}\right)^{\beta-1} db - v \int_v^{b(0)} \left(1 - \frac{b(0)-b}{\lambda_\varepsilon \xi}\right)^{\beta-1} db \right] dv \\
&= \frac{1}{\left[1 - \left(1 - \bar{k}\right)^\beta \right]} \int_0^{b(0)} \left\{ b(0) - \frac{\lambda_\varepsilon \xi}{\beta+1} \left[1 - \left(1 - \frac{b(0)-v}{\lambda_\varepsilon \xi}\right)^{\beta+1} \right] - v \right\} dv \\
&= \frac{1}{\left[1 - \left(1 - \bar{k}\right)^\beta \right]} \left\{ b(0) \left(b(0) - \frac{\lambda_\varepsilon \xi}{\beta+1} \right) + \int_0^{b(0)} \left(\frac{\lambda_\varepsilon \xi}{\beta+1} \left(1 - \frac{b(0)-v}{\lambda_\varepsilon \xi}\right)^{\beta+1} \right) dv - \frac{b(0)^2}{2} \right\} \\
&= \frac{1}{\left[1 - \left(1 - \bar{k}\right)^\beta \right]} \left\{ b(0) \left(b(0) - \frac{\lambda_\varepsilon \xi}{\beta+1} \right) - \frac{b(0)^2}{2} + \frac{\lambda_\varepsilon \xi}{\beta+1} \left[\frac{\lambda_\varepsilon \xi}{\beta+2} \left(1 - \frac{b(0)-v}{\lambda_\varepsilon \xi}\right)^{\beta+2} \right]_0^{b(0)} \right\} \\
&= \frac{1}{\left[1 - \left(1 - \bar{k}\right)^\beta \right]} \left\{ \frac{b(0)^2}{2} - \frac{\lambda_\varepsilon \xi}{\beta+1} b(0) + \frac{\lambda_\varepsilon \xi}{\beta+1} \frac{\lambda_\varepsilon \xi}{\beta+2} \left[1 - \left(1 - \frac{b(0)}{\lambda_\varepsilon \xi}\right)^{\beta+2} \right] \right\}
\end{aligned}$$

Using $b(k) = \lambda_\varepsilon \frac{1-\lambda-k\lambda_\varepsilon}{2(1-\lambda+\lambda_\varepsilon^2)}$, $\xi = \frac{\lambda_\varepsilon}{2(1-\lambda+\lambda_\varepsilon^2)}$ and $\bar{k} = \frac{1-\lambda}{\lambda_\varepsilon}$, we then have:

$$\begin{aligned}
S_p &= \frac{1}{\left[1 - \left(1 - \frac{1-\lambda}{\lambda_\varepsilon}\right)^\beta\right]} \left\{ \frac{\left(\lambda_\varepsilon \frac{1-\lambda}{2(1-\lambda+\lambda_\varepsilon^2)}\right)^2}{2} + \frac{\lambda_\varepsilon \frac{\lambda_\varepsilon}{2(1-\lambda+\lambda_\varepsilon^2)}}{\beta+1} \left[\frac{\lambda_\varepsilon \frac{\lambda_\varepsilon}{2(1-\lambda+\lambda_\varepsilon^2)}}{\beta+2} \left(1 - \left(1 - \frac{\lambda_\varepsilon \frac{1-\lambda}{2(1-\lambda+\lambda_\varepsilon^2)}}{\lambda_\varepsilon \frac{\lambda_\varepsilon}{2(1-\lambda+\lambda_\varepsilon^2)}}\right)^\beta\right) \right] \right\} \\
&= \frac{1}{\left[1 - \left(1 - \frac{1-\lambda}{\lambda_\varepsilon}\right)^\beta\right]} \left\{ \frac{\lambda_\varepsilon^2 (1-\lambda)^2}{8(1-\lambda+\lambda_\varepsilon^2)^2} + \frac{\lambda_\varepsilon^3}{4(\beta+1)(1-\lambda+\lambda_\varepsilon^2)^2} \left[\frac{\lambda_\varepsilon}{(\beta+2)} \left(1 - \left(1 - \frac{1-\lambda}{\lambda_\varepsilon}\right)^\beta\right) \right] \right\} \\
&= \frac{\lambda_\varepsilon^2}{\left[1 - \left(1 - \frac{1-\lambda}{\lambda_\varepsilon}\right)^\beta\right]} \frac{(\beta+1)(1-\lambda)^2 + \frac{2\lambda_\varepsilon^2}{(\beta+2)} \left(1 - \left(1 - \frac{1-\lambda}{\lambda_\varepsilon}\right)^\beta\right) - 2\lambda_\varepsilon(1-\lambda)}{4(\beta+1)(1-\lambda+\lambda_\varepsilon^2)^2}
\end{aligned}$$

A.8.3 Dealers' surplus

If we take into account that expected demand and supply are $D(a) = (1 - a(k)) f_{\bar{k}}(k)$ and $S(b(k)) = b(k) f_{\bar{k}}(k)$ then expected profits are $\pi(k; \lambda, \varepsilon) = \beta \frac{(1-k)^{\beta-1}}{1-(1-\bar{k})^\beta} \frac{(1-\lambda-k\lambda_\varepsilon)^2}{4(1-\lambda+\lambda_\varepsilon^2)}$. Either way we know that the calculation of aggregate dealers' surplus is the same regardless of which interpretation we give (match-

ing or probability). Therefore aggregate dealers' surplus is:

$$\begin{aligned}
S_d(\varepsilon) &= \int_0^{\bar{k}} \Pi(k; \lambda, \varepsilon) dk \\
&= \int_0^{\bar{k}} \left(\beta \frac{(1-k)^{\beta-1}}{1-(1-\bar{k})^\beta} \frac{(1-\lambda-k\lambda_\varepsilon)^2}{4(1-\lambda+\lambda_\varepsilon^2)} \right) dk \\
&= \frac{\beta}{\left(1-(1-\bar{k})^\beta\right) 4(1-\lambda+\lambda_\varepsilon^2)} \int_0^{\bar{k}} (1-k)^{\beta-1} [(1-\lambda)^2 + k^2\lambda_\varepsilon^2 - 2(1-\lambda)k\lambda_\varepsilon] dk \\
&= \frac{\beta(1-\lambda)^2}{\left(1-(1-\bar{k})^\beta\right) 4(1-\lambda+\lambda_\varepsilon^2)} \left[\frac{-1}{\beta} (1-k)^\beta \right]_0^{\bar{k}} + \\
&\quad + \frac{\beta\lambda_\varepsilon^2}{\left(1-(1-\bar{k})^\beta\right) 4(1-\lambda+\lambda_\varepsilon^2)} \left\{ \left[-\frac{1}{\beta} (1-k)^\beta k^2 \right]_0^{\bar{k}} + \frac{2}{\beta} \int_0^{\bar{k}} ((1-k)^\beta k) dk \right\} - \\
&\quad - \frac{\beta 2(1-\lambda)\lambda_\varepsilon}{\left(1-(1-\bar{k})^\beta\right) 4(1-\lambda+\lambda_\varepsilon^2)} \left\{ \left[\frac{-1}{\beta} (1-k)^\beta k \right]_0^{\bar{k}} + \frac{1}{\beta} \int_0^{\bar{k}} (1-k)^\beta dk \right\}
\end{aligned}$$

and using $\bar{k} = \frac{1-\lambda}{\lambda_\varepsilon}$, we then have:

$$\begin{aligned}
S_d &= \frac{(1-\lambda)}{\left(1-(1-\bar{k})^\beta\right) 4(1-\lambda+\lambda_\varepsilon^2)} \left\{ (1-\lambda) - \frac{2\lambda_\varepsilon}{\beta+1} + \left((1-\lambda) + \frac{2\lambda_\varepsilon}{\beta+1} \left(\frac{\varepsilon}{\lambda_\varepsilon} \right) \right) \left(\frac{\varepsilon}{\lambda_\varepsilon} \right)^\beta \right\} + \\
&\quad + \frac{\lambda_\varepsilon^2}{\left(1-(1-\bar{k})^\beta\right) 4(1-\lambda+\lambda_\varepsilon^2)} \left\{ \frac{2}{(\beta+1)(\beta+2)} - \left(\frac{\varepsilon}{\lambda_\varepsilon} \right)^\beta \left[\left(\frac{1-\lambda}{\lambda_\varepsilon} \right)^2 + \frac{2\left(\left(\frac{\varepsilon}{\lambda_\varepsilon} \right)^2 + (\beta+2) \left(\frac{\varepsilon}{\lambda_\varepsilon} \right) \frac{1-\lambda}{\lambda_\varepsilon} \right)}{(\beta+1)(\beta+2)} \right] \right\} \\
&= \frac{(1-\lambda)}{\left(1-(1-\bar{k})^\beta\right) 4(1-\lambda+\lambda_\varepsilon^2)} \left\{ (1-\lambda) - \frac{2\lambda_\varepsilon}{\beta+1} + \left((1-\lambda) + \frac{2\varepsilon}{\beta+1} \right) \left(\frac{\varepsilon}{\lambda_\varepsilon} \right)^\beta \right\} + \\
&\quad + \frac{\lambda_\varepsilon^2}{\left(1-(1-\bar{k})^\beta\right) 4(1-\lambda+\lambda_\varepsilon^2)} \left\{ \frac{2}{(\beta+1)(\beta+2)} - \left(\frac{\varepsilon}{\lambda_\varepsilon} \right)^\beta \left(\frac{1-\lambda}{\lambda_\varepsilon} \right)^2 - \left(\frac{\varepsilon}{\lambda_\varepsilon} \right)^\beta \frac{2\varepsilon(\varepsilon+(\beta+2)(1-\lambda))}{\lambda_\varepsilon^2(\beta+1)(\beta+2)} \right\}
\end{aligned}$$

Some comments on this expression in footnote²².

²²Notice that

$$\begin{aligned}
\frac{\partial \left(\frac{\varepsilon}{\lambda_\varepsilon} \right)}{\partial \varepsilon} &= \frac{\lambda_\varepsilon - \varepsilon}{\lambda_\varepsilon^2} \\
&= \frac{1-\lambda}{\lambda_\varepsilon^2} > 0
\end{aligned}$$

so that for $\beta < 1$ then $\left(\frac{\varepsilon}{\lambda_\varepsilon}\right)^\beta$ is decreasing in ε .

Also notice that

$$\begin{aligned}\frac{\partial}{\partial \varepsilon} \left(\frac{\lambda_\varepsilon^2}{4(1-\lambda+\lambda_\varepsilon^2)} \right) &= \frac{4(1-\lambda+\lambda_\varepsilon^2)2\lambda_\varepsilon - 8\lambda_\varepsilon^3}{[4(1-\lambda+\lambda_\varepsilon^2)]^2} \\ &= \frac{8\lambda_\varepsilon(1-\lambda)}{[4(1-\lambda+\lambda_\varepsilon^2)]^2} > 0\end{aligned}$$

Therefore for $\beta < 1$ and taking on smaller and smaller values, the term $\frac{\lambda_\varepsilon^2}{(1-(1-\bar{k})^\beta)^{4(1-\lambda+\lambda_\varepsilon^2)}}$ is larger and larger when ε increases.

And for the term $\frac{2\varepsilon(\varepsilon+(\beta+2)(1-\lambda))}{\lambda_\varepsilon^2(\beta+1)(\beta+2)}$ we have that

$$\begin{aligned}\frac{\partial}{\partial \varepsilon} \left(\frac{\varepsilon(\varepsilon+(\beta+2)(1-\lambda))}{\lambda_\varepsilon^2} \right) &= \frac{\lambda_\varepsilon^2[(\varepsilon+(\beta+2)(1-\lambda))+\varepsilon] - 2\lambda_\varepsilon\varepsilon(\varepsilon+(\beta+2)(1-\lambda))}{\lambda_\varepsilon^4} \\ &= \frac{\lambda_\varepsilon^2[2\varepsilon+(\beta+2)(1-\lambda)] - 2\lambda_\varepsilon\varepsilon^2 - 2\lambda_\varepsilon\varepsilon(\beta+2)(1-\lambda)}{\lambda_\varepsilon^4} \\ &= \frac{\lambda_\varepsilon^2 + \lambda_\varepsilon^2[\varepsilon+(\beta+2)(1-\lambda)] - 2\lambda_\varepsilon\varepsilon[\varepsilon+(\beta+2)(1-\lambda)]}{\lambda_\varepsilon^4} \\ &= \frac{\lambda_\varepsilon^2 + (\lambda_\varepsilon^2 - 2\lambda_\varepsilon\varepsilon)[\varepsilon+(\beta+2)(1-\lambda)]}{\lambda_\varepsilon^4} \\ &= \frac{\lambda_\varepsilon + (1-\lambda-\varepsilon)[\varepsilon+(\beta+2)(1-\lambda)]}{\lambda_\varepsilon^3}\end{aligned}$$

which is positive if and only if

$$\begin{aligned}\lambda_\varepsilon + (1-\lambda-\varepsilon)[\varepsilon+(\beta+2)(1-\lambda)] &> 0 \\ (1-\lambda) + (2-\lambda)\varepsilon + (\beta+2)(1-\lambda)^2 &> \varepsilon[\varepsilon+(\beta+2)(1-\lambda)] \\ (1-\lambda) + (\beta+2)(1-\lambda)^2 &> \varepsilon[\varepsilon+(\beta+2)(1-\lambda) - (2-\lambda)] \\ (1-\lambda)[1+(\beta+2)(1-\lambda)] &> \varepsilon^2 + \varepsilon[\beta(1-\lambda) - \lambda]\end{aligned}$$

so that

$$\begin{aligned}-\varepsilon^2 - \varepsilon[\beta(1-\lambda) - \lambda] + (1-\lambda)[1+(\beta+2)(1-\lambda)] &> 0 \\ -\frac{1}{2} \left[[\beta(1-\lambda) - \lambda] \pm \sqrt{(\beta(1-\lambda) - \lambda)^2 + 4(1-\lambda)[1+(\beta+2)(1-\lambda)]} \right] &= \varepsilon_{1,2} \\ -\frac{1}{2} \left[[\beta(1-\lambda) - \lambda] \pm \sqrt{(\beta^2 + 4(\beta+2))(1-\lambda)^2 + 2(1-\lambda)(2-\beta\lambda) + \lambda^2} \right] &= \varepsilon_{1,2}\end{aligned}$$

So if $\varepsilon_2 = -\frac{1}{2} \left[[\beta(1-\lambda) - \lambda] - \sqrt{(\beta(1-\lambda) - \lambda)^2 + 4(1-\lambda)[1+(\beta+2)(1-\lambda)]} \right] > 0$ then $\frac{2\varepsilon(\varepsilon+(\beta+2)(1-\lambda))}{\lambda_\varepsilon^2(\beta+1)(\beta+2)}$ is increasing in ε for all $\varepsilon \in [0, \varepsilon_2]$ and decreasing otherwise. If instead $\varepsilon_2 < 0$ then it is decreasing for all ε .