

HEALTH INSURANCE OVER THE LIFE CYCLE WITH ADVERSE SELECTION

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ABSTRACT. This paper studies health insurance over the life-cycle with adverse selection to analyze the welfare implications of the Affordable Care Act of 2010 that targets at improving the access to health insurance for the uninsured. For this purpose, this study develops a life-cycle model following Huggett [19] and incorporates health insurance and private information similar to Chatterjee [8]. In particular, the model has: (i) individuals with privately known health type that is persistent and that stochastically affects their health expenses; (ii) competitive insurers that offer contracts against health expenditure risk; and (iii) government sponsors uncompensated care for individuals without private health insurance coverage. The insurers learn from an individual's history of health outcomes and insurance market behavior about his type and encapsulate his likelihood of being a healthy type in a health score. For this economic environment, I establish the existence of competitive equilibrium. Quantitative analysis takes the model to data choosing the parameters of the model to match key data moments such as the fraction of the uninsured non-elderly. The model is broadly consistent with the characteristics of the uninsured: they are usually in low income and poor health. It is also consistent with the persistence of being uninsured. The model is then used to evaluate the potential welfare consequences of the policy proposal that restricts the use of detailed medical information beyond age by the insurers and that extends subsidies to low income individuals for health insurance. A conservative evaluation is that the individuals are willing to forgo 0.3% of their consumption to live in an environment with that policy.

1. INTRODUCTION

Health care now constitutes about 15%, of GDP in the US. Although health insurance is a major way of receiving affordable health care in the current system, in 2011, 50 million people in the U.S. under age 65 (19%) lacked health insurance. Most of these individuals come from working families and have low incomes¹. To study the lack of health insurance provision,

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¹Statistics are Kaiser Family Foundation (2011) "The Uninsured: A Primer"

this paper develops a life-cycle model following Huggett [19] and incorporates private information about health status similar in vein to Chatterjee [8]. The model incorporates the main characteristics of U.S. health care system and replicates some of the key empirical characteristics of health insurance behavior in the the U.S. Specifically, we construct a model consistent with the following stylized facts:

- Health status is persistent and health insurance is subject to adverse selection due to private information about health status.²
- A considerable fraction of non-elderly adult population is uninsured. The uninsured households are typically in low income. About 70% of the uninsured earn below 2.5 times of federal poverty level income (\$22,050 for a family of four).
- Being uninsured is persistent (about 3 years).³
- Young adults have the highest uninsured rate (30%) of any age group. Almost half of all uninsured nonelderly adults have a chronic condition.⁴ Moreover, the uninsured non-elderly are in poor financial shape.
- Insurers can use information on pre-existing conditions for individuals working in small firms. For large firms, since individuals are insured as a pool, if the firm contracts with an insurer, each worker's medical history is not investigated in the offering of a health insurance contract.⁵
- Government plays an active role in health care system. It insures low income households subject to eligibility requirements and pays for uncompensated health care bills.⁶

We present some more important details on the characteristic of the uninsured and important facts about the institutional arrangement of health insurance, laws governing the nature of contracts and the practices by the insurers. According to a study by Kaiser Foundation, in 2010, 56.2% of nonelderly adults, those at aged between 19 and 65, have employer-sponsored health insurance, 19.2% are covered through Medicaid, 5.5% obtain health

²Cutler [10] reviews a substantial literature that suggest the importance of asymmetric information in the health insurance market. Although Cardon and Hendel [6] does not reject the null-hypotheses of symmetric information, Finkelstein [13] suggest that the inability to reject the hypotheses using data only on the payments in the event of the insured risk is not conclusive since the asymmetric information can potentially reveal itself in different dimensions of the contract such as the comprehensiveness of contract.

³This is documented in Haley *et al.* [16].

⁴Davidoff A and Kenney G. 2005. "Uninsured Americans with Chronic Health Conditions: Key Findings from the National Health Interview Survey" The Urban Institute and the University of Maryland.

⁵Source: <http://www.nahu.org/consumer/groupinsurance.cfm>.

⁶The details of this are documented in Kaiser (2011).

insurance in Private Non-Group Market and 18.5% are uninsured.⁷ The uninsured are usually in poor health and in poor financial condition.

The uninsured are not as healthy as those with health insurance coverage. They are more likely to experience episodes of preventable hospitalizations and missed diagnoses of serious health conditions.⁸ Moreover, after diagnosis of a chronic condition, they are less likely to receive follow-up care and more likely to have their health decline.⁹ Furthermore, the uninsured have significantly higher mortality rates than those with insurance.¹⁰

Most of the uninsured come from low- or medium-income households. About 70% of the uninsured earn below 2.5 times of federal poverty level income (\$22,050 for a family of four).¹¹ They often pay higher amounts for similar health services than the insured.¹² The uninsured household on average has no net assets.¹³ Twenty-seven percent of the uninsured report having used up all or most of their savings due to medical bills. They also accumulate debt faster, which deteriorates their credit ratings. About one-third of the uninsured individual are contacted by a collection agency about their medical bills compared to 8% of insured adults. Medical expenses are the major reason for bankruptcy filings.¹⁴

The nature of employment in term of the size of employer and the presence of government are important aspects of health insurance system in the US. Most non-elderly adults, 56% obtain subsidized health insurance through their employer. Health insurance is issued differently for different types of employers. For health insurance purposes, small employers are those with number of workers between 3 and 50. The small employers are significant source of employment and they employ 60% of the working population in 2010 - this fraction is about this level within the last decade. The remainder work in large employers. While most of the workers in a large employer are insured, the uninsured tend to work for small employers.¹⁵

The laws about how health insurance coverage can be issued to large groups are different than those to small groups, and the way that premium rates are determined is also different.¹⁶ In particular, federal law allow health insurance companies to look back at individual group applicants medical

⁷Source: Kaiser/HRET Survey of Employer-Sponsored Health Benefits, 2011.

⁸Institute of Medicine. 2009. *America's Uninsured Crisis, Consequences for Health and Health Care*. Washington, DC: National Academies Press. p 60-63.

⁹J. Hadley, 2007.

¹⁰Institute of Medicine. 2009. *America's Uninsured Crisis, Consequences for Health and Health Care*. Washington, DC: National Academies Press. p 60-63.

¹¹Kaiser Foundation, the Uninsured: Premier, 2011.

¹²G. Anderson, 2007, "From 'Soak The Rich' to 'Soak the Poor': Recent Trends in Hospital Pricing." *Health Affairs* 26(4):780-789.

¹³Kaiser Foundation, 2011.

¹⁴Chatterjee *et al.* [8]

¹⁵These statistics are reported in 2011 Kaiser/HRET Employer Health Benefits Survey.

¹⁶National Association of Health Underwriters <http://www.nahu.org/consumer/groupinsurance.cfm>

histories for pre-existing conditions and may decide not to cover certain conditions for a specified period of time. By federal law, this period is up to a year. In 38 states, the law allows small group health insurance companies to determine their premium rates for each company through a practice of medical underwriting.¹⁷ When small group plans use medical underwriting, employees are asked to provide detailed information about their medical and claim use history. Insurance companies use the medical information when determining rates. Furthermore, if an individual changes an employer, a new group health insurance requires him to provide a Certificate of Creditable Coverage, which is a document issued from the previous health plan that states how long the individual were insured.¹⁸ Therefore, in practice insurers use information for small groups both on medical and insurance history.

Large group health insurance (for employers with 50 or more workers) differs in coverage requirements and how premium rates are determined. Unlike small group health insurance contracts, health insurance company can reject a large employer group for coverage based on its claims history. If the insurer issues a policy to a large employer, then all of its eligible employees must be issued coverage as a group. Employees are not generally asked to fill out a medical questionnaire prior to obtaining coverage. The health insurance company bases annual premium changes for large employer groups primarily on the claims experience of the group in past years.¹⁹

A small fraction, 3.6%, of non-elderly individuals obtain health insurance in non-group private insurance market.²⁰ The individual health insurance plan uses information on medical history, age, and smoking habits to decide whether or not to cover an individual. If a policy is offered, the premium reflects that information. Individuals with chronic illnesses and specific conditions including, but not limited to, AIDS, cancer under treatment, diabetes and heart disease can be denied coverage.²¹ In informational aspects, non-group private insurance market is therefore similar to small-group insurance market and they are treated the same way in our model. It is worth noting that individuals face higher premiums and more cost sharing than their employer-covered counterparts.²² A study of contractual differences in relation to information asymmetries that can account for the small size of the private health insurance market is beyond the scope and is left for future research.

It is important for insurers to assess the risk profile of individuals applying for coverage. In practice, insurers use various statistical methods,

¹⁷For other states, the practice is that health plans use community rate for a given geographic location and incorporate information on age, gender or smoker status.

¹⁸[http://www.dmhc.ca.gov/dmhc_consumer/hp/hp.group.aspx](http://www.dmh.ca.gov/dmhc_consumer/hp/hp.group.aspx).

¹⁹<http://www.nahu.org/consumer/groupinsurance.cfm>

²⁰Kaiser Foundation, the Uninsured Premier, 2010

²¹http://www.dmh.ca.gov/dmhc_consumer/hp/hp.individual.aspx

²²For more detail see, <http://www.chcf.org/publications/2009/07/facts-and-findings-for-policy-makers-individual-health-insurance-in-california>

information on individual characteristics and expense information on various medical conditions to formulate the expected expenses for an individual with particular risk characteristics. This formulation is called as risk assessment and is an important feature of health insurance market.²³

In the light of this empirical facts, I propose a model to account for the characteristics of the uninsured and study potential welfare gains from a policy targeted at improving health insurance access for these individuals. In particular, the model features individuals with a finitely long planning horizon make a health insurance decision and choose how much to consume and how much to save or borrow to smooth consumption. Individuals face health expenditure risk against which they seek health insurance. Health expenses are governed by the health type which has full persistence and is privately observed. The individual can be employed in a small or a large firm. Income realizations and assets are privately observed by the individual. Insurers offer one-period health insurance contracts at premiums that depend on medical history, age and the size of the employer. Government provides health insurance, in the form of Medicaid, to eligible low-income individuals and make transfers to health care providers for medical bills that individuals are not able to pay. This contractual arrangement for health insurance broadly reflects the nature of contracts offered in the health insurance system in the US.

A key contribution of our paper is to connect the recent facts on health insurance with the theory of household behavior in macroeconomics literature. In particular, we expand the life-cycle model of Huggett [19] by incorporating health insurance and private information. A theoretical contribution of the paper is to prove the existence of a competitive equilibrium in which the premium on the full-insurance contract charged to a household with a given set of observable characteristics, namely, age, health score, and employment in various sizes of firms, exactly compensates the insurers for the objective health expense frequency for households with those characteristics. The proof requires constructing new approaches for two reasons: the state space involves endogenous *functions*; and non-convexities in allocation space arise from the presence of private information, and these render inapplicable the competitive equilibrium allocation characterization as a solution to the planning problem. We extend and apply the techniques developed by Chatterjee *et al.* [8].

The model is broadly successful in matching the key facts about the uninsured. Good types are less likely to be hit by health expense shocks. Under competition, the premium for full-health insurance could be expected to positively related to the probability of a person being of type g . Further, posterior probability for the insured will be lower because type b values

²³For more information risk assessment practices, see <http://www.ama-assn.org/resources/doc/psa/risk-assessment.pdf>

insurance more other characteristics being the same. This provides an explanation for why individuals with better medical history are offered lower premiums. Moreover, individuals with good type remain uninsured when young. Over the life-cycle bad type receive health expenditures more often relative to the good type that deteriorates their health score, which makes health insurance more costly for them. Being unable to afford health insurance, the bad type individuals have to pay health expenditures out-of-pocket and hence cannot accumulate assets. This implies that among the working age individuals, later in life-cycle the uninsured are usually in poor health and have low assets.

We use our framework to evaluate relevant counterfactuals. We find that the ACA policy which, among its various other provisions, restricts the use of medical information in contract offerings and mandates insurance coverage increases the overall welfare by 0.03% in consumption equivalent terms. This measure is considered as a lower bound for the welfare gain as the model currently does not include the effects of preventive medicine on health shocks, which is shown by several studies to have positive effect. Since the proposed insurance system does not discriminate individuals on the basis of medical history, individuals are risk-pooled and premiums are uniform. In practice, this resembles a single-payer system that has been implemented in most European countries.

The rest of the paper is organized as follows. Section 2 present the related literature. Section 3: specifies a life-cycle model where information on medical history is used in health insurance; defines an equilibrium; and discusses existence. Section 4 describes a model economy where there are restrictions on the nature of information that can be used in health insurance contracts and there is government subsidy for low-income individuals for health insurance. More specifically, we assume that insurers cannot use medical history or pre-existing conditions in insurance contract and that the government extend Medicaid eligibility to individuals with income less than 133% of Federal Poverty Level. Information restriction is consistent with the policy proposal that precludes insurers from using medical information in their health insurance offerings. Section 5 estimates the parameters of the model of Section 2 to match certain key moments in the data. Section 6 studies the properties of the model. Section 7 evaluates the potential performance of the proposed policy change in terms of welfare gains and the fraction of individuals insured. The counterfactual exercise in this section informs about the consequences of “The 2010 Patient Protection and Affordable Care Act” (hereafter, ACA). Section 8 concludes. Section 9 contains: Theoretical Appendix for the detailed proof for existence of equilibrium; and Algorithm that numerically computes equilibrium.

2. RELATED LITERATURE

There are two strands of existing literature to which this study is closely related. The first strand relates to the studies that build on the general equilibrium life-cycle model of Huggett [19] and that, in this context, allow heterogeneity in household's health shocks in addition to the differences in income across. Various studies (Palumbo [26], Jeske and Kitao [20], Attanasio *et al.* [3], De Nardi *et al.* [11], Jung and Tran [23], Halliday *et al.* [17], and Ozkan [25]) incorporated health shocks into Huggett's framework and study consumption saving decisions. Among these studies more recent ones explicitly study endogenous demand for health insurance. Besides differences in the environment (e.g. health shocks are governed by health capital in Ozkan, for instance, and health type is private information here), the main difference here is that health insurance premiums are endogenously determined as reflecting the role of information on medical history. In extending Huggett's framework to incorporate private information, the relation to Chatterjee *et al.* [8] is noted earlier.

The second strand relates to Cochrane [9] on long-term health insurance. He argues that the short-run contracts are poor devices for insuring long-term health risks. Our model confirms this logic: with only one-period contracts there is a significant number of uninsured who incur high health expenses. He further notices the defects of the long-run contracts however carefully designed as a sequence of short-run contracts respecting time-consistency and self-enforcement (from contract theories of Malcomson and Spinnewyn [24]; Fudenberg, Holmstrom, and Milgrom [14]; Rey and Salanie [27]). He proposes Pareto-optimal, time consistent contract providing both health insurance against health expenditure shocks *and* insurance against "premium increases" due to adverse health outcomes. In his proposed system, an individual pays premium that is actuarially fair for the population. This is also the same under the ACA. In the health insurance system proposed by Cochrane, there is a role for an intermediary to provide "premium insurance" to shield against premium increases resulting from adverse health outcomes. He argues that a private intermediary would assume such a role without government intervention. However, such an institution has not arisen. More recently, Hendel and Lizzeri [18] in the context of life-insurance and Finkelstein *et al.* [12] in the context of long-term care insurance argue that the market failure in the system proposed by Cochrane is due to one-sided commitment problem on the part of individuals. In this study, health insurance mandate and subsidies to low income individuals for health insurance together with the restrictions on medical information seems to approximate the allocation implied by Pareto-optimal time-consistent contract. Moreover, the framework here provides quantitative estimates about the severity of market failures that are pointed out by the empirical studies mentioned.

In the context of employer-sponsored health insurance, Cebul *et al.* [7] construct a search theoretic model to study small-group insurance markets. They argue that search frictions in that market induced by the competitive insurers lead to inefficiencies in the form of: less coverage for preventive medicine and higher premium relative to the frictionless benchmark economy. Their analysis suggests that publicly-financed insurance option can help to reduce inefficiencies induced by search frictions. Our model abstracts from search frictions and instead focuses on welfare consequences of the use of medical information in small-group insurance markets.

3. MODEL ECONOMY 1

3.1. People, Preferences and Endowments. Time is discrete and indexed by $t = 0, 1, 2, \dots, T$. There is a unit measure of people alive at each date. A person has a permanent health type $h \in \{g, b\}$.²⁴ Health type is privately known to the agent. Let μ denote the unconditional probability that an individual is of type g . At each age, an individual faces a health expense shock $x_t \in X := \{0, x_1, \dots, \bar{x}\}$ that takes finite number of elements. Health expense shock determines the fraction of income to expended on health care. Health type and age determine the distribution of health expenses $\pi(\cdot|h, t) \in \Delta(X)$. In particular, for individuals at the same age, relative to good health bad health generates high health expenses with higher probability. Moreover, for each type, with age the distribution worsens in a first-order stochastically manner. An individual draws her income e_t independently from her health type from a probability space $(E, \mathcal{B}(E), \Phi_t)$ where $E = [e, \bar{e}] \subset \mathbb{R}_{++}$ is a strictly positive closed interval and $\mathcal{B}(E)$ is the Borel sigma algebra generated by E . Alternatively, e_t can be interpreted as productivity. Probability measure Φ_t has a persistence in that high income realization today makes high income realizations tomorrow more likely. We allow the probability measures to depend on the period t to be able to incorporate life-cycle considerations. For example, an economy where agents work for T_{RET} periods and then retire for $T - T_{RET}$ periods is captured by setting $e_t > 0$ for $t \leq T_{RET}$ and $e_t = 0$ for $T_{RET} \leq t \leq T$. It is assumed that the earnings distribution does not depend on the size of employer. The government taxes total income progressively with average tax rate $\tau(\cdot)$.

Denote the life-time non-negative stream of consumption by a sequence $\{c_1, \dots, c_T\}$. Her ex ante utility is

²⁴This assumption is motivated by the empirical studies that find evidence for the strong association between the differences in biological and socioeconomic conditions in which a child grows up and difference in various health outcomes experienced by that individual in adulthood. In particular, low birth weight has a strong correlation with the occurrence of chronic diseases, such as asthma, hypertension, diabetes, health diseases and cancer, in adulthood. For this evidence and various other biological pathways linking early life experiences and incidence of diseases later in adulthood, see Barker [5] and references therein. For the pathways linking the socioeconomic factors early in life to the outset of diseases in adulthood, see Johnson and Schoeni [22] and references therein.

$$(1) \quad \mathbb{E}_0 = \sum_{t=1}^T \beta^{t-1} u(c_t)$$

The vonNeumann-Morgenstern utility function u is bounded, continuous, twice differentiable and concave. We have a full support assumption for incomes $\Phi(e'|e)$ for all $e, e' \in E$

More explicitly, an agent's utility from $c = \{c_t\}_{t=1}^T$ is given by

$$(2) \quad U(\{c\}) = \sum_{t=1}^T \beta^{t-1} \int u(c_t; e_t) \Phi_t(e_t|e_{t-1}) \Phi_{t-1}(e_{t-1}|e_{t-2}) \cdots \Phi_1(e_1) de_t de_{t-1} \cdots de_1$$

3.2. Market for health insurance. There is a competitive health insurance market that accepts premiums and makes benefit payments for individuals. The health insurance contract is issued for one-period and pays health care providers for health expenses incurred by the insured individual in that period. If the insured individual does not experience a health shock in that period, premiums do not accumulate for future health insurance.

A probability of payment by an insurer reflects the likelihood of health expenditure shock experienced by the individual. We model the health insurance arrangement to resemble the important facts about the employer-sponsored health insurance, especially the use of medical history in the offerings of health insurance contracts. In particular, insurance contracts are in *short-term* nature and are renewed annually. Moreover, there is heterogeneity in employer-sponsored insurance. For an individual working for small firms, with the number of workers less than 50, an insurer offers a premium based on the medical history of that individual. On the other hand, individuals in a large firm receives a uniform insurance contract. The premium for a large firm reflects health expense risk of the entire population of its workers.

There is a competitive market in health insurance contracts. A contract pays a fraction, θ , of medical expenses after deductibles and copayment and it is offered at a price $p_t(s_t; l)$. Since health type is private information, an insurer forms belief s_t at age t that a person is of healthy type. The role of the size of employer is noted by the presence of its size l as an argument in the price. Beliefs about an individual's type are important to insurers because the probability of payments for benefits vary across types. An important aspect of the market arrangement is that an insurer uses information on medical history of an individual when possible and estimates the probability s_t that a given individual is of type good at time t . We call estimated probability of being good type the individual's *health score*.

Insurers are assumed competitive and they offer the insurance at a price that is actuarially fair for an individual with a given age and health score

ν_t , interim probability is of good type computed from health score s_t , insurance decision and health expense process using Bayes' rule. This implies a presence of *premium function* $p_t(\cdot; l)$. For an individual who is unemployed or work for a small firm, the premium is priced actuarially fair relative to his risk score and age, that is, $p_t(nu_t; S) = \theta \sum_{x \in X} (s_t \pi(x|g, t) + (1 - s_t) \pi(x|b, t)) x$. The presence of S as an argument indicates that the individual in question seeks health insurance in the small-group market. We assume further that the formulation for individuals who are unemployed or self-employed is the same.

Individuals in a given large firm are treated uniformly by insurers. Each of the individuals working at a large firm has the same health score. This score is the probability μ_t that an individual is of good type agents among the population at age t who choose to be insured. The actuarially fair full insurance contract is priced as $p_t(s_t; L) = \theta \sum_{x \in X} (\mu_t \pi(x|g, t) + (1 - \mu_t) \pi(x|b, t)) x$. This formulation assumes that each of the large firms holds a representative population of agents. For individuals employed at a large firm the price for insurance contract does not depend on individual's health score.

The use of information on medical history implies the presence of *health score updating function* $\psi_{t+1}(s_t, \iota_t, x_t)$ as part of the market arrangement. It gives an individual's health score at the start of next period conditional on having begun the current period with type score s_t , making health insurance choice ι_t and experiencing health expense outcome x_t .

3.3. Financial Market Structure. Financial markets are modeled in a partial equilibrium. The interest rate q is taken as given by individuals. Individuals can save in the financial market at the interest rate q to cope with health expense risk beyond health insurance. They are not allowed to borrow. They can default in the event of high health expenditure shocks if the total of their earnings and assets is not high enough to pay for medical bills (i.e., $x_t > e_t + b_t - c_{\min}$), this is termed as uncompensated care. In the event of default due to medical expenditures: the government makes transfers to the health care provider through Medicaid for the non-elderly or Medicare for the elderly in the amount in excess of individual's financial resources; the individual receives transfers from the government for consumption that gives her a positive minimal consumption for one period; and her assets are set to zero. In future periods, she can accumulate asset. It is assumed here that the government can costlessly verify an individual's resources when it determines the eligibility for transfers.

3.4. Government. The government provides health insurance to individuals who satisfy certain eligibility conditions. Among the non-elderly who has income below FPL and have dependent children are eligible for Medicaid.

All of the elderly, those aged 65 and over are covered through Medicare.²⁵ The government also makes transfers to the health care providers for uncompensated care. It makes the Social Security transfer, ζ , to the elderly. The government imposes a progressive income tax $\tau(\cdot)$ to finance its expenditures.

3.5. Decision Problems.

3.5.1. *People.* Let $\mathbb{Y} = \{(\iota, b') | (\iota, b') \in \{0, 1\} \times Y\}$ denote the set of possible choices for health insurance and asset accumulation. A variable with superscript $'$ denotes the value of that variable in the next period.

An individual is born with a health type h that is permanent and is privately known. At age t , he has health score s_t and assets b_t and receives income endowment e_t that is drawn from a distribution $\Phi_t(\cdot, e_{t-1})$. Decisions for insurance ι_t and savings $b' - b$ are made after the realization of income. After then health expense shock x_t occurs, which has to be expended in that period. If the sum of earnings and assets is less than the health expense, the government makes transfers to the health care provider for the difference. A period finalizes with consumption.

Each individual takes as given

- the premium function $p : T \times [0, 1] \times \{0, 1\} \rightarrow R$,
- the health scoring function $\psi : T \times [0, 1] \times \{0, 1\} \times X \rightarrow [0, 1]$, and
- the interest rate q .

An individual's decision problem is formulated recursively. The state variables are (t, h, e, b, s, z, k) whose components are the individual's age, health type, current earnings, assets, health score, the size of the employer, and whether the individual has a dependent children, respectively. For insurance choice problem we distinguish between individuals depending on whether they are retired or in working age.

Retirement Years: Individuals retire at age T_{RET} and live until age T . In retirement years, individuals are covered by Medicare so they do not face an insurance choice problem. The government pays for fraction θ of the medical expenses. They receive constant social security transfers ζ from the government and decides on consumption and savings allocation. Therefore, the relevant state space for the individuals collapses to $(h, \zeta, b; q)$. For $t \geq T_{RET}$ their decision problem is simply

$$V_t(h, \zeta, b; q) = \sum_{\{(1-\theta)x \leq \zeta + b\}} \pi(x|h, t) \bar{W}_t(h, \zeta, b, x; q) + \sum_{\{(1-\theta)x > \zeta + b\}} \pi(x|h, t) \bar{W}_t(h, \zeta, b; q)$$

²⁵ Nearly all of the elderly, those aged 65 and over, are covered through Medicare. In 2010, 729,000 of those individuals aged 65 and over uninsured bringing the total uninsured to 49.9 million.

where

$$\begin{aligned}\overline{W}_t(h, \zeta, b, x; q) &= \max_{\{b': \zeta + b - q \cdot b' - (1 - \theta)x \geq 0\}} \{u(\zeta + b - q \cdot b' - (1 - \theta)x) + \\ &\quad \beta V_{t+1}(h, \zeta, b'; q)\} \text{ and} \\ \widehat{W}_t(h, \zeta, b; q) &= u(c_{\min}) + \beta V_{t+1}(h, \zeta, 0; q)\end{aligned}$$

for all $t = T_{RET}, \dots, T$ with $V_{T+1} \equiv 0$.

Working Years: In working years, individuals face a non-trivial insurance choice problem. We start by describing the set of feasible actions.

Definition 1. *[Feasible Set] For $t \leq T_{RET}$ and given (t, e, b, s, z, k) , the set of feasible actions is a set $B(t, e, b, s, z, k; q, p, \psi) \subset \mathbb{Y}$ that contains:*

- (1) for $k = 0$
 - (i) all $(1, b')$ such that $c = (1 - \tau(e + b))(e + b) - q \cdot b' - p_t(s; f) \geq 0$
 - (ii) all $(0, b')$ such that $c = (1 - \tau(e + b))(e + b) - q \cdot b' \geq 0$
- (2) for $k = 1$
 - if $e + b \geq FPL$ the elements analogous to that in the case $k = 0$
 - if $e + b < FLP$, all $(1, b')$ such that $e + b - q \cdot b' \geq 0$

Individuals with dependent children, $k = 1$, are eligible for Medicaid if their total of earnings and savings are below FPL. Note also that the feasible action set does not depend on individual's health type since this is not directly known to health insurers. Observe also the dependence of the feasible set of actions on health score and health insurance premium functions. Consistent with the empirical observations in Introduction, for individuals working for a large firm their health score is the health score for the entire group of employees in the firm. Under the assumption that a large firm has a representative population, for an individual working in a large firm, her health score equals to the fraction of the healthy type in the population, that is, for those individuals $s = \mu_0$. For individuals seeking insurance in the small-group market, the health score is computed individually.²⁶ Non-elderly individuals whose total of earnings and assets is less than FPL and have dependent children are covered by Medicaid.

We impose a technical restriction that the actions of being insured and being uninsured are chosen with at least a positive yet small probability. This is a trembling hand perfection argument in the context of refinements of Nash equilibrium for dynamic games. We require tremble in our model to ensure in our analysis that the health score updating function, which is

²⁶Very large firms (with number of workers 500 or more) usually manage their own health insurance plans that are usually administered by health insurance firms. However, such firms are not numerous and the analysis here abstracts from them. As noted in Introduction, most of the working individuals are employed by smaller firms and they seek health insurance in the non-group or group market depending on their employment status and the size of their employers if they are employed.

Bayesian updating, be well-defined. With this restriction, we define below the feasible set of choices as a randomization over the set of feasible actions.

Definition 2 (Feasible choice set). *Given (t, e, b, s, z, k) the feasible choice set $M(t, e, b, s, z, k)$ is the set of all $m \geq 0$ such that: (i) for all $(\iota, y) \notin B(t, e, b, s, z, k; q, p, \psi)$; (ii) $m^{(\iota, y)} \geq \epsilon$ for all $(\iota, y) \in B(t, e, b, s, z, k; q, p, \psi)$; and (iii) $\sum_{(\iota, y) \in \mathbb{Y}} m^{(\iota, y)} = 1$.*

In addition we assume that $\bar{e} + b_{\min} - b_{\max} - \bar{x} > 0$. With this assumption, being insured is always a feasible choice for some earnings and hence it will be chosen with positive probability in any equilibrium. Being uninsured is always feasible.

In this formulation, the feasible set consists of a pair of insurance and savings choices. If an individual is uninsured and he is faced with a sufficiently high health expense shock, then he defaults and the government pays for the uncompensated care. This assumption is reflected below in computing utility for each of the feasible choice.

Given the states (t, e, b, s, z, k) and the functions p, q and ψ , an individual's contemporaneous utility from choosing self-insurance and asset b' for next period is

$$R_x^{(0, b')}(t, e, b, s, z, k; q, p, \psi) = u(e + b - q \cdot b' - x);$$

if the health expense shock is not too large that $x < e + b - q \cdot b'$; otherwise his utility is

$$R_x^{(0, b')}(t, e, b, s, z, k; q, p, \psi) = u(c_{\min}),$$

because in this case the government pays for the uncompensated care in excess of the individuals resources, which amounts to $x - (e + b)$ and the individual receives the transfer $c_{\min}$ from the government.

Contemporaneous utility from being insurance and saving b' , if this choice is feasible, is

$$R_x^{(1, b')}(t, e, b, s, z, k; q, p, \psi) = u(e + b - q \cdot b' - p_t(s; f)).$$

The continuation utility, $V_t(h, e, b, s, z, k; q, p, \psi)$, is the unique solution to

$$(3) \quad V_t(h, e, b, s, z, k; q, p, \psi) = \max_{m \in M_h(t, e, b, s, z, k; q, p, \psi)} \sum_{(\iota, b')} \sum_x \pi(x|h, t) \left(\begin{array}{c} R_x^{(\iota, b')}(t, e, b, s, z, k; q, p, \psi) + \\ \beta W_x^{(\iota, b')}(t+1, h, e, s, z, k; q, p, \psi) \end{array} \right) \cdot m^{(\iota, b')}$$

where

$$W_x^{(\iota, b')}(t+1, h, e, s, z, k; q, p, \psi) = \int_E V_{t+1}(h, e', b', \psi_{t+1}(s, \iota, x), z, k; q, p, \psi) \Phi_t(de'|e)$$

for all $t = 1, \dots, T_{RET}$.

Let $M_h(t, e, b, s, z, k; q, p, \psi)$ denote the optimal decision correspondence and $m_h^*(t, e, b, s, z, k; q, p, \psi)$ a given selection from this correspondence.

3.5.2. *Insurers.* A representative insurer operates in a competitive market and takes the premium function $p_t(\nu; l)$ as given. The insurer uses information about an individual's age t and health score s to form interim belief ν that an individual is of type g . The profit $W(t, l, \nu)$ on an insurance contract of type (t, l, ν) is given by

$$(4) \quad W(t, \iota, \nu; p) = \begin{cases} p_t(\nu; l) - \theta \left(\frac{\sum_{x \in X} (\nu \cdot \pi(x|g, t) + (1 - \nu) \cdot \pi(x|b, t)) x}{(1 - \nu) \cdot \pi(x|b, t)} \right) & \text{if } \iota = 1 \\ 0 & \text{if } \iota = 0 \end{cases}$$

Denote by $\mathcal{B}(C)$ the Borel sets of $C := T \times \{0, 1\} \times [0, 1]$ and $\mathcal{A}$ the set of all probability measures defined on the measurable space $(C, \mathcal{B}(C))$. If $\alpha \in \mathcal{A}$, $\alpha(t, l, \nu)$ is the measure of insurance contracts of type (t, l, ν) . Therefore, the decision problem of the insurer is

$$\max_{\alpha \in \mathcal{A}} \int W(t, \iota, s') d\alpha(t, \iota, s').$$

Another important function of the insurer is to compute health scores. We do not provide an explicit model of how health score is computed by the insurer. Our approach is to impose restrictions on the outcome of this process. This is the rational expectations hypothesis in equilibrium. In particular, we assume that (i) the interim belief $\nu_t(s, \iota)$ is the fraction of type g individuals who at age t start health score s and choose ι and (ii) $\psi_t(s, \iota, x)$ is the fraction of type g individuals who at age t start with health score s , make health insurance choice ι , and experience health outcome x . We name ν as type scoring function and ψ as health score function.

Let $P_{h,t}^\iota$ denote the fraction of type h agents who at age t make insurance decision ι . It is computed as follows

$$(5) \quad P_{h,t}^\iota(s; q, p, \psi) = \sum_{b \in Y, z \in \{0,1\}, k \in \{0,1\}} \int_E m_h^{(\iota, y)}(t, e, b, s, z, k; q, p, \psi) \Gamma(t, de, h, b, z, k)$$

where Γ is the endogenous joint distribution of individuals over health type, earnings, asset holdings, size of employer, and presence of dependent children in the population. Since earnings e and assets b are unobservable to insurers, they are integrated out using endogenous population distribution implied by decision rules and the stochastic processes governing the primitives.

Then, the condition (i) implies

$$(6) \quad \nu_t(s, \iota; q, p, \psi) = \frac{P_{g,t}^\iota(s; q, p, \psi) \cdot s}{P_{g,t}^\iota(s; q, p, \psi) \cdot s + P_{b,t}^\iota(s; q, p, \psi) \cdot (1 - s)}.$$

This is the interim belief that an individual is of type g given prior belief s and observed insurance choice ι .

Further conditioning on health shock realization the condition (ii) together with (6) imply the posterior belief

$$(7) \quad \psi_t(s, \iota, x; q, p, \psi) = \frac{\nu_t(s, \iota; q, p, \psi) \cdot \pi(x|g, t)}{\nu_t(s, \iota; q, p, \psi) \cdot \pi(x|g, t) + (1 - \nu_t(s, \iota; q, p, \psi)) \cdot \pi(x|b, t)}.$$

3.6. Equilibrium. As a solution concept, we define a stationary recursive competitive equilibrium.

Definition 3. A stationary recursive competitive equilibrium consists of (i) a premium function $p_t^*(\nu; l)$; (ii) health scoring function $\psi_t^*(s, \iota, x)$; and (iii) decision rules $m_t^*(h, e, b, s, z, k; q, p^*, \psi^*)$ such that for each t

- (D1) $m_t^*(h, e, b, s, z, k; q, p^*, \psi^*)$ is a selection from $M_t^*(h, e, b, s, z, k; q, p^*, \psi^*)$,
- (D2) $p_t^*(\nu; l)$ is such that $W(t, l, s') = 0$ in (4) $\forall (t, l, s')$,
- (D3) $\psi_t^*(s, \iota, x)$ satisfies (7) for $m_t^*(h, e, b, s, z, k; q, p^*, \psi^*)$, $h \in \{g, b\}$.

3.7. Existence. To simplify the analysis we start with the observation that for each t the premium function p_t and health scoring function ψ_t being continuous functions of interim belief ν_t share the continuity properties of the type scoring function ν_t .

Lemma 1. If the type scoring function $\nu_t(s, \iota)$ is continuous in s for each t and ι then the premium function $p_t(\nu_t)$ and health scoring function $\psi_t(s, \iota, x)$ is continuous in the current health score s for each t, ι , and x .

Proof. Follows from the assumed continuity of ν_t and for continuity of p_t from the zero profit condition in (4), and for continuity of ψ_t from Bayesian updating in (7) which is continuous in ν_t . $\square$

Given Lemma 1, the equilibrium analysis reduces to finding a function $\nu_t(s, \iota)$ such that the conditions for the equilibrium (D1)-(D3) hold. To establish existence we take the following steps.

- (S1) The function ν is defined on $\Omega = T \times [0, 1] \times \{0, 1\}$. Let F denote the set of functions on Ω that takes values in $[0, 1]$ and let K be the subset of functions in F that are continuous in s . Since ι takes only finite number of elements, the continuity with respect to that element trivially holds and it is suppressed. Endowed with the sup-norm topology, K is a closed, bounded, and convex subset of F .
- (S2) Define an operator $T : K \rightarrow F$ as follows. Given $f \in K$, solve the individual's problem to get $m_t^*(h, e, b, s, z, k; f)$. Then use (5)-(7) to get $T(f)$.
- (S3) Establish that the operator T and the set K has the following properties: (i) $T(K) \subset K$, (ii) T is a continuous operator, and (iii) $T(K)$ is equicontinuous.
- (S4) Apply Schauder's fixed point theorem to prove the existence of a fixed point $f \in K$ such that $T(f) = f$.

To apply Schauder's fixed point theorem we must verify the conditions on the operator T and the family $T(K)$ given in step (S3). In a series of results

we verify these conditions for our model economy. Inspecting the conditions (5) and (6), which define the operator T , suggests that the continuity of the operator is closely related to that of the decision rule $m_t^*(h, e, b, s, z, k; f)$ through (5).

In Appendix 9.1 we prove the main existence result.

Theorem 1. *A recursive competitive equilibrium specified in Definition 3 exists.*

We briefly sketch the proof here. The structure of the proof mirrors the structure of the proof in Chatterjee *et al.* [8] generalizing to allow for life-cycle in decision problems and persistence in continuum variable, namely earnings.

Lemma 2 applies a generalization of the Theorem of the Maximum by Ausubel and Deneckere [4] to show that the decision correspondence $M_t^*(\cdot; f)$ is non-empty, compact valued, upper hemi-continuous in e and s for any given $f \in K$. The generalized version requires only upper hemi-continuity of the feasible choice set and allows the objective function to have “upward-jumps”. Lemma 3 uses results in Araujo and Mas-Colell [2] to show that $M_t^*(\cdot; f)$ is single valued and continuous almost everywhere in E for a given $f \in K$. To establish equicontinuity of $T(K)$, we will use a Lipschitz argument. As an input into this argument, Lemma 4 proves a local Lipschitz property of decision rules $m_t^*(\cdot; f)$ in s . This result mainly follows from the fact that the action set is finite, which implies that for a small enough change in s there is no change in actions except at a countable number of earnings levels. Lemma 5 establishes that $P_{h,t}^\ell(s; f)$ is well-defined and continuous in s . Intuitively, integrating over an atomless distribution for earnings e in equation (5) “smooths out” any discontinuities in the selection $m_t^*(\cdot; f)$. Lemma 6 establishes that $\nu(s, \ell; f)$ is continuous in s , which follows from equations (5) and (7) and Lemma 5. Lemma 7 establishes that $P_{h,t}^\ell(s; f)$ has Lipschitz constant 1 in s for any f . The lemma extends the local Lipschitz property of decision rules in Lemma 4 globally to $P_{h,t}^\ell(\cdot; f)$. Intuitively, if $P_{h,t}^\ell(\cdot; f)$ fails to be globally Lipschitz there must be an open set of s where the local Lipschitz property is contradicted. Since this holds for any f , the family of functions $\{P_{h,t}^\ell(\cdot; f)\}_{f \in K}$ is uniformly Lipschitz continuous. After establishing some algebraic properties of Lipschitz functions in Lemma 8, Lemma 9 proves that $\{\nu_t(\cdot; f)\}_{f \in K}$ is also uniformly Lipschitz family. Using this allows us to prove equicontinuity in Lemma 10. Finally, Theorem 1 establishes that the conditions for equilibrium in Definition 3 are satisfied.

4. MODEL ECONOMY 2

Now we specify a model economy where the insurers are restricted by the detail of information they can use in their contract offerings, and where subsidies for insurance is expanded. In particular the insurers cannot deny coverage based on pre-existing conditions and are allowed to vary rating

based only on age. There is a mandate for individuals to be insured. All working age individuals with earnings less than 133% of FPL are eligible for Medicaid regardless of dependent children status. All other elements of Model Economy 2 are the same as in Model Economy 1.

This policy change affects decision problems. Since the informational restrictions are only on the insurers, the individual's problem is identical to what we had in the benchmark model. In particular, we substitute s_t for μ_t the fraction of the good type among the individuals at age t . This also reduces the importance of the size of employer, since the premiums are computed in the same way independent of the employer size. Moreover, with the expansion in Medicaid the role of dependent children in determining eligibility reduces to null.

The informational restrictions on the insurer affect the updating function in (5)-(6). In particular, interim belief function is now given by

$$(8) \quad \nu_t(\mu_t, \iota; q, p, \psi) = \frac{P_{g,t}^\iota(\mu_t; q, p, \psi) \cdot \mu_t}{P_{g,t}^\iota(\mu_t; q, p, \psi) \cdot \mu_t + P_{b,t}^\iota(\mu_t; q, p, \psi) \cdot (1 - \mu_t)}.$$

For this updated belief, premiums are computed using the premium function in (4). Notice that the key difference from (5)-(6) arises from the measurability restrictions in (8). Similar as before we use information on the distribution of agents in the economy to construct the “prior” likelihood that an agent is of type g .

5. MOMENT MATCHING

A model period corresponds to one year. The discount rate β for both types is set to be 0.99. The risk-free interest rate r is set to satisfy $\beta(1+r) = 1$. The utility function takes the form $u(c) = \frac{c^{1-\sigma}}{(1-\sigma)}$. Thus, in this calibration we will abstract from preference differences between types. The constant relative risk aversion parameter is set $\sigma = 3$. This follows from the estimate reported in De Nardi *et al.* [11] for a structural model with uncertain medical expenditures.²⁷

The FPL in the data corresponds is 57% of median income for a family of four and this statistics is used in our model economy. We assume that the “tremble” parameter $\epsilon = 0.0001$. This is the probability that agents will play a suboptimal but feasible action by mistake. Consistent with data facts, we exogenously set that 60% of workers are employed in small firms. Moreover, the exogenous rate of working age individuals with dependent children is taken 70%.

Insurance plans available are private health insurance and Medicaid. These insurance plans vary regarding deductible and co-payment features. From Ozkan's [25] findings on Medical Expenditure Panel Survey (MEPS) the benefits of these plans are very similar and therefore in the analysis here

²⁷This coefficient is usually assumed $\sigma = 2$. The results qualitatively hold with this parameter value is used instead.

these contracts are treated the same way. His estimates also suggest that insurance benefit coverage is about 75%. Using this estimate, the benefit fraction for an insurance contract is set $\theta = 0.75$.

We use the earning process calibrated by Huggett [19]. In working age, each period an agent makes a random draw from an earning distribution. In particular, this earning distribution does not depend on health type. In retirement, individuals receive pension, that pays them each period average life-time earnings.²⁸ Health expense process $\pi(\cdot|h, t)$ is governed by health type and age.

We use data from the PSID 2001-2009 to construct the health expense process. An important part of the quantitative analysis is to determine health type of an individual in the data. This is challenging since the health type is assumed to be privately known and is not immediately identifiable from the data. To determine an individual's health type this study follows Johnson [21]. In particular, using PSID, Johnson [21] estimates the various measures on the strength of the association between socioeconomic status in childhood and the outset of chronic diseases in adulthood. Relevant to the estimation problem here, he reports that the individual who spent their childhood residing in poor socioeconomic status subsequently experienced one-quarter of their years between 35 and 55 in fair or poor health. For individuals born into better socioeconomic status, the corresponding proportion of adulthood spent in fair or poor health is roughly 10 percent. Using these statistics as criteria together with data on health status and on chronic conditions, individuals are classified in two types: good health and poor health. This yields a distribution over non-elderly adult population where 65% of individuals are type good.

Given externally set parameter values and health types, an important part of the quantitative analysis is to estimate the stochastic process $\pi(x|h, t)$ governing the health expenditures over the life-cycle. Here we first describe the calibration exercise including: the values of the parameters for the health expense process, targeted statistics and untargeted features of the uninsured, then we discuss the performance of the model.

The calibration exercise here chooses

- (Cal1) The support of the ratio of health expenditures to average earnings for the cohort $X = \{0, 0.1, 0.5, 1, 1.5\}$
- (Cal2) Age cohorts for non-elderly individuals: 20-29, 30-39, 40-49, 50-59 and 60-65

Given health type, X and age cohorts, the probabilities $\pi(x|h, t)$ are just identified from empirical frequencies. For a workable state space, we compute five point distribution for the fraction health expenditure to cohort's average wage earnings. The values are

²⁸A more realistic model of the retirement system would be incorporated. However, this additional complication would not add further insight for information problems considered here for the insurance decisions made during working age.

TABLE 1. Health expenditures of good type relative to income by cohort

x	Ages				
	20-29	30-39	40-49	50-59	60-65
0.1	0.1430	0.2380	0.2730	0.3580	0.3850
0.5	0.0367	0.0425	0.0433	0.0520	0.0579
1	0.0541	0.0613	0.0625	0.0815	0.1036
1.5	0.0303	0.0403	0.0714	0.1067	0.1228

TABLE 2. Health expenditures of bad type relative to income by cohort

x	Ages				
	20-29	30-39	40-49	50-59	60-65
0.1	0.2167	0.1238	0.1279	0.1033	0.0946
0.5	0.1388	0.2511	0.2194	0.2167	0.3347
1	0.0556	0.0773	0.0636	0.1200	0.2975
1.5	0.0200	0.0310	0.0967	0.1912	0.3250

Notice that for each health type expense distribution becomes worse with age in the first-order stochastic sense. Furthermore, for any given age group the distribution for individuals with poor health type put more weights on higher expenditures than that for the good type.

The parameter values in (Cal1) and (Cal2) for the support of the distribution X and age cohorts in minimize the distance between the key statistics on the characteristics of the uninsured by age and by income and those implied by the model. In particular, the key characteristics that the model targets at matching and its performance to their values in the data are given in the following table.

TABLE 3. The characteristic of the non-elderly uninsured

Statistic	Data	Model
Fraction the non-elderly	18%	15%
By age		
19-34	39%	25%
35-54	34%	28%
55-64	11%	29%
By income		
< 100 FPL	41%	52%
100-250% FPL	37%	33%
251-399% FPL	13%	8%

In quantitative analysis the model identifies 9 parameters, 4 of them for the support of X given in (Cal1) and the remainder for age cohorts in (Cal2). It uses 47 moments, 7 is on the characteristics of the uninsured in Table 4 and the remaining 40 from health expenditure given in Table 1 and Table 2.

Persistence of being uninsured and the share of top spenders are the statistics that are not directly targeted by the calibration and they provide overidentifying restrictions for the fitness of the model to the data. The comparisons of the data and the model along the values of these statistics are given in the following table.

TABLE 4. The characteristic of the non-elderly uninsured

Statistic	Data	Model
Average duration of being uninsured (in years)	2.3	3.1
Fraction of total health expenditures by top 5% spenders	0.49	0.46

In Table 4 the statistics for the top spenders in the data are taken from the Medical Expenditure Survey conducted in 1996-97 and 2002-03.²⁹

6. MODEL PROPERTIES

Since health score are based on observed health outcomes, we start by analyzing the equilibrium decision rules of individuals.

Individuals with a given health score are uninsured if their earnings and savings are not sufficiently high to afford health insurance. With higher health score individuals afford health insurance for a larger set of high earnings and savings. This holds true for any health type, age group and employment status. Among young individuals who afford insurance and are with a given health score, type g agents will be uninsured if they have sufficiently high earnings and savings while type b agents will be uninsured for a smaller set of high earnings and savings. Young agents who afford insurance will be uninsured for a smaller set of low earnings and savings if they have higher health scores. Young individuals with type g have lower health score if they work for a large firm, reflecting that the pool provided by a large employer to an insurer is disadvantageous for type g agents.

Relative to young individuals, for any given health score, old agents afford insurance for a smaller set of high earning and savings. This is due to life-cycle effect of deteriorating health status on increased health expenses, which increases the premiums for actuarially fair insurance contract. Relative to their younger counterparts, among old individuals who afford insurance will be uninsured for a larger set of high earnings and savings. Similar to young individuals, among the old who afford insurance those with type g will be

²⁹M.W. Stanton, 2006, “The high concentration of US health care expenditures.” US Department of Health and Human Services, Public Health Services, Agency for Healthcare Research and Quality.

uninsured if they have sufficiently high earnings and savings while type b agents will be uninsured for a smaller set of high earnings and savings. Old individuals who afford insurance will be uninsured for a smaller set of high earning and savings if they have higher health score. In equilibrium, every action is taken by some agents.

To provide a test of the model, we compare its predictions to the key facts about the uninsured. For this purpose, the dynamic of health scores are constructed.

- (1) The decision rules for the insurance choice imply the distribution of the uninsured. The key statistics implied by the model is similar to those in the data. The comparison of the model to the data is presented in Table 4.

Moreover, as in the data, distribution for the uninsured puts more weight on type g among young individuals, while the type b have more weight in older ages. This property qualitatively remains unchanged under various alternative classifications of individuals into health types.

- (2) Young agents have low earnings and therefore are net dissavers. In middle ages, individuals have better earning distributions and they save for any health type. In old ages, they are again dissavers. For any health score, earnings and savings, individuals with type b would save higher than those with type g , reflecting the more health expense risk the former faces. For any age group, asset accumulation for the type g is higher than the type b . Among the agents whose medical expenses are uncompensated and paid by the government, the distribution puts more weight on type b individuals. This is consistent with the data that payments for expensive chronic conditions by the government constitutes a significant fraction of the total government expenditures for uncompensated care.
- (3) Given earnings distributions, the decision rules also imply certain properties for the health scoring function via (5) - (7). Consistent with the findings documented by Johnson [21], as individuals age their health score deteriorates, reflecting the adverse life-cycle effect of aging; and for agents with type b health score worsen at a faster manner.
- (4) The model makes predictions about insurance premiums. Since the premium function depends linearly on the health scoring function via (4), the premium is decreasing in health score. For any given age, type g has higher health score, they can have health insurance at lower premiums. For a given type, as health score decreases over time, the premium increases.
- (5) Furthermore, the model makes predictions about the duration of being uninsured. The average duration of uninsured is 2.3 years. This statistics in NSAF data reported by Haley *at al.* [16] is about 3

years. This is an important data moment about the uninsured and it is not directly targeted at by the calibration procedure. Therefore, it provides a test for the model.

7. POLICY EXPERIMENT

Here we use the model to address a question about the welfare consequences of the ACA. This policy mandates that individuals have health insurance. Moreover, it requires that insurers cannot use information on medical history in their offerings of health insurance to individuals and that they are allowed to vary ratings by age. It also creates public health insurance entity, named as Health Insurance Exchange, to provide health insurance in the individual and small-group health insurance market.³⁰ This proposal in effect creates pool of individuals to insurers who cannot screen individuals using information on medical histories. Since the pool health score is the average over the participating individuals, its health score is higher than that of bad type individuals, while it is lower than that of good type. Consequently, the bad type are implicitly subsidized by the good type. Therefore, the pool induced by the policy, on average, raises welfare for bad type, while it reduces for the good type. Therefore, it is not clear a priori whether the net effect is positive or negative or whether it has significant magnitude.

To answer this question, we compute consumption equivalents using the following formulas. The expected present discounted value of utility for type i individual at age t with earnings e_t , savings b_t and health expenditure x_t is given by

$$W_{i,\tau}(e_\tau, b_\tau, x_\tau) = E_{i,\tau} \left[\sum_{t=\tau}^T \beta^t \frac{c_{t,i}(e_t, x_t)^{1-\sigma}}{1-\sigma} \right]$$

To assess how much a type i agent would be willing to pay throughout her life-time to be in regime governed by the ACA, for each i we compute the consumption equivalent, $\lambda_{i,\tau}(e_\tau, b_\tau, x_\tau)$, such that

$$\begin{aligned} W_i^{ACA}(e_\tau, b_\tau, x_\tau) &= E_i \left[\sum_{t=\tau}^T \beta^t \frac{(1 + \lambda_i)c_{t,i}(e_t, b_t, x_t)^{1-\sigma}}{1-\sigma} \right] \\ &= (1 + \lambda_{i,t}(e_t, b_t, x_t))^{1-\sigma} W_{i,t}(e_t, b_t, x_t) \end{aligned}$$

$$\text{or } \lambda_i(e_t, b_t, x_t) = \left[\frac{W_i(e_t, b_t, x_t)}{W_i^{ACA}(e_t, b_t, x_t)} \right]^{1/(1-\sigma)} - 1$$

Then the total welfare gain/loss is given by

$$\sum_{t=1}^T \lambda_{i,t}(e_t, b_t, x_t) \Gamma(i, t, e_t, b_t, x_t)$$

³⁰For more detailed information on the policies proposed under the ACA see <http://www.kff.org/healthreform/8061.cfm>

We use the parameterization in the calibrated model for the Model Economy 1 with no restrictions on the use of medical information by the insurers and with restrictions on Medicaid eligibility. As a whole, the economy is better off with the information restriction and increased Medicaid subsidies. Specifically, the welfare gain is 0.003. The average consumption equivalents for good types is -0.0004, while it is +0.006 for bad types. Type g on average would not prefer the restriction on information and subsidies and must be compensated, while type b on average are better off under the new policy. The consumption equivalents are on average positive for older individuals while it is negative for the younger agents.

An important implication of the welfare analysis is that the ACA is not chosen under the majority voting rule. The one hand, the good types are in significant majority, 65%, of the population and are made worse-off by the policy change. On the other hand, the bad types are in minority and would prefer it. Since the rule chooses the alternative most preferred by the majority, it does not implement the information restriction proposed in the ACA.

8. CONCLUSION

The empirical facts point to (1) the persistent and private information nature of health status, (2) the prevalence of the uninsured among the non-elderly adults who are usually in poor health and in poor financial shape, (3) the use of information on pre-existing conditions in health insurance contracts, and (4) the heterogeneity in health insurance contracts with respect to the size of employer. Recognizing these observations, the ACA aims to increase access to health insurance for the insured.

We propose a framework to analyze the potential welfare consequences of this policy. The framework uses a dynamic life-cycle model with heterogeneous agents of Huggett [19] and as a novel feature incorporates health insurance and private information similar to Chatterjee *et al.* [8]. Given the persistent nature of private information that governs an individual's health outcomes over the life-cycle, the insurers use information on the histories of health expense realizations and insurance decisions to infer the individual's likelihood of experiencing health expense shock. A similar inference problem is incorporated in a quantitative dynamic model by Chatterjee *et al.* [8] in their study of credit scoring in credit markets in relation to default behavior. There is virtually no existing work on health insurance in a dynamic life-cycle model with heterogeneous agents that incorporates asymmetric information despite its empirical prevalence. Therefore, this study is a step forward to meet this challenge.

The model is broadly successful in matching the key characteristics of the uninsured: that they are usually in poor health and in poor financial shape. It also provides a conservative estimate for the welfare gains from the ACA.

Among the various other provisions of the ACA, this study analyzes the welfare implications of the policy that restricts the use of information on pre-existing conditions by insurers and expands subsidies for health insurance. We find that the individuals are willing to forgo 0.3% of their consumptions to live in an environment with that policy. The estimate is conservative for two reasons: (1) the analysis currently does not incorporate the positive effects of preventive medicine that the health insurance offers; and (2) the analysis assumes that all individuals have the uniform life-expectancy and therefore it does not incorporate positive effects that would result from increased longevity for the uninsured. Incorporating these positive effects of insurance is a part of future research. The welfare analysis also points out a potential problem on the viability of the information restrictions under the ACA: the good types do not prefer it and they are in majority.

In studying the characteristics of the uninsured, this paper focuses only on the key features of the health insurance provided by the employers and the government. In practice, there is more variation in the nature of health insurance contracts in terms of co-insurance, deductibles and coverage. Better understanding of the nature of health insurance contracts, especially in relation to almost non-existence of individual health insurance market, is left for future research. Furthermore, our analysis takes as given the organization of the provision of health insurance in which the insurers act as an intermediaries between individuals and health care providers. It therefore provides a positive analysis while abstracting from the optimality of the organization of health insurance in economy. There has been an increase in intermediation of health insurance since its start in early 1930s. A natural question is how health insurance should be provided.

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9. APPENDIX

9.1. Existence Proof. Let $\nu \in K$ and using it from (4) and (7) to conserve on notation define $f := (q, p, \psi)$. The first result proven in Lemma 2 is that the decision correspondence $m_t^*(h, e, b, s, z, k; f)$, which is supported on the feasible set $B(t, e, b, s, z, k; f)$ is well-defined and has the standard properties. It uses a generalization of Theorem of the Maximum by Ausubel

and Deneckere [4]. The generalized theorem relaxes the standard continuity assumption of the feasible choice correspondence and allows a particular form of discontinuity so that the choice correspondence can be upper-hemi continuous (uhc). We show that the feasible choice set is uhc, although the presence of strictly positive trembles fails the standard continuity assumption. In particular, the feasible choice set is not necessarily lower hemi-continuous in e, s, f . This is because given a sequence p_n converging to p , such that there is an action $\hat{y}$ that delivers strictly positive consumption for all p_n but delivers zero consumption for p . Then every $m(\cdot; p_n) \in M(\cdot; p_n)$ assigns at least ϵ probability weight to $\hat{y}$ but there exists at least one feasible $\tilde{m}(\cdot; p) \in M(\cdot; p)$ which assigns zero probability to $\hat{y}$ that is $\tilde{m}(\cdot; p) = 0$. This shows that there does not exist any feasible sequence of $m(\cdot; p_n)$ that converges to $\tilde{m}(\cdot; p)$ and hence the failure of lower-hemi continuity.

Lemma 2. *The decision correspondence $M_t^*(h, e, b, s, z, k; f)$ is non-empty, compact valued, upper hemi-continuous correspondence in e, s , and f , for each t, h, b and z .*

Proof. We verify that the hypotheses of the Generalized Theorem of the Maximum by Ausubel and Deneckere [4] are satisfied in our model.

Claim 1: For each $t = 1, \dots, T$ the feasible choice set $M_t(h, e, b, s, z, k; f)$ is non-empty for each t, h, e, b, s, z, f . This follows from the fact that being uninsured and having some positive consumption are always affordable.

Claim 2: $M_t(h, e, b, s, z, k; f)$ is compact valued for each $t = 1, \dots, T$. This follows from the observation that it is finite dimensional closed simplex.

Claim 3: To show that $M_t(h, e, b, s, z, k; f)$ is uhc for each $t = 1, \dots, T$. We restrict attention to the set of arguments with continuum values, since uhc holds trivially with respect to the arguments that take finite set of values, namely t, h, b , and z . Pick an arbitrary convergent sequence $(e_n, s_n, f_n) \rightarrow (e, s, f)$ and $m_n \in M(h, e_n, b, s_n, z, k; f_n)$ and find a subsequence $(e_{n_k}, s_{n_k}, f_{n_k})$ such that $m_{n_k} \rightarrow m$ and $m \in M_t(h, e, b, s, z, k; f)$, where the convergence of f_n to f is in the sup-norm metric. To prove the claim, suppose that m does not belong in $M(h, e, b, s, z, k; f)$. Then there exists a pair (ι, y) such that $m^{(\iota, y)} > 0$ for some (ι, y) not in $B(h, e, b, s, z, k; f)$. Since every element at the prelimit is a probability vector on the finite dimensional simplex and the latter is a closed space, the limit is a well-defined probability vector. Therefore, for the limit vector m to become infeasible it must assign positive weight to some insurance and savings pair that yield negative consumption, that is $c = e + b - q \cdot y - p(\nu(s, \iota; f)) \cdot \iota - x \cdot (1 - \iota) < 0$. But this implies that for sufficiently large n , $c_n = e_n + b - q \cdot y - p(\nu(s_n, \iota; f_n)) \cdot \iota - x \cdot (1 - \iota) < 0$. Therefore, for all n sufficiently large we must have $m_n^{(\iota, y)} = 0$, which is a contradiction.

Claim 4: The current expected utility function $R_{t,x}^{\iota,y}(h, e, b, s, z, k; f)$ is continuous in e, s , and f , for each t, h, b and z . Since the premium function p is continuous in type score s' , by Lemma 1, and type scoring function $\nu \in K$ is a continuous function of s , R is continuous in s . Moreover, R is

also continuous in e since the temporal utility function u is continuous in current consumption and the latter is linear in e .

For $t = T$, since there is no continuation after T , $V_{T+1} \equiv 0$. Consumption/savings choice is trivial so that $y = 0$ and hence the optimization problem becomes

$$V_T(h, e, b, s, z, k; f) = \max_{m \in M_T(e, b, s, z, k; f)} \sum_{(\iota, y)} \left[\sum_x \pi(x|h, T) R_x^{(\iota, y)}(T, e, b, s, z, k; f) \right] \cdot m^{(\iota, 0)}$$

Together Claims 1 to 4 for $t = T$ constitute the hypotheses of the Theorem of Maximum in Ausubel and Deneckere [4]. Therefore, $V_T(h, e, b, s, z, k; f)$ is continuous in e, s for each (h, b, z, f) . The other important consequence of the theorem is that the choice correspondence $M_T^*(h, e, b, s, z, k; f)$ is a non-empty, compact valued and upper hemi-continuous (uhc) correspondence in e, s for each (h, b, z, f) .

We construct a backward induction argument for the rest of the proof. For $t = 1, \dots, T - 1$ the optimization problem becomes

$$V_t(h, e, b, s, z, k; f) = \max_{m \in M_t(e, b, s, z, k; f)} \sum_{(\iota, y)} \left[\sum_x \pi(x|h, t) R_x^{(\iota, y)}(t, e, b, s, z, k; f) + \beta \int_E V_{t+1}(h, e', y, \psi(\iota, x, s), z, k; f) \Phi_t(de'|e) \right] \cdot m^{(\iota, y)}$$

From Claim 4 for $t = T - 1$ the temporal utility function is continuous in e and s for each (h, b, z, f) . The expected continuation utility is also continuous in these arguments. This is because V_T is an integral of a continuous function of e and s each of which takes values in a compact set and hence uniformly bounded in these variables and probability distribution is continuous in e . The required continuity claim follows from an application of Bounded Convergence Theorem. The objective function is a sum of continuous functions and hence continuous in e and s . This together with Claims 1 to 3 for $t = T - 1$ constitute the hypotheses of the Theorem of Maximum. Therefore, for each (h, b, z, f) , $V_T(h, e, b, s, z, k; f)$ is continuous function in e, s ; and the choice correspondence $M_T^*(h, e, b, s, z, k; f)$ is a non-empty, compact valued and upper hemi-continuous (uhc) correspondence in e, s . From the analogous argument made for $t = T - 2, \dots, 1$ in a backwards manner the claim of the lemma follows. $\square$

We next use some results by Araujo and Mas-Colell [2] to show that M^* is single-valued and continuous except possibly for a set of points in E that has Lebesgue measure zero. Formally, we establish

Lemma 3. $M_t^*(h, e, b, s, z, k; f)$ single-valued and continuous almost everywhere (a.e.) in E_i for each $(t, h, b, z, k; f)$.

Proof. For any given $(t, h, b, s, z, k; f)$ we verify that Assumptions 1 to 4 and the Sondermann Condition on pages 115 – 116 of Araujo and Mas-Colell [2] are satisfied for the objective function corresponding to the individual optimization problem given by

$$F(m, e) := \sum_{(\iota, y)} \left[\sum_x \pi(x|h, t) \left[R_x^{(\iota, y)}(t, e, b, s, z, k; f) + \beta \int_E V_{t+1}(h, e', y, \psi(\iota, x, s), z, k; f) \Phi_t(de') \right] \right] \cdot m^{(\iota, y)}$$

with m and e in the roles of $x \in X$ and $a \in \mathcal{E}$, respectively, as well as Φ_t in the role of the probability measure ν , in Theorem 1 of Araujo and Mas-Colell. If the conditions hold for our model economy, $M_t^*(h, e, b, s, z, k; f)$ is single-valued almost everywhere in e .

- (AM1) Assumption 1: that $(X \times X) \setminus \Delta$ is a Lindelöf space, where $\Delta = \{(x, y) \in X \times X : x = y\}$, which is necessary for a countable open cover, holds. In our case the set of all feasible choices is $X := \overline{M}$ where $\overline{M} := M(h, \bar{e}, b, s, z, k; f)$. To see this note that $e \leq e'$ implies that $M(e, b, s, z, k; f) \subseteq M(e', b, s, z, k; f)$. Note also that X is compact and so is the product space $X \times X$. Moreover, an open cover of $X \times X \setminus \Delta$ has a countable subcover. Consequently, $X \times X \setminus \Delta$ is a Lindelöf space, since the latter is a weakening of compactness (See section 7.2 of Gemignani [15]).
- (AM2) Assumption 2: that $F : X \times \mathcal{E} \rightarrow R$ is a continuous function, holds. As shown in the proof of Lemma 2 the objective function is continuous.
- (AM3) Assumption 3: that for every i , $x \in X$ and $a \in \mathcal{E}$, $\partial_{a_i} F(x, a)$ exists and depends continuously on x and a , holds. Notice that a small change in e has two effects through: (1) its direct effect on the consumption set in the current period; and (2) its direct effect on the probability distribution $\Phi(\cdot, e)$ from which earnings in the next period are drawn. Therefore, the differential with respect to e is

$$\partial_e F(m, e) := \sum_{(\iota, y, x)} u'(c(\iota, y, x)) \cdot m^{(\iota, y, x)}$$

where, to conserve on notation, $m^{(\iota, y, x)}$ denotes $\pi(x|h, t) \cdot m^{(\iota, y)}$. This function varies continuously in e and m by the continuous differentiability hypothesis on $u(c)$, and linearity in m .

More formally, to show continuity of the derivative, pick a convergent sequence, say, $(e_n, m_n) \rightarrow (e, m)$ where $m_n \in M(t, e_n, b, s, z, k; f)$ and $m \in M(t, e, b, s, z, k; f)$. This is possible since $M(t, \cdot, b, s, z, k; f)$ is an uhc correspondence by arguments analogous to those used in Claim 3 of Lemma 2. Therefore, for n sufficiently large, the support

of m_n and m are the same and each element of m_n converges point-wise to that of m . Together with the fact that consumption is linear in e and hence $c_n(\iota, y) \rightarrow c(\iota, y)$ for each (d, y) in the support of m as $e_n \rightarrow e$. Moreover, we have that

$$\begin{aligned}\partial_e F(m_n, e_n) &:= \sum_{(\iota, y, x)} u'(c_n(\iota, y, x)) \cdot m_n^{(\iota, y, x)} \\ &\rightarrow \sum_{(\iota, y, x)} u'(c(\iota, y, x)) \cdot m^{(\iota, y, x)} =: \partial_e F(m, e)\end{aligned}$$

showing the continuity of the derivative.

- (AM4) Assumption 4: that ν is a product probability measure, each factor being absolutely continuous with respect to Lebesgue measure, holds. By assumption our probability measure Φ is absolutely continuous with respect to the Lebesgue measure.
- (AM5) Sondermann Condition (SC): that if $F(x, a) = F(y, a)$, $x \neq y$, then $\partial_{a_i}(F(x, a) - F(y, a)) \neq 0$ for some i , holds. Suppose, to the contrary that $F(m, e) = F(\hat{m}, e)$ and $m \neq \hat{m}$ implies that $\partial F_e(m, e) = \partial F_e(\hat{m}, e)$ for all e . The latter implies that

$$\sum_{(\iota, y, x)} u'(c(\iota, y, x)) \cdot (m^{(\iota, y, x)} - \hat{m}^{(\iota, y, x)}) = 0.$$

From strict positivity of marginal utility this implies that $m^{(d, y)} - \hat{m}^{(d, y)} = 0$ for all (d, y) in the common support or $m = \hat{m}$, which is a contradiction.

Together, items (AM1) to (AM5) verify the hypotheses for Theorem 1 of Araujo and Mas-Colell [2] are satisfied. Consequently, $M_t^*(h, e, b, s, z, k; f)$ is a single-valued correspondence *a.e.* in E .

Next we show that the uhc correspondence M^* that is *a.e.* single-valued in E is continuous *a.e.* in E . To see this, fixing the arguments $(h, b, s, z, k; f)$ and suppressing them, we pick an arbitrary convergent sequence in the domain $e_n \rightarrow e$ and show that $M^*(e_n) \rightarrow M^*(e)$ in e a.e. Let M^* be single-valued at e and $M^*(e)$ be the value. By upper hemi-continuity there exists a subsequence such that $e_{n_k} \rightarrow e$ and $M^*(e_{n_k}) \rightarrow M^*(e)$. Since the limit of any such subsequence is unique, the original sequence converges to the same limit, that is, $M^*(e_n) \rightarrow M^*(e)$. This shows that $M^*(e)$ is continuous at the set of points where it is single-valued. But the latter set has probability one. Therefore, $M^*(e)$ is continuous in e *a.e.*

Hence, the claim of the lemma follows. $\square$

Since we will establish equicontinuity using a Lipschitz condition, the next lemma proves that small changes in s satisfy a Lipschitz condition on decision rules with Lipschitz constant 1 almost everywhere. Given upper hemicontinuity in Lemma 2 and single-valuedness *a.e.* in e in Lemma 3, the result follows from the finiteness of the action set.

Lemma 4. *For a given $(h, b, z, k; f)$ and any s , there exists a $\delta_s(f) > 0$ such that for any $s' \neq s$ with $|s' - s| \leq \delta_s(f)$, $|m^{*(\iota, y)}(h, e, b, s, z, k; f) - m^{*(\iota, y)}(h, e, b, s', z, k; f)| \leq |s - s'|$ *a.e.* in E .*

Proof. Fixing a given (h, b, z, k) and suppressing it to conserve the notation, suppose to the contrary that there exists s such that for any $\delta_s(f) > 0$ and for some $s' \neq s$ with $|s - s'| < \delta_s(f)$ we can find a positive measure set $E_i(s, s'; f)$ of earnings that has the property

$$(9) \quad |m_t^{*(\iota, y)}(e, s; f) - m_t^{*(\iota, y)}(e, s'; f)| > |s - s'| \text{ for each } e \in E_i(s, s'; f),$$

where $E(s, s'; f)$ is the set of e for which both $m_t^{*(\iota, y)}(e, s; f)$ and $m_t^{*(\iota, y)}(e, s'; f)$ are single-valued. This is possible since $m_t^{*(\iota, y)}(\cdot; f)$ is single-valued a.e. in e at both s and s' by Lemma 3.

The following observations lead to the desired contradiction.

(O1) Since $s \neq s'$, the condition in (9) implies that $m_i^{*(\iota, y)}(e, s; f) \neq m_i^{*(\iota, y)}(e, s'; f)$ for each $e \in E_i(s, s'; f)$. This further implies that

$$(10) \quad |m_t^{*(d, y)}(e, s; f) - m_t^{*(d, y)}(e, s'; f)| \geq \epsilon \text{ for each } e \in E_i(s, s'; f),$$

where ϵ is the tremble parameter. This follows since $m_t^{*(\iota, y)}(e, s; f)$ and $m_t^{*(\iota, y)}(e, s'; f)$ are single valued for each $e \in E_i(s, s'; f)$, the action set has a finite number of elements, and the smallest possible difference in probability mass assigned to actions is ϵ .

(O2) By Lemma 3 $m_t^{*(\iota, y)}(e, s; f)$ is single-valued in e . This together with uhc of $m_t^{*(\iota, y)}(e, s; f)$ at s by Lemma 2 imply that for an open ball of radius $\epsilon/2$ around $m_t^{*(\iota, y)}(e, s; f)$ there exists an open ball of radius $\hat{\delta}_s(f) > 0$ around s such that

$$(11) \quad |m_t^{*(\iota, y)}(e, s; f) - m_t^{*(\iota, y)}(e, s'; f)| \leq \epsilon/2$$

for every s' such that $|s - s'| < \hat{\delta}_s(f)$.

(O3) Since condition (9) must hold for any $\delta_s(f) > 0$, if we pick s' in (O1) satisfying $|s - s'| < \hat{\delta}_s(f)$ then (11) in (O2) contradicts (10). $\square$

Next we prove that $P_{h,t}^\iota(s; f)$ in equation (5) is well-defined. Given the continuity result of Lemma 3, the next lemma also establishes the continuity of $P_i(\cdot)$ in s . Intuitively, the integral “smooths out” the discontinuities in $m_t^{*(\iota, y)}(h, \cdot, b, s, z, k; f)$.

Lemma 5. *Given the measure Φ_t and the initial probability μ_0 of good type, the measure $P_{h,t}^\iota(s; f)$ of individuals choosing ι given in equation (5) is well defined for each h, t, ι and $f \in K$. Further, $P_{h,t}^\iota(s; f)$ is continuous in s .*

Proof. For the first part of the lemma, we know that $M_t^*(h, e, b, s, z, k; f)$ is a compact valued and uhc correspondence by Lemma 2. From the Measurable Selection Theorem (see, for example, [28, Theorem 7.6]), there exists a function $m_t^*(h, e, b, s, z, k; f)$, measurable with respect to $\mathcal{B}(E)$, such that $m_t^*(h, e, b, s, z, k; f) \in M_t^*(h, e, b, s, z, k; f)$. Furthermore, $m_i^*(e, \cdot) \leq 1$

and Γ_t is a probability measure. Therefore $m_t^{*(\iota, y)}$ is Γ_t integrable and $\sum_{y \in \mathbb{Y}} \int m_t^{*(\iota, y)}(h, e, b, s, z, k; f) \Gamma_t(de, h, b, z, k)$ exists.

For the second part of the proof, fix t, h, b, z and f , and suppress them to conserve the notation. Pick $\hat{e}$ and $\hat{s}$. Assume that $M^*(\hat{e}, \hat{s})$ is single-valued. Therefore $m_t^*(h, \hat{e}, b, \hat{s}, z, k; f) = M^*(\hat{e}, \hat{s})$. Let $s_n \rightarrow \hat{s}$. We claim that $m^*(\hat{e}, s_n) \rightarrow m^*(\hat{e}, \hat{s})$. Suppose not, then for any $\epsilon > 0$ there exists a subsequence $m^*(\hat{e}, s_{n_k})$ such that $|m^*(\hat{e}, s_{n_k}) - m^*(\hat{e}, \hat{s})| > \epsilon$ for all n_k . But $m^*(\hat{e}, s_{n_k})$ is a selection from $M^*(\hat{e}, s_{n_k})$. So, by the uhc of M_i^* , the subsequence must contain a subsequence converging to a point in $M_i^*(\hat{e}, \hat{s})$. But the latter contains only $m^*(\hat{e}, \hat{s})$. Thus there must be some N such that $|m^*(\hat{e}, s_N) - m^*(\hat{e}, \hat{s})| < \epsilon$, a contradiction. Therefore, $m_t^*(h, \hat{e}, b, s_n, z, k; f) \rightarrow m_t^*(h, \hat{e}, b, \hat{s}, z, k; f)$.

Now consider $P_{h,t}^\iota(s_n; f) = \sum_{y \in \mathbb{Y}} \int m_t^{*(\iota, y)}(h, e, b, s_n, z, k; f) \Gamma_t(de, h, b, z, k)$. Notice that (i) $m_t^{*(\iota, y)}(h, \hat{e}, b, s_n, z, k; f) \rightarrow m_t^{*(\iota, y)}(h, \hat{e}, b, \hat{s}, z, k; f)$ for all e for which $m_t^{*(\iota, y)}(h, \hat{e}, b, \hat{s}, z, k; f)$ is single-valued, and therefore, for e a.e., and (ii) $m_t^{*(\iota, y)}(h, \hat{e}, b, \hat{s}_n, z, k; f) \leq 1$. Therefore, by the Lebesgue Dominated Convergence Theorem that allows to interchange limit and integration

$$\begin{aligned} \lim_n P_{h,t}^\iota(s_n; f) &= \lim_n \sum_{y \in \mathbb{Y}} \int m_t^{*(\iota, y)}(h, \hat{e}, b, s_n, z, k; f) \Gamma_t(de, h, b, z, k) \\ &= \sum_{y \in \mathbb{Y}} \int \lim_n m_t^{*(\iota, y)}(h, \hat{e}, b, s_n, z, k; f) \Gamma_t(de, h, b, z, k) \\ &= \sum_{y \in \mathbb{Y}} \int m_t^{*(\iota, y)}(h, \hat{e}, b, \hat{s}, z, k; f) \Gamma_t(de, h, b, z, k) = P_{h,t}^\iota(\hat{s}; f) \end{aligned}$$

□

Let $\nu^{new} := T(\nu^{old})$ denote the new scoring function obtained from the old scoring functions $f^{old} := (\nu^{old}, \psi^{old})$ by applying the operator T as defined in equations (6) and (7). The updated new function is continuous in s because $P_{h,t}^\iota(s; f^{old})$ is continuous in s by Lemma 5 and from (6) and (7) the new scoring function is continuous in $P_{h,t}^\iota$.

Lemma 6. ν^{new} and ψ^{new} is continuous in s for each ι, x , and $f := (\nu^{old}, \psi^{old})$.

The next lemma establishes that $P_{h,t}^\iota(\cdot; f)$ has Lipschitz constant 1. The proof uses the fact from Lemma 4 that small changes in s yield small changes in decision rules a.e. for any s . We show in the first part of the proof that this implies that small changes in s yield small changes in $P_{h,t}^\iota(\cdot; f)$ at any s and then extend this to all changes in s via an argument similar to a nondifferentiable version of the Mean Value Theorem. In particular, the standard Mean Value Theorem is often used to prove theorems that make global conclusions about a function on an interval starting from local hypotheses about derivatives at points of the interval. Here we extend that

idea to make global conclusions about the Lipschitz constant without assuming differentiability of $P(\cdot; f)$. Further, the lemma establishes that the family of functions $\{P(\cdot; f)\}_{f \in K}$ is uniformly Lipschitz continuous, which uses the following:

Definition 4 (Uniform Lipschitz Continuity). *We say that the family of functions $\{P(\cdot; f)\}_{f \in K}$ is uniformly Lipschitz continuous if each function in the family is Lipschitz continuous and has the same Lipschitz constant for any $f \in K$.*

Lemma 7. *For any given (h, t, ι) and any $f \in K$, $|P_{h,t}^\iota(s; f) - P_{h,t}^\iota(s'; f)| \leq |s - s'|$ whenever $s \neq s'$.*

Proof. First we establish that for any given s and f , there exists a $\delta_s(f)$ such that $|P_{h,t}^\iota(s; f) - P_{h,t}^\iota(s'; f)| \leq |s - s'|$ whenever $|s - s'| \leq \delta_s(f)$ and $s \neq s'$. To see this, fix an f . For any s, s' with $s \neq s'$,

(12)

$$\begin{aligned} & |P_{h,t}^\iota(s; f) - P_{h,t}^\iota(s'; f)| \\ &= \left| \sum_{y \in \mathbb{Y}} \left(\int_E \left(m_{h,t}^{*(\iota, y)}(e, s; f) - m_{h,t}^{*(\iota, y)}(e, s'; f) \right) \Phi_t(de|h, b, z, k) \right) \hat{\Gamma}_t(h, b, z, k) \right| \\ &\leq \sum_{y \in \mathbb{Y}} \left(\int_E \left| m_{h,t}^{*(\iota, y)}(e, s; f) - m_{h,t}^{*(\iota, y)}(e, s'; f) \right| \Phi_t(de|h, b, z, k) \right) \hat{\Gamma}_t(h, b, z, k), \end{aligned}$$

where the equality follows from the definition of $P_{h,t}^\iota(\cdot; f)$ and the decomposition of the joint probability into independent parts: (i) the distribution $\Phi(de)$ over earnings for any other characteristics of an individual is the unconditional distribution induced by Markov process for earnings: and (ii) the distribution over the rest of the state variables $\hat{\Gamma}(\cdot)$. The inequality follows from the Jensen's inequality since $|\cdot|$ is a convex function.

The Lemma 4 gives the existence of a $\delta_s(f) > 0$ such that $\left| m_{h,t}^{*(\iota, y)}(e, s; f) - m_{h,t}^{*(\iota, y)}(e, s'; f) \right| \leq |s - s'|$ a.e. whenever $|s - s'| \leq \delta_s(f)$. For such s and s' , integrating over earnings yields $\int_E \left| m_{h,t}^{*(\iota, y)}(e, s; f) - m_{h,t}^{*(\iota, y)}(e, s'; f) \right| \Phi_t(de) \leq |s - s'|$, which from (12) implies that $|P_{h,t}^\iota(s; f) - P_{h,t}^\iota(s'; f)| \leq |s - s'|$ whenever $|s - s'| \leq \delta_s(f)$.

Next we extend the argument to all $s \neq s'$ and not just those where $|s - s'| \leq \delta_s(f)$. In particular, for any given f , fix $s, s' \in [0, 1]$ with $s \neq s'$ and assume without loss of generality that $s' > s$. For an arbitrary $z \in \mathfrak{R}$ define a function $g_z : [0, 1] \rightarrow \mathfrak{R}$ as a product in the following way:

$$g_z(\tilde{s}) := z \cdot \left(P_{h,t}^\iota(\tilde{s}; f) - P_{h,t}^\iota(s; f) - (\tilde{s} - s) \frac{P_{h,t}^\iota(s'; f) - P_{h,t}^\iota(s; f)}{s' - s} \right).$$

Note that by construction, $g_z(s) = g_z(s') = 0$. Since by Lemma 5 we have $P_{h,t}^\iota(s; f)$ is a continuous function of s , g_z is continuous in $\tilde{s}$. Moreover, restricted to a compact subset $[s, s']$ of $[0, 1]$, g_z is also continuous in $\tilde{s}$ on that subset. Therefore, by the Weierstrass Theorem (see, for example, Aliprantis and Border [1], page 40), there exists an interior point $\xi \in (s, s')$ at which g_z attains a maximum or a minimum. Therefore, there are two cases to consider depending on whether ξ is a minimum or a maximum.

(Case1) g_z attains a minimum at ξ . If ξ is a minimum,

$$(13) \quad \liminf_{\substack{\tilde{s}_n > \xi \\ \tilde{s}_n > \xi}} \frac{g_z(\tilde{s}_n) - g_z(\xi)}{\tilde{s}_n - \xi} \geq 0.$$

This holds because the $\liminf$ is well-defined and for each $\tilde{s}_n$, the numerator is non-negative since ξ is a minimum and the denominator is non-negative since the sequence of $\{\tilde{s}_n\}$ was chosen such that $\tilde{s}_n > \xi$. Using the definition of the function g_z the latter implies that

$$\begin{aligned} g_z(\tilde{s}) - g_z(\xi) &= z \cdot (P_{h,t}^\iota(\tilde{s}; f) - P_{h,t}^\iota(s; f)) \\ &- z \cdot \left((\tilde{s} - \xi) \frac{P_{h,t}^\iota(s'; f) - P_{h,t}^\iota(s; f)}{s' - s} \right) \geq 0 \end{aligned}$$

or, for any s'

$$z \cdot (P_{h,t}^\iota(\tilde{s}; f) - P_{h,t}^\iota(\xi; f)) \geq z \cdot \left((\tilde{s} - \xi) \frac{P_{h,t}^\iota(s'; f) - P_{h,t}^\iota(s; f)}{s' - s} \right).$$

In particular, this condition for any $\tilde{s} > \xi$ and linearity imply that

$$z \cdot \left(\frac{P_{h,t}^\iota(\tilde{s}; f) - P_{h,t}^\iota(\xi; f)}{\tilde{s} - \xi} \right) \geq z \cdot \left(\frac{P_{h,t}^\iota(s'; f) - P_{h,t}^\iota(s; f)}{s' - s} \right).$$

Since this condition is true for any $\tilde{s} > \xi$, then for any sequence of $\tilde{s}_n$ converging to ξ from above we know

$$(14) \quad \liminf_{\substack{\tilde{s}_n > \xi \\ s'_n > \xi}} z \cdot \left(\frac{P_{h,t}^\iota(\tilde{s}_n; f) - P_{h,t}^\iota(\xi; f)}{\tilde{s}_n - \xi} \right) \geq z \cdot \left(\frac{P_{h,t}^\iota(s'; f) - P_{h,t}^\iota(s; f)}{s' - s} \right).$$

Moreover, by the definition of the absolute value function $|\cdot|$,

$$(15) \quad z \cdot \left(\frac{P_{h,t}^\iota(\tilde{s}_n; f) - P_{h,t}^\iota(\xi; f)}{\tilde{s}_n - \xi} \right) \leq |z| \frac{|P_{h,t}^\iota(\tilde{s}_n; f) - P_{h,t}^\iota(\xi; f)|}{|\tilde{s}_n - \xi|}.$$

Since $\tilde{s}_n > \xi$, for sufficiently large n 's we have $|\tilde{s}_n - \xi| < \delta_\xi^f$ and hence by the first part of this proof we know $\frac{|P_{h,t}^\iota(\tilde{s}_n; f) - P_{h,t}^\iota(\xi; f)|}{|\tilde{s}_n - \xi|} \leq 1$. From (15), the latter implies that for all sufficiently large n 's,

$z \cdot \left(\frac{P_{h,t}^\ell(\tilde{s}_n; f) - P_{h,t}^\ell(\xi; f)}{\tilde{s}_n - \xi} \right) \leq |z|$, which combined with (14) yields the desired inequality

$$(16) \quad |z| \geq z \cdot \left(\frac{P_{h,t}^\ell(s'; f) - P_{h,t}^\ell(s; f)}{s' - s} \right).$$

(Case2) If, on the other hand, ξ is a maximum, then by an analogous argument we can show (13) for a sequence $\tilde{s}_n$ converging to ξ from below,

$$\liminf_{\substack{\tilde{s}_n > \xi \\ \tilde{s}_n < \xi}} \frac{g_z(\tilde{s}_n) - g_z(\xi)}{\tilde{s}_n - \xi} \geq 0.$$

Using this condition and following an analogous argument made in (Case1) establishes that (16) also holds in (Case2).

We have established that for an arbitrary z the condition in (16) holds. Therefore, the condition holds for any z . In particular, it holds for $z = \left(\frac{P_{h,t}^\ell(s'; f) - P_{h,t}^\ell(s; f)}{s' - s} \right)$; that is,

$$\left| \frac{P_{h,t}^\ell(s'; f) - P_{h,t}^\ell(s; f)}{s' - s} \right| \geq \left(\frac{P_{h,t}^\ell(s'; f) - P_{h,t}^\ell(s; f)}{s' - s} \right)^2$$

or, equivalently, $\left| \frac{P_{h,t}^\ell(s'; f) - P_{h,t}^\ell(s; f)}{s' - s} \right| \leq 1$. Rearranging now shows that for any given $f \in K$, $\left| P_{h,t}^\ell(s'; f) - P_{h,t}^\ell(s; f) \right| \leq |s' - s|$. The uniform Lipschitz property follows from the independence of this condition from a particular f . $\square$

Having established that $P_{h,t}^\ell(\cdot; f)$ is Lipschitz for any f , we now need to establish that $T(f)$, or in particular ν , which are functions of $P_{h,t}^\ell(\cdot; f)$, are also Lipschitz in s . The next lemma establishes certain properties of functions of Lipschitz functions.

Lemma 8. *[Algebra of lipschitz functions] If $g : [0, 1] \rightarrow [0, 1]$ and $\hat{g} : [0, 1] \rightarrow [0, 1]$ are Lipschitz continuous functions with the same Lipschitz constant Z , then: (i) their product $h := g\hat{g}$ is also Lipschitz continuous with Lipschitz constant is $2Z$; and (ii) their sum $h := g + \hat{g}$ is also Lipschitz continuous and its Lipschitz constant is $2Z$.*

Proof. Part (i). We must show that there exists a $\hat{Z} > 0$ such that for any s, s' with $s \neq s'$ $|h(s) - h(s')| \leq \hat{Z}|s - s'|$ and $\hat{Z} = 2Z$. Note that

$$\begin{aligned}
|h(s) - h(s')| &= |g(s)\hat{g}(s) - g(s')\hat{g}(s')| \\
&= |g(s)\hat{g}(s) - g(s)\hat{g}(s') + g(s)\hat{g}(s') - g(s')\hat{g}(s')| \\
&\leq |g(s)\hat{g}(s) - g(s)\hat{g}(s')| + |g(s)\hat{g}(s') - g(s')\hat{g}(s')| \\
&= g(s)|\hat{g}(s) - \hat{g}(s')| + \hat{g}(s')|g(s) - g(s')| \\
&\leq (g(s) + \hat{g}(s'))Z|s - s'| \\
&\leq \hat{Z}|s - s'|, \text{ where } \hat{Z} = 2Z.
\end{aligned}$$

The first equality follows from the definition of h . The second by adding and subtracting a term. The third equality follows since $g(s)$ and $\hat{g}(s)$ are non-negative. The first inequality uses the triangle inequality, the second uses the fact that g and $\hat{g}$ are Lipschitz continuous with Lipschitz constant Z . The last inequality uses the fact that g and $\hat{g}$ take values in $[0, 1]$. Part (ii). A similar (even simpler) argument to above. $\square$

Lemma 9. $\{\nu_t(s, \iota; f)\}_{f \in K}$ is uniformly Lipschitz continuous.

Proof. Fix an arbitrary (h, t, ι) . By Lemma 7, $P_{h,t}^\iota(s; f)$ is uniformly Lipschitz continuous. By Lemma 8, $g(s; f) := P_{h,t}^\iota(s; f) \cdot s$ is Lipschitz continuous since both $P_{h,t}^\iota(s; f)$ and the identity map $s \rightarrow s$ are both Lipschitz continuous. Note that from the definition of $\nu_t(s, \iota; f)$ in (6), it is of the form $\frac{g(s; f)}{\hat{g}(s; f)}$ where $g(s; f)$ and $\hat{g}(s; f)$ are Lipschitz continuous (by Lemma 8 being finite sums of functions of the form $P_{h,t}^\iota(s; f) \cdot s$). Moreover, $g(s; f)$ belongs to a family that is uniformly Lipschitz continuous. This is because $g \in \{P_{h,t}^\iota(s; f) \cdot s\}_{\{f \in K\}}$ where $\{P_{h,t}^\iota(\cdot; f)\}_{\{f \in K\}}$ is a uniformly Lipschitz continuous family and the mapping $s \rightarrow P_{h,t}^\iota(s; f)s$ is Lipschitz continuous. Therefore, $g(\cdot)$ and $\hat{g}(\cdot)$ with Lipschitz constants κ and $\hat{\kappa}$ belong to uniformly Lipschitz continuous families with a constant, say, $\bar{\kappa} \geq \max\{\kappa, \hat{\kappa}\}$.

Consider

$$\begin{aligned}
|\nu(s) - \nu(s')| &= \left| \frac{g(s)}{\hat{g}(s)} - \frac{g(s')}{\hat{g}(s')} \right| \\
&= \left| \frac{g(s)\hat{g}(s') - \hat{g}(s)g(s')}{\hat{g}(s)\hat{g}(s')} \right| \\
&\leq \frac{|g(s)\hat{g}(s') - \hat{g}(s)g(s')|}{D} \\
&= \frac{|g(s)\hat{g}(s') - g(s')\hat{g}(s') + g(s')\hat{g}(s') - \hat{g}(s)g(s')|}{D}
\end{aligned}$$

$$\begin{aligned}
&\leq \frac{\hat{g}(s') |g(s) - g(s')| + g(s') |\hat{g}(s) - \hat{g}(s')|}{D} \\
&\leq \frac{(\hat{g}(s') + g(s')) \bar{\kappa} |s - s'|}{D} \\
&\leq \tilde{\kappa} |s - s'|, \text{ where } \tilde{\kappa} = \frac{2\bar{\kappa}}{D}
\end{aligned}$$

where $D := \inf_{\{(h,t,\iota) \in \{0,1\} \times T \times \{0,1\}\}} \{P_{h,t}^\iota(s; f)\}$. The first and second equalities are effectively by definition and the third follows by adding and subtracting a term. The first inequality uses the fact that $P_{h,t}^\iota(s; f) > 0$ since all actions are feasible given the assumption that $\bar{e} + \ell_{\min} - \ell_{\max} > 0$. The second inequality is a consequence of applying the triangle inequality and recognizing that g and $\hat{g}$ are non-negative. The third inequality results from Lipschitz continuity of g and $\hat{g}$ with the same constant. Finally, the last inequality follows from the fact that g and $\hat{g}$ take values in $[0, 1]$. Since (h, t, ι) is arbitrary, this shows that $\{\nu_t(s, \iota; f)\}_{\{f \in K\}}$ is a uniformly Lipschitz continuous family. $\square$

We now establish the properties of the operator T in step (S4). These properties are required by Schauder's fixed point theorem which is the key ingredient of the main existence result in Theorem 1 below.

Lemma 10. *For the operator $T : K \rightarrow F$ defined in step 3: (i) $T(K) \subseteq K$ (ii) $T(K)$ is continuous in the sup-norm; and (iii) $T(K)$ is an equicontinuous family.*

Proof. To see part (i), starting with a continuous function $\nu^{old} \in K$, the application of the operator T through (5) and (6) updates to a new interim type scoring function $\nu^{new} = T(\nu^{old})$ which is continuous by Lemma 6 and has the properties of the scoring function ν given in step (S2). Therefore, $T(\nu^{old}) = \nu^{new} \in K$. Since $\nu^{old} \in K$ is arbitrary, we have that $T(K) \subseteq K$.

To see part (ii), pick an arbitrary sequence of functions that converges in K , say, $f_n \rightarrow f$ in the sup-norm. We need to show that $f_n^{new} := T f_n$ converges to $f^{new} := T f$ in the sup-norm, that is $\sup_{s \in [0,1]} |f_n^{new}(s) - f^{new}(s)| \rightarrow 0$ as $n \rightarrow \infty$. By the definition of convergence in the sup-norm, for an arbitrary $s \in [0, 1]$, $f_n(s) \rightarrow f(s)$.

Observe that for an arbitrary s a variation in $f(s)$ as f changes in K and a variation in s for a given f has the same effects on the feasible choice set and on the objective function for the individual decision problem given by (3). More formally, from Definition 1 (i.e. the condition that defines the feasible action set B) and Lemma 1, we know that the budget set varies continuously with f for a given s in a similar way as it varies continuously with s for a given f by the continuity of ν in s and that of ψ and p in ν . It therefore follows from Definition 2 that these variations have the same continuous effect on the feasible choice set. By analogous arguments, they have the same effect on objective function. Therefore, the arguments made

in Lemmas 2, 3, 5, and 6 for s for an arbitrary f work analogously for $f \in K$ for an arbitrary s . This shows, in particular, that $f_n^{new}(s) \rightarrow f^{new}(s)$ for each s . Moreover, since the domain of the functions is compact, the convergence is uniform and hence $Tf_n \rightarrow Tf$ in the sup-norm, showing the continuity of T .

To see part (iii), since $Tf(s) = \nu(s; f)$ and $T(K) = \{\nu(\cdot; f)\}_{f \in K}$, it follows that $T(K)$ is a uniformly Lipschitz continuous family by lemmas 9. But establishing uniform Lipschitz continuity is sufficient for establishing equicontinuity. In particular, by definition, a family of functions K is equicontinuous if given an $\epsilon > 0$, there exists a (single) $\delta > 0$ such that $|f(s) - f(s')| < \epsilon$ whenever $|s - s'| < \delta$ for all $f \in K$ (see, for example, Kolmogorov and Fomin, page 102). But this condition is implied by uniform Lipschitz property. To see this, let K be a uniformly Lipschitz continuous family, with a Lipschitz constant, say, κ . Therefore, $|f(s) - f(s')| < \kappa|s - s'|$ for all $f \in K$. For a given ϵ , choosing $\delta = \frac{\epsilon}{\kappa}$ shows that the equicontinuity property is satisfied. $\square$

Having established the key properties of the operator T , we end with the main existence result.

Theorem 1. *A recursive competitive equilibrium specified in Definition 3 exists.*

Proof. The set of functions K as specified in step (S1) and step (S2) is a convex and closed subset of a continuous functions defined on Ω . These properties of K together with the properties of the operator T as defined in step (S3) that are established in Lemma 10 constitute sufficient conditions for Schauder's fixed point theorem. Consequently there exists a type scoring function ν^* that is a fixed point of the operator T , i.e., $T(\nu^*) = (\nu^*)$. The existence of a fixed point to this operator establishes the existence of a competitive equilibrium as specified in Definition 3. The claim then follows by verifying conditions (D1) to (D3) in Definition 3.

Given a scoring function ν^* that is a fixed point of the operator T , a pricing function p^* is found by solving the zero profit condition for each (t, ι, s') : $W(t, \iota, \nu^*; p) = 0$, verifying condition (D2) in Definition 3. Moreover, given ν^* health scoring function ψ^* is a solution to (7). Given these price and score functions (p^*, ψ^*) and individual characteristics (t, h, e, b, s) , from Definition 1 and 2, $M(t, e, b, s; q, p^*, \psi^*)$ defines the feasible choice set. From Lemma 2, the selection $m_t^*(h, e, b, s, z, k; q, p^*, \psi^*)$ is feasible and solves the decision problem in (3), which verifies condition (D1). Finally, by the definition of the operator T and the existence of a fixed point of that operator, for $m_t^*(h, e, b, s, z, k; q, p^*, \psi^*)$, ν^* solves (6) which in turn determines and ψ^* from (7), verifying the condition (D3). $\square$

9.2. Algorithm to compute equilibrium.

- (1) Set grid points for endowments, savings and scores.

- (a) There are 18 endowment grid points equally spaced between the bounds of the earning distribution.
 - (b) There are 200 savings grid points that are spaced densely for low values and sparsely for high values.
 - (c) There are 20 score grid points equally spaced between ϵ and $1 - \epsilon$.
- (2) Start iteration $j = 1$ with a set of initial guesses for the score function ν^j . Given this function from (4) and (6)-(7) compute premium and score updating functions, respectively.
 - (3) Given the score and premium functions and the individual state (t, h, e, b, s, z, k) solve for the feasible actions set $B^j(t, e, b, s, z, k; q, p^j, \psi^j)$.
 - (4) Solve for $V_t(h, e, b, s, z, k; q, p^j, \psi^j)$ starting at $t = T$ and going backwards until $t = 1$. If s' is not on grids, linear interpolation is used for $W_{t+1}(h, \iota, y, s')$ where

$$W_{t+1}(h, \iota, y, s') = \sum_x \pi(x|h, t) \left[\beta \int_E V_{t+1}(h, e', y, \psi(\iota, x, s), z, k; q, p^j, \psi^j) \Phi_t(de'|e) \right]$$

- (5) The solution gives the set of optimal decision rule $m_i^j(t, e, b, s, z, k; q, p^j, \psi^j) \in M_i^j(t, e, b, s, z, k; q, p^j, \psi^j)$.
- (6) Given $m_i^j(t, e, b, s, z, k; q, p^j, \psi^j)$, calculate $\nu_t^{j+1}(s, \iota; q, p^j, \psi^j)$ via (5) and (6).
- (7) Given $\nu_t^{j+1}(s, \iota; q, p^j, \psi^j)$, calculate $p_t^{j+1}(s')$ and $\psi_t^{j+1}(s, \iota, x; q, p^j, \psi^j)$ via (4) and (7), respectively.
- (8) Start new iteration $j + 1$ by using $q, p_t^{j+1}(s')$, and $\psi_t^{j+1}(s, \iota, x)$ as the new set of initial guesses. Repeat until they converge.