Implications of Labor Market Frictions for Risk Aversion and Risk Premia

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Abstract

A flexible labor margin allows households to absorb shocks to asset values with changes in hours worked as well as changes in consumption. This ability to absorb shocks along both margins can greatly alter the household's attitudes toward risk, as shown by Swanson (2012a). The present paper analyzes how those results are affected by labor market frictions and shows that: 1) risk aversion is higher in recessions, 2) risk aversion is higher in more frictional labor markets, and 3) risk aversion is higher for households that are less employable. Quantitatively, labor market flow rates in the U.S. and other OECD countries are large relative to the discount rate, implying that the cost of labor market frictions is small because frictions only delay adjustment. Thus, the frictionless formulas in Swanson (2012a,b) appear to be very good approximations in frictional labor markets as well.

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1. Introduction

A number of recent studies have focused on bringing standard dynamic macroeconomic models into closer agreement with basic asset pricing facts, such as the equity premium or the long-term bond premium.[1] In these models—indeed, in any consumption-based asset-pricing model—a crucial parameter is risk aversion, the compensation that households require to bear an increase in risk to their assets. At the same time, a key feature of standard dynamic macroeconomic models is that households have some ability to vary their labor supply. A fundamental difficulty with this line of research, then, is that much of what is known about risk aversion has been derived under the assumption that household labor is exogenously fixed. For example, Arrow (1964) and Pratt (1965) define absolute and relative risk aversion, $-u''(c)/u'(c)$ and $-c\,u''(c)/u'(c)$, in a static model without labor (or, equivalently, in which labor is exogenously fixed). Similarly, Epstein and Zin (1989) and Weil (1989) define risk aversion for generalized recursive preferences in a dynamic model without labor (or in which labor is fixed).

Swanson (2012a) derives closed-form expressions for risk aversion in a general dynamic equilibrium framework in which households face a perfectly frictionless market for their labor. Intuitively, a flexible labor margin gives households the ability to absorb shocks to asset values with changes in hours worked as well as changes in consumption. This ability to absorb shocks along either or both margins can greatly alter the household's attitudes toward risk. For example, with period utility function $u(c_t, l_t) = c_t^{1-\gamma}/(1-\gamma) - \eta l_t$, the quantity $-c\,u_{11}/u_1 = \gamma$ is often referred to as the household's coefficient of relative risk aversion, but in fact the household is *risk neutral* with respect to gambles over asset values or wealth (Swanson, 2012a). Intuitively, the household is indifferent at the margin between using labor or consumption to absorb a shock to asset values, and the household in this example is clearly risk neutral with respect to gambles over hours. More generally, when $u(c_t, l_t) = c_t^{1-\gamma}/(1-\gamma) - \eta l_t^{1+\chi}/(1+\chi)$, risk aversion equals $(\gamma^{-1} + \chi^{-1})^{-1}$, a combination of the parameters on the household's consumption and labor margins, reflecting the fact that the household absorbs shocks along both margins.

The present paper analyzes how those results are affected when labor markets are frictional, as in Shimer (2005, 2010). In that case, risk aversion lies somewhere in between the fixed-labor and flexible-labor cases (between γ and $(\gamma^{-1} + \chi^{-1})^{-1}$ in the example above). The present paper derives the corresponding closed-form expressions for risk aversion with frictional labor markets

[1] See, e.g., Boldrin, Christiano, and Fisher (2001), Tallarini (2000), Rudebusch and Swanson (2008, 2012), Uhlig (2007), Van Binsbergen et al. (2012), Backus, Routledge, and Zin (2008), and Gourio (2011, 2012).

and shows that those expressions depend on the ratio of labor market flow rates to the household's discount rate. Intuitively, labor market frictions only *delay*, and do not prevent, the household's labor adjustment; thus, a lower discount rate implies that frictions are less of a constraint on the household's ability to use labor supply to insure itself from asset value fluctuations.

The closed-form expressions for risk aversion derived in the present paper have three main implications: First, risk aversion is higher in recessions, when unemployment is higher. Second, risk aversion is greater in more frictional labor markets, such as Continental Europe. And third, risk aversion is higher for households that are less employable, such as retired households, households with less education, and households that face discrimination in the labor market. In all of these cases, it is more difficult for the household to vary its employment in response to shocks, and so more of the burden of asset fluctuations passes through to consumption.

Quantitatively, labor market flow rates in and out of employment and unemployment are large in the U.S. and other OECD countries relative to the discount rate. This implies that households are patient enough for labor market frictions to be only a minor constraint on the household's ability to use labor supply to insure itself from asset fluctuations. In other words, risk aversion in frictional labor markets is quantitatively very similar to risk aversion in frictionless labor markets, considered in Swanson (2012a,b). As a result, the formulas for risk aversion derived in those papers seem to be very good approximations more generally.

There are a few previous studies that extend the Arrow-Pratt definition beyond the one-good, one-period case. Kihlstrom and Mirman (1974) provide an early example of the difficulties involved. In a static, multiple-good setting, Stiglitz (1969) measures risk aversion using the household's indirect utility function rather than utility itself, essentially a special case of Proposition 1 of the present paper. Constantinides (1990) measures risk aversion in a dynamic endowment economy (i.e., with fixed labor) using the household's value function, another special case of Proposition 1. Boldrin, Christiano, and Fisher (1997) apply Constantinides' definition to some very simple endowment economy models for which they can compute closed-form expressions for the value function, and hence risk aversion. The present paper builds on these studies by deriving closed-form solutions for risk aversion in dynamic equilibrium models in general, demonstrating the importance of the labor margin, and showing how labor market frictions affect those results.

The remainder of the paper proceeds as follows. Section 2 defines a general dynamic equilibrium framework with labor market frictions faced by a household. Section 3 derives closed-form expressions for risk aversion in that framework. Section 4 discusses and proves the implications of labor market frictions for risk aversion described above. The quantitative importance of these results is explored in Section 5. Section 6 considers some numerical examples that explore risk aversion away from the steady state, and the equity premium. Section 7 extends the analysis to the case of balanced growth. Section 8 reviews the general implications and concludes. An Appendix provides details of the model, proofs, and numerical solution methods that are outlined in the main text.

2. Dynamic Equilibrium Framework with Labor Market Frictions

2.1 The Household's Optimization Problem and Value Function

Time is discrete and continues forever. At each time t, the household seeks to maximize the expected present discounted value of utility flows,

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[U(c_\tau) - V(l_\tau + u_\tau) \right], \tag{1}$$

where E_t denotes the mathematical expectation conditional on the household's information set at time t, $\beta \in (0,1)$ is the household's discount factor, and c_τ, l_τ, and u_τ denote the household's state-contingent plans for future consumption, labor, and unemployment at time τ, with the explicit state-dependence of these plans suppressed to reduce notation. Let Ω_c denote the domain of c_τ and Ω_l the domain of $l_\tau + u_\tau$.

Assumption 1. *The function $U\colon \Omega_c \to \mathbb{R}$ is increasing, twice-differentiable, and strictly concave, and $V\colon \Omega_l \to \mathbb{R}$ is increasing, twice-differentiable, and strictly convex.*

The labor market is characterized by search and matching. Household labor l_t is a state variable rather than a choice variable, evolving at each time t according to

$$l_{t+1} \;=\; (1-s)l_t + f(\theta_t)u_t, \tag{2}$$

where $s \in [0,1]$ denotes a constant exogenous rate of job destruction, θ_t is a Markovian state vector that is exogenous to the household and characterizes the state of the aggregate economy at time t, and f is a function of the aggregate state that gives the propensity of finding a job per unit of household unemployment.

A detailed microfoundation for the household's preferences in (1) is tangential to the present paper, but is provided in the Appendix. Briefly, the household consists of a continuum of individuals of unit mass who pool their income. At each time t, an individual who is not employed can either search for a job by entering the unemployment pool, or stay home and produce nonmarket goods and services (which can be thought of as including "leisure"). The household's home production function is increasing and concave in the number of individuals staying at home; as a result, V in (1) is increasing and convex in the number of workers not at home, $l_t + u_t$.[2]

In each period t, the household chooses c_t and u_t (and a state-contingent plan for future c_τ and u_τ) to maximize (1), subject to the labor market friction (2), the flow budget constraint

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t, \tag{3}$$

in each period t, and the no-Ponzi-scheme condition

$$\lim_{T\to\infty} \prod_{\tau=t}^{T} (1 + r_{\tau+1})^{-1} a_{T+1} \geq 0, \tag{4}$$

where a_t denotes the household's beginning-of-period assets and w_t, r_t, and d_t denote the real wage, interest rate, and net transfer payments to the household in each period t, respectively.

The exogenous Markov state vector θ_t governs the processes for w_t, r_t, and d_t. Before choosing c_t and u_t in each period t, the household observes θ_t and hence w_t, r_t, and d_t. The state vector and information set of the household's optimization problem at each date t is thus $(a_t, l_t; \theta_t)$, where a_t and l_t are endogenous and θ_t is exogenous to the household. Let X denote the domain of $(a_t, l_t; \theta_t)$, and $\Gamma : X \to \Omega$ describe the set-valued correspondence of feasible choices for (c_t, u_t) for each given $(a_t, l_t; \theta_t)$, where Ω denotes the domain of (c_t, u_t).

In addition to Assumption 1, a few more technical conditions are required to ensure the value function for the household's optimization problem exists and satisfies the Bellman equation (Stokey and Lucas (1990), Alvarez and Stokey (1998), and Rincón-Zapatera and Rodríguez-Palmero (2003) give different sets of such sufficient conditions). The details of these conditions are tangential to the present paper, so we simply assume:

Assumption 2. *The value function $W : X \to \mathbb{R}$ for the household's optimization problem exists and satisfies the Bellman equation*

$$W(a_t, l_t; \theta_t) \; = \max_{(c_t, u_t)\in\Gamma(a_t, l_t;\theta_t)} U(c_t) - V(l_t + u_t) \; + \; \beta E_t W(a_{t+1}, l_{t+1}; \theta_{t+1}), \tag{5}$$

where l_{t+1} is given by equation (2), and a_{t+1} by equation (3).

[2] The labor market search literature often assumes that household leisure or home production is linear in the number of workers staying at home (e.g., Shimer, 2010). We require strict convexity of V in Assumption 1 to guarantee the uniqueness of the household's optimal choice of (c_t, u_t) at each time t, discussed below. Intuitively, the case of a linear V can be approximated with a V having infinitesimal convexity.

Together, Assumptions 1–2 guarantee the existence of a unique optimal choice for (c_t, u_t) at each point in time, given $(a_t, l_t; \theta_t)$. Let $c_t^* \equiv c^*(a_t, l_t; \theta_t)$ and $u_t^* \equiv u^*(a_t, l_t; \theta_t)$ denote the household's optimal choices of c_t and u_t as functions of the state $(a_t, l_t; \theta_t)$. Then W can be written as

$$W(a_t, l_t; \theta_t) \ = \ U(c_t^*) - V(l_t^* + u_t^*) \ + \ \beta E_t W(a_{t+1}^*, l_{t+1}^*; \theta_{t+1}), \tag{6}$$

where $a_{t+1}^* \equiv (1 + r_t)a_t + w_t l_t^* + d_t - c_t^*$ and $l_{t+1}^* = (1 - s)l_t + f(\theta_t)u_t^*$. These solutions are also assumed to be interior:

Assumption 3. *For any $(a_t, l_t; \theta_t) \in X$, the household's optimal choice (c_t^*, u_t^*) exists, is unique, and lies in the interior of $\Gamma(a_t, l_t; \theta_t)$.*

Intuitively, Assumption 3 requires the partial derivatives of U and V to grow sufficiently large toward the boundary that only interior solutions for c_t^* and u_t^* are optimal for all $(a_t, l_t; \theta_t) \in X$.

Assumptions 1–3 guarantee that W is continuously differentiable with respect to a and satisfies the Benveniste-Scheinkman equation, but we will require slightly more than this below:

Assumption 4. *For any $(a_t, l_t; \theta_t)$ in the interior of X, the second derivative of W with respect to its first argument, $W_{11}(a_t, l_t; \theta_t)$, exists.*

Assumption 4 also implies differentiability of the optimal policy functions, c^* and u^*, with respect to a_t. Santos (1991) provides relatively mild sufficient conditions for this assumption to be satisfied; intuitively, U and V must be strongly concave.

2.2 Representative Household and Steady State Assumptions

Up to this point, the analysis has focused on a single household in isolation, leaving the other households of the model and the production side of the economy unspecified. Implicitly, the other households and production sector jointly determine the process for θ_t (and hence w_t, r_t, and d_t), and much of the analysis below does not need to be any more specific about these processes than this. However, to move from general expressions for risk aversion to more concrete, closed-form expressions, three standard assumptions from the macroeconomics literature are adopted:[3]

Assumption 5. *The household is* infinitesimal.

Assumption 6. *The household is* representative.

[3] Alternative assumptions about the nature of the other households in the model or the production sector may also allow for closed-form expressions for risk aversion. However, the assumptions used here are standard and thus the most natural to pursue.

Assumption 7. *The model has a* nonstochastic steady state, *at which* $x_t = x_{t+k}$ *for all* $k = 1, 2, \ldots,$ *and* $x \in \{c, u, l, a, w, r, d, \theta\}$.

Assumption 5 implies that an individual household's choices for c_t and u_t have no effect on the aggregate quantities w_t, r_t, d_t, and θ_t. Assumption 6 implies that, when the economy is at the nonstochastic steady state, any individual household finds it optimal to choose the steady-state values of c and u given a and θ. Throughout the text, a variable without its time subscript t is used to denote its steady-state value.[4]

It is important to note that Assumptions 6–7 do not prohibit offering an individual household a hypothetical gamble of the type described below. The steady state of the model serves only as a reference point around which the *aggregate* variables w, r, d, and θ and the *other households'* choices of c, u, a and l can be predicted with certainty. This reference point is important because it is there that closed-form expressions for risk aversion can be computed.

Finally, many dynamic models do not have a steady state *per se,* but rather a balanced growth path. The results below carry through essentially unchanged to the case of balanced growth. For ease of exposition, Sections 3–5 restrict attention to the case of a steady state, while Section 6 shows the adjustments required under the more general:

Assumption 7′. *The model has a* balanced growth path *that can be renormalized to a nonstochastic steady state after a suitable change of variables.*

3. Risk Aversion

3.1 The Coefficient of Absolute Risk Aversion

The household's attitudes toward risk at time t generally depend on the household's state vector at time t, $(a_t, l_t; \theta_t)$. Given this state, the household's aversion to a hypothetical one-shot gamble in period t of the form

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma\varepsilon_{t+1} \tag{7}$$

can be considered, where ε_{t+1} is a random variable representing the gamble, with bounded support $[\underline{\varepsilon}, \overline{\varepsilon}]$, mean zero, unit variance, independent of θ_τ for all times τ, and independent of a_τ, l_τ, c_τ, and u_τ for all $\tau \leq t$. A few words about (7) are in order: First, the gamble is dated $t + 1$ to

[4] Let the exogenous state θ_t contain the variances of any shocks to the model, so that $(a, l; \theta)$ denotes the nonstochastic steady state, with the variances of any shocks (other than the hypothetical gamble described in the next section) set equal to zero; $c(a, l; \theta)$ corresponds to the household's optimal consumption choice at the nonstochastic steady state, etc.

clarify that its outcome is not in the household's information set at time t. Second, c_t cannot be made the subject of the gamble without substantial modifications to the household's optimization problem, because c_t is a choice variable under control of the household at time t. However, (7) is clearly equivalent to a one-shot gamble over net transfers d_t or asset returns r_t, both of which are exogenous to the household. Indeed, thinking of the gamble as being over r_t helps to illuminate the connection between (7) and the price of risky assets, which will be discussed further in Section 4. As shown there, the household's aversion to the gamble in (7) is directly linked to the premium households require to hold risky assets.

Following Arrow (1964) and Pratt (1965), one can ask what one-time fee μ the household would be willing to pay in period t to avoid the gamble in (7):

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - \mu. \tag{8}$$

The quantity μ that makes the household just indifferent between (7) and (8), for infinitesimal σ and μ, is the household's coefficient of absolute risk aversion.[5] Formally, this corresponds to the following definition:

Definition 1. *Let $(a_t, l_t; \theta_t)$ be an interior point of X. Let $\widetilde{W}(a_t, l_t; \theta_t; \sigma)$ denote the value function for the household's optimization problem inclusive of the one-shot gamble* (7)*, and let $\mu(a_t, l_t; \theta_t; \sigma)$ denote the value of μ that satisfies $W(a_t - \frac{\mu}{1+r_t}; \theta_t) = \widetilde{W}(a_t, l_t; \theta_t; \sigma)$. The household's* coefficient of absolute risk aversion *at $(a_t, l_t; \theta_t)$, denoted $R^a(a_t, l_t; \theta_t)$, is given by $R^a(a_t, l_t; \theta_t) = \lim_{\sigma \to 0} \mu(a_t, l_t; \theta_t; \sigma)/(\sigma^2/2)$.*

In Definition 1, $\mu(a_t, l_t; \theta_t; \sigma)$ denotes the household's "willingness to pay" to avoid a one-shot gamble of size σ in (7). As in Arrow (1964) and Pratt (1965), R^a denotes the limit of the household's willingness to pay per unit of variance as this variance becomes small. Note that $R^a(a_t, l_t; \theta_t)$ depends on the economic state because $\mu(a_t, l_t; \theta_t; \sigma)$ depends on that state. Proposition 1 shows that $\widetilde{W}(a_t, l_t; \theta_t; \sigma)$, $\mu(a_t, l_t; \theta_t; \sigma)$, and $R^a(a_t, l_t; \theta_t)$ in Definition 1 are well-defined and that $R^a(a_t, l_t; \theta_t)$ equals the "folk wisdom" value of $-W_{11}/W_1$:[6]

Proposition 1. *Let $(a_t, l_t; \theta_t)$ be an interior point of X. Given Assumptions 1–6, $\widetilde{W}(a_t, l_t; \theta_t; \sigma)$, $\mu(a_t, l_t; \theta_t; \sigma)$, and $R^a(a_t, l_t; \theta_t)$ exist and*

$$R^a(a_t, l_t; \theta_t) \;=\; \frac{-\,E_t W_{11}(a^*_{t+1}, l^*_{t+1}; \theta_{t+1})}{E_t W_1(a^*_{t+1}, l^*_{t+1}; \theta_{t+1})}\,, \tag{9}$$

[5] Discussion of relative risk aversion is deferred until the next subsection because defining total household wealth is complicated by the presence of human capital—that is, the household's labor income.

[6] See, e.g., Constantinides (1990), Farmer (1990), Campbell and Cochrane (1999), and Flavin and Nakagawa (2008). For the more general case of Epstein-Zin (1990) preferences, equation (9) no longer holds and there is no folk wisdom; see Swanson (2012b) for the more general formulas corresponding to that case.

where W_1 and W_{11} denote the first and second partial derivatives of W with respect to its first argument. Given Assumptions 6–7, (9) *can be evaluated at the steady state to yield*

$$R^a(a, l; \theta) \;=\; \frac{-W_{11}(a, l; \theta)}{W_1(a, l; \theta)}\,. \tag{10}$$

Proof: See Appendix.

Equations (9)–(10) are essentially Constantinides' (1990) definition of risk aversion, and have obvious similarities to Arrow (1964) and Pratt (1965). Here, of course, it is the curvature of the value function W with respect to assets that matters, rather than the curvature of U with respect to consumption.[7]

A practical difficulty with Proposition 1 is that closed-form expressions for W do not exist in general, even for the simplest dynamic models with labor. One can solve this problem by observing that W_1 and W_{11} often can be computed even when closed-form solutions for W cannot be. For example, the Benveniste-Scheinkman equation,

$$W_1(a_t, l_t; \theta_t) = (1 + r_t)\, U'(c_t^*), \tag{11}$$

states that the marginal value of a dollar of assets equals the marginal utility of consumption times $1 + r_t$ (the interest rate appears here because beginning-of-period assets in the model generate income in period t). In (11), U' is a known function. Although a closed-form solution for the function c^* is not known in general, the point c_t^* often is known—for example, when it is evaluated at the nonstochastic steady state, c. Thus, one can compute W_1 at the nonstochastic steady state by evaluating (11) at that point.

The second derivative W_{11} can be computed by noting that equation (11) holds for general a_t; hence it can be differentiated to yield

$$W_{11}(a_t; \theta_t) \;=\; (1 + r_t)\, U''(c_t^*)\, \frac{\partial c_t^*}{\partial a_t}\,. \tag{12}$$

All that remains is to find the derivative $\partial c_t^* / \partial a_t$.

Intuitively, $\partial c_t^* / \partial a_t$ should not be too difficult to compute: it is just the household's marginal propensity to consume today out of a change in assets, which we can deduce from the household's Euler equation and budget constraint. Differentiating the Euler equation,

$$U'(c_t^*) \;=\; \beta E_t (1 + r_{t+1})\, U'(c_{t+1}^*), \tag{13}$$

[7] Arrow (1964) and Pratt (1965) occasionally refer to utility as being defined over "money", so one could argue that they always intended for risk aversion to be measured using indirect utility or the value function.

with respect to a_t yields[8]

$$U''(c_t^*)\,\frac{\partial c_t^*}{\partial a_t} \;=\; \beta E_t(1+r_{t+1})\,U''(c_{t+1}^*)\,\frac{\partial c_{t+1}^*}{\partial a_t}\,. \tag{14}$$

Evaluating (14) at steady state, $\beta = (1+r)^{-1}$ and the $U''(c)$ factors cancel, giving

$$\frac{\partial c_t^*}{\partial a_t} \;=\; \frac{\partial c_{t+1}^*}{\partial a_t} \;=\; \frac{\partial c_{t+k}^*}{\partial a_t}\,, \quad k = 1, 2, \ldots \tag{15}$$

In other words, starting from steady state, whatever is the change in the household's consumption today, it must be the same as the change in consumption tomorrow, and the change in consumption at each future date $t+k$.[9]

The household's budget constraint is implied by asset accumulation equation (3) and no-Ponzi condition (4). Differentiating (3) with respect to a_t, evaluating at steady state, and applying (4) gives

$$\sum_{k=0}^{\infty} \frac{1}{(1+r)^k} \left[\frac{\partial c_{t+k}^*}{\partial a_t} - w \frac{\partial l_{t+k}^*}{\partial a_t} \right] \;=\; 1+r\,. \tag{16}$$

In other words, the present discounted value of the change in consumption equals the change in assets plus the present discounted value of the change in labor income.

To solve for $\partial c_t / \partial a_t$ using equations (15)–(16), it only remains to solve for $\partial l_{t+k}^* / \partial a_t$. This is done in two steps, using the household's Euler equation for unemployment,

$$\frac{V'(u_\tau^* + l_\tau^*)}{f(\theta_t)} \;=\; \beta E_\tau \left[w_{\tau+1} U'(c_{\tau+1}^*) \;+\; V'(u_{\tau+1}^* + l_{\tau+1}^*)\,\frac{1-s-f(\theta_{\tau+1})}{f(\theta_{\tau+1})} \right], \tag{17}$$

and the transition equation (2) for labor. Equation (17) is derived in the same way as the consumption Euler equation and is standard in the labor market search literature. The left-hand side of (17) represents the household's marginal cost of finding a job, while the right-hand side is the discounted marginal benefit of having one more employed worker, given by the wage times the marginal utility of consumption less the marginal disutility of work, plus the job retention rate $(1-s)$ times the marginal benefit of not having to search for a job next period.

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[8] The notation $\frac{\partial c_{t+1}^*}{\partial a_t}$ is taken to mean $\frac{\partial c_{t+1}^*}{\partial a_{t+1}} \frac{da_{t+1}^*}{da_t} = \frac{\partial c_{t+1}^*}{\partial a_{t+1}} \left[1 + r_{t+1} - \frac{\partial c_t^*}{\partial a_t} \right]$, and analogously for $\frac{\partial c_{t+2}^*}{\partial a_t}$, $\frac{\partial c_{t+3}^*}{\partial a_t}$, etc. Note that $\frac{da_{t+2}^*}{da_t} = (1+r_{t+2}) \frac{da_{t+1}^*}{da_t} + w_{t+1} \frac{\partial l_{t+1}}{\partial u_t} \frac{\partial u_t^*}{\partial a_t} - \frac{\partial c_{t+1}^*}{\partial a_t}$.

[9] Note that this equality does not follow from the steady state assumption. For example, in a model with internal habits, considered in Swanson (2009), the individual household's optimal consumption response to a change in assets increases with time, even starting from steady state.

Equation (17) holds at each date $\tau \geq t$ for any initial asset stock a_t. Differentiating (17) with respect to a_t yields

$$\frac{V''(u^*_\tau + l^*_\tau)}{f(\theta_\tau)} \left(\frac{\partial l^*_\tau}{\partial a_t} + \frac{\partial u^*_\tau}{\partial a_t} \right) = \beta E_\tau \Bigg[w_{\tau+1} U''(c^*_{\tau+1}) \frac{\partial c^*_{\tau+1}}{\partial a_t} + V''(l^*_{\tau+1} + u^*_{\tau+1}) \frac{1 - s - f(\theta_{\tau+1})}{f(\theta_{\tau+1})} \left(\frac{\partial l^*_{\tau+1}}{\partial a_t} + \frac{\partial u^*_{\tau+1}}{\partial a_t} \right) \Bigg]. \tag{18}$$

Evaluating (18) at steady state, we have

$$\frac{\partial l^*_\tau}{\partial a_t} + \frac{\partial u^*_\tau}{\partial a_t} = \beta w \frac{U''(c)}{V''(l+u) f(\theta)} \frac{\partial c^*_{\tau+1}}{\partial a_t} + \beta \big(1 - s - f(\theta)\big) \left(\frac{\partial l^*_{\tau+1}}{\partial a_t} + \frac{\partial u^*_{\tau+1}}{\partial a_t} \right). \tag{19}$$

Substituting for $\partial c^*_{\tau+1} / \partial a_t$ from (15), and for w from (17) evaluated at steady state, this becomes

$$\frac{\partial l^*_\tau}{\partial a_t} + \frac{\partial u^*_\tau}{\partial a_t} = -\frac{\gamma}{\chi} \frac{l+u}{c} \left[1 - \beta\big(1 - s - f(\theta)\big) \right] \frac{\partial c^*_t}{\partial a_t} + \beta\big(1 - s - f(\theta)\big) \left(\frac{\partial l^*_{\tau+1}}{\partial a_t} + \frac{\partial u^*_{\tau+1}}{\partial a_t} \right), \tag{20}$$

where $\gamma \equiv -cU''(c)/U'(c)$ denotes the elasticity of U' with respect to c, evaluated at steady state, and $\chi \equiv (l+u)V''(l+u)/V'(l+u)$ the elasticity of V' with respect to $l+u$, at steady state. Assuming $|1 - s - f(\theta)| < 1$, equation (20) can be solved forward to yield

$$\frac{\partial l^*_\tau}{\partial a_t} + \frac{\partial u^*_\tau}{\partial a_t} = -\frac{\gamma}{\chi} \frac{l+u}{c} \frac{\partial c^*_t}{\partial a_t}. \tag{21}$$

The transition equation (2) for labor implies

$$\frac{\partial l^*_{t+k}}{\partial a_t} = (1-s) \frac{\partial l^*_{t+k-1}}{\partial a_t} + f(\theta_{t+k-1}) \frac{\partial u^*_{t+k-1}}{\partial a_t}. \tag{22}$$

Evaluating (22) at steady state and applying (21) gives

$$\frac{\partial l^*_{t+k}}{\partial a_t} = \big(1 - s - f(\theta)\big) \frac{\partial l^*_{t+k-1}}{\partial a_t} - \frac{\gamma}{\chi} \frac{l+u}{c} f(\theta) \frac{\partial c^*_t}{\partial a_t}. \tag{23}$$

Solving (23) backward to the initial condition $\partial l_t / \partial a_t = 0$ gives

$$\frac{\partial l^*_{t+k}}{\partial a_t} = -\frac{\gamma}{\chi} \frac{l+u}{c} \frac{f(\theta)}{s + f(\theta)} \left[1 - \big(1 - s - f(\theta)\big)^k \right] \frac{\partial c^*_t}{\partial a_t}. \tag{24}$$

In response to a change in assets, household consumption moves instantly to a new steady-state level, but l responds only gradually, approaching its new level asymptotically as $k \to \infty$. Moreover, labor and consumption move in opposite directions (and unemployment and consumption move in opposite directions, combining (24) with (21)). Intuitively, it is natural to think of both consumption and time spent at home as normal goods, so that $\partial c^*_t / \partial a_t > 0$, $\partial l^*_t / \partial a_t < 0$, and $\partial u^*_t / \partial a_t < 0$, although we do not require this assumption below.

Combining (16), (21), and (24) yields

$$\frac{\partial c_t^*}{\partial a_t} = \frac{r}{1 + w\,\dfrac{\gamma}{\chi}\,\dfrac{l+u}{c}\,\dfrac{f(\theta)}{r+s+f(\theta)}}. \tag{25}$$

In response to a unit increase in assets, the household raises consumption in every period by the extra asset income, r (the "golden rule"), adjusted downward by the amount $1+w\,\frac{\gamma}{\chi}\,\frac{l+u}{c}\,\frac{f(\theta)}{r+s+f(\theta)}$, which takes into account the household's decrease in hours worked and labor income. Thus, equation (25) represents a "modified golden rule" that takes into account the household's labor margin. Equation (25) is also very similar to the modified golden rule derived in Swanson (2012a) for the frictionless labor case. The difference here is the additional factor of $f(\theta)/(r+s+f(\theta))$ in the denominator, which captures the effect of labor market frictions on the household's ability to vary its labor supply.

We can now express the household's coefficient of absolute risk aversion in terms of known quantities. Substituting (11), (12), and (25) into (10), we have proved the following:

Proposition 2. *Given Assumptions 1–7, the household's coefficient of absolute risk aversion, $R^a(a_t, l_t; \theta_t)$, evaluated at steady state, satisfies*

$$R^a(a,l;\theta) = \frac{-U''(c)}{U'(c)}\,\frac{r}{1 + w\,\dfrac{\gamma}{\chi}\,\dfrac{l+u}{c}\,\dfrac{f(\theta)}{r+s+f(\theta)}}. \tag{26}$$

There are several features of Proposition 2 worth noting. If labor supply is exogenously fixed, corresponding to $s = f(\theta) = 0$, then risk aversion in (26) reduces to $-rU''/U'$, which is just the usual Arrow-Pratt definition multiplied by r, a scaling factor that translates assets into units of current-period consumption.[10] More generally, when $f(\theta) > 0$, households can partially offset shocks to assets through changes in hours worked, as discussed by Swanson (2012a). Even though consumption and labor are additively separable in (1), the household's consumption process is connected to the labor market through the budget constraint; as a result, the household's aversion to a gamble over assets is related to its ability to self-insure asset fluctuations in the labor market.

Note that the presence of any flexibility in the labor margin implies that risk aversion is less than or equal to the fixed-labor case:

[10] A gamble over a lump sum of \$$X$ is equivalent here to a gamble over an annuity of \$$X/r$. Thus, even though W_{11}/W_1 is different from U''/U' by a factor of r, this difference is exactly the same as a change from lump-sum to annuity units. Thus, the difference in scale is essentially one of units.

Corollary 3. *The coefficient of absolute risk aversion,*

$$R^a(a, l; \theta) \;\leq\; \frac{-rU''(c)}{U'(c)}. \tag{27}$$

If $r < 1$*, then* (25) *is also less than* $-U''/U'$*.*

Since r is the net interest rate, $r \ll 1$ in typical calibrations.

We will turn to the relationship between labor market flexibility and risk aversion after first defining relative risk aversion.

3.2 The Coefficient of Relative Risk Aversion

The distinction between absolute and relative risk aversion lies in the size of the hypothetical gamble faced by the household. If the household faces a one-shot gamble of size A_t in period t,

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + A_t \sigma \varepsilon_{t+1}, \tag{28}$$

or the household can pay a one-time fee $A_t \mu$ in period t to avoid this gamble, then it follows from Proposition 1 that $\lim_{\sigma \to 0} 2\mu(\sigma)/\sigma^2$ for this gamble is given by

$$\frac{-\,A_t E_t W_{11}(a^*_{t+1}, l^*_{t+1}; \theta_{t+1})}{E_t W_1(a^*_{t+1}, l^*_{t+1}; \theta_{t+1})}\,. \tag{29}$$

The natural definition of A_t, considered by Arrow (1964) and Pratt (1965), is the household's wealth at time t. The gamble in (28) is then over a fraction of the household's wealth and (29) is referred to as the household's coefficient of relative risk aversion.

In models with labor, however, household wealth can be more difficult to define because of the presence of human capital. Note that the household's financial assets a_t are *not* a good measure of wealth A_t, since a_t for an individual household may be zero or negative at some points in time. Swanson (2012a,b) discusses the issue of human capital and household wealth in more detail; however, the details of this discussion are tangential to the present paper, so we simply define household wealth to be the present discounted value of consumption:

Definition 2. *Let* $(a_t, l_t; \theta_t)$ *be an interior point of* X*. The household's* consumption-wealth coefficient of relative risk aversion, *denoted* $R^c(a_t, l_t; \theta_t)$*, is given by* (29) *with wealth* $A_t \equiv (1 + r_t)^{-1} E_t \sum_{\tau=t}^{\infty} m_{t,\tau} c^*_\tau$*, the present discounted value of household consumption, where* $m_{t,\tau} = \beta U'(c_\tau)/U'(c_t)$ *denotes the household's stochastic discount factor.*

The factor $(1+r_t)^{-1}$ in the definition expresses wealth A_t in beginning- rather than end-of-period-t units, so that in steady state $A = c/r$ and $R^c(a, l; \theta)$ is given by

$$R^c(a,l;\theta) \;=\; \frac{-A\, W_{11}(a,l;\theta)}{W_1(a,l;\theta)} \;=\; \frac{\gamma}{1 + w\,\dfrac{\gamma}{\chi}\,\dfrac{l+u}{c}\,\dfrac{f(\theta)}{r+s+f(\theta)}}\,, \tag{30}$$

where we have used the definition $\gamma = -cU''(c)/U'(c)$. Note that if labor is exogenously fixed, so that $s = f(\theta) = 0$, equation (30) reduces to the usual Arrow-Pratt definition. As long as the household has some ability to vary its hours of work, however, risk aversion will typically be reduced by the factor in the denominator of (30).

4. Implications of Labor Market Frictions for Risk Aversion

We are now ready to consider the implications of labor market frictions for risk aversion. Intuitively, labor market frictions make it more difficult for the household to self-insure against asset fluctuations by varying hours of work. Thus, a greater degree of labor market frictions should imply higher risk aversion, all else equal.

The transition equation (2) for labor, evaluated at steady state, implies

$$sl = f(\theta)u. \tag{31}$$

We can use (31) to substitute out $f(\theta)u/l$ in (30) to obtain

$$R^c(a,l;\theta) \;=\; \frac{\gamma}{1 + \dfrac{\gamma}{\chi}\,\dfrac{wl}{c}\,\dfrac{s+f(\theta)}{r+s+f(\theta)}}\,. \tag{32}$$

If labor is exogenously fixed, so $s = f(\theta) = 0$, equation (32) reduces to the usual Arrow-Pratt definition, γ. As the ratio $(s + f(\theta))/\big(r + s + f(\theta)\big)$ approaches 1, equation (32) converges to the formula for risk aversion derived by Swanson (2012a) for the flexible-labor case. Thus, $(s + f(\theta))/\big(r + s + f(\theta)\big)$ ranges between 0 and 1 and can be thought of as an index of labor market flexibility, with 0 corresponding to perfect labor market rigidity and 1 to perfect flexibility. The interest rate r appears in this index because labor frictions only *delay* the household's labor adjustment, they do not prevent it, and r is related to the cost of this delay. Households that are very patient (have a low r) view labor market frictions as less costly, because they can adjust their labor supply as desired given enough time to do so. We will return to this index of labor market flexilibity in our quantitative analysis, below.

We can also use (31) to substitute out $f(\theta)$ in (30), giving

$$R^c(a,l;\theta) \;=\; \frac{\gamma}{1+\dfrac{\gamma}{\chi}\,\dfrac{wl}{c}\,\dfrac{s(1+(l/u))}{r+s(1+(l/u))}}\,. \tag{33}$$

$R^c(a,l;\theta)$ is decreasing in $s\big(1+(l/u)\big)$, holding fixed the other quantities in equation (33). This suggests that greater labor market frictions (lower s) or a recession (lower l/u) should correspond to higher levels of risk aversion. The remainder of this section makes these two points more rigorously and investigates their quantitative importance.

The following assumption is not strictly necessary, but helps simplify the discussion and intuition in the analysis below:

Assumption 8. *The elasticity γ is constant with respect to c, χ is constant with respect to $l+u$, and $wl = c$.*

The assumption that γ and χ are constant is equivalent to assuming U and V each have the isoelastic functional form. The assumption $wl = c$ is equivalent to $ra+d = 0$, from the household's flow budget constraint (3); this will be the case, for example, if there are no assets in steady state and no transfers, or alternatively if lump-sum taxes offset the household's asset income.

The crucial feature of Assumption 8 is that γ, χ, and wl/c in (33) can be regarded as stable compared to $s\big(1+(l/u)\big)$. The intuition and basic results in the analysis below continue to hold if Assumption 8 is satisfied only approximately rather than exactly. However, if any of γ, χ, or wl/c vary substantially more than $s\big(1+(l/u)\big)$, then the analytical results below will not hold and we must resort to numerical solutions of the model to determine the corresponding variation in (33).

4.1 Risk Aversion Is Higher in Recessions

Intuitively, a recession is a period in which employment is low and unemployment is high, or the ratio l/u is low. The following proposition characterizes the relationship between l/u and risk aversion:

Proposition 3. *Given Assumptions 1–8 and fixed values for the parameters s, β, γ, and χ, $R^c(a,l;\theta)$ is decreasing in l/u.*

Proof: Since $1+r = 1/\beta$, r is independent of l/u. Assumption 8 then implies $R^c(a,l;\theta)$ in equation (33) is decreasing in l/u.

Proposition 3 shows that risk aversion is higher in recessions. A lower ratio of employment to unemployment implies that it is harder for unemployed workers to find a job, because $f(\theta) = sl/u$. As a result, it is more difficult for households to use the labor market to insure themselves from asset fluctuations.

Although l/u is the ratio of *steady-state* employment to unemployment, low l/u is the standard way to model a recession in labor market search models. Gross flows in and out of employment and unemployment are large in the U.S., so calibrated labor market search models imply that employment and unemployment converge very rapidly to their steady state levels, in a matter of weeks rather than quarters (e.g., Shimer, 2012, and Elsby, Hobijn, and Şahin, 2013). Thus, the interpretation of low l/u in Proposition 3 as being a recession is consistent with this literature.

The finding that risk aversion is countercyclical is potentially important because there is a great deal of evidence that risk premia in financial markets are countercyclical (e.g., Campbell and Cochrane 1999, Campbell 1999, and the references therein). Indeed, one of the main points of Campbell-Cochrane (1999) was to generate countercyclical risk aversion in a model to better match the observed countercyclicality of risk premia in the data. Proposition 3 here shows that labor market frictions can provide an alternate mechanism for generating countercyclical risk aversion. Even without consumption habits, households' reduced ability to insure themselves from risk in a recession makes them more risk averse and thus will tend to raise risk premia. The quantitative importance of this effect will be discussed below.

Proposition 3 is also noteworthy in that the source of the change in l/u is irrelevant. The ratio l/u could be lower because of a decrease in the efficiency of the matching function f, or because firm productivity is lower, or government purchases are lower, or some other element of θ is lower. Regardless of the source of the labor market weakness, the household is more risk averse.

It is also interesting that Proposition 3 holds no matter what the details of the production side of the economy, so long as Assumptions 1–8 for the household's problem are satisfied. The details of the production function and matching technology will generally affect the stochastic process for θ_t and the functional form of f, but do not affect the conclusions of the proposition.

4.2 Risk Aversion Is Higher in More Frictional Labor Markets

Intuitively, labor market frictions are greater when $f(\theta)$ is lower—when it is harder for an unemployed worker to find a job. The following proposition characterizes the relationship between $f(\theta)$ and risk aversion:

Proposition 4. *Let $f_1, f_2 : \Theta \to \mathbb{R}$, and hold the other parameters of the household's problem fixed. Given Assumptions 1–8, let $(a_1, l_1; \theta_1)$ and $(a_2, l_2; \theta_2)$ denote the steady-state values of $(a_t, l_t; \theta_t)$ corresponding to f_1 and f_2, respectively, and $R_1^c(a_1, l_1; \theta_1)$ and $R_2^c(a_2, l_2; \theta_2)$ the corresponding values of* (32)*. If $f_1(\theta_1) < f_2(\theta_2)$, then $R_1^c(a_1, l_1; \theta_1) > R_2^c(a_2, l_2; \theta_2)$.*

Proof: Since $1 + r = 1/\beta$, r is independent of $f(\theta)$. Assumption 8 then implies $R^c(a, l; \theta)$ in equation (32) is decreasing in $f(\theta)$.

Consistent with the intuition presented earlier, Proposition 4 shows that a more rigid labor market implies greater risk aversion, because households are less able to use their labor supply to insure themselves from asset fluctuations. The quantitative importance of this effect will be discussed below. The case $f(\theta) = s = 0$ corresponds to complete labor market rigidity and leads to the maximal level of household risk aversion, γ.

Proposition 4 is interesting because it suggests that economies with more frictional labor markets should also have higher risk premia. For example, if European and Japanese labor markets are characterized by lower job-finding probabilities $f(\theta)$ than in the U.S., then Propostion 4 implies household risk aversion in those countries should be higher than in the U.S. Risk premia in those countries should be higher as well, all else equal, although other differences across countries (such as in productivity or financial intermediation) may make it very difficult to hold all else equal in practice for this last comparison.

As with Proposition 3, Proposition 4 holds regardless of the details of the production side of the economy. The production function and matching technology may affect the stochastic process for θ_t and the functional form of f, but so long as the conditions of Proposition 4 are satisfied, its conclusions remain valid.

Finally, it might be tempting to conclude from (32) that R^c is decreasing in s as well as $f(\theta)$, but that is not necessarily the case. Changes in s may affect the steady-state level of θ and thus $f(\theta)$—for example, a lower value of s tends to increase l/u, which increases $f(\theta)$ in standard labor market search models. Thus, the effect of s on R^c in (32) can be ambiguous. The following corollary characterizes the relationship between s and risk aversion:

Corollary 5. *Let $s_1, s_2 \in [0, 1]$, $f_1, f_2 : \Theta \to \mathbb{R}$, and hold the other parameters of the household's problem fixed. Given Assumptions 1–8, let $(a_1, l_1; \theta_1)$ and $(a_2, l_2; \theta_2)$ denote the steady-state values of $(a_t, l_t; \theta_t)$ corresponding to (s_1, f_1) and (s_2, f_2), respectively, and $R_1^c(a_1, l_1; \theta_1)$ and*

$R_2^c(a_2, l_2; \theta_2)$ the corresponding values of (32). If $s_1 < s_2$ and $f_1(\theta_1) \leq f_2(\theta_2)$, then $R_1^c(a_1, l_1; \theta_1) > R_2^c(a_2, l_2; \theta_2)$. More generally, if $s_1 + f_1(\theta_1) < s_2 + f_2(\theta_2)$, then $R_1^c(a_1, l_1; \theta_1) > R_2^c(a_2, l_2; \theta_2)$.

PROOF: Since $1 + r = 1/\beta$, r is independent of s and f. Assumption 8 then implies $R^c(a, l; \theta)$ in equation (32) is increasing in $s + f(\theta)$.

4.3 Risk Aversion Is Higher for Households that Are Less Employable

Up until now, we have maintained the assumption of a representative household. In this section, we consider the case where the economy is populated by a unit continuum of households divided into two types: a set of measure one of households of type 1, and a set of measure zero of households of type 2. Obviously, the aggregate equilbrium of this economy is determined by the type 1 households, which can be thought of as "representative". We will think of the type 2 households as being "less employable" households, by which we mean $f_2(\theta) < f_1(\theta)$—that is, it is harder for an individual in a type 2 household to find a job than it is for an individual in a type 1 household. (Note that the aggregate equilibrium θ is the same for both types of households.) The other parameters of the two households are assumed to be the same, for simplicity.

The aggregate equilibrium of the economy is determined by the type 1, "representative" households. It is straightforward to check that Propositions 1–4 continue to hold in this economy for both type 1 and type 2 households. In particular, Propositions 1 and 2 give the formulas for risk aversion for both type 1 and type 2 households, and the risk aversion of the two types of households can be compared using Proposition 4. Since $f_2(\theta) < f_1(\theta)$, Proposition 4 implies that $R_2^c(a_2, l_2; \theta) > R_1^c(a_1, l_1; \theta)$.

In other words, risk aversion is higher for less employable households. These households face greater labor market frictions than the type 1 households, so it is more difficult for them to insure themselves from asset fluctuations in the labor market, and they are correspondingly more risk averse. The quantitative importance of this effect is explored below.

Some examples of households that might fit the type 2, "less employable" classification are households nearing retirement age, households with less education, and households that suffer from discrimination in the labor market. According to the theory above, these households should be more risk averse than regular, type 1 households, and hold a smaller fraction of their wealth in risky assets such as stocks.

This result provides a formal justification for why retired households are typically advised to hold a greater fraction of their wealth in bonds rather than stocks. All else equal, retired households should be more risk averse than younger households, who have much greater labor market flexibility. Of course, households nearing retirement also have a greater fraction of their wealth concentrated in financial assets rather than human capital, which provides an additional rationale for retirees to diversify their financial holdings. Bodie, Merton, and Samuelson (1992) consider the question of optimal portfolio balance over the life cycle in great detail, and find that labor market flexibility is an important factor in the household's willingness to hold risky assets. However, their analysis considers only the cases of perfectly flexible and perfectly rigid labor markets, for tractability. The possibility that older households face greater labor market frictions than younger households is acknowledged by Bodie et al. (1992) as being potentially important, but is beyond the scope of their analysis.[11] The present paper shows how their analysis can be extended to the case of frictional labor markets, and provides immediate insights confirming their intuition about this effect.

5. Quantitative Importance of Labor Market Frictions for Risk Aversion

Labor market frictions can potentially have a very large effect on risk aversion. Swanson (2012a) considers the two extreme cases of perfect labor market rigidity and perfect labor market flexibility. In the present paper, the former case corresponds to $f(\theta) = s = 0$, while perfect labor market flexibility corresponds to $s+f(\theta)$ being much larger than r, so that $\big(s+f(\theta)\big)/\big(r+s+f(\theta)\big) \to 1$. When the labor market is perfectly rigid, $R^c(a, l; \theta) = \gamma$, while $R^c(a, l; \theta) = [\gamma^{-1} + \chi^{-1}]^{-1}$ in the case of perfect labor market flexibility. As discussed in Swanson (2012a), the difference between these two expressions can be very large for reasonable parameterizations. For example, no matter how large is γ, R^c can be arbitrarily small in the flexible-labor case as $\chi \to 0$. Intuitively, a household with linear disutility of work ($\chi = 0$) is risk neutral with respect to gambles over hours, and optimally chooses to absorb shocks to asset values along the hours margin.

In the present paper, we are interested in intermediate degrees of labor market frictions and intermediate values of $s+f(\theta)$, as well as the extreme cases discussed above. In other words, over a range of empirically plausible estimates of $s + f(\theta)$, how much does risk aversion vary?

[11] For example, "The ability to vary labor supply ex post induces the individual to assume greater risks in his portfolio ex ante." (Bodie et al. 1992, p. 427); "Obviously, the opportunity to vary continuously one's labor supply without cost is a far cry from the workings of actual labor markets. A more realistic model would allow limited flexibility in varying labor and leisure." (ibid., p. 448); and "It is reasonable to hypothesize that, for most individuals, the degree of labor flexiliby diminishes over the life cycle. For this reason, the effective human capital on which the individual can draw also declines." (ibid., p. 446).

Table 1: Quantitative Importance of Labor Market Frictions for Risk Aversion

	s	$f(\theta)$	$\frac{s+f(\theta)}{r+s+f(\theta)}$	Relative Risk Aversion R^c $\gamma = 2$ $\chi = 1.5$	$\gamma = 5$ $\chi = 0.5$	$\gamma = 10$ $\chi = 2.5$	$\gamma = 20$ $\chi = 10$
Theoretical labor market benchmarks:							
perfect rigidity	0	0	0	2	5	10	20
near-perfect flexibility	1	1	.998	0.86	0.46	2.00	6.68
International comparison (based on Hobijn and Şahin, 2007):							
United States	.006	.282	.986	0.86	0.46	2.02	6.73
Germany	.006	.035	.911	0.90	0.49	2.15	7.09
France	.007	.033	.909	0.90	0.50	2.16	7.10
Spain	.012	.020	.889	0.92	0.51	2.20	7.20
Italy	.004	.013	.810	0.96	0.55	2.36	7.64
Business cycle variation (based on Shimer, 2012):							
United States, expansion	.017	.35	.989	0.86	0.46	2.02	6.71
United States, recession	.022	.20	.982	0.87	0.46	2.03	6.75

Relative risk aversion R^c from equation (32) for different values of γ and χ, using estimated values of s and $f(\theta)$ from Hobijn and Şahin (2007) and Shimer (2012). In each row, s is sum of monthly flow rates from employment (E) to unemployment (U) and to nonemployment (N); $f(\theta)$ is monthly flow rate from U to E. Monthly interest rate r is set to .004. See text for details.

Table 1 provides a first answer to this question. The top two rows of Table 1 report values of s and $f(\theta)$ corresponding to the two extreme cases just discussed, perfect labor market rigidity and flexibility. The third column reports the corresponding value of $(s+f(\theta))/\big(r+s+f(\theta)\big)$, taking the monthly interest rate r to be .004, as in Shimer (2010). The numbers in the third column must lie between 0 and 1 and provide an index of labor market flexibility, with 0 corresponding to perfect rigidity and 1 to perfect flexibility.[12] The last four columns of Table 1 report the corresponding values of relative risk aversion R^c from equation (32), for different combinations of values of γ and χ, the curvature of utility with respect to consumption and hours. When labor markets are perfectly rigid, $R^c = \gamma$, the traditional Arrow-Pratt measure, while when labor markets are perfectly flexible, R^c is much smaller—by a factor of about three to five—and corresponds to the formulas and values in Swanson (2012a).

Intermediate values of s and $f(\theta)$ are considered in the subsequent rows of Table 1. Hobijn and Şahin (2007) estimate average values of s and $f(\theta)$ over time for a variety of OECD countries, and values for five of the largest of these countries are reported in Table 1. In the model of the

[12] The benchmark values of $s = f(\theta) = 1$ yield a labor market flexibility index of .998 rather than 1. We cannot generate an index value higher than this without assuming a lower value for r or without violating the constraint $s + f(\theta) < 2$ that was used to solve equation (20) forward.

present paper, individuals may be out of the labor force, employed, or unemployed, so s in Table 1 is defined to be the sum of the monthly flow rates from employment into unemployment and from employment into nonemployment. The job-finding rate $f(\theta)$ is the monthly flow rate from unemployment into employment. A striking feature of these estimates is that the job-finding rate in Continental Europe is an order of magnitue lower than in the U.S.—about .02 or .03 compared to .28.

A remarkable feature of Table 1, however, is that the index of labor market flexibility in the third column, $(s + f(\theta))/\big(r + s + f(\theta)\big)$, is quite close to one for all of the countries listed. According to this index, which measures labor market flows relative to households' discount rate, labor markets in all of these countries are very close to being perfectly flexible. Mechanically, even though the job-finding probabilities $f(\theta)$ are much smaller in Europe than in the U.S., they are still much larger than the monthly interest rate r, which is taken to be .004, as for the U.S. In other words, households' discount rate is low enough that, in all of these countries, the time it takes for them to vary their employment is not a significant constraint on their ability to self-insure fluctuations in asset values.

The labor market in the U.S. is estimated to be almost perfectly flexible, with an index value $(s + f(\theta))/\big(r + s + f(\theta)\big)$ of about .99. The index values for Continental Europe are lower than for the U.S., but even Italy's labor market flexibility measures about 81 percent by this metric. Households would either have to be much more impatient, or labor markets would have to be much more rigid, for risk aversion in these countries to be more affected by labor market frictions.

The last four columns of Table 1 report the coefficient of relative risk aversion R^c across countries for given values of γ and χ. Labor market frictions in Italy cause risk aversion to be about 10 to 20 percent higher than in the U.S., but are still far lower than what would be implied if labor was completely fixed. The other European countries are even more similar to the U.S.

Thus, one of the main takeaways from Table 1 is that the flexible-labor expressions for risk aversion derived in Swanson (2012a,b) are good approximations to risk aversion in all of these countries. In no country are labor markets rigid enough for the traditional, fixed-labor measure of risk aversion, γ, to be appropriate. The fixed-labor measure of risk aversion typically overstates risk aversion in these countries by a factor of three or more, depending on the exact parameterization of U and V. And as shown in Swanson (2012a,b) it is risk aversion as derived in the present paper that is related to asset prices in the model, rather than the hypothetical fixed-labor measure.

The last two rows of Table 1 consider the variation in U.S. labor market frictions over the business cycle. Shimer (2012) estimates the flow rates in and out of employment and unemployment over time. As for the international comparison, above, the job separation rate s is taken to be the sum of the monthly flow rate from employment to unemployment and from employment to nonemployment. The job-finding rate $f(\theta)$ is taken to be the monthly flow rate from unemployment to employment. The job separation rate increases slightly in recessions, while the job finding rate falls by almost half.

Nevertheless, the U.S. labor market is so flexible on average that even a fall in $f(\theta)$ by almost half has very little effect on the labor market flexibility index in the third column. Labor market flexilibity falls from about .99 in expansions to about .98 in recessions, a tiny change. The resulting differences in risk aversion R^c in the last four columns of Table 1 are correspondingly tiny. Although risk aversion is higher in recessions in this model, labor markets in the U.S. are just too flexible—even in recessions—for the implied change in R^c to be quantitatively important.

Of course, the importance of labor market frictions and business cycle variation in other countries could be more relevant quantitatively. For example, if the job-finding rate in Italy were to fall by half in recessions, the implied change in risk aversion would be much more substantial. More research on the cyclicality of job separation and finding rates in other countries would be needed to obtain better estimates of the importance of this effect. Countries where households face higher interest rates r will also have greater risk aversion, and more cyclical risk aversion, because the ratio $(s + f(\theta))/\big(r + s + f(\theta)\big)$ is smaller and more variable when r is larger.

The quantitative importance of labor market frictions for risk aversion in less employable households is harder to assess. There is relatively little data on the job-finding rate faced by a retiree, for example. However, a back-of-the-envelope calculation suggests that the effects for less educated or minority households in the U.S. should not be particularly large. If we assume that the steady-state unemployment rate is twice as large for these households as for the representative household, which is roughly consistent with the data, and job separation rates are about the same, then the job-finding rate $f(\theta)$ for these households would be roughly half as large as for the representative household, perhaps one-third as large if we allow for the fact that l might also be lower in steady state for these households. But labor markets in the U.S. are so close to frictionless already, that even a fall in $f(\theta)$ by a factor of three in Table 1 would have relatively little effect on $(s + f(\theta))/\big(r + s + f(\theta)\big)$ or on risk aversion R^c. It would take a much more extreme fall in

$f(\theta)$—which may be empirically plausible in some cases—to generate an appreciable difference in risk aversion for those households.

Finally, Assumption 8 imposed the condition that $wl = c$, so one might wonder whether variation in wl/c across countries or over the business cycle might increase the variation in risk aversion R^c in equation (32). For the U.S., at least, variation in wl/c does not seem to be a promising source of variation in R^c. Labor's share of income, wl/y, tends to increase in recessions, by roughly 4 percent according to the estimates in Gomme and Rupert (2004). At the same time, the ratio of consumption to output in the U.S. rises by only a few percent in the U.S. in recessions. Thus, if anything, the data suggest that wl/c rises slightly in recessions in the U.S., which would tend to push risk aversion R^c down. However, the ratio wl/c may differ across countries both in its average level and its cyclicality, so further research could suggest a more important role for the ratio wl/c in risk aversion outside the U.S.

6. Numerical Examples

6.1 Risk Aversion Away from the Steady State

The simple, closed-form expressions for risk aversion derived above hold exactly only at the model's nonstochastic steady state. For values of $(a_t, l_t; \theta_t)$ away from steady state, these expressions are only approximations. In this section, we evaluate the accuracy of those approximations by computing risk aversion numerically for a standard real business cycle model with labor market search and matching, as in Shimer (2010).

There is a unit continuum of representative households in the model, each with optimization problem (1)–(4). There is also a unit continuum of perfectly competitive firms in the model, each with production function

$$y_t = A_t k_t^{1-\alpha} l_t^{\alpha}, \tag{34}$$

where y_t, l_t, and k_t denote firm output, labor, and beginning-of-period capital, and A_t denotes an exogenous technology process that follows

$$\log A_t = \rho \log A_{t-1} + \varepsilon_t, \tag{35}$$

where ε_t is i.i.d. with mean zero and variance σ_ε^2. Capital is supplied by households at the competitive rental rate r_t^k. Capital is the only asset in the economy, which households accumulate according to

$$k_{t+1} = (1 + r_t)k_t + w_t l_t - c_t, \tag{36}$$

where $r_t = r_t^k - \delta$, δ is the capital depreciation rate, and c_t denotes household consumption.

To hire labor, firms post vacancies v_t at a cost of κ per vacancy per period. Labor evolves according to

$$l_t = (1-s)l_{t-1} + h_t, \tag{37}$$

where new hires h_t are given by the Cobb-Douglas matching function,

$$h_t = \bar{\mu}\, u_t^{1-\eta} v_t^{\eta}. \tag{38}$$

Thus,

$$f(\theta_t) = \frac{h_t}{u_t} = \bar{\mu}\left(\frac{v_t}{u_t}\right)^{\eta}. \tag{39}$$

I calibrate the model to monthly data, as in Shimer (2010, p. 84). I set $\beta = .996$, $\gamma = 1$, and $\chi = 1.5$, corresponding to an intertemporal elasticity of substitution of 1 and Frisch elasticity of 2/3. I set $\alpha = .7$, $\delta = .0028$, $\rho = .98$, $\sigma_\varepsilon = .005$, $\bar{\mu} = 2.32$, and $\eta = 0.5$. Following Shimer (2012), I set $s = .02$.

The state variables of the model are k_t, l_t, and A_t.[13] At the steady state, relative risk aversion is given by (33), which for the parameter values above implies $R^c(k,l,A) =$ TBD, less than half the traditional measure of $\gamma = 1$. Away from steady state, (8) and (10)–(17) remain valid, and we use them to compute $R^c(k_t, l_t, A_t)$ by solving for W_1, W_{11}, and $\partial c_t^*/\partial a_t$ numerically (see the Appendix for details). Figure 1 graphs the result over a wide range of values for k_t, l_t, and A_t. (Figure 1 to be completed).

To be completed.

6.2 Risk Aversion and the Equity Premium

To be written.

7. Balanced Growth

The results in the previous sections carry through essentially unchanged to the case of balanced growth. We collect the corresponding expressions here in Lemma 6, Proposition 7, and Corollary 8, and provide proofs in the Appendix.

[13] The endogenous state variables are k_t and l_t, while the exogenous state variables are A_t, K_t, and L_t, the aggregate capital and labor stocks. In equilibrium, $k_t = K_t$ and $l_t = L_t$, so we write the state vector as (k_t, l_t, A_t), although it would be written as $(k_t, l_t; A_t, K_t, L_t)$ for the analysis in Section 3.

Along a balanced growth path, $x \in \{l, r, u\}$ satisfies $x_{t+k} = x_t$ for $k = 1, 2, \ldots$, and we drop the time subscript to denote the constant value. For $x \in \{a, c, w, d\}$, we have $x_{t+k} = G^k x_t$ for $k = 1, 2, \ldots$, for some $G \in (0, 1+r)$, and we let x_t^{bg} denote the balanced growth path value. We denote the balanced growth path value of θ_t by θ_t^{bg}, although the elements of θ may grow at different constant rates over time (or remain constant). The job-finding rate $f(\theta_t^{bg})$ must be constant along the balanced growth path because $f(\theta_t^{bg}) = sl/u$, and we write this value as $f(\theta)$. Additional details regarding balanced growth are provided in King, Plosser, and Rebelo (1988, 2002).

Lemma 6. *Given Assumptions 1–6 and 7′, for $k = 1, 2, \ldots$, along the balanced growth path, $\partial c^*_{t+k}/\partial a_t = G^k \partial c^*_t/\partial a_t$ and*

$$\frac{\partial c_t^*}{\partial a_t} = \frac{1 + r - G}{1 + \dfrac{\gamma}{\chi} \dfrac{w_t^{bg} l}{c_t^{bg}} \dfrac{s + f(\theta)}{(\frac{1+r}{G} - 1) + s + f(\theta)}}. \tag{40}$$

The larger is G, the smaller is $\partial c_t^*/\partial a_t$, since the household chooses to absorb a greater fraction of asset shocks in future periods.

Proposition 7. *Given Assumptions 1–6 and 7′, absolute risk aversion satisfies*

$$R^a(a_t^{bg}, l; \theta_t^{bg}) = \frac{-W_{11}(a_{t+1}^{bg}, l_{t+1}^{bg}; \theta_{t+1}^{bg})}{W_1(a_{t+1}^{bg}, l_{t+1}^{bg}; \theta_{t+1}^{bg})} \tag{41}$$

and

$$R^a(a_t^{bg}, l; \theta_t^{bg}) = \frac{-U''(c_t^{bg})}{U'(c_t^{bg})} \frac{\frac{1+r}{G} - 1}{1 + \dfrac{\gamma}{\chi} \dfrac{w_t^{bg} l}{c_t^{bg}} \dfrac{s + f(\theta)}{(\frac{1+r}{G} - 1) + s + f(\theta)}}. \tag{42}$$

Note that (42) agrees with Proposition 2 when $G = 1$. The larger is G, the smaller is R^a, since larger G implies greater household wealth and ability to absorb asset shocks.

Corollary 8. *Given Assumptions 1–6 and 7′, relative risk aversion satisfies*

$$R^c(a_t^{bg}, l; \theta_t^{bg}) = \frac{\gamma}{1 + \dfrac{\gamma}{\chi} \dfrac{w_t^{bg} l}{c_t^{bg}} \dfrac{s + f(\theta)}{\left(\frac{1+r}{G} - 1\right) + s + f(\theta)}}. \tag{43}$$

Thus, the expressions for relative risk aversion are largely unchanged by balanced growth. The labor market flexibility index takes the form $\dfrac{s + f(\theta)}{(\frac{1+r}{G} - 1) + s + f(\theta)}$ rather than $\dfrac{s + f(\theta)}{r + s + f(\theta)}$, which has the effect of decreasing the importance of r and amplifying the importance of $s + f(\theta)$ in the index, but other than that the expression is unchanged.

8. Conclusions

Traditional studies of risk aversion, such as Arrow (1964), Pratt (1965), Epstein and Zin (1989), and Weil (1989), assume that household labor supply is fixed. However, this assumption ignores the household's ability to partially offset shocks to asset values by varying hours of work. As a result, these fixed-labor measures of risk aversion are not representative of the household's aversion to holding risky assets when labor supply can vary. For reasonable parameterizations, traditional, fixed-labor measures of risk aversion can overstate the household's actual aversion to holding a risky asset by a factor of three or more, as in Table 1. Moreover, as shown in Swanson (2012a,b), fixed-labor measures of risk aversion are essentially unrelated to the equity premium in a standard RBC model, while the risk aversion measure R^c derived in the present paper is closely related.

The present paper analyzes whether realistic levels of labor market frictions significantly affect the conclusions in Swanson (2012a). The closed-form expressions for risk aversion derived in the present paper lie in between the fixed-labor and flexible-labor cases. Quantitatively, the degree of labor market frictions in the U.S. and other OECD countries estimated by Shimer (2012) and Hobijn and Şahin (2007) imply that risk aversion should be very close to the flexible-labor expressions derived in Swanson (2012a), and far away from the traditional, fixed-labor measure.

The closed-form expressions derived in the present paper imply that risk aversion is higher when labor market frictions are greater. In particular, risk aversion is higher in recessions, risk aversion is higher in more frictional labor markets, and risk aversion is higher for households that are less employable. In each of these cases, it is more difficult for the household to use the labor market to insure itself from asset value fluctuations.

Intuitively, labor market frictions delay, but do not prevent, the household from adjusting its employment in response to shocks. If households are sufficiently patient, then labor market frictions do not represent a significant constraint on the household. In the data, monthly flow rates in and out of employment and unemployment are high relative to the discount rate, implying that labor market frictions are a relatively small concern for the household, at least as far as its attitudes toward risk are concerned.

Appendix: Proofs of Propositions and Numerical Solution Details

Proof of Proposition 1

Since $(a_t, l_t; \theta_t)$ is an interior point of X, $W(a_t + \frac{\sigma\underline{\varepsilon}}{1+r_t}, l_t; \theta_t)$ and $W(a_t + \frac{\sigma\overline{\varepsilon}}{1+r_t}, l_t; \theta_t)$ exist for sufficiently small σ, and $W(a_t + \frac{\sigma\underline{\varepsilon}}{1+r_t}, l_t; \theta_t) \leq \widetilde{W}(a_t, l_t; \theta_t; \sigma) \leq W(a_t + \frac{\sigma\overline{\varepsilon}}{1+r_t}, l_t; \theta_t)$, hence $\widetilde{W}(a_t, l_t; \theta_t; \sigma)$ exists. Moreover, since $W(\cdot, \cdot; \cdot)$ is continuous and increasing in its first argument, the intermediate value theorem implies there exists a unique $-\mu(a_t, l_t; \theta_t; \sigma) \in [\sigma\underline{\varepsilon}, \sigma\overline{\varepsilon}]$ satisfying $W(a_t - \frac{\mu}{1+r_t}, l_t; \theta_t) = \widetilde{W}(a_t, l_t; \theta_t; \sigma)$.

For a sufficiently small fee μ in (8), the change in household welfare (6) is given to first order by

$$\frac{-W_1(a_t, l_t; \theta_t)}{1+r_t}\, d\mu\,. \tag{A1}$$

Using the envelope theroem, we can rewrite (A1) as

$$-\beta E_t W_1(a^*_{t+1}, l^*_{t+1}; \theta_{t+1})\, d\mu\,, \tag{A2}$$

where $a^*_{t+1} \equiv (1+r_t)a_t + w_t l_t + d_t - c^*_t$ and $l^*_{t+1} = (1-s)l_t + f(\theta_t)u^*_t$.

Turning now to the gamble in (7), note that the household's optimal choices for consumption and unemployment in period t, c^*_t and u^*_t, will generally depend on the size of the gamble σ—for example, the household may undertake precautionary saving when faced with this gamble. Thus, in this section we write $c^*_t \equiv c^*(a_t, l_t; \theta_t; \sigma)$ and $u^*_t \equiv u^*(a_t, l_t; \theta_t; \sigma)$ to emphasize this dependence on σ. The household's value function, inclusive of the one-shot gamble in (7), satisfies

$$\widetilde{W}(a_t, l_t; \theta_t; \sigma) = U(c^*_t) - V(l_t + u^*_t) + \beta E_t W(a^*_{t+1}, l^*_{t+1}; \theta_{t+1}). \tag{A3}$$

Because (7) describes a one-shot gamble in period t, it affects assets a^*_{t+1} and l^*_{t+1} in period $t+1$ but otherwise does not affect the household's optimization problem from period $t+1$ onward; as a result, the household's value-to-go at time $t+1$ is just $W(a^*_{t+1}, l^*_{t+1}; \theta_{t+1})$, which does not depend on σ except through a^*_{t+1} and l^*_{t+1}.

Differentiating (A3) with respect to σ, the first-order effect of the gamble on household welfare is

$$\left[U' \frac{\partial c^*_t}{\partial \sigma} - V' \frac{\partial u^*_t}{\partial \sigma} + \beta E_t W_1 \cdot \big(-\frac{\partial c^*_t}{\partial \sigma} + \varepsilon_{t+1}\big) + \beta E_t W_2 f(\theta_t) \frac{\partial u^*_t}{\partial \sigma}\right] d\sigma, \tag{A4}$$

where the arguments of U', V', W_1, and W_2 are suppressed to reduce notation. Optimality of c^*_t and u^*_t implies that the terms involving $\partial c^*_t/\partial\sigma$ and $\partial u^*_t/\partial\sigma$ in (A4) cancel, as in the usual envelope theorem (these derivatives vanish at $\sigma = 0$ anyway, for the reasons discussed below). Moreover, $E_t W_1(a^*_{t+1}, l^*_{t+1}; \theta_{t+1})\varepsilon_{t+1} = 0$ because ε_{t+1} is independent of θ_{t+1}, l^*_{t+1}, and a^*_{t+1}, evaluating the latter two at $\sigma = 0$. Thus, the first-order cost of the gamble is zero, as in Arrow (1964) and Pratt (1965).

To second order, the effect of the gamble on household welfare is

$$\begin{aligned}\Bigg[& U'' \left(\frac{\partial c^*_t}{\partial \sigma}\right)^2 - V'' \left(\frac{\partial u^*_t}{\partial \sigma}\right)^2 + U' \frac{\partial^2 c^*_t}{\partial \sigma^2} - V' \frac{\partial^2 u^*_t}{\partial \sigma^2} \\ & + \beta E_t W_{11} \cdot \left(-\frac{\partial c^*_t}{\partial \sigma} + \varepsilon_{t+1}\right)^2 + \beta E_t W_{12} \cdot \left(-\frac{\partial c^*_t}{\partial \sigma} + \varepsilon_{t+1}\right) f(\theta_t) \frac{\partial u^*_t}{\partial \sigma} + \beta E_t W_1 \cdot \left(-\frac{\partial^2 c^*_t}{\partial \sigma^2}\right) \\ & + \beta E_t W_{22} f(\theta_t)^2 \left(\frac{\partial u^*_t}{\partial \sigma}\right)^2 + \beta E_t W_{12} \cdot \left(-\frac{\partial c^*_t}{\partial \sigma} + \varepsilon_{t+1}\right) f(\theta_t) \frac{\partial u^*_t}{\partial \sigma} + \beta E_t W_2 f(\theta_t) \frac{\partial^2 u^*_t}{\partial \sigma^2} \Bigg] \frac{d\sigma^2}{2}\,. \end{aligned} \tag{A5}$$

The terms involving $\partial^2 c^*_t/\partial\sigma^2$ and $\partial^2 u^*_t/\partial\sigma^2$ cancel due to the optimality of c^*_t and u^*_t. The derivatives $\partial c^*_t/\partial\sigma$ and $\partial u^*_t/\partial\sigma$ vanish at $\sigma = 0$ (there are two ways to see this: first, the linearized version of the model is certainty equivalent; alternatively, the gamble in (7) is isomorphic for positive and negative σ, hence c^* and u^* must be symmetric about $\sigma = 0$, implying the derivatives vanish). Thus, for infinitesimal gambles, (A5) simplifies to

$$\beta E_t W_{11}(a^*_{t+1}, l^*_{t+1}; \theta_{t+1})\, \varepsilon^2_{t+1} \frac{d\sigma^2}{2}\,. \tag{A6}$$

Finally, ε_{t+1} is independent of θ_{t+1}, l^*_{t+1}, and a^*_{t+1}, evaluating the latter two at $\sigma = 0$. Since ε_{t+1} has unit variance, (A6) reduces to

$$\beta E_t W_{11}(a^*_{t+1}, l^*_{t+1}; \theta_{t+1}) \frac{d\sigma^2}{2}\,. \tag{A7}$$

Equating (A2) to (A7) allows us to solve for $d\mu$ as a function of $d\sigma^2$. Thus, $\lim_{\sigma\to 0} 2\mu(a_t, l_t; \theta_t; \sigma)/\sigma^2$ exists and is given by

$$\frac{-\,E_t W_{11}(a^*_{t+1}, l^*_{t+1}; \theta_{t+1})}{E_t W_1(a^*_{t+1}, l^*_{t+1}; \theta_{t+1})}\,. \tag{A8}$$

To evaluate (A8) at the nonstochastic steady state, set $a_{t+1} = a$, $l_{t+1} = l$, and $\theta_{t+1} = \theta$ to get

$$\frac{-\,W_{11}(a, l; \theta)}{W_1(a, l; \theta)}\,. \tag{A9}$$

Proof of Lemma 6

The household's Euler equation (13), evaluated along the (nonstochastic) balanced growth path, implies

$$U''(c^{bg}_t) \;=\; \beta(1+r)U''(c^{bg}_{t+1}) \;=\; \beta(1+r)U''(Gc^{bg}_t). \tag{A10}$$

As in King, Plosser, and Rebelo (2002), assume that preferences U are consistent with balanced growth for all initial asset stocks and wages in a neighborhood of a^{bg}_t and w^{bg}_t, and hence for all initial values of (c_t, l_t) in a neighborhood of (c^{bg}_t, l). Thus, (A10) can be differentiated to yield

$$U''(c^{bg}_t) \;=\; \beta(1+r)G\,U''(Gc^{bg}_t). \tag{A11}$$

Differentiating (A10) with respect to a_t yields

$$U''(c^{bg}_t)\frac{\partial c^*_t}{\partial a_t} \;=\; \beta(1+r)\,U''(c^{bg}_{t+1})\frac{\partial c^*_{t+1}}{\partial a_t}\,. \tag{A12}$$

Solving for $\partial c^*_{t+1}/\partial a_t$ and using (A10) yields $\partial c^*_{t+1}/\partial a_t = G\,\partial c^*_t/\partial a_t$.

Next, use the household's budget constraint (1)–(2) and the above to solve for $\partial c^*_t/\partial a_t$, following along the lines of (16)–(25).

Proof of Proposition 7

Proposition 1 implies (41). Assumptions 1–6 imply (11)–(12). Substituting (11)–(12) and Lemma 6 into (41) gives

$$R^a(a^{bg}_t, l^{bg}_t; \theta^{bg}_t) \;=\; \frac{-U''(c^{bg}_{t+1})}{U'(c^{bg}_{t+1})}\; \frac{1+r-G}{1+\dfrac{\gamma}{\chi}\,\dfrac{w^{bg}_t l}{c^{bg}_t}\,\dfrac{G(s+f(\theta))}{(1+r-G)+G(s+f(\theta))}}. \tag{A13}$$

Using (A10)–(A11) completes the proof.

Proof of Corollary 8

As in Definition 2, define wealth A^{bg}_t in beginning- rather than end-of-period-t units; this requires multiplying by $(1+r)^{-1}G^{-1}$ rather than just $(1+r)^{-1}$. Then the present discounted value of consumption along the balanced growth path is given by $A^{bg}_t = c^{bg}_t/(\frac{1+r}{G} - 1)$. Substituting these into Proposition 7 completes the proof.

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