

Mortgage Guarantees and House Price Inflation: A Quantitative Analysis

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Abstract

The U.S. economy witnessed dramatic and widespread house price inflation during the previous decade. We develop a quantitative model with incomplete markets and heterogeneous agents to investigate the impact of implicit mortgage guarantees and bailouts on house prices. In this environment, we show that the mispricing of risk can have a potentially enormous impact on house prices even in an environment with limited financial instruments. The quantitative results from the calibrated version of the model indicate that distortions stemming from implicit or explicit mortgage guarantees and bailouts of too-big-to-fail financial institutions likely played a significant role in the housing boom of the early 2000s.

1 Introduction

One of the major empirical puzzles to emerge from the housing boom and bust of the prior decade is the dramatic movements in house prices, both absolutely and relative to rents. A number of studies have attempted to explain this apparent deviation from fundamentals by reductions in credit market frictions. (See for example, Favilukis, Ludvigson, and Nieuwerburgh (2010); Kiyotaki, Michaelides, and Nikolov (2011)) Other explanations include various forms of non-standard beliefs (e.g. Burnside, Eichenbaum, and Rebelo (2011)) or speculative bubbles (Barlevy and Fisher (2010)). Kahn (2010) suggests some role for fundamentals in at least igniting the boom and triggering the bust.

This paper quantifies the impact on house prices of credit market distortions stemming from implicit loan guarantees and bailouts. Such interventions amount to implicit subsidies for borrowers, as in Jeske,

Krueger, and Mitman (2012). In our model, crucially (and in contrast with Jeske, Krueger, and Mitman (2012)), these subsidies affect house prices, and consequently, price-rental ratios as well. To generate a realistic model of housing credit, we model heterogeneous consumers deciding on housing and non-housing consumption in an incomplete markets setting. We find that the distortions stemming from implicit or explicit guarantees could have played a significant role in the housing boom of the early 2000s. In particular, we show that under plausible parameterizations of the model, such guarantees, along with relaxed lending standards (represented by higher loan-to-value ratios) could easily generate 10 to 30 percent increases in house prices even for intermediate values of the likelihood or extent of bailouts. At more extreme values, where mortgages are nearly fully guaranteed by taxpayers, the impact on house prices can be astronomical. In addition to the impact on prices, the policies also generate large increases in borrowing on the extensive margin (in addition to the intensive) and, of course, in default rates.

2 Background: Mortgage Guarantees and Bailouts

2.1 Federal Housing Policy in the United States

Direct federal intervention in the mortgage market dates back to the 1930s with the creation of the Federal Housing Administration (FHA) and Federal National Mortgage Association (FNMA, or “Fannie Mae”), among other Government-Sponsored Enterprises (GSEs). The goal was increased home ownership, and over the years the primary mechanisms were a system of mortgage insurance—essentially fairly priced loan guarantees, but with implicit backing by taxpayers—and the creation of a secondary market for mortgages via securitization.

Fannie Mae evolved by 1968 into a privately owned corporation, having spun off the Government National Mortgage Association (GNMA), which was explicitly backed by the government, and shortly thereafter competing with the newly created Federal Home Loan Mortgage Corporation (FHLMC, known as “Freddie Mac”). Originally serving as a buyer of FHA insured loans, Fannie and Freddie were allowed, beginning in 1970 to purchase privately issued mortgages, and the first mortgage passthrough securities were developed.

Over the years, through a combination of political pressure to extend credit to lower income communities, and the profit motive, Fannie and Freddie began to expand their reach and capitalize on a perceived Federal guarantee of their solvency. This implicit guarantee allowed them to borrow at close to risk-free rates to finance the purchases of mortgages that yielded higher rates of return, at least in expectation. This, along with rising house prices and a perception that large pools of mortgages were therefore relatively safe, helped to feed the lending craze that developed in the late 1990s and continued into the subsequent decade. Not only did the quantity of mortgages increase, but as is well-known, lending standards relaxed, with loan-to-

value ratios commonly exceeding the old norm of 80 percent, sometimes reaching 100 percent or higher, along with lax verification of income and credit histories.

At the same time, Fannie and Freddie's share of outstanding mortgages rose from 25.7 percent in 1990 to over 45 percent in 2002 ¹. A case can be made that these GSEs were driving the mortgage market not only with their actual purchases, but also as a potential buyer for many more privately-issued mortgages than they actually purchased. Certainly their growth increased the likelihood that they would be considered "too big to fail" in the event of a market collapse. Many policymakers and politicians recognized the looming problem but the GSEs were politically very influential and staved off any meaningful preemptive measures. Ultimately of course the housing market collapsed and Fannie and Freddie were indeed bailed out by taxpayers, thereby justifying the belief in Federal backing.

2.2 Modeling Housing Policy

We argue in this paper that one element of this boom and bust was an unsustainable implicit promise of taxpayer backing of the GSEs' mortgages. We model this as a simple (partial) guarantee, with a parameter ranging from zero to one that can be thought of either as the probability of a 100 percent bailout, or a certain partial bailout.

To avoid needless complexity we assume that in order to consume housing, each agent must purchase a house (that is, renting is not allowed, or is prohibitively costly).² Given realistic distributions of asset-income ratios, the ownership requirement induces many agents to borrow to finance their purchase. As a baseline, we compute equilibrium house prices in this setting without guarantees, or, equivalently, with guarantees that are fairly priced and therefore built into loan rates.

Introducing underpriced loan guarantees, in this setting, turns out to have a substantial impact on house prices. An important component of the model is the limit on loan-to-value (LTV) ratios. Absent any guarantees, default costs typically imply an interior optimum for borrowers, and LTV limits are not necessarily binding. The guarantees make it privately optimal to borrow as much as possible, so it makes sense for policy to combine even an implicit guarantee with LTV limits that will typically be binding. As a consequence, whatever impact the guarantees themselves have on house prices are also a function of this exogenously specified limit. To the extent this limit is increased, the impact will be magnified.

Finally, it is important to distinguish systemic risk and bailouts from individual mortgage guarantees. In the absence of aggregate risk, mortgage-backed securities can be fairly priced and by definition will exhibit

¹Federal Housing Finance Association, "Updates to Single-Family Mortgages Originated and Outstanding, 1990 - 2011 Q2," available at <http://www.fhfa.gov/Default.aspx?Page=70>

²It is straightforward to allow renting with an assumption that it is more costly than the implicit rental cost of owner-occupied housing, but this would require additional heterogeneity.

essentially no risk. There will be predictable credit and prepayment risk, but these will be priced into the borrowing rate and not a risk for financial institutions holding diversified portfolios. Thus only aggregate risk should in principle be a concern, and under any reasonable regulatory scheme, bailouts would only occur as a result of an aggregate shock.

Nonetheless, for reasons of tractability, and as a first step, our model has only idiosyncratic risk, so we compare across steady states with and without guarantees. Nonetheless, we argue that our results, properly interpreted, still provide insight into the impact of implicit backing of too-big-to-fail financial institutions on house prices. The mechanism is essentially the same: the probability of a bailout, coupled with competition among institutions or even just profit maximization of monopolistic institutions results in an implicit lending subsidy that drives up asset prices, which feeds further borrowing. While a comparison across steady states obviously does not capture all aspects of this phenomenon, we argue that it does realistically convey the potential quantitative impact of the implicit guarantees. In a sense, the model is more precisely one of underpriced or subsidized mortgage insurance, with the idea that such underpricing results in vulnerability to an aggregate shock. We model the positive impact on house prices of the implicit subsidy, but not the aggregate shock itself. We plan to explore more fully dynamic models with aggregate risk, but view this as a first required step.

3 A Simple Static Model

Because the fully dynamic model with heterogeneous agents is complex and inevitably more of a black box, we begin with a model of a simple static setting in which ex ante identical consumers are endowed with some “housing” and some non-stochastic income. The house receives a shock in its consumption-equivalent value, after which it is consumed along with the income. Since agents are ex ante identical, the price of the risky house must be such that the agent is willing to hold it. While complete risk-sharing is not permitted, the agent can imperfectly share risk through issuing secured debt.

Even this very stylized model does not have a simple closed-form solution except in special cases, but it does more transparently illustrate the mechanism that operates in the fully dynamic heterogeneous agent model that follows.

3.1 Baseline with No Guarantees

Suppose a continuum of identical agents of unit measure, each endowed with a tangible asset h , plus claims on consumption y . They have utility $u(c)$. h will turn into consumption, but first will be hit with an idiosyncratic shock z with unit mean and distribution function F . Thus in autarky, agents will have uncertain

consumption $y + hz$.

Markets are incomplete. There are only two trades available to reduce risk. One is to sell h to other agents at some market price P in exchange for y . Since agents are ex ante identical, in equilibrium such trades will not occur. The other is to issue debt on h via a financial intermediary that is able to pool risks. Specifically, each agent can deposit some or all of y in a “bank.” He can also trade his h for an asset that pays

$$\max \{zh - b, 0\} + \Phi b - \gamma h \mathbf{1} \{b > zh\}. \quad (1)$$

where $\mathbf{1} \{b > zh\}$ takes on the value one if $b > zh$, zero otherwise. Here b is debt secured by h . $\Phi \leq 1$ is the discount on b , so that Φb is the amount of claims on c the agent receives in exchange for giving up $\min \{b, zh\}$ after z is revealed. γh is a default cost, assumed to be proportional to h , and incurred by the borrower.³ We assume z has bounded support, and that b/h does not exceed that upper bound.

To streamline the exposition, let $\bar{z} \equiv b/h$. With only these trading possibilities, final consumption is

$$c = h \max \{z - \bar{z}, 0\} + h\Phi\bar{z} - \gamma h \mathbf{1} \{\bar{z} > z\} + y, \quad (2)$$

Φ is set so that lenders make zero profits, so

$$\Phi\bar{z} = E \{ \min \{ \bar{z}, z \} \} = \int_0^{\bar{z}} x dF(x) + \bar{z}(1 - F(\bar{z})). \quad (3)$$

For example, suppose z is distributed uniformly on $[0, 2]$. If $\gamma = 0$, risk-aversion implies that agents will want choose $\bar{z}$ as large as possible, i.e. $\bar{z} = 2$. Then $\Phi = 0.5$, so agents will get a “loan” of h , with repayment $2h$, which they will default on with probability one.⁴ Lenders will break even, since the mean of zh is h . This is the case of perfect risk-sharing, because without any default costs the debt functions as a pure insurance contract. The financial intermediary provides claims on consumption of h , which it can do because of its claim on zh . The market-clearing price of h will be one, since that is the rate at which h can be converted in to consumption.

The case with $\gamma = 0$ does not really look like borrowing, as agents essentially surrender their ownership in exchange for a certain claim. In the more interesting case with $\gamma > 0$, however, borrowing up to the natural limit will not generally be optimal because of the lumpy default cost. If we substitute for $\Phi\bar{z}$, and let $\psi(\bar{z}; h, y) \equiv h \int_0^{\bar{z}} x dF(x) + h\bar{z}(1 - F(\bar{z})) + y$ be the certain component of consumption, agents will choose

³Similar results would obtain if the cost were incurred by the lender.

⁴If we thought of the gap between the initial decision about h and the resolution of uncertainty as occurring over some discrete time interval, and there was a positive real interest rate r , then the zero-profit condition would apply to $\Phi\bar{z}(1 + r)$. This would not change the essential flavor of the example.

$\bar{z}$ to solve

$$V(h, y) = \max_{\bar{z}} E \left\{ u \left(h \max \{z - \bar{z}, 0\} - \gamma h \mathbf{1} \{ \bar{z} > z \} + h \int_0^{\bar{z}} x dF(x) + \bar{z} h (1 - F(\bar{z})) + y \right) \right\}$$

or

$$V(h, y) = \max_{\bar{z}} u(\psi(\bar{z}; h, y) - \gamma) F(\bar{z}) + \int_{\bar{z}}^{\infty} u(h(z - \bar{z}) + \psi(\bar{z}; h, y)) dF(z).$$

The first term in the maximand is consumption in the default state multiplied by the probability of that state, which conditional on default does not depend on z . The second term is the risky but larger consumption in the non-default state.

First-order conditions (noting that $\psi' = h(1 - F(\bar{z}))$):

$$\begin{aligned} 0 &= u'(\psi(\bar{z}; h, y) - \gamma h) (1 - F(\bar{z})) F(\bar{z}) h + u(\psi(\bar{z}; h, y) - \gamma h) f(\bar{z}) \\ &\quad - u(\psi(\bar{z}; h, y)) f(\bar{z}) - h F(\bar{z}) \int_{\bar{z}}^{\infty} u'(h(z - \bar{z}) + \psi(\bar{z}; h, y)) dF(z). \end{aligned}$$

As discussed above, we know that for $\bar{z}$ such that $F(\bar{z}) = 1$, $\psi(\bar{z}; h, y) = h + y$, so we are left with $u(\psi(\bar{z}; h, y) - \gamma) - u(\psi(\bar{z}; h, y)) < 0$ for $\gamma > 0$, implying that $\bar{z}$ should be reduced.

The shadow price of housing P is just the marginal impact of h on V relative to that of y . Noting that $\psi_h = \int_0^{\bar{z}} x dF(x) + \bar{z}(1 - F(\bar{z}))$, we have

$$\begin{aligned} V_h &= u'(\psi(\bar{z}; h, y) - \gamma h) \psi_h F(\bar{z}) + \int_{\bar{z}}^{\infty} (z - \bar{z} + \psi_h) u'(h(z - \bar{z}) + \psi(\bar{z}; h, y)) dF(z) \\ &= E \left\{ u'(c) \frac{dc}{dh} \right\} \end{aligned}$$

We also have

$$\begin{aligned} V_y &= u'(c_0) F(\bar{z}) + \int_{\bar{z}}^{\infty} u'(h(z - \bar{z}) + \psi(\bar{z}; h, y)) dF(z) \\ &= E \{ u'(c) \} \end{aligned}$$

So we have

$$\begin{aligned} P &= \frac{E \{ u'(c) \frac{dc}{dh} \}}{E \{ u'(c) \}} = \frac{\text{cov}(u'(c), \frac{dc}{dh}) + E \{ u'(c) \} E \{ \frac{dc}{dh} \}}{E \{ u'(c) \}} \\ &= E \left\{ \frac{dc}{dh} \right\} + \frac{\text{cov}(u'(c), \frac{dc}{dh})}{E \{ u'(c) \}} \end{aligned}$$

Now

$$\frac{dc}{dh} = \begin{cases} \psi_h - \gamma & z < \bar{z} \\ \psi_h + z - \bar{z} & z > \bar{z} \end{cases}$$

so

$$\begin{aligned} E \left\{ \frac{dc}{dh} \right\} &= (\psi_h - \gamma) F(\bar{z}) + \int_{\bar{z}}^{\infty} (z - \bar{z} + \psi_h) dF(z) \\ &= 1 - \gamma F(\bar{z}). \end{aligned}$$

Also $cov(u'(c), \frac{dc}{dh}) < 0$ if u is concave, since $u'(c)$ is decreasing in c , and both c and dc/dh are non-decreasing (and sometimes increasing) in z . Consequently $P < 1 - \gamma F(\bar{z})$. This is no surprise, since hz is risky, so there would be a risk premium. That is, modulo default costs, h trades at $P = 1$ ex post, so agents get a “return” of $1 - P$ as compensation for holding h . Of course P is closer to one than it would be if borrowing were not possible, as borrowing reduces (in absolute value) the covariance of $u'(c)$ and dc/dh .

3.2 Guarantees

Suppose the government guarantees some portion η of the loan b , so that the lender will receive

$$h \min \{ \bar{z}, (1 - \eta) z + \eta \bar{z} \} = h \eta \bar{z} + h (1 - \eta) \min \{ \bar{z}, z \}.$$

Consequently

$$\Phi \bar{z} = E \{ \eta \bar{z} + (1 - \eta) \min \{ \bar{z}, z \} \} = \eta \bar{z} + (1 - \eta) \int_0^{\bar{z}} z dF(z) + (1 - \eta) \bar{z} (1 - F(\bar{z}))$$

and the consumer’s problem is generalized to

$$\begin{aligned} &V(h, y, \eta) \\ &= \max_{\bar{z}} E \left\{ u \left(h \max \{ z - \bar{z}, 0 \} + h \eta \bar{z} + h (1 - \eta) \int_0^{\bar{z}} x dF(x) + h (1 - \eta) \bar{z} (1 - F(\bar{z})) - \gamma h \mathbf{1} \{ \bar{z} > z \} + \tilde{y} \right) \right\} \end{aligned}$$

where $\tilde{y}$ is after-tax income (since the guarantees will be funded by lump-sum taxes). This can be written as

$$V(h, y, \eta) = \max_{\bar{z}} u(\psi(\bar{z}; h, y, \eta) - \gamma h) F(\bar{z}) + \int_{\bar{z}}^{\infty} u(h(z - \bar{z}) + \psi(\bar{z}; h, y, \eta)) dF(z)$$

where

$$\begin{aligned}\psi(\bar{z}; h, y, \eta) &\equiv h\eta\bar{z} + h(1-\eta) \int_0^{\bar{z}} x dF(x) + h(1-\eta)\bar{z}(1-F(\bar{z})) + \tilde{y} \\ &= h(1-\eta) \int_0^{\bar{z}} x dF(x) + h\bar{z}(1-(1-\eta)F(\bar{z})) + \tilde{y}.\end{aligned}$$

As for the taxes, it is straightforward to see that the guarantees will result in expenditures of

$$y - \tilde{y} \equiv G = h\eta \int_0^{\bar{z}} (\bar{z} - x) dF(x) = h\eta \left(\bar{z}F(\bar{z}) - \int_0^{\bar{z}} x dF(x) \right)$$

We will require $\tilde{y} \geq 0$, though in principle the government could tax housing as well as income. Note that to the consumer, $\tilde{y}$ depends on η and everyone else's choice of $\bar{z}$, but not on the consumer's own choice of $\bar{z}$, so we will write $\tilde{y}(\eta)$ where relevant.

Let $z_{\max} \equiv \inf \{z | F(z) = 1\}$, and let $\hat{z}$ denote the privately optimal choice of $\bar{z}$. The first-order condition for the maximization problem is

$$\begin{aligned}0 &= u'(\psi(\hat{z}; h, y, \eta) - \gamma h) \psi' F(\hat{z}) + u(\psi(\hat{z}; h, y, \eta) - \gamma h) f(\hat{z}) \\ &\quad - u(\psi(\hat{z}; h, y, \eta)) f(\hat{z}) + (\psi' - h) \int_{\hat{z}}^{\infty} u'(h(z - \hat{z}) + \psi(\hat{z}; h, y, \eta)) dF(z)\end{aligned}$$

with

$$\begin{aligned}\psi &= h(1-\eta) \int_0^{\hat{z}} x dF(x) + h\hat{z}(1-(1-\eta)F(\hat{z})) + \tilde{y}. \\ \psi' &= h(1-(1-\eta)F(\hat{z})). \\ \psi' - h &= -h(1-\eta)F(\hat{z})\end{aligned}$$

This can be expressed as

$$\begin{aligned}&hF(\hat{z}) \left((1-(1-\eta)F(\hat{z})) u'(\psi(\hat{z}; h, y, \eta) - \gamma h) - (1-\eta) \int_{\hat{z}}^{\infty} u'(h(z - \hat{z}) + \psi(\hat{z}; h, y, \eta)) dF(z) \right) \\ &= f(\hat{z}) (u(\psi(\hat{z}; h, y, \eta)) - u(\psi(\hat{z}; h, y, \eta) - \gamma h))\end{aligned}$$

The left side is the gain in utility terms from any increase in consumption from higher $\bar{z}$. The right side is the marginal loss from higher default costs. This implicitly defines $\hat{z}$ as a function of η , given h and y and the distribution of z .

We can see various special cases in this condition. If $\gamma = 0$, then the right-hand side is zero, and we have,

provided u is strictly concave,

$$(1 - (1 - \eta) F(\hat{z})) u'(\psi(\hat{z}; h, y, \eta)) \geq (1 - \eta) \int_{\hat{z}}^{\infty} u'(h(z - \hat{z}) + \psi(\hat{z}; h, y, \eta)) dF(z) \quad \forall \hat{z}$$

which implies that $\hat{z} = z_{\max}$, and $F(\hat{z}) = 1$. For $\gamma > 0$ and $\eta = 0$, assuming $f(z) > 0 \quad \forall z \in [0, z_{\max}]$, then the right side is positive while the left goes to zero as $\bar{z} \rightarrow z_{\max}$, suggesting that $\hat{z} < z_{\max}$. We likely need to check second-order conditions as well, however.

At the other extreme, if $\eta = 1$, $\psi' = h$, $\psi = h\bar{z} + \tilde{y}(1)$, and

$$c = h \max\{z, \bar{z}\} - \gamma h \mathbf{1}\{\bar{z} > z\} + \tilde{y}(1)$$

As $\bar{z} \rightarrow z_{\max}$, $c \rightarrow y + h - \gamma h$. We have at the optimum

$$hF(\hat{z}) u'(h\hat{z} + \tilde{y}(1) - \gamma h) = f(\hat{z}) (u(h\hat{z} + \tilde{y}(1)) - u(h\hat{z} + \tilde{y}(1) - \gamma h))$$

This may result in complete risk-sharing, but at a cost of γh for everyone. Since this is suboptimal for the private sector, it is suboptimal as policy. Nonetheless, it might be individually optimal. We have

$$\begin{aligned} \frac{d}{d\bar{z}} E\{u(h \max\{z, \bar{z}\} - \gamma h \mathbf{1}\{\bar{z} > z\} + \tilde{y})\} = \\ hF(\bar{z}) u'(h\bar{z} - \gamma h + \tilde{y}) + f(\bar{z}) (u(h\bar{z} - \gamma h + \tilde{y}) - u(h\bar{z} + \tilde{y})). \end{aligned}$$

If $f(z_{\max}) = 0$, then the privately optimal $\bar{z}$ is $z_{\max}$. Otherwise, for γ sufficiently large, the privately optimal $\bar{z} < z_{\max}$.

For the price, once again we have

$$P = \frac{V_h}{V_y} = \frac{E\{u'(c) \frac{dc}{dh}\}}{E\{u'(c)\}} = E\left\{\frac{dc}{dh}\right\} + \frac{\text{cov}(u'(c), \frac{dc}{dh})}{E\{u'(c)\}}$$

where, again

$$\begin{aligned} \psi &= h(1 - \eta) \int_0^{\bar{z}} x dF(x) + h\bar{z}(1 - (1 - \eta) F(\bar{z})) + \tilde{y}(\eta). \\ c &= \begin{cases} \psi - \gamma h & z < \bar{z} \\ \psi + h(z - \bar{z}) & z > \bar{z} \end{cases} \end{aligned}$$

$$\begin{aligned}\frac{dc}{dh} &= \begin{cases} \psi_h - \gamma & z < \bar{z} \\ \psi_h + z - \bar{z} & z > \bar{z} \end{cases} \\ \psi_h &= (1 - \eta) \int_0^{\bar{z}} x dF(x) + \bar{z} (1 - (1 - \eta) F(\bar{z}))\end{aligned}$$

so

$$E \left\{ \frac{dc}{dh} \right\} = \psi_h - \gamma F(\bar{z}) + \int_{\bar{z}}^{z_{\max}} (z - \bar{z}) dF(z).$$

We have (letting $\hat{z}$ denote the consumer's optimized choice of $\bar{z}$):

$$P = \frac{F(\hat{z}) u'(\psi - \gamma h) \psi_h + \int_{\hat{z}}^{\infty} u'(\psi + h(z - \hat{z})) (\psi_h + z - \hat{z}) dF(z)}{F(\hat{z}) u'(\psi - \gamma h) + \int_{\hat{z}}^{\infty} u'(\psi + h(z - \hat{z})) dF(z)} \quad (4)$$

along with the FOC for the choice of $\hat{z}$:

$$\begin{aligned} & hF(\bar{z}) \left((1 - (1 - \eta) F(\bar{z})) u'(\psi(\bar{z}; h, y, \eta) - \gamma h) - (1 - \eta) \int_{\bar{z}}^{\infty} u'(h(z - \bar{z}) + \psi(\bar{z}; h, y, \eta)) dF(z) \right) \\ &= f(\bar{z}) (u(\psi(\bar{z}; h, y, \eta)) - u(\psi(\bar{z}; h, y, \eta) - \gamma h)) \end{aligned}$$

Example: Suppose $F(x) = x/2$ for $x \in [0, 2]$. Then

$$G = h\eta\hat{z}^2/4$$

and

$$c = \begin{cases} h\hat{z} - h(1 - \eta)\hat{z}^2/4 + \tilde{y}(\eta) - \gamma h & z \leq \hat{z} \\ hz - h(1 - \eta)\hat{z}^2/4 + \tilde{y}(\eta) & z \geq \hat{z} \end{cases}.$$

Suppose $u(c) = \ln(c)$. The objective function as a function of $\hat{z}$ (and letting $\bar{z} = x$ in the $\tilde{y}(\eta)$ function, since it is not a choice variable):

$$\begin{aligned} V &= \hat{z} \ln(\hat{z} - (1 - \eta)\hat{z}^2/4 + y/h - \eta x^2/4 - \gamma) - \\ & (\hat{z} - (1 - \eta)\hat{z}^2/4 + y/h - \eta x^2/4) \ln(\hat{z} - (1 - \eta)\hat{z}^2/4 + y/h - \eta x^2/4) \\ & - 2 + \hat{z} + (2 - (1 - \eta)\hat{z}^2/4 + y/h - \eta x^2/4) \ln(2 - (1 - \eta)\hat{z}^2/4 + y/h - \eta x^2/4) \end{aligned}$$

The consumer's choice of $\hat{z}$ satisfies the first-order condition (setting $x = \hat{z}$)

$$\begin{aligned} & \frac{\hat{z}(1 - 0.5(1 - \eta)\hat{z})}{\hat{z} - \hat{z}^2/4 + y/h - \gamma} - 0.5(1 - \eta)\hat{z} \ln \left(\frac{2 - \hat{z}^2/4 + y/h}{\hat{z} - \hat{z}^2/4 + y/h} \right) \\ & \geq \ln \left(\frac{\hat{z} - \hat{z}^2/4 + y/h}{\hat{z} - \hat{z}^2/4 + y/h - \gamma} \right) \end{aligned}$$

since $\tilde{y}(\eta) = y - h\eta\hat{z}^2/4$, where the strict inequality implies $\hat{z} = 2$.

If $\eta = 0$, we have

$$\begin{aligned} & (1 - 0.5\hat{z}) \frac{\hat{z}}{\hat{z} - \hat{z}^2/4 + y/h - \gamma} - 0.5\hat{z} \ln \left(\frac{2 - \hat{z}^2/4 + y/h}{\hat{z} - \hat{z}^2/4 + y/h} \right) \\ & \geq \ln \left(\frac{\hat{z} - \hat{z}^2/4 + y/h}{\hat{z} - \hat{z}^2/4 + y/h - \gamma} \right). \end{aligned}$$

Clearly in this case $\hat{z} > 0$, provided $\gamma < y/h$, but also $\hat{z} < 2$ for $\gamma > 0$.

If $\eta = 1$, $\hat{z}$ satisfies

$$\frac{\hat{z}}{\hat{z} - \hat{z}^2/4 + y/h - \gamma} \geq \ln \left(\frac{\hat{z} - \hat{z}^2/4 + y/h}{\hat{z} - \hat{z}^2/4 + y/h - \gamma} \right)$$

subject to $\hat{z} \leq 2$. It's clear that $\hat{z} > 0$. At the other extreme, $\hat{z} = 2$ requires

$$\frac{2}{1 + y/h - \gamma} \geq \ln \left(\frac{1 + y/h}{1 + y/h - \gamma} \right)$$

which can hold for γ sufficiently small and/or h/y sufficiently large. For example, if $h \geq y$, this condition holds for any $\gamma < 1 + y/h$, since $\ln(x) < x \forall x > 0$. In fact, though, $h > y$ is not feasible with $\eta = 1$, because it would imply $\tilde{y} < 0$. But clearly the above condition can hold with $h < y$, and if it does, then

$$P = \frac{dc}{dh} = 2 - \gamma$$

and

$$c = h + y - \gamma h.$$

Thus even though in equilibrium, taking taxes into account, higher h only increases c by $1 - \gamma$, $P = 2 - \gamma$ because of a sort of free rider problem where people bid up the price and borrow up to the natural limit, not taking into account the impact on taxes from the loan guarantees.

Of course loan guarantees would normally come with a loan-to-value ratio (LTV) limit, which we will denote by ζ , i.e. a constraint $\bar{z} < \zeta P$. In the above example with $\eta = 1$, $\hat{z} = 2 = P + \gamma$, so any $\zeta < 1$ is a binding constraint. More generally, for any η and ζ such that a borrowing limit of the form $\bar{z} \leq \zeta P$ will be

binding, we can generalize (4) for P to

$$\begin{aligned}
P &= \frac{F(\hat{z}) u'(\psi - \gamma h) \psi_h + \int_{\hat{z}}^{\infty} u'(\psi + h(z - \hat{z})) (\psi_h + z - \hat{z}) dF(z)}{F(\hat{z}) u'(\psi - \gamma h) + \int_{\hat{z}}^{\infty} u'(hz + y - h\hat{z}^2/4) dF(z)} \\
&= \frac{\frac{\hat{z}(\hat{z} - (1-\eta)\hat{z}^2/4)}{\hat{z} - \hat{z}^2/4 + y/h - \gamma} + 2 - \hat{z} + (\eta\hat{z}^2/4 - y/h) \ln\left(\frac{2 - \hat{z}^2/4 + y/h}{\hat{z} - \hat{z}^2/4 + y/h}\right)}{\frac{\hat{z}}{\hat{z} - \hat{z}^2/4 + y/h - \gamma} + \ln\left(\frac{2 + y/h - \hat{z}^2/4}{\hat{z} + y/h - \hat{z}^2/4}\right)}.
\end{aligned}$$

and substitute ζP for $\hat{z}$. Rearranging terms, we get an implicit expression for P ,⁵

$$\begin{aligned}
0 &= P\zeta + \frac{(P^2\zeta)(1 - \zeta(1 - (1 - \eta)(P\zeta)/4))}{(P\zeta) + y/h - (P\zeta)^2/4 - \gamma} \\
&\quad + \left(P + y/h - \eta(P\zeta)^2/4\right) \ln\left(\frac{2 + y/h - (P\zeta)^2/4}{(P\zeta) + y/h - (P\zeta)^2/4}\right) - 2.
\end{aligned}$$

Figure 1 depicts $P(\eta)$ for $\zeta = 0.8$ and 0.95 , with $y/h = 2$ and $\gamma = .05$. We see that increasing η can increase P as much as 50 percent, and the impact on P of an increase in ζ from 0.8 to 0.95 can be as much as 25 percent, depending on η , and of course the impact of η is substantial as well for any given ζ .

Of course this is just a static example with a convenient distribution for z and many other simplifying assumptions. The remainder of the paper will be devoted to computing a fully dynamic version of the model and finding a realistic calibration to gauge the impact of bailouts and loan guarantees on house prices.

4 A Dynamic Model with Heterogeneous Agents

Time t is discrete. There is a continuum of infinitely-lived households. Households derive utility from consumption c_t and housing services h_t , discounting future at rate $0 < \beta < 1$. The preferences are represented by

$$\sum_{t=0}^{\infty} \beta^t u(c_t, h_{t+1})$$

where

$$u(c_t, h_{t+1}) = u(\tilde{c}_t) = \frac{\tilde{c}_t^{1-\sigma} - 1}{1 - \sigma} \text{ and } \tilde{c}_t = c_t^\theta h_{t+1}^{1-\theta}$$

Households derive housing services at time t from the stock of housing h_{t+1} they own. Price of consumption is normalized to 1, and price of housing at date t is P_t . In addition, households supply labor inelastically at

⁵More generally, P should be written as a function of $\min\{P\zeta, \hat{z}\}$, as for some parameters the borrowing limit $\bar{z} \leq P\zeta$ is not binding. In practice, for a wide range of plausible parameter choices, the constraint is binding even for high values of ζ .

post-tax wage rate $\bar{w}_t = (1 - \tau)w_t$.

The households are subject to i.i.d. quality shocks $z_t \geq 0$ to housing, which is the only source of uncertainty in the economy. The shocks have a cdf $F(z)$ with support $[0, z_{max}]$ and $\mathbb{E}(z) = 1$. In addition we assume that $F(z)$ is continuously differentiable everywhere in $(0, z_{max})$, and $F(0) = 0$.

Households can borrow and/or lend, but borrowing is essentially a mortgage contract secured against housing they own. We use b_{t+1} to denote mortgage debt and a_{t+1} to denote the holdings of capital-backed risk-free assets acquired at time t . Due to a “no unsecured borrowing” assumption, we have $b_{t+1} \geq 0$ and $a_{t+1} \geq 0$ for all time periods. Households borrow and lend through financial intermediaries and we model both as one-period contracts. Asset markets are incomplete since these two assets are the only ways in which households can smooth their consumption over time.

Households can default on their mortgages, in which case their house is “foreclosed.” That is, they cede the house to the lender, who gets the value less default costs. This is the only punishment to defaulting, as there is no real discretion or moral hazard on the part of the borrower.

Here is the timing within a period t :

1. Households observe z , and default if $z < \bar{z}$.⁶
2. Given prices, households choose c_t , a_{t+1} , b_{t+1} , h_{t+1} . Housing can be used immediately for housing services at time t .

These assumptions imply that the household will choose to default at time $t + 1$ if and only if

$$z_{t+1}P_{t+1}h_{t+1} - b_{t+1} < 0 \quad (5)$$

This defines a threshold value of shock, $\bar{z}_{t+1} \equiv \frac{b_{t+1}}{P_{t+1}h_{t+1}}$. If the household draws a value $z_{t+1} < \bar{z}_{t+1}$ next period, default occurs.

Working on the steady-state of the economy, we drop the time subscripts from all prices assuming $P_t = P$, $r_t = r$, $\bar{w}_t = w$ for all t . Discount rates on debt contracts, Φ , are contingent on housing h_{t+1} and the amount borrowed b_{t+1} . We will show that at the equilibrium, discounts rates can be written as a function of $\bar{z}_{t+1}$ only. We also require that the amount borrowed should be strictly less than the current value of housing, $b_{t+1}\Phi(\bar{z}_{t+1}) < P_t h_{t+1}$, or equivalently, $\bar{z}_{t+1}\Phi(\bar{z}_{t+1}) < 1$ for all feasible $\bar{z}$.⁷ This is a technical requirement to ensure that choice of housing is well-defined. We defer the discussion of this requirement until the next section. The budget constraint of a household is

⁶Strictly speaking, if z is within γ of $\bar{z}$ there would be some gains from renegotiation, but for convenience we ignore this possibility.

⁷If this condition is violated, marginal cost of housing is non-positive, since housing a normal good, the demand for housing would be infinitely large.

$$c_t + Ph_{t+1} + a_{t+1} \leq I_t + \Phi(\bar{z}_{t+1})b_{t+1}$$

$$c_t, h_{t+1}, b_{t+1}, a_{t+1} \geq 0$$

where

$$I_t = \bar{w} + a_t(1+r) + \max\{0, Pz_th_t - b_t\}$$

This is equivalent to

$$c_t + Ph_{t+1}(1 - \Phi(z_{t+1})z_{t+1}) + a_{t+1} \leq \bar{w}_t + a_t(1+r) + Ph_t \max\{0, z_t - \bar{z}_t\}$$

$$c_t, h_{t+1}, \bar{z}_{t+1}, a_{t+1} \geq 0$$

For policy experiments, we also impose an exogenous loan-to-value constraint of the form

$$\bar{z}_{t+1} \leq \bar{\zeta}$$

where $\bar{\zeta} \leq z_{max}$. Here $\bar{\zeta}$ is chosen such that $\bar{z}_{t+1}\Phi(\bar{z}_{t+1}) < 1$ holds for all feasible $\bar{z}$.

We will focus on a stationary recursive competitive equilibrium, therefore we drop the time scripts for the rest of the presentation below. The recursive formulation of the household's problem can be written as

$$V(I) = \max_{c, h', a', \bar{z}'} u(c, h') + \beta \mathbb{E}_z V(I') \quad (6)$$

subject to

$$c + Ph'[1 - \Phi(\bar{z}')\bar{z}'] + a' \leq I \quad (7)$$

$$c, \bar{z}', a', h' \geq 0$$

$$\bar{z}' \leq \bar{\zeta}$$

where

$$I' \equiv I'(z, a', h', \bar{z}') = \bar{w} + a'(1+r) + Ph' \max\{0, z - \bar{z}'\}$$

It is clear that a household that chooses the ex-ante loan-to-value ratio $\bar{z}'$ defaults with probability $F(\bar{z}')$.

4.1 Production

There is a representative firm that uses capital K and labor N , producing consumption and capital goods. The output by the representative firm is

$$Y = AK^\alpha N^{1-\alpha}$$

For convenience, we also define $f(K, N) \equiv AK^\alpha N^{1-\alpha} - \delta K$, the output net of depreciation. The firm rents capital at rate r and labor at rate w .

4.2 Financial Markets

Financial markets are competitive and financial intermediaries are risk-neutral. This ensures that profits are zero for a financial intermediary for each mortgage contract, assuming that a law of large numbers holds.

If a household defaults on mortgage debt, the mortgage provider seizes the house, getting $(1 - \gamma)zPh'$ while paying a fraction γ of the total value of the house as cost. This is modeled as a deadweight loss measured in terms of consumption goods. In addition, the government can guarantee a fraction of the debt. With the guarantee, conditional on the realization z of the shock, if a household defaults on the mortgage, the contract (h', b') yields

$$(1 - \eta)(1 - \gamma)zPh' + \eta b'$$

The two extreme values of η , 0 and 1, represent the case of no guarantee and full guarantee respectively. In case of no default, the intermediary gets b' .

Due to perfect competition, the interest rates must satisfy the following zero-profit condition:

$$\bar{z}'\Phi(\bar{z}') = \frac{1}{1+r} \left[\bar{z}'[1 - (1 - \eta)F(\bar{z}')] + (1 - \eta)(1 - \gamma) \int_0^{\bar{z}'} x dF(x) \right] \quad (8)$$

Using the assumptions on $F(\cdot)$, it is easy to show that

1. Function $\bar{z}'\Phi(\bar{z}')$ is continuously differentiable in $\bar{z}' \in (0, z_{max})$.
2. $\lim_{\bar{z}' \rightarrow 0+} \bar{z}'\Phi(\bar{z}') = 0$
3. For $0 < \eta < 1$, function $\Phi(\bar{z}')$ is strictly decreasing for all $\bar{z}' > 0$, and in particular $\Phi(\bar{z}') < \frac{1}{1+r}$ for all $\bar{z}' > 0$.

4.3 Government

Government taxes labor income linearly at rate τ to finance bail-outs. Since labor is inelastic, this is a lump-sum tax equal to τw . We assume that the government runs a balanced budget in each period.

5 Equilibrium

We investigate the long-run effects of policy on house prices, therefore we use the notion of stationary recursive competitive equilibrium in this economy.

To clarify the notation, we let $\mathcal{I} = [\bar{w}, \bar{I}] \subset \mathbb{R}$ be the compact set of all values I can take and let Σ denote the Borel σ -algebra on $\mathcal{I}$. Let $\mu(I)$ represent the steady-state measure of households defined on the measurable space $(\mathcal{I}, \Sigma)$.

Definition 1 A *stationary recursive competitive equilibrium* is a set of prices $(P, r, w, \Phi(\bar{z}'))$; policy functions $c(I), h'(I), \bar{z}'(I), a'(I)$; steady-state distribution $\mu(I)$; fiscal policy $0 < \tau < 1$ such that

1. Given prices and fiscal policy, policy functions solve the households' problem.
2. Given factor prices (r, w) , firms maximize profits, therefore

$$f_K(K, N) = r$$

$$f_N(K, N) = w$$

3. Intermediaries maximize profits, bond discounts satisfy equation (8).
4. Given policy functions, prices clear all markets:

(a) Labor market

$$N = 1$$

(b) House market

$$H' = H = \int h'(I) d\mu = 1$$

(c) Capital markets

$$K' = K = \int a'(I) d\mu - P \int \bar{z}'(I) h'(I) \Phi(\bar{z}'(I)) d\mu$$

(d) *Goods market*

$$C + K' + G + DWL = Y + (1 - \delta)K$$

where dead-weight loss DWL equals

$$DWL = \gamma P \int h'(I) \int_0^{\bar{z}'(I)} x dF(x) d\mu$$

5. *Government runs a balanced budget.*

$$\tau wN = G' = G = \eta P \int h'(I) \left[F(\bar{z}'(I)) \bar{z}'(I) - (1 - \gamma) \int_0^{\bar{z}'} x dF \right] d\mu$$

6. *The measure of households μ is generated by policy functions.*

$$\mu(\mathcal{I}_0) = \int \left[\int \chi_{I'(z, a'(I), h'(I), \bar{z}'(I)) \in \mathcal{I}_0} dF(z) \right] d\mu \text{ for each } \mathcal{I}_0 \in \Sigma$$

where χ_p is an indicator function, taking value 1 if p is true and 0 otherwise.

6 Analysis of the Model

We first present the necessary conditions for the solution to the household's problem, assuming that the value function is differentiable at the optimum. For what is to follow, we let μ_{z1} , μ_{z2} and μ_a represent the Lagrange/Kuhn-Tucker multipliers associated with $\bar{z} \geq 0$, $\bar{z} \leq \bar{\zeta}$ and $a' \geq 0$ respectively.

$$\bar{z} : Ph' \left[u_1(c, h') \frac{d(\bar{z}\Phi(\bar{z}))}{d\bar{z}} - \beta \int_{\bar{z}}^{z_{max}} V'(I') dF(z) \right] + (\mu_{1z} - \mu_{2z}) = 0 \quad (9)$$

$$h' : -u_1(c, h')P(1 - \bar{z}\Phi(\bar{z})) + u_2(c, h') + \beta P \int_{\bar{z}}^{z_{max}} V'(I')(z - \bar{z}) dF(z) = 0 \quad (10)$$

$$a' : u_1(c, h') - \mu_a = \beta(1 + r)\mathbb{E}V'(I') \quad (11)$$

where (8) can be used to show that

$$\frac{d(\bar{z}\Phi(\bar{z}))}{d\bar{z}} = \frac{1}{1+r} \left[1 - (1-\eta)[F(\bar{z}) + \gamma\bar{z}f(\bar{z})] \right] \quad (12)$$

These necessary conditions can be used to rationalize some of the important institutional assumptions we have made earlier.

Claim: An equilibrium exists only if $\bar{z}\Phi(\bar{z}) < 1$ for all feasible $\bar{z}$.

The proof of this claim is immediate from the first-order condition with respect to housing choice. Suppose that, for some feasible $\bar{z}$, $\bar{z}\Phi(\bar{z}) \geq 1$ holds. If the household chooses such a $\bar{z}$, for any given house price $P > 0$, the first term in the equality is non-negative. Therefore, left-hand side of the necessary condition is strictly positive, which implies the household would demand infinite housing since it can be acquired costlessly. By doing so, it can increase the instantaneous payoff indefinitely.

This result indicates that an upper bound on $\bar{z}$ is essential under some circumstances. It is easy to verify that provided that there is cost to defaulting ($\gamma > 0$), if there is no subsidy (when $\eta = 0$), $\sup \bar{z}\Phi(z) < 1$ holds, therefore there is no need to impose an upper bound on $\bar{z}$. However, this result breaks down as soon as we introduce government subsidies. We will therefore define government subsidy as a pair $(\eta, \bar{\zeta})$ where $\bar{\zeta}$ restricts the choice set in such a way that $\sup \bar{z}\Phi(z) < 1$ holds over all feasible $\bar{z}$.

7 Calibration and Results

Our model period is a year. Following the literature, we choose the discount rate $\beta = 0.96$ and capital depreciation rate of $\delta = 0.1$. The share of capital in production is set to $\alpha = 1/3$. TFP is normalized to unity, $A = 1$. The intertemporal elasticity of substitution is set to $\sigma = 1$, so that the utility function is logarithmic.

We assume that the distribution of shocks follow Kumaraswamy distribution. Kumaraswamy is a two-parameter class of distributions that are as flexible as the Beta distribution, but with the additional benefit of having simple closed-form expressions for pdf and cdf⁸. For our quantitative exercise, we use the

⁸The pdf and cdf for a random variable $X \in [0, 1]$ following Kumaraswamy distribution with shape parameters a, b are

$$f(x) = abx^{a-1}(1-x^a)^{b-1}$$

$$F(x) = 1 - (1-x^a)^b.$$

The distribution can be defined for any interval $z_{min} \leq Z \leq z_{max}$ by transforming the random variable as

$$x = \frac{z - z_{min}}{z_{max} - z_{min}}$$

Kumaraswamy distribution with bounds $[0, 2]$. We use default cost γ to match 0.5% default rate under no-subsidy. This is reported by Jeske, Krueger, and Mitman (2012) to be the approximate percent of defaults (in “normal times”) that end up being liquidated. Because we do not have a clear basis for calibrating all aspects of the distribution, we undertake two alternative calibrations, each with different distributional parameters. The shape parameters a and b are chosen to match $\mathbb{E}(z) = 1$ and two different left-tail probabilities, $Pr(z \leq 0.8) \in \{0.05, 0.15\}$. All parameter values are summarized in table 1, where the first part of the table includes the common parameter values and the second part of the table includes two sets of parameters for the alternative calibration targets.

A policy reform in our quantitative exercise consists of a subsidy rate-LTV limit pair $(\eta, \bar{\zeta})$. Indeed, a LTV limit is a *requirement* when there is large enough government subsidy: As discussed earlier, if there is no limit, for any value of housing, agents can borrow an amount higher than the ex-ante value of housing (i.e. $\bar{z}\Phi(\bar{z}) \geq 1$), this in turn would lead to an infinite demand for housing. To get a good idea of the impact of alternative policies, we report results for nine different combinations of these policy variables: Three alternative values for the loan-to-value limits $\bar{\zeta} \in \{0.8, 0.85, 0.9\}$ and three different values for subsidy rate $\eta \in \{0.3, 0.6, 0.9\}$. Along with two different calibrations, we report the impact of 18 policy changes in total. Tables 2 and 3 summarize the findings for low left-tail probability and high left-tail probability cases respectively.

The impact of the implicit subsidy on house prices is rather dramatic, even for modest policy values. The values in parantheses right next to house prices in tables 2 and 3 represent the percent increase over the no-subsidy benchmark.

For the first calibration (low left-tail probability), these values range from 2.3% (with 30% subsidy and 80% LTV limit) to 77.25% (with 90% subsidy and 90% LTV limit). Essentially, all agents borrow up to the limit for all subsidy rates. An increase in subsidy rate unambiguously leads to an appreciation of house prices, and an increase in tax rates. An increase in LTV limit has the same effect on these variables qualitatively. In addition, default rates go up significantly as LTV limit goes up.

The second set of results (high left-tail probability) indicates that there is no direct relationship between subsidy rates and house prices for relatively low subsidy rates, in particular when agents do not borrow up to the limit. For example, as table 3 suggests, house prices decline by a negligible amount (-0.7%) when the subsidy rate is 30% and the LTV limit is 0.8. This is presumably due to a small general equilibrium effect induced by the interest rate movements. However, for higher values of η , the movement in house prices is reversed. Indeed, for this calibration, the house price appreciates by more than 350% when the subsidy is about 90% and the LTV limit is 90%!

Overall these quantitative results suggest a potentially large, and in any case non-negligible impact of

the guarantees on house prices. There are some aspects of the calibration that need more refinement. In the baseline case the average LTV is somewhat on the low side. The increases in default rates for η equal to 0.6 are too large. Most important, narrowing down a reasonable range of values for η is crucial for narrowing down the range of price impacts. While high values of η are reasonable for individual mortgages, we would argue that a more appropriate interpretation of η is the likelihood of a systemic shock that results in bailouts of financial institutions. This is presumably much smaller, though arguably grew over time up to the onset of the crisis.

8 Conclusions

This paper quantifies the impact on house prices of credit market distortions stemming from implicit loan guarantees and bailouts. Such interventions amount to implicit subsidies for borrowers. We specifically model the impact of these implicit subsidies on house prices, and, consequently, on price-rental ratios as well. We find that the distortions stemming from implicit or explicit guarantees could have played a significant role in the housing price boom of the early 2000s. In particular, we show that under plausible parameterizations of the model, such guarantees, along with relaxed lending standards (represented by higher loan-to-value ratios) could easily generate 10 to 30 percent increases in house prices even for intermediate values of the likelihood or extent of bailouts. At more extreme values, where mortgages are nearly fully guaranteed by taxpayers, the impact on house prices can be astronomical. In addition to the impact on prices, the policies also generate large increases in borrowing on the extensive margin (in addition to the intensive) and, of course, in default rates.

At present we regard these results as suggestive, and more work is needed on the calibration. Borrowing rates are somewhat low in the baseline, and with guarantees the default rates increase by too much even relative to the large increases observed during the crisis. Nonetheless we would argue that the exercise successfully demonstrates the enormous potential distortions of even moderate policy interventions given the very large changes in behavior (notably leverage) and valuations the policies induce. Future work will consider aggregate shock, not simply to model the actual collapse, but to distinguish between policies of mortgage guarantees and bailouts of financial institutions. The latter would more likely be triggered only in the event of a systemic event. In our model, the systemic event is “in the background,” and we only quantify the positive impact on prices of the implicit subsidies.

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Table 1: Parameter Values

Parameters	Values	Target Moments	Target values
<i>Common Parameters</i>			
Discount rate (β)	0.960	Standard Annual Rate	
Depreciation rate (δ)	0.100	Standard Annual Rate	
Intertemporal Elast. of Subst. (σ)	1.000		
Capital share in production (α)	0.333	Standard	
<i>Two Alternative Calibrations</i>			
Distribution shape Parameter (a)	{11.042, 6.066}	$Pr(z \leq 0.8)$	{0.05, 0.15}
Distribution shape Parameter (b)	{1270.916, 42.085}	$\mathbb{E}(z) = 1$ (Normalization)	
Share of Housing (θ)	{0.092, 0.093}	Flow expenditure share	25%
Cost of Default (γ)	{0.022, 0.137}	Equilibrium default	0.5%

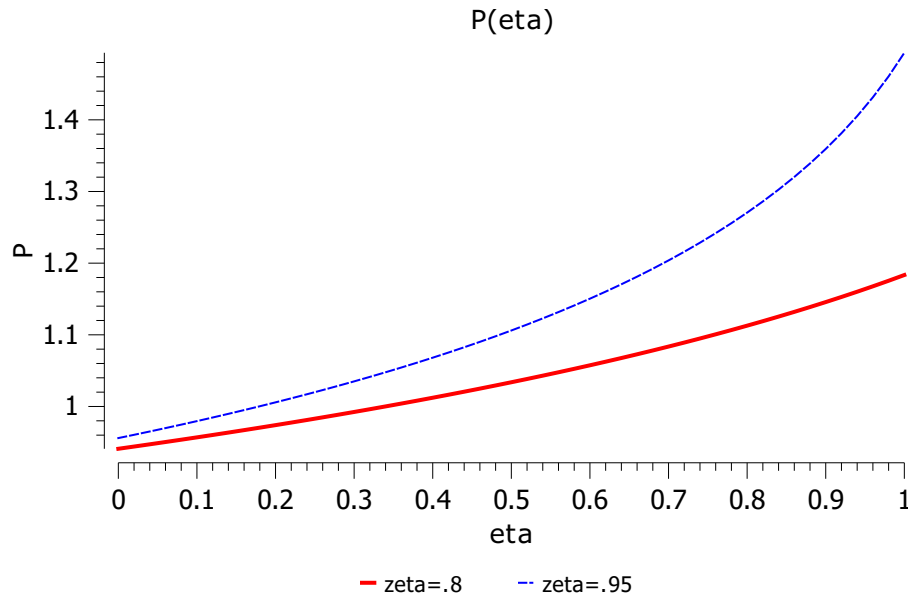
Table 2: Results for Low Left-Tail Probability

Variable	No subsidy	$\eta = 0.3$	$\eta = 0.6$	$\eta = 0.9$
$\bar{\zeta} = 0.80$				
House Price (P)	2.74	2.80 (2.30%)	2.94 (7.42%)	3.10 (13.07%)
Risk-free Rate (r)	4.14%	4.14%	4.14%	4.14%
Tax Rate (τ)	0.00%	0.56%	1.17%	1.85%
Percent Default	0.50%	7.71%	7.71%	7.71%
Percent Borrowing	33.11%	100%	100%	100%
Average loan-to-value ratio ($\bar{z}(1 + r)$)	0.35	0.83	0.83	0.83
$\bar{\zeta} = 0.85$				
House Price (P)	2.74	2.86 (4.26%)	3.17 (15.67%)	3.56 (29.83%)
Risk-free Rate (r)	4.14%	4.14%	4.14%	4.14%
Tax Rate (τ)	0.00%	1.15%	2.55%	4.30%
Percent Default	0.50%	14.52%	14.52%	14.52%
Percent Borrowing	33.11%	100%	100%	100%
Average loan-to-value ratio ($\bar{z}(1 + r)$)	0.35	0.89	0.89	0.89
$\bar{\zeta} = 0.90$				
House Price (P)	2.74	2.95 (7.57%)	3.68 (34.40%)	4.86 (77.25%)
Risk-free Rate (r)	4.14%	4.15%	4.14%	4.13%
Tax Rate (τ)	0.00%	2.76%	5.67%	11.20%
Percent Default	0.50%	25.55%	25.53%	25.51%
Percent Borrowing	33.11%	100%	100%	100%
Average loan-to-value ratio ($\bar{z}(1 + r)$)	0.35	0.94	0.94	0.94

Table 3: Results for High Left-Tail Probability

Variable	No subsidy	$\eta = 0.3$	$\eta = 0.6$	$\eta = 0.9$
$\bar{\zeta} = 0.80$				
House Price (P)	2.78	2.76 (-0.7%)	3.19 (14.60%)	4.54 (63.34%)
Risk-free Rate (r)	4.05%	4.06%	4.08%	4.01%
Tax Rate (τ)	0.00%	0.13%	7.74%	16.44%
Percent Default	0.50%	0.64%	18.72%	18.65%
Percent Borrowing	25.36%	62.05%	100%	100%
Average loan-to-value ratio ($\bar{z}(1+r)$)	0.29	0.13	0.83	0.83
$\bar{\zeta} = 0.85$				
House Price (P)	2.78	2.78 (0.06%)	3.44 (23.77%)	6.51 (134.06%)
Risk-free Rate (r)	4.05%	4.06%	4.09%	3.93%
Tax Rate (τ)	0.00%	0.13%	12.41%	34.70%
Percent Default	0.50%	0.74%	25.90%	25.71%
Percent Borrowing	25.36%	62.11%	100%	100%
Average loan-to-value ratio ($\bar{z}(1+r)$)	0.29	0.13	0.88	0.88
$\bar{\zeta} = 0.90$				
House Price (P)	2.78	2.76 (-0.5%)	3.88 (39.72%)	12.80 (360.43%)
Risk-free Rate (r)	4.05%	4.06%	4.09%	3.44%
Tax Rate (τ)	0.00%	0.11%	20.09%	93.31%
Percent Default	0.50%	0.77%	34.61%	33.56%
Percent Borrowing	25.36%	62.19%	100%	100%
Average loan-to-value ratio ($\bar{z}(1+r)$)	0.29	0.14	0.94	0.93

Figure 1: House Price as a function of Subsidy



9 Appendix

Proposition 1 *If $V(\cdot)$ is strictly increasing, full subsidy ($\eta = 1$) induces borrowing up to the limit $\bar{z} = \bar{\zeta}$.*

Proof. When $\eta = 1$, $\frac{d(\bar{z}\Phi(\bar{z}))}{d\bar{z}} = \frac{1}{1+r}$ due to (12). Rearranging (9), noting $h'(I) > 0$, we have

$$\begin{aligned} u_1(c, h') - \beta(1+r) \int_{\bar{z}}^{z_{max}} V'(\cdot) dF(z) &= \frac{1+r}{Ph'} (\mu_{2z} - \mu_{1z}) \\ &\geq \beta(1+r) [\mathbb{E}V'(\cdot) - \int_{\bar{z}}^{z_{max}} V'(\cdot) dF(z)] \end{aligned}$$

where the inequality follows from $u_1(c, h') \geq \beta(1+r)\mathbb{E}V'(\cdot)$, which must be satisfied for all parameters. This is derived from (11).

We claim that $\bar{z} = 0$ cannot hold at an optimum. If $\bar{z} = 0$ were true, then $\mu_{2z} = 0$, $\mu_{1z} > 0$ and $[\mathbb{E}V'(\cdot) - \int_{\bar{z}}^{z_{max}} V'(\cdot) dF(z)] = 0$. These three conditions lead to the following contradiction.

$$0 > \frac{1+r}{Ph'} (\mu_{2z} - \mu_{1z}) \geq \beta(1+r) [\mathbb{E}V'(\cdot) - \int_{\bar{z}}^{z_{max}} V'(\cdot) dF(z)] = 0.$$

Since $\bar{z} > 0$ at the optimum and $V(\cdot)$ is strictly increasing, $[\mathbb{E}V'(\cdot) - \int_{\bar{z}}^{z_{max}} V'(\cdot) dF(z)] > 0$. Due to the inequality above, $\mu_{2z} > \mu_{1z}$ must be satisfied at the optimum which implies $\bar{z} = \bar{\zeta}$ must hold.

Proposition 2 *If $V(\cdot)$ is strictly increasing, strictly concave, and default is not costly ($\gamma = 0$), all agents borrow up to the limit, i.e. $\bar{z} = \bar{\zeta}$.*

Proof. When $\gamma = 0$, $\frac{d(\bar{z}\Phi(\bar{z}))}{d\bar{z}} = \frac{1}{1+r} (1 - (1-\eta)F(\bar{z})) \geq \frac{1}{1+r} (1 - F(\bar{z}))$. Rearranging (9), noting $h' > 0$ and $F(\bar{z}) < 1$, we have

$$\begin{aligned} \frac{1+r}{Ph'(1-F(\bar{z}))} (\mu_{2z} - \mu_{1z}) &\geq u_1(c, h') - \beta(1+r)\mathbb{E}(V'(\cdot)|z \geq \bar{z}) \\ &\geq \beta(1+r) [\mathbb{E}(V'(\cdot)) - \mathbb{E}(V'(\cdot)|z \geq \bar{z})] \end{aligned}$$

where the second inequality follows from $u_1(c, h') \geq \beta(1+r)\mathbb{E}V'(\cdot)$, which must be satisfied for all parameters. This is derived from (11).

We claim that $\bar{z} = 0$ cannot hold at an optimum. If $\bar{z} = 0$ were true, then $\mu_{2z} = 0$, $\mu_{1z} > 0$ and $[\mathbb{E}V'(\cdot) - \mathbb{E}(V'(\cdot)|z \geq \bar{z})] = 0$. These three conditions lead to the following contradiction.

$$0 > \frac{1+r}{Ph'(1-F(\bar{z}))}(\mu_{2z} - \mu_{1z}) \geq \beta(1+r)[\mathbb{E}V'(\cdot) - \mathbb{E}(V'(\cdot)|z \geq \bar{z})] = 0.$$

Since $\bar{z} > 0$ at the optimum, $V(\cdot)$ is strictly increasing and strictly concave, $[\mathbb{E}V'(\cdot) - \mathbb{E}(V'(\cdot)|z \geq \bar{z})] > 0$. Due to the inequality above, $\mu_{2z} > \mu_{1z}$ must be satisfied at the optimum which implies $\bar{z} = \bar{\zeta}$ must hold.

Proposition 3 *If $V(\cdot)$ is strictly increasing and strictly concave, there is an equilibrium under full subsidy only if $\bar{\zeta} < 1+r < \frac{1}{\beta}$.*

Proof. The fact that $1+r < \frac{1}{\beta}$ follows by applying the standard arguments from Aiyagari (1994) on equation (11). We need to show that $\bar{\zeta} < 1+r$. Suppose, to get a contradiction, that $\bar{\zeta} \geq 1+r$ at an equilibrium. Under full subsidy, Proposition 1 implies $\bar{z} = \bar{\zeta}$. Marginal utility of housing is equal to

$$\begin{aligned} & -u_1(c, h')P(1 - \bar{z}\Phi(\bar{z})) + u_2(c, h') + \beta P \int_{\bar{z}}^{z_{max}} V'(\cdot)(z - \bar{z})dF(z) \\ & = -u_1(c, h')P(1 - \frac{\bar{\zeta}}{1+r}) + u_2(c, h') + \beta P \int_{\bar{\zeta}}^{z_{max}} V'(\cdot)(z - \bar{\zeta})dF(z) \\ & = P \left[-u_1(c, h')(1 - \frac{\bar{\zeta}}{1+r}) + \beta \int_{\bar{\zeta}}^{z_{max}} V'(\cdot)(z - \bar{\zeta})dF(z) \right] + u_2(c, h') \quad (13) \\ & > 0 \end{aligned}$$

where the first equality follows from $\bar{z}\Phi(\bar{z}) = \frac{\bar{z}}{1+r}$ under full subsidy and the last strict inequality follows from our assumption that $(1 - \frac{\bar{\zeta}}{1+r}) \leq 0$ and that the value function is strictly increasing. Since this inequality is true for any state, house demand is unbounded for all agents, a contradiction to house market clearing.