Screening as a Unified Theory of Delinquency, Renegotiation, and Bankruptcy[*]

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Abstract

We propose a simple model with adverse selection where delinquency, renegotiation, and bankruptcy all occur in equilibrium as a result of a simple screening mechanism. A borrower has private information about her cost of bankruptcy, and a lender may use random contracts to screen different types of borrowers. In equilibrium, some borrowers choose not to repay, and thus become delinquent. The lender renegotiates with some delinquent borrowers. In absence of renegotiation, delinquency leads to bankruptcy. Presence of competition may induce the incumbent lender to renegotiate even when he would not do so in the monopoly setting. We apply the model to analyze effects of a government intervention in debt restructuring. A mortgage modification program aimed at limiting foreclosures that fails to take into account private debt restructuring may have the opposite effect from the one intended.

Keywords: Default, Delinquency, Bankruptcy, Renegotiation, Adverse Selection, Screening, Consumer Credit

JEL Codes: D14, D82, D86, G18, G21

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1 Introduction

Default in the consumer credit market is not a simple binary event, but rather has multiple stages and possible outcomes. The first ‘stage’ is delinquency, which is defined as being overdue on loan payments for a specified period of time (usually at least 60 days). Some, but not all, delinquent borrowers end up in bankruptcy. Lenders sometimes renegotiate with delinquent borrowers to prevent bankruptcy and achieve debt settlement.

We propose a very simple model where a single friction generates all three phenomena — delinquency, renegotiation, and bankruptcy — as a part of an optimal arrangement. The single friction is adverse selection: a borrower has private information about her cost of bankruptcy. The borrower is indebted to a single lender. To keep the model as simple as possible, we abstract from how the debt was acquired; we later discuss how an endogenously determined debt level can be incorporated into the model. The lender offers repayment options to the borrower. The borrower may choose to file for bankruptcy rather than making the repayment. We study a simple case where the borrower's cost of bankruptcy, unknown to the lender, can take one of two values, high or low.

Faced with adverse selection, the lender has two basic options when restricted to offering deterministic contracts. First, by asking for repayment that does not exceed the low-cost borrower's willingness to pay, the lender can guarantee repayment from both types. Second, by asking for a greater repayment, the lender can extract more from the high-cost borrowers but loses the low-cost type to bankruptcy.

The lender may be able to do better if he can extract different repayments from different types of borrowers. However, he cannot separate the two types of borrowers by offering a menu of deterministic contracts. The reason is that both types of borrowers have the same utility if they make the same repayment, so naturally every borrower will choose a lower repayment. However, different types do have different utilities if they do *not* repay and end up in bankruptcy. The lender can thus utilize this feature and attempt to separate the two types of borrowers by using lotteries over repayments and bankruptcy. That is, the separation is possible because the two types value lotteries in a different way, as their utility from bankruptcy is different.

We show that the optimal (expected repayment maximizing) mechanism that separates the two types involves the lender offering a menu of random contracts that consists of a deterministic repayment aimed at the high cost borrowers, and a lottery aimed at the low-cost borrowers. The lottery for the low type is over a repayment that is lower than the deterministic one, and a very high repayment that exceeds the willingness to pay of both types. That is, the high-cost type makes a higher repayment, while the low-cost type is offered a better deal with some probability, but is forced into bankruptcy with the complementary probability.

One of the central points of the paper is that such a mechanism has a natural economic interpretation and delivers the three phenomena — delinquency, renegotiation, and bankruptcy — described above. Indeed, offering the aforementioned menu is equivalent to making the following sequential offers. First, the lender offers a high repayment, which only high-cost borrowers accept. We interpret the borrowers who have agreed to make the high payment as having repaid the loan, while the borrowers who refuse to do it as becoming *delinquent*. Next, the lender offers a lower repayment to the fraction of delinquent borrowers. We interpret the event of offering the lower repayment as *renegotiation*. The delinquent borrowers with whom the lender does not renegotiate declare *bankruptcy*.[1]

Renegotiation allows the lender to extract some repayment from the low-cost borrowers who reject the high repayment. However, the possibility of renegotiation makes delinquency more attractive and thus limits the amount that can be extracted from the high-cost borrowers. It is for this reason that the lender does not renegotiate with all delinquent borrowers. Thus our paper also addresses the question of why we see some renegotiation in the consumer credit market but not all bankruptcies are avoided.

Having characterized interaction between the borrower and the monopolistic lender, we then analyze an environment with competing lenders. The debt is owed to one lender — the incumbent, and both the incumbent and outsiders offer contracts to the borrower. We assume that old debt is senior, so that if the borrower accepts an outsider's contract, the outsider has to repay the debt to the incumbent.

We show that the outsiders never renegotiate with the borrower, although the incumbent lender might. But the incumbent who did not renegotiate in the monopoly setting may choose to do so under competition. The intuition is as follows. Presence of competition limits the ability of the incumbent lender to extract repayments from borrowers. When repayment from the high-cost borrowers is sufficiently limited, the lender can renegotiate with the low-cost borrowers without distorting the decision of the high-cost ones. Hence the probability of renegotiation under competition is higher than that under the monopoly.

[1]An implicit assumption for the sequential offers to be equivalent to the (simultaneous) menu offer is that the lender can commit not to renegotiate with all delinquent borrowers, for otherwise the high-cost borrowers will never agree to the initial high repayment.

We also illustrate that our model generates reasonable comparative statics predictions. In particular, we show that the bankruptcy rate is increasing in the debt level and is decreasing in the borrower's income.

Our model puts us in the unique position to analyze effects of a government intervention in consumer debt restructuring, such as a mortgage modification program aimed at limiting foreclosures. Indeed, for an individual borrower, such an intervention is triggered by *delinquency*, offers debt restructuring (i.e., involves *renegotiation*) with the goal of avoiding *bankruptcy*, or foreclosure. Not only does our model capture all these stages of default, but, most importantly, it allows us to explicitly analyze the response of private lenders to the government intervention. We show that a government program that fails to take into account private debt restructuring may have the opposite effect from the one intended — rather than limiting foreclosures, it may lead to a greater number of foreclosures in equilibrium.

1.1 Related Literature

Theoretical analysis of default in consumer credit markets has largely focused on bankruptcy and abstracted from delinquency, and especially renegotiation. (See, for example, Chatterjee, Corbae, Nakajima, and Ríos-Rull, 2007, Livshits, MacGee, and Tertilt, 2007, and many others.) Notable exceptions are Chatterjee (2010) and Benjamin and Mateos-Planas (2012). While Chatterjee makes a distinction between delinquency and bankruptcy, he does not allow for renegotiation. Most notably, Benjamin and Mateos-Planas (2012) propose a quantitative model where all three "phases" of default are present. The mechanics of their model are very different from ours, however. Renegotiation in Benjamin and Mateos-Planas is an exogenously imposed process, but the possibility of renegotiation leads to an endogenous distinction between delinquency and bankruptcy. They incorporate this mechanism into a quantitative model with incomplete markets and symmetric information, which allows them to study quantitative effects of policy changes. We instead offer a very simple mechanism that generates all three phenomena as *endogenous* features (stages) of an optimal screening contract in the presence of adverse selection. As will be clear from Section 6, the endogeneity of renegotiation can be very important for policy analysis.

Sovereign debt literature has paid much more attention to renegotiation — see the seminal work by Bulow and Rogoff (1989) and more recent contributions by Kovrijnykh and Szentes (2007), Benjamin and Wright (2009), Yue (2010), Arellano and Bai (2012), etc. Unlike those papers, we study the 'extensive margin' of renegotiation as the fraction of borrowers with whom the lender renegotiates is determined endogenously.

From the modeling standpoint, our analysis is very similar to the problem of a monopolistic seller facing risk-averse buyers with unknown valuations. In fact, our random contracts are akin to optimal pricing schemes described in Matthews (1983), Maskin and Riley (1997), and Miller, Piankov, and Zeckhauser (2005). In their settings, the seller uses risk to screen high-valuation buyers, just like the lender in our model uses lotteries to screen high-cost borrowers.

2 The Environment

We begin with a simple one-period model with one lender and one borrower.[2] The key friction in the model is adverse selection, which arises from the lender's inability to observe the borrower's cost of default.

The borrower is risk averse, and derives utility from consumption according to the utility function $u(c)$. The function u is continuous, strictly increasing and strictly concave. The borrower has endowment I, which is observable by everyone. The borrower is sufficiently indebted to the lender so that he can demand arbitrarily large repayments from her. For simplicity, we abstract from where the debt comes from; we discuss a possible way of endogenizing debt in the Appendix.

As an alternative to making repayments to the lender, the borrower has an option of declaring bankruptcy. The borrower's cost of filing for bankruptcy can be low or high, $\theta \in \{\theta_L, \theta_H\}$, where $\theta_L < \theta_H$. This cost is known to the borrower, but is unobservable to the lender. The prior belief of the lender that the bankruptcy cost is high is denoted by γ, where $0 \leq \gamma \leq 1$. Alternatively, γ can be interpreted as the fraction of high-cost borrowers. If the borrower declares bankruptcy, she receives utility $v(I, \theta)$, while the lender receives nothing. The function $v(I, \theta)$ is strictly increasing in I for each θ, and $v(I, \theta_H) < v(I, \theta_L)$. Moreover, $u(0) < v(I, \theta) < u(I)$ for all I and θ.

The lender is risk neutral and maximizes the expected repayment that he extracts from the borrower. We assume that the lender makes a take-it-or-leave-it offer to the borrower. An offer consists of the menu of contracts, where each contract — which can be deterministic or random — specifies how much the borrower should repay to the lender. A deterministic contract is simply the amount R the borrower is asked to repay; a random contract is

[2]We can alternatively assume that there are many borrowers.

a lottery over repayments. The borrower chooses one contract from the offered menu or rejects all contracts. In the latter case, or if the borrower does not make the repayment specified in the contract he chose, she has to declare bankruptcy and bear the bankruptcy cost.

3 Equilibria with the Monopolistic Lender

Before proceeding to the analysis of possible contracts that the lender can offer in equilibrium, it will be useful to define the $R_j(I)$ — the largest amount that a borrower with income I and bankruptcy cost θ_j is willing to repay:

$$u(I - R_j(I)) = v(I, \theta_j), \tag{1}$$

$j \in \{L, H\}$. By construction, the 'willingness to repay' of the low-cost borrowers is lower than that of the high-cost borrowers: $R_L \leq R_H$.

3.1 Deterministic Contracts

Suppose first that the lender is restricted to offering a single deterministic contract, R. Depending on the level of R, three situations may arise. If $R \leq R_L$, then both types of borrowers will accept the contract. If $R \in (R_L, R_H]$, then only the high-cost borrowers will accept the contract. Finally, if $R > R_H$, no borrower will accept it. Therefore, to maximize the expected repayment, the lender will either offer $R = R_L$ or $R = R_H$. We will refer to the first alternative as 'pooling', as it attracts both types of borrowers, and to the second alternative as 'exclusion', as it excludes (forces to bankruptcy) the low-cost borrowers.

Which of the two contracts generates higher profits to the lender will depend on the parameters of the model, in particular, on the fraction of high-cost borrowers, γ, and the extent to which the bankruptcy cost parameters, θ_H and θ_L, are different from each other. We elaborate on this more later.

3.2 Random Contracts

Since a deterministic contract specifies only the repayment, it is impossible to offer a menu of deterministic contracts and have different types of borrowers accepting different contracts. However, the lender may be able to achieve this by offering a menu of random contracts, as we will demonstrate below. We will refer to this case as 'screening', as the lender uses lotteries to screen the borrowers based on their cost of bankruptcy. As we only have two types of borrowers, we can limit the analysis to just two random contracts without loss of generality.

It is straightforward to see that the expected repayment is maximized by offering the following pair of contracts. The first contract is deterministic with repayment R_S that attracts only high-cost borrowers. The second contract is a lottery that offers a lower repayment with probability p and an implausibly large repayment (anything above R_H) with probability $1 - p$. To maximize the lender's expected profit, the repayment in the second contract must be set to R_L, as it makes the low-cost type just indifferent between accepting the lottery and declaring bankruptcy, and also minimizes the attractiveness of this contract to the high-cost borrowers. We denote this lottery by (R_L, p).

At the same time, the deterministic repayment R_S must be set to make the high-cost type just indifferent between the two contracts:

$$u(I - R_S) = pv(I, \theta_L) + (1 - p)v(I, \theta_H) = pu(I - R_L) + (1 - p)u(I - R_H), \qquad (2)$$

where the second equality follows from (1). Clearly, R_S is lower than R_H as long as $p > 0$, as the presence of the second contract will prevent extracting the full surplus from the high type. Also, R_S is higher than R_L as long as $p < 1$, for otherwise the high-cost borrowers' incentive constraint is lax and the lender could increase repayment by increasing R_S It is worth noting that the only reason why p is set strictly below one is to keep the high-cost type from accepting the contract meant for the low-cost type. Indeed, if p were equal to one, the high-cost borrowers would never opt for the higher payment offered to them.

The lender's problem is then simply to choose p to maximize the expected repayment:

$$\max_{p \in [0,1]} \gamma R_S(p) + (1 - \gamma)pR_L, \qquad (3)$$

where $R_S(p)$ is defined in (2). Notice that choosing $p = 1$ and $p = 0$ corresponds to the pooling and exclusion scenarios described above, respectively. Therefore the lender's problem is fully captured by (2)−(3). Strict concavity of the utility function immediately implies that this problem has a unique solution, which we denote by p^*. We summarize the above discussion in the following proposition.

Proposition 1 *The repayment scheme that maximizes the lender's profits is to offer a* *menu consisting of a deterministic repayment* R_S *and a lottery* (R_L, p^*)*, where* p^* *solves (3) subject to (2), and* R_S *is the corresponding* $R_S(p^*)$*.*

3.3 Sequential Interpretation of the Optimal Contract

One of the central points of the paper is that the simple screening mechanism described above generates the three stages of default in consumer credit: delinquency, renegotiation, and bankruptcy. In what follows, we use a sequential setting to illustrate that these stages are captured by the optimal contract described above.

Suppose that instead of offering the two contracts simultaneously, the lender offers them sequentially. We assume that the lender can commit to making certain offers ahead of time, i.e., he can commit to make the second offer before the first offer is made. Under this assumption, the simultaneous and sequential settings are equivalent. Indeed, the problem (2)−(3) fully captures the lender's problem in the sequential setting as well, and thus the same optimal pair of contracts is derived.

In the sequential setting, the optimal contract described earlier has the following interpretation. First, the lender offers a higher repayment, which only high-cost borrowers accept. We interpret the low-cost borrowers who refuse to make the specified repayment are as *delinquent*. Next, the lender offers a lower repayment to — i.e., *renegotiates* with — delinquent borrowers, but only with some probability. The borrowers with whom the lender renegotiates reach debt settlement, while the rest declare *bankruptcy*.

Notice that the assumption of commitment is crucial here. Without it, the lender would want to renegotiate with all borrowers who refused to make the initial high repayment. Of course, anticipating this, no one would make the high repayment to begin with.

3.4 Screening and Risk Aversion

We have described three possible strategies that the lender may follow: pooling, exclusion, and screening. Given the focus of the paper, the screening scenario is the most interesting of the three. Then the question arises: does the lender ever use screening (i.e., $p \in (0, 1)$) in equilibrium?

Interestingly, if borrowers were risk neutral, lotteries (and hence screening) would never be utilized in equilibrium. To see this, notice that with a linear utility function equation (2) reduces to

$$R_S = pR_L + (1 - p)R_H,$$

and the lender's problem becomes

$$\max_{p\in[0,1]} pR_L + (1-p)\gamma R_H,$$

which is simply a linear combination of the profits under pooling and exclusion. That is, screening is always dominated by either pooling or exclusion. Thus, the lender does not benefit from using random contracts.

With risk-averse borrowers, there are parameter values for which screening via random contracts gives the lender a strictly higher payoff than the pooling or exclusion alternatives. This happens, for example, when pooling and exclusion generate identical profits, i.e., when $R_L = \gamma R_H$. Indeed, in this case the low-cost borrowers are not affected by the riskiness of the lottery, as both outcomes generate the same utility for those borrowers (equal to their value of bankruptcy). However, if the high-cost borrowers accepted the lottery, they would receive $u(R_L)$ if renegotiation is successful and $u(R_H)$ otherwise. Thus they are willing to make a higher deterministic repayment to avoid the risk associated with the lottery: $R_S > pR_L + (1-p)R_H$. As a result, screening yields the maximum profit to the lender.

Of course, there are parameter values for which either pooling or exclusion would be the lender's optimal strategies. In particular, exclusion (pooling) is attractive when γ is high (low) enough.

4 Competition among Lenders

So far we considered an environment with a single monopolistic lender. Now we will assume that there is an incumbent lender and a competitive fringe of outside lenders (or, equivalently, just one outside lender), and the borrower owes the amount D to the incumbent. The lenders simultaneously offer a menu of contracts to the borrower. The borrower either chooses a contract from these menus or refuses all of them. We assume that the old debt is senior, which means that if the borrower accepts an outsider's contract and does not declare bankruptcy, the outsider must pay D to the incumbent.[3] Notice that the level of debt only plays a role in the presence of competition, by affecting the behavior of outsiders.

The first result in this competitive setting is that outsiders never offer lotteries in equilibrium. In other words, an outsider offers the same terms to all borrowers, and never

[3] Our key results do not change if we assume instead that an outsider needs to repay D even if the borrower declares bankruptcy.

renegotiates with borrowers who reject his offer.

Proposition 2 *Outsider lenders never offer random contracts in equilibrium. They always offer a deterministic repayment equal to the debt level D.*

Proof. Suppose on the contrary that an outsider offers R for the high-cost type and R' with probability p for the low type. If both contracts are accepted, the resulting profit to the lender is $\gamma(R-D)+(1-\gamma)p(R'-D)$. It must be the case that the outsider generates zero profits, for otherwise the incumbent (or another outsider) could offer slightly lower repayments and earn positive profits. An even stronger result must hold: the outsider must generate zero profits on each contract he offers, that is, $R-D=R'-D=0$. Indeed, if the higher repayment exceeded D, then an outsider could earn positive profits by offering only that repayment. On the other hand, if the lower repayment was less than D, the lender could increase his profits by not offering that repayment. Therefore outsiders offer a single repayment equal to D. $\square$

The argument behind the proof is very simple. Due to competition, outsiders must earn zero profits. Moreover, if an outsider offered a pair of contracts, he must generate zero profits on each of the contracts. If this was not the case, the outsider could only offer the most profitable contract and earn a positive payoff.

Even though the outsiders never renegotiate with the borrower, the incumbent lender might. The reason is that for the incumbent, debt is sunk cost, so he can earn less than D on some borrowers. As we will demonstrate later in this section, this is indeed what happens if the incumbent renegotiates with the borrower in the presence of competition: he earns exactly D on the high-cost borrowers, and strictly less than D on each of the low-cost borrowers. Furthermore, as we show below, the presence of competition may induce the incumbent to renegotiate, even when a monopolistic lender would not have (would have chosen exclusion).

Notice that if $D \leq R_L$, then outsiders' offers generate pooling, while if $D \in (R_L, R_H]$, they correspond to exclusion. If $D > R_H$, outsiders can never generate positive profits, and therefore their presence is irrelevant. But when D is low enough, outsiders make sufficiently attractive offers, which puts a restriction on what the incumbent can offer.

In light of Proposition 2, the incumbent's problem under competition can be easily obtained from problem the monopolistic problem (2)−(3) by simply imposing additional constraints that the offered repayments cannot exceed D. Specifically, let $R_L^C = \min\{R_L, D\}$.

Then the incumbent's problem becomes

$$\max_{p\in[0,1],R_S^C} \gamma R_S^C + (1-\gamma)pR_L^C, \tag{4}$$

$$\text{s.t.} \qquad u(I - R_S^C) = pu(I - R_L^C) + (1-p)u(I - R_H), \tag{5}$$

$$R_S^C \leq D. \tag{6}$$

Let p_C^* denote the incumbent's optimal choice of p in the competitive environment. Note that when constraint (6) binds, $R_S^C = D$, and p_C^* is pinned down by equation (5):

$$p_C^* = \frac{u(I-D) - u(I-R_H)}{u(I-R_L^C) - u(I-R_H)}. \tag{7}$$

In particular, when $D \leqslant R_L$, the above problem simply delivers simply $R_S^C = D$ and $p_C^* = 1$, which is the same (pooling) offer as that offered by outsiders.

The following proposition describes the types of contracts offered by the incumbent depending on the level of debt and on what he would have offered in absence of competitors. These types of contracts as well as the equilibrium bankruptcy rate, $(1-\gamma)(1-p_C^*)$, are illustrated on Figure 1.

Proposition 3 *(i) For $D \geq \overline{D}$, where $\overline{D} = R_H$, R_S, or R_L, if the monopolistic offer involves exclusion, screening, or pooling, respectively, the incumbent behaves as a monopolist.*

(ii) For $D \leq R_L$, the incumbent and outsiders make the same pooling offers (where the repayment equals D).

(iii) If the incumbent chooses screening under monopoly, he also performs screening under competition for $D > R_L$.

(iv) If the incumbent chooses exclusion under monopoly, he switches to screening under competition for $D \in (R_L, R_H)$.

Part (i) follows immediately since $\overline{D}$ corresponds to the highest repayment offered by the monopolist. By Proposition 2, outsiders offer D. Therefore, when $D \geq \overline{D}$, the outsiders' offers do not restrict the incumbent's choices, and thus he behaves as a monopolist.

When $D \leq R_L$, the offer of repayment equal to D is accepted by both types of borrowers. Clearly, the incumbent cannot offer a higher repayment, and would not find it profitable to offer a lower one either. Thus in this case the incumbent and outsiders making the same pooling offers, as stated in part (ii) of the proposition, and all borrowers avoid bankruptcy.

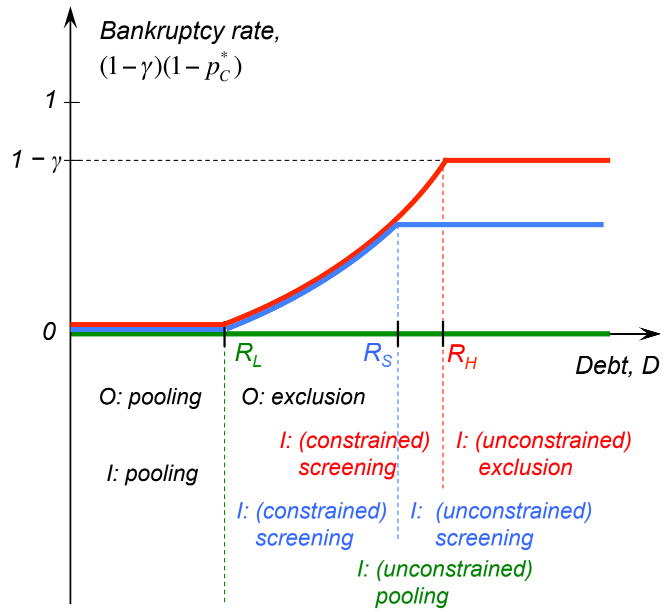

Figure 1: Types of equilibria and the bankruptcy rate depending on the level of debt. I and O stand for contracts offered by the incumbent and outsiders, respectively.

If the incumbent uses screening under monopoly, he would be restricted to offer D to the high-cost type under competition when $D > R_L$. But this allows the incumbent to increase the probability of renegotiation to the point where the high-cost type again becomes indifferent between the deterministic repayment and the lottery. Therefore the incumbent will still perform screening under competition in this case, as shown in part (iii). The lower the level of debt, the higher the probability of renegotiation, and the lower the bankruptcy rate. (Since $D > R_L$, the probability of renegotiation will always remain strictly smaller than one.)

To understand part (iv), recall that under exclusion the monopolist extracts R_H from the high-cost borrowers. He does not find screening attractive when the expected repayment from the low-cost borrowers is not enough to offset the decrease in the repayment from the high-cost borrowers. But with competition, the incumbent can only extract $D(\in (R_L, R_H))$ from them anyway. Therefore he might as well offer R_L to the low-cost type, by picking the probability of renegotiation to make the high-cost type just indifferent between the two offers.[4]

Proposition 3 contains an important result: presence of competition induces the incumbent to renegotiate more often (i.e., with a higher probability). In particular, the incumbent

[4]Equation (2) provides a simple way to see this: if $R_S \in (R_L, R_H)$, then $p \in (0, 1)$.

may start to renegotiate even when he would not do so in the monopoly setting. Intuitively, the only reason why the incumbent does not renegotiate with (all) delinquent borrowers is that the high-cost borrowers would then refuse to make the high repayment. By restricting this repayment, the presence of competition reduces the cost of renegotiation, causing the incumbent to renegotiate more often.

When competition is binding (which happens when $D \leq \bar{D}$) constraint (6) holds with equality so that $R_S = D$, and thus high-cost borrowers fully repay their debt. But when the debt level is large enough ($D > \bar{D}$), the incumbent asks for a repayment that is strictly below the debt level: $R_S < D$. Thus there is initial debt forgiveness for all borrowers. Low-cost borrowers refuse to make this payment, and we consider them delinquent. The lender renegotiates with a fraction of the delinquent borrowers, while the rest declare bankruptcy.

As argued above and as shown on Figure 1, the equilibrium bankruptcy rate is increasing in the level of debt. Moreover, the bankruptcy rate equals zero for if the debt level is low enough. Therefore the model generates reasonable comparative statics with respect to the amount of debt. We elaborate on this result as well as discuss other comparative statics results in the next section.

5 Comparative Statics

In this section, we establish some key comparative statics of the optimal contracts. We focus on how the equilibrium bankruptcy rate varies with the borrower's income and debt.

5.1 Comparative Statics with Respect to Debt

As we already discussed in the previous section, the model generates very reasonable comparative statics with respect to the debt level — the bankruptcy rate is (weakly) increasing in the amount of debt.

As can be seen from Figure 1, there are three 'regions' of debt levels for a given income level. When debt is sufficiently low ($D < R_L$), it is always repaid in full, and no bankruptcy is observed in equilibrium. When the face value of debt is sufficiently high ($D > \overline{D}$), competition is irrelevant, and thus so are marginal changes to the debt level. The incumbent behaves as a monopolist, and the bankruptcy rate is invariant to the debt level within this region. For intermediate levels of debt, the bankruptcy rate is strictly increasing in debt. We summarize the above results in the following proposition.

Proposition 4 *The equilibrium bankruptcy rate is increasing in the level of debt, strictly increasing for $D \in (R_L, \overline{D})$.*

5.2 Comparative Statics with Respect to Income

We now turn to the analysis of how the equilibrium contracts as well as the bankruptcy rate change with the level of the borrower's income. In order to derive analytical results, we turn to specific functional forms of the utility function and bankruptcy cost. In particular, we restrict our attention to the CRRA utility function $u(c) = c^{1-\sigma}/(1-\sigma)$, and the case where the bankruptcy cost is proportional to income: $v(I, \theta) = u((1-\theta)I)$.

Lemma 1 *Suppose that $u(c) = c^{1-\sigma}/(1-\sigma)$ and $v(I, \theta) = u((1-\theta)I)$. Then in the case with a monopolistic lender,*

(i) Repayments R_L, R_H, and R_S are proportional to I;

(ii) The probability of bankruptcy, $(1-\gamma)(1-p^*)$, is independent of the borrower's income.

Proof. First consider the case of $v(I, \theta) = u((1-\theta_j)I)$. Equation (1) becomes

$$u(I - R_j) = u((1-\theta_j)I),$$

which immediately implies that $R_j = \theta_j I$ for $j \in \{L, H\}$. Furthermore, substituting $u(c) = c^{1-\sigma}/(1-\sigma)$ into equation (2) and rearranging terms yields

$$\frac{(1-R_S/I)^{1-\sigma}}{1-\sigma} = p\frac{(1-\theta_L)^{1-\sigma}}{1-\sigma} + (1-p)\frac{(1-\theta_H)^{1-\sigma}}{1-\sigma}.$$

Since the right-hand side of the above equation does not vary with I, the left-hand side must not either. Thus R_S must be proportional to I for any p. This implies that we can simply factor I out of the objective function (3), and thus p^* does not depend on the borrower's income. This implies that the optimal level of R_S is still proportional to I. $\square$

Lemma 1 shows that if a monopolistic lender faces a borrower with higher income, he simply scales up the repayment(s) proportionally, but does not change the probability of renegotiation. Thus, the bankruptcy rate is invariant to the borrower's income. This last result might sound undesirable at first glance as in reality high-income borrowers are presumably less likely to declare bankruptcy. Notice, however, that this result is established for 'over-indebted' borrowers whose debt exceeds the level $\overline{D}$ defined in Proposition 3 so that the incumbent behaves as a monopolist. And as part (i) suggests, this debt level is strictly increasing in the borrower's income. Since the bankruptcy rate is lower under competition than under monopoly, increasing income for a fixed level of debt eventually lowers the bankruptcy rate. Moreover, the bankruptcy rate under competition is itself decreasing in income, as Proposition 5 below establishes.

The results of Lemma 1 and Proposition 5 are illustrated on Figure 2.

Proposition 5 *Suppose that* $u(c) = c^{1-\sigma}/(1-\sigma)$ *and* $v(I,\theta) = u((1-\theta)I)$*. Then in the presence of competition the probability of bankruptcy,* $(1-\gamma)(1-p_C^*)$*, is decreasing in the borrower's income* I *for any debt level* D*.*

Proof. If competition is non-binding ($D \geq \overline{D}$) or sufficient to preclude bankruptcy altogether ($D \leq R_L$), then the statement holds trivially as the bankruptcy rate does not change with income – see Lemma 1. What we want to establish is that the bankruptcy rate decreases with income in the region of "constrained screening", where $D \in (R_L, \bar{D})$. Note that in that region constraint (6) is binding, and hence constraint (7) with $R_L^C = R_L$ becomes

$$p_C^* = \frac{u(I-D) - v(I,\theta_H)}{v(I,\theta_L) - v(I,\theta_H)}.$$

Note that p_C^* is strictly decreasing in D, just as Proposition 4 suggests.

For $u(c) = c^{1-\sigma}/(1-\sigma)$ and $v(I,\theta) = u((1-\theta)I)$, the above equation becomes

$$p_C^* = \begin{cases} [(1-D/I)^{1-\sigma} - (1-\theta_H)^{1-\sigma}]/[(1-\theta_L)^{1-\sigma} - (1-\theta_H)^{1-\sigma}], & \text{if } \sigma \neq 1, \\ [\ln(1-D/I) - \ln(1-\theta_H)]/[\ln(1-\theta_L) - \ln(1-\theta_H)], & \text{if } \sigma = 1. \end{cases}$$

The numerator in the first expression is strictly increasing (strictly decreasing) in I and the denominator is strictly positive (strictly negative) when $\sigma < 1$ ($\sigma > 1$). Thus the first expression is strictly increasing in I, and the same is true for the second expression. Thus for all σ, $(1-\gamma)(1-p_C^*)$ is strictly decreasing in I for a given level of debt D such as $D \in (R_L(I), \overline{D}(I))$. Finally, the bounds of this interval are themselves strictly increasing in I, as part (i) of Lemma 1 and the definition of $\overline{D}$ suggest. $\square$

Note that the above proof implies that the bankruptcy rate in this case is invariant to proportional changes in both income and debt levels, as it only depends on I and D through their ratio.

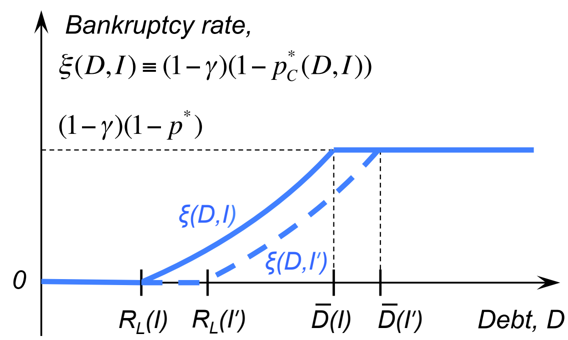

Figure 2: The probability of bankruptcy for two levels of income, I and I', where $I' > I$.

To summarize, this section shows that our model generates reasonable comparative statics of the bankruptcy rate with respect to debt and income levels.

6 Application: Government Intervention in Debt Restructuring

In this section, we will demonstrate that modeling private sector debt restructuring is crucial for understanding the effects of government intervention. One example of such an intervention is a mortgage modification program, such as HAMP (Home Affordable Mortgage Program) introduced in 2009 that aims at lowering the foreclosure rate (in order to limit the associated deadweight loss and/or out of concern for spill-over through depressed house prices etc.). We will analyze effects of such a program through the lens of our model and show that the intervention may have unintended consequences if its design is naive and ignores the effect on private restructuring.

Within our framework, we will assume that the government steps in if bankruptcy (foreclosure) is initiated, that is, if private renegotiation has been unsuccessful. In what follows, we restrict the government policy to simply offering repayment R_G with probability p_G. If the borrower accepts the offer and makes the repayment, the repayment is transferred to the incumbent lender.

Before proceeding with illustrating the unintended consequences of the policies, it is important to point out that in our model intervention is never Pareto improving, since equilibrium is constrained Pareto efficient (we assume that the government is subject to the same frictions as private lenders). In our analysis, we simply take the intervention as

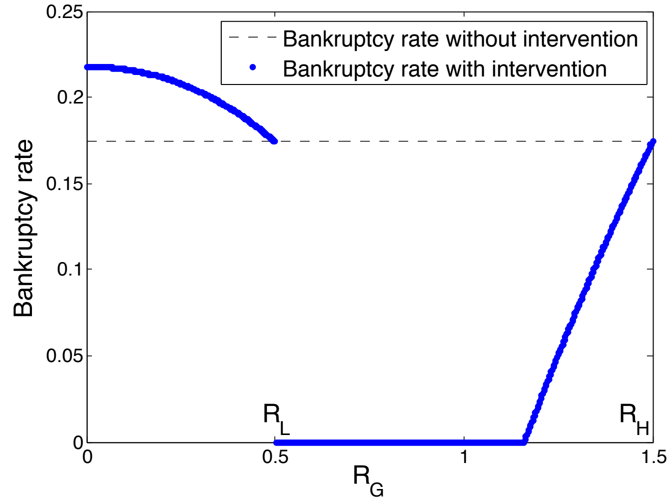

Figure 3: The bankruptcy (foreclosure) rate as a function of R_G. $p_G = p^*/2$.

exogenous and focus on its effects.

For simplicity, we restrict our attention to the case when the laissez-faire outcome is monopolistic screening, and will analyze the effects of the government intervention on the foreclosure (bankruptcy) rate. However, all of the results outlined below also hold in the presence of competition, in the constrained screening case.

In what follows, we will demonstrate three scenarios. In the first scenario, the policy is totally ineffective, although government is busy preventing foreclosures. In the second case, the policy is effective, while government appears to be inactive. In the third case the policy has the opposite effect from the one intended, that is, the foreclosure rate rises instead of falling.

The three cases are illustrated on Figure 3. It plots the bankruptcy (foreclosure) rate as a function of R_G. The probability of the government offer p_G is set to $p^*/2$, where p^* is the monopolistic probability of renegotiation in the absence of the intervention. The solid and dashed lines correspond to the bankruptcy (foreclosure) rate with and without the government intervention, respectively. Notice that when $R_G = R_H$, the government intervention is irrelevant. Thus the solid and the dashed line intersect at that point.

First, we analyze an example where the intervention is ineffective although the government is busy preventing foreclosures. Suppose that $R_G = R_L$ and $p_G \leq p^*$. Without the intervention, the lender sets p equal to p^*. Anticipating the intervention, the lender simply adjust p to offset the intervention, i.e., $p + (1-p)p_G = p^*$. The resulting foreclosure rate, $(1-\gamma)(1-p^*)$, is same as without the laissez-faire one, and thus the intervention is ineffective. This case is illustrated on Figure 3 where the solid line meets the dashed line at $R_G = R_H$. (On the figure $p_G = p^*/2 < p^*$.) In this case the government is busy preventing foreclosures, but its net effect is exactly nil.

Next, we describe the situation where the program is effective at reducing the foreclosure rate, although the government appears to be inactive. Suppose that $R_G \in (R_L, R_H)$, that is, the government offers a higher repayment to delinquent borrowers than the lender does. Clearly, the low-cost borrowers would rather declare bankruptcy than make this repayment, and thus the government renegotiation will be unsuccessful. However, the possibility of receiving the government offer will be attractive to the high-cost borrowers. Thus the lender has to reduce R_S to keep the high-cost type from rejecting his offer and opting for the chance of receiving a more attractive offer. Renegotiation might become more attractive causing the lender to increase p. If R_G is low enough (or p_G is high enough), screening becomes too costly for the lender, and he may increase p all the way to one and switch to pooling. As a result, the foreclosure rate drops to zero, and both types of borrowers are better off (the high type strictly so). While the government does not renegotiate with borrowers in equilibrium, its policy puts pressure on the lender to offer better repayment terms to borrowers. This scenario is illustrated on on Figure 3 to the right of the point $R_G = R_L$. As can be seen from the graph, in this case the foreclosure rate is lower is below the laissez-faire one, so the government policy is effective even though the government appears to be inactive.

Finally, we illustrate the most interesting scenario where the government program 'backfires' as it leads to a higher rather than lower foreclosure rate. Suppose that the government offers $R_G < R_L$. In this case, the government offer affects the lender's ability to extract repayment not only from the high type, but also from the low type.Screening becomes costly, and the lender may decrease p and move closer to exclusion. This may lead to a higher rather than lower equilibrium foreclosure rate, as the figure illustrates.

The above results indicate that explicit modeling of the private sector debt restructuring is key for analyzing the effects of government intervention. In particular, the failure of understanding how private lenders renegotiate with delinquent borrowers can lead to the opposite effect from the one intended.

7 Conclusions

We propose a simple model of consumer credit where a lender demands repayments from an indebted borrower. The borrower's alternative to making a repayment is to declare bankruptcy. The main friction in the model is that the borrower's cost of bankruptcy is her private information.

We characterize the optimal contract in this environment. We show that the lender may choose to screen different types of borrowers using lotteries over repayments. The optimal screening contract has a natural economic interpretation as it generates three stages of default — delinquency, bankruptcy, and renegotiation. Specifically, the lender first offers a high repayment that only borrowers with a high cost of bankruptcy accept. The low-cost borrowers refuse to make this payment, and are thus considered delinquent. The lender then renegotiates by offering a lower repayment, but only with a fraction of the delinquent borrowers, while the rest end up in bankruptcy.

We also show that in a setting with competitive lenders where old debt is senior, outsiders never renegotiate with the borrower. However, the pressure of competitors may induce the incumbent lender to renegotiate even when he would not do so under the monopoly.

We apply the model to analyze government intervention such as a mortgage modification program. We show that a program aiming at reducing foreclosures that overlooks the way the private debt restructuring works, can lead to an increase rather than a reduction in the foreclosure rate.

Appendix: A Two-Period Model of Debt

This section presents a simple two-period model that endogenizes the debt level acquired in the first period and has our basic mechanism at work in the second period.

Consider an incomplete market environment with one borrower and several identical lenders. There are two periods: $t = 1, 2$. Assume for simplicity that the borrower's endowment if period 1 is zero. Her endowment in period 2, as well as her cost of declaring bankruptcy in that period, are random and unknown to everybody in period 1. Once the uncertainty is realized in period 2, the realization of the borrower's endowment is public knowledge, while her cost of bankruptcy is her private information.

We assume that the markets are incomplete in that the period 1 contracts cannot be made contingent on the realization of endowment (as well as on the cost of bankruptcy). The contracting in period 1 is restricted to specifying a transfer of resources to the borrower, c_1, and the face value of the debt in period 2, D. Lenders compete in contracts (c_1, D) that they offer to the borrower, and the borrower picks one contract or rejects all contracts (the latter option means living in autarky). The lender whose contract is accepted becomes the incumbent. In period 2, once uncertainty is realized, the borrower, the incumbent lender, and the other lenders interact in the environment described in the main text given the level of debt D and the endowment realization I.

The borrowers preferences are represented by

$$u(c_1) + \beta E[(1 - \chi)u(I - R(D, I)) + \chi v(I, \theta)],$$

where $R(D, I)$ is the repayment in period 2 and χ is the indicator of bankruptcy. The expectation in the above expression is then taken over both realizations of the endowment in the second period, realization of the bankruptcy costs, and, if applicable, any contractual randomness.

The lenders have deep pockets and maximize $\frac{1}{1+r}E[R(D, I)] - c_1$, where r denotes the risk-free interest rate. Competing lenders earn zero profits in period 1, and thus (c_1, D) maximizes the borrower's expected utility in the first period subject to the lenders' zero profit condition. The problem of finding the equilibrium level of D can then be written as follows:

$$\max_D u\left(\frac{1}{1+r}E[R(D, I)]\right) + \beta E[(1 - \chi)u(I - R(D, I)) + \chi v(I, \theta)].$$

or

$$\max_D u\left(\frac{1}{1+r}E_I[\gamma R_S^C(D,I)+(1-\gamma)p_C^*(D,I)R_L^C(D,I)]\right)$$
$$+\beta E_I[\gamma u(R_S^C(D,I))+(1-\gamma)(p_C^*(D,I)u(R_L^C)+(1-p_C^*(D,I))v(I,\theta_L))].$$

In this model, the type of contract offered by the incumbent in the second period will vary depending on the income realization. For a given level of debt D, there is a threshold $\bar{I}$ such that $R_L(I) > D$ for all $I > \bar{I}$, so that the equilibrium in the corresponding states is competitive pooling, where all borrowers repay their debt. There is also a second threshold $\underline{I} \in (0, \bar{I}]$ such that $R_H(I) < D$ for all $I < \bar{I}$, so that the corresponding income states lead to a monopoly situation. In this case, the incumbent lender grants partial debt forgiveness to all borrowers, and then might renegotiate further with a fraction of low-cost borrowers who refuse to repay the specified payment. For $I \in (\underline{I}, \bar{I})$, we have $D \in (R_L(I), \bar{D}(I))$, which implies that the incumbent lender performs constrained screening and renegotiates with a fraction of delinquent borrowers. In this region, only the high-cost borrowers fully repay their debt, and the incumbent renegotiates with some low-cost borrowers. We can easily construct a distribution of second-period incomes that results in all three cases occurring in equilibrium with positive probability.

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