Search-Based Endogenous Illiquidity, Business Cycles and Monetary Policy

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Abstract

Illiquid secondary asset markets have adverse effects on firms' funding conditions and investment decisions. Yet, the micro-foundations of asset liquidity and the impact on aggregate business cycle, and the transmission of (unconventional) monetary policy interventions are scarcely explored. We introduce endogenous asset liquidity into a dynamic macroeconomic model. The endogenously varying asset liquidity is due to search frictions in secondary asset markets. Asset illiquidity creates a role for a liquid asset as a buffer for future funding needs. The liquidity differential between assets gives scope to non-standard monetary policy. The central bank may, for instance, control the supply of liquid assets and change the portfolio composition held by the private sector through asset purchase programs. Our analysis suggests, however, that such policies need to be carefully designed in order not to crowd out private investment.

Keywords: endogenous asset liquidity; search frictions; unconventional policy.

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1 Introduction

Asset illiquidity arises from impediments to the sale of assets in secondary markets. Empirical evidence points to pro-cyclical variation in the market liquidity of a wide range of assets.[1] By making asset liquidations more costly, such variation in asset illiquidity affects the funding conditions of both financial and nonfinancial firms. The variation thus may significantly affect the business cycle. At the same time, assets illiquidity creates a role for liquid assets as an insurance against future funding constraints. This gives scope to unconventional monetary policy which can control the supply of liquid assets and change the portfolio composition held by firms.

Endogenous secondary market illiquidity has profound implications for the financing conditions of firms. Nonfinancial US firms on average fund 35% of their total expenditures on physical capital (net of working capital) through primary (debt and equity) and secondary markets (portfolio liquidations).[2] While debt and equity issuance cover 75.67% of this financing gap, sales of liquid reserves make up a sizeable 20.74% and illiquid reserves 3.69%. Firms are thus directly affected by lower collateral values and higher liquidation costs.

We develop a dynamic general equilibrium model in which aggregate productivity fluctuations generate endogenous fluctuations in asset liquidity. Asset liquidity in the model is measured by the resaleable fraction of the underlying assets. The model shows how the variation in liquidity interacts with the funding conditions of firms over the business cycle.

To that end, we extend a conventional real business cycle model with search frictions in asset markets.[3] Our model economy is populated by three types of agents: a household sector whose members are either entrepreneurs (firms) or workers (investors), final goods producers, and the government. Entrepreneurs hold claims on corporate sector cash flows (including their own firms') on their balance sheets, which we interpret as a catch-all for privately issued assets such as commercial paper, corporate bonds, bank loans, and mortgages. Firms face idiosyncratic investment opportunities which cannot be fully financed by outside funding. In order to take full advantage of idiosyncratic investment opportunities, firms want to liquidate their asset portfolios. However, the private claims on their balance sheets may be illiquid because they have to be sold on a search market. Search for appropriate investments is assumed to be costly for investors. This market structure intends to capture the features of OTC markets, in which a large fraction of corporate bonds, asset-backed securities and private equity is traded.[4]

A shock that decreases the return to capital, such as a negative productivity

[1]See Huberman and Halka (2001), Chordia et al. (2001) and Chordia et al. (2005).
[2]See Ajello (2012) for detail documentation.
[3]By endogenizing asset liquidity we extend the foundational work of Kiyotaki and Moore (2012).
[4]See Dick-Nielsen et al. (2012), Bao et al. (2011), Anderson and Gascon (2009).

shock, makes investment into capital goods less attractive. This crowds out buyers from the equity market, which endogenously decreases the liquidity of equity claims. Lower liquidity of equity is tantamount to a tighter resaleability constraint. This liquidity-effect restricts the amount of funding available to entrepreneurs and, thereby, aggregate investment. These dynamics enable the model to generate procyclial asset liquidity and a counter-cyclical liquidity premium in line with the data. In other words, in our economy the value of liquid assets is high precisely when aggregate liquidity is low. This explains the demand for a liquid asset as an instrument to hedge illiquidity risks.

The main contribution is building search-base asset illiquidity into a dynamic macroeconomic model. The search-based approach differs from an information-based approach to asset illiquidity. In the latter, assets are either sold because of low quality or low cash flow. As income shrinks in bad economic conditions, more high-quality assets may be sold in such conditions to smooth consumption. This effect would improve asset liquidity, i.e. make it counter-cyclical. Information-based approach focuses on the selling pressures of claims holders. In the search-based approach, on the other hand, asset liquidity is driven by the market participation of investors, which depends on the productivity of assets. The search-based approach results in a straightforward positive correlation between productivity and liquidity.

Studies by Huberman and Halka (2001), Chordia et al. (2001) and Chordia et al. (2005) assert that market liquidity is time-varying and highly correlated across asset classes such as bonds and stocks in the US. This observation implies that common factors drive liquidity. Moreover, the observed time variation is pro-cyclical. Further, Chordia et al. (2005) find evidence that expansionary monetary policy improves stock liquidity during recessions.

The view that asset liquidity dries up during recessions has been reinforced by the 2007-2009 financial crisis. In this period, illiquidity problems were most pronounced in markets for short-term corporate finance such as the corporate bond, commercial paper, and repo. Using data that includes the majority of secondary over-the-counter corporate bond transactions in the US, Dick-Nielsen et al. (2012) identify a break in the market liquidity of corporate bonds at the onset of the subprime crisis. The liquidity component of spreads of all but AAA rated bonds increased and turnover rates declined, making refinancing on the corporate bond market more difficult. Similarly, Anderson and Gascon (2009) report that commercial paper (CP), which is largely traded on a search market with dealers as match-makers, experienced illiquidity. Asset-backed commercial paper, which in 2008 accounted for more than 40% all commercial paper outstanding, saw a sharp increase in spreads and a decline in accepted maturities resulting in 90% overnight issuances. At the same time, money market mutual funds, the main investors in the CP market, shifted away from commercial paper to government securities reflecting both a flight-to-safety and a flight-to-liquidity motive. Finally, as emphasized by Gorton and Metrick (2012), the repo market has registered strongly increasing haircuts during the crisis. These are attributed largely to concerns about the market liquidity of securities, i.e. claims on private sector cash flows, which are used as underlying collateral in repo agreements.

1.1 Related Literature

Our work is closely related to Kiyotaki and Moore (2012) (henceforth, KM), who motivate assets' market liquidity by an exogenous constraint on the resaleability of private paper. Their basic idea is that divestment takes time (due to unmodelled frictions), such that only some fraction of asset holdings can be sold in a given period to finance new investment. Money or government bonds, on the other hand, can be readily sold when needed and thus provide a liquidity service. A "spectrum of returns" emerges as a result of differences in asset liquidity. As a consequence, the irrelevance result of Wallace (1981), which stipulates that non-standard open-market operations in private assets are irrelevant for prices and allocations, no longer holds. In fact, the portfolio composition between liquid and illiquid assets in the private sector affects the equilibrium. To the extent that open-market operations can change this composition by controlling the supply of liquid assets they have real effects. Del Negro et al. (2011) analyse the stabilizing potential of such non-standard liquidity policy after an exogenous fall in liquidity. With standard monetary policy constrained by the Zero Lower Bound the authors show that liquidity injections effectively dampen the liquidity shortfall.[5] Asset illiquidity may further interact with financing constraints to induce delays in asset sales as in Cui (2013). This interaction prolongs shocks to the financing conditions of the private sectors and results in countercyclical productivity dispersion.

Treatment of illiquidity in these papers abstracts from feedback effects between macroeconomic conditions and asset market liquidity. In particular, it cannot account for endogenous cyclical variations in asset illiquidity. The search literature, on the other hand, provides a natural theory of endogenous liquidity. It has been applied to a wide range of markets such as housing (Wheaton, 1990; Ungerer, 2012), bank loans (Haan et al., 2003; Wasmer and Weil, 2004) as well as OTC markets for asset-backed securities, corporate bonds, US federal funds, private equity or real estate (Duffie et al., 2005, 2007; Ashcraft and Duffie, 2007; Lagos et al., 2009; Feldhutter, 2011). The bottom line of this research is that search frictions can explain

[5] Shi (2012) demonstrates that exogenous liquidity shocks induce asset price booms, because the liquidity shock essentially amounts to a supply shock on equity. Demand for equity, on the other hand, does not decrease substantially as investment projects' quality does not decrease. This counterfactual result carries over to our analysis of endogenous asset liquidity for plausible parameterizations.

substantial variation in a range of measures of liquidity, such as bid-ask spreads, trade volume (market depth) and trading delays.

Endogenous asset market liquidity has been the subject of a number of related studies. Eisfeldt (2004) provides a partial equilibrium model of endogenous asset market liquidity driven by information frictions. The model shows how investment and trading volume are amplified if asset liquidity endogenously varies with productivity. In Bigio (2011), dispersion shocks to capital quality endogenously decrease the liquidity of private assets due to information asymmetries. Such shocks translate into substantial fluctuations in hours, investment and output when private assets are used as collateral in working capital loan contracts. However, the model does not consider alterantive assets with different information properties, such as government bonds. In Eisfeldt and Rampini (2009), entrepreneurs endogenously accumulate liquid assets in the form of a riskfree store of value to fund investment opportunities. Because liquidity is accumulated out of retained earnings, the supply of liquidity correlates positively with productivity; there is no secondary market for private assets, however, and no alternative liquid asset whose supply can be controlled exogenously. Our approach differs from these papers, in that we motivate endogenous liquidity differentials across assets and preserve the role of a liquid asset as a lubricant of financial flows as in KM.

The rest of the paper is organized as follows: Section 2 presents the model and characterizes the equilibrium of our economy. Section 3 discusses impulse responses [and policy experiments]. In section 4, we conclude and outline avenues for further research.

2 The Model

Time is discrete and with infinite horizon. The economy comprises three sectors: a representative household, whose members are either entrepreneurs or workers, consumption good producers and the government. There are two deviations from the real business cycle framework: Limited enforcement in new claims to investment goods and search frictions afflicting the sale of existing claims. Government bonds, on the other hand, are assumed to be information-insensitive assets which can be traded freely on a spot market. Thus, such assets carry value when in the hands of entrepreneurs because they relax their financing constraints and improve aggregate investment. In this framework, non-standard open market operations that affect the supply of liquid assets have real effects.

2.1 Households

Representative household structure. The economy comprises a continuum of representative households of measure one with a unit measure of members each. Each period household members receive an idiosyncratic shock that determines their type during the period. With probability π household members become entrepreneurs (type i) and with probability $(1-\pi)$ they become workers (type s). Entrepreneurs have productive investment opportunities, while workers earn income by supplying their labour. Type shocks are *iid* across members and time. By the law of large numbers, each household thus consists of a fraction π of entrepreneurs and a fraction $(1-\pi)$ of workers. Both groups are temporarily separated such that resources cannot be re-allocated among household members during the period. Only at the very end of each period, both types come together again to share their consumption goods as well as their accumulated assets. This implies that all members enter the next period with an equal share of the household's assets.[6]

Timing. Each period is split into four phases: household's decisions, production, investment, and consumption.

Portfolio. Physical capital is held by households. There is a claim to the return of every unit of capital, which is either retained by households or sold to outside investors. In addition, households invest into riskless government bonds. Hence, at the onset of each period households have a portfolio of government bonds, equity claims on other households' return on capital and own physical capital. These assets are financed by equity claims issued on the return to own physical capital and net worth. This financing structure gives rise to the beginning-of-period balance sheet in Table 1.

Table 1: Household's Balance Sheet

Assets		Liabilities	
liquid bonds	b_t	equity issued	$q_{m,t}s_t^I$
ohter's equity	$q_{m,t}s_t^O$		
capital stock	$q_{m,t}k_t$	net worth	$q_{m,t}s_t + b_t$

Portfolio adjustments are affected by search frictions: Claims to private cash flows are sold on an OTC market. The rate at which these sales are matched to appropriate buyers is $P_{S,t}$. Hence, only a fraction $P_{S,t}$ of other households' equity can be sold per period due to search frictions. Similarly, the same fraction of previously uncommitted returns to physical capital can be mortgaged, i.e. $P_{S,t}(k_t - s_t^I)$. This simplification ensures that both types of assets not only yield the same return, but are equally liquid. They will thus yield the same price $q_{m,t}$ on the search market

[6]The representative household structure with temporarily separated agents has been introduced in Lucas (1990) and applied to the KM framework in Shi (2012) and Del Negro et al. (2011).

and can be treated as perfect substitutes, such that we only need to keep track of net equity, defined as

$$s_t = \underbrace{s_t^O}_{\text{other's equity}} + \underbrace{k_t - s_t^I}_{\text{unmortgaged capital}}$$

Preferences. The household determines entrepreneurs' and workers' choices in order to maximize

$$\mathbb{E}_t \sum_{k=0}^{\infty} \beta^{t+k} \left[u(c_{t+k}) - \frac{\mu}{1+\nu} l_{t+k}^{1+\nu} \right] \tag{1}$$

subject to resource and incentive constraints of its members, a law of motion of equity and non-negativity constraints. The decision problem is developed in section 2.1.5. Note that since both types of agents lump their consumption goods together at the end of the period, the household optimizes over household-wide consumption $c_t = \pi c_t^i + (1-\pi) c_t^s$.

2.1.1 Entrepreneurs

At the beginning of any period, members of a household are endowed with the same share of previously accumulated assets, i.e. net equity λs and bonds b. Once household members are separated, entrepreneurs receive investment opportunities. In order to finance investment projects, entrepreneurs can issue equity claims against newly installed capital and sell (or mortgage) claims to undepreciated net equity on the OTC market. However, only a fraction P_S of offered assets will be matched to a buyer.[7] Thus, the amount of net equity retained by entrepreneurs until the end of the period is endogenously bounded from below due to search frictions. Retained equity is bounded from above by the value of depreciated previous equity claims plus potential purchases of new equity $P_B e^i$. Hence, entrepreneurs' equity falls in the interval

$$(1 - P_S)(\lambda s + i) \leq s^{i'} \leq \lambda s + P_B e^i \tag{2}$$

Entrepreneurs' income consists of dividends to equity rs, net sales of equity claims $q_m \left((\lambda s + i) - s^{i'} \right)$ and returns on nominal government bonds $R^b \frac{b}{P}$. These resources can be spent on consumption, search effort, investment and new bonds. The gov-

[7] In contrast to KM and Shi (2012), this fraction will be endogenously determined on a search market.

ernment may levy lump-sum taxes τ. Hence, the flow-budget constraint reads

$$rs + q_m\left((\lambda s + i) - s^{i'}\right) + R^b\frac{b}{P} \geq c^i + \kappa e^i + i + \frac{b^{i'}}{P} + \tau \tag{3}$$

For the rest of the paper we focus on equilibria where entrepreneurs go short on the equity market up to the resaleability constraint, i.e. the lower bound of their future equity holdings. In these equilibria investment is profitable such that households have entrepreneurs devote all their resources to it. Accordingly, entrepreneurs do not purchase equity ($e^i = 0$), do not consume and ($c^i = 0$) and do not invest in liquid assets ($b^{i'} = 0$). From the binding resaleability constraint, entrepreneurs' net equity position evolves according to $s^{i'} = (1 - P_S)(\lambda s + i)$.

Plugging these equalities into the budget constraint yields entrepreneurs' financing constraint

$$rs + q_m P_S \lambda s + R^b\frac{b}{P} - \tau \geq (1 - q_m P_S)i \tag{4}$$

If the marginal utility from consumption is strictly positive, the budget constraint will hold with equality. Then, entrepreneurs' resaleability constraint is binding if and only if the financing constraint is binding.

2.1.2 Workers

Workers do not have investment opportunities and go long in the equity market. Their next-period equity consists of depreciated previous equity claims, purchases of new equity and purchases of claims to existing capital. To acquire the latter claims, workers have to exert costly search effort e^s, which will result in matches with appropriate sellers at the rate P_B. Hence, workers' net equity at the end of the period equals

$$s^{s'} = \lambda s + P_B e^s \tag{5}$$

Workers' flow-budget constraint is

$$wl + rs + q_m\left(\lambda s - s^{s'}\right) + R^b\frac{b}{P} \geq c^s + \kappa e^s + \frac{b^{s'}}{P} + \tau \tag{6}$$

Income derives from labour, dividends to equity and nominal bonds. This is spent on consumption, search effort, new equity claims, new bonds and lump-sum taxes.

2.1.3 Search and Matching

The matching function on the search market is a Cobb-Douglas aggregator taking as inputs aggregate effort $E = \pi e^i + (1-\pi)e^s + e^g$ and the aggregate amount of assets which are up for sale $S = \pi(\lambda s + i)$. The number of matches is

$$M = E^{(1-\gamma)}(S)^{\gamma} \tag{7}$$

where $\gamma \in (0,1)$ is the elasticity of matches w.r.t. assets up for sale.

The endogenous rate at which buyers encounter matching sellers (purchase rate) is

$$P_B \equiv \frac{M}{E} = \left(\frac{E}{S}\right)^{-\gamma} \tag{8}$$

while the endogenous sales rate is

$$P_S \equiv \frac{M}{S} = \left(\frac{E}{S}\right)^{1-\gamma} \tag{9}$$

2.1.4 Bargaining

Matched assets are valued from the household perspective, since the objective function is the utility of the representative household. Hence, there is only one valuation of matched equity independent of buyer type.[8] The value of a purchase on the search market equals the expected future marginal value of matched equity to the household, i.e. $\beta\mathbb{E}[V_{s'}]$.[9] The marginal value to the household of an entrepreneur not selling equity on the search market is given by $V_{s^{i\prime}}(X) = \pi\left\{-\eta q_m + \beta\mathbb{E}\left[V_{s'}(X')\right]\right\}$.

Matching surpluses for sellers and buyers, respectively, consist in the difference between the value of a transaction and the opportunity cost.

- seller surplus from match: $v^S = q_m - V_{s^{i\prime}}$
- buyer surplus from match: $v^B = \beta\mathbb{E}[V_{s'}] - q_m$
- total match surplus: $v = v^S + v^B = \beta\mathbb{E}[V_{s'}] - V_{s^{i\prime}}$

The price of assets in the search market is implicitly determined via Nash bargaining, i.e. maximizes the weighted sum of log-surpluses.

$$\max_{q_m} \omega \ln(v^S) + (1-\omega)\ln(v^B) \tag{10}$$

[8]Additionally, in the equilibria considered here only workers who value equity claims homogeneously go long in equity.

[9]This valuation is determined by FOC (A.8) and envelope condition (A.9) in Appendix A.

The FOC for this problem reads

$$\omega \frac{1}{v^S} \frac{\partial v^S}{\partial q_m} + (1-\omega) \frac{1}{v^B} \frac{\partial v^B}{\partial q_m} = 0 \tag{11}$$

which implies

$$(1 + \pi\eta) q_m = \left[\omega(1+\pi\eta) + (1-\omega)\pi\right] \beta \mathbb{E}\left[V_{s'}\right] \tag{12}$$

where η is the Lagrange multiplier on the households' budget constraint.

2.1.5 Decision Problem

The households' value function $V(s, b; K, Z)$ depends on the individual states net equity s and bond holdings b, the endogenous aggregate state variable capital K and exogenous states Z. Let $X = (s, b; K, Z)$. The household's choices each period consist of $(c^i, e^i, b^{i'}, i)$ for entrepreneurs, l for workers and (c, e, b') for the household. These choices implicitly pin down workers' consumption, equity and bond holdings $(c^s, e^s, b^{s'})$. Hence, the household solves

$$V(s, b; K, Z) = \max_{\{c^i, e^i, b^{i'}, i, l, c, e, b'\}} \left\{ u(c) - \frac{\mu}{1+\nu} l^{1+\nu} + \beta \mathbb{E}\left[V(s', b'; K', Z')\right] \right\} \tag{13}$$

subject to the financing constraint of entrepreneurs (4), the flow-budget constraint of the household, which is simply the sum of the members' budget constraints,

$$rs + q_m(\lambda s - s') + (q_m - 1)i + (1-\pi)wl + R^b \frac{b}{P} \geq c + \kappa e + \frac{b'}{P} + \tau \tag{14}$$

where $e = \pi e^i + (1-\pi)e^s$, the transition equation for equity

$$s' = \pi s^{i'} + (1-\pi)\lambda s + P_B e \tag{15}$$

and non-negativity constraints. When the financing constraint (4) binds, entrepreneurs' optimal choices of $(c^i, e^i, b^{i'})$ are $c^i = e^i = b^{i'} = 0$.

Let η be the Lagrange multiplier on the household resource constraint and η^i the multiplier on the financing constraint. Then the first-order conditions characterizing a solution to the household's optimization problem read[10]

$$u_c = \eta \tag{16}$$

$$w = \frac{\mu}{1-\pi}\frac{l^{\nu}}{u_c} \tag{17}$$

$$\eta\pi(q_m - 1) \leq \eta^i(1 - q_m P_S) \quad \text{and} \quad i \geq 0 \tag{18}$$

$$\eta = \beta\mathbb{E}\left[(\eta' + \eta^{i'})\frac{R^{b'}}{\pi'}\right] \tag{19}$$

$$\eta(q_m + \frac{\kappa}{P_B}) = \beta\mathbb{E}\left[\eta'\left(r' + q_m'\lambda + (1-\pi)\lambda\left(q_m' + \frac{\kappa}{P_B'}\right)\right) + \eta^{i'}(r' + q_m' P_S'\lambda)\right] \tag{20}$$

where the inequalities in (18) satisfy complementary slackness.

2.1.6 Value of Liquidity and the Equity Premium

With a binding financing constraint, (18) serves to manipulate (19) and (20) in order to obtain familiar asset-pricing equations for bonds

$$1 = \beta\mathbb{E}\left[\frac{\eta'}{\eta}\left(1 + \frac{\eta^{i'}}{\eta'}\right)R^b\right] \tag{21}$$

and for illiquid assets (taking into account search costs)

$$q_m + \frac{\kappa}{P_B} = \beta\mathbb{E}\left\{\frac{\eta'}{\eta}\left[r' + q_m'\lambda + (1-\pi)\lambda\left(q_m' + \frac{\kappa}{P_B'}\right) + \frac{\eta^{i'}}{\eta'}(r' + q_m' P_S'\lambda)\right]\right\} \tag{22}$$

The returns to both assets can be decomposed into two component: The first is the standard return that captures the asset's value for relaxing the future budget constraint. The second is the liquidity service provided by the asset by relaxing the future investment financing constraint. The multiplier that captures the marginal value of liquidity to the household is given by

$$\frac{\eta^i}{\eta} = \frac{\pi(q_m - 1)}{1 - q_m P_S} \tag{23}$$

This captures the (negative) liquidity premium that bonds pay in an economy with liquidity frictions relative to a frictionless economy. While the entirety of bond returns is liquid, only the resaleable fraction of private assets provides liquidity

[10] For the derivation see Appendix A.

services.[11] The difference in the relative liquidity services of both assets is, therefore,

$$
\begin{aligned}
\Delta &\equiv \frac{\eta^i}{\eta}\left(1 - \frac{r + q_m P_S \lambda}{r + q_m \lambda}\right) \\
&= \frac{\eta^i}{\eta}\frac{(1 - P_S) q_m \lambda}{r + q_m \lambda}
\end{aligned}
\tag{24}
$$

The liquidity differential implies that private assets pay a premium relative to riskless bonds in order to compensate for their lower liquidity service. This drives a wedge between the returns to both assets that increases as P_S drops, i.e. when the liquidity friction becomes tighter. From (21) and (22), this liquididty component of the realized equity premium next period amounts to[12]

$$
\Xi \equiv \frac{\eta^{i'}}{\eta'}\frac{(1 - P_S') q_m' \lambda}{q_m} \tag{25}
$$

2.2 The Government

The government conducts unconventional open market operations to affect the distribution of assets in the economy.[13] In particular, the government exerts effort to buy assets on the search market according to the rule

$$
e^g = \phi_e \left(\frac{P_S}{\bar{P}_S} - 1\right) \tag{26}
$$

Accordingly, the stock of private assets owned by the government evolves according to

$$
s^{g'} = \lambda s^g + P_B e^g \tag{27}
$$

Aside asset purchases, the government maintains a constant level of government consumption g and needs to redeem maturing government bonds B at rate R^b. These expenditures are financed by lump sum taxes τ, newly issued bonds B' and returns on private assets. The corresponding budget constraint is

$$
(\kappa + q_m P_B) e^g + R^b B + g = \tau + B' + r s^g \tag{28}
$$

[11] This is similar to Del Negro et al. (2011), where the resaleable fraction of private assets is constant, however.

[12] The premium must also compensate for the search cost incurred for bying the asset today less the costs saved by reduced search tomorrow. In order to isolate the variation in the premium due to illiquidity, we isolate the above liquidity component from

$$
0 = \beta \mathbb{E}\left\{ \frac{\eta'}{\eta}\left[\left(1 + \frac{\eta^{i'}}{\eta'}\right)\left(\frac{r' + q_m'\lambda}{q_m} - R^b\right) - \frac{\eta^{i'}}{\eta'}\frac{(1 - P_S')q_m'\lambda}{q_m}\right] + \frac{\eta'}{\eta}\frac{(1-\pi)\lambda(q_m' + \kappa/P_B')}{q_m} - \frac{\kappa/P_B}{q_m}\right\}
$$

[13] Our modelling of unconventional government policy is based on Del Negro et al. (2011).

In order to ensure intertemporal solvency, the government follows a tax rule.[14]

$$\tau - (\bar{\tau}) = \phi_\tau \left(R^b B - \bar{R}^b \bar{B} - q_m s^g \right) \tag{29}$$

2.3 Final Goods Producers

Final good producers operate on a competitive, frictionless market. They use capital (K) rented from households and labour from workers (L) as inputs into production and operate a technology which is subject to stochastic total factor productivity ($\exp(z)$).

$$Y = \exp(z) F(K, L) \tag{30}$$

Factors earn their marginal product, such that the interest rate on capital is $r = \exp(z) F_K(K, L)$ and wages are given by $w = \exp(z) F_L(K, L)$.

2.4 Aggregation and Market Clearing

Goods market clearing.

$$Y = c + g + \kappa E + \pi i \tag{31}$$

Labour market clearing.

$$L = (1 - \pi) l \tag{32}$$

Search market clearing.

$$\begin{aligned} P_S \pi(\lambda s + i) &= P_B (1 - \pi) e^s + P_B e^g \\ &= P_B E \end{aligned} \tag{33}$$

Clearing of the equity market implies that aggregate equity evolves according to

$$\begin{aligned} s' + s^{g'} &= \pi s^{i'} + (1 - \pi) s^{s'} + s^{g'} \\ &= \pi((1 - P_S)(\lambda s + i) + (1 - \pi)(\lambda s + P_B e^s) + \lambda s^g + P_B e^g \\ &= \pi((1 - P_S)(\lambda s + i) + (1 - \pi)(\lambda s) + \lambda s^g + \pi P_S (\lambda s + i) \\ &= \lambda(s + s^g) + \pi i \end{aligned} \tag{34}$$

[14] A low parameter value of ϕ_τ ensures that open market operations are primarily financed by government bond issuances.

From market clearing on the capital market it follows that there is an equity claim to every unit of aggregate capital.

$$K = s + s^g \tag{35}$$

The law of motion of the aggregate capital stock is consistent with the aggregation of the individual laws of motion

$$K' = \lambda K + \pi i \tag{36}$$

The market for government bonds clears such that individual holdings add up to the aggregate supply

$$b' = B \tag{37}$$

3 Results

3.1 Technology Shock

Responses to a negative technology shock are displayed in Figure 1. A negative technology shock depresses the return on capital, i.e. makes search for investment into entrepreneurs less attractive. As a result, the search effort exerted by workers drops. Less buyer effort is equivalent to a tightening of the search market from the perspective of asset sellers, i.e. entrepreneurs. This is reflected in the sharp 2.5% drop in the sales rate. As asset resaleability decreases the asset market becomes endogenously less liquid following a negative TFP shock. This *procyclical* asset liquidity is mirrored in a *countercyclical* response of the marginal value of liquidity to the household. The latter is measured by the shadow price of entrepreneurs' financing constraint as in equation (22), which tightens. Accordingly, the liquidity component of the equity premium from equation (25) rises in par with the marginal value of liquidity. As the financing constraint on new investment tightens, the amount of new investment falls.

At the same time, the drop in asset resaleability acts as a strong supply shock on the asset market. As this supply shock outweighs the decline in demand for assets, their price increases. This result persists despite the concurrent drop in productivity.

In the absence of search frictions, the liquidity effects do not occur. Figure 2 compares our model to a real business cycle benchmark, which is calibrated to yield the same volatility of investment. Here, the transmission channel is different: The negative TFP shock affects primarily the demand for capital goods and thus reduces the optimal level of investment. In contrast, entrepreneurs in our framework are financing constraint and would find it profitable to decrease investment by less.

3.2 Unconventional Policy

By changing the mix of assets held by private agents open market operations by the government may have real effects. Figure 3 displays the sensitivity of the model dynamics to asset purchases by the public sector on the search market. By increasing the supply of government bonds, the policy succeeds in dampening the marginal value of liquidity and the liquidity component of the equity premium. This also reduces the price impact of the shock on private assets. However, the government intervention has two important drawbacks: When the government has to purchase assets on the search market, this will take time. Accordingly, the amount of publicly held assets and the amount of liquid bonds issued to finance these purchases only rise very gradually. Secondly, the government intervention appears to crowd out search effort by private investors. Although the policy succeeds in reducing the need for liquidity in the private sector, the net effect on investment and output is negligible because of the crowding-out effect.

These results suggest that asset purchases could be more effective if conducted on a centrally cleared market - even if this entails efficiency losses due to matching with bad sellers.

[To be completed.]

3.3 Sensitivity Analysis

The key parameters that drive the behaviour of the search market are γ and ω. Figure 4 shows the sensitivity of our results to different values for the supply share γ in the matching function. The lower this value the higher is the sensitivity of matches to search effort, i.e. the demand side. A lower value of γ thus implies a sharper drop in the sales rate. Hence, illiquidity problems are more severe and the investment financing constraint becomes tighter. This amplifies the fall in investment. Moreover, the illiquidity of private assets induces a flight into liquid government bonds, such that their return becomes negative.

Similarly, a higher bargaining weight of sellers ω makes asset purchases less attractive for investors, since a larger share of the match surplus goes to the seller. This deters investors, such that search effort and the sales rate drop more severely. The impact as shown in Figure 4 is similar to that of a fall in γ in Figure 5.

4 Conclusion

We illustrate how asset liquidity (resaleable fraction) can co-move with aggregate productivity through secondary asset markets with search frictions. Liquidity of assets is important for the financing of future investment. Therefore, aggregate productivity affects not only resources produced but also financing conditions of firms through endogenous liquidity fluctuations.

We model the search costs on the buyer side for simplicity and tractability. One possible future extension is to include costs on both buyer and seller sides. The important difference is that there will be resale price discounts that induce another type of asset illiquidity. Such price discounts of assets will interact with the resaleable fraction of assets and induce rich dynamics of asset prices and quantities traded in secondary markets.

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A Households' Optimality Conditions Benchmark Model

State variables:

- individual states: $\{s, b\}$
- aggregagte states: $\{K, Z\}$

Choice variables:

- for entrepreneurs: $\{c^i, e^i, b^{i'}, i\}$
- for workers: $\{l\}$
- for the household: $\{c, e, b'\}$

Value function of the representative household

$$V(s, b; K, Z) = \max_{\{c^i, e^i, b^{i'}, i, l, c, e, b'\}} \left\{ u(c) - \frac{\mu}{1+\nu} l^{1+\nu} + \beta \mathbb{E}\left[V(s', b'; K', Z')\right] \right\} \quad \text{(A.1)}$$

subject to

- the flow-budget-constraint of the household (Lagrange multiplier η):

$$rs + q_m(\lambda s - s') + (q_m - 1)i + (1-\pi)wl + R^b \frac{b}{P} \geq c + \kappa e + \frac{b'}{P} + \tau \quad \text{(A.2)}$$

- entrepreneurs' financing constraint (Lagrange multiplier η^i):

$$rs + q_m P_S \lambda s + \frac{R^b b}{P} - \tau \geq (1 - q_m P_S) i \quad \text{(A.3)}$$

  where we have used the fact that under a binding equity constraint, the investor will use all available resources to finance investment, i.e. $s^{i'} = (1 - P_S)(\lambda s + i)$, $e^i = 0$, $c^i = 0$, $b^{i'} = 0$

- the transition equation of aggregate equity

$$s' = \pi s^{i'} + (1-\pi)\lambda s + P_B e \quad \text{(A.4)}$$

- non-negativity constraints (comprising the liquidity constraint)

Let $X = \{s, b; K, Z\}$. First Order Conditions:

- FOC wrt i

$$V_i(X) = \eta\pi(q_m - 1) - \eta^i(1 - q_m P_S) \leq 0 \tag{A.5}$$
$$i \geq 0, \text{ complementary slackness}$$

- FOC wrt l

$$V_l(X) = -\mu l^\nu + \eta(1 - \pi)w \leq 0 \tag{A.6}$$
$$l \geq 0, \text{ complementary slackness}$$

- FOC wrt c

$$V_c(X) = u_c - \eta \leq 0 \tag{A.7}$$
$$c \geq 0, \text{ complementary slackness}$$

- FOC wrt e

$$V_e(X) = -\eta(q_m P_B + \kappa) + \beta\mathbb{E}\left[V_{s'}(X')\frac{\partial s'}{\partial e}\right] \leq 0 \tag{A.8}$$
$$e \geq 0, \text{ complementary slackness}$$

where $\frac{\partial s'}{\partial e} = P_B$ and the Envelope Condition yields

$$V_s(X) = \eta(r + q_m\lambda) + \eta^i(r + q_m P_S\lambda) + \beta\mathbb{E}\left[V_{s'}(X')\frac{\partial s'}{\partial s}\right] \tag{A.9}$$

where $\frac{\partial s'}{\partial s} = (1 - \pi)\lambda$

- FOC wrt b'

$$V_{b'}(X) = -\eta/P + \beta\mathbb{E}\left[V_{b'}(X')\right] \leq 0 \tag{A.10}$$
$$b' \geq 0, \text{ complementary slackness}$$

Envelope Condition

$$V_b(X) = (\eta + \eta^i)\frac{R^b}{P} \tag{A.11}$$

B Proof: Binding Fincaning Constraing

Let the household buget constraint bind, i.e. $\eta > 0$. Then (18) implies that the financing constraint binds, i.e. $\eta^i > 0$, if and only if

$$\begin{aligned} & q_m > 1 > q_m P_S \\ \iff & 1 < q_m < \frac{1}{P_S} \end{aligned}$$

C Equilibrium

The competitive equilibrium is a collection of decision rules, constraints and market clearing conditions that pin down the choice variables $\{c_t, s_t, s^g, i_t, E_t, e^g_t, l_t, M_t, P_{S,t}, P_{B,t}, \tau_t, b_t, Y_t, L_t\}$, the endogenous and exogenous state variables $\{K_t; z_t\}$, prices $\{q_{m,t}, r_t, w_t, R^b_t\}$ and the Lagrange multiplier on the financing constraint $\{\eta^i_t\}$.

From the households optimization problem we have the financing constraint

$$\left(r_t + q_{m,t} P_{S,t} \lambda\right) s_t + R^b_{t-1} b_t - \tau_t = (1 - q_{m,t} P_{S,t}) i_t \tag{C.1}$$

the labour supply equation

$$w_t = \frac{\mu}{1-\pi} \frac{l_t^\nu}{c_t^{-\sigma}} \tag{C.2}$$

an equation pinning down the marginal value of relaxing the financing constraint

$$\frac{\eta^i_t}{c_t^{-\sigma}} = \frac{\pi(q_{m,t} - 1)}{1 - q_{m,t} P_{S,t}} \tag{C.3}$$

an asset-pricing equation for bonds

$$1 = \beta \mathbb{E}\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma} \left(1 + \frac{\eta^i_{t+1}}{c_{t+1}^{-\sigma}}\right) R^b_t \right] \tag{C.4}$$

an asset-pricing equation for illiquid assets (or, equivalently, effort)

$$q_{m,t} + \frac{\kappa}{P_{B,t}} = \beta \mathbb{E}\left\{ \left(\frac{c_{t+1}}{c_t}\right)^{-\sigma} \left[r_{t+1} + r^c_{t+1} + \frac{\eta^i_{t+1}}{c_{t+1}^{-\sigma}} (r_{t+1} + q_{m,t+1} P_{S,t+1} \lambda) \right] \right\} \tag{C.5}$$

where $r^c_{t+1} = q_{m,t+1}\lambda + (1-\pi)\lambda\left(q_{m,t+1} + \frac{\kappa}{P_{B,t+1}}\right)$.

The matching technology determines the number of matches in the search market, the sales-rate of entrepreneurs and the purchase-rate of workers

$$M_t = E_t^{(1-\gamma)}[\pi(\lambda s_t + i_t)]^\gamma \tag{C.6}$$

$$P_{S,t} = \frac{M_t}{\pi(\lambda s_t + i_t)} \tag{C.7}$$

$$P_{B,t} = \frac{M_t}{E_t} = P_{S,t}^{\frac{\gamma}{\gamma-1}} \tag{C.8}$$

From (A.8) we know that in equilibrium $\beta\mathbb{E}\left[V_{s'}\right] = \eta(q_m + \frac{\kappa}{P_B})$. Substituting this into the bargaining solution, the unique equilibrium Nash bargaining price becomes

$$q_{m,t} = \frac{\omega(1+\pi c_t^{-\sigma}) + (1-\omega)\pi}{c_t^\sigma + \omega(\pi - (1+\pi c_t^{-\sigma}))}\frac{\kappa}{P_{B,t}} \tag{C.9}$$

Final goods production is frictionless. The Cobb-Douglas-technology

$$Y_t = \exp(z_t) K_t^\alpha L_t^{1-\alpha} \tag{C.10}$$

implies that factors earn their marginal products

$$r_t = \exp(z_t)\alpha\left(\frac{K_t}{L_t}\right)^{\alpha-1} \tag{C.11}$$

$$w_t = \exp(z_t)(1-\alpha)\left(\frac{K_t}{L_t}\right)^{\alpha} \tag{C.12}$$

From the government policy and balanced budget we have

$$(\kappa + q_{m,t}P_{B,t})e_t^g + R_t^b B_t + g = \tau + B_{t+1} + r_t s_t^g \tag{C.13}$$

$$e_t^g = \phi_e\left(\frac{P_{S,t}}{\bar{P}_S} - 1\right) \tag{C.14}$$

$$s_{t+1}^g = \lambda s_t^g + P_{B,t}e_t^g \tag{C.15}$$

Equilibrium prices clear the markets for consumption, labour, capital, liquid assets, and illiquid assets

$$Y_t = c_t + g + \kappa E_t + \pi i_t \tag{C.16}$$

$$L_t = (1-\pi)l_t \tag{C.17}$$

$$K_t = s_t + s_t^g \tag{C.18}$$

$$b_t = B_t \tag{C.19}$$

$$s_{t+1} + s_{t+1}^g = \lambda(s_t + s_t^g) + \pi i_t \tag{C.20}$$

where the latter implies $K_{t+1} = \lambda K_t + \pi i_t$.

Exogenous shocks follow an AR(1) process in levels with white-noise disturbances

$$z_t = \rho_z z_{t-1} + \epsilon_t \tag{C.21}$$
$$z_t^{liq} = \rho_{liq} z_{t-1}^{liq} + \epsilon_t^{liq} \tag{C.22}$$

D Steady State

From (C.20) the steady state ratio of investment to illiquid assets is

$$\frac{i}{s} = \frac{1-\lambda}{\pi} \tag{D.1}$$

(C.10), (C.11), and market clearing for illiquid assets imply

$$\frac{Y}{s} = r/\alpha \tag{D.2}$$

Since the government does not conduct unconventional policy in the steady state, we get from the budget constraint

$$\frac{\tau}{s} = \frac{g}{s} - \left(1 - R_t^b\right) \frac{B}{s} \tag{D.3}$$

Combining (C.3) and (C.4) gives q_m as a function of the sales rate P_S

$$\begin{aligned} q_m &= \frac{1 + \left((\beta R^b)^{-1} - 1\right) \pi^{-1}}{1 + \left((\beta R^b)^{-1} - 1\right) \pi^{-1} P_S} \\ &\equiv \frac{1+A}{1+AP_S} \end{aligned} \tag{D.4}$$

where we set R^b exogenously and endogenize the parameter κ. Substituting equations (D.1)-(D.4) into the the steady state financing constraint yields the return on capital goods r as a function of the sales rate only

$$r = \left[1 - \lambda - (1 + \lambda(\pi - 1)) \frac{1+A}{1+AP_S} P_S\right] \left[\pi \left(1 + \left(\frac{b}{Y} - \frac{g}{Y}\right)/\alpha\right)\right]^{-1} \tag{D.5}$$

Next, we express the endogenous parameter κ as a function of r and P_S by substituting (C.3) and (C.8) into (C.5)

$$\kappa = \left\{\frac{r}{R^b} + \left[\beta\lambda\left(2 - \pi + A\pi P_S\right) - 1\right] \frac{1+A}{1+AP_S}\right\} (1 - \beta\lambda(1-\pi))^{-1} P_S^{\frac{\gamma}{\gamma-1}} \tag{D.6}$$

From (C.9) we derive another expression that links κ, r, P_S by expanding $c^{-\sigma}$ to $\left(\frac{c}{s}\right)^{-\sigma} s^{-\sigma}$ and substituting (C.8).

$$\begin{aligned}&\left[\left(\frac{c}{s}\right)^{\sigma} s^{\sigma} + \omega(\pi-1)\right] \frac{1+A}{1+AP_S} \\ &= [\omega + (1-\omega)\pi] \frac{\kappa}{P_S^{\frac{\gamma}{\gamma-1}}} + \omega\pi \left(\frac{c}{s}\right)^{-\sigma} s^{-\sigma} \left(\frac{\kappa}{P_S^{\frac{\gamma}{\gamma-1}}} + \frac{1+A}{1+AP_S}\right)\end{aligned} \tag{D.7}$$

Now, $s^{-\sigma}$ can be gleaned from the labour supply equation (C.2) together with the factor rents (C.11) and (C.12)

$$s^{-\sigma} = \left[(1-\alpha)(1-\pi)\mu^{-1} \left(\frac{r}{\alpha}\right)^{\frac{\alpha+\nu}{\alpha-1}} \left(\frac{c}{s}\right)^{-\sigma}\right]^{\frac{-\sigma}{\sigma+\nu}} \tag{D.8}$$

while from the resource constraint (C.16) we have

$$\frac{c}{s} = \left(1 - \frac{g}{Y}\right) r/\alpha - \kappa\frac{e}{s} - (1-\lambda) \tag{D.9}$$

where we have used that $E = e$ in the steady state; and (C.7) implies

$$\frac{e}{s} = (1 + \lambda(\pi - 1))P_S^{\frac{1}{1-\gamma}} \tag{D.10}$$

Inserting these equations into (D.7) delivers another equation in κ, r, P_S. Equations (D.5), (D.6) and (D.7) now jointly solve for κ, r, P_S. Once we retrieve these values, we can solve for all other steady state values by substituting backwards through the above steady-state relationships.

Finally, the steady state level of exogenous processes is zero, $z = z^{liq} = 0$.

E Tables

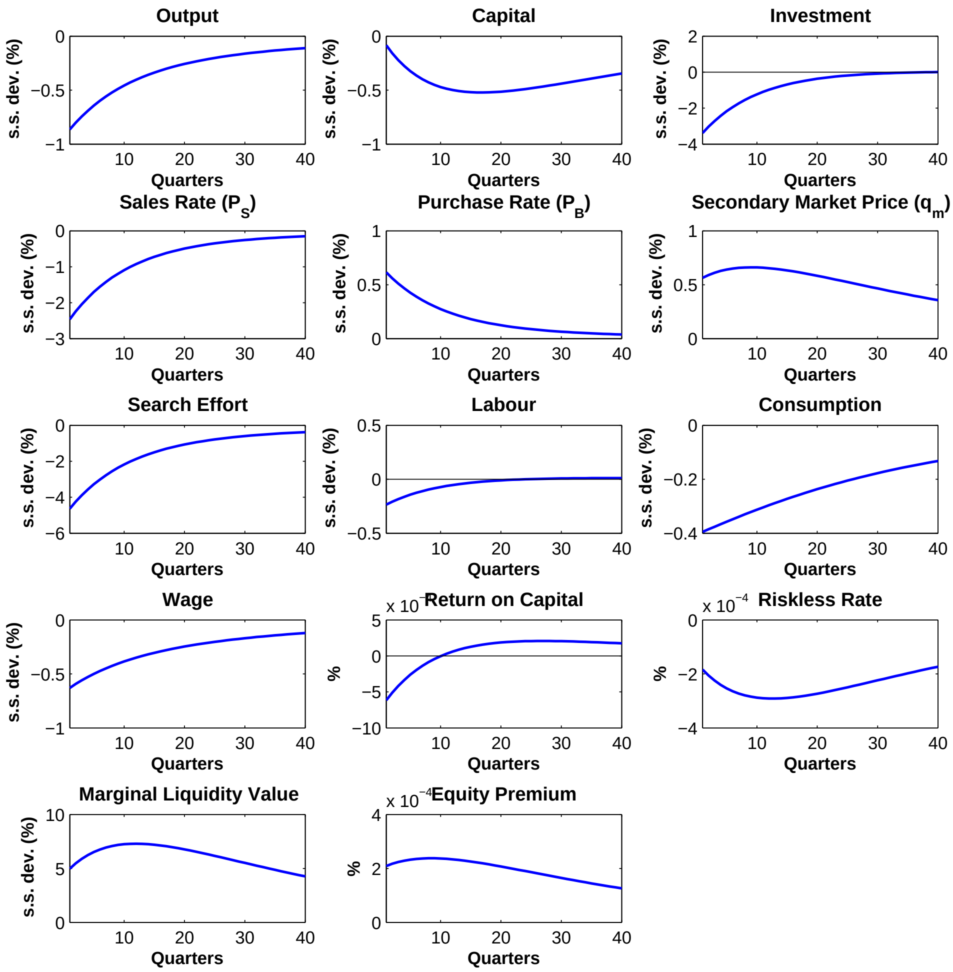
F Figures

Table 2: Baseline calibration

	Parameter	Value	Target/Source
Preferences			
Households' discount factor	β	0.99	riskless interest rate: 1%
Relative Risk aversion	σ	1	log utility
Utility weight on leisure	μ	2.7	working time: 30%
Inverse Frisch elasticity of labour supply	ν	1	Del Negro et al. (2011)
Mass of entrepreneurs	π	0.03	Shi (2012)
Final goods production			
Capital share of output	α	0.3	standard
Capital goods production			
Depreciation rate of capital	$1-\lambda$	0.025	standard
Search and Matching			
Search costs	κ	0.7591	target: liquidity premium
Supply sensitivity of matching	γ	0.2	exogenous
Bargaining weight of sellers	ω	0.5	exogenous
Government			
Government consumption	g	0.1634	target: $g/Y = 0.15$
Sensitivity of unconv. policy rule	ϕ_e	0	
Sensitivity of tax rule	ϕ_τ	0.1	
Shock processes			
Persistence, productivity shock	ρ_z	0.9	standard
Std. dev., productivity shock	σ_z	0.007	standard

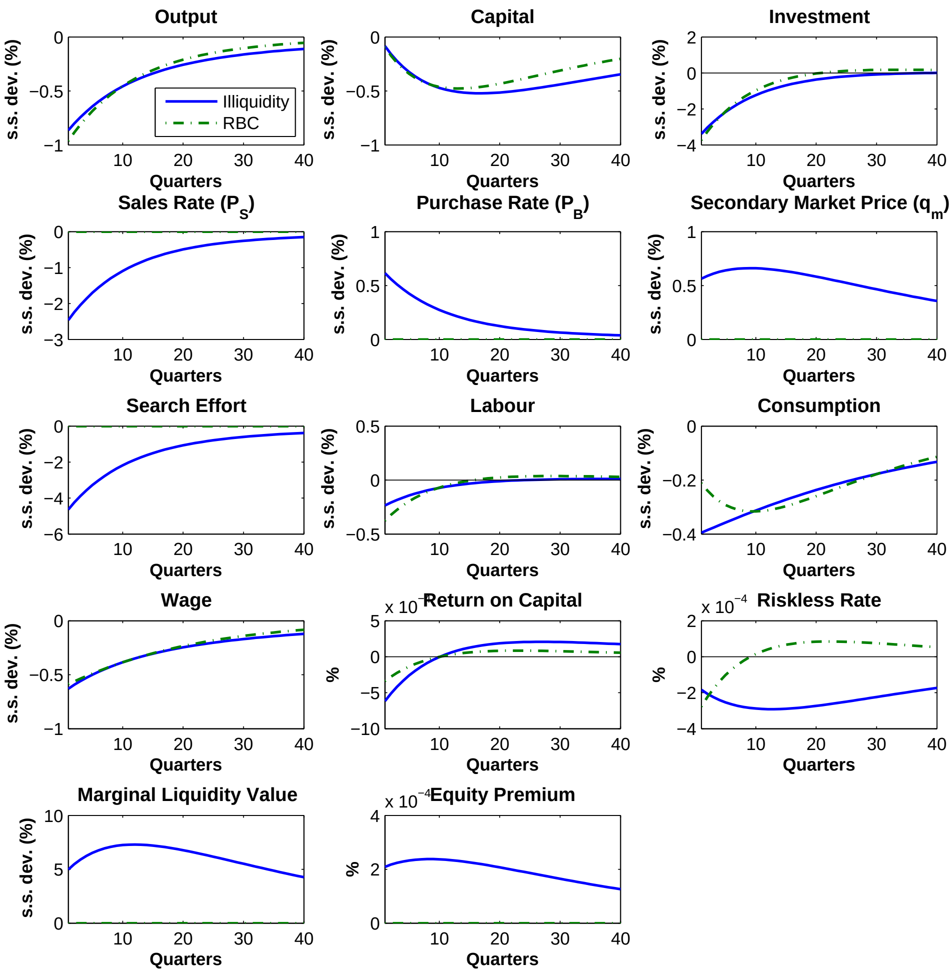
Notes: The model is calibrated for quarterly data.

Figure 1: Responses to a Technology Shock



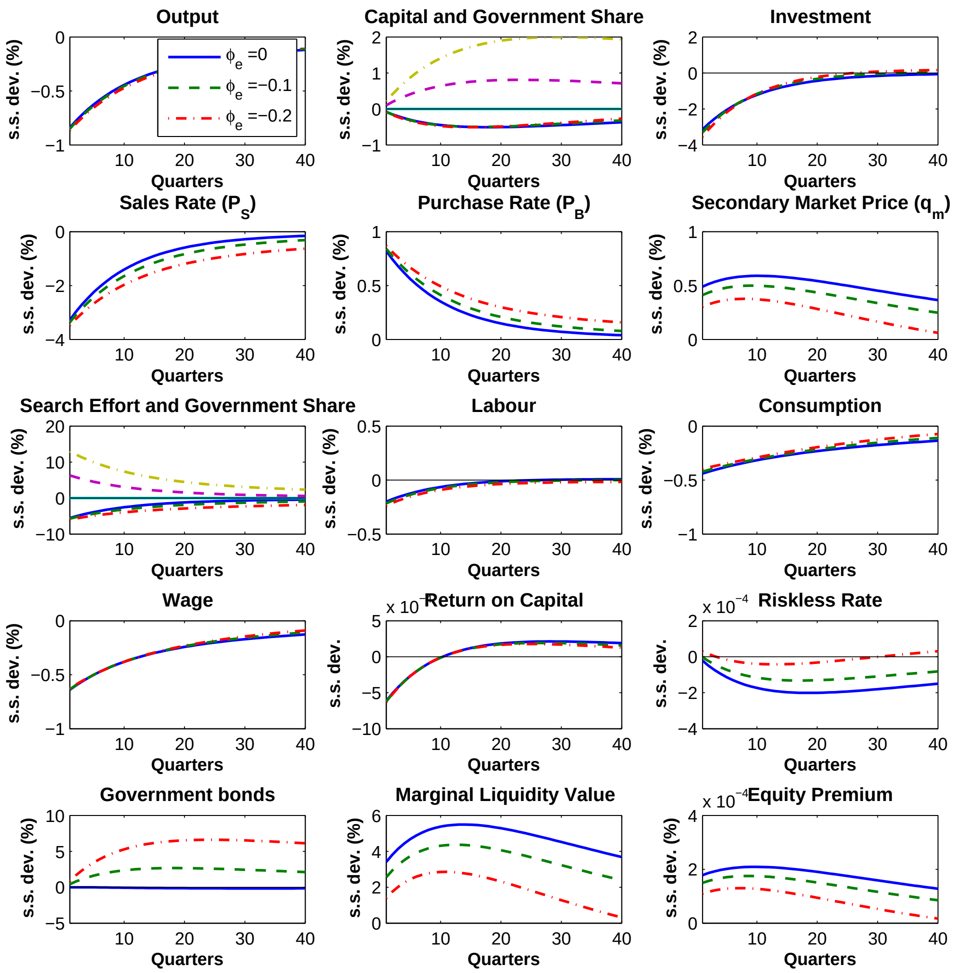
Notes: Impulse responses to a negative one-standard-deviation technology shock. No policy.

Figure 2: Benchmark Analysis: Illiquidity vs. RBC



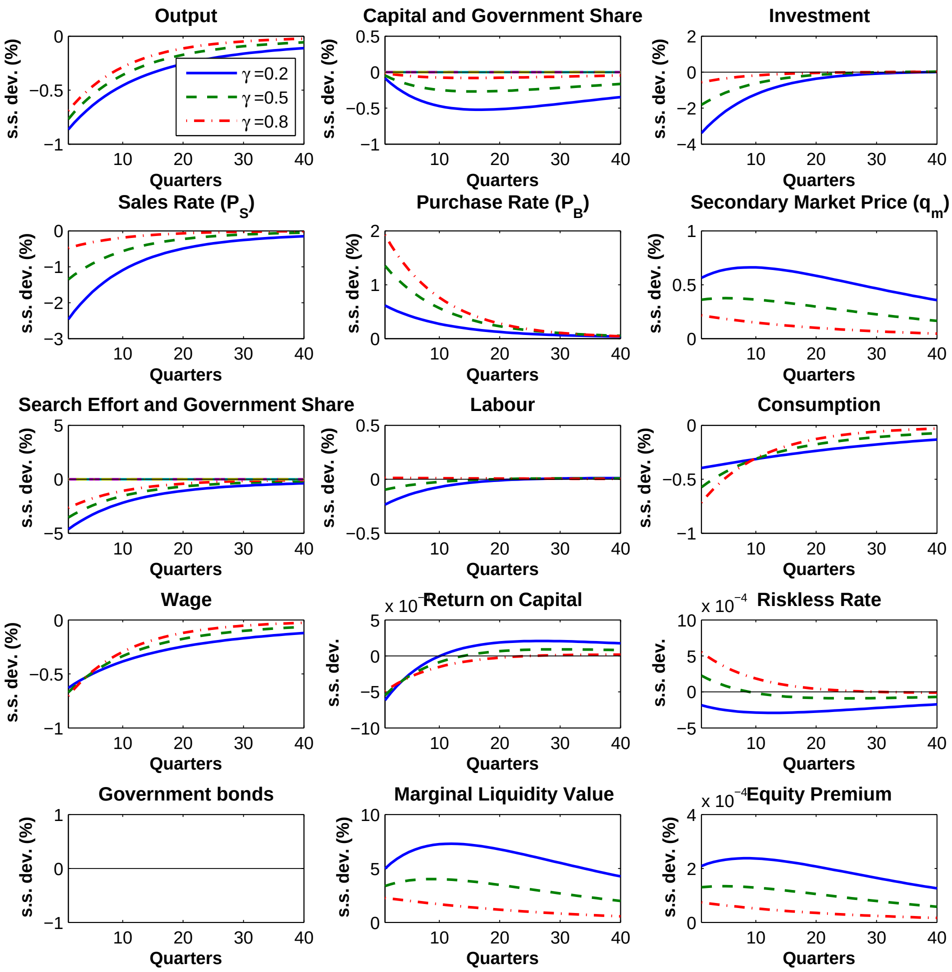
Notes: Impulse responses to a negative one-standard-deviation technology shock. Model with asset illiquidity due to search frictions versus RBC benchmark (dashed lines). No policy.

Figure 3: Unconventional Policy



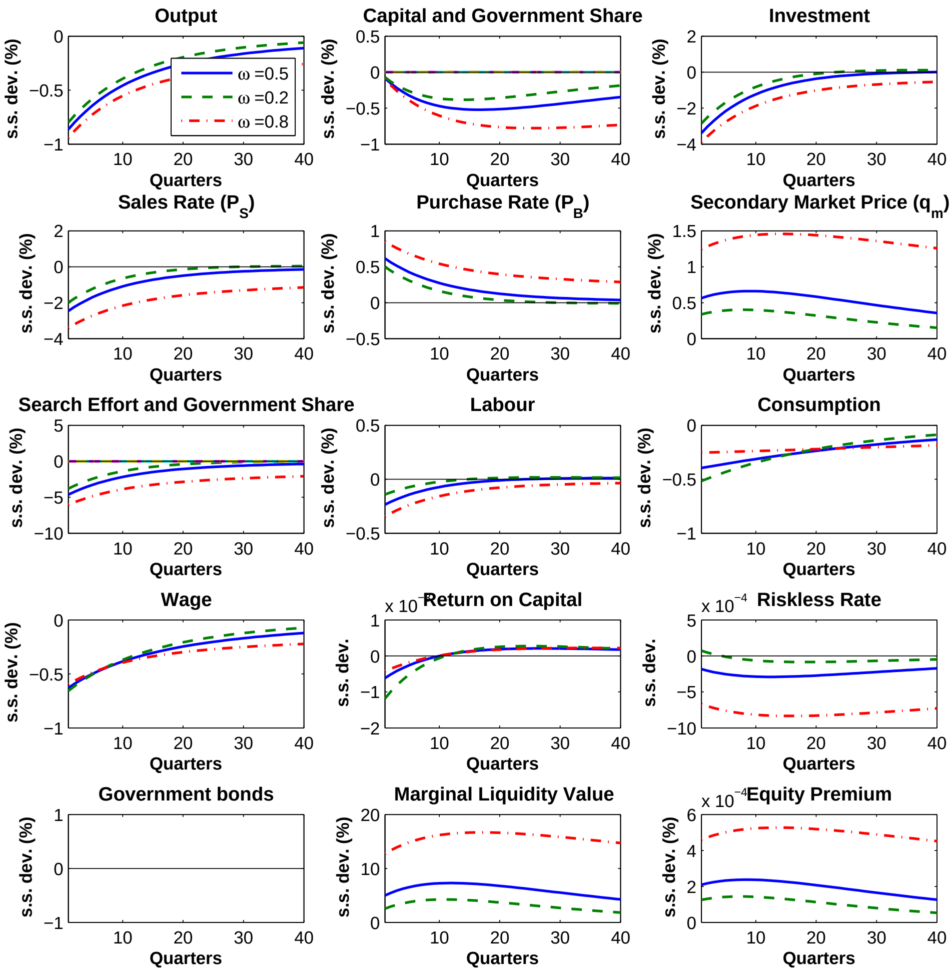
Notes: Impulse responses to a negative one-standard-deviation technology shock with different degrees of government open market interventions.

Figure 4: Sensitivity to γ



Notes: Impulse responses to a negative one-standard-deviation technology shock. No policy.

Figure 5: Sensitivity to ω



Notes: Impulse responses to a negative one-standard-deviation technology shock. No policy.