Math and Gender: What if Girls Do Math?*

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Abstract:
There is a large gender gap in advanced math coursework in high school that many believe exists because girls are discouraged from taking math courses. In this paper, we exploit an institutional change that reduced the costs of acquiring advanced high school math, in particular for girls at the top of the math ability distribution, to determine if access is, in fact, the mechanism. By estimation of marginal treatment effects of acquiring advanced math qualifications, we document substantial beneficial wage effects from encouraging even more females to opt for these qualifications. Our analysis suggests that the beneficial effect comes from accelerating graduation and attracting females to traditionally male-dominated career tracks and to CEO positions. Our results may be reconciled with experimental and empirical evidence suggesting there is a pool of unexploited math talent among high ability girls that may be retrieved by changing the institutional set-up of math teaching.

JEL Classification: I21, J24.

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1. Introduction

Although the college gender gap has evaporated, women are still underrepresented in high-powered careers as CEOs, Ph.D.s, and more generally in finance, business, science, technology, engineering and mathematics (Bertrand, Goldin and Katz, 2010; Ginther and Kahn, 2004; 2009; Smith, Smith and Verner, 2011). More generally, women are doing well on average, but there still exists substantial inequality at the top of the income distribution. But why is this?

Recent studies indicate that the teaching environment of math is important for this achievement pattern (Pope and Sydnor, 2010; Niederle and Vesterlund, 2010). As a consequence, there may be a lost pool of talent among girls with high math abilities. Changing the learning environment towards fostering, identifying and attracting girls with high math qualifications would help retrieving this pool of talent. In this paper, we exploit a pilot scheme that exogenously reduced the costs of acquiring advanced high school math for high ability girls. The pilot scheme allowed for a more flexible combination of advanced math with other courses. We investigate whether these advanced math qualifications do indeed trigger high-powered careers.

In the previous literature, the focus has been on describing the gender differences in career outcomes and the gender gap in math qualifications over the distribution, while so far no attempt has been made to combine the two and identify the causal effect of math on gender differences in career outcomes. However, it is evident that the gender differences in math qualifications at the top of the distribution may explain a substantial part of the gender gap in wages and in career outcomes more broadly, because the positive causal effect of advanced math on earnings is sizeable (Joensen and Nielsen, 2009).

Goldin, Katz and Kuziemko (2006) find that better female college preparedness, as measured by grades and test scores as well as completion of advanced math and science courses from high school, is an important factor in the reversal from male-dominated colleges to female-dominated colleges in the US. Paglin and Rufolo (1990), Rose and Betts (2004) and Goldin, Katz and Kuziemko (2006) touch upon the link between lack of math qualifications and the gender wage gap, but due to the lack of exogenous variation in the choice of math, it is still not fully established. In this paper, we exploit exogenous variation in the choice of math combined with rich panel data of career outcomes in order to better address this issue.

There is a wide consensus that the gender gap in math performance increases gradually as we move from the mean to the top of the performance distribution, and that the ratio of males to females who score at the top 5% of the distribution is around two to one (Pope and Sydnor, 2010; Machin and Pekkarinen, 2008; Hyde et al., 2008; Niederle and Vesterlund, 2010; Ellison and Swanson, 2010). The literature indicates that this gap in performance is to a large extent driven by cultural and environmental factors. One line of reasoning stems mainly from the experimental literature, while another line of reasoning draws on spatial variation in the gender gap at the top of the math performance distribution.

In a survey of the experimental literature on gender differences in preferences, Croson and Gneezy (2009) conclude that women tend to have higher risk aversion and lower preference for competition than men, and they suggest that these gaps are related to a gender gap in self-confidence. Below, we explain why these preference parameters and self-confidence may be closely related to the gender gap in math performance. Niederle and Yestrumskas (2008) find that women are less likely to seek challenges than men with the same abilities and that this is because they have a higher risk aversion or higher uncertainty about their own abilities than men do. If advanced math courses are relatively challenging, this explains why women often opt out of those courses. Niederle and Vesterlund (2010) regard math tests and math teaching as a type of mixed-sex competition in which women are known to be less willing to enter and – according to some studies[1] – to perform worse than men with similar abilities. The underlying experiments indicate that this pattern is to a large extent explained by lower self-confidence and less taste for competition among women than among men.[2] Niederle and Vesterlund argue that these two factors play a substantial role in math – and more so at the right tail of distribution.[3] One reason is that girls have particularly little faith in their own math abilities – conditional on actual abilities – due to extensive gender-stereotyping in math working through parents and teachers, and that this lower self-perception is exacerbated among the most able girls. Another reason is that math tends to be a very competitive discipline because the answers to exercises are either right or wrong. A natural consequence of this reasoning is that the environment surrounding math teaching is important for girls' performance in math.

Now we turn to empirical evidence from studies of spatial variation across schools, geographic regions and countries. Ellison and Swanson (2010) analyze high school students participating in a

[1] In their survey, Croson and Gneezy (2009) conclude that there are still many open questions regarding performance in mixed-sex competition.
[2] See Niederle and Vesterlund (2007).
[3] Hill et al. (2010) survey a large amount of related psychological experiments showing the importance of self-assessed math ability and stereotypes in the environment. They report that interest and performance in math are shaped by the environment.

range of elite math contests, and they find that the female contestants come from a small set of super-elite schools (>99th percentile in the school distribution), while the male contestants come from a variety of backgrounds. This indicates that some schools are better at identifying, cultivating or attracting female talent than others.

Pope and Sydnor (2010) study geographical variations across the US in gender gaps in the stereotypical male-dominated tests of science and math and the stereotypical female-dominated tests of reading among eighth graders. They find that some states appear to be more gender-equal across all tests, while other states appear to be more gender-unequal across all tests. For instance, New England experiences the lowest ratios of males to females at the 95th percentile in the science and math tests (1.5 and 1.3, respectively) and the lowest ratio of females to males in the reading test (2.1), while East South Central census divisions have gaps twice as large. Studying how these gaps correlate with state characteristics, they find that the gap is significantly correlated with the fraction agreeing that "Math is for boys" and "Women are better suited for home". Thus, it appears that some areas adhere more or less strongly to prevailing gender stereotypes rather than just favoring one sex over the other. The authors interpret their findings as evidence that social forces are very important for creating gender disparity at the top of the distribution without, though, being able to point at which aspects of the cultural and environmental differences play a role.

Several authors identify country-specific cultural factors as main contributors to the gender gap in math achievement. Bedard and Cho (2010) find large variation in gender gaps across OECD countries and argue that this is correlated with variation in educational institutions such as pro-female sorting and academic streaming. Guiso et al. (2008) find smaller gender gaps in countries with higher gender-equality according to a variety of measures. Fryer and Levitt (2010) observe smaller gender gaps in Muslim countries and speculate that single-sex education may be the reason why these countries stand out. Focusing specifically at the top end of the distribution, Andreescu et al. (2008) find that some countries and ethnic groups (e.g. Asian and Eastern Europe) are much better at identifying and nurturing females with a very high math talent than others (e.g. the US). This is evidenced by the variation in the proportion of female participants in the International Math Olympiad, the proportion of PhD degrees granted to women in math-related subjects and the proportion of women among mathematics faculty at the universities.

All these studies hint at cultural differences across countries, across education systems and across individual schools as important explanations for the gender gap in math qualifications at the top of the distribution. Therefore, it seems obvious that there is scope for improving female math qualifications and subsequent career outcomes by understanding these environmental differences in the costs of achieving advanced math qualifications: How is math taught? How is math marketed? And - in our case – how is math packaged with other courses?

We use Danish register data for the three cohorts of high school students of 1984-86. We exploit exogenous variation from a high school pilot scheme to identify the channels through which advanced high school math causes more favorable career outcomes. The pilot scheme reduced the costs of choosing advanced math - in particular for girls at the top of the math ability distribution - because it allowed for a more flexible combination of math with other courses. Only one out of ten female high school students chooses advanced math without the pilot scheme, and this fraction almost doubled after introduction of the pilot scheme. It is this exogenous cost variation that we exploit in order to understand the potential of advanced math to attract women to high-powered careers. We specifically analyze the effect of advanced high school math on wages, college enrollment and graduation, PhD graduation, field of study, promotion to top-corporate jobs, choice of sector and industry, and age at birth of first child.

Consistent with earlier work, we find strong evidence of a causal effect of math on earnings for students who are induced to choose math after being exposed to the pilot scheme. Studying marginal treatment effects, we cannot reject that the returns to advanced math are equal across gender for two individuals with an identical propensity to choose advanced math. This indicates that the underlying math ability distribution is also equal. To reconcile this finding with the fact that males dominate at the top of the math test score distribution, we refer to Örs, Palomino and Peyrache (2008) and Jurajda and Münich (2011), who find that women underperform in high-stake tests relative to men with similar abilities, and to Niederle and Vesterlund (2010), who conclude that the gap at the top of the math test score distribution does not necessarily imply a gap in the underlying math abilities. Even though the marginal treatment effects are equal, the fact that the proportion of girls in our data set who choose math is significantly lower than the proportion of boys means that there is indeed an unexploited math talent to be retrieved.

In addition, we find that advanced math accelerates graduation and moves females from the female-dominated field of Humanities to high-paid and partly more male-dominated career tracks in Health and Technical Sciences. Furthermore, it increases the probability of becoming a CEO. Thus, it is to some extent a route to a high-powered career.

The rest of the paper is organized as follows: Section 2 presents the institutional framework and the identification strategy. Section 3 describes the data. Section 4 presents the empirical analysis of the impact of advanced math on earnings and investigates whether there is an unexploited pool of math talent, while section 5 presents the empirical analysis of the impact of advanced math on education and other career outcomes and investigates whether advanced math is a route to a high-powered career. Section 6 concludes the paper.

2. Using a High School Pilot Scheme for Identification

In this section, we briefly describe the environment of the high school pilot scheme and the applied identification strategy. In the first subsection, we present the relevant Danish high school regime. Then we describe the pilot scheme which forms the basis for the instrumental variable approach.

2.1. The Pre-1988 High School[4]

In the period 1961-1988, the Danish high school system was a "branch-based" high school system in which courses were grouped into restrictive course packages.[5]

We focus on this period for two reasons. First, the supply of course packages gives us a useful exogenous variation in the cost of acquiring advanced math. Second, when we focus on students who enter high school prior to 1988, the data set includes completed education spells as well as labor market outcomes when the individuals are in their thirties.

This system implied that students upon high school graduation would have achieved one of three math levels available: advanced, intermediate, or basic level. The difference between the three levels is reflected in the number of lessons per week as well as in the content of the courses. For instance, the extent of geometry and algebra increases as the level becomes more advanced. In the empirical analysis, we focus on whether students choose the advanced math course or not, meaning that the intermediate and basic level math courses are lumped together. The decision about which math level to opt for is taken at the end of the first year in high school. The only way to obtain the advanced

[4] Consult Joensen and Nielsen (2009) for additional details on the Danish high school regime during the relevant period.

[5] Available course packages were labelled: Social Science and Languages, Music and Languages, Modern Languages, Classical Languages, Math-Social Science, Math-Natural Science, Math-Music, Math-Physics and Math-Chemistry.

math course was in combination with advanced physics, unless the student was enrolled at a pilot school, where the advanced math course could also be obtained in combination with advanced chemistry. It is exactly this increased course flexibility which some students were unexpectedly exposed to at pilot schools that constitutes the natural experiment we exploit in this paper.

2.2. The Pilot Scheme

The pilot scheme was implemented as an experimental curriculum at about half of the high schools prior to the 1988-reform. Table 1 gives an overview of the gradual implementation of the pilot scheme from 1984-86. The table is divided by types of high schools: schools with no pilot scheme (*PilotSchool=0*), schools where the pilot scheme was introduced after enrollment of the relevant cohort (*PilotSchool=1* & *PilotIntro=1*), and schools where the pilot scheme was implemented prior to enrollment of the relevant cohort (*PilotSchool=1* & *PilotIntro=0*).

Schools were not randomly assigned to become pilot schools. Instead, from 1984-86, they could apply to the Ministry of Education for permission to adopt the experimental curriculum, whereas in 1987 the high school principals could make this decision without approval from the ministry. It is not possible to check whether the pilot schools represent a sample of schools which is essentially random with respect to math ability.[6]

It is clear, however, that students with a preference for advanced math and chemistry may self-select into schools that are known to offer the pilot program before entrance. This is why we distinguish between students at pilot schools where the pilot scheme was unexpectedly introduced after they had enrolled in the high school (*PilotSchool=1* & *PilotIntro=1*), and those who knew that the school was a pilot school before they applied for entering the school (*PilotSchool=1* & *PilotIntro=0*).

The instrumental variable exploits the fact that the pilot scheme reduces the psychological cost of choosing advanced math since the students exposed to the scheme are not required to take the physics course together with advanced math. Hence, first-year high school students enrolled at a school when it decided to introduce the pilot scheme were exposed to an exogenous cost shock, which induced more students to choose advanced math compared to students at non-pilot schools. If the selection of newly participating schools is exogenous with respect to student ability, which Joensen and Nielsen (2009) argue it is, the pilot scheme provides exogenous variation in students'

[6] Joensen and Nielsen (2009) elaborate more on the entrance procedures and the essential randomness of pilot school status.

math qualifications without influencing the outcomes of interest except through the effect on math qualifications.

Table 1. Introduction of the Pilot Scheme

Females

Cohort starting in high school	PilotSchool=0		PilotSchool=1 PilotIntro=0		PilotSchool=1 PilotIntro=1		All
	#schools	#students	#schools	#students	#schools	#students	#students
1984	120	7,296	0	0	22	1,808	9,104
1985	105	5,931	22	1,617	15	983	8,531
1986	90	4,676	37	2,465	15	882	8,023
All		**17,903**		**4,082**		**3,673**	**25,658**

Males

Cohort starting in high school in	PilotSchool=0		PilotSchool=1 PilotIntro=0		PilotSchool=1 PilotIntro=1		All
	#schools	#students	#schools	#students	#schools	#students	#students
1984	121	5,348	0	0	22	1,399	6,747
1985	105	4,434	22	1,315	15	731	6,480
1986	90	3,521	37	1,961	15	697	6,179
All		**13,303**		**3,276**		**2,827**	**19,406**

The instrumental variable, *PilotIntro*, is equal to one if the individual enrolled in a high school which afterwards decided to introduce the experimental curriculum for the first time, and it takes the value zero otherwise. This instrument is valid if the pilot scheme is randomly assigned to schools and if individuals are randomly distributed across schools that have not yet decided to introduce the experimental curriculum. This assumption is violated only if the school decides to participate in the program based on the math abilities of local students. Joensen and Nielsen (2009) check this by testing overidentifying restrictions. They let *PilotIntro* interact with each of the cohort dummy variables and find that each of the interaction terms between cohorts 1984–86 and *PilotIntro* may be left out of the outcome equation, while the interaction term between cohort 1987 and PilotIntro cannot. The schools which introduced the program in 1987 tend to be negatively selected in terms of the students' math abilities, while no similar concerns are raised regarding the other cohorts. Therefore, we disregard the cohort starting in high school in 1987 in the present study. In section 3 below, we test for similarities of the student and parent bodies across school status, and we find almost no significant differences.

The instrument is strong if the unexpected introduction of the scheme induces students to choose advanced math, which is directly tested in the empirical section. The instrument satisfies the monotonicity (or uniformity) condition if individuals who chose advanced math when it could only be combined with physics also would have chosen advanced math if they had unexpectedly had the option of also combining it with advanced chemistry. We are confident that the monotonicity assumption is reasonable in our application since all the options available at non-pilot schools were also available at schools that introduced the pilot scheme.

Our instrument exploits the exogenous variation in the exposure of students to the possibility of combining advanced math courses with advanced chemistry. Hence, the "treatment" that we investigate is the combined treatment of advanced math and advanced chemistry. Because advanced math and advanced chemistry are combined in a course package, we cannot separate the effect of advanced math from that of advanced chemistry or from the potential synergy effect of the combination of math and chemistry. However, the earlier literature suggests that if any specific course work matters it is math rather than Science courses; see e.g. Rose and Betts (2004) and Altonji (1995).

Figure 1 illustrates the distribution of students across course packages (“branches”). The number of pupils in mathematically based branches is larger in schools which had announced their pilot status (*PilotSchool=1* & *PilotIntro=0*) than in both non-pilot schools (*PilotSchool=0*) and schools implementing the pilot scheme for the first time for the relevant cohort (*PilotSchool=1* & *PilotIntro=1*). The choice between a mathematically based branch and a language-based branch was made at entry, and therefore this pattern indicates some degree of self-selection into pilot schools by pupils potentially interested in the pilot scheme. Figure 1 suggests that more girls than boys have self-selected into the schools which had announced their pilot status, because the proportion of girls choosing Math-Chemistry is higher (14%) at these schools than at the schools unexpectedly introducing the pilot scheme (12%). Figure 1 also suggests that the schools who unexpectedly announce the pilot program have slightly fewer students who choose the language-based branches.

At the relevant point in time, the Danish high school attracted a little less than half of a cohort.[7] About 80% of male high school students chose math-based branches, while roughly 50% of females chose math-based branches. At non-pilot schools, 39% of males and 11% of females chose advanced math. At schools where the pilot scheme was unexpectedly introduced, 51% of males and 20% of females chose advanced math. Apparently, the combination of advanced math and chemistry

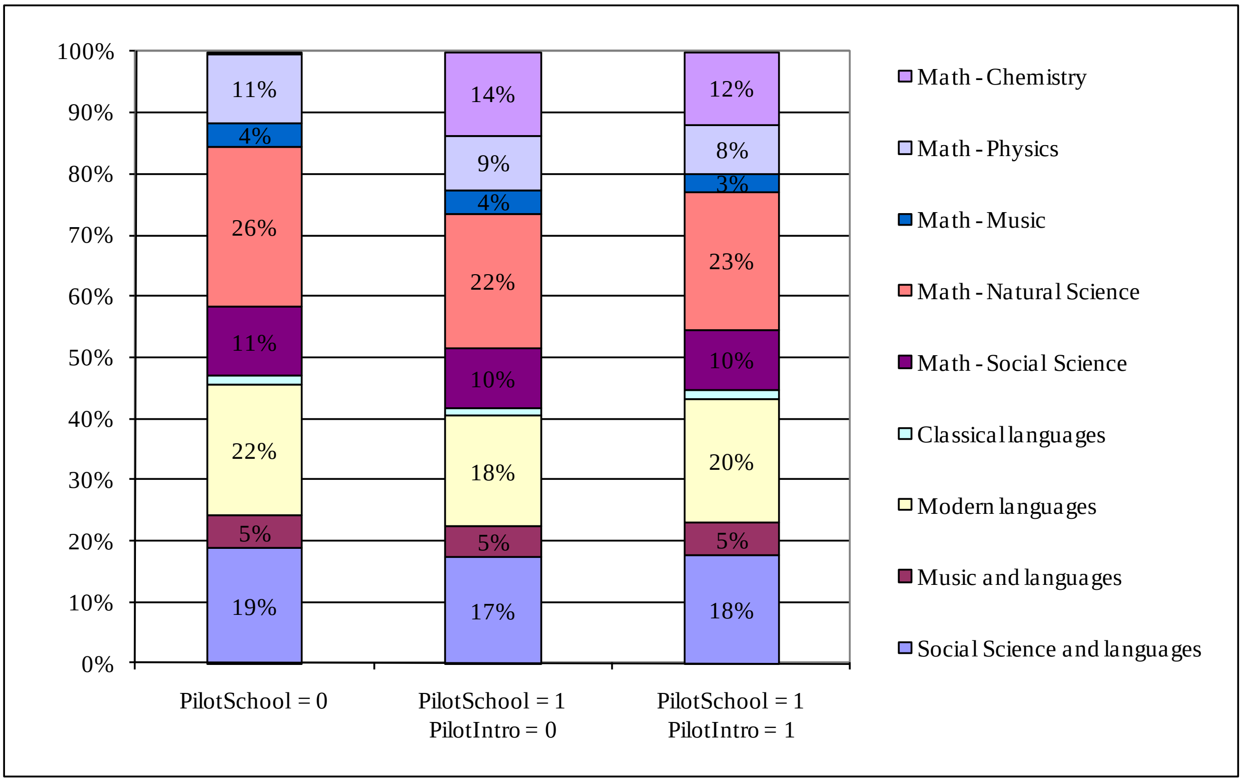
[7] Among the cohort graduating in 1989/90, 45% graduated from high school while another 35% completed vocational education, see Statistics Denmark (2002).

attracted relatively more females than males, since the relative difference across pilot status is much higher for females (82%) than for males (31%). This is because of the content of chemistry classes compared to physics, but there may also be a spill-over effect because the expected gender composition of the Math-Chemistry branch is more equal compared to the Math-Physics branch. While the girls constitute 25% of students at the Math-Physics branches, they constitute 44% of students at the Math-Chemistry branches in the schools unexpectedly introducing this option. The proportion of girls was 48% when the schools announced their pilot status prior to enrollment of the relevant students.

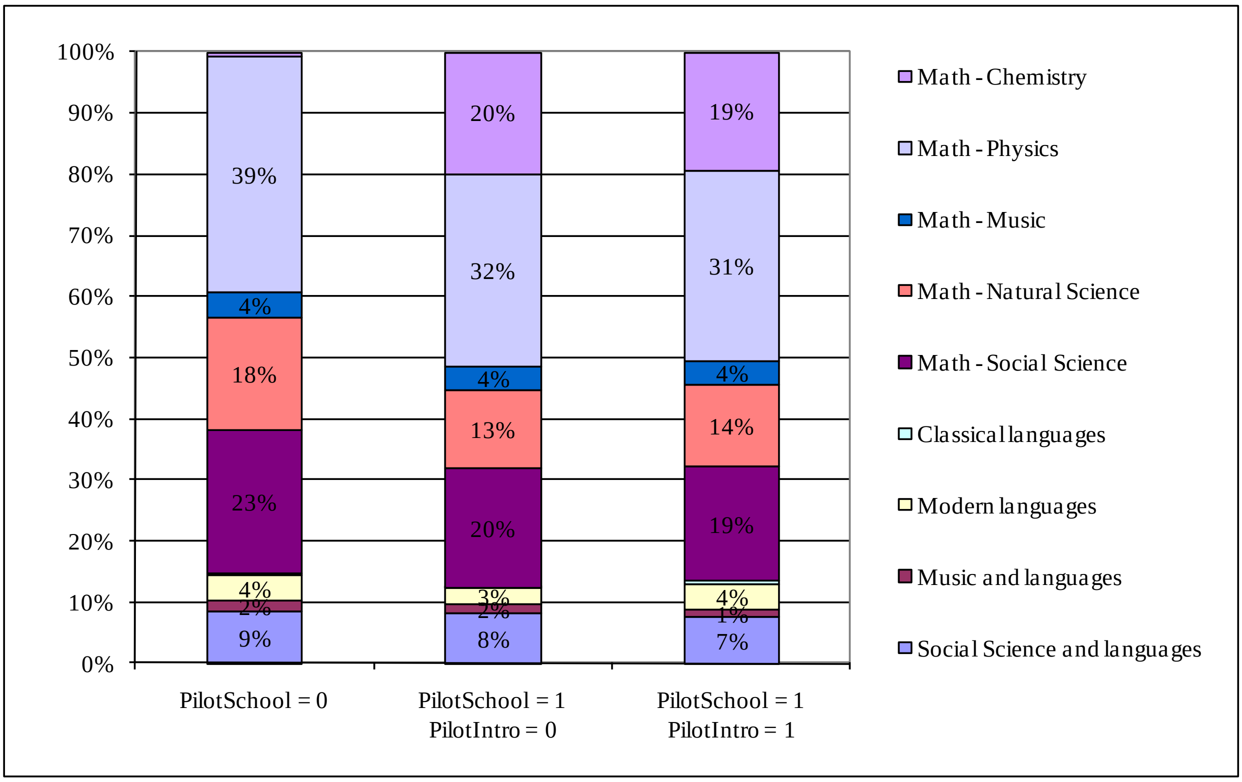
If we take into account that the traditional high school attracted less than half of a cohort at this point in time, the 11 and 20% of female high school students who chose advanced math are probably drawn from the top 5-10% of the math ability distribution.

Figure 1. Branch Choices in the Pre-1988 High School

Females



Males



3. Data Description

3.1. Sample Selection

For our empirical analysis we use a panel data set comprising the population of individuals starting high school from 1984-86 in Denmark. The data are administered by Statistics Denmark, which has gathered the data from administrative registers. For each individual, we have information about complete detailed educational histories, including detailed codes for the type of education attended (level, subject, and educational institution) and the dates for entering and exiting the education, along with an indication of whether the individual completed the education successfully, dropped out, or is still enrolled as a student. Furthermore, we have information on the choice of branch in high school and on high school GPAs. The GPA is a weighted average of final exam grades for each course. In addition, we have the standard battery of information about the post-high school labor market career such as occupation, industry, self-employment, part-time and child bearing.

Among the gross population of high school entrants for 1984-86, only high school graduates who finished in three years are selected.[8] Furthermore, we require at least one year of employment in order to have an earnings observation.[9] After these constraints, the sample contains observations on about 25,658 females and 19,406 males from about 140 different high schools; see Table 1. Some of the career outcomes such as occupation codes are not observed for all individuals, and in these cases the sample is reduced to 24,201 females and 18,051 males.

3.2. Outcome and Control Variables

In order to obtain a complete picture of the impact of math on career, we construct a wide range of outcome variables. In addition to log earnings, we study the impact of math on outcome variables describing college graduation[10] as well as field of study. Furthermore, we investigate the effect of math on occupation, self-employment, part-time vs. full-time work, industry and fertility outcomes.

As control variables we include county indicators, entry cohort fixed effects, high school-specific controls and parental background. The latter includes a set of mutually exclusive indicator variables

[8] About 10% do not complete in three years. The main part of drop out takes place before the choice of advanced math. Drop out is uncorrelated with pilot school status.

[9] The overall labor force participation in the sample is 94% for males and 92% for females, and less than 1% do not participate at least one of the years.

[10] College may be either short-cycle (e.g. diplomas in health assistance, computer programming), medium-cycle (e.g. BA and BSc obtained at nursery college, teachers' college), long-cycle higher education (MA and MSc degrees) or research education (PhD).

for the level of highest completed education of the mother and father, respectively, and their income as observed at the end of the year before the individual started high school. We leave out post-graduation control variables and thus estimate the total effect of advanced math.

3.3. Data Description

Summary statistics related to educational achievement are shown in Table 2 and so are differences by math level. Table 2 reveals that more than 80% of our three cohorts of high school graduates enroll in college. The outcomes are more favorable for males than for females with only one minor exception, namely the rate of college graduation. Furthermore, students who have chosen advanced math perform significantly better on all dimensions compared to other students: they enroll more often and graduate from college educations at all levels, they graduate faster and they earn higher wages.

Summary statistics of all outcome variables as measured 18 years after graduation are shown in Table A1 in the Appendix, while summary statistics for the control variables are shown in Table A2. Table A1 shows that students with advanced math qualifications more often than others complete an education in Health Sciences, Natural Sciences and Technical Sciences. They more often end up in the private sector and in managerial positions. Their industry of employment is more often in Industry, Building and Construction or Real Estate, Renting and Business Services, while Education as well as Social and Health Services are less frequent. Looking at fertility outcomes, it is seen that these cohorts were around 30 when they had their first child, and that women have their first child around half a year later when they have chosen the advanced math course.

Table A2 shows that the most favorable characteristics exist for students at schools which have advertised their pilot status (*PilotSchool=1* & *PilotIntro=0*), while the least favorable characteristics are found for individuals at non-pilot schools (*PilotSchool=0*). However, only few characteristics are significantly different. Female students at schools which unexpectedly introduced the pilot scheme (*PilotSchool=1* & *PilotIntro=1*) more often have parents with only basic schooling, and their mothers have significantly lower income, while other slight differences are found for male students. We control for these differences in our empirical analysis. Most importantly, the mean parental background at the high school in 1983 was very similar across pilot school status. We see only two statistically significant differences between the characteristics: The cohort of 1983 at schools which are classified as unexpectedly introducing the program (*PilotSchool=1* & *PilotIntro=1*) more often had fathers with medium-cycle higher education (e.g. teachers), and their mothers had lower income compared to the students at schools classified as non-pilot schools. There are no statistically significant differences in the regions in which the schools are situated.

Table 2. Summary Statistics (measured 18 years after high school graduation)

	Sample means			
	Female		**Male**	
	Overall mean	**Mean difference by *Math* A**	**Overall mean**	**Mean difference by *Math* A**
Enrollment				
College Enrollment	0.834	0.058 ***	0.856	0.077 ***
Master's Enrollment	0.394	0.184 ***	0.562	0.100 ***
Graduation				
College Degree	0.760	0.049 ***	0.723	0.049 ***
Master's Degree	0.297	0.189 ***	0.427	0.101 ***
Phd	0.021	0.042 ***	0.048	0.037 ***
Time to Graduation				
Years from HS to Master's Graduation	8.86	-1.128 ***	8.41	-1.106 ***
Labor Market Outcomes				
Log earnings (2000 DKK)	12.12	0.251 ***	12.47	0.230 ***
Number of Individuals	**24,201**		**18,051**	

Note: Significance at a 1%-, 5% and 10%-level are indicated by ***, ** and *, respectively. We only have 23,999 and 17,892 observed earnings for females and males, respectively.

4. Impact of Math on Earnings - Is There a Pool of Unexploited Math Talent?

In this section, we scrutinize the impact of math on earnings, while in the next section we focus on the impact of math on the other career outcomes. First, we present an empirical analysis of the effect of math on earnings. Second, we investigate the distribution of treatment effects over the math preference distribution.

4.1. Estimating the Impact of Math on Earnings

Let $MathA_i$ be an indicator of whether individual i chooses the advanced math course, and let Y_i be log earnings for individual i. We estimate the following equation:

$$Y_i = \beta_0 + \beta_1 X_i + \delta MathA_i + \varepsilon_i, \quad (1)$$

where X_i is a vector of background characteristics of individual i, including parental background, cohort, and regional as well as high school characteristics. To keep notation simple, we write $X_i = X_{ist} = (X_i, X_s, D_t)$, thus suppressing the fact that we do indeed account for both high school and cohort characteristics. Importantly, we always control for pilot school status, *PilotSchool*$_i$, to allow for potential selection into pilot schools. The indicator variable for whether individual *i* chose the advanced math course in high school, *MathA*$_i$, is potentially endogenous since unobserved variables most likely affect both earnings and the choice of advanced math. We use *PilotIntro*$_i$ as the instrumental variable that exogenously affects the costs of choosing advanced math without affecting future earnings through other channels than the likelihood of choosing advanced math. *PilotIntro*$_i$ is equal to one for individuals who were unexpectedly exposed to the experimental curriculum, because the school introduced it for the first time just before they chose math level, and it takes the value of zero otherwise (see Table 1). Assuming that the selection of schools into the pilot program is as good as random (conditional on X_i), the instrumental variable affects earnings only through its effect on the costs of choosing advanced math. The outcome variable, Y_i, is log earnings 9-18 years after high school graduation, at which point in time individuals are likely to be settled into their careers. The preferred income measure would be lifetime income, which is impossible, however, to compute for our sample of individuals in their thirties.

In Table 3, we present the results from estimating this earnings equation by OLS and IV.[11] The table lists results from the outcome equation as well as the first stage equation and indicates which control variables are included in both equations. First stage shows a very strong effect of unexpected exposure to the pilot scheme on the probability of choosing advanced math. The marginal effect is around 10 percentage points for both genders no matter which specification is used.

For females, OLS shows a significantly positive association between earnings and advanced math in the range 0.23-0.25. The point estimates from IV are slightly higher (around 0.30) and they are also significantly different from zero.[12] For males, OLS coefficients are estimated to be 0.21-0.23. The IV estimates vary and adding high school-specific controls reduces the coefficients and increases the p-values. This could indicate that the high school-specific controls pick up some systematic characteristics among the male pupils in the schools introducing the pilot scheme.[13] Notice that the reported coefficient estimates should be estimated *conditional on* participating in the labor force,

[11] We estimate a Heckman-type model for binary treatment and continuous outcome, but our results are robust to alternative estimation methods.

[12] The IV estimates are not statistically significantly different from the OLS estimates. However, if we take the differences between the point estimates at face value, they are reflective of compliers having a higher return to advanced math than the average student. Figure 1 indicates that the compliers would alternatively have chosen Math-Natural Sciences or Math-Social Sciences.

[13] Results are very similar if we replace high-school specific controls by high-school fixed effects.

which is in itself also positively affected by advanced math.[14] We regard specification (6) as our preferred specification which is used in the following analyses.

Table 3. Estimation of the Impact of Math on Earnings

	Parameter estimates (standard errors) [marginal effects]					
	OLS			IV		
	(1)	(2)	(3)	(4)	(5)	(6)
Females						
Effect of High level Math on Outcome:						
Earnings (average income 9-18 years after starting hs)	0.247 ***	0.248 ***	0.246 ***	0.309 ***	0.318 ***	0.313 ***
	(0.01)	(0.01)	(0.01)	(0.06)	(0.05)	(0.05)
Earnings (income with pooled cross section)	0.229 ***	0.230 ***	0.228 ***	0.287 ***	0.292 ***	0.287 ***
	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)
High level Math First-Stage:				0.351 ***	0.337 ***	0.324 ***
Pilot School Intro				(0.03)	(0.03)	(0.03)
				[0.09]	[0.09]	[0.08]
Number of individuals/observations	**23,999/224,113**					
Males						
Effect of High level Math on Outcome:						
Earnings (average income 9-18 years after starting hs)	0.228 ***	0.228 ***	0.226 ***	0.370 ***	0.361 ***	0.340 ***
	(0.01)	(0.01)	(0.01)	(0.07)	(0.07)	(0.09)
Earnings (income with pooled cross section)	0.207 ***	0.207 ***	0.206 ***	0.289 ***	0.267 ***	0.110 *
	(0.01)	(0.01)	(0.01)	(0.04)	(0.06)	(0.07)
High level Math First-Stage:				0.276 ***	0.260 ***	0.241 ***
Pilot School Intro				(0.03)	(0.03)	(0.03)
				[0.11]	[0.10]	[0.09]
Number of individuals/observations	**17,892/170,653**					
Additional control variables:						
Parental variables (for mother and father):						
Highest completed education and income		+	+		+	+
Regional controls:						
County indicators		+	+		+	+
Cohort controls:						
Entry cohort fixed effects		+	+		+	+
High School specific controls:						
Average parental beckground in 1983			+			+

Note: Significance at a 1%-, 5%-level and 10%-level are indicated by ***, ** and *, respectively. For the pooled cross sections, standard errors are clustered by individuals.

[14] We have also estimated the impact of advanced math on labor force participation (not reported), and find significantly positive effects of 3.3 and 4.3 percentage points for females and males, respectively.

4.2. *Propensity Scores and Marginal Treatment Effects*

In this section, we assess whether encouraging even more students to opt for advanced math would still lead to beneficial outcomes. In order to answer this question, we need to better understand the connection between the selection into the advanced math course and individual returns to taking the course. To this end and to better understand our IV estimates, we estimate the marginal treatment effect (MTE) by using local instrumental variables (LIV).[15] We take a parametric approach and assume that the errors for the two potential outcomes and the selection equation are trivariate normal. First, we estimate the propensity score, *P(Z)*, using a standard probit model (identical to the first stage reported in Table 3). Second, we estimate the frequencies of the predicted propensity scores in the samples with $MathA = 1$ and $MathA = 0$, respectively, in order to identify the common support region. Note that the MTE is only identified over the common support of the propensity score. Hence, the stronger the instrument the larger the region over which we can identify the MTE. Third, we estimate the MTE by using the parametric two-step procedure suggested by Heckman, Urzua and Vytlacil (2006). Lastly, the IV weights are calculated from data in order to better understand how our IV estimator is obtained as a weighted average of MTEs.

Individuals choose advanced math if the expected gains exceed the expected costs; i.e. if $Y_{i1} - Y_{i0} - C_i \geq 0$. We specify the choice of advanced math to be given by the following selection equation:

$$(2)\quad MathA_i = \mathbf{1}[\alpha_0 + \alpha_1 X_i + \theta PilotIntro_i + V_i \geq 0] = \mathbf{1}[\mu_M(Z_i) - V_i \geq 0] = \mathbf{1}[P(Z_i) > U_M],$$

in which $\mu_M(Z_i) - V_i$ denotes the net utility of advanced math for individuals with observable characteristics Z_i and unobservable characteristics V_i. The last equality simply follows from using the standard normalization of taking the CDF of V, F_V, on both sides of the inequality. Therefore, U_M (on the horizontal axis in Figures 2 and 3) is uniformly distributed by construction. A higher U_M means a higher cost of choosing advanced math relative to the return of the choice. Note that it thus takes a high $P(Z)=p$ to compensate for a high $U_M = u_M$ and bring the individual to indifference between choosing advanced math or not. Hence, individuals with a higher U_M will be less likely to choose advanced math, and high values of the propensity score identify returns for individuals whose unobservables make them less likely to choose advanced math.

[15] See e.g. Carneiro, Heckman and Vytlacil (forthcoming) and Heckman, Urzua and Vytlacil (2006) for additional details and a discussion of alternative approaches.

The probability of choosing math is predicted based on the Z vector corresponding to specification (5) in Table 3. The common support region for females is 0.08-0.36 and for males it is a propensity score in the region 0.25-0.63.[16] Thus, we can estimate MTEs for both females and males with a probability of choosing advanced math in the range 0.25-0.35, although there are too few observations in this region to make precise inferences about gender differences in treatment effects.

Figure 2 shows MTEs based on specification (5) for pooled incomes for the regions of full support. The figure illustrates marginal treatment effects for different values of the math preference (U_M) required to attract the individual to do the advanced math course. At the left hand side of the figure, individuals have observable characteristics associated with a high probability of choosing math, and therefore they only need a low math preference, U_M, in order for them to be induced to choose advanced math, and vice versa at the right hand side of the figure.

The MTE curve for females has a positive slope, meaning that the females with the higher MTE tend to be less likely to choose advanced math (i.e. a large value of U_M is needed to induce them to choose advanced math). On the other hand, as one would expect, males who are more likely to choose advanced math (i.e. a moderate value of U_M is needed to induce them to choose advanced math) also have significantly higher MTEs. In other words, females at the margin of choosing advanced math (i.e. at the right hand side of the 0.11-0.20 region) have an MTE which is at least as high as the one those choosing math under the studied regime have, while males at the margin of choosing advanced math (i.e. at the right hand side of the 0.39-0.51 region) have a significantly lower MTE than those choosing math under the studied regime. However, in both cases the MTEs are significantly positive. This indicates that there are indeed potentially beneficial effects from encouraging even more high school students to opt for more advanced math qualifications.

For males the treatment effect for the marginal attendant is significantly lower (around 0.1) than it is for females (around 0.3). Thus, the marginal benefit from attracting more girls to advanced math is substantial. For the regions in which the range of full support overlaps across gender (the 0.25-0.35 region), it seems that the MTEs are equal for equal probabilities of choosing math. This suggests that the underlying math preference or math ability distribution is identical across gender. If we take into account that just below half of a cohort complete high school, this concerns roughly the 90th– 95th percentile of the distribution. This would be consistent with the suggestion by Niederle and

[16] This corresponds well with Figure 1 in which 0.11 and 0.20 of females and 0.39 and 0.51 of males choose advanced math at non-pilot and pilot schools, respectively.

Vesterlund (2010) that women underperform at the top of the distribution due to less taste for mixed-sex competition and low self-confidence in relation to math.

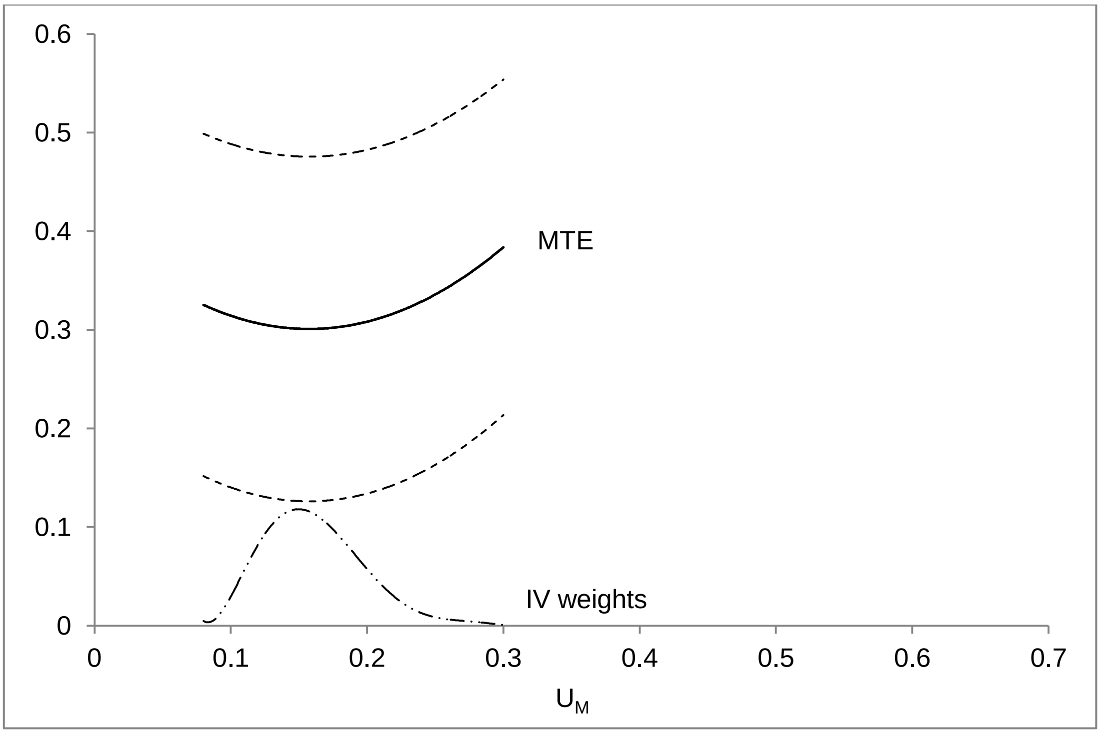
After the high school reform in 1988 (see section 2), advanced math could be combined with any other advanced course – Physics, Chemistry, Biology, Social Sciences or a Linguistic course. This reform increased the proportion of a high school cohort who chose advanced math to 75% for males and 40% for females.[17] Based on (extrapolation of) the estimated distribution of MTEs in Figure 2, the marginal return for the males would then be approaching nil for the marginal attendant while there may still be a positive return for females.

The variation of MTEs tends to be higher at the top of the distribution (at the left hand side of the diagram), in particular for males. This makes sense if the unobserved variables are more diverse and potentially more important at the top of the preference/ability distribution.

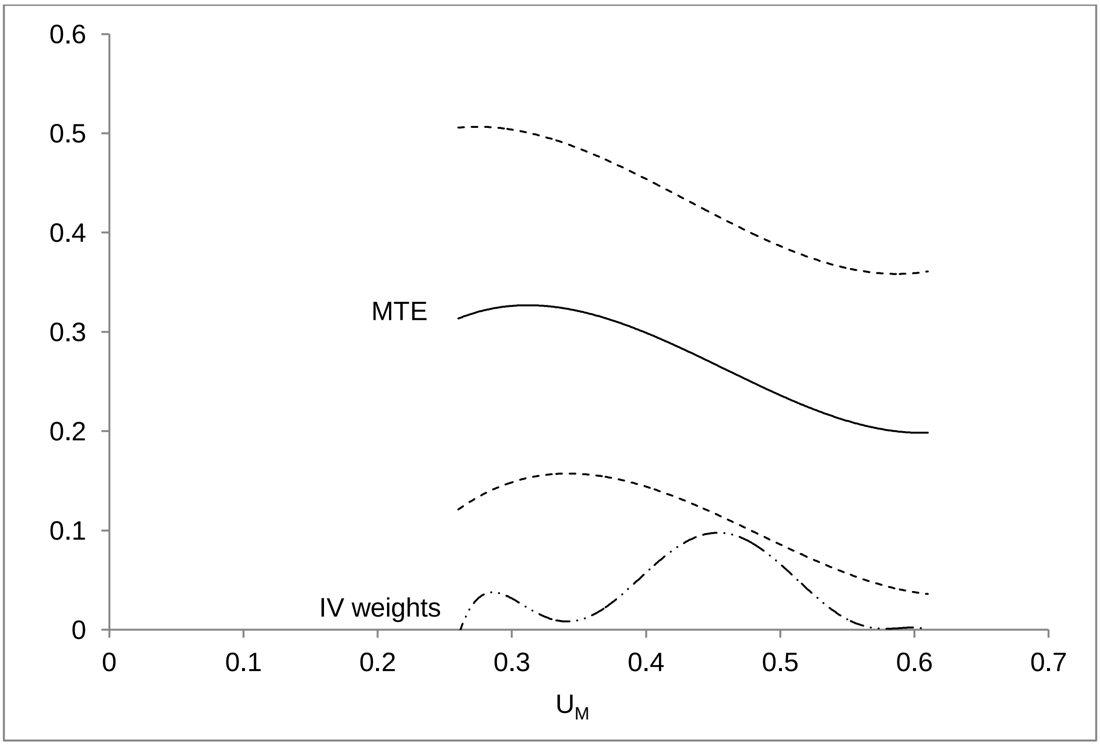
[17] Other changes took place at the same time. For instance, advanced math was given fewer lessons per week and a reduced curriculum. Therefore, the reform would not be informative as to the causal impact of advanced math.

Figure 2. Impact of Math on Earnings: Marginal Treatment Effects (MTE) and IV Weights

Females



Males



5. Impact of Math on Career Outcomes - Is Math the Route to a High-Powered Career?

In this section, we present results of estimating the effect of advanced high school math on a range of career outcomes: length and type of education, occupation, industry, sector, self-employment, and time to birth of first child. We present the results from using OLS and IV. [18]

In Table 4, we present the results from estimating the effect of advanced math on different measures of college graduation 8-18 years after leaving high school. We exclude individuals who have already graduated at a higher level in order to capture the effect at each specific margin.

The upper part of Table 4 shows the results for 2- or 4-year colleges: OLS indicates strong positive associations between advanced math and graduating from 2- or 4-year colleges, while IV estimates indicate that there are no significant effects for females and only borderline significant effects 16 and 18 years after leaving high school for males. The middle part of the table shows results for Master's education. We find that math influences graduation from Master's education for females but not for males. The effects are mainly significant early after high school graduation (8, 10 and 12 years after), which indicates that the effect is one of acceleration of graduation rather than one of increasing lifetime completion rates. We also studied effects on enrollment, but none of those came out significantly different from zero. This pattern of results indicates that advanced math influences *productivity* at college – i.e. the capability of completing an education – and not just preferences for entering college. The lower part of Table 4 shows results for graduate studies. We find some scattered significant effects of math on obtaining a PhD degree for females, while strong and robust effects - as large as 10 percentage points - are seen for males from 12 years after high school graduation and onwards.

In Table 5, we investigate the causal impact of advanced math on field of study. We see that advanced math increases the probability of obtaining a long-cycle higher education in Health and Technical Sciences for females, while it decreases the probability of obtaining a long-cycle higher education in Humanities or a medium-cycle higher education in Humanities or Teaching. For males, advanced math increases the probability of obtaining a higher education in Social Sciences. Thus, advanced math draws females away from standard female education and towards high-paid long

[18] We estimate a Heckman-type model for binary treatment and continuous outcome. The conclusions are generally robust to using probit and bivariate probit instead.

college education. While Health Sciences (i.e. medical school and dentistry) are not particularly male-dominated, Technical Sciences (i.e. Engineering) are clearly male-dominated.

Above we concluded that math seems to render higher productivity in the education system for females. However, now we see that clearly it also affects the attraction of choosing certain fields of education – in particular for females. Drawing on Niederle and Vesterlund (2010) and Pope and Sydnor (2010), we expect that the pilot scheme worked by reducing the extent of male-stereotypicality of the course packages involving advanced math. We suspect that our results indicate that math changes women's preferences for long-cycle college education and education in traditionally male-dominated subject areas, or that it tears down some of the psychological barriers in terms of self-perceived academic and math abilities among women which have earlier been shown important at these margins (see also Humlum, Kleinjans and Nielsen, 2012).

Table 4. Results from Estimation of the Impact of Math on Graduation Outcomes

	Parameter estimates and (standard errors)									
	Females					Males				
		OLS		IV			OLS		IV	
	Nobs	(1)		(2)		Nobs	(3)		(4)	
College Graduation										
within 8 years of hs graduation	21388	0.038	***	0.005		14351	0.083	***	0.117	
		(0.01)		(0.12)			(0.01)		(0.12)	
within 10 years of hs graduation	19230	0.045	***	-0.033		12435	0.082	***	0.143	
		(0.01)		(0.14)			(0.01)		(0.13)	
within 12 years of hs graduation	18116	0.041	***	0.073		11415	0.081	***	0.187	
		(0.01)		(0.13)			(0.01)		(0.15)	
within 14 years of hs graduation	17509	0.047	***	0.014		10836	0.080	***	0.181	
		(0.01)		(0.13)			(0.01)		(0.15)	
within 16 years of hs graduation	17158	0.052	***	-0.077		10495	0.081	***	0.235	*
		(0.01)		(0.12)			(0.01)		(0.14)	
within 18 years of hs graduation	16961	0.051	***	-0.090		10309	0.079	***	0.235	*
		(0.01)		(0.13)			(0.01)		(0.14)	
Master's Graduation										
within 8 years of hs graduation	24642	0.170	***	0.223	***	18730	0.145	***	0.073	
		(0.01)		(0.07)			(0.01)		(0.09)	
within 10 years of hs graduation	24312	0.180	***	0.224	***	18183	0.120	***	0.135	
		(0.01)		(0.08)			(0.01)		(0.11)	
within 12 years of hs graduation	24045	0.172	***	0.190	**	17724	0.096	***	0.205	*
		(0.01)		(0.08)			(0.01)		(0.12)	
within 14 years of hs graduation	23813	0.161	***	0.122		17433	0.088	***	0.206	*
		(0.01)		(0.09)			(0.01)		(0.12)	
within 16 years of hs graduation	23727	0.161	***	0.111		17260	0.086	***	0.177	
		(0.01)		(0.09)			(0.01)		(0.11)	
within 18 years of hs graduation	23608	0.158	***	0.155	*	17118	0.082	***	0.163	
		(0.01)		(0.09)			(0.01)		(0.11)	
PhD Graduation										
within 8 years of hs graduation	24670	0.007	***	0.017	**	18818	0.008	***	0.010	***
		(<0.01)		(0.01)			(<0.01)		(0.02)	
within 10 years of hs graduation	24379	0.013	***	0.006		18445	0.021	***	0.038	
		(<0.01)		(0.01)			(<0.01)		(0.03)	
within 12 years of hs graduation	24208	0.022	***	0.033	**	18207	0.033	***	0.093	**
		(<0.01)		(0.02)			(<0.01)		(0.04)	
within 14 years of hs graduation	24125	0.031	***	0.019		18092	0.035	***	0.083	*
		(<0.01)		(0.02)			(<0.01)		(0.04)	
within 16 years of hs graduation	24151	0.037	***	0.048	*	18051	0.037	***	0.098	**
		(<0.01)		(0.03)			(<0.01)		(0.05)	
within 18 years of hs graduation	24125	0.041	***	0.033		17985	0.037	***	0.098	**
		(<0.01)		(0.03)			(<0.01)		(0.05)	

Note: College includes education at 2- or 4-year colleges. Significance at a 1%-, 5%- and 10%-level are indicated by ***, ** and *, respectively. Controls: parental background variables, county indicators, entry cohort fixed effects, high school specific controls and pilot school indicator. First stage results are similar to those in Table 3, and are not reported.

Table 5. Results from Estimation of the Impact of Math on Choice of Educational Field

	Parameter estimate (standard error) [marginal effects]			
	Females		Males	
	OLS	IV	OLS	IV
	(1)	(2)	(3)	(4)
Effect of High Level Math on Educational Length and Field:				
LHE Health	0.040 ***	0.078 ***	0.000 ***	-0.029
	(0.003)	(0.035)	(0.002)	(0.031)
LHE NatSci	0.036 ***	0.026	0.028 ***	0.028
	(0.003)	(0.031)	(0.003)	(0.046)
LHE Technical	0.070 ***	0.093 ***	0.143 ***	-0.001
	(0.003)	(0.030)	(0.004)	(0.065)
LHE Humanities	-0.050 ***	-0.109 **	-0.040 ***	-0.016
	(0.005)	(0.051)	(0.003)	(0.044)
LHE SocSci	0.036 ***	0.054	-0.058 ***	0.123
	(0.005)	(0.059)	(0.005)	(0.085)
MHE Teacher	-0.072 ***	-0.074	-0.047 ***	-0.004
	(0.006)	(0.066)	(0.003)	(0.050)
MHE Health	-0.036 ***	0.006	-0.007 ***	0.026
	(0.006)	(0.068)	(0.002)	(0.026)
LHE + MHE Technical	0.137 ***	0.049	0.243 ***	-0.086
	(0.004)	-0.046	(0.006)	(0.095)
LHE + MHE Health	0.004	0.084	-0.007 ***	-0.004
	(0.007)	(0.074)	(0.003)	(0.039)
LHE + MHE SocSci	0.046 ***	-0.025	-0.075 ***	0.247 **
	(0.006)	(0.069)	(0.006)	(0.099)
LHE + MHE Humanities + MHE Teacher	-0.172 ***	-0.202 **	-0.112 ***	-0.064
	(0.086)	(0.086)	(0.070)	(0.070)
High level Math First-Stage:		0.328 ***		0.253 ***
Pilot School Intro		(0.029)		(0.029)
		[0.082]		[0.100]
Number of observations	24201		18051	

Note: LHE=long cycle higher education (MSc degree), MHE= medium cycle higher education (4-year college). Significance at a 1%-, 5%-level and 10%-level are indicated by ***, ** and *, respectively. Controls: parental background variables, county indicators, entry cohort fixed effects, high school specific controls and pilot school indicator.

In Table 6, we study the impact of advanced math on other career outcomes reflecting type of employment and occupation. In OLS regressions, most outcomes are significantly associated with advanced math. Using IV, we find that advanced math strongly influences the probability of being employed in the private sector 18 years after graduation. The magnitude of the effect of advanced math is 0.28, which is large compared to the mean; 47% of females and 70% of males are employed in the private sector (see Table 2). Advanced math also increases the probability of being a chief executive officer (CEO) regardless of whether this is measured by using raw occupation codes or the more detailed Danish version of the international standard classification of occupations (D-ISCO) which more appropriately identifies CEOs in the private sector. No significant effects on these career outcomes are found for males.

Looking at nine industry indicators as outcomes (not shown), we find that advanced math increases the probability of going into transportation, mail and communication for females, while probabilities of going into the other industries are unaffected. No significant effects are found for males. We have also investigated the effect of math on career outcomes *conditional on* being employed in one of those nine specific industries 18 years after high school graduation. For females, we find that math influences the probability of possessing managerial positions at various levels in two industries: Building and Construction and Financial Institutions, Insurance and Financing, both of which are typically in the private sector. For males, we find that a positive effect on the probability of possessing managerial positions is seen conditional on being employed in Social and Health Services, which are typically part of the public sector. Thus, advanced math has different impacts on the position on the hierarchical ladder of the firm across genders.

Table 6. Results from Estimation of the Impact of Math on other Career Outcomes

	Parameter estimate and (standard error)			
	Females		Males	
	OLS	IV	OLS	IV
	(1)	(2)	(3)	(4)
Type of employment				
Fulltime	0.009 ***	0.017	0.009 ***	0.011
	(0.003)	(0.029)	(0.002)	(0.039)
Self-Employed	-0.007 **	0.050	-0.003	-0.035
	(0.003)	(0.034)	(0.003)	(0.048)
Private sector	0.100 ***	0.277 ***	0.134 ***	0.037
	(0.009)	(0.099)	(0.007)	(0.103)
Occupation codes				
CEO	0.007 ***	0.048 *	0.002	0.022
	(0.003)	(0.029)	(0.004)	(0.057)
Manager	0.006 *	0.044	0.000	0.008
	(0.003)	(0.034)	(0.004)	(0.063)
Managerial Employee	0.155 ***	0.072	0.096 ***	0.143
	(0.009)	(0.094)	(0.008)	(0.113)
D-ISCO				
Top CEO	0.001 *	0.019 **	0.002	-0.026
	(0.001)	(0.008)	(0.002)	(0.024)
Vice-Director	-0.001	0.017	0.000	-0.018
	(0.002)	(0.019)	(0.003)	(0.040)
Middle Manager	0.006 **	0.044	0.002	-0.003
	(0.003)	(0.029)	(0.004)	(0.057)
Number of observations	**24201**		**18051**	

Note: Significance at a 1%-, 5%-level and 10%-level are indicated by ***, ** and *, respectively. The following variables are included as controls: parental background variables, county indicators, entry cohort fixed effects, high school-specific controls and pilot school indicator. The first stage results are identical to those reported in Table 5.

In Table 7, we present results from estimation of the effect of math on fertility outcomes. The OLS estimates indicate that individuals who did advanced math have the same number of children as others, but they tend to have them later. Looking at the IV estimates, the delay of childbearing is not significant at conventional levels, although the point estimates indicate that advanced math tends to delay childbearing for females. This is most likely a derived effect of the changed career patterns potentially working through marriage (to a different partner or marrying later).

Table 7. Results from Estimation of the Impact of Math on Fertility Outcomes

	Parameter estimate (standard error)			
	Females		Males	
	OLS	IV	OLS	IV
	(1)	(2)	(3)	(4)
Duration to birth of first child	0.379 ***	1.322	0.173 ***	-0.516
	(0.079)	(0.809)	(0.068)	(1.004)
Age at birth of first child	0.269 ***	1.107	0.082	-0.859
	(0.079)	(0.806)	(0.069)	(1.008)
Completed fertility	-0.008	0.272	0.005	-0.323
	(0.019)	(0.207)	(0.016)	(0.250)

Note: Significance at a 1%-, 5%-level and 10%-level are indicated by ***, ** and *, respectively. The following variables are included as controls: parental background variables, county indicators, entry cohort fixed effects, high school specific controls and pilot school indicator. The first stage results are identical to those reported in Table 5.

6. Conclusion

We confirm the large effects of math on earnings as already documented by Joensen and Nielsen (2009). The objective of this paper is to investigate the mechanisms behind these large math effects separately by gender. We focus on the impact of math on educational choices as well as on other career outcomes such as occupation, industry, type and sector of employment and time to birth of the first child.

We analyze the distribution of treatment effects of math on earnings and ask whether there is an unexploited pool of math talent among young women. We find that this is indeed the case because the marginal treatment effects are large and positive even for those women not choosing math under the studied regime. We find higher treatment effects for women than for men, and our study indicates that this is because the marginal woman considering doing advanced math has a greater math talent, because women have traditionally been reluctant to choose math.

We investigate the effect of math on a range of career outcomes and ask whether math is a route to high-powered careers. We find that doing advanced high school math is to a large extent a route to a high-powered career and mostly so for women. It speeds up acquirement of Master's degrees and attracts females to high-paid or male-dominated careers. It increases the probability of being employed in the private sector and to some extent it increases the probability of obtaining a PhD degree and climbing the hierarchical ladder to top-executive positions.

We interpret the results as an indication that advanced math improves productivity of females in the education system, because it accelerates graduation without influencing enrollment. However, the fact that advanced math makes females drift away from traditional female education in Humanities and teaching towards high-paid education in Health Sciences and Technical Sciences also indicates that preferences or self-perception may be affected by succeeding with advanced math. This is mere speculation, and we leave it for future research to find hard evidence for the exact economic mechanism of the large effects of advanced math on female careers.

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Appendix

Table A1. Summary Statistics of Outcome Variables (measured 18 years after graduation)

	Female			Male		
	Overall mean	Mean difference by *Math* A		Overall mean	Mean difference by *Math* A	
Log earnings (2000 DKK)	12.12	0.251	***	12.47	0.230	***
Level of education:						
College Enrollment	0.834	0.058	***	0.856	0.077	***
Master's Enrollment	0.394	0.184	***	0.562	0.100	***
College Degree	0.760	0.049	***	0.723	0.049	***
Master's Degree	0.297	0.189	***	0.427	0.101	***
Phd Degree	0.021	0.042	***	0.048	0.037	***
Field of education:						
SHE Technical	0.022	0.001		0.017	0.001	
SHE Other	0.050	0.009	*	0.055	-0.019	***
MHE Teacher	0.129	-0.074	***	0.052	-0.047	***
MHE Humanities	0.055	-0.049	***	0.02	-0.026	***
MHE Health	0.137	-0.037	***	0.013	-0.007	***
MHE SocSci	0.043	0.011	**	0.058	-0.017	***
MHE TechSci	0.035	0.065	***	0.114	0.101	***
MHE Other	0.012	-0.006	***	0.016	0.002	
LHE Health	0.031	0.042	***	0.018	0.000	
LHE NatSci	0.026	0.037	***	0.043	0.028	***
LHE Technical	0.024	0.072	***	0.09	0.143	***
LHE Humanities	0.074	-0.047	***	0.039	-0.039	***
LHE SocSci	0.1	0.040	***	0.157	-0.057	***
LHE Other	0.021	0.004		0.031	-0.010	***
Career outcomes:						
Type of employment						
Fulltime Employment	0.978	0.008	***	0.977	0.009	***
Self-Employed	0.029	-0.007	**	0.045	-0.004	
Private	0.469	0.103	***	0.705	0.132	***
Occupation codes						
Manager	0.029	0.006	*	0.082	0.001	
Managerial Employee	0.375	0.163	***	0.515	0.098	***
CEO	0.022	0.008	**	0.067	0.004	
D-ISCO						
Top-CEO	0.002	0.002	*	0.011	0.002	
Vice-Director	0.010	-0.001		0.032	0.001	
Middle Manager	0.022	0.006	**	0.068	0.004	
Industry outcomes:						
Industry, building and construction activities	0.112	0.062	***	0.181	0.057	***
Trade, hotel and catering	0.087	-0.003		0.107	-0.006	
Transport company, mail, and telecommunication	0.031	-0.001		0.059	-0.008	**
Financial institutions, insurance, and financing	0.037	0.016	***	0.061	0.001	
Real estate, renting, and business service	0.126	0.051	***	0.251	0.119	***
Public administration, defence, and social security	0.074	0.018	***	0.076	-0.029	***
Education	0.149	-0.037	***	0.104	-0.050	***
Social and health services	0.259	-0.070	***	0.058	-0.029	***
Organizations, entertainment, and sports	0.049	-0.016	***	0.051	-0.037	***
Fertility outcomes:						
Duration to first birth after high-school entry	13.155	0.450	***	14.547	0.165	**
Age at first birth	29.391	0.330	***	30.814	0.075	
Completed fertility	1.693	-0.016		1.404	0.012	
Number of Individuals	**24,201**			**18,051**		

Note: Log income is measured 8-18 years after HS graduation. Significance at a 1%-, 5%-level and 10%-level are indicated by ***, ** and *, respectively.

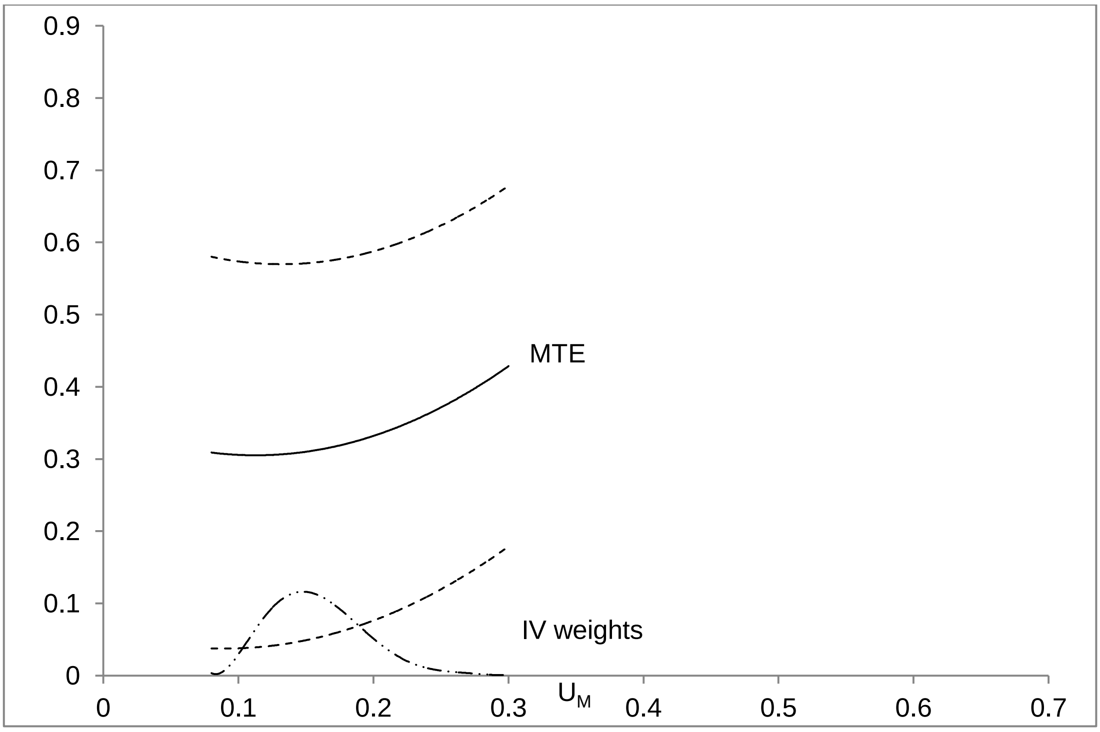
Table A2. Summary Statistics of Background Variables

	Means and (standard deviations)					
	Female			Male		
Control variables:	Overall mean	Mean difference between *PilotSchool=1 PilotIntro=1* and *PilotSchool=0*	Mean difference between *PilotSchool=1 PilotIntro=1* and *PilotSchool=1 PilotIntro=0*	Overall mean	Mean difference between *PilotSchool=1 PilotIntro=1* and *PilotSchool=0*	Mean difference between *PilotSchool=1 PilotIntro=1* and *PilotSchool=1 PilotIntro=0*
Parental background variables:						
Father's income (year 2000 DKK)	8.92 (5.45)	0.04	0.16 **	8.61 (5.64)	-0.14	-0.14
Mother's income (year 2000 DKK)	8.25 (5.15)	-0.17 *	-0.10	7.99 (5.29)	-0.41 **	-0.27 ***
Father basic school	0.17	-0.02 **	0.01 **	0.13	-0.02 **	0.01
Father high school	0.01	0.00	0.00	0.01	0.00	0.00
Father vocational training	0.29	0.01	0.02 **	0.26	0.00	0.00
Father short higher education	0.03	0.00	0.00	0.03	0.00	-0.01 ***
Father medium higher education	0.13	0.01 *	0.00	0.14	0.01	0.00
Father long higher education	0.10	0.00	-0.01	0.12	-0.01	0.00
Mother basic school	0.25	-0.02 **	0.03 ***	0.20	0.00	0.04 ***
Mother high school	0.01	0.00	0.00	0.02	-0.01 ***	-0.01 **
Mother vocational training	0.29	0.01	0.00	0.28	-0.01	-0.02 **
Mother short higher education	0.03	0.01 **	0.00	0.04	0.01	0.00
Mother medium higher education	0.15	0.00	0.00	0.17	-0.01	-0.01
Mother long higher education	0.03	0.00	0.00	0.04	0.00	-0.01 **
Mean parental background at the high school in 1983:						
Father's income (year 2000 DKK)	8.59 (0.54)	0.02	0.07	8.57 (0.54)	0.02	0.07
Mother's income (year 2000 DKK)	7.59 (0.62)	-0.27 ***	0.03	7.59 (0.63)	-0.29 ***	0.01
Father basic school	0.18	-0.02	0.01	0.17	-0.02	0.01
Father high school	0.01	0.00	0.00	0.01	0.00	0.00
Father vocational training	0.27	0.01	0.01	0.27	0.01	0.01
Father short higher education	0.03	0.00	0.00	0.03	0.00	0.00
Father medium higher education	0.12	0.01 **	0.00	0.12	0.01 ***	0.00
Father long higher education	0.10	0.00	-0.01	0.11	0.00	-0.01
Mother basic school	0.29	-0.02	0.02	0.28	-0.01	0.02
Mother high school	0.01	0.00	0.00	0.02	0.00	0.00
Mother vocational training	0.26	0.01	0.00	0.27	0.00	0.00
Mother short higher education	0.03	0.00	0.00	0.04	0.00	0.00
Mother medium higher education	0.13	0.00	0.00	0.13	0.00	-0.01
Mother long higher education	0.03	0.00	0.00	0.03	0.00	0.00
County indicators:						
Region 2	0.13	-0.04	-0.01	0.15	-0.04	-0.01
Region 3	0.09	-0.01	-0.01	0.10	-0.01	0.00
Region 4	0.05	0.05	-0.01	0.05	0.07	-0.01
Region 5	0.05	0.04	-0.01	0.05	0.02	0.00
Region 6	0.05	-0.02	0.02	0.05	-0.02	0.01
Region 7	0.08	0.03	0.04	0.08	0.02	0.02
Region 8	0.05	0.02	-0.04	0.04	0.02	-0.03
Region 9	0.04	0.02	0.02	0.04	0.03	0.02
Region 10	0.07	0.02	0.01	0.06	0.04	0.02
Region 11	0.05	-0.05	0.02	0.04	-0.05	0.02
Region 12	0.11	-0.02	0.03	0.12	-0.04	0.01
Region 13	0.05	-0.03	-0.02	0.04	-0.02	-0.02
Region 14	0.10	0.06	-0.02	0.09	0.06	-0.01
High school cohort:						
Startyear 85	0.33	-0.07	-0.12	0.33	-0.07	-0.13
Startyear 86	0.34	-0.02	-0.36 ***	0.33	-0.03	-0.36 ***
Number of Individuals		**24201**			**18051**	

Note: Significance at a 1%-, 5%-level and 10%-level are indicated by ***, ** and *, respectively.

Figure A1. Marginal Treatment Effects (MTE) and IV Weights, Average Income

Females



Males