

Smoothing shocks and balancing budgets in a currency union*

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December 2011

Abstract: We study simple fiscal rules for stabilizing the government debt level in response to asymmetric demand shocks in a country that belongs to a currency union. We compare debt stabilization through tax rate adjustments with debt stabilization through expenditure changes. While rapid and flexible adjustment of public expenditure might seem institutionally or informationally infeasible, we discuss one concrete way in which this might be implemented: setting salaries of public employees, and social transfers, in an alternative unit of account, and delegating the valuation of this numeraire to an independent fiscal authority.

Using a sticky-price DSGE matching model of a small open economy in a currency union, we compare the business cycle implications of several different fiscal rules that all achieve the same reduction in the standard deviation of the public debt. In our simulations, compared with rules that adjust tax rates, a rule that stabilizes the budget by adjusting public salaries and transfers reduces fluctuations in consumption, employment, and private and public after-tax real wages, thus bringing the market economy closer to the social planner's solution.

Keywords: Fiscal authority, public wages, sovereign debt, monetary union

JEL classification: E24, E32, E62, F41

*Views expressed in this paper are those of the authors. Readers should not assume that this paper reflects the views of the Bank of Spain or the Eurosystem. The authors are grateful for helpful comments from Thijs van Rens, Jordi Galí, Pablo Burriel, Rubén Segura, Aitor Erce, Javier Pérez García, Pablo Hernández de Cos, Henrique Basso, Giovanni Lombardo, Harris Dellas, Henry Siu, Alessia Campolmi, seminar participants at the ECB, and participants at ESSIM 2010, the Vienna Macroeconomics Workshop 2011, and the Simposio de Moneda y Crédito 2011. Beatriz de Blas thanks the Spanish Ministry of Education and Science for financial support under grant number ECO2008-04073. The authors take responsibility for any errors.

1 Introduction

Beginning in the spring of 2010, speculative attacks on European public debt, starting in Greece but rapidly spreading to other Mediterranean countries and to Ireland, have called into question the design of the Economic and Monetary Union. The crisis has been accompanied by calls for a more complete fiscal union to backstop the monetary union, for example:

“... it is clear that a minimal fiscal Europe is necessary to make up for the loss of independent monetary policy as a sovereign default prevention mechanism.” (Buiter and Rahbari, 2010)

Calls like these seemed somewhat fanciful even a year ago. But at the time of this writing (October 2011) the ECB has stepped in to back the sovereign bonds of several member states, and European governments are in almost daily negotiations to strengthen and expand the European Financial Stability Facility, opening the door to possible large fiscal transfers across countries. The moral hazard this implies is leading European leaders to impose strict reforms on member states seeking support, and there is renewed talk of constitutional changes to further integrate fiscal policy.

Regardless of whether a fiscal union is eventually established or not, European countries will need to find mechanisms to better control their public debt levels. Inside a fiscal union, this will be necessary to prevent moral hazard problems; in the absence of a fiscal union, it will be necessary to avoid debt levels that imply a risk of speculative attack. There are many possible margins, some more effective than others, that could be adjusted to control the public debt; moreover, different margins of adjustment will have different effects on the rest of the macroeconomy. Therefore, this paper compares the budgetary and macroeconomic implications of several fiscal rules that could be employed to reduce fluctuations in public debt. We consider both tax adjustments and spending adjustments as possible instruments for controlling debt. In particular, we discuss how high-frequency public spending adjustments might serve to stabilize the government budget without amplifying business cycle fluctuations. This might seem counterintuitive, since it is commonly assumed that discretionary spending cannot be adjusted quickly, and since “automatic stabilizers” of the business cycle increase deficits in recessions (Friedman 1948), but we believe this conventional wisdom fails to consider a sufficiently wide set of possible fiscal instruments. In the next subsection, we discuss in detail how to design a fiscal framework in which public spending could serve as a high-frequency stabilizer both for the budget and for the business cycle.

1.1 A framework for rapid fiscal adjustments

According to the textbook Mundell-Fleming model (Mundell 1961), asymmetric demand shocks are amplified inside a currency union. Independent monetary policies can no longer offset these shocks; this gives individual countries an incentive to use fiscal policy to offset them. However, large macroeconomic shocks like those of the recent crisis can lead rapidly to fiscal difficulties even if the previous fiscal stance was responsible. Figures 1-4 show how recently Spain seemed to be a prudent, low-debt country, whereas it is now perceived to be near the front line of attack in the ongoing sovereign debt crisis.¹ This problem is aggravated by the fact that when a country emits debt in a currency it does not control, the probability of a self-fulfilling attack on the currency is increased (Eichengreen and Hausmann 2005; De Grauwe 2010). It is further aggravated by the instability of cross-border banking flows in a monetary union (Bruche and Suárez, 2010).

On the other hand, membership of a monetary union implies that in principle, resources could be transferred from other union members to fight a speculative attack on any particular country's debt. That is, fiscal stabilization could be simplified by creating a fiscal union to accompany the monetary union. Obviously though, moral hazard issues arise: there is less incentive for countries to run responsible fiscal policies if they expect to be bailed out by partners when they have solvency difficulties. This moral hazard problem has the potential to deepen political tensions within the Eurozone as countries that transfer resources demand the imposition of reforms on recipient countries. The failure of the proposed European Constitution already showed growing skepticism about further integration among the European public, and tension across EU members is now openly visible as peripheral countries protest against imposed reforms and core countries protest against the dangers of a "transfer union".

If political tensions make substantial cross-border transfers impossible, fiscal union may not be sufficient to maintain stability of the monetary union. On the other hand, since *any* mechanism capable of guaranteeing long-term budget balance at the national level could prevent the speculative attacks that are destabilizing the monetary union today, fiscal union is in principle not logically *necessary* either. So, does any

¹Figures 1 and 2 report the debt and deficit situation for an aggregate of the Euro area, Belgium, Germany, Ireland, Greece, Spain, France, Italy, Portugal, Sweden, and the United Kingdom for 2007 and 2009. All data displayed in figures come from Eurostat. Quantities in Figs. 1-2 correspond to government consolidated gross debt as a percentage of GDP, and net lending (+) or borrowing (-) in ESA 1995 as a percentage of GDP, respectively. Note that most of these countries went from surpluses or balance in 2007 into deficits in 2009 (Greece is exceptional in that it was already running a large deficit in 2007).

Figure 1: *International comparison of debt-to-GDP ratios.*

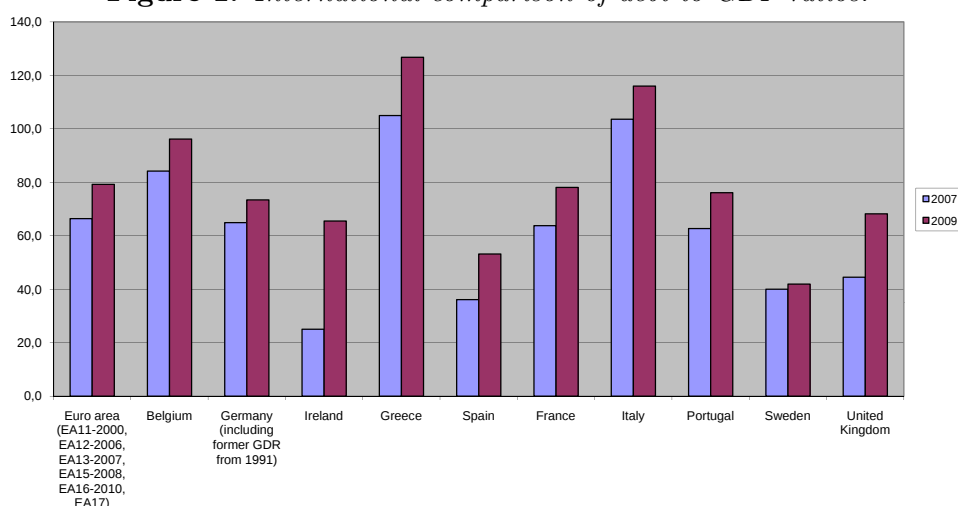
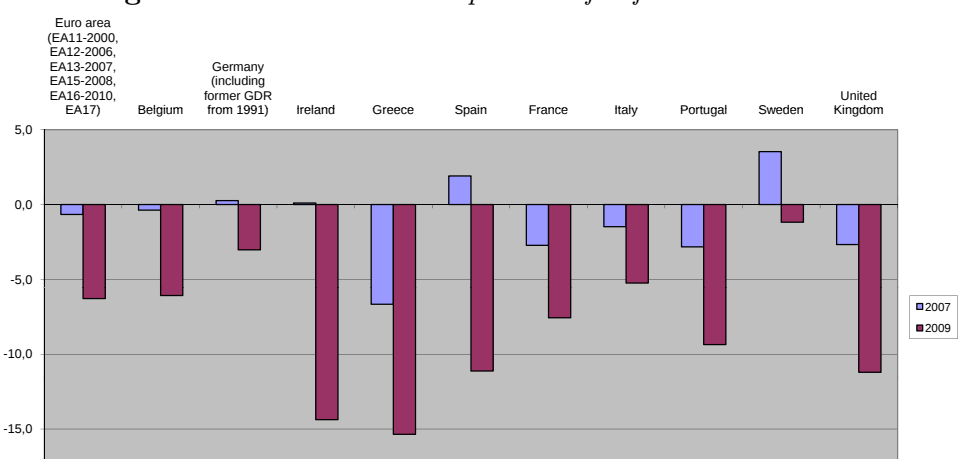


Figure 2: *International comparison of deficit-to-GDP ratios.*



institutional arrangement for ensuring national solvency exist? In recent years, many economists have proposed that budget stability could be greatly enhanced by delegating some fiscal decisions to an independent authority, modeled along the lines of an independent central bank.² This paper also advocates fiscal delegation. But unlike previous literature on independent fiscal authorities, our paper also addresses the question: does budget stabilization necessarily destabilize output over the business cycle? After all, the Stability and Growth Pact has been widely criticized for its potential to amplify recessions by requiring fiscal retrenchment when output falls. And any tax-based mechanism for maintaining budget stability seems likely to have similar implications.

²Prominent examples include von Hagen and Harden (1995), Blinder (1997), Eichengreen et al. (1999), Calmfors (2003), Wyplosz (2005), and Leeper (2009). The theory of independent fiscal authorities is surveyed by Debrun, Hauner, and Kumar (2009).

Figure 3: *Evolution of Spanish public debt: 1995-2009.*

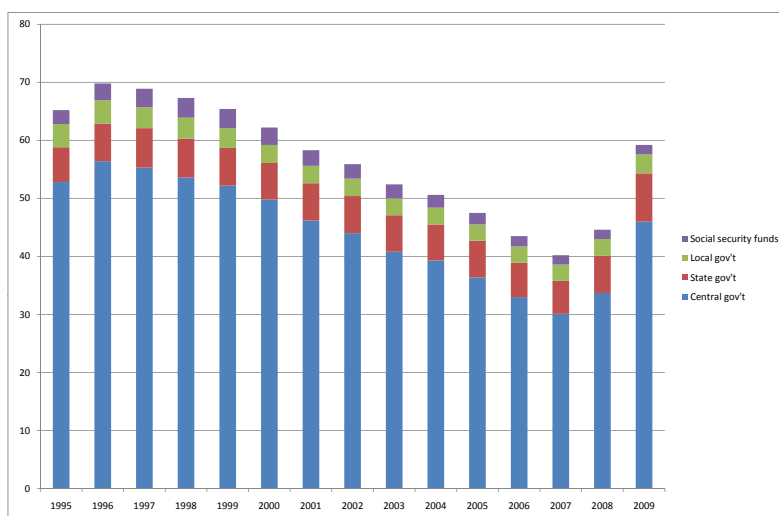
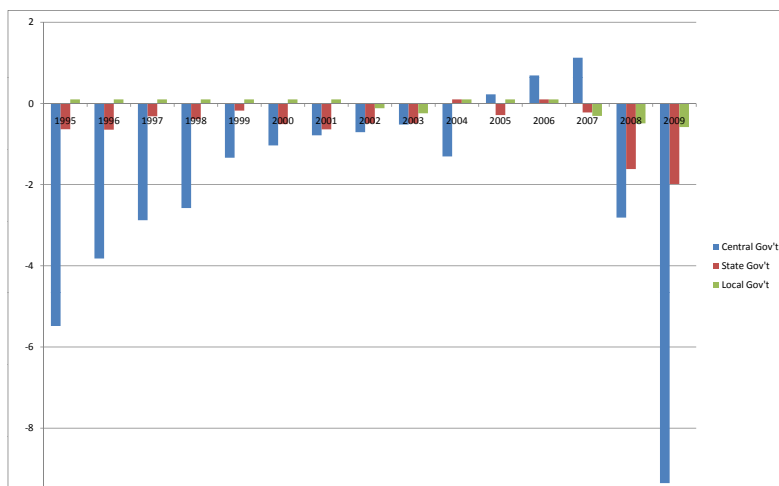


Figure 4: *Evolution of Spanish public deficit: 1995-2009.*



Yet handing control of spending to an independent authority seems harder to do, since public spending decisions are complex, multi-dimensional, and inherently political.

However, rapid and flexible control of the spending side could be facilitated by setting up a formal framework for adjusting public salaries and transfers.³ Empirical evidence suggests that these margins of fiscal policy have a particularly durable budgetary effect (Alesina and Perotti, 1996; Corsetti, Meier, and Mueller, 2009). Moreover, focusing budgetary retrenchment on public salaries and transfers has the advantage of

³Gomes (2009) shows that the government would optimally decrease public sector wages in recessions, or when public spending needs rise. Our model has similar implications, even though we assume that the public sector wages are subject to a sticky bargaining process, instead of being freely chosen by the government.

promoting competitiveness by fostering a real devaluation when the economy falls into recession. These margins have recently featured in many governments' *ad hoc* responses to the crisis; here we advocate a more permanent mechanism for adjusting them.

Rapid adjustment of the many different items of salaries and transfers requires the definition of some across-the-board shift parameter affecting all these items simultaneously. A possible way to do this would be to set public salaries, and various public transfer payments, in an alternative unit of account.⁴ An independent fiscal authority at the national level would set the exchange rate X : the euro value of one unit of the alternative currency. The fiscal authority would take into account tax policy, government expenditure policy, and the state of the business cycle, resetting X regularly for consistency with a long-run target level for public debt. The fiscal authority would also have the mandate, at most once per year, to shift up or down the schedule of income taxes or VAT rates.

For a concrete example, consider the following scenario for the case of Spain.⁵ Imagine that the government legally defines one "civil service unit" (CSU) as the base salary of civil servants of type "Funcionario A". Civil servants and transfer recipients would be paid in euros, but contracts would be negotiated in CSU; for example, the unions and the government might agree to set the base salary of a "Funcionario B" to 1.2 CSU. The government would legally delegate to an independent agency the choice of X , the value of one CSU in euros. The base salary of each Funcionario A would therefore be X euros, and the base salary of each Funcionario B would be $1.2X$ euros. This fiscal authority could be based in Madrid, for better access to local fiscal information; or it could be based in Brussels, where it would be more clearly sheltered from local political pressures. It could be set up as one of many national fiscal agencies as part of a European system; but it would also be beneficial to any solvent EMU member country to set up such an agency for itself. The fiscal authority would publish the new value of X , monthly, at a date sufficiently early to allow the processing of that month's paychecks. The authority would choose X at a level consistent with long-term balance of the government budget. Nonetheless, it would permit deficit spending during recessions, to be compensated by surpluses during economic expansions.

Notice that on one hand, this system helps protect the economy from the amplification of shocks associated with a currency union. It does so by directly attacking the

⁴Buiter (2007) studies monetary policy when the medium of exchange differs from the unit of account.

⁵Political economy issues related to this framework, and more institutional details about how it could be implemented, are discussed in a companion paper, Costain and de Blas (2011).

Figure 5: *Evolution of components of Spanish public expenditure: 2007-2009.*

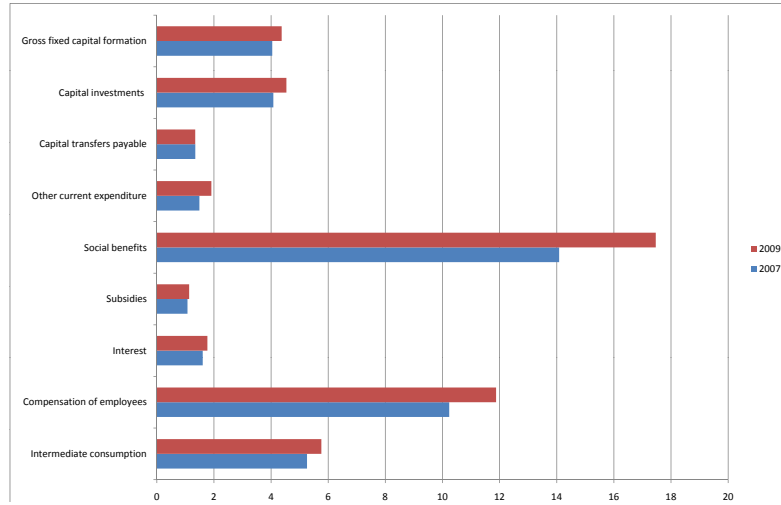
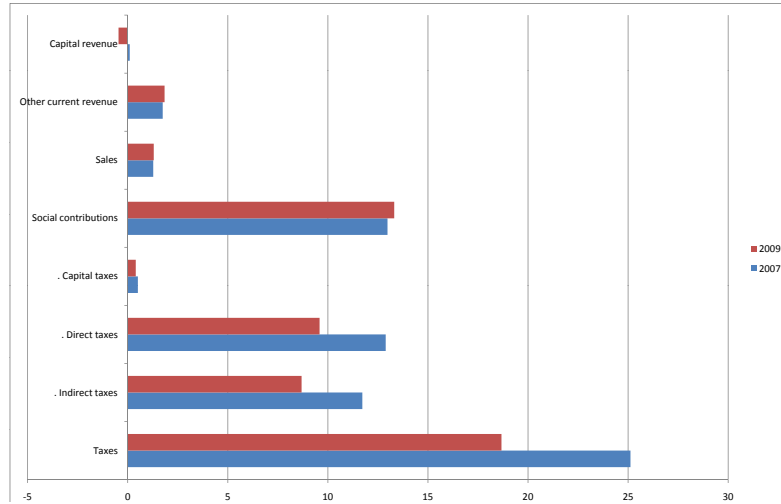


Figure 6: *Evolution of components of Spanish public revenues: 2007-2009.*



root cause of the problem: nominal rigidity in the common currency.⁶ Obviously, the standard textbook analysis of asymmetric demand shocks in a currency union assumes nominal rigidity: there would be no cost to giving up a flexible exchange rate if a country could frictionlessly adjust its internal price and wage levels.⁷ The implications of

⁶One example of the importance of nominal rigidity in public spending is the continued increase in compensation of Spanish public employees (by 2% of Spanish GDP) in 2007-09 in the midst of the financial crisis, as seen in Fig. 5. The figure also shows that compensation of public employees and social transfers make up roughly 60% of total government expenditure in Spain. Thus an adjustment along these two margins represents an across-the-board shift in the large majority of expenditure.

⁷Like us, Farhi, Gopinath, and Itskhokhi (2011) show that the loss of an independent monetary instrument in a currency union may be undone by considering policy instruments ignored in the

the New Keynesian DSGE model we develop here are similar to those of the traditional analysis, and the details of our proposed mechanism are also based on a similar logic. Public spending is distributed over a huge number of distinct items; if we assumed it were informationally feasible for a monopsonistic government to readjust its prices for these items quickly and cheaply, we would be assuming away the problem of nominal stickiness (in fact, we would be assuming central planning works). Therefore we restrict the problem of the fiscal authority to choosing one or two numbers: an exchange rate between public and private numeraires, and a shift parameter for the tax code. The forecasting expertise necessary to make these decisions would be similar to the expertise required by an independent central bank setting an interest rate.

Note also that as a purely nominal shift, setting an exchange rate between the public and private units of account would be *neutral* in the absence of nominal rigidities. Nonetheless, our numerical model will show that for realistic degrees of wage stickiness, the CSU mechanism has a powerful effect on the government budget. And while some alternative mechanisms might at first seem equivalent— for example, sector-specific tax rates for private and public employees— we do not see any alternatives that would be as informationally cheap as the one we analyze here. For example, since some workers are simultaneously employed by the private and public sectors, implementing sector-specific tax rates would require rewriting the tax code to define how marginal rates interact across the two types of income. In contrast, our mechanism for adjusting the value of the public unit of account would require no changes in Spanish tax regulations or labor market regulations.

Analysis of the CSU mechanism brings up a variety of issues, but above all it brings up two very different questions which we are addressing in *separate* papers. First, is it desirable to delegate some aspects of fiscal policy to an independent authority, and if so, is it politically feasible? We emphasize that this question is *not* addressed here; we review the relevant political economy literature in our companion paper, Costain and de Blas (2011).⁸ Second, if control of some fiscal decisions is to be delegated to an independent agency, precisely which fiscal instruments should the authority control in order to stabilize both the business cycle and the government finances? This is a question of macroeconomic dynamics rather than political economy, and it is the question we address here. We do so by constructing a sticky-price DSGE matching

standard textbook analysis. However, their proposal requires coordinated changes in four different tax rates in response to shocks; the mechanisms discussed here may be simpler for policy makers to implement.

⁸For a skeptical discussion of independent fiscal authorities, see Wren-Lewis (2010). For a more optimistic viewpoint, see Wyplosz (2008).

model of a small open economy in the context of a currency union. Bargaining in the private sector takes place in euros; bargaining in the public sector is assumed to take place (by law) in an alternative unit of account, called the CSU. Note that since workers may find jobs in either sector, a decrease in real public wages due to a decrease in the value of the CSU will feed across to the private sector as well. The model represents the action of the fiscal authority by two rules. The first rule sets the exchange rate X —the euro value of one CSU—on a period-by-period basis, following a rule that reacts to the level of public debt, the level of output, and the level of government spending (relative to steady-state output). The second rule sets the tax rate on labor income, τ , as a function of the same variables.

Our analysis compares various rules designed to stabilize the government budget. That is, starting from a baseline fiscal policy in which the public debt exhibits large fluctuations, we compare several different fiscal rules that all reduce the standard deviation of the public debt by the same amount. We focus on shocks to foreign prices, an export demand shifter, government spending, and the interest rate in the rest of the union, since these are asymmetric shocks to which an argument like that of Mundell (1961) could apply. In order to get some initial intuition about the rigidities implied by sharing a single currency, we first analyze how the economy reacts to shocks when the degree of nominal rigidity in the economy varies. Lower stickiness (in either prices or wages) stabilizes both total debt and labor, and brings consumption closer to the social planner’s solution. Then we fix the degree of nominal rigidity and study the macroeconomic implications of policy rules that adjust the tax rate τ or the public-sector exchange rate X in response to macroeconomic and/or budgetary conditions. We find that if debt is stabilized by varying the tax rate, fluctuations in consumption and labor are amplified by a nontrivial amount. We find that imposing public wage bargaining in a fictitious currency, whose value is determined by a fiscal authority, can stabilize debt while reducing business cycle fluctuations in consumption and labor. In fact, even after-tax public wages fluctuate less in the latter case than they would if debt were stabilized by means of tax instruments.

The remainder of this paper builds a model and uses it to analyze the budgetary and cyclical implications of different fiscal rules. Section 2 defines preferences and technologies and states the social planner’s problem. Section 3 describes a decentralized market equilibrium. Section 4 analyzes how the various inefficiencies of the decentralized equilibrium alter the economy’s response to shocks. Finally, Section 5 studies different fiscal rules that help stabilize the government budget in the market economy, comparing how each rule stabilizes or destabilizes the cyclical fluctuations of several macroeconomic variables. Section 6 concludes.

2 A small open economy facing asymmetric demand shocks

In this section we describe a small open economy that suffers business cycles driven by asymmetric demand shocks. Before considering a decentralized market version of the economy, we study the social planner's solution, as a benchmark for comparison.

Since we wish to analyze the effects of adjusting public sector wages, our model distinguishes private- from public-sector employment. We assume that employment cannot be instantly reallocated from one sector to the other; we therefore model employment in terms of a matching technology. The reason we consider a matching model, instead of a competitive labor market without adjustment frictions, is that it allows us to study an equilibrium in which wages differ (but interact) across sectors.

2.1 Preferences over final goods

There is a unit mass of households in the home country. Their lifetime utility function is

$$E_t \sum_{l=0}^{\infty} \beta^{l-t} \left(\frac{c_t^{1-\eta_0} - 1}{1 - \eta_0} - \chi_n \frac{n_t^{1+\psi_n}}{1 + \psi_n} \right), \quad (1)$$

where $0 < \beta < 1$ is a constant discount factor, c_t denotes consumption, and n_t denotes total labor supply.

Following Galí and Monacelli (2005), the final consumption good is a CES aggregate of home and foreign goods:

$$c_t = \left[(1 - \alpha_f)^{\frac{1}{\eta_1}} (c_t^h)^{\frac{\eta_1-1}{\eta_1}} + \alpha_f^{\frac{1}{\eta_1}} (c_t^f)^{\frac{\eta_1-1}{\eta_1}} \right]^{\frac{\eta_1}{\eta_1-1}}. \quad (2)$$

Home consumption c_t^h is itself a CES aggregate with elasticity of substitution η_2 , across a unit mass of differentiated home retail goods, indexed by $j \in [0, 1]$:

$$c_t^h = \left(\int_0^1 (c_{j,t}^h)^{\frac{\eta_2-1}{\eta_2}} dj \right)^{\frac{\eta_2}{\eta_2-1}}. \quad (3)$$

We assume $\eta_2 > \eta_1$: demand is more elastic across differentiated retail goods than it is between domestic and foreign aggregates.

Foreigners also value the same aggregate of home retail goods that the home households do. Their aggregate purchases of these goods are denoted c_t^x :

$$c_t^x = \left(\int_0^1 (c_{j,t}^x)^{\frac{\eta_2-1}{\eta_2}} dj \right)^{\frac{\eta_2}{\eta_2-1}}. \quad (4)$$

Finally, the government also demands a CES aggregate of differentiated goods, which we will call public goods, to distinguish them from the retail goods consumed by the households. For simplicity, we assume the same functional form and the same elasticity in the public sector as in the retail sector. Thus aggregate government consumption g_t is given by

$$g_t = \left(\int_0^1 (g_{j,t})^{\frac{\eta_2-1}{\eta_2}} dj \right)^{\frac{\eta_2}{\eta_2-1}}, \quad (5)$$

2.2 Production and matching technologies

Differentiated goods are indexed both by sector, $i \in \{h, g\}$, where h indicates the retail sector and g indicates the public sector, and by $j \in [0, 1]$, which refers to a specific good produced in that sector. Production $y_{j,t}^i$ of good j in sector i is given by a trivial linear production function:

$$y_{j,t}^i = k_{j,t}^i, \quad (6)$$

where $k_{j,t}^i$ is the quantity of intermediate inputs used to produce $y_{j,t}^i$.

Intermediate inputs k_t^i are sector-specific. They are produced linearly, using labor as the only input, according to the production function

$$k_t^i = a n_t^i, \quad (7)$$

where a represents labor productivity and n_t^i is labor supplied to sector $i \in \{h, g\}$.

Households can only supply labor by finding appropriate jobs, which occurs through a matching technology. The number of jobs filled in period t is given by the matching function

$$a_m (u_t^s)^{\alpha_u} v_t^{1-\alpha_u}, \quad (8)$$

where v_t is the number of vacant jobs in the economy, and u_t^s is the number of effective job searchers, with $a_m > 0$ and $\alpha_u \in (0, 1)$. The ratio $\theta_t = v_t/u_t^s$ will be called “labor market tightness”.

We assume all searchers have the same probability of finding a job, so this probability is

$$s_t = a_m \theta_t^{1-\alpha_u} \equiv s(\theta_t), \quad (9)$$

Similarly, the probability of a vacancy being filled is

$$q_t = a_m \theta_t^{-\alpha_u} \equiv q(\theta_t), \quad (10)$$

implying $s(\theta_t) = \theta_t q(\theta_t)$.

Matching is random, so a searcher may find a job in either sector. Thus, if we write vacancies in the private and public sectors as v_t^h and v_t^g , respectively, with $v_t^h + v_t^g = v_t$, then a searcher's probability of finding a job in sector $i \in \{h, g\}$ is

$$s_t^i = s(\theta_t) \frac{v_t^i}{v_t}. \quad (11)$$

The dynamics of employment in sector i are

$$n_t^i = (1 - \delta_n) n_{t-1}^i + q(\theta_t) v_t^i, \quad (12)$$

where δ_n is an exogenous match separation rate.

Both the workers unemployed at time $t - 1$, and those who just lost their jobs at the end of $t - 1$, are included in the search pool for time t jobs. That is,

$$u_t^s = 1 - n_{t-1} + \delta_n n_{t-1} = 1 - (1 - \delta_n) n_{t-1}. \quad (13)$$

This timing implies (as in Blanchard and Galí, 2009) that some workers and vacancies find matches immediately, without spending a full period unemployed, so that employment can adjust to shocks immediately along the extensive margin.⁹ Following Thomas (2008), the cost of creating v_t^i vacancies in sector i at time t is¹⁰

$$\frac{\chi_v}{1 + \psi_v} \left(\frac{v_t^i}{n_{t-1}^i} \right)^{\psi_v} v_t^i. \quad (14)$$

This cost is denominated in units of the sector- i intermediate good.

2.3 Aggregate consistency conditions

Total intermediate goods used in sector i must equal the total quantity produced:¹¹

$$\int_0^1 k_{j,t}^i dj = k_t^i = a n_t^i - \frac{\chi_v}{1 + \psi_v} \left(\frac{v_t^i}{n_{t-1}^i} \right)^{\psi_v} v_t^i \quad (15)$$

⁹Given our quarterly calibration, this timing assumption avoids imposing an unrealistic lower bound on the length of an unemployment spell.

¹⁰This cost function is applicable at all scales. Thus, in the decentralized economy, if a firm l in sector i with lagged employment $n_{l,t-1}^i$ creates $v_{l,t}^i$ vacancies, it pays $\frac{\chi_v}{1 + \psi_v} \left(\frac{v_{l,t}^i}{n_{l,t-1}^i} \right)^{\psi_v} v_{l,t}^i$ units of the sector- i intermediate good to do so.

¹¹Eq. (15) assumes vacancy costs are paid by the planner. When we define the market economy we must integrate these costs across decentralized producers of intermediate goods.

Likewise, for each differentiated good j in each sector, the quantity used must equal the quantity produced:

$$c_{j,t}^h + c_{j,t}^x = y_{j,t}^h = k_{j,t}^h, \quad (16)$$

$$g_{j,t} = y_{j,t}^g = k_{j,t}^g, \quad (17)$$

respectively. Also, total labor supplied must equal employment in the two sectors:

$$n_t = n_t^h + n_t^g. \quad (18)$$

Total income and spending need not be equalized in the small open economy, because it can borrow and lend from the rest of the world. Transactions with the rest of the world are denominated in a currency which we will call the euro. P_t^f will represent the price of the imported consumption good c_t^f , and $P_{j,t}^x$ will be the price of exports of retail good j , so the nominal trade balance will equal $\int_0^1 P_{j,t}^x c_{j,t}^x dj - P_t^f c_t^f$. The price of home country bonds emitted in euros at time t will be $\frac{1}{R_t I_t}$, where $R_t - 1$ is the world interest rate, and $I_t - 1$ is a risk premium on home country bonds. Therefore the dynamics of the debt of the small open economy (in euros) are

$$\frac{D_t}{R_t I_t} = D_{t-1} + P_t^f c_t^f - \int_0^1 P_{j,t}^x c_{j,t}^x dj \quad (19)$$

where D_t is the euro face value of debt emitted at t , to be paid off at time $t + 1$.

The risk premium that makes foreign lenders willing to accept home country debt is assumed to vary with the level of debt:¹²

$$I_t = \exp \left(\psi_I \left(\frac{D_t}{PY_{ss}} - \bar{d} \right) \right) \quad (20)$$

Here PY_{ss} represents the steady-state value, in euros, of home country output; $\psi_I > 0$ and $\bar{d}$ are parameters.

2.4 Shock processes

The home country is regarded as a small open economy; the rest of the world is a monetary union, of which the home country is an infinitesimal member. Since our focus is on the home country, we treat the behavior of the rest of the world as exogenous. Thus, the rest of the world determines the real interest rate, which follows an exogenous stochastic process:

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*} \right)^{\rho_R} \exp(\epsilon_t^R), \quad (21)$$

¹²Up to a first-order approximation, this premium is equivalent to that assumed by Schmitt-Grohe and Uribe (2003).

where ϵ_t^R is a mean-zero, normal shock.

Likewise, the foreign price level follows an exogenous stochastic process:

$$\frac{P_t^f}{P^{f*}} = \left(\frac{P_{t-1}^f}{P^{f*}} \right)^{\rho_f} \exp(\epsilon_t^f), \quad (22)$$

where ϵ_t^f is a mean-zero normal shock. Gross exports depend negatively on the price of the local good, as well as an exogenous shifter:

$$c_t^x = \chi_t^x \left(\frac{P_t^x}{P_t^f} \right)^{-\eta_3}, \quad (23)$$

$$\frac{\chi_t^x}{\chi^{x*}} = \left(\frac{\chi_{t-1}^x}{\chi^{x*}} \right)^{\rho_x} \exp(\epsilon_t^x), \quad (24)$$

where ϵ_t^x is a mean-zero normal shock. It is natural to assume $\eta_2 > \eta_3 > \eta_1$, that is, the elasticity of demand for any given firm's output is greater than that for any given country's output, which is greater than that between domestic and foreign aggregates.

Finally, since our paper focuses mainly on the financing of government purchases, rather than their level, we treat government demand as an exogenous process:

$$\frac{g_t}{g^*} = \left(\frac{g_{t-1}}{g^*} \right)^{\rho_g} \exp(\epsilon_t^g), \quad (25)$$

where ϵ_t^g is a mean-zero normal shock.

2.5 Social planner's problem

In order to have an efficient benchmark against which to compare the effects of policy in the decentralized equilibrium, we now consider the problem of a planner who maximizes the welfare (1) of the representative household of the small open economy, taking as given the technology and constraints described in Section 3, including the behavior of the rest of the world.

The planner must respect the exogenous process for government purchases, but can costlessly reassign financial resources across domestic agents (in other words, the planner can levy lump sum taxes). While the planner controls economic activity in the small open economy, it takes as given the behavior of the monetary union (the rest of the world), including the exogenous shock processes governing the interest rate r_t , the foreign price level P_t^f , and the level of demand for domestic exports χ_t^x , as well as foreigners' Dixit-Stiglitz preferences across domestic retail goods. Finally, the planner can borrow and save in international markets in euro-denominated bonds, subject to a risk premium I_t that depends on the real debt-to-output ratio.

In the absence of sticky prices and wages that will affect the decentralized version of this economy, all firms are identical in this model. Therefore the social planner prefers a symmetric solution across all differentiated products. Imposing symmetry, so that $c_{j,t}^h \equiv c_t^h$, $c_{j,t}^x \equiv c_t^x$, and so forth, the social planner's problem is:

$$\max_{D_t, I_t, c_t, c_t^h, c_t^x, P_t^x, v_t^h, v_t^g, n_t^h, n_t^g, \theta_t} E_t \sum_{l=0}^{\infty} \beta^{l-t} \left(\frac{c_t^{1-\eta_0} - 1}{1 - \eta_0} - \chi_n \frac{(n_t^h + n_t^g)^{1+\psi_n}}{1 + \psi_n} \right) \quad (26)$$

$$\text{subject to:} \quad (27)$$

$$\frac{D_t}{R_t I_t} = D_{t-1} + P_t^f c_t^f - P_t^x c_t^x \quad (28)$$

$$c_t = \left[(1 - \alpha_f)^{\frac{1}{\eta_1}} (c_t^h)^{\frac{\eta_1-1}{\eta_1}} + \alpha_f^{\frac{1}{\eta_1}} (c_t^f)^{\frac{\eta_1-1}{\eta_1}} \right]^{\frac{\eta_1}{\eta_1-1}} \quad (29)$$

$$c_t^x = \chi_t^x \left(\frac{P_t^x}{P_t^f} \right)^{-\eta_3} \quad (30)$$

$$c_t^h + c_t^x = a n_t^h - \frac{\chi_v}{1 + \psi_v} \left(\frac{v_t^h}{n_{t-1}^h} \right)^{1+\psi_v} n_{t-1}^h \quad (31)$$

$$g_t = a n_t^g - \frac{\chi_v}{1 + \psi_v} \left(\frac{v_t^g}{n_{t-1}^g} \right)^{1+\psi_v} n_{t-1}^g \quad (32)$$

$$n_t^h = (1 - \delta_n) n_{t-1}^h + a_m \theta_t^{-\alpha_u} v_t^h \quad (33)$$

$$n_t^g = (1 - \delta_n) n_{t-1}^g + a_m \theta_t^{-\alpha_u} v_t^g \quad (34)$$

$$\theta_t = \frac{v_t^h + v_t^g}{1 - (1 - \delta_n)(n_{t-1}^h + n_{t-1}^g)} \quad (35)$$

$$\ln I_t = \psi_I \left(\frac{D_t}{P_{ss} Y_{ss}} - \bar{d} \right) \quad (36)$$

and subject to the four shock processes (21), (22), (24), and (25).

Equation (28) states the budget constraint for the social planner: it describes the evolution of total national debt, in euros, as a function of the trade balance. Note that (30) implies that the social planner acts as if the local economy were a monopolistic competitor—it chooses the price of the exported good, P_t^x , taking into account the world demand curve for those exports. Equations (33)-(35) describe the matching technology that restricts the planner's ability to reassign labor across sectors; equations (31)-(32) describe total production in the private and public sectors, taking into account the linear production technology. Two key implications of the planner's solution are the Euler equations that govern vacancies in sectors $i \in \{h, g\}$:

$$\lambda_t^{ni} = a \lambda_t^{yi} - \chi_n (n_t)^{\psi_n} + \beta E_t \left[\lambda_{t+1}^{yi} \frac{\chi_v \psi_v}{1 + \psi_v} (z_{t+1}^i)^{1+\psi_v} + (1 - \delta_n) (\lambda_{t+1}^{ni} - \lambda_{t+1}^{\theta} \theta_{t+1}) \right] \quad (37)$$

where λ_t^{yi} represents the multipliers on constraints (31)-(32), λ_t^{ni} represents the multipliers on constraints (33)-(34), and λ_t^θ is the multiplier on (35).

Shortly, we will analyze impulse responses of the planner's problem with respect to the shocks in the model. But first, we define a decentralized equilibrium of this economy in the context of a monetary union, so that we can compare the two side by side.

3 The small open economy in a currency union

Now that we have defined an optimal allocation in this economic environment, we next consider how a market economy responds to these shocks. In particular, we assume that the small open economy defined above forms part of a monetary union. Unlike the planner's economy, we assume that the market economy faces nominal rigidities in price and wage adjustment. Since the labor market is subject to search frictions, we will assume that wages are intermittently adjusted through a Nash bargaining process.

3.1 Decision makers in the decentralized economy

In the decentralized economy, *households* choose consumption over time. They supply labor through a random matching technology, and borrow or lend as necessary to finance consumption in response to their fluctuating income.

Retailers purchase intermediate goods, and resell them, monopolistically, as differentiated final goods. One set of retailers acts in the private sector, and another in the public sector.

Intermediate goods producers hire workers through a random matching technology to produce intermediate goods, which they sell in a competitive market. The intermediate good used in the private sector differs from that used in the public sector; one set of producers serves the private sector, while another serves the public sector (we sometimes refer to these public sector producers as "government agencies").

The government demands an aggregate of differentiated final goods. The total quantity demanded is simply treated as an exogenous stochastic process.

Control of one or more parameters of fiscal policy is assumed to be delegated to a *fiscal authority*. The fiscal authority's decision is represented by a rule that determines the delegated parameter as a function of observable macroeconomic data.

Behavior of the *rest of the world* is exogenous from the point of view of the home economy. It affects the home economy by determining interest rates, determining an interest premium on home country debt as a function of the debt level, supplying an import good, and demanding the home export good.

3.2 Households

Households maximize the utility function (1) subject to the following period budget constraint:

$$\frac{D_t^h}{R_t I_t^h P_t^c} = \frac{D_{t-1}^h}{P_t^c} + c_t - (1 - \tau_t) \left[\int_0^1 w_{l,t}^h n_{l,t}^h dl + \int_0^1 w_{k,t}^g n_{k,t}^g dk \right] - (1 - n_t) b_t - \text{Div}_t. \quad (38)$$

Here D_t^h is nominal household debt emitted in period t , and P_t^c is the consumer price index. $R_t I_t^h$ is the gross nominal interest rate, where R_t is the world interest rate, and I_t^h is a risk premium. The household owns the firms, receiving a dividend payment Div_t . The household pays a flat tax rate τ_t on its labor income, which is derived from employment at a continuum of private firms l and a continuum of government agencies k . Employment of household members at firm l is $n_{l,t}^h$, and these workers earn real wage $w_{l,t}^h$; the employment and real wage at agency k are $n_{k,t}^g$ and $w_{k,t}^g$. Unemployed members of the household receive a subsidy b_t ; $n_t = \int_0^1 n_{l,t}^h dl + \int_0^1 n_{k,t}^g dk = n_t^h + n_t^g$ is the fraction of household members employed.

Notice that this budget constraint implies that household members insure one another by sharing labor income between the employed and unemployed. The household does not choose employment directly; instead, it is determined in equilibrium by a process of matching and bargaining.

Since the household knows that the interest premium rises with debt, its consumption Euler equation is

$$\left(1 - \frac{D_t^h (I_t^h)'(D_t^h)}{I_t^h} \right) c_t^{-\eta_0} = R_t I_t^h P_t^c \beta E_t \frac{c_{t+1}^{-\eta_0}}{P_{t+1}^c}, \quad (39)$$

where $(I_t^h)'(D_t^h)$ is the derivative of I_t^h with respect to D_t^h . The household allocates expenditure across imported goods c_t^f and the home aggregate c_t^h to minimize the cost of attaining its total aggregate consumption c_t , given its aggregation preferences (2). The resulting demand functions are:

$$c_t^h = (1 - \alpha_f) \left(\frac{P_t^h}{P_t^c} \right)^{-\eta_1} c_t, \quad c_t^f = \alpha_f \left(\frac{P_t^f}{P_t^c} \right)^{-\eta_1} c_t, \quad (40)$$

Here P_t^h and P_t^f are price aggregates for domestic and foreign goods, and the domestic consumer price index P_t^c is defined as

$$P_t^c = \left[(1 - \alpha_f) (P_t^h)^{1-\eta_1} + \alpha_f (P_t^f)^{1-\eta_1} \right]^{\frac{1}{1-\eta_1}}. \quad (41)$$

Likewise, its preferences (3) across differentiated retail goods imply that the demand function for good j is:

$$c_{j,t}^h = \left(\frac{P_{j,t}^h}{P_t^h} \right)^{-\eta_2} c_t^h, \quad (42)$$

where $P_{j,t}^h$ denotes the price of the intermediate good j , and P_t^h is a price index for domestically-produced private-sector goods: i

$$P_t^h = \left(\int_0^1 (P_{j,t}^h)^{\eta_2-1} dj \right)^{\frac{1}{\eta_2-1}}. \quad (43)$$

3.3 Final goods producers

Allocation of demand across differentiated products by foreigners and the government is analogous to (42). Therefore, the private retailers and government agencies that produce the differentiated goods j and k , respectively, act as monopolistic competitors facing the following demand curves:

$$Y_{j,t}^h = \left(\frac{P_{j,t}^h}{P_t^h} \right)^{-\eta_2} Y_t^h, \quad \text{where } Y_t^h = c_t^h + c_t^x, \quad (44)$$

$$Y_{k,t}^g = \left(\frac{P_{k,t}^g}{P_t^g} \right)^{-\eta_2} g_t, \quad \text{where } Y_t^g \equiv g_t. \quad (45)$$

Each of these final producers, in sectors $i \in \{h, g\}$, operates a linear production function that depends on a homogeneous intermediate input, purchased at nominal price $P_t^c \phi_t^i$. As in Calvo (1983), prices are “sticky” for a random number of periods. Each period, a firm gets the chance to adjust its price with probability $1 - \xi_p$, or with probability ξ_p its price remains constant: $P_{j,t}^i = P_{j,t-1}^i$. If it gets to reset its price in period t , it sets its new price $\tilde{P}_{j,t}^i$ to maximize expected discounted profits over the periods in which this price remains fixed. Thus its decision problem is:

$$\max_{\tilde{P}_{j,t}^i} E_t \sum_{s=0}^{\infty} \beta_{t,t+s} (\xi_p)^s \left(\tilde{P}_{j,t}^i - P_{t+s}^c \phi_{t+s}^i \right) \left(\frac{\tilde{P}_{j,t}^i}{P_{t+s}^i} \right)^{-\eta_2} Y_{t+s}^i \quad (46)$$

where $\beta_{t,t+s} = \beta \left(\frac{c_{t+s}}{c_t} \right)^{-\eta_0}$ is the household’s stochastic discount factor for aggregate consumption. This implies the following price setting equation:

$$\frac{\eta_2 - 1}{\eta_2} \tilde{P}_{j,t}^i E_t \sum_{s=0}^{\infty} (\beta \xi_p)^s \frac{c_{t+s}^{-\eta_0}}{c_t^{-\eta_0}} \left(\frac{\tilde{P}_{j,t}^i}{P_{t+s}^i} \right)^{-\eta_2} Y_{t+s}^i = E_t \sum_{s=0}^{\infty} (\beta \xi_p)^s \frac{c_{t+s}^{-\eta_0}}{c_t^{-\eta_0}} \left(\frac{\tilde{P}_{j,t}^i}{P_{t+s}^i} \right)^{-\eta_2} P_{t+s}^c \phi_{t+s}^i Y_{t+s}^i, \quad (47)$$

from which it is clear that all firms which reset their price in period t set it at the same level ($\tilde{P}_{j,t}^i = \tilde{P}_t^i$, for all $j \in (0, 1)$).

Given probability ξ_p of updating in any given period, we average over all contracts to express the dynamics of the aggregate price level as follows:

$$P_t^i = \left((1 - \xi_p) \left(\tilde{P}_t^i \right)^{\eta_2 - 1} + \xi_p \left(P_{t-1}^i \right)^{\eta_2 - 1} \right)^{\frac{1}{\eta_2 - 1}}. \quad (48)$$

Combining (47) and (48) and log-linearizing around a zero inflation steady state yields the New Keynesian Phillips curve for sector i :

$$\hat{\pi}_t^i = \beta E_t \hat{\pi}_{t+1}^i + \frac{(1 - \xi_p)(1 - \beta \xi_p)}{\xi_p} (\hat{\phi}_t^i + \hat{P}_t^c - \hat{P}_t^i). \quad (49)$$

Also, given prices and quantities in the two sectors, we can define nominal GDP as

$$P_t Y_t = P_t^h (c_t^h + c_t^x) + P_t^g g_t$$

where P_t is the GDP deflator, and Y_t is real GDP. Note that in what follows, we never need to solve for the GDP deflator and real GDP separately to solve the model; only nominal GDP enters the analysis.

3.4 Intermediate goods producers

In the private sector, a continuum of intermediate goods firms, indexed by l , sell their output competitively to the retailers at price $P_t^c \phi_t^h$. For simplicity, we make an analogous assumption for the public sector: a continuum of government agencies produce intermediate goods for the government, valuing their production at the price $P_t^c \phi_t^g$. Each of these producers operates in only one sector (private or public). Without loss of generality, we set the mass of producers in each sector to one.¹³

In both sectors ($i \in \{h, g\}$), intermediate goods are produced according to a linear production function with labor as the only input:

$$y_{l,t}^i = a n_{l,t}^i, \quad (50)$$

where $n_{l,t}^i$ denotes the number of members of the household employed by producer l in sector i ; and a is the constant level of productivity. We assume that each producer

¹³The reason this implies no loss of generality is that we assume a technology with constant returns to scale. Thus the model determines the total amount produced in each sector, but has nothing to say about how it is distributed across producers.

l posts vacancies to hire workers, and bargains with workers to determine the wage. Posting vacancies entails the costs (14), scaled to the level of employment in the firm:

$$\frac{\chi_v}{1 + \psi_v} \left(\frac{v_{l,t}^i}{n_{l,t-1}^i} \right)^{1+\psi_v} n_{l,t-1}^i, \quad (51)$$

in units of the intermediate good.

At time t , the firm takes as given its previous employment stock $n_{l,t-1}^i$, and chooses $v_{l,t}^i$ to maximize the present discounted value of current and future profits. In doing so, it takes as given its nominal wage, which is constrained by a Calvo friction, intermittently updated by a Nash bargain that is set in nominal terms. If we write the value of a match with an individual worker as $\mathcal{J}_t^i(W_{l,t}^i)$, then optimal hiring implies:

$$\chi_v \left(\frac{v_{l,t}^i}{n_{l,t-1}^i} \right)^{\psi_v} \phi_t^i = q_t \mathcal{J}_t^i(W_{l,t}^i). \quad (52)$$

3.5 Wage bargaining

Following Gertler, Sala, and Trigari (2008), we assume that wages are subject to Calvo stickiness at the firm level. That is, all workers in a given firm, including new hires, are paid that firm's previous period's wage with a probability ξ_w , while with probability $1 - \xi_w$ the firm is free to renegotiate nominal wages with its workforce.¹⁴ Those firms adjusting to the new nominal wage will in equilibrium set the same wage, W_t^{i*} , given that we have no firm-specific shocks other than the Calvo adjustment process itself.

To model our proposed mechanism for fiscal adjustments on the expenditure side, we allow for the possibility that public and private wages are bargained in different units of account. Therefore, we define X_t^i as the exchange rate between the euro and the bargaining currency. For private firms, $X_t^h \equiv 1$ — meaning simply that private-sector wages are set in euros. For public firms, X_t^g represents euros per CSU, which will vary over time depending on the decisions of the fiscal authority.

Written in the bargaining currency, the law of motion of the average nominal wage is given by:

$$W_t^i = \int_0^1 W_{l,t}^i dl = \xi_w W_{t-1}^i + (1 - \xi_w) W_t^{i*}, \quad (53)$$

since fraction $1 - \xi_w$ of firms adjust wages each period. But to fully describe wage dynamics, we must also find an equation for the reset wage W_t^{i*} . To do so, we calculate match surplus for workers and firms, and then derive a Nash bargaining equation.

¹⁴We focus on the case of equal Calvo parameters in the public and private sectors. This assumption is not crucial for our results.

Given the probabilities of labor market transitions, the value in consumption units of a worker with a job in sector i is:

$$\begin{aligned} \mathcal{W}_{l,t}^i = & (1 - \tau_t) \frac{X_t^i W_{l,t}^i}{P_t^c} - \chi_n n_t^{\psi_n} c_t^{\eta_0} + E_t \beta_{t,t+1} \left[\delta_n (1 - s(\theta_{t+1})) \mathcal{U}_{t+1} + (1 - \delta_n) \mathcal{W}_{l,t+1}^i \right. \\ & \left. + \delta_n s(\theta_{t+1}) \int_0^1 \frac{v_{L,t+1}^h \mathcal{W}_{L,t+1}^h + v_{L,t+1}^g \mathcal{W}_{L,t+1}^g}{v_{t+1}} dL \right]. \end{aligned}$$

On the right-hand side, we see that a worker employed at time t stays in the same producer at time $t+1$ with probability $1 - \delta_n$, and becomes unemployed with probability $\delta_n (1 - s(\theta_{t+1}))$. With probability $\delta_n s(\theta_{t+1})$, the worker finds a job in a new producer, which may either be private (h) or public (g). Here $\mathcal{U}$ represents the value of an unemployed worker, which satisfies:

$$\mathcal{U}_t = b_t + E_t \beta_{t,t+1} \left[(1 - s(\theta_{t+1})) \mathcal{U}_{t+1} + s(\theta_{t+1}) \int_0^1 \frac{v_{L,t+1}^h \mathcal{W}_{L,t+1}^h + v_{L,t+1}^g \mathcal{W}_{L,t+1}^g}{v_{t+1}} dL \right]. \quad (54)$$

Subtracting these two equations, the household's surplus, $\mathcal{H}_{l,t}^i \equiv \mathcal{W}_{l,t}^i - \mathcal{U}_t$ will satisfy:

$$\begin{aligned} \mathcal{H}_{l,t}^i = & (1 - \tau_t) \frac{X_t^i W_{l,t}^i}{P_t^c} - b_t - \chi_n n_t^{\psi_n} c_t^{\eta_0} + (1 - \delta_n) E_t \beta_{t,t+1} \mathcal{H}_{l,t+1}^i \\ & - E_t \beta_{t,t+1} (1 - \delta_n) s(\theta_{t+1}) \int_0^1 \frac{v_{L,t+1}^h \mathcal{H}_{L,t+1}^h + v_{L,t+1}^g \mathcal{H}_{L,t+1}^g}{v_{t+1}} dL. \end{aligned} \quad (55)$$

Here $s(\theta_{t+1}) \frac{v_{L,t+1}^i}{v_{t+1}}$ denotes the probability of becoming matched with producer L in sector i at time $t+1$.

A similar Bellman equation governs the producer's surplus, $\mathcal{J}_{l,t}^i$.¹⁵

$$\mathcal{J}_{l,t}^i = a \phi_t^i - \frac{X_t^i W_{l,t}^i}{P_t^c} + E_t \beta_{t,t+1} \max_z \left[(1 - \delta_n + q_{t+1} z) \mathcal{J}_{l,t+1}^i - \frac{\chi_v}{1 + \psi_v} z^{1+\psi_v} \phi_{t+1}^i \right] \quad (56)$$

or equivalently

$$\mathcal{J}_{l,t}^i = a \phi_t^i - \frac{X_t^i W_{l,t}^i}{P_t^c} + E_t \beta_{t,t+1} \left[(1 - \delta_n) + \frac{\psi_v}{1 + \psi_v} z_{l,t+1}^i q_{t+1} \right] \mathcal{J}_{l,t+1}^i. \quad (57)$$

In each period, the wage is renegotiated, in nominal terms, with probability $1 - \xi_w$. Renegotiation is assumed to solve a Nash bargaining problem, with bargaining power σ for the worker. The first-order condition is

$$\sigma \mathcal{J}_t^{i*} \frac{\partial \mathcal{H}_t^{i*}}{\partial W_t^{i*}} = (1 - \sigma) \mathcal{H}_t^{i*} \left| \frac{\partial \mathcal{J}_t^{i*}}{\partial W_t^{i*}} \right| \quad (58)$$

¹⁵As is standard in matching models, here we have written the producer's surplus per match *after* hiring at time t .

The notation reflects the fact that any firm in sector i that renegotiates at time t sets the same nominal wage W_t^{i*} .

Note that as Gertler, Sala, and Trigari pointed out, a marginal change in the nominal wage is more valuable to the firm than it is to the worker, because it is applicable not only to the current workforce, but also to new employees hired prior to the next wage adjustment. Moreover, the worker's bargaining share is also lowered by the factor $(1 - \tau_t)$, reflecting the fact that given proportional taxes, the worker receives less benefit from one euro of wages than this euro costs to the firm. Using Bellman equations (55) and (57), we can calculate the marginal value of the sticky wage to each bargaining party as follows:

$$\frac{\partial \mathcal{H}_{l,t}^i}{\partial W} = (1 - \tau_t) \frac{X_t^i}{P_t^c} + (1 - \delta_n) \xi_w E_t \beta_{t,t+1} \frac{\partial \mathcal{H}_{l,t+1}^i}{\partial W}, \quad (59)$$

$$\left| \frac{\partial \mathcal{J}_{l,t}^i}{\partial W} \right| = \frac{X_t^i}{P_t^c} + \xi_w E_t \beta_{t,t+1} (1 - \delta_n + q_{t+1} z_{l,t+1}^i) \left| \frac{\partial \mathcal{J}_{l,t+1}^i}{\partial W} \right|. \quad (60)$$

Solving a matching model with sticky wages is complicated by the fact that the surpluses of workers and firms differ across matches— we need to evaluate the surpluses as a function of the current wage. In the appendix, we explain how all surpluses can be approximated as linear functions of the current wage, and how we can then rewrite the dynamics in terms of average wages (and average surplus functions) only.

3.6 Monetary and fiscal policy

Three public authorities determine monetary and fiscal policy in the home economy: the central bank of the monetary union, the home country government, and the home country fiscal authority.

3.6.1 The central bank

The nominal interest rate is determined by the central bank of the monetary union, and is therefore exogenous from the point of view of the small open economy under consideration. It follows the stochastic process (21), as described earlier.

3.6.2 The government

We assume the home-country government undertakes a number of complex economic tasks, including various forms of public expenditure, and designing a tax code to finance this expenditure. However, the content of these choices is not crucial for our

argument, and will not be modeled. Therefore, for simplicity we just treat real government consumption expenditure as an exogenous stochastic process, given by (25), and we will treat all taxation as if it were just a flat labor income tax.

Finally, for realism, we assume the government makes transfer payments to the unemployed. The benefit level b_t is set to a constant fraction $\underline{b}$ of the public sector wage. Given these budget items, the nominal government debt, in euros, evolves as

$$\frac{D_t^g}{R_t I_t^g} = D_{t-1}^g + P_t^g g_t - \tau_t (W_t^h n_t^h + X_t W_t^g n_t^g) + (1 - n_t) X_t W_t^g \underline{b}. \quad (61)$$

Here D_{t-1}^g is the government's debt, in euros, at the beginning of t , and I_t^g is a risk premium on government debt; $D_t^g/(R_t I_t^g)$ is the amount of new euro debt that must be issued at t to roll over the existing debt and to finance the deficit. $P_t^g g_t$ is government spending, in euros. $W_t^h n_t^h h_t^h$ is the labor income of private sector workers, in euros, and $X_t W_t^g n_t^g h_t^g$ is the labor income of public sector workers, in euros. Here the private sector wage W_t^h is denominated directly in euros, whereas the public sector wage W_t^g is denominated in CSU, so it must be multiplied by the “exchange rate” X_t to convert to euros. The last term in the budget constraint represents the payment of the real benefit $b_t = X_t W_t^g \underline{b}/P_t^c$ to the unemployed.

3.6.3 The fiscal authority

Whereas the government is assumed to make some complex but unmodeled decisions, the fiscal authority is assumed to make at most two simple decisions: adjusting a parameter that shifts tax rates, and/or adjusting a parameter that affects the value of public salaries and transfer payments.

We model these two decisions as rules that depend on observable macroeconomic data. The tax rule determines an additive shift of all labor income taxes as a function of deviations of nominal GDP from its trend level, nominal public spending from its trend level, and public debt, as a fraction of GDP, from a target level:

$$\tau_t - \tau^* = \rho^\tau (\tau_{t-1} - \tau^*) + (1 - \rho^\tau) \left[\tau^y \left(\frac{P_t Y_t - P_{ss} Y_{ss}}{P_{ss} Y_{ss}} \right) + \tau^g \left(\frac{P_t^g g_t - P_{ss}^g g_{ss}}{P_{ss} Y_{ss}} \right) + \tau^d \left(\frac{D_t^g}{P_{ss} Y_{ss}} - d^* \right) \right] \quad (62)$$

Formally, in this expression, τ^* represents a steady-state flat tax rate. However, it should be interpreted as a stand-in for the whole tax code. That is, the deviations from τ^* determined by the rule (62) should be interpreted as additive shifts of the whole tax code chosen by the government.

Second, the fiscal authority determines each period's exchange rate X_t between the public sector and private sector numeraire (the price of one CSU, in euros). We model

this decision as a rule that reacts to deviations of nominal GDP and nominal public spending from their trend levels.¹⁶

$$\frac{X_t}{X^*} = \left(\frac{X_{t-1}}{X^*} \right)^{\rho^x} \left[\exp \left(\zeta^y \left(\frac{P_{t-1}Y_{t-1} - P_{ss}Y_{ss}}{P_{ss}Y_{ss}} \right) \right) \exp \left(\zeta^g \left(\frac{P_t^g g_t - P_{ss}^g g_{ss}}{P_{ss}Y_{ss}} \right) \right) \right]^{1-\rho^x} \quad (63)$$

Note that the exchange rate is determined as a function of *lagged* nominal output rather than current nominal output; this helps avoid indeterminacy.

3.7 Market clearing conditions

Several market clearing conditions that were previously stated in the context of the planner's problem must be restated here to properly aggregate the decisions of decentralized decision-makers.

Aggregating vacancy costs at the firm level, the market clearing conditions for the private and public sector are:

$$c_t^h + c_t^x = an_t^h - \int_0^1 \frac{\chi_v}{1 + \psi_v} (z_{l,t}^h)^{1+\psi_v} n_{l,t-1}^h dl \quad (64)$$

$$g_t = an_t^g - \int_0^1 \left[\frac{\chi_v}{1 + \psi_v} (z_{l,t}^g)^{1+\psi_v} n_{l,t-1}^g \right] dl \quad (65)$$

As with our treatment of wages, aggregation of these heterogeneous hiring expenditures can be simplified by linearizing around a zero-inflation steady state.

In the decentralized market, market willingness to hold the debt of public and private decision makers will depend on the current debt level of each borrower. Thus we assume the following specification for the risk premia I_t^h and I_t^g :

$$\ln I_t^i = \psi_I^i \left(\frac{D_t^i}{PY_{ss}} - \bar{d}^i \right) \quad (66)$$

in sectors $i \in \{h, g\}$. For consistency with the risk premium on the aggregate debt of the planner we assume $\bar{d}^g = \alpha_g \bar{d}$, $\bar{d}^h = (1 - \alpha_g) \bar{d}$, $\psi_I^g = \psi_I / \alpha_g$, and $\psi_I^h = \psi_I / (1 - \alpha_g)$, where α_g represents the fraction of debt capacity attributed to the government.

¹⁶This functional form is largely analogous to that used for the tax rate, except that it is multiplicative rather than additive. The multiplicative nature of the exchange rate rule ensures that X_t is always a positive number, whereas the additive tax rule allows us to consider possibilities such as positive and negative tax rates, or an efficient steady-state tax rate $\tau^* = 0$.

3.8 Steady state

In steady state, the relation between employment and tightness is

$$\theta_{ss} = \left(\frac{\delta_n n_{ss}}{a_m(1 - (1 - \delta_n)n_{ss})} \right)^{\frac{1}{1-\alpha_u}} \quad (67)$$

Given the steady-state flow relation $\delta_n n_{ss} = q(\theta_{ss})v_{ss}$, we see that the vacancy-to-employment ratio z is equalized across sectors in steady state:

$$z_{ss}^i = \delta_n / q(\theta_{ss}), \quad i \in \{h, g\}. \quad (68)$$

Furthermore, in each sector, the real marginal cost ϕ_{ss}^i , the real wage $X_{ss}^i W_{ss}^i / P_{ss}^c$, and the surpluses $\mathcal{J}_{ss}^i$ and $\mathcal{H}_{ss}^i$ are determined by linear equations: the zero profit condition on vacancies, the Nash bargaining equation, and the Bellman equations for the surpluses:

$$\chi_v z^{\psi_v} \phi_{ss}^i = q(\theta_{ss}) \mathcal{J}_{ss}^i, \quad (69)$$

$$(1 - \beta \xi_w)(1 - \tau_{ss}) \sigma \mathcal{J}_{ss}^i = (1 - \beta \xi_w(1 - \delta_n))(1 - \sigma) \mathcal{H}_{ss}^i, \quad (70)$$

$$(1 - \beta(1 - \delta_n)) \mathcal{H}_{ss}^i = (1 - \tau_{ss}) X_{ss}^i W_{ss}^i / P_{ss}^c - \underline{w}_{ss}, \quad (71)$$

$$(1 - \beta(1 - \delta_n / (1 + \psi_v))) \mathcal{J}_{ss}^i = a \phi_{ss}^i - X_{ss}^i W_{ss}^i / P_{ss}^c. \quad (72)$$

While these equations are sector-specific, note that no other sector-specific variables appear in them. Therefore the steady-state solutions for these four variables must be equalized across sectors: $\phi_{ss}^h = \phi_{ss}^g$, $X_{ss}^h W_{ss}^h / P_{ss}^c = X_{ss}^g W_{ss}^g / P_{ss}^c$, $\mathcal{J}_{ss}^h = \mathcal{J}_{ss}^g$, and $\mathcal{H}_{ss}^h = \mathcal{H}_{ss}^g$. Analogous symmetry holds in the social planner's solution too.

This has an important consequence for our proposed fiscal mechanism. The fact that real wages are equalized across sectors in steady state, regardless of the value X^g of the CSU, implies that *changes in the value of the CSU are neutral in the long run* in this model. Thus, any policy effects obtained from setting the value of the CSU (a purely nominal change) will be only short run effects. This is why we consider a CSU rule that depends only on short-run fluctuations in output and spending, with responses to the slow-moving fluctuations in public debt limited to the tax side.¹⁷

¹⁷Actually, our discussion of the CSU framework in Section 1.1 attempts to create a nominal anchor by defining the CSU in terms of a specific quantity of public sector labor. If this anchor is sufficiently strong, the CSU will not be neutral in the long run, and it could be useful to include a term in the exchange rate rule that devalues the CSU when debt increases. However, the anchor might eventually fail if those civil servants whose base salary is defined as one CSU manage to renegotiate their contracts to place less emphasis on base salary and more emphasis on other forms of compensation. Therefore we ignore this possible anchor in our model, and show that the CSU mechanism can strongly enhance stability of the budget and the business cycle even without an explicit nominal anchor.

4 Comparing planner and market solutions

Comparing the first-order conditions of the planner's problem in any t with the equations that govern the market economy shows five possible sources of inefficiency in the market equilibrium:

1. existence of labor market inefficiency wedges such as distorting taxes or unemployment subsidies;
2. possible violation of Hosios' (1990) condition for efficient bargaining in a matching economy;
3. monopolistic competition in final goods production;
4. failure to exploit monopoly power in supply of home country exports;
5. nominal rigidities in price and wage setting.

Several of the effects listed above can be seen by adding the workers' and firms' surplus equations to obtain an equation for total match surplus in the decentralized economy. Defining $\mathcal{S}_{l,t}^i \equiv \mathcal{H}_{l,t}^i + \mathcal{J}_{l,t}^i$, we can show that the dynamics of total match surplus are given by

$$\begin{aligned} \mathcal{S}_{l,t}^i &= a\phi_t^i - \tau_t \frac{X_t^i W_{l,t}^i}{P_t^c} - b_t - \chi_n n_t^{\psi_n} c_t^{\eta_0} + E_t \beta_{t,t+1} \frac{\psi_v \chi_v}{1 + \psi_v} z^{1+\psi_v} \phi_{t+1}^i \\ &\quad + (1 - \delta_n) E_t \beta_{t,t+1} \left[\mathcal{S}_{l,t+1}^i - s(\theta_{t+1}) \sigma \int_0^1 \frac{v_{L,t+1}^h \mathcal{S}_{L,t+1}^h + v_{L,t+1}^g \mathcal{S}_{L,t+1}^g}{v_{t+1}} dL \right]. \end{aligned} \quad (73)$$

This equation can be compared to the planner's Euler equation (37) if we interpret the match value $\mathcal{S}_{l,t}^i$ as a quantity analogous to the planner's marginal value of employment λ_t^{ni} , and marginal cost ϕ_t^i as analogous to planner's marginal value of intermediate goods λ_t^{yi} .¹⁸

We first notice that the planner's Euler equation contains no terms analogous to the labor tax and unemployment insurance terms that appear in (73). Thus benefits and labor income taxes should be zero in order to make (37) and (73) equivalent. In turn, this requires some other source of tax revenues to finance government expenditure; to avoid introducing other distortions in the model, these would have to be lump sum taxes. Next, the last term in (73) is the worker's lost search gains upon accepting

¹⁸To compare the equations, note that (37) is written in utility units, whereas (73) is written in units of consumption goods.

a match; the last term in (37) is analogous, and can be shown to be algebraically equivalent if the elasticity of the matching function with respect to search, α_u , equals workers' share of match surplus, σ . This is Hosios' (1990) condition for efficiency in matching models.

A third source of inefficiency in the decentralized economy is the fact that the monopolistic producers of differentiated final goods mark them up relative to marginal cost. In steady state, the markup is $\frac{\eta_2}{\eta_2-1}$ (that is, the steady state of the price-setting equation (47) reduces to $P_j^i = \frac{\eta_2}{\eta_2-1} P^c \phi^i$.) This inefficiency could be eliminated by subsidizing the production of differentiated goods at rate $1/\eta_2$. Of course, this remedy is unlikely to be very beneficial unless it can be financed through nondistortionary taxes.

Another monopolistic incentive arises from the fact that we have defined the planner as an agent that represents the well-being of the home economy only. Therefore, the planner has an incentive to restrict trade to exploit the home economy's market power as a monopolistic producer of its export good. Tariff policy could be used to achieve this objective. However, this possibility is not very relevant in the context of this paper, since we are modeling one member of a monetary union which also functions as a free trade area. Therefore we will ignore this difference between the market and planner solutions from here on.

Finally, the market equilibrium differs from the planner's solution because the planner, by assumption, is not subject to any nominal rigidities. We will see that price and wage stickiness and the inefficiency wedges in the value of a job both imply large differences between the planner and market impulse responses.

4.1 Quantifying the inefficiencies in the decentralized economy

To have some idea of the quantitative importance of the various inefficiencies in the market economy, we now provide a rough calibration of the model and calculate impulse response functions comparing the planner's solution and the benchmark decentralized economy to intermediate cases in which some inefficiencies are eliminated.

The parameterization is stated in Table 1. The period is quarterly. We set the worker's bargaining power equal to the unemployment elasticity of matching, thus satisfying Hosios' condition and thereby eliminating one possible source of inefficiency in the model. Vacancy cost parameters are calibrated so that vacancy costs amount to one percent of output, a standard calibration. The separation rate is set at $\delta_n = 0.07$ per quarter, which is high, but is intended to capture the high share of temporary

workers in the Spanish economy. Wage and price persistence are set to 0.9 and 0.6, respectively, which are unlikely to be overestimates, given the high nominal rigidity that exists in Spain. The size of government and the replacement ratio are both set lower than the corresponding figures for Spain, so the overall tax wedge τ^* is also low; we choose these conservative figures because our model becomes somewhat unstable if we raise them further.

Consumption openness is set to 25%, which roughly corresponds to the average of Spanish imports and exports in recent years. Elasticities of substitution are similar to those used in Galí and Monacelli (2005). The debt elasticity of the risk premium is a rounding-off of the one assumed in Schmitt-Grohe and Uribe (2003). The level of debt $\bar{d}$ at which the risk premium factor begins to exceed one is set to 60% of GDP; we attribute half of this debt to the private sector and half to the public sector, so effectively we are assuming that the risk premium kicks in when public debt reaches half of the Maastricht limit. The parameterization of the fiscal authority's rules, which for the benchmark economy sets $\tau^d = 0.01$ and all other fiscal parameters to zero, will be discussed in detail in the next section.

As a first illustration of the impact of the inefficiencies of the market economy, we solve the benchmark market equilibrium and the planner's problem and we compare them to an economy identical to the market benchmark except that all taxes are lump sum. There are large differences in steady state employment: the planner's solution has a 97.6% employment rate, whereas the lump-sum economy has a 96.5% employment rate, and the market benchmark has an 88.7% employment rate. The tax rule in the benchmark case is calculated around a base rate of $\tau^* = 30.95\%$.

Figure 7 reports impulse responses for these three economies with respect to an export demand shock χ^x .¹⁹ One notable feature of the impulse responses is that private- and public-sector employment are virtually unchanged by the shock under the planner's solution (seen as a thick dashed line). This results from our matching technology, in which vacancies can be filled immediately, and vacancy costs are paid in units of the intermediate goods firms' output. Blanchard and Galí (2009) show that under this technology, the optimal response to a technology shock is for labor to stay fixed. This result carries over approximately in our more complex model to the case of export demand shocks.

In contrast, in the market benchmark economy (thick line), export demand shocks cause large procyclical swings in private sector employment. While the planner's solution, with unchanged employment, implies that home consumption falls to accomodate

¹⁹The time unit is quarterly in these impulse responses and all other dynamic results in the paper.

Table 1: *Benchmark parameterization (quarterly)*

Parameter	Initial value
<i>Preference parameters</i>	
Elasticities	$\eta_0 = 2, \eta_1 = 2, \eta_2 = 8, \eta_3 = 3$
Consumption openness	$\alpha_f = 0.25$
Labor supply	$\chi_n = 0.7, \psi_n = 1$
<i>Output and matching technologies</i>	
Labor productivity	$a = 1$
Vacancy costs	χ_v calibrated: vacancy costs are 1% of output; $\psi_v = 1$
Matching function	a_m calibrated: so that $q_{ss}=1$; $\alpha_u = 0.5$
Separation rate	$\delta_n = 0.08$
<i>Price adjustment parameters</i>	
Worker bargaining share	$\sigma = \alpha_u$
Adjustment probabilities	$1 - \xi_p = 0.4; 1 - \xi_w = 0.1$
<i>Debt demand parameters</i>	
Level parameters:	$\bar{d} = 2.4, \alpha_g = 0.5$
Debt elasticity of risk premium	$\psi_I = 0.001$
<i>Policy parameters</i>	
Replacement rate	$\underline{b} = 0.35$
Level parameters:	$d^* = \bar{d}, X^* = 1$
Level parameter:	τ^* calibrated: so that $D_{ss}^g/P_{ss}Y_{ss} = \bar{d}$
Elasticities of tax rate	$\tau^y = 0$ (varies), $\tau^g = 0$ (varies), $\tau^d = 0.01$ (varies)
Elasticities of public salaries	$\zeta^g = 0$ (varies), $\zeta^y = 0$ (varies)
Policy persistence parameters	$\rho^x = 0, \rho^\tau = 0$
<i>Shock processes</i>	
	<i>Steady state Persistence Std dev of innovation</i>
Government spending	$g_{ss} = 0.2 \quad \rho_g = 0.95 \quad \sigma_g = 0.01$
Foreign price level	$P_{ss}^f = 1 \quad \rho_{pf} = 0.95 \quad \sigma_{pf} = 0.01$
Export demand level	$\chi_{ss}^x = \alpha_f \quad \rho_{cx} = 0.95 \quad \sigma_{cx} = 0.01$
Interest rate (quarterly)	$r_{ss} = 0.003 \quad \rho_r = 0.95 \quad \sigma_r = 0.001$

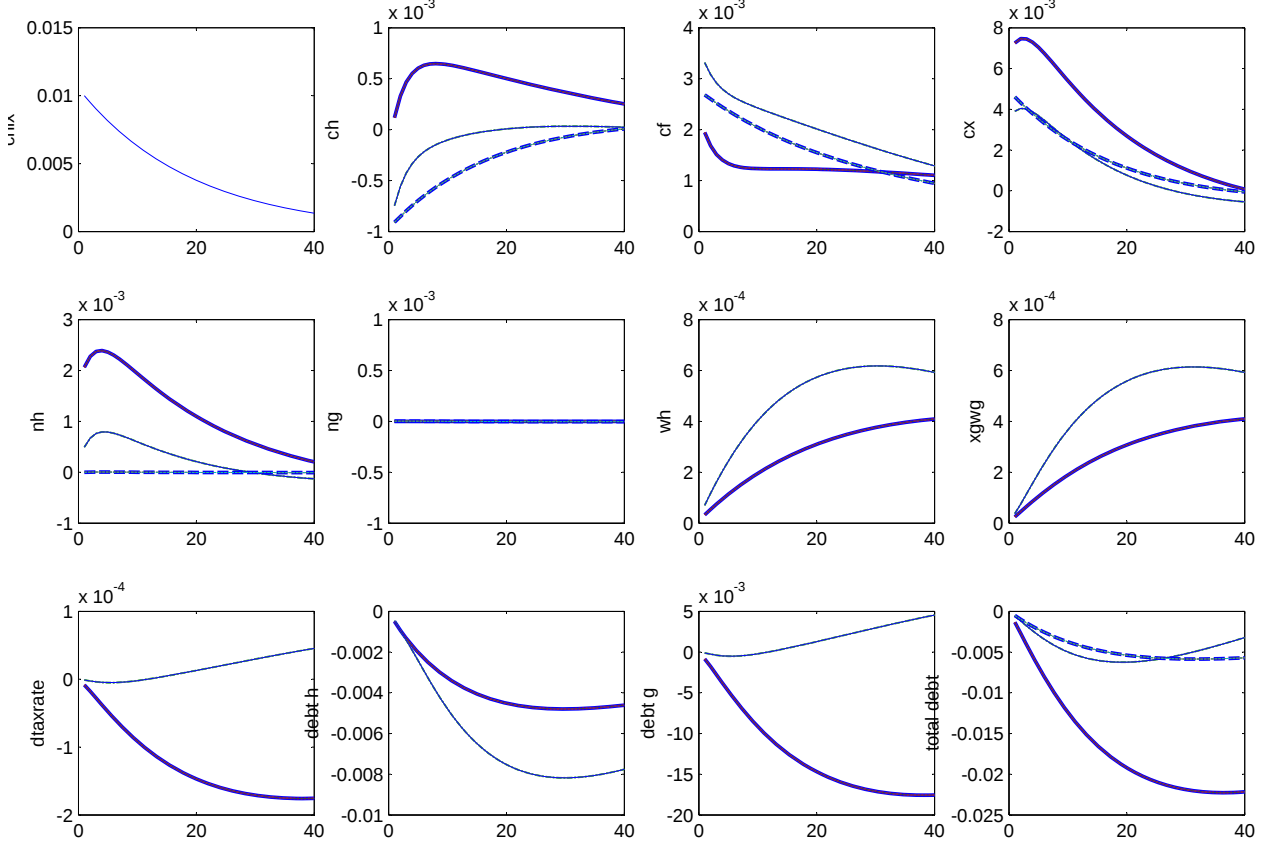
increased exports, employment in the market economy expands so much that home consumption *rises*. We also note that the trade balance fluctuates excessively in response to the shock, as exports c^x rise too much but imports c^f do not rise enough.

Replacing distorting labor income taxes by lump sum taxes (thin line) eliminates much of these inefficient fluctuations. The rise in n^h is sufficiently reduced so that home consumption now falls, though less than it should under the planner's solution; the fluctuation in exports is more or less on target with respect to the planner's solution. Likewise, for the other three shocks considered in our model, eliminating distorting labor taxes in favor of lump sum taxes gets rid of most of the inefficient fluctuations of the market economy. Graphs of these cases are available from the authors.

Next, we look at how nominal rigidities affect the fluctuations of the market economy. This is particularly interesting in our context since knowing the effects of nominal

Figure 7: *Effects of export demand shock: Planner vs. benchmark vs. lump sum taxes.*

Comparative dynamics: shock to c_x . Dark blue dash=PLANNER. Thick blue line=BENCHMARK.



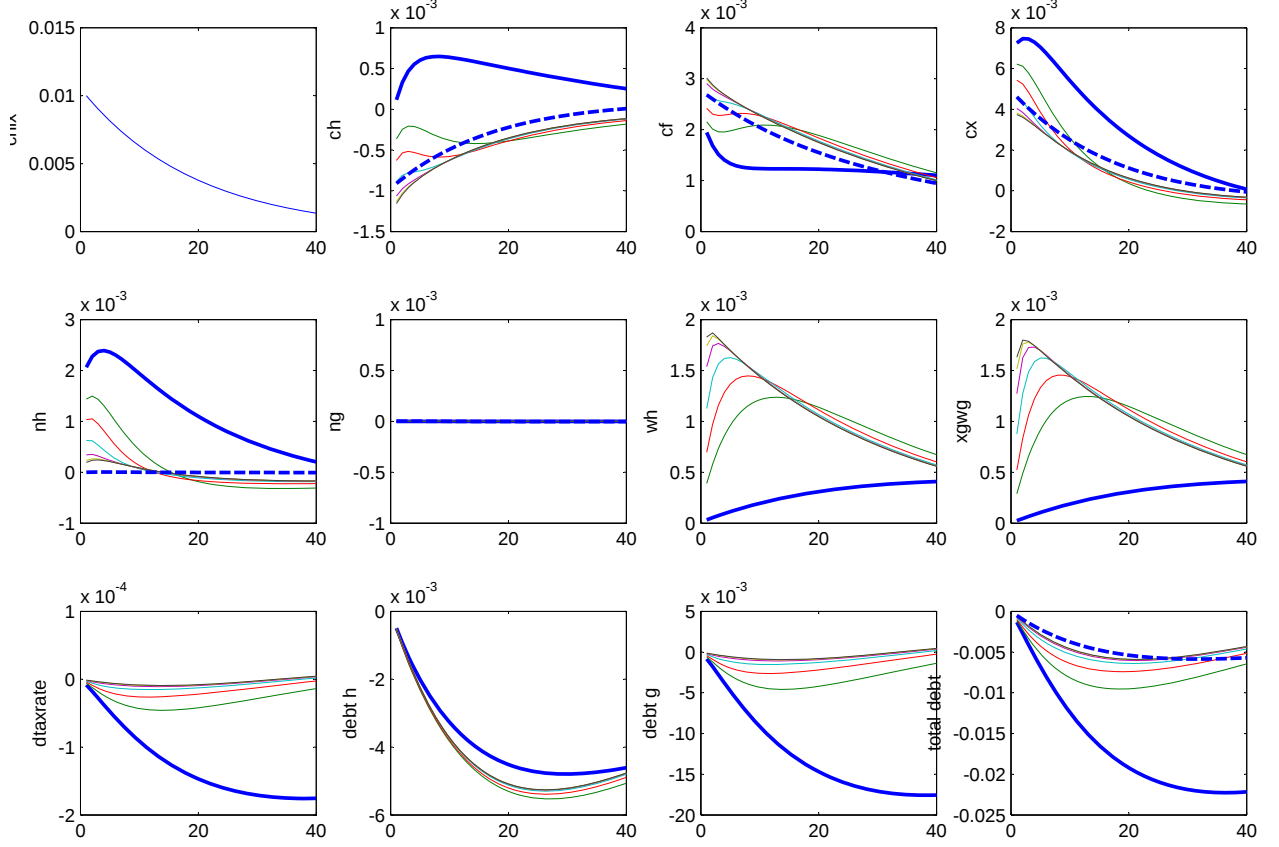
rigidities will help us see whether our proposed mechanism for expenditure adjustment acts also, as intended, as a mechanism for reducing the effective nominal rigidities in the economy. Figure 8 compares the impulse responses to an export demand shock across different degrees of nominal rigidity (the same benchmark and planner solutions are shown again, as in the previous figure). The benchmark economy (thick line) has wage persistence $\xi_w = 0.9$ at a quarterly rate and price persistence $\xi_p = 0.6$. The thinner lines report alternative parameterizations of stickiness: $\xi_w \in \{0.75, 0.6, 0.4, 0.2, 0.1, 0.06\}$, with $\xi_p = \frac{2}{3}\xi_w$ in all cases.²⁰

The effects of eliminating nominal rigidity are very powerful here. Since there are still other inefficiencies in the model, fully eliminating nominal rigidity drives the impulse responses to a limit that is not exactly equal to the responses of the planner's

²⁰Varying either Calvo parameter separately has effects qualitatively similar to varying both jointly; quantitatively the more powerful of the two is ξ_w .

Figure 8: *Effects of export demand shock: decreasing Calvo stickiness.*

Comparative dynamics: shock to cx . Dark blue dash=PLANNER. Thick blue line=BENCHMARK.



solution. Nonetheless, the responses are quite close. Just reducing stickiness by 20%, from 0.9 to 0.75, eliminates more than half the deviation between the market equilibrium and the planner's solution, for all the consumption and labor components. Eliminating stickiness also pushes the dynamics of total debt near to the planner's solution, and almost completely stabilizes government debt.²¹

²¹While our simulations will treat most variables in log-linear terms, debt will be treated in levels, since its sign need not be positive. Thus our impulse response functions all show debt fluctuations in percentage points of GDP; likewise, they show tax rate fluctuations in percentage points. All other variables are shown in log deviations.

5 Dynamic effects of simple fiscal rules

5.1 Rules to stabilize public debt

We saw in Section 3.8 that the exchange rate between CSU and euros, being a purely nominal change, is neutral in steady state. Therefore, while adjustments to the value of the CSU may potentially be a powerful fiscal and/or macroeconomic instrument, they can only have “short-run” effects, with a horizon related to the degree of nominal rigidity. This is why our rule (3.6.3) for X depends on current output and/or government spending but is not conditioned on the debt level, which is a slow-moving variable.

Since in our model budgetary adjustments are *only* possible insofar as they are given by the rules, the only way to ensure a “Ricardian” fiscal policy (one in which the public sector commits to pay off any debt it incurs, through future fiscal adjustments) is to ensure that at least one fiscal rule adjusts sufficiently strongly to changes in the debt level.²²

A sufficiently strong fiscal adjustment to any increase in debt would be one that adjusts other budget items by enough to pay off the increased interest. Since the tax base in this model is labor income, as a rough approximation near the steady state we can say that fiscal policy is Ricardian if

$$X_{ss}W_{ss}n_{ss}\tau^d > R_{ss}I_{ss}^g - 1 \quad (74)$$

($X_{ss}W_{ss}$ needs no superscript since it is equalized across sectors in steady state).

At our steady state, (74) requires $\tau^d > 0.00255$. On the other hand, optimal borrowing and lending requires that debt behave approximately as a random walk; therefore we wish to keep τ^d relatively close to zero, implying that shocks to revenues and expenditure are smoothed out over time by accumulation and decumulation of debt. Therefore, in order to achieve a high degree of tax smoothing while being entirely sure that we keep out of the Ricardian regime, we focus on tax rules satisfying $\tau^d = 0.01$, unless specified otherwise.

Our policy analysis will compare the macroeconomic effects of several different fiscal rules that stabilize the public debt. We compare debt stabilization by adjusting tax rates with debt stabilization by means of shifts in the value of public sector wages and transfers, asking how each one affects the volatility of other macroeconomic variables.

²²Kirsanova and Wren-Lewis (2011) analyze interactions across monetary and fiscal policy across both Ricardian and non-Ricardian regimes. While equilibrium may exist in the non-Ricardian “fiscal” regime, that regime involves a large decrease in welfare, so we wish to avoid it.

In particular, we start from a benchmark equilibrium characterized by $\tau^d = 0.01$, with all other fiscal parameters set to zero. We then compare alternative rules that decrease the variability of debt relative to the benchmark equilibrium. All rules leave steady state variables unchanged, including the steady-state level of public debt, so our analysis focuses only on the cyclical consequences of each rule.

Now, since a fully credible government would like to pursue a debt policy as close as possible to a random walk, one might wonder why we are interested in policies that stabilize debt at all. The reason, of course, is that no government is fully credible, and that European governments in particular seem to have lost a great deal of credibility recently, so that many are now asking themselves which fiscal margins should be adjusted to reduce the level and standard deviation of their debt. This is the spirit of the exercises we will perform in this section. If for some reason it becomes necessary to reduce public debt fluctuations, what are the macroeconomic consequences of various different policies by which this could be achieved?

5.2 Cyclical effects of different budget stabilization rules

Figure 9 studies the effects of an export demand shock under several different tax rules. It compares the planner's economy and the benchmark economy (where $\tau^y = 0$) against four different rules with $\tau^y < 0$ —that is, rules in which taxes are decreased (increased) in a boom (recession).²³

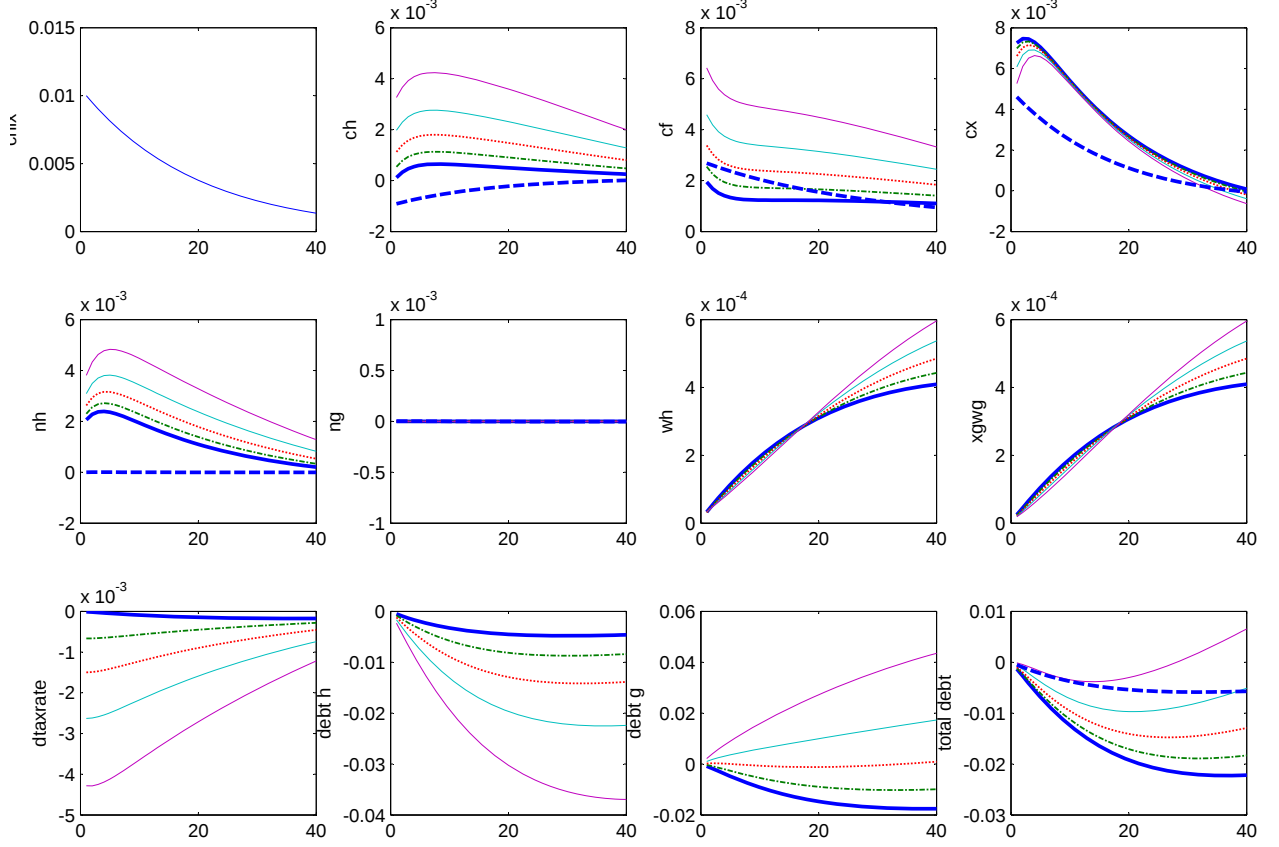
The shock to export demand stimulates output, so without any adjustment in the tax rate the tax base would rise and government debt would begin to fall, as we see in the benchmark case (thick blue line). Thus a rule with $\tau^y < 0$ can stabilize debt in response to a shock of this type, by lowering tax rates as output rises. The figure compares the values $\tau^y \in \{-0.25, -0.5, -0.75, -1\}$; in the latter case when nominal GDP is one percentage point above its trend, the tax rate falls by one percentage point. Such a large adjustment in tax rates is clearly undesirable as it actually goes beyond debt stabilization to make debt *countercyclical*. But a more moderate adjustment, around $\tau^y = -0.5$, (thin red line) serves to keep government debt approximately unchanged in response to this shock, and helps stabilize total national debt too, as we see in the third row of the figure.

However, while these rules may help stabilize debt in this context, they destabilize business cycle fluctuations. By lowering taxes when demand is rising, the rules with $\tau^y < 0$ amplify the fluctuations in n^h and c^h , pushing them even further away from

²³We repeat for clarity: all rules set the coefficient $\tau^d = 0.01$, unless otherwise stated.

Figure 9: *Effects of export demand shock: Tax rate falls with output.*

Comparative dynamics: shock to c_x . Dark blue dash=PLANNER. Thick blue line=BENCHMARK.



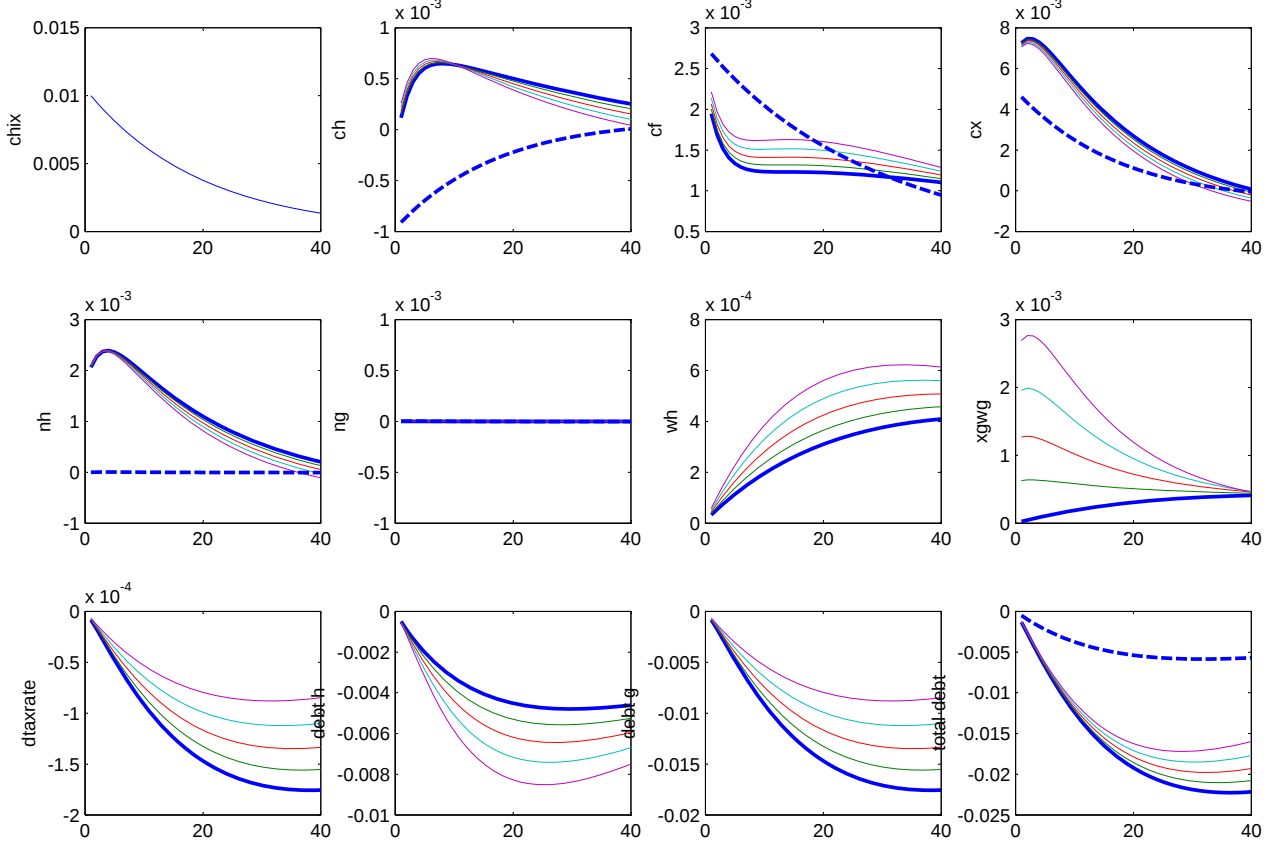
the planner's solution. Indeed, the deviation of c^h from the planner's solution roughly doubles with $\tau^y = -0.5$, though this is partially offset by an increase in the inefficiently small response of c^f .

The obvious destabilizing tendencies of countercyclical tax adjustments as a tool for budget balance motivate adjustments on the spending side instead. Figure 10 shows the effects of implementing a rule that adjusts the value of the CSU procyclically, considering the values $\zeta^y \in \{0.25, 0.5, 0.75, 1\}$. Thus the thin magenta line in the figure shows what happens when public salaries and unemployment benefits are boosted by one percent when nominal GDP is one percentage point above its trend. This is a quantitatively weaker budget adjustment than a one-percentage-point change in tax rates,²⁴ so setting $\zeta^y = 1$ only removes about half the variation in government debt

²⁴In other words, since X is centered around one whereas τ is centered around 0.31 in this simulation, a one-percentage-point change in the tax rate is a larger proportional change than a one-percentage-

Figure 10: *Effects of export demand shock: Value of CSU rises with output.*

Comparative dynamics: shock to c_x , $x_{multy} > 0$. Dark blue dash=CSU constant; Thick blue line=BENCHMARK.



seen in the impulse response function. But in contrast with the effects of the tax rule, the exchange rate rule pushes n^h and c^h and both sides of the trade balance moderately in the direction of the social planning solution as it stabilizes debt.

Intuitively, these CSU rules are stabilizing because they mean that in the event of an expansion, the fiscal authority will implement an *internal revaluation* by raising the wage of public workers (that is, increasing X_t). This potentially introduces volatility into public sector wages, but brings the dynamics of the model closer to the planner's solution while allowing for tax smoothing.

Figures 12-15 (in the appendix) compare the effects of stabilization through the tax parameter τ^y and the public wage parameter ζ^y in the case of shocks to foreign prices and interest rates (the diagrams show the same range of values for the parameters that

point change in X .

we saw in the previous figures). Our previous results are only reinforced by the shock to P^f , which simultaneously destabilizes n^h , c^h , c^f , and c^x . The results are somewhat muddier in the case of the shock to interest rates, where X has nontrivial budget effects but very little effect on labor and consumption, but our central finding remains true in this case too. Namely, stabilizing the debt level via the tax parameter τ^y further destabilizes the already-inefficient fluctuations in n^h and c^h , whereas stabilizing debt via public wages does not.

Finally, Figs. 16-17 study the case of shocks to government spending. The shape of the impulse responses is very different in this case, but they still have the property that eliminating nominal rigidities by shutting down the Calvo frictions brings the responses of employment and consumption closer to the social planners' solution. However, the budgetary implications are reversed with respect to the other demand shocks considered up to now: an expansion caused by an increase in government spending is likely to increase rather than decrease the debt. Therefore coefficients $\tau^y < 0$ and $\zeta^y > 0$ are completely counterproductive from a budgetary perspective in the context of public spending shocks.

Since government spending shocks are an important source of cyclical fluctuations, we see that if tax and public wage rules include terms that react to output, they should include terms that react to public spending as well. Assuming a fiscal multiplier on the order of magnitude of one, offsetting the counterproductive effects of $\tau^y < 0$ and $\zeta^y > 0$ requires coefficients τ^g and ζ^g of at least the same absolute magnitude, but of opposite sign.

Clearly, just looking at shocks one-by-one, and examining individual impulse responses over a limited time horizon, gives only a partial analysis of the volatility implications of the rules we are considering. Therefore Table 2 reports the standard deviation of selected variables in the presence of all four shocks, under several alternative scenarios for the rules.

First, we compare volatilities of consumption, labor, and fiscal and wage variables under the social planner solution and under the benchmark economy. We then consider the same volatilities under three fiscal rules that stabilize debt more aggressively than the benchmark economy does. The first rule chooses the tax rule of the form $0 > \tau^y = -\tau^g$ which minimizes the standard deviation of public debt. The second chooses a CSU rule of the form $0 < \zeta^y = -\zeta^g$ that achieves the same standard deviation of public debt that the tax rule does. Finally, the third is a minimalist tax rule based only on the level of debt ($\tau^d > 0$, with all other rule parameters set to zero) which also achieves the same standard deviation of public debt as the other two rules do.

Table 2: *Macroeconomic variability as a function of fiscal regime*

Standard deviations of key variables under different fiscal regimes					
	Planner	Benchmark	$\tau^y = -0.28$ $\tau^g = 0.28$	$\zeta^y = 0.49$ $\zeta^g = -0.49$	$\tau^d = 0.0155$
c^h	0.0169	0.0290	0.0407	0.0286	0.0324
c^f	0.0527	0.0452	0.0447	0.0434	0.0467
c^x	0.0687	0.0858	0.0872	0.0822	0.0862
n	0.0107	0.0314	0.0380	0.0305	0.0328
n^h	0.0186	0.0397	0.0484	0.0386	0.0416
n^g	0.0302	0.0320	0.0320	0.0320	0.0320
τ		0.0087*	0.0167*	0.0078*	0.0121*
D^h/PY_{ss}		0.2019*	0.2886*	0.2155*	0.2143*
D^g/PY_{ss}		0.8699*	0.7775*	0.7775*	0.7775*
D/PY_{ss}	0.1928*	0.9434*	0.8814*	0.8612*	0.8567*
$(1 - \tau)w^h$		0.0261	0.0342	0.0254	0.0307
$(1 - \tau)Xw^g$		0.0261	0.0341	0.0295	0.0307
$(1 - \tau)w^h n^h$		0.0501	0.0735	0.0487	0.0567
$(1 - \tau)Xw^g n^g$		0.0416	0.0473	0.0441	0.0442

Note: quantities with asterisks are stated in levels. All other variables are expressed in logs. Simulation assumes economy is driven by shocks ϵ^{cx} , ϵ^{pf} , ϵ^g , and ϵ^r with standard deviations 0.01, 0.01, 0.01, and 0.001 respectively.

Among the class of tax rules considered, the one that minimizes public debt fluctuations is $\tau^y = -0.28 = -\tau^g$. This reduces the standard deviation of debt from 87% of GDP (the benchmark figure) to 78% of GDP. A CSU rule with $\zeta^y = 0.49 = -\zeta^g$ implies the same variability of public debt, but implies smaller standard deviations of all the utility-relevant variables (all components of consumption and labor) than the tax rule achieves. The difference is especially strong for c^h and n^h , which are roughly one-third and one-fourth less volatile under the CSU rule compared with the tax rule.

The CSU rule also reduces the variability of consumption and labor with respect to the benchmark economy (which in turn has less variability of these variables than the economy with $\tau^y = -0.28 = -\tau^g$). The volatility ranking across these economies can be understood in terms of tax smoothing: the benchmark policy smoothes taxes by bringing the economy closer to a random walk in debt; making taxes contingent on output and government spending restrains debt variation at the cost of lower tax smoothing and therefore greater variation in consumption and labor. The CSU rule, by smoothing the cost of the government's purchases, achieves a higher degree of tax smoothing in equilibrium for any given setting of the tax rule parameters.

Motivated by these tax-smoothing considerations, the last column of the table reports another rule that achieves the same reduction in debt volatility as the two rules

considered previously, by simply raising the coefficient τ^d of debt on tax rates. Note that this coefficient has a much more powerful effect on debt variability than τ^y and τ^g : it suffices to raise τ^d to 0.0155 to reduce the standard deviation of public debt to 78% of GDP. Thus this rule amplifies c^h and n^h less than $\tau^y = -0.28 = -\tau^g$ does. However, it still raises volatility of all utility-relevant variables above their benchmark level, which is in turn more volatile than the equilibrium under the CSU rule.

In the last four rows of the table, we also report after-tax wages and after-tax labor income for public and private employees. Remarkably, in spite of the fact that the CSU rule acts directly by varying public sector wages, after-tax wages and after-tax labor income are actually less variable under the CSU rule than they are under either of the tax rules that achieve the same level of debt variability. Intuitively, the reduction in the volatility of the economy as a whole suffices to offset the direct impact of the CSU rule on public wages. Of the four specifications considered, the only one with lower volatility of public-sector after-tax wages is the benchmark policy. Thus, public sector workers might prefer an equilibrium with greater public debt variation. But, under the parameters considered here, if a reduction in debt volatility becomes inevitable, rational public sector workers should, in principle, welcome a policy that achieves this reduction by varying their wages rather than varying tax rates.

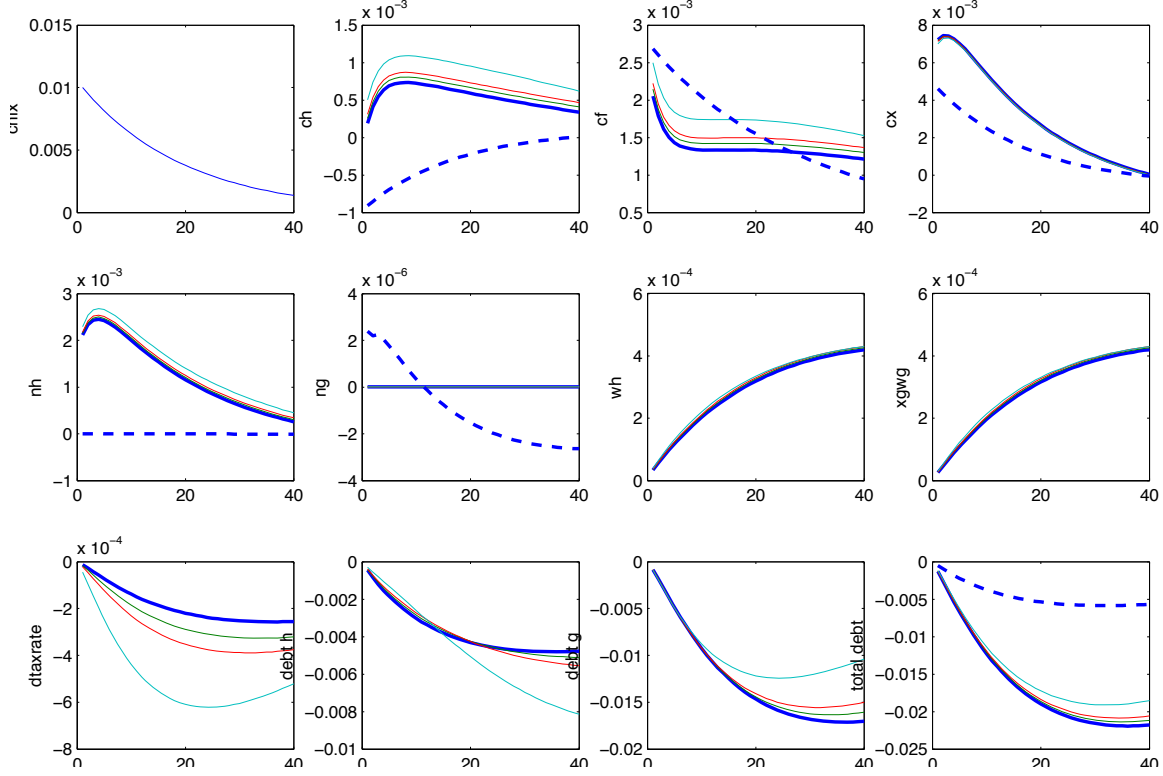
The results in Table 2 demonstrate that τ^d is a powerful instrument for reducing debt fluctuations. This is intuitive, since it responds to debt directly, instead of responding to output and spending. Therefore, Fig. 11 analyzes the effects of debt stabilization by means of rules with $\tau^d > 0$, with all other coefficients set to zero. The figure compares the benchmark policy ($\tau^d = 0.01$) with several alternatives up to a maximum of $\tau^d = 0.05$. With $\tau^d = 0.05$ the standard deviation of public debt falls to roughly 0.4, much lower than the values seen in Table 2. However, the figure shows that τ^d substantially amplifies the fluctuations in home consumption and private-sector labor, just like the tax rule analyzed previously in Fig. 9. Thus, while τ^d is a better stabilizer for public debt than τ^y , both amplify the cycle in a qualitatively similar way. Regardless of whether tax adjustments respond to the debt level or to output, achieving some budget stabilization through the CSU exchange rate is likely to be welfare-improving, given its stabilizing effect on consumption and labor.

6 Conclusions

This paper has studied simple fiscal rules for stabilizing the government debt level in response to asymmetric demand shocks in a small open economy that forms part of a currency union. In particular, starting from a baseline fiscal policy with large

Figure 11: *Effects of export demand shock: Taxes rise with debt.*

Comparative dynamics: shock to cx . Dark blue dash=PLANNER. Thick blue line=BENCHMARK.



fluctuations in the public debt, we compare the business cycle effects of several different rules that reduce the fluctuations in the public debt (the steady state level of public debt is held fixed across all cases we compare). We consider stabilizing public debt either by adjustments on the revenue side, or on the expenditure side. We have argued that an informationally feasible way of adjusting expenditure frequently and flexibly would be to make across-the-board shifts in public salaries and transfer payments.

Our simulations are based on a DSGE matching model with nominal rigidities, with business cycles driven by several types of asymmetric demand shocks. We show that stabilizing public debt by adjusting public salaries and transfers slightly reduces the fluctuations in consumption and labor, whereas stabilizing public debt by adjusting taxes strongly amplifies the fluctuations in both variables. Intuitively, this way of stabilizing debt on the expenditure side amounts to an internal devaluation when the economy enters into a recession. Thus, this particular expenditure-based mechanism for stabilizing the government debt keeps the economy closer to the first-best social planner's solution than adjusting taxes does. Even after-tax public-sector labor income

is stabilized by this mechanism, because the reduction in fluctuations of employment and tax rates is sufficient to offset the direct impact of the rule on public wages.

While this study is a first step towards understanding how policy-making could be improved in the context of a monetary union, there is a great deal still to be explored. It would be interesting to explore a wide variety of other fiscal instruments not considered here, such as consumption taxes, as well as alternative versions of the fiscal rules. More generally, it would be interesting to find ways of aggregating the model we have used here beyond a linear approximation, in order to calculate welfare gains and/or study Ramsey optimal policy.

In the current context, it is possible that sudden progress towards a fiscal union in Europe will make the issue of sovereign debt seem less urgent. But even if an agreement on fiscal union is reached soon, countries will have to face the issue of which fiscal margins to adjust in order to meet their commitments under the new agreement. Also, countries that wish to maintain as much sovereignty as possible over their own fiscal decisions may prefer to undertake institutional reforms to guarantee that their public debts never reach levels that would imply European sanctions. Thus the policy innovations discussed here may be considered complementary to fiscal union, rather than mutually exclusive.

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A Wage dynamics with Calvo stickiness

To solve the model, we need to eliminate all firm-specific variables and write the dynamics in terms of aggregate variables only. The only source of firm-specific differences is wage variation across firms. We assume inflation is zero in steady state. Therefore, cross-sectional wage differences are driven *only* by differences in aggregate conditions at the time of last adjustment; thus cross-sectional variation in wages can be treated by a first (or n th) order approximation if and only if time variation in macro quantities can be treated by a first (or n th) order approximation.

The surplus equations for workers and firms, (55) and (57), can be rewritten as

$$\mathcal{H}_{l,t}^i = (1 - \tau_t) \frac{X_t^i W_{l,t}^i}{P_t^c} - \underline{w}_t + (1 - \delta_n) E_t \beta_{t,t+1} \mathcal{H}_{l,t+1}^i, \quad (75)$$

$$\mathcal{J}_{l,t}^i = \bar{w}_{l,t}^i - \frac{X_t^i W_{l,t}^i}{P_t^c} + (1 - \delta_n) E_t \beta_{t,t+1} \mathcal{J}_{l,t+1}^i, \quad (76)$$

where

$$\underline{w}_t = b_t + \chi_n n_t^{\psi_n} c_t^{\eta_0} + (1 - \delta_n) E_t \beta_{t,t+1} s(\theta_{t+1}) \int_0^1 \frac{v_{L,t+1}^h \mathcal{H}_{L,t+1}^h + v_{L,t+1}^g \mathcal{H}_{L,t+1}^g}{v_{t+1}} dL, \quad (77)$$

and

$$\bar{w}_{l,t}^i = \phi_t^i a + E_t \beta_{t,t+1} \left[\frac{\chi_v \psi_v}{1 + \psi_v} (z_{l,t+1}^i)^{1+\psi_v} \phi_{t+1}^i \right]. \quad (78)$$

Notice that $\underline{w}_t$ is equal across all producers, since it represents the worker's outside option and thus is independent of a worker's current match, whereas $\bar{w}_{l,t}^i$ differs both across sectors i and producers l .

All producers renegotiating wages at time t in sector i will set the same wage. Thus, let $\mathcal{H}_{t|t-j}^i$ denote workers' time- t surplus in a match that last renegotiated wages at time $t-j$. Since adjustment occurs with probability ξ_w each period, $\mathcal{H}_{t|t-j}^i$ satisfies

$$\mathcal{H}_{t|t-j}^i = (1 - \tau_t) \frac{X_t^i W_{t-j}^{i*}}{P_t^c} - \underline{w}_t + (1 - \delta_n) E_t \beta_{t,t+1} (\xi_w \mathcal{H}_{t+1|t-j}^i + (1 - \xi_w) \mathcal{H}_{t+1}^{i*}), \quad (79)$$

where $\mathcal{H}_{t+1}^{i*} \equiv \mathcal{H}_{t+1|t+1}^i$ denotes the surplus of a worker in a firm that renegotiates the wage at time $t+1$. Proceeding in the same way for firms, we have

$$\mathcal{J}_{t|t-j}^i = \bar{w}_{t|t-j}^i - \frac{X_t^i W_{t-j}^{i*}}{P_t^c} + (1 - \delta_n) E_t \beta_{t,t+1} [\xi_w \mathcal{J}_{t+1|t-j}^i + (1 - \xi_w) \mathcal{J}_{t+1}^{i*}]. \quad (80)$$

Here, $\bar{w}_{t|t-j}^i$ is the quantity $\bar{w}_{l,t}^i$ defined by (78), evaluated at time t for a firm l which last renegotiated wages at time $t-j$ and therefore has nominal wage W_{t-j}^{i*} in its bargaining currency (or $X_t^i W_{t-j}^{i*}$ in euros). It equals

$$\bar{w}_{t|t-j}^i = \phi_t^i a + \frac{\chi_v \psi_v}{1 + \psi_v} E_t \beta_{t,t+1} \phi_{t+1}^i [\xi_w (z_{t+1|t-j}^i)^{1+\psi_v} + (1 - \xi_w) (z_{t+1}^{i*})^{1+\psi_v}], \quad (81)$$

where we see that the vacancy-to-lagged-employment ratio $z_{l,t+1}^i$ will depend on whether or not the nominal wage of firm l remains stuck at W_{t-j}^{i*} .

As we have emphasized, the only idiosyncratic state variable of producer l is its nominal wage $W_{l,t}^i$. Hence, in a log-linear approximation (indicated by hats), all produce-specific endogenous quantities must depend log-linearly on the nominal wage. That is, for any two producers l and m , worker and producer surpluses must be related by

$$\hat{\mathcal{H}}_{l,t}^i - \hat{\mathcal{H}}_{m,t}^i = \gamma_H (\hat{W}_{l,t}^i - \hat{W}_{m,t}^i), \quad (82)$$

$$\hat{\mathcal{J}}_{l,t}^i - \hat{\mathcal{J}}_{m,t}^i = -\gamma_J (\hat{W}_{l,t}^i - \hat{W}_{m,t}^i), \quad (83)$$

where γ_H and γ_J are unknown constants. To evaluate the Nash bargaining equation, we will also need the derivatives of surpluses with respect to the wage. Equations (59)

and (60) show that $\frac{\partial \mathcal{H}}{\partial W}$ is independent of W , whereas $\frac{\partial \mathcal{J}}{\partial W}$ varies with W . Therefore we must also solve for the constant γ_{dJ} such that

$$\left| \frac{\partial \widehat{\mathcal{J}}_{l,t}^i}{\partial W} \right| - \left| \frac{\partial \widehat{\mathcal{J}}_{m,t}^i}{\partial W} \right| = \gamma_{dJ}(\hat{W}_{l,t}^i - \hat{W}_{m,t}^i). \quad (84)$$

Likewise, for the other variables that vary across matches, we must have

$$\hat{z}_{l,t}^i - \hat{z}_{m,t}^i = -\gamma_z(\hat{W}_{l,t}^i - \hat{W}_{m,t}^i), \quad (85)$$

$$\hat{w}_{l,t}^i - \hat{w}_{m,t}^i = -\gamma_w(\hat{W}_{l,t}^i - \hat{W}_{m,t}^i). \quad (86)$$

Here, for convenience, we have signed (82)-(86) in a way which will imply that the unknown constants γ_H , γ_J , γ_{dJ} , γ_z , and γ_w are all positive.

Next, we solve for γ_H using the Bellman equation for worker surplus. In particular, consider a worker employed by a producer l that last adjusted its wage at time $t - j$. Log-linearizing (79), and indicating steady states by the subscript ss ,

$$\begin{aligned} \mathcal{H}_{ss}^i \hat{\mathcal{H}}_{t|t-j}^i &= (1 - \tau_{ss}) \frac{X_{ss}^i W_{ss}^i}{P_{ss}^c} \left(\hat{X}_t^i + \hat{W}_{t-j}^i - \hat{P}_t^c - \frac{\tau_{ss}}{1 - \tau_{ss}} \hat{\tau}_t \right) - \underline{w}_{ss} \underline{\hat{w}}_t^i \\ &\quad + (1 - \delta_n) \beta \mathcal{H}_{ss}^i E_t \left(\eta_0(\hat{c}_t - \hat{c}_{t+1}) + \xi_w \hat{\mathcal{H}}_{t+1|t-j}^i + (1 - \xi_w) \hat{\mathcal{H}}_{t+1}^{i*} \right). \end{aligned} \quad (87)$$

Now consider two producers that last adjusted their wages at times $t - j$ and $t - k$. Evaluating (87) for each one, and subtracting, we have

$$\mathcal{H}_{ss}^i (\hat{\mathcal{H}}_{t|t-j}^i - \hat{\mathcal{H}}_{t|t-k}^i) = (1 - \gamma_w) w_{ss}^i (\hat{W}_{t|t-j}^i - \hat{W}_{t|t-k}^i) + (1 - \delta_n) \beta \mathcal{H}_{ss}^i \xi_w E_t \left(\hat{\mathcal{H}}_{t+1|t-j}^i - \hat{\mathcal{H}}_{t+1|t-k}^i \right), \quad (88)$$

where w_{ss}^i is the steady-state *real* wage $X_{ss}^i W_{ss}^i / P_{ss}^c$. Note that the log difference between the wages of two firms that last adjusted at times $t - j$ and $t - k$ is constant in real terms, so it is equivalent regardless of the future period in which it is evaluated: $\hat{W}_{t|t-j}^i - \hat{W}_{t|t-k}^i = \hat{W}_{t+1|t-j}^i - \hat{W}_{t+1|t-k}^i$. Since $\hat{\mathcal{H}}_{t|t-j}^i - \hat{\mathcal{H}}_{t|t-k}^i = \gamma_H (\hat{W}_{t|t-j}^i - \hat{W}_{t|t-k}^i)$, we can cancel wages out of the last equation to solve for the unknown constant:

$$\gamma_H = \frac{(1 - \tau_{ss}) w_{ss}^i}{(1 - (1 - \delta_n) \beta \xi_w) \mathcal{H}_{ss}^i} \quad (89)$$

Similarly, consider the surplus $\mathcal{J}$ of a producer that last adjusted its wage at time $t - j$. Log-linearizing (80),

$$\begin{aligned} \mathcal{J}_{ss}^i \hat{\mathcal{J}}_{t|t-j}^i &= a \phi_{ss}^i \hat{\phi}_{t|t-j}^i - \frac{X_{ss}^i W_{ss}^i}{P_{ss}^c} \left(\hat{X}_t^i + \hat{W}_{t-j}^i - \hat{P}_t^c \right) \\ &\quad + (1 - \delta_n) \beta \mathcal{J}_{ss}^i E_t \left(\eta_0(\hat{c}_t - \hat{c}_{t+1}) + \xi_w \hat{\mathcal{J}}_{t+1|t-j}^i + (1 - \xi_w) \hat{\mathcal{J}}_{t+1}^{i*} \right) \end{aligned} \quad (90)$$

$$+\beta \frac{\psi_v}{1+\psi_v} z_{ss}^i q_{ss} \mathcal{J}_{ss}^i E_t \left(\eta_0(\hat{c}_t - \hat{c}_{t+1}) + \hat{q}_{t+1} + \xi_w(\hat{z}_{t+1|t-j}^i + \hat{\mathcal{J}}_{t+1|t-j}^i) + (1 - \xi_w)(\hat{z}_{t+1}^{i*} + \hat{\mathcal{J}}_{t+1}^{i*}) \right)$$

Now consider two producers that last adjusted their wages at times $t - j$ and $t - k$.

Evaluating (90) for each of these producers, and subtracting, we have

$$\begin{aligned} \mathcal{J}_{ss}^i(\hat{\mathcal{J}}_{t|t-j}^i - \hat{\mathcal{J}}_{t|t-k}^i) &= -w_{ss}^i(\hat{W}_{t|t-j}^i - \hat{W}_{t|t-k}^i) + (1 - \delta_n)\beta \mathcal{J}_{ss}^i \xi_w E_t \left(\hat{\mathcal{J}}_{t+1|t-j}^i - \hat{\mathcal{J}}_{t+1|t-k}^i \right) \\ &\quad + \beta \xi_w \frac{\psi_v}{1+\psi_v} z_{ss}^i q_{ss} \mathcal{J}_{ss}^i E_t \left((\hat{z}_{t+1|t-j}^i + \hat{\mathcal{J}}_{t+1|t-j}^i - \hat{z}_{t+1|t-k}^i - \hat{\mathcal{J}}_{t+1|t-k}^i) \right) \end{aligned}$$

Using $\hat{\mathcal{J}}_{t|t-j}^i - \hat{\mathcal{J}}_{t|t-k}^i = -\gamma_J(\hat{W}_{t|t-j}^i - \hat{W}_{t|t-k}^i)$ and $\hat{z}_{t|t-j}^i - \hat{z}_{t|t-k}^i = -\gamma_z(\hat{W}_{t|t-j}^i - \hat{W}_{t|t-k}^i)$, the minus sign cancels out as we solve for γ_J :

$$\mathcal{J}_{ss}^i \gamma_J = w_{ss}^i + (1 - \delta_n)\beta \xi_w \mathcal{J}_{ss}^i \gamma_J + \beta \xi_w \frac{\psi_v z_{ss}^i q_{ss}}{1+\psi_v} \mathcal{J}_{ss}^i (\gamma_z + \gamma_J). \quad (91)$$

A log-linearization of the zero-profit condition (52) implies $\gamma_J = \psi_v \gamma_z$, and in steady state we have $z_{ss}^i q_{ss} = \delta_n$, so this equation simplifies to

$$\gamma_J = [1 - \beta \xi_w (1 - \delta_n(1 - \psi_v))]^{-1} \frac{w_{ss}^i}{\mathcal{J}_{ss}^i}. \quad (92)$$

The absolute value of the derivative of surplus, $|\frac{\partial \mathcal{J}}{\partial W}|$, can be analyzed similarly. After some tedious algebra, we obtain

$$\gamma_{dJ} = (1 - \delta_n)\beta \xi_w \gamma_{dJ} + \beta \xi_w \delta_n (\gamma_{dJ} - \gamma_z) = -(1 - \beta \xi_w)^{-1} \beta \xi_w \delta_n \psi_v \gamma_J. \quad (93)$$

We can now write all the surplus equations in terms of cross-sectional averages, such as the *average* worker surplus $\hat{\mathcal{H}}_t^i$. We can find a recursive equation for $\hat{\mathcal{H}}_t^i$ by calculating the cross-sectional average at time $t + 1$ for employees whose wages are *not* renegotiated at time $t + 1$. This average is

$$\hat{\mathcal{H}}_{t+1}^i + \gamma_H(\hat{W}_t^i - \hat{W}_{t+1}^i), \quad (94)$$

since the log-linear solution for $\hat{\mathcal{H}}$ must be corrected for the change in the average wage, $\hat{W}_{t+1}^i - \hat{W}_t^i$. Rewriting (87) in terms of cross-sectional averages, we have

$$\begin{aligned} \mathcal{H}_{ss}^i \hat{\mathcal{H}}_t^i &= (1 - \tau_{ss}) \frac{X_{ss}^i W_{ss}^i}{P_{ss}^c} \left(\hat{X}_t^i + \hat{W}_t^i - \hat{P}_t^c - \frac{\tau_{ss}}{1 - \tau_{ss}} \hat{\tau}_t \right) - \underline{w}_{ss} \underline{\hat{w}}_t^i \\ &\quad + (1 - \delta_n) \beta \mathcal{H}_{ss}^i E_t \left[\eta_0(\hat{c}_t - \hat{c}_{t+1}) + \hat{\mathcal{H}}_{t+1}^i + \gamma_H \left(\xi_w(\hat{W}_t^i - \hat{W}_{t+1}^i) + (1 - \xi_w)(\hat{W}_{t+1}^{i*} - \hat{W}_{t+1}^i) \right) \right]. \end{aligned} \quad (95)$$

This equation can be simplified further by log-linearizing the average wage dynamics (53) of sector i . Bearing in mind that in steady state, the average nominal wage equals the nominal reset wage, ($W_{ss}^i = W_{ss}^{i*}$), the log-linearized average wage dynamics are

$$\hat{W}_t^i = \xi_w \hat{W}_{t-1}^i + (1 - \xi_w) \hat{W}_t^{i*}. \quad (96)$$

Rearranging, (96) becomes

$$(1 - \xi_w)(\hat{W}_t^{i*} - \hat{W}_t^i) = \xi_w(\hat{W}_t^i - \hat{W}_{t-1}^i) \quad (97)$$

Thus the log deviation between the reset wage and the sectoral average wage is proportional to sectoral wage inflation, $\hat{W}_t^i - \hat{W}_{t-1}^i$. Note that (97) can be cancelled out of (95), leaving a Bellman equation in terms of cross-sectional averages only:

$$\mathcal{H}_{ss}^i \hat{\mathcal{H}}_t^i = (1 - \tau_{ss}) \frac{X_{ss}^i W_{ss}^i}{P_{ss}^c} \left(\hat{X}_t^i + \hat{W}_t^i - \hat{P}_t^c - \frac{\tau_{ss}}{1 - \tau_{ss}} \hat{\tau}_t \right) - \underline{w}_{ss} \hat{w}_t^i + (1 - \delta_n) \beta \mathcal{H}_{ss}^i E_t \left(\eta_0(\hat{c}_t - \hat{c}_{t+1}) + \hat{\mathcal{H}}_{t+1}^i \right).$$

Likewise, the firm's surplus can be rewritten in terms of cross-sectional averages by using (97) to eliminate the terms relating to wage dispersion:

$$\begin{aligned} \mathcal{J}_{ss}^i \hat{\mathcal{J}}_t^i &= a \phi_{ss}^i \hat{\phi}_t^i - \frac{X_{ss}^i W_{ss}^i}{P_{ss}^c} \left(\hat{X}_t^i + \hat{W}_t^i - \hat{P}_t^c \right) + (1 - \delta_n) \beta \mathcal{J}_{ss}^i E_t \left(\eta_0(\hat{c}_t - \hat{c}_{t+1}) + \hat{\mathcal{J}}_{t+1}^i \right) \\ &\quad + \beta \frac{\psi_v}{1 + \psi_v} z_{ss}^i q_{ss} \mathcal{J}_{ss}^i E_t \left(\eta_0(\hat{c}_t - \hat{c}_{t+1}) + \hat{q}_{t+1} + \hat{z}_{t+1}^i + \hat{\mathcal{J}}_{t+1}^i \right). \end{aligned}$$

Similarly, for the derivative of the producer's surplus, we obtain

$$\begin{aligned} \left| \frac{\partial \mathcal{J}^i}{\partial W} \right|_{ss} \left| \frac{\partial \widehat{\mathcal{J}}_t^i}{\partial W} \right| &= \frac{X_{ss}^i}{P_{ss}^c} \left(\hat{X}_t^i - \hat{P}_t^c \right) + (1 - \delta_n) \beta \left| \frac{\partial \widehat{\mathcal{J}}_t^i}{\partial W} \right| E_t \left(\eta_0(\hat{c}_t - \hat{c}_{t+1}) + \left| \frac{\partial \widehat{\mathcal{J}}_{t+1}^i}{\partial W} \right| \right) \\ &\quad + \beta z_{ss}^i q_{ss} \left| \frac{\partial \mathcal{J}^i}{\partial W} \right|_{ss} E_t \left(\eta_0(\hat{c}_t - \hat{c}_{t+1}) + \hat{q}_{t+1} + \hat{z}_{t+1}^i + \left| \frac{\partial \widehat{\mathcal{J}}_{t+1}^i}{\partial W} \right| \right). \end{aligned}$$

We can now use these log-linearized surplus equations to state the Nash bargaining equation in terms of cross-sectional averages. The equation

$$\sigma \mathcal{J}_t^{i*} \frac{\partial \mathcal{H}_t^{i*}}{\partial W_t^{i*}} = (1 - \sigma) \mathcal{H}_t^{i*} \left| \frac{\partial \mathcal{J}_t^{i*}}{\partial W_t^{i*}} \right| \quad (98)$$

can be rewritten as

$$\hat{\mathcal{J}}_t^i - \gamma_J(\hat{W}_t^{i*} - \hat{W}_t^i) + \frac{\partial \widehat{\mathcal{H}}_t^{i*}}{\partial W_t^{i*}} = \hat{\mathcal{H}}_t^i + \gamma_H(\hat{W}_t^{i*} - \hat{W}_t^i) + \left| \frac{\partial \widehat{\mathcal{J}}_t^i}{\partial W_t^i} \right| + \gamma_{dJ}(\hat{W}_t^{i*} - \hat{W}_t^i). \quad (99)$$

This equation suffices to determine the reset wage W_t^{i*} .

Figure 12: *Effects of foreign price shock: Value of CSU rises with output.*

Comparative dynamics: shock to pf, xmulty>0. Dark blue dash=CSU constant; Thick blue line=BENCHMARK.

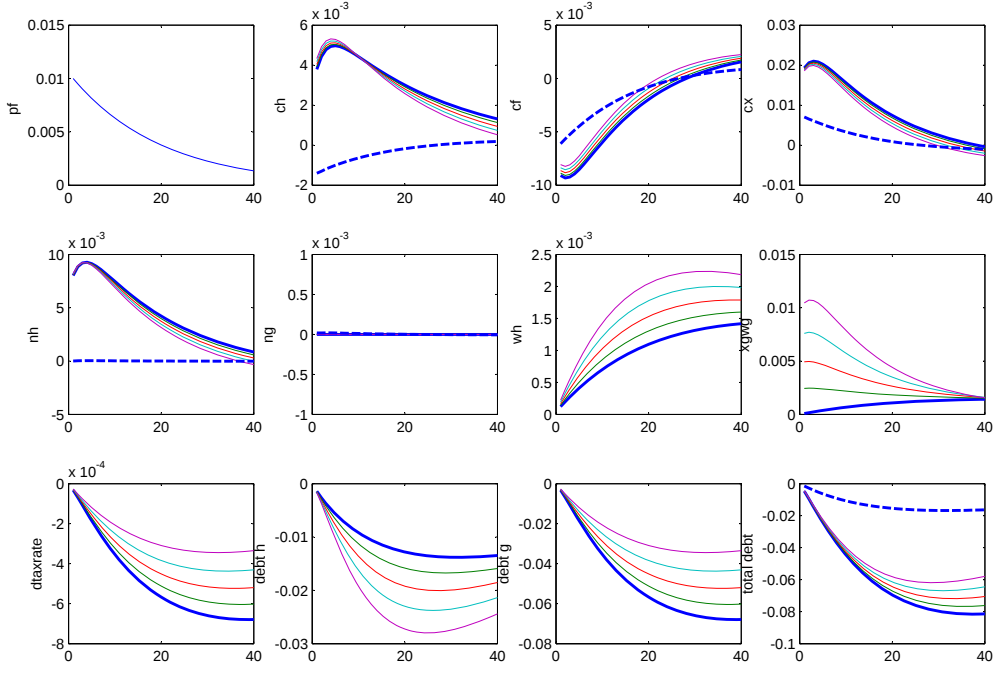


Figure 13: *Effects of foreign price shock: Tax rate falls with output.*

Comparative dynamics: shock to pf. Dark blue dash=PLANNER. Thick blue line=BENCHMARK.

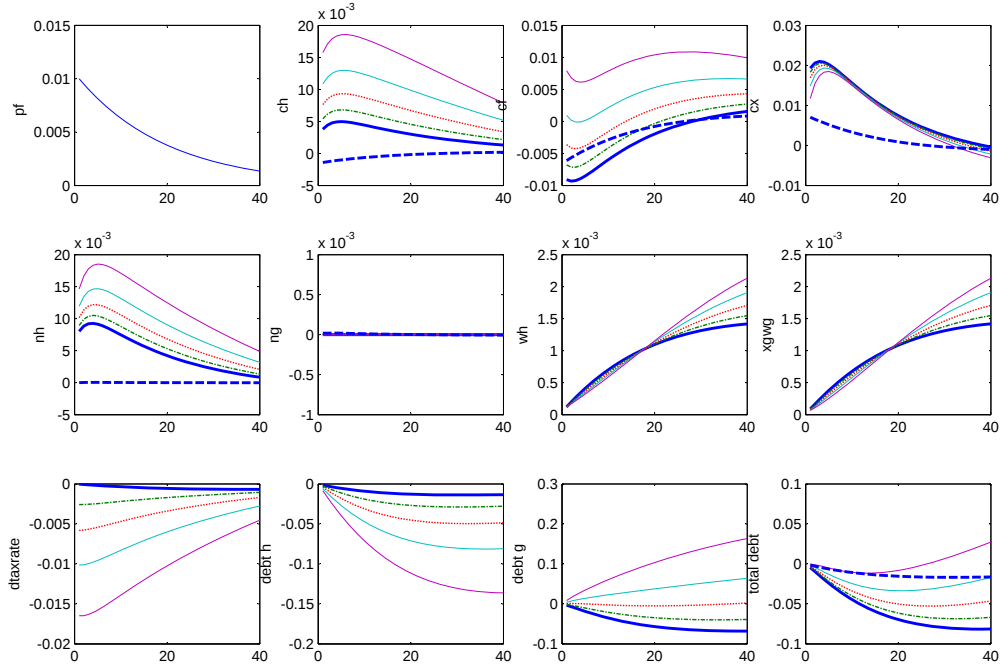


Figure 14: *Effects of interest rate shock: Value of CSU rises with output.*

Comparative dynamics: shock to r , $x_{multy} > 0$. Dark blue dash=CSU constant; Thick blue line=BENCHMARK.

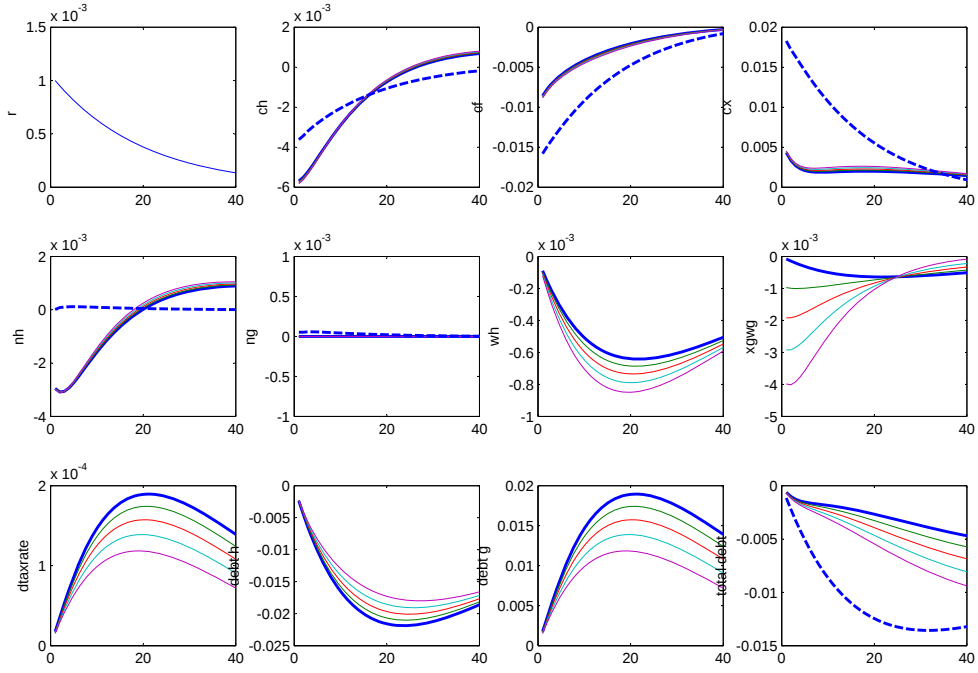


Figure 15: *Effects of interest rate shock: Tax rate falls with output.*

Comparative dynamics: shock to r . Dark blue dash=PLANNER. Thick blue line=BENCHMARK.

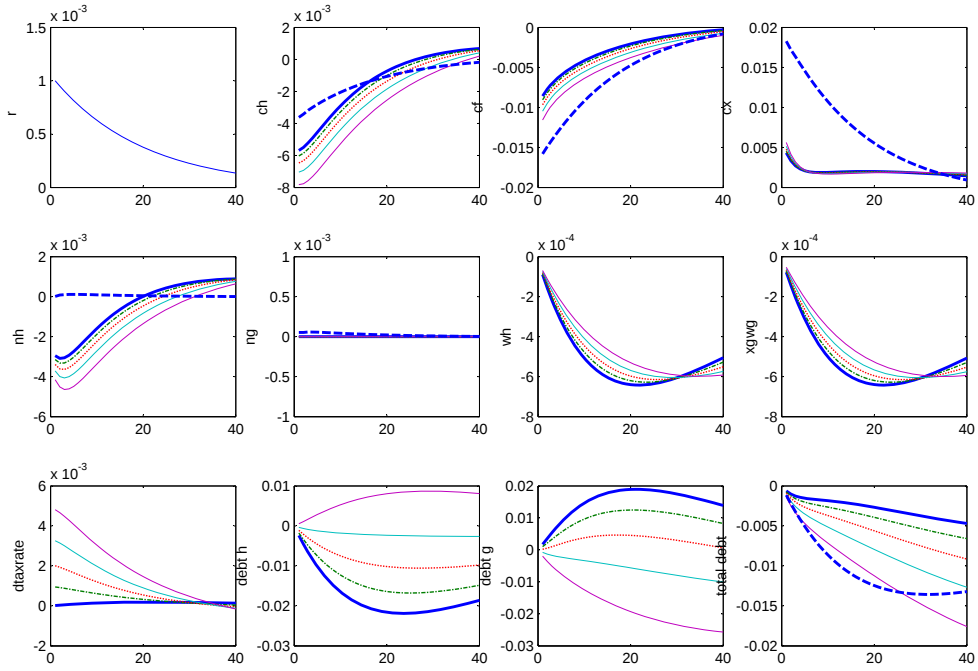


Figure 16: *Effects of government spending shock: Value of CSU rises with output.*

Comparative dynamics: shock to g , $x_{multy} > 0$. Dark blue dash=CSU constant; Thick blue line=BENCHMARK.

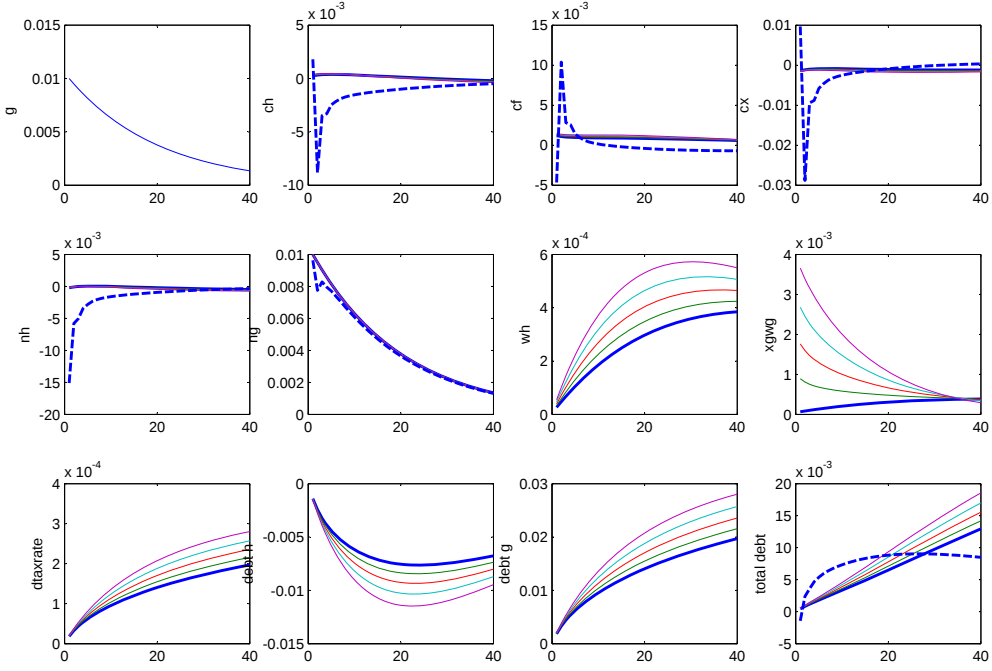


Figure 17: *Effects of government spending shock: Tax rate falls with output.*

Comparative dynamics: shock to g . Dark blue dash=PLANNER. Thick blue line=BENCHMARK.

