On the optimal design of a Financial Stability Fund*

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Abstract

A financial stability fund set by a union of sovereign countries (e.g. the European Stability Mechanism), can improve countries's ability to borrow and lend, and to share risks, with respect to debt financing. Efficiency gains arise from the ability of the fund to offer long-term financial contracts, subject to limited enforcement and moral hazard constraints. In contrast, debt contracts are subject to untimely roll-over and default risk. We develop a model of the fund as a long-term partnership - with alternative regimes, depending on the constraints it accounts for - and quantitatively compare economies where a country can only use debt contracts with economies within which it has access to different fund contracts. In particular, we characterize how (implicit) interest rates and asset holdings vary across these different regimes. We also study how different regimes react to crisis, the feasibility and possible gains of entering the fund with different levels of accumulated debt, and we contrast episodes of partial default with state-contingent realizations of the long-term fund contract. Of special interest is the case of multi-sided limited enforcement, since the resulting constrained-efficient policies take into account the limited capacity, or political will, of sovereign countries to redistribute funds on a persistent manner. Our simulations show how, even with multi-sided limited enforcement, there are efficiency gains in establishing a well-designed financial stability fund. Hence, our theory provides a basis for its design.

JEL classification: E43, E44, E47, E63, F34, F36

Key words: Recursive contracts, debt contracts, partnerships, limited enforcement, debt restructuring, sovereign funds.

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1 Introduction

There is ample experience on how funds, such as the IMF, can help to resolve sovereign debt problems. Almost surprisingly, little theoretical research has been done on how a fund set by a group of countries – such as the European Stability Mechanism – should be designed and operated. We build on the literature on sovereign debt in economies with incomplete markets (e.g. Arellano (2008), Aguiar and Gopinath (2006)) and on the literature on dynamic optimal contracts with enforcement and moral-hazard constraints (e.g. Marcet and Marimon (2011), Atkeson (1991)) and the related one on price decentralization of optimal contracts (e.g. Alvarez and Jermann (2000), Krueger *et al.* (2008)), to develop a model of long-term contracts with state contingent debt as a prototype contract for a stability fund. The fund is an alternative to standard debt-financing, but it can operate in different regimes, depending under which different enforcement and informational constraints must operate. A central theme of this paper is to characterize (constrained) efficient contracts for these different regimes and to show their contrasting empirical implications and, in particular, to compare them with standard debt financing. Our model adresses three key aspects of sovereign debt: i) debt contracts, while being optimal in anonymous relationship are inefficient in the context of long-term relationships among partner countries; ii) even in the context of a fiscal and monetary union, limited enforceability is a characteristic of sovereign debt and places limits on the amount of persistent redistribution that member countries are willing to tolerate; iii) moral hazard must also be accounted for in guaranteeing solvency. A main advantage for countries to participate in the fund is that their – possibly, non sustainable – short-term and not state-contingent debt is transformed into a state-contingent long-term debt. In this sense, the fund provides a technology of maturity transformation.

Nevertheless, even if the basic mechanism of maturity transformation is understood, one needs to asses how a Financial Stability Fund should, and would, perform. In particular, since as said, several constraints need to be taken into account. This is precisely the inquiry and contribution of this paper. We develop two basic model economies, one with incomplete markets, where agents ability to borrow and lend and insure is severely mitigated by the fact that the unique financial instrument is one-period debt. The other with a Financial Stability Fund (FSF) and in this one we consider two basic regimes: in one there is one-sided limited commitment (i.e. the lender is fully committed) and in the other both the borrower and the lender may quit the fund at any time. In particular, the lender may be willing to quit when providing insurance and lending is perceived as a burden, beyond the *quid-pro-quo* for which the fund is created. Yet, in spite of these constraints the two-sided limited commitment FSF economy dominates the incomplete markets economy.

We model the fund as a long-term contract between the borrower and the lender and build on the theory or Recursive Contracts to characterize the (constrained) optimal contract. Then, in order to properly compare these FSF economies with the incomplete markets economy we 'decentralize' the contract generating the appropriate prices. This gives testable implications to our theory and helps our diagnosis. For example, while the two-sided limited commitment outperforms the incomplete markets economy without default, there may be significant interest rates spreads (with respect to the risk free rate) in this FSF economy; more precisely, the source of these spreads is not the borrower in stress, but the lender not willing to contribute to the fund beyond a certain (redistribution) limit. In other words, our normative exercise also has positive implications. In particular, debt restructuring episodes can be seen as an *ex-post* way of making non-contingent debt contracts more contingent: our theory and corresponding calibrations can help to understand how successful such interventions are. The Financial Stability Fund developed and analyzed here is not design as a 'rescue fund' just to address severe debt crisis, on the contrary it is an efficient design to avoid these extreme – but recurrent – situations; nevertheless, the inquire that this paper opens can also help the study rescue events, as the European Stability Fund will have to face from its beginnings.

2 Economies with different borrowing possibilities

We consider a standard infinite-horizon representative agent economy, where the agent has preferences for current leisure, $l = 1 - n$, and consumption, c, represented by $u(c, 1-n)$ and discounts the future at the rate β. The agent has access to a decreasing returns labor technology $y = \theta f(n)$, where $f' > 0$, $f'' < 0$ and θ is a productivity shock, assumed to be Markovian; $\theta \in (\theta_1, ..., \theta_N)$, $\theta_i < \theta_{i+1}$. The economy is a small open economy in a world with no uncertainty with interest rate r satisfying $1/(1+r) \geq \beta$; an inequality that, in general, we will assume to be strict. In order to borrow and save, the agent, which we also identify with the government of the country, may have access to different financial technologies, which will define different regimes, which we also call different economies. Since we also consider other shocks, in general we denote by s^t is the history of the realizations of shocks up to period t and denote $\theta_t = \theta(s^t)$,the Markovian productivity component of s^t when needed.

2.1 The short-term debt contract

In the *incomplete markets* (IM) economy the financial technology is one of one-period non-contingent debt. We consider three different variants of the IM economy: *i)* when there is no possibility of default, in the sense that there are borrowing limits that need to be respected; *ii)* when there is default risk – as in Arellano (2008),– and *iii)* when default risk has a moral hazard component, since the agent, or sovereign government, can influence productivity shocks. In general, the agent's problem has the following recursive form:

$$\begin{aligned} V^{bi}(b,\theta) &= \max_{c,I,n}\left\{u(c)+U(1-n)+\beta\sum_{i=1}^{N}p_i(I,\theta)V^{bi}(b',\theta_i)\right\} \\ & \quad \text{s.t.} \\ c+q(\theta,b')b' &\le \theta f(n)-I+b, \end{aligned}$$

where c is consumption, n is labour and $q(\theta,b')$ is the price of debt and I is good government policy which makes bad shocks less likely to occur. I is unobservable and it satisfies:

$$\frac{\partial p_i(I,\theta)}{\partial I}/p_i(I,\theta) \text{ is increasing in } i, \text{ for all } I \text{ and } \theta.$$

We assume that the country can always default on its debt. In this case, there is a probability $1-\lambda$ that the country stays in autarky and a probability of λ that the country comes back to the market. In this case, we assume that it returns with no debt. We will assume that λ is low enough as to not to deter borrowing (i.e. to avoid Bulow-Rogoff's problem). The value of autarky is:

$$V^{ai}(\theta)=\max_{I,n}\left\{u(\theta f(n)-I,1-n)+\beta\sum_{i=1}^{N}p_i(I,\theta)\left[(1-\lambda)V^{ai}(\theta_i)+\lambda V^{bi}(0,\theta_i)\right]\right\}$$

The choice of default is given by:

$$D(\theta,b)=1 \text{ if } V^{ai}(\theta)>V^{bi}(b,\theta) \text{ and } 0 \text{ otherwise,}$$

which determines the price of borrowing as

$$\begin{aligned} q(\theta,b') &= \frac{1}{1+r}\sum_{i=1}^{N}p_i(I,\theta)\left(1-D(\theta_i,b')\right) \\ &= \frac{1-d(\theta,b')}{1+r}, \end{aligned}$$

where $d(\theta,b')$ is the expected default rate in state θ and roll-over debt b'. The corresponding interest rate is $r^i(\theta,b')=1/q(\theta,b')-1$ and the *spread*: $r^i(s^t)-r$.

2.2 The *Financial Stability Fund* as a long-term contract

An economy with a *financial stability fund* (FSF) is modeled as a long-term contract with a risk-neutral lender, who can freely borrow and lend in the international market, and a borrower, who is 'the representative agent' of the small open economy. In the fund contract, the country (borrower) gets c and the FSF (lender) gets $\theta f(n)-c$. We consider four (2x2) variants of the FSF economy: *i)* one-sided limited enforcement, or commitment, in which case the borrower may default on the contract, keeping all the output $\theta f(n)$ and with some probability is admitted later into the fund, while the lender participates in the contract as long as it has a present value of $Z \leq 0$ at the beginning of the contract; *ii)* Two-sided limited enforcement, in which there is limited enforcement on the side of the borrower but also on the lender side, since the present value lower bound, $Z \leq 0$, must be satisfied in every period, *iii)* we also study both forms of limited commitment when moral hazard is introduced and the long-term contract mus also satisfy incentive-compatibility constraints. For the rest of this Section we consider economies without moral hazard problems (i.e. productivity shocks are exogeneous and there is no policy I decision).

The outside option of the lender is $Z \leq 0$ and the outside option of the borrower is $V^{af}(s^t)$. This is either financial autarky (if $\lambda = 0$) or the value of returning to the fund[1] with probablity $\lambda \in (0,1)$ and of staying in autarky with probability $1-\lambda$:

$$
\begin{aligned}
&V^{af}(s^t) = \max_n \{u(\theta(s^t)f(n)) + U(1-n) \\
&+\beta \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t)\left[(1-\lambda)\, V^{af}(s^{t+1}) + \lambda V^{bf}(s^{t+1})\right]\},
\end{aligned}
$$

where $V^{bf}(s^{t+1})$ is the borrower's value of starting a new fund contract in state s^{t+1}, which we will now define.

With two-sided limited enforcement, an optimal fund contract is a solution to the following problem:

$$
\begin{aligned}
&\max_{\{c(s^t),n(s^t)\}} \{\mu_{b0} \sum_{t=0} \sum_{s^t} \beta^t \pi\left(s^t\right)\left[u(c(s^t)) + U(1-n(s^t))\right] + \\
&+\mu_{l0} \sum_{t=0} \sum_{s^t} \left(\frac{1}{1+r}\right)^t \pi\left(s^t\right)\left[\theta(s^t) f\left(n(s^t)\right) - c(s^t)\right]\} \\
&\text{s.t.} \sum_{r=t} \sum_{s^r|s^t} \beta^{r-t} \pi\left(s^r|s^t\right)\left[u(c(s^r)) + U(1-n(s^r))\right] \geq V^{af}\left(s_t\right) \qquad (1) \\
&\text{and} \sum_{r=t} \sum_{s^r|s^t} \left(\frac{1}{1+r}\right)^{r-t} \pi\left(s^r|s^t\right)\left[s_r f\left(n(s^r)\right) - c(s^r)\right] \geq Z, \quad t \geq 0, \ (2)
\end{aligned}
$$

where (1) and (2) are the intertemporal participation constraints for the borrower and the lender, respectively, and (μ_{b0}, μ_{l0}) are initial Pareto weights that, without loss of generality, we normalize $\mu_{b0} = 1$ and μ_{l0} as to satisfy (2) in period zero with equality. In fact, the fund contract in an economy with one-sided limited commitment is a solution to the same problem with (2) only being a constraint in period zero.

As it is known (see Marcet and Marimon (2011)) we can rewritte the fund contract problem as:

[1]It could also be a different, but equally modeled, fund. Similarly, one could consider the alternative to going back to the economy with incomplete markets at some penalty cost.

$$\min_{\{\gamma_b(s^t),\gamma_l(s^t)\}} \max_{c(s^t),n(s^t)} \left\{ \sum_{t=0} \sum_{s^t} \beta^t \pi\left(s^t\right) \mu_b\left(s^{t+1}\right) \left[u(c(s^t)) + U(1-n(s^t))\right] \right.$$
$$+ \sum_{t=0} \sum_{s^t} \left(\frac{1}{1+r}\right)^t \pi\left(s^t\right) \mu_l\left(s^{t+1}\right) \left[s_t f\left(n(s^t)\right) - c(s^t)\right]$$
$$\left. - \sum_{t=0} \sum_{s^t} \beta^t \pi\left(s^t\right) \gamma_b\left(s^t\right) V^A\left(s_t\right) - \sum_{t=0} \sum_{s^t} \left(\frac{1}{1+r}\right)^t \pi\left(s^t\right) \gamma_l\left(s^t\right) Z \right\}$$
$$\mu_b\left(s^{t+1}\right) = \mu_b\left(s^t\right) + \gamma_b\left(s^t\right), \ \mu_b\left(s_0\right) = \mu_{b0}$$
$$\mu_l\left(s^{t+1}\right) = \mu_l\left(s^t\right) + \gamma_l\left(s^t\right), \ \mu_l\left(s_0\right) = \mu_{l0},$$

where $\gamma_b\left(s^t\right)$ and $\gamma_l\left(s^t\right)$ are the Lagrange multipliers of the enforcement constraints (1) and (2), respectively, in state s^t. That is, with one-sided limited commitment $\gamma_l\left(s^t\right) = 0$, $\forall t \geq 0$. Let $\eta \equiv \beta(1+r)$, then the first-order conditions of this problem are:

$$u'(c(s^t)) = \frac{\mu_l\left(s^{t+1}\right)}{\mu_b\left(s^{t+1}\right)\eta^t}, \tag{3}$$
$$\frac{U'(1-n(s^t))}{u'(c(s^t))} = \theta(s^t) f'\left(n\left(s^t\right)\right). \tag{4}$$

We can normalize the multipliers by defining

$$x(s^{t+1}) = \frac{\mu_l\left(s^{t+1}\right)}{\mu_b\left(s^{t+1}\right)\eta^t} \text{ and } v_i(s^t) = \frac{\gamma_i\left(s^t\right)}{\mu_i\left(s^t\right)}, i = l, b;$$

$x(s^{t+1})$ is the temporary relative pareto weight of the lender, and $v_b(s^t)$ and $v_l(s^t)$ are the normalized Lagrange multipliers of the agents. Notice that with this normalization (3) simplifies to: $u'(c(s^{t+1})) = x(s^t)$ and that, furthermore:

$$x(s^{t+1}) = \frac{1 - v_b(s^t)}{1 - v_l(s^t)} \frac{x\left(s^t\right)}{\eta}.$$

Since we have assumed that exogeneous shock processes are Markovian, it is easy to see that the first order conditions have a recursive structure, corresponding to the recursive formulation of the fund contract, which defines optimal policies – $c(x, s), n(x, s)$ and $v_b(x, s), v_l(x, s)$ – satisfying:

$$u'(c(x, s)) = x' = \frac{1 - v_b(x, s)}{1 - v_l(x, s)} \frac{x}{\eta} \tag{5}$$

and

$$\frac{U'(1 - n(x, s))}{u'(c(x, s))} = \theta f'(n(x, s)). \tag{6}$$

The value function of the fund contract saddle-point problem takes the form: $W(x,s) = xV^{lf}(x,s) + V^{bf}(x,s)$, where

$$V^{bf}(x,s) = u(c(x,s)) + U(1-n(x,s)) + \beta \sum_{s' \in S} \pi(s'|s) V^{bf}(x',s') \tag{7}$$

and

$$V^{lf}(x,s) = \theta f(n(x,s)) - c(x,s) + \frac{1}{1+r} \sum_{s' \in S} \pi(s'|s) V^{lf}(x',s') \tag{8}$$

The first order conditions (5) and (6), together with the following slackness conditions

$$V^{bf}(x,s) \geq V^{af}(s) \text{ and } v_b(x,s)\left[V^{bf}(x,s) - V^{af}(s)\right] = 0 \tag{9}$$

and

$$V^{lf}(x,s) \geq Z \text{ and } v_l(x,s)\left[V^{lf}(x,s) - Z\right] = 0, \tag{10}$$

define a recursive solution to obtain $c(x,s), n(x,s)$ and $v_b(x,s), v_l(x,s)$. Furthermore, with $\pi(s'|s)$ they also define (x',s). Recall that, (x_0,s_0) is defined by $x_0 = \mu_{l0} = \mu_l(s_0)$ such that $V^{lf}(x_0,s_0) = Z$.

3 Decentralization of the fund contract

In this section we show how to decentralize the optimal contract as a competitive equilibrium with endogenous borrowing constraints, which will allow us to compare the different fund contracts with the debt contract of the economy with incomplete markets. We build on the work of Alvarez and jermann (2000) and Krueger, Lustig and Perri (2008).

3.1 The competitive equilibrium

In the market equilibrium, the borrower has a home technology that produces $\theta(s^t) f(n(s^t))$ with his own labor. The borrower has access to a complete set of one period Arrow securities and solves the following problem:

$$\max_{\{c_b(s^t), n(s^t), a_b(s^{t+1})\}} \sum_{t=0} \sum_{s^t} \beta^t \pi\left(s^t\right) \left[u(c_b(s^t)) + U(1-n(s^t))\right]$$

$$\text{s.t.} \qquad c_b(s^t) + \sum_{s^{t+1}|s^t} q\left(s^{t+1}|s^t\right) a_b\left(s^{t+1}\right) = \theta(s^t) f\left(n(s^t)\right) + a_b(s^t)$$

$$a_b\left(s^{t+1}\right) \geq A_b\left(s^{t+1}\right)$$

where $q\left(s^{t+1}|s^t\right)$ is the price of the one period state contingent claim and $a_b\left(s^{t+1}\right)$ represents the amount of state contingent claims chosen by the borrower and $A_b\left(s^{t+1}\right)$ is a borrowing limit defined below. The Euler and transversality conditions imply that:

$$q\left(s^{t+1}|s^t\right) \geq \beta^t \pi\left(s^{t+1}|s^t\right) \frac{u'\left(c_b\left(s^{t+1}\right)\right)}{u'\left(c_b\left(s^t\right)\right)}$$

with equality if $a_b\left(s^{t+1}\right) > A_b\left(s^{t+1}\right)$ and

$$\lim_{t\to\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)u'\left(c_b\left(s^t\right)\right)\left[a_b\left(s^t\right)-A_b\left(s^t\right)\right]\leq 0$$

The lender also has access to a complete set of Arrow securities and he solves:

$$\begin{aligned}
&\max_{\{c_l(s^t),a_l(s^{t+1})\}}\sum_{t=0}\sum_{s^t}\left(\frac{1}{1+r}\right)^t\pi\left(s^t\right)c_l(s^t)\\
\text{s.t.}\qquad &c_l(s^t)+\sum_{s^{t+1}|s^t}q\left(s^{t+1}|s^t\right)a_l\left(s^{t+1}\right)=a_l(s^t)\\
a_l\left(s^{t+1}\right)\quad &\geq\quad A_l\left(s^{t+1}\right)
\end{aligned}$$

As above, the Euler and transversality conditions imply:

$$q\left(s^{t+1}|s^t\right)\geq\beta^t\pi\left(s^{t+1}|s^t\right)$$

with equality if $a_l\left(s^{t+1}\right) > A_l\left(s^{t+1}\right)$ and

$$\lim_{t\to\infty}\sum_{s^t}\left(\frac{1}{1+r}\right)^t\pi\left(s^t\right)\left[a_l\left(s^t\right)-A_l\left(s^t\right)\right]\leq 0$$

Market clearing implies that:

$$\begin{aligned}
c_b\left(s^t\right)+c_l\left(s^t\right) &= \theta(s^t)f\left(n(s^t)\right)\ \text{for all } s^t\\
a_b\left(s^{t+1}\right)+a_l\left(s^{t+1}\right) &= 0\ \text{for all } s^{t+1}
\end{aligned}$$

The values for the borrower and the lender in the trading arrangement can be writen recursively as

$$\begin{aligned}
W^b(a_b\left(s^t\right),\theta(s^t)) &= u(c_b(s^t))+U(1-n(s^t))+\\
&\quad\beta\sum_{s^{t+1}|s^t}\pi(s^{t+1}|s^t)W^b(a_b\left(s^{t+1}\right),\theta\left(s^{t+1}\right)) \qquad (11)\\
W^l(a_l\left(s^t\right),\theta(s^t)) &= c_l(s^t)+\\
&\quad\frac{1}{1+r}\sum_{s^{t+1}|s^t}\pi(s^{t+1}|s^t)W^l(a_l\left(s^{t+1}\right),\theta\left(s^{t+1}\right)) \qquad (12)
\end{aligned}$$

We assume that the borrowing limits are properly tight in the sense that satisfy:

$$\begin{aligned}
W^b(A_b\left(s^t\right),\theta(s^t)) &= V^{af}(s^t)\\
W^l(A_l\left(s^t\right),\theta(s^t)) &= Z
\end{aligned}$$

Let

$$Q\left(s^t|s_0\right)=q\left(s^1|s_0\right)q\left(s^2|s^1\right)...q\left(s^t|s^{t-1}\right)$$

We consider allocations that satisfy the *high implied interest rate condition*, namely:

$$\sum_{t=0}^{\infty}\sum_{s^t} Q\left(s^t|s_0\right)\left[c_b\left(s^t\right)+c_l\left(s^t\right)\right]<\infty$$

3.2 Decentralization

Let $\left\{c_b^*\left(s^t\right), c_l^*\left(s^t\right), n^*\left(s^t\right)\right\}$ be the allocations in the optimal fund contract. We now show that we can decentralize them as a competitive equilibrium with endogenous borrowing constraints that are not too tight. First, we use the allocations to define the Arrow security price as follows:

$$\begin{aligned}
q^*\left(s^{t+1}|s^t\right) &= \beta\pi\left(s^{t+1}|s^t\right)\frac{u'\left(c_b^*\left(s^{t+1}\right)\right)}{u'\left(c_b^*\left(s^t\right)\right)} \text{ if } v_b\left(s^{t+1}\right)=0 \text{ and } v_l\left(s^{t+1}\right)>0 \\
\text{or } v_b\left(s^{t+1}\right) &= v_l\left(s^{t+1}\right)=0 \\
q^*\left(s^{t+1}|s^t\right) &= \frac{1}{1+r}\pi\left(s^{t+1}|s^t\right) \text{ if } v_l\left(s^{t+1}\right)=0 \text{ and } v_b\left(s^{t+1}\right)>0
\end{aligned}$$

Note that the price can be expressed alternatively as follows (see Appnedix for details):

$$q^*\left(s^{t+1}|s^t\right)=\max\left\{\beta\pi\left(s^{t+1}|s^t\right)\frac{u'\left(c_b^*\left(s^{t+1}\right)\right)}{u'\left(c_b^*\left(s^t\right)\right)},\left(\frac{1}{1+r}\right)\pi\left(s^{t+1}|s^t\right)\right\} \quad (13)$$

Since we impose borrowing limits that bind exactly when the participation are binding in the optimal fund contract, $q\left(s^{t+1}|s^t\right)=q^*\left(s^{t+1}|s^t\right)$ satisfies the Euler conditions in the competitive equilibrium. In what follows, we let

$$Q\left(s^t|s_0\right)=q\left(s^1|s_0\right)q\left(s^2|s^1\right)...q\left(s^t|s^{t-1}\right)$$

Note also that $n^*\left(s^t\right)$ clearly satisfies the optimality condition in the competitive equilibrium with respect to labor.

We can use the intertemporal budget constraints to construct the **asset holdings** that make the allocations in the optimal contract satisfy the present value budget, namely:

$$\begin{aligned}
a_b\left(s^t\right) &= \sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t} Q\left(s^{t+n}|s^t\right)\left[c_b^*\left(s^{t+n}\right)-\theta(s^{t+n})f\left(n^*(s^{t+n})\right)\right] \\
a_l\left(s^t\right) &= \sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t} Q\left(s^{t+n}|s^t\right)c_l^*\left(s^{t+n}\right)
\end{aligned}$$

In this economy, binding participation constraints provide us with the borrowing limits; hence they are only binding when the enforcement constraints are binding. More precisely,

$$
\begin{aligned}
A_b\left(s^{t+1}\right) &= -\sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t} Q\left(s^{t+n}|s^t\right)\left[\theta(s^{t+n})f\left(n^*(s^{t+n})\right)\right] \\
A_l\left(s^{t+1}\right) &= Z,
\end{aligned}
$$

where the first equality refers for future histories s^{t+1} when the enforcement constraint binds.

The prices $q\left(s^{t+1}|s^t\right)$ derived from the allocation of consumption and labor of the optimal fund contracct, also define a price for a **one-period riskless bond**:

$$
q^f\left(s^t\right) = \sum_{s^{t+1}|s^t} q\left(s^{t+1}|s^t\right)
$$

This is the implicit price that we can use to compare with the one-period bond price in the incomplete markets economy. Alternatively, we can use the implicit interest rate: $r^f\left(s^t\right) = 1/q^f\left(s^t\right) - 1$ or the *spread*: $r^f\left(s^t\right) - r$.

Finally, since the technological shock is the only exogenous shock and it is Markovian, we can identify s^t with θ_t and rewritte the value functions of the fund contract (7) and (8) as:

$$
\begin{aligned}
V^{bf}(x,\theta) &= u(c(x,\theta)) + U(1-n(x,\theta)) + \beta\sum_{\theta'}\pi(\theta'|\theta)V^{bf}(x',\theta') \\
V^{lf}(x,\theta)(x,\theta) &= \theta f(n(x,\theta)) - c(x,\theta) + \frac{1}{1+r}\sum_{\theta'}\pi(\theta'|\theta)V^{lf}(x,\theta)(x',\theta').
\end{aligned}
$$

but, similarly, since the θ is also the only shock in the decentralized FSF economy and agents' decisions are recursive, we can redefine (11) and (12) as

$$
\begin{aligned}
W^{bf}(a_b,\theta) &= u(c_b(a_b,\theta)) + U(1-n(a_b,\theta)) + \beta\sum_{\theta'}\pi(\theta'|\theta)W^{bf}(a_b',\theta') \\
W^{lf}(a_l,\theta) &= c_l(a_l,\theta) + \frac{1}{1+r}\sum_{\theta'}\pi(\theta'|\theta)W^{lf}(a_l',\theta'),
\end{aligned}
$$

and, being an equilibrium, $c_l(a_l,\theta) = \theta f((a_b,\theta)) - c_b(a_b,\theta)$ and $a_l = -a_b$.These recursive equations clearly show how the value functions (W^{bf}, W^{lf}) are the mirror image of the value functions (V^{bf}, V^{lf}) on a different domain, and why we can use the derived prices to diagnose the properties of the optimal fund contract. However, notice that, while we have identfied s^t with θ_t, $q^f\left(s^t\right)$ is misspecified as $q^f(\theta)$, since it exclusive dependence on θ does not convey the binding participation constraits – that is, the events in which $q^*\left(s^{t+1}|s^t\right) \neq \left(\frac{1}{1+r}\right)\pi\left(s^{t+1}|s^t\right)$ in (13). In (W^{bf}, W^{lf}) this information is captured by x, in (V^{bf}, V^{lf}) by $a_l = -a_b$.

4 Solution Method and Parameterization

The short term contract is a standard incomplete market economy with a fixed borrowing constraints and we solve it using a standard policy iteration algorithm. In what follows, we discuss the solution method that we use for the long term contract with two sided lack of commitment as well as the parameterization for all our economies.

4.1 Solution Method

To solve for the long term contract with two sided lack of commitment, we assume the following utility for the borrower: $\log(c)+\frac{\gamma(1-n)^{1-\sigma}}{1-\sigma}$ and the following production function $f(n)=n^{\alpha}$. In this version of the model, there is only one shock so that $s=\theta$. Also, to simplify the algebra, we use as a state the relative pareto weight of the borrower $z=\frac{1}{x}$. Using the functional forms, the equilibrium conditions can be rewriten as:

$$\begin{aligned}
\frac{1}{c(z,\theta)} &= \frac{1-\upsilon_b(z,\theta)}{1-\upsilon_l(z,\theta)}\frac{1}{\eta z}=\frac{1}{z'} \\
z' &= \frac{1-\upsilon_l(z,\theta)}{1-\upsilon_b(z,\theta)}\eta z
\end{aligned}$$

$$c(z,\theta)\gamma(1-n(z,\theta))^{-\sigma}=\theta\alpha n(z,\theta)^{\alpha-1}$$

$$V^{bf}(z,\theta)=\log(c(z,\theta))+\frac{\gamma(1-n(z,\theta))^{1-\sigma}}{1-\sigma}+\beta\sum_{\theta'\in S}\pi(\theta'|\theta)V^{bf}(z',\theta').$$

$$V^{lf}(z,\theta)=\theta n(z,\theta)^{\alpha}-c(z,\theta)+\frac{1}{1+r}\sum_{\theta'\in S}\pi(\theta'|\theta)V^{lf}(z',\theta').$$

$$V^{af}(\theta)=\max_{n}\left\{\begin{array}{c}\log(\theta n^{\alpha})+\frac{\gamma(1-n)^{1-\sigma}}{1-\sigma}+\beta\sum_{\theta'\in S}\pi(\theta'|\theta)(1-\lambda)V^{af}(\theta') \\ +\beta\sum_{\theta'\in S}\pi(\theta'|\theta)\lambda V^{lf}(z^*(\theta'),\theta')\end{array}\right\}$$

To solve the problem, we use a policy function iteration algorithm that we describe in the appendix. We also implement the decentralization on the computer. To do this, we first recover the Arrow security prices from the allocations as follows:

$$\begin{aligned}
q(\theta'|\theta) &= \max\left\{\beta\pi(\theta'|\theta)\frac{u'(c(z',\theta'))}{u'(c(z,\theta))},\left(\frac{1}{1+r}\right)\pi(\theta'|\theta)\right\} \\
&= \max\left\{\beta\pi(s'|s)\frac{c(z,\theta)}{c(z',\theta')},\left(\frac{1}{1+r}\right)\pi(\theta'|\theta)\right\}
\end{aligned}$$

The price of a **one period bond** is then equal to:

$$q^f(\theta) = \sum_{\theta' \in S} q(\theta'|\theta)$$

which in turn implies a risk free rate of $r^f(\theta) = \frac{1}{q^f(\theta)}$. Finally, we can recover the asset holdings numerically by iterating to find the **asset holding** function that satisfies:

$$\begin{aligned} a_b(z,\theta) &= \sum_{\theta' \in S} q(\theta',\theta)\, a_b(z',\theta') + c(z,\theta) - \theta f(n(z,\theta)) \\ a_l(z,\theta) &= -a_b(z,\theta) \end{aligned}$$

Moreover, we define **the repayment** as:

$$a_b(z',\theta') - \sum_{\theta' \in S} q(\theta',\theta)\, a_b(z',\theta')$$

4.2 Parameterization

The model period is assumed to be one quarter. To make the different contracts comparable, we choose the same parameter values across economies. The technology shock θ is assumed to be a Markov chain with the following values and transition matrix:

$$\begin{aligned} \pi &= \begin{bmatrix} 0.9 & 0.1 & 0 & 0 & 0 \\ 0.1 & 0.8 & 0.1 & 0 & 0 \\ 0 & 0.1 & 0.8 & 0.1 & 0 \\ 0 & 0 & 0.1 & 0.8 & 0.1 \\ 0 & 0 & 0 & 0.1 & 0.9 \end{bmatrix} \\ \theta &= \begin{bmatrix} 0.1 & 0.3 & 0.5 & 0.7 & 0.9 \end{bmatrix} \end{aligned}$$

The discount factor of the borrower is assumed to be equal to $\beta = 0.96$ and the interest rate is set to $r = 0.01$, implying a different discount factor for the lender of $\frac{1}{1+r} = 0.9901$, as well as a growth rate for the relative pareto weight of the borrower of $\eta = 0.9696$ in the optimal contract. As stated earlier, we assume that the utility of the borrower is additively separable in consumption and leisure, with $\sigma = 2$ and $\gamma = 1$. The labor share in the technology of the borrower is set to $\alpha = 0.67$. The participation constraint of the lender in the optimal contract is set to $Z = -0.5$, and we choose the same value for the borrowing limit in the short term contract. Finally, the probability that the borrower comes back to the market upon default in the optimal contract is set to $\lambda = 0.01$.

5 Numerical Results

This section discusses the numerical results from our preliminary solution. For now, we present the results of the optimal contract with both one sided and two sided lack of commitment, as well as the results from the short term contract with incomplete markets but no default.

5.1 Policy Functions

This section briefly discusses the policy functions from our numerical solution. Figure 1 displays the policy functions for the main variables in the optimal contract with two sided lack of commitment. The policies are ploted for all the values of the technology shock θ as a function of the relative pareto weight of the borrower, which we denote *pareto weight* in what follows.

The left panel on the first row depicts the policy function for the borrower's consumption. As shown earlier, this follows exactly the law of motion of the pareto weight z'. The graph reflects that the borrower's consumption is increasing in the technology shock θ. For a given value of the shock, consumption is also increasing in the pareto weight inside the region where none of the participation constraints are binding, while it is flat when the participation constraint binds for the borrower (for low values of the pareto weight) or for the lender (for high values of the pareto weight). We also see that the borrower's participation constraint is binding for higher values of the pareto weight if the technology shock is higher, since this increases his autarky value. However, the opposite happens for the lender, since the lending contract is able to generate more revenue with a higher technology shock even if the borrower consumes more. The value function of the borrower, which is depicted in the left panel of the second row, follows closely the borrower's consumption, with flat regions whenever the participation constraints are binding.

The right panel of the first row depicts the labor supply of the borrower. For a given value of the shock, the borrower consumes more and thus works less as the pareto weight increases. Unlike the consumption, however, labor supply is not necessarily monotone in the technology shock. This is because of the interaction of two different effects. On the one hand, it is of efficient from he production point of view, for more productive workers to work more. This effect dominates for high levels of the pareto weight, which deliver a relatively high consumption level to the borrower. On the other hand, for low values of the pareto weight, the borrower might work more even if he is less productive in order to compensate for the relatively low consumption.

The value of the lender is depicted in the right panel of the second row. It is decreasing in the pareto weight but it is not monotone in the technology shock for similar reasons to the ones just discussed. An interesting observation is that the value of the lender is almost a mirror image of the implicit borrower's asset holdings derived in the decentralization, which are depicted in the right panel of the third row. To see why this is the case, consider first the case with one sided lack of committment. As shown in the decentralization of the optimal contract, the asset holdings of the borrower are equal to the negative of the present value of the lender's consumption, using the market discount factor:

$$a_b\left(s^t\right) = -\sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t} Q\left(s^{t+n}|s^t\right)\left[\theta(s^{t+n})f\left(n^*(s^{t+n})\right) - c_b^*\left(s^{t+n}\right)\right]$$

Since the lender is always unconstrained with one sided lack of committment, the market discount factor is always equal to his own. Therefore, the asset holdings of the borrower are exactly equal to the negative of the lender's value function, $a_b\left(s^t\right) = -V^{lf}\left(s^t\right)$. While the market discount factor is not always equal to the one of the lender with two sided lack of commitment, the borrower's asset holdings and the lender's value function still follow each other pretty closely.

Finally, the left panel of the last row depicts the equilibrium bond prices. As shown before, the bond price is equal to $\frac{1}{1+r}$ for low values of the pareto weight, implying that the participation constraint is only binding for the borrower. When the participation constraint of the lender starts binding, the bond price may increase above $\frac{1}{1+r}$, dependind on the dynamics of optimal consumption. Note that, similarly to other (endogenous or exogenous) incomple market model, the implied risk-free rate can even take negative values.

5.2 Simulations.

In this section, we discuss the simulation results from three different experiments. In the first, denoted by *business cycles* (Figures 2a and 2b), we assume that the economy is hit by a series of consecutive expansions and recessions that last for ten periods, corresponding to two and a half years with our quarterly calibration. In the second, denoted by *crisis* (Figures 3a and 3b), we assume that the economy is hit by the lowest income shock forever. In the third, denoted by *negative impulse response* (Figures 4a and 4b), we compute the average impulse response from 2000 simulations of 100 periods each that start with the lowest technology shock. In Figures 2a, 3a and 4a, we compare an Autarkic economy, with two FSF economies, one with one-sided limited commitment (i.e. there the lender has full commitment) and one with two-sided limited commitment, while in Figures 2b, 3b and 4b, we compare the same FSF economy with two-sided limited commitment with the Incomplete Markets economy without default.

The three experiments have several common patterns, that discriminate across economies, or regimes. The economy with one-sided lack of commitment – the closest to the economy with complete markets and full commitment, not shown in the figures – is the one with constant bond prices and higher welfare (borrower's welfare is negatively related to his asset holdings), it is also the one with smoother consumption over the business cycle and with a lower drop of consumption when there is a negative (impulse response) shock. In contrast, autarky is the economy with the highest volatility of consumption over the business cycle, the one with the largest drop of consumption after a negative productivity shock and the one with lowest welfare in the three experiments, as expected.

Of particular interest is the FSF economy with two-sided limited commitment. In Figures Figures 2a, 3a and 4a, it shows as an intermediate case between autarky and one-sided limited commitment, but with an important difference with respect to the latter: bond prices are not constant and, in particular, respond to the tightening of the fund credit conditions when the lender hits the participation constraint. In other words, lender's asset holdings (and, correspondingly, welfare) fluctuates in the one-sided limited commitment FSF economy, however bond prices are constant since the lender never hits a binding participation constraint after period zero (see Subsection 3.2). The lender's unwillingness to accommodate to the needs of the borrower (e.g. of getting into a path of persistent transfers), translates into a higher volatility in the two-sided limited commitment technology. Nevertheless, when this latter economy is compared with the incomplete markets economy (Figures 2b, 3b and 4b) it shows the two-sided limited commitment economy displays better performance, in terms of consumption smoothing and welfare and resilience to crisis.

FIGURES

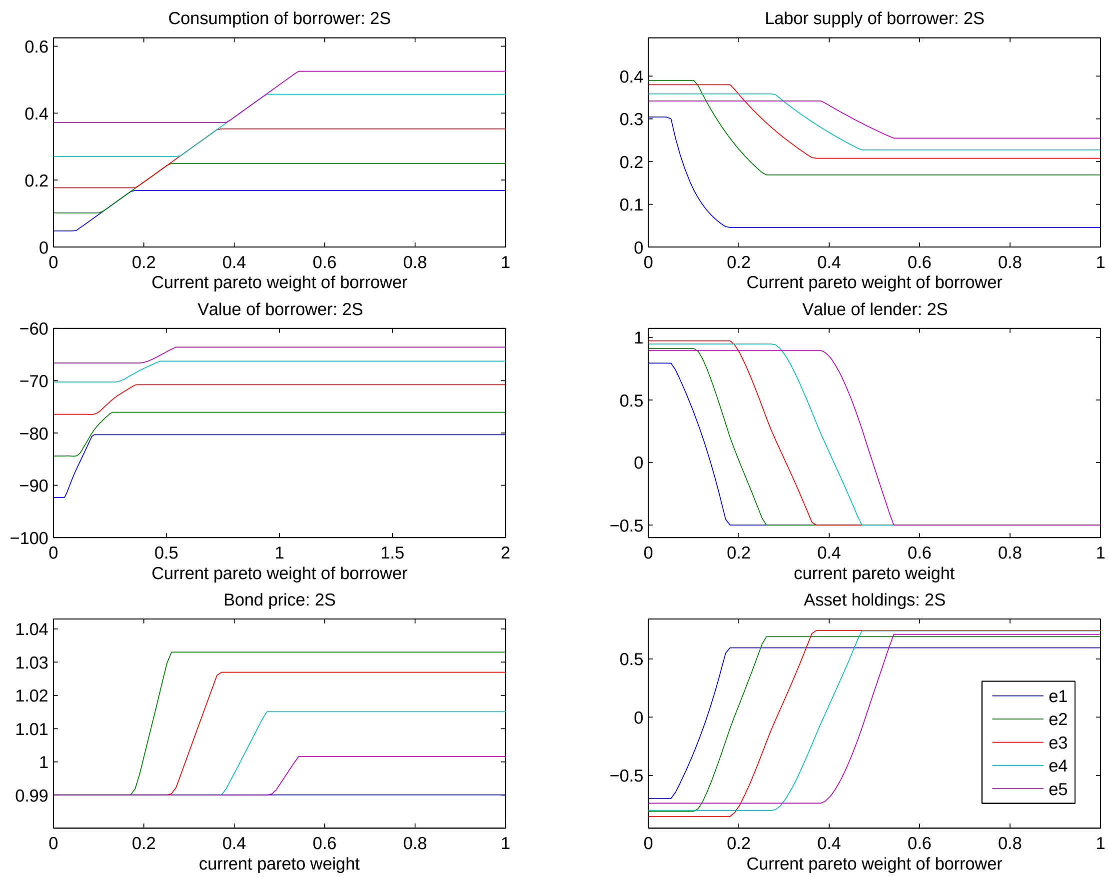

Fig 1: Policy Functions in the optimal contract with two sided lack of committment.

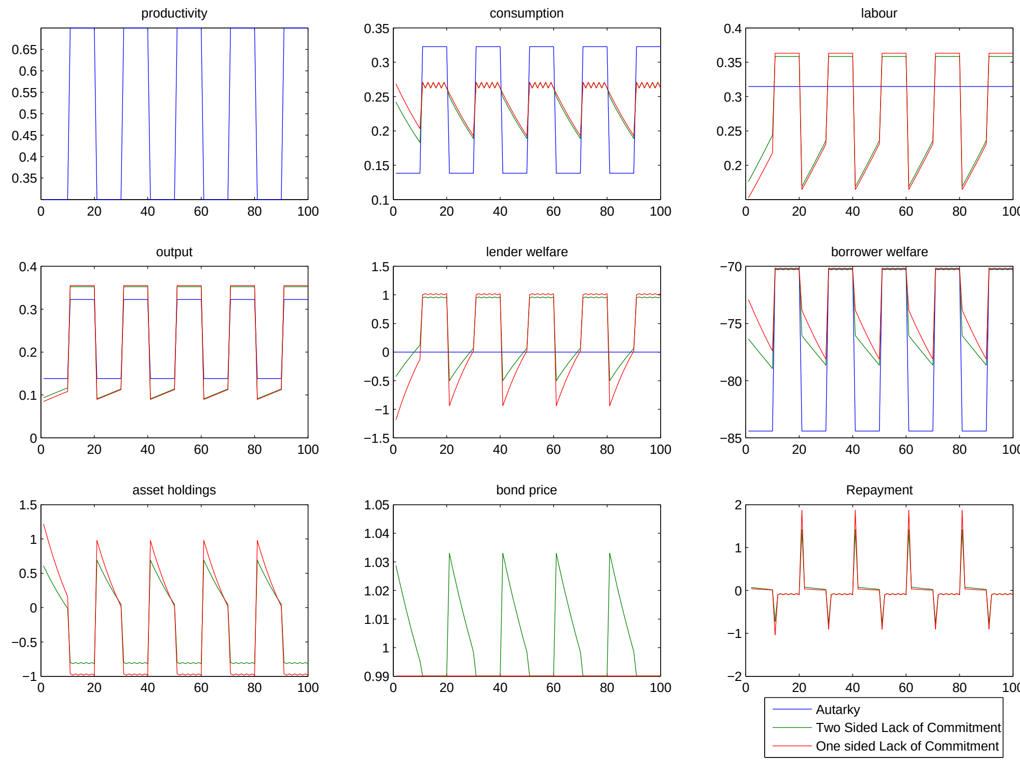

Fig. 2a: Business Cycle simulation for the optimal contract

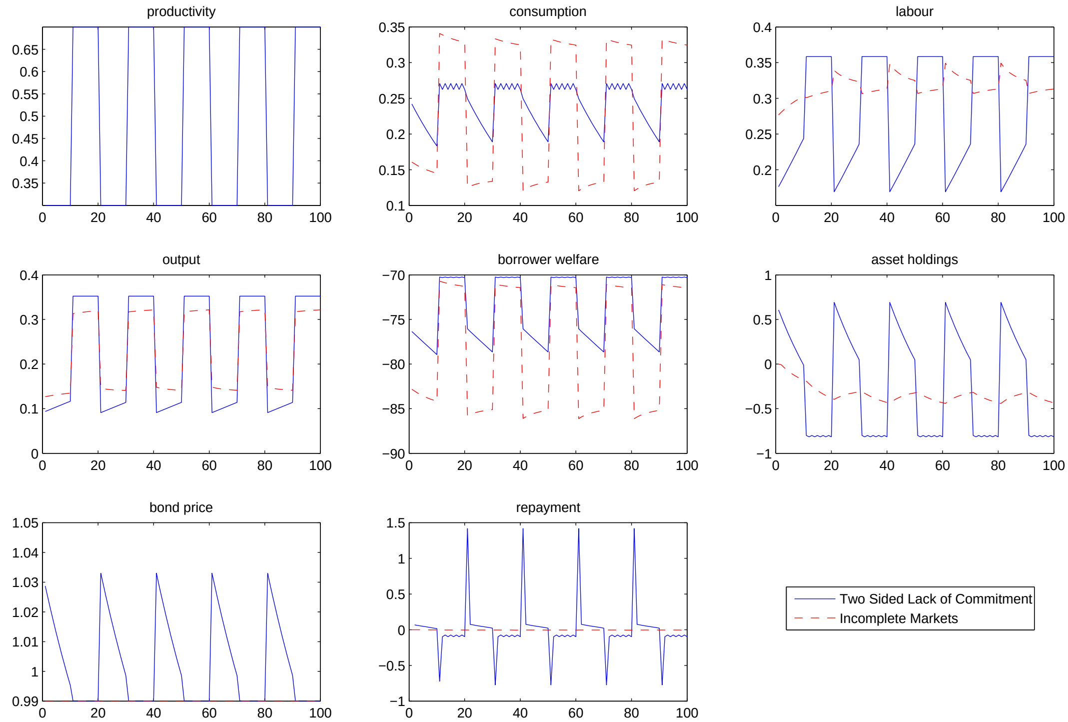

Fig. 2b: Business Cycle simulation in the optimal contract with two sided lack of committment and in the short term contract.

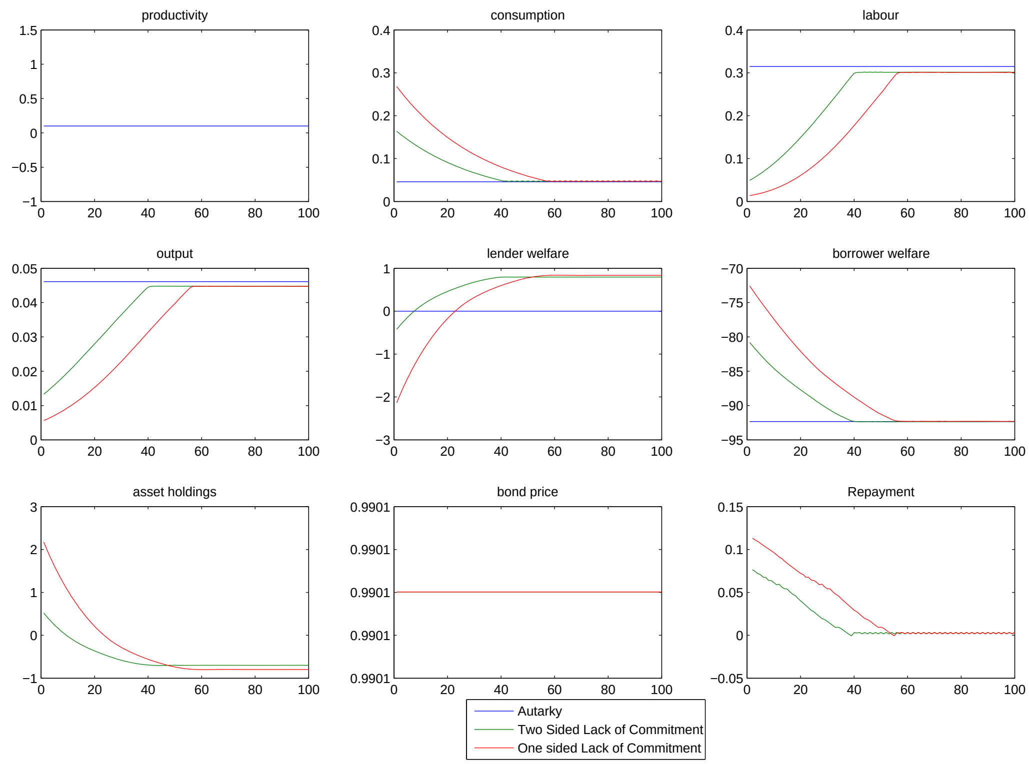

Fig. 3a: Crisis Simulation in the optimal contract

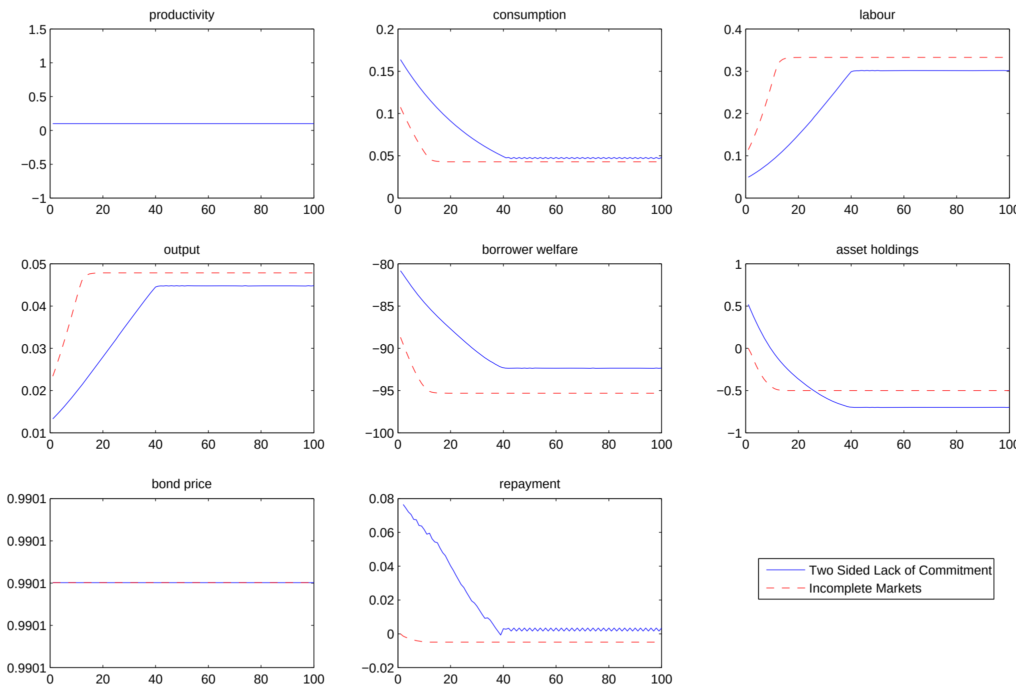

Fig. 3b: Crisis Simulation in the optimal contract with two sided lack of committment and in the short term contract.

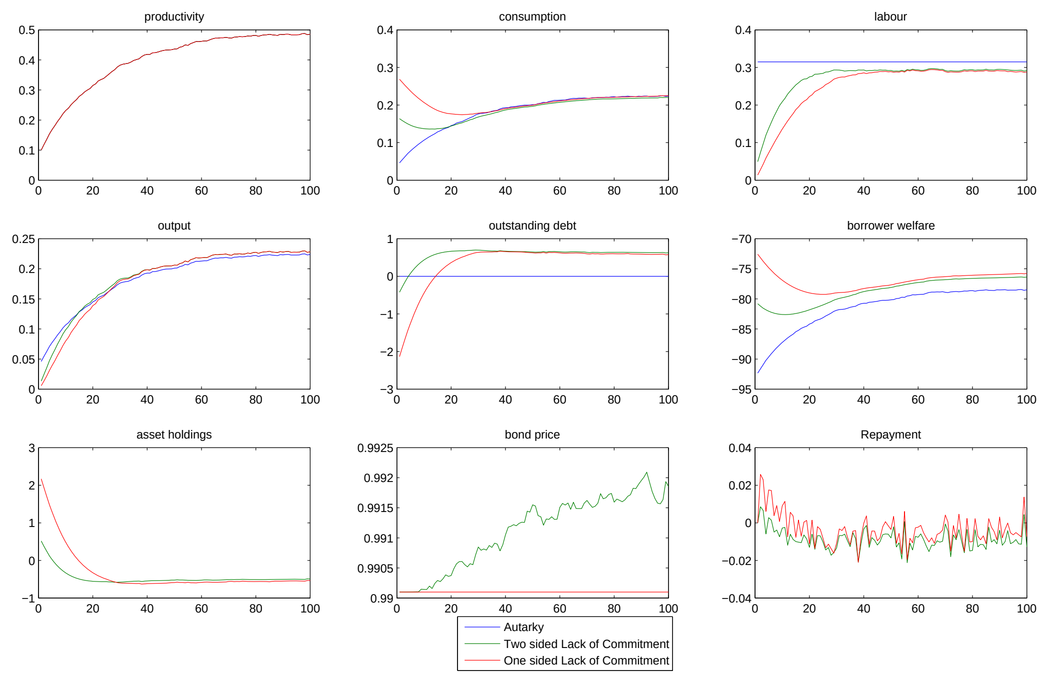

Fig. 4a: Average negative impulse responses from optimal contract

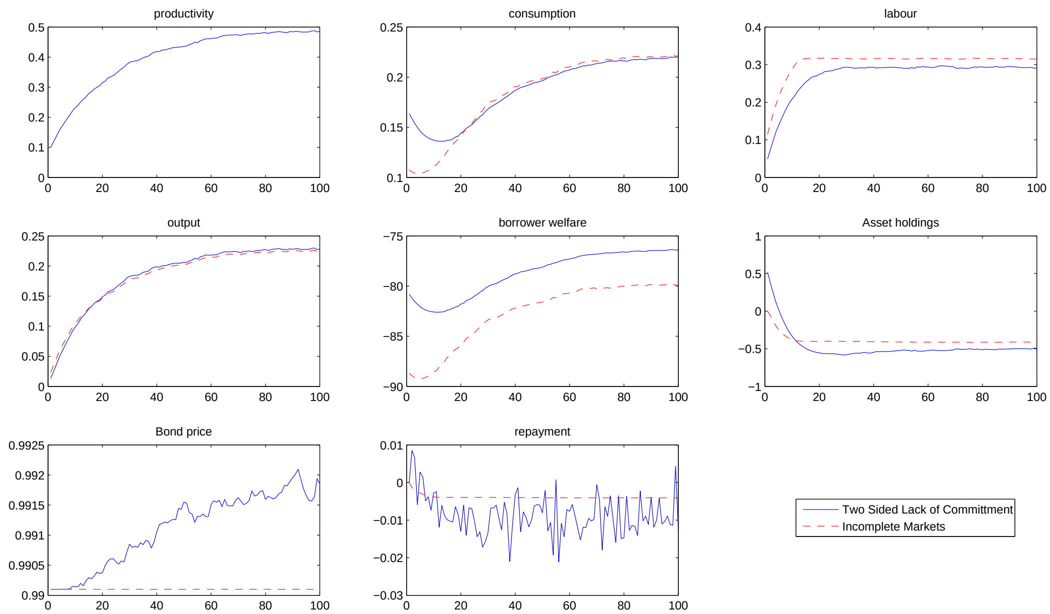

Fig. 4b: Average negative impulse responses for the optimal contract with two sided lackof committment and the short term contract.

6 Appendix

6.1 Asset prices and transverslty conditions

We first show that asset prices can be defined by (5).To see this, notice that the optimality conditions of the fund contract imply:

$$\frac{\beta u'(c_b(s^t))}{u'(c_b(s^{t-1}))} = \frac{1}{(1+r)} \frac{(1-v_b(s^t))}{(1-v_l(s^t))}$$

If the participation constraint is not binding for either agent then $v_b\left(s^t\right) = v_l\left(s^t\right) = 0$ and

$$\frac{\beta u'(c_b(s^t))}{u'(c_b(s^{t-1}))} = \frac{1}{(1+r)}$$

If the participation constraint is binding only for the lender, then $v_b\left(s^t\right) = 0$ and $v_l\left(s^t\right) > 0$ so that:

$$\frac{\beta u'(c_b(s^t))}{u'(c_b(s^{t-1}))} = \frac{1}{(1+r)} \frac{1}{(1-v_l(s^t))} > \frac{1}{(1+r)}$$

If the participation constraint is binding only for the borrower then $v_b\left(s^t\right) > 0$ and $v_l\left(s^t\right) = 0$ so that:

$$\frac{\beta u'(c_b(s^t))}{u'(c_b(s^{t-1}))} = \frac{1}{(1+r)} \left(1-v_b(s^t)\right) < \frac{1}{(1+r)}$$

It follows that (5) properly defines the price of the one period state contingent claim.

We now show that the transversality conditions in the competitive equilibrium are satisfied:

$$\begin{aligned}
& \lim_{t\to\infty} \sum_{s^t} \beta^t \pi\left(s^t\right) u'\left(c_b^*\left(s^t\right)\right) \left[a_b\left(s^t\right) - A_b\left(s^t\right)\right] \\
\leq \quad & \lim_{t\to\infty} \sum_{s^t} \beta^t \pi\left(s^t\right) u'\left(c_b^*\left(s^t\right)\right) \left[\sum_{n=0}^{\infty} \sum_{s^{t+n}|s^t} Q\left(s^{t+n}|s^t\right) \left[c_b^*\left(s^{t+n}\right)\right]\right] \\
\leq \quad & u'\left(c_b^*\left(s_0\right)\right) \lim_{t\to\infty} \sum_{s^t} \beta^t \pi\left(s^t\right) \frac{u'\left(c_b^*\left(s^t\right)\right)}{u'\left(c_b^*\left(s_0\right)\right)} \left[\sum_{n=0}^{\infty} \sum_{s^{t+n}|s^t} Q\left(s^{t+n}|s^t\right) \left[c_b^*\left(s^{t+n}\right) + c_l^*\left(s^{t+n}\right)\right]\right] \\
\leq \quad & u'\left(c_b^*\left(s_0\right)\right) \lim_{t\to\infty} \sum_{s^t} Q\left(s^t|s_0\right) \left[\sum_{n=0}^{\infty} \sum_{s^{t+n}|s^t} Q\left(s^{t+n}|s^t\right) \left[c_b^*\left(s^{t+n}\right) + c_l^*\left(s^{t+n}\right)\right]\right] = 0
\end{aligned}$$

and

$$
\begin{aligned}
& \lim_{t\to\infty}\sum_{s^t}\left(\frac{1}{1+r}\right)^t \pi\left(s^t\right)\left[a_l\left(s^t\right)-A_l\left(s^t\right)\right] \\
\leq \quad & \lim_{t\to\infty}\sum_{s^t}\left(\frac{1}{1+r}\right)^t \pi\left(s^t\right)\left[\sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t} Q\left(s^{t+n}|s^t\right)\left[c_l^*\left(s^{t+n}\right)\right]\right] \\
\leq \quad & \lim_{t\to\infty}\sum_{s^t}\left(\frac{1}{1+r}\right)^t \pi\left(s^t\right)\left[\sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t} Q\left(s^{t+n}|s^t\right)\left[c_b^*\left(s^{t+n}\right)+c_l^*\left(s^{t+n}\right)\right]\right] \\
\leq \quad & \lim_{t\to\infty}\sum_{s^t} Q\left(s^t|s_0\right)\left[\sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t} Q\left(s^{t+n}|s^t\right)\left[c_b^*\left(s^{t+n}\right)+c_l^*\left(s^{t+n}\right)\right]\right]=0
\end{aligned}
$$

The first inequalities follow from the definitions of $a\left(s^t\right)$ and $A\left(s^t\right)$. The second inequalities follow from the fact that individual consumption is less than aggregate consumption. The third inequalities follow from the relationship between q and Q as well as the relationship between q and the marginal rates of substitution of the agents. The last inequalities follow from the assumption of high implied interest rates.

6.2 Solution Method

6.2.1 Long term contracts

To solve the model, we assume the following utility for the borrower: $\log(c)+\frac{\gamma(1-n)^{1-\sigma}}{1-\sigma}$and the following production function θn^{α}. With these functional forms, the equilibrium conditions are:

$$
\frac{1}{c\left(x,s\right)}=\frac{1-\upsilon_b(x,s)}{1-\upsilon_l(x,s)}\frac{x}{\eta}=x'
$$

$$
c\left(x,s\right)\gamma\left(1-n\left(x,s\right)\right)^{-\sigma}=\theta\alpha n\left(x,s\right)^{\alpha-1}
$$

$$
V^{bf}(x,s)=\log(c(x,s))+\frac{\gamma(1-n(x,s))^{1-\sigma}}{1-\sigma}+\beta\sum_{s'\in S}\pi(s'|s)V^{bf}(x',x').
$$

$$
V^{lf}(x,s)=\theta n(x,s)^{\alpha}-c(x,s)+\frac{1}{1+r}\sum_{s'\in S}\pi(s'|s)V^{lf}(x',s').
$$

$$
V^{af}(s)=\max_n\left\{\begin{array}{c}\log(\theta n^{\alpha})+\frac{\gamma(1-n)^{1-\sigma}}{1-\sigma}+\beta\sum_{s'\in S}\pi(s'|s)\left(1-\lambda\right)V^{af}(s') \\ +\beta\sum_{s'\in S}\pi(s'|s)\lambda V^{bf}(x^*(s'),s')\end{array}\right\}
$$

For simplicity, we rewrite the model differently using as a state variable the relative pareto weight for the borrower $z = \frac{1}{x}$, the system of equations above can then be rewriten as:

$$\begin{aligned}
\frac{1}{c(z,\theta)} &= \frac{1-\upsilon_b(z,\theta)}{1-\upsilon_l(z,\theta)} \frac{1}{\eta z} = \frac{1}{z'} \\
z' &= \frac{1-\upsilon_l(z,\theta)}{1-\upsilon_b(z,\theta)} \eta z
\end{aligned}$$

$$c(z,\theta)\gamma(1-n(z,\theta))^{-\sigma} = \theta\alpha n(z,\theta)^{\alpha-1}$$

$$V^{bf}(z,\theta) = \log(c(z,\theta)) + \frac{\gamma(1-n(z,\theta))^{1-\sigma}}{1-\sigma} + \beta \sum_{\theta'\in S} \pi(\theta'|\theta)V^{bf}(z',\theta').$$

$$V^{lf}(z,\theta) = \theta n(z,\theta)^{\alpha} - c(z,\theta) + \frac{1}{1+r} \sum_{\theta'\in S} \pi(\theta'|\theta)V^{lf}(z',\theta').$$

$$V^{af}(\theta) = \max_{n} \left\{ \begin{array}{c} \log(\theta n^{\alpha}) + \frac{\gamma(1-n)^{1-\sigma}}{1-\sigma} + \beta \sum_{\theta'\in S} \pi(\theta'|\theta)(1-\lambda)V^{af}(\theta') \\ +\beta \sum_{\theta'\in S} \pi(\theta'|\theta)\lambda V^{lf}(z^*(\theta'),\theta') \end{array} \right\}$$

To solve the problem, we use a policy function iteration algorithm that we describe in what follows:

- We discretize the shock for the borrower θ (S values) and the relative pareto weight for the borrower z (N values).
- For each grid point, we can calculate the value of autarky as follows:
  - find the optimal labor in autarky, which solves $\frac{\alpha}{n} = \gamma(1-n)^{-\sigma}$.
  - find $V^{bf}(z^*(\theta'),\theta')$ by first solving for $z^*(\theta')$ such that $V^{lf}(z^*(\theta'),\theta') = 0$.
  - calculate $V^{af}(\theta)$ from the previous equation
- We then define the region of pareto weights between which none of the participation constraints are binding as $[z(j+1), z(l-1)]$. In that region, the solution is characterized by the first best:

$$\begin{aligned}
\upsilon_b(z,\theta) &= \upsilon_l(z,\theta) = 0 \\
z' &= \eta z \\
c(z,\theta) &= \eta z \text{ and } c_l(z,\theta) = \theta n(z,\theta)^{\alpha} - \eta z \\
\eta z\gamma(1-n(z,\theta))^{-\sigma} &= \theta\alpha n(z,\theta)^{\alpha-1} \\
V^{lf}(z,\theta) &= \theta n(z,\theta)^{\alpha} - \eta z + \frac{1}{1+r} \sum_{\theta'\in S} \pi(\theta'|\theta)V^{lf}(z',\theta') \\
V^{bf}(z,\theta) &= \log(\eta z) + \frac{\gamma(1-n(z,\theta))^{1-\sigma}}{1-\sigma} + \beta \sum_{\theta'\in S} \pi(\theta'|\theta)V^{bf}(z',\theta').
\end{aligned}$$

- For a given shock θ, if the value function for the borrower at the worst possible pareto weight $z(1)$ is higher than his autarky value, then his participation constraint is never binding and we set $j = 0$.
- Similarly, for a given shock θ, if the value function for the lender at the worst possible pareto weight $z(N)$ is higher than his outside option then his participation constraint is never binding and we set $l = N + 1$.
- To find the region for which the participation constraint binds for the borrower, for each shock θ, we find $c(z_b, \theta) = \eta x_b$ such that $V^{bf}(z_b, \theta) = V^{af}(\theta)$.

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