

Life, Death and World Inequality

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Abstract

Life expectancy around the world has increased substantially since 1970. In contrast, consumption per capita has fallen in some countries, remained stagnant, or sharply increased in others. What are the welfare gains of the systematic increase in life expectancy around the world? How does a "full measure" of per capita income, one that adjusts for life expectancy, compare to standard measures of world inequality that only consider income? This paper documents how standard models used to answer these questions give rise to a number of predictions that are inconsistent with well-documented evidence, particularly on the value of statistical life. It then proposes a generalized model with non-separable preferences that exhibits a low elasticity of intertemporal substitution and a low degree of mortality aversion. The non-separable model reverts the counterfactual predictions of the standard model, and it also provides plausible measures of changes in welfare and inequality around the world.

Keywords: life expectancy, value of statistical life, mortality aversion, Epstein-Zin-Weil preferences, welfare, AIDS.

JEL Classification: D10, D64, D91, J1.

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1 Introduction

Life expectancy around the world has increased substantially since 1970. In contrast, consumption per capita has fallen in some countries, remained stagnant, or sharply increased in others. For instance, between 1970 and 2005 life expectancy went up by 20 years in Bangladesh, 22 in Indonesia, 18 in Nicaragua and 15 years in Gambia. However, consumption per capita remained almost unchanged in Bangladesh, was multiplied by a factor of 5 in Indonesia, increased by about 70% in Gambia and fell around 40% in Nicaragua. What are the welfare gains of the systematic increase in life expectancy around the world? How does a "full measure" of per capita income, one that adjusts for life expectancy, compare to standard measures of world inequality that only consider income?

A recent literature addressing these questions has found significant welfare gains from reduced mortality around the world, and a corresponding decrease in world inequality (Becker, Phillipson and Soares (2005) and Jones and Klenow (2011), more notably). However, the model that has been used in this literature has the counterfactual prediction that a number of poor countries do not value additional life. Figure 1 illustrates this observation in a cross-section of countries in 2005.¹ The figure plots the value of statistical life (VSL) normalized by consumption per capita against life expectancy. According to the figure, the VSL in a good number of countries is actually negative, implying that a decrease in life expectancy will bring a welfare gain! Although most of these countries are in Africa, interestingly they also include Bangladesh and Gambia. As war and AIDS remain a reality in Africa, the model portrayed in Figure 1 predicts that these disasters are actually beneficial because in these poor countries people would prefer a shorter life.

The origin of these predictions is that the standard model requires to assume a value of a minimum level of consumption. This value is not binding for rich countries but it is fundamental for countries in the middle, and specially the bottom, of the world income distribution. In this literature, the minimum level of consumption is computed in order to match the VSL in the United States. The minimum consumption calibrated in Figure 1 is \$1,204, which implies that countries with per capita consumption below this number would display a negative VSL. But the evidence on the VSL surveyed in Viscusi and Aldy (2003) indicates that in the poorest country for which data is available, India, the VSL is positive. Based on wage data for Indian manufacturing workers in 1990, whose average annual income was \$778, estimates of the VSL are around \$1 million, and they go up to \$4 million when correcting for self-selection. Figure 2 displays available estimates of the VSL normalized by annual income for different countries from Viscusi and Aldy (2003). Three different estimates for India appear on the top left corner of the plot. The figure suggests that the ratio of VSL to annual income is decreasing in income: poorer people value life relative to their annual income more than richer people do. Even excluding the data points from India, Figure 2 suggest that the ratio of the VSL to annual income hovers around 300 for a wide range of income levels. Additional evidence casting doubt on the prediction that poor people prefer a shorter life is offered in Figure 3, which is similar to Figure 2 but corresponds to estimates for different levels of

¹All the details on the construction of this figure are provided below in Section 2.

income within the US. Figure 3 suggests a wide variation of the VSL at lower income levels, but does not support the idea that the poor value life relatively less.

This paper proposes an alternative model to assess the welfare gains of life expectancy around the world. The model departs from the standard in this literature by considering non-separable preferences that explicitly separate risk or mortality aversion from intertemporal substitution as in Epstein and Zin (1989) and Weil (1990). This reformulation does not require the introduction of a minimum level of consumption above zero, and therefore does not imply that the poorest countries do not value life. In fact, separating mortality aversion from intertemporal substitution allows us to directly tie the VLS with the degree of mortality aversion. The standard separable model can be seen as a special case of the non-separable model that equalizes the parameters that govern intertemporal substitution and mortality aversion. Our analysis suggests that standard separable and non-separable preferences predict similar VSL for rich countries, but quite different for poor countries. In fact, we show that the predictions of the non-separable model are consistent with the patterns displayed by the data in Figures 1, 2 and 3. Specifically, in the non-separable model the ratio of the VSL to per capita consumption is decreasing in consumption, so gains in life expectancy are quite valuable for these countries.

In addition to its consistency with available data on the VSL, our non-separable model yields interesting predictions on the welfare effects of changes in life expectancy in poorer countries. Consider first the welfare changes across time between 1970 and 2005 for each country in the sample. The separable model implies that countries that lost life expectancy during this period experienced a welfare *gain*. This is the case because since the VSL is negative in these countries, then shorter life spans across time increase welfare. In contrast, the non-separable model predicts the opposite: a full measure of income that incorporates both changes in income and life expectancy indicates a welfare loss for these (mostly poor) countries. Turning now to countries that gained life expectancy between 1970 and 2005, which are mainly poor and middle-income countries, we show that the separable model computes a welfare *loss* for these countries. The reason is that the separable model heavily penalizes gains in life expectancy of even as much as 20 to 25 years because per capita consumption remained mostly stagnant, or even decreased in these countries. In contrast, as the VSL is positive in all countries under the non-separable model, large gains in life expectancy do show up as welfare gains even for those countries whose per capita income remained stagnant over the 1970-2005 period. The differences between the separable and non-separable models in evaluating the welfare gains across time are dramatic.

Consider now the welfare measures across countries in 2005. In this case we are comparing each country's income and full income against that of the US in 2005. We show that in a cross-section, the separable model implies that using a full measure of income that includes life expectancy reduces the world "full income inequality." In other words, once life expectancy is taken into account, poor countries fair better compared to the US than when only per capita consumption is considered. The reason is that even though life expectancy is much lower in these countries, under the separable model life is also worth less there (shorter life spans are preferred). Things are almost the opposite according to the non-separable model. For almost all countries the full measure of income implies

an increase in world inequality. In the non-separable case, poorer countries fair worse relative to the US when a full measure of income is taken into account because life expectancy there is too low. More interestingly, there are a number of countries with a life expectancy higher than the US for whom the full measure of income in the non-separable model ranks them above the US. These are plausible predictions and are reminiscent of the effects of ranking countries according to the Human Development Index (HDI) of the United Nations, rather than ranking them only by per capita income. Under the HDI ranking, the US is penalized due to its relatively lower life expectancy. These predictions do not follow from the separable model.

The model we propose can also be used to assess the welfare effects of positive events like the end of wars and devastating events like AIDS. For this purpose we compute full measures of income to compare 1990 and 2005, the relevant dates to the AIDS pandemic. Countries like Rwanda, Bhutan and Nepal gained 16, 12 and 11 years of life respectively between 1990 and 2005. Again, the welfare calculation from the separable model indicates that taking into account these gains in life expectancy makes these countries worse off relative to a measure that only takes income into account! In Bhutan, not only life expectancy increased by 12 years, but per capita consumption was also multiplied by 2 between 1990 and 2005. Having gained both life expectancy and income, it is hard to believe the prediction of the separable model. In contrast, the non-separable model more intuitively predicts that taking into account time variation, Bhutan is better off under a full measure of income. A number of countries lost years of life between 1990 and 2005, mostly due to AIDS: Central Africa (3 years), South Africa (9), Botswana (13) and Zimbabwe (19). Interestingly, Zimbabwe not only lost years of life but also 10% of their income. However, the separable model implies that taking into account the 19-year life loss in this country, welfare has actually increased. In contrast, the non-separable model intuitively reflects a tremendous welfare loss in Zimbabwe.

The remainder of the paper is organized as follows. Section 2 presents the standard separable case used in the literature, calibrates the model and derives the implications for the VSL across countries. Section 3 proposes an alternative model with non-separable preferences, calibrates it and discusses how this model overcomes the shortcoming of the separable model. In Section 4 we compute full measures of income in order to perform welfare evaluations across time and across countries. A special application to the effects of ending wars and the AIDS pandemic is presented in Section 5. Concluding comments are in Section 6.

2 The separable case

This section sets up a benchmark perpetual youth model used in the literature and highlights some of its shortcomings. A key feature of this standard model is that utility is time separable, so the expected utility framework applies when computing the lifetime utility of individuals. We show that this separable model calibrated to match US evidence generates values of statistical life for low and middle income countries that are inconsistent with available evidence. In particular, the model significantly under-predicts the willingness to pay for mortality reduction programs in those countries, and predicts that life span is a bad rather than a good for a large number of countries.

Similarly, the model predicts that a significant fraction of the world population would be better off dead rather than alive. The model is in the spirit of Yaari (1965), Usher (1973), Blanchard (1985), Rosen (1988), Murphy and Topel (2006) and particularly Becker, Phillipson and Soares (2005) –BPS henceforth.

2.1 A model of perpetual youth

Consider the problem of an individual who faces a constant survival probability, π , lifetime income Y , and preferences described by $\sum_{t=0}^{\infty} \beta^t [\pi^t u(c_t) + (1 - \pi^t) u(\omega)]$ where $\beta < 1$ is a pure discount factor, c_t is consumption at time t , ω is an imputed consumption level if dead and $u(c)$ is a standard per-period utility function satisfying $\lim_{c \rightarrow 0} u'(c) = \infty$. Markets are complete and the interest rate is assumed to be $1/\beta$. The individual's problem can be described as:

$$V(Y, \pi) = \max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta\pi)^t (u(c_t) - u(\omega)) \quad \text{subject to } Y \geq \sum_{t=0}^{\infty} (\beta\pi)^t c_t. \quad (1)$$

Given the stated assumptions, the first order condition of the problem implies a constant consumption path. The budget constraint can then be used to solve for consumption:

$$c_t = c \equiv (1 - \beta\pi) Y. \quad (2)$$

The model predicts that higher survival reduces consumption per-period. Substituting (2) into the utility function results in the indirect utility function:

$$V(Y, \pi) = \frac{u((1 - \beta\pi) Y) - u(\omega)}{1 - \beta\pi}. \quad (3)$$

A continuous time version of this model is used by BPS to assess the economic value of mortality reductions. Key for the exercise is to choose parameter values that match empirical evidence on the willingness to pay for mortality reduction programs. The most common target is the value of statistical life.

2.2 Value of statistical life

Consider a program that *permanently* increases the survival probability in p basis points. From the individual's perspective the program increases the chances to live longer while from the society's perspective the program saves certain number of lives each period. Let $WTP(p)$ be the willingness of an individual to pay for such program. $WTP(p)$ is implicitly defined by:

$$V(Y_0, \pi_0) = V(Y_0 - WTP(p), \pi_0 + p). \quad (4)$$

where the subscript 0 is used to identify the time when the program is introduced.

Let $P(p) \equiv (1 - \beta(\pi + p)) WTP(p)$ be the associated annual willingness to pay for the program.² The value of statistical life, VSL, can then be defined as:

$$VSL(p) \equiv P(p)/p = (1 - \beta(\pi + p)) WTP(p)/p. \quad (5)$$

A standard interpretation of the VSL is that it measures the society's willingness to pay to save a life. To motivate this interpretation, consider a pool of N identical individuals. The willingness to pay for the program by the entire pool is $P(p)N$ per period while the number of lives saved by the program in a period is pN . Therefore, $P(p)N/(pN) = P(p)/p$ measures the overall willingness to pay to save a life in a given period.

The value of statistical life is the key concept that we use to assess the model below. The VSL defined by (5) depends on p . A commonly used approximation of the VSL that does not depend on p considers only marginal variations in the survival probability. For small p , $WTP(p)$ can be approximated by

$$WTP(p) \simeq MRS \cdot p. \quad (6)$$

where $MRS = |\partial Y / \partial \pi|_V$ is the marginal rate of substitution between income and survival probability obtained from (3). Using (3), (2) and (6), the value of statistical life can then be approximated by

$$VSL \equiv (1 - \beta\pi) MRS = \beta Y \times \left[\frac{u(c) - u(\omega) - u'(c)c}{u'(c)c} \right] \quad (7)$$

The first term in the expression, βY , is the lifetime income associated to a life saved by the program. The term in the second bracket is an adjustment factor, a death aversion coefficient, that captures the gains from living longer. Living one more period entails $u(c) - u(\omega)$ more utils but it cost $u'(c)c$ utils to finance those additional utils, resources that otherwise could be used to finance higher consumption during previous periods. Therefore $c_s \equiv (u(c) - u(\omega) - u'(c)c)/u'(c)$ is the surplus of living one additional period measured in units of consumption while c_s/c , the adjustment factor in the second bracket, is the fraction of consumption that actually corresponds to net gains from living longer.

For the standard CRRA utility function with risk aversion parameter $\sigma > 0$, VSL can be written as:

$$VSL = \beta Y \times \left[\frac{\sigma - (\omega/c)^{1-\sigma}}{1 - \sigma} \right] = \beta Y \times \left[\frac{(c/\omega)^{\sigma-1} - \sigma}{\sigma - 1} \right]. \quad (8)$$

The expression in the middle of (8) helps to better understand the case $\sigma \in (0, 1)$, the low curvature case, while the expression at the right helps for the case $\sigma > 1$, the most common case in macro. Notice first that if $\sigma > 1$ then $VSL \rightarrow \infty$ as $\omega \rightarrow 0$. This result shows that ω is the key parameter for matching empirical evidence on the value of statistical life and cannot be set arbitrarily to zero. A second observation is that a positive VSL , which also means that longevity is a good rather than

²In other words, $c = (1 - \beta(\pi + p))(Y - WTP(p)) = (1 - \beta(\pi + p))Y - (1 - \beta(\pi + p))WTP(p) = (1 - \beta(\pi + p))Y - P(p)$.

a bad, requires

$$c > c_{\min} \equiv \omega \sigma^{\frac{1}{\sigma-1}} > \omega. \quad (9)$$

Consider the BPS parameters: $\omega = \$353$ and $\sigma = 0.8$. In this case, $c_{\min} = \$1,077$. This means that the model predicts that individuals with consumption below \$1,077 in 1990 prices, would not value life extensions but would rather prefer shorter life spans. As we show below, a significant number of countries and individuals fall below such minimum consumption. This prediction is not only counter-intuitive but also inconsistent with empirical evidence on the willingness to pay for life extensions, as we show below. A third observation is that VSL increases with σ . This is clear for the case $\sigma \in (0, 1)$ and $\omega = 0$ but follows true for the general case given that $c > \omega \sigma^{\frac{1}{\sigma-1}}$.

Notice that equation (7) is an approximation of (5) for small p . The empirical estimates of the VSL typically considers small changes in p . However, it is important to recognize that (7) is not a good approximation for large p . To construct an appropriate measure of VSL for large p , let $\overline{WTP}$ be the willingness to pay for eternal life. $\overline{WTP}$ is defined implicitly by $V(Y, \pi) = V(Y - \overline{WTP}, 1)$. Given $\overline{WTP}$, then $P(p)$ is given by

$$P(p) \simeq (1 - \beta) \overline{WTP} \frac{p}{1 - \pi}, \quad (10)$$

case in which the approximation for VSL for large p , $\overline{VSL}$, would be

$$\overline{VSL} = \frac{1 - \beta}{1 - \pi} \overline{WTP} = \frac{1 - \beta}{1 - \pi} Y \left[1 - \frac{1 - \beta \pi}{1 - \beta} \frac{1}{c} u^{-1} \left(u(c) \frac{1 - \beta + \beta(1 - \pi) u(\omega)/u(c)}{1 - \beta \pi} \right) \right], \quad (11)$$

which follows from (5), (10), (3) and (2). For CRRA utility (11) becomes

$$\overline{VSL} = \frac{1 - \beta}{1 - \pi} Y \left[1 - \left(\frac{1 - \beta}{1 - \beta \pi} \right)^{\frac{\sigma}{1 - \sigma}} \left[1 + \left(\frac{\omega}{c} \right)^{1 - \sigma} \frac{(1 - \pi)}{(1/\beta - 1)} \right]^{\frac{1}{1 - \sigma}} \right]. \quad (12)$$

Finally, a useful concept is the willingness to pay for an extra year of life, P_T , or the value of a year of life. Since life expectancy is given by $T = 1/(1 - \pi)$, then P_T can be approximated by:

$$P_T = \left| \frac{\partial Y}{\partial T} \right|_V = MRS \cdot \frac{\partial \pi}{\partial T} = \frac{MRS}{T^2} = \frac{VSL}{1 - \beta \pi} \frac{1}{T^2}.$$

2.3 Evidence on the VSL

There is a large literature estimating the VSL. Estimation is often based on wage differential across occupations with different mortality risks, or from market prices for products that reduce fatal injuries. For example, suppose a worker requires an annual premium of \$600 per year in order to accept an increase in the annual probability of accidental death of 1/10000. In a pool of 10,000 workers, one worker is expected to die and the aggregate compensation for such dead is

$VSL = \$600 * 10,000 = \6 million. Actual estimates of the VSL range between \$4 to \$9 million in 2004 dollars for a 40 year old male (Viscusi 1993, Viscusi and Aldy 2003). The Environmental Protection Agency has used \$6,3 million in cost-benefit analysis since 1993. A similar value is also used by Murphy and Topel (2006) in assessing the value of health and longevity.

2.4 Calibration and results

We now assess the ability of the benchmark model to match empirical estimates of the VSL. The following parameters and functional forms are needed: β , c , π , $u(\cdot)$ and ω . The strategy is to choose parameters and functional forms standard in the macroeconomic literature, compute the implied VSL for each country in the sample, assess their plausibility and perform multiple robustness checks.

For those parameters common across countries we proceed as follows. As in BPS, we set the risk free rate to 3%, which implies $\beta = 0.97$, and let $u(c) = c^{1-\sigma}/(1-\sigma)$ where $1/\sigma$ is the intertemporal elasticity of substitution (EIS). For our benchmark calibration we set $EIS = 0.8$ as in Murphy and Topel (2006), which is also a standard value used in the macroeconomic literature, but as a robustness check we also consider the alternative value of $EIS = 1.25$ used by BPS. The level of consumption imputed in the dead state, ω , is set to \$493, which is the same value used by BPS but in 2005 prices rather than 1990 prices.³ This value of ω implies a minimum level of consumption $c_{\min}$ in (9) of \$1,204 in 2005 prices, a little above \$3 a day. Alternative values of ω are also considered for robustness checks.

Countries in our sample differ in their survival probability π and per capita consumption c . For the US, π in 2005 was 98.7%, which corresponds to a life expectancy at birth of 78 years. In addition, $c = \$32,230$ is the value of US per capita consumption in 2005 according to the Penn World Tables Version 7.0. The calibrated model implies a $VSL = \$4,8$ million for the US in 2005. This value is in the lower end of the range of reliable estimates according to Viscusi and Aldy (2003), but other relevant papers such as BPS and Hall and Jones (2007) also use VSL in the lower range.

We use a sample of 144 countries, larger than the one used in BPS and Jones and Klenow (2011). For each country in the sample we collect data on life expectancy between 1970 and 2005 from the World Development Indicators. In addition, data on per capita consumption at 2005 prices between 1970 and 2005 is collected from the Penn World Tables 7.0. For each country, the model implies a VSL for each year in 2005 prices. Figure 4 portrays the ratio of the VSL to per capita consumption (VSL/c) for all countries in the sample in 2005. Similar to Figure 1 discussed above, Figure 4 indicates that for a number of poorer countries, the VSL implied by

³BPS use the following procedure to obtain ω . They argue that $\varepsilon = \frac{v'c}{v} = 0.346$ is what the empirical estimates from the VSL literature find, where $v(c) = \frac{c^{1-\sigma}}{1-\sigma} + \alpha$ and $\alpha = c^{1-\sigma} \left(\frac{1}{\varepsilon} - \frac{1}{1-\sigma} \right)$. In our notation $v(c) = u(c) - u(\omega)$ so that $u(\omega) = \frac{\omega^{1-\sigma}}{1-\sigma} = -\alpha = c^{1-\sigma} \left(\frac{1}{1-\sigma} - \frac{1}{\varepsilon} \right)$. Therefore,

$$\omega = c \left(\frac{\varepsilon + \sigma - 1}{\varepsilon} \right)^{\frac{1}{1-\sigma}}.$$

the separable model is negative. These are countries for which annual per capita consumption in 2005 is below $c_{\min} = \$1,204$. There are 36 countries satisfying this criteria in our sample. All are African countries, except for Afganistan, Bangladesh, Nepal and Papua New Guinea. The calibrated version of this separable model implies that in these countries people would prefer a shorter life. In fact, as more than half of the people in the world live on \$2 a day, the model has dramatic implications for at least half of humanity.

Of course, it is in principle feasible to choose a lower value of ω so that the VSL is positive for all countries. This is the case if $\omega = \$50$, which implies a corresponding $c_{\min} = \$122$. However, such a value of ω implies a much larger VSL for the US of around \$11.4 million, outside of the range of available empirical estimates. But more importantly, even this would not be enough to improve the performance of the separable model. A lower value of ω would shift up the curve in Figure 4, but it would still imply that the ratio VSL/c is increasing in c , a feature that is inconsistent with the evidence provided by Viscusi and Aldy (2003) and displayed in Figures 2 and 3. As discussed before, if anything these figures imply that ratio VSL/c is decreasing in c .

2.5 Robustness

The results above are robust to different assumptions: $1 + r \neq 1/\beta$, life-cycle components (Murphy and Topel, 2006), and incomplete markets. [To be completed]

3 An alternative model

This section describes a model that solves the shortcomings of the standard separable model presented above. Our approach is to disentangle the EIS from a parameter that controls risk or mortality aversion using Epstein-Zin-Weil type of preferences. This non-separable utility representation allows the EIS and parameter ω to take on any plausible values, while letting the mortality aversion parameter to be the one matching the VSL. We use the estimates of the VSL in the US to identify the parameter that governs mortality aversion. Section 4 then uses the non-separable model to reassess the economic value of mortality reductions observed over time and across countries, and arrives at different conclusions from the existing literature.

3.1 A model with non-separable utility

Consider the following Epstein-Zin-Weil (EZW) preferences:

$$V_t = \left\{ c_t^{1-\sigma} + \beta[\pi V_{t+1}^{1-\gamma} + (1-\pi) D^{1-\gamma}]^{\frac{1-\sigma}{1-\gamma}} \right\}^{\frac{1}{1-\sigma}}, \sigma \geq 0, \gamma \geq 0.$$

where V and D are the lifetime utilities of being alive and dead respectively, $1/\sigma$ is the intertemporal elasticity of substitution (while alive) and γ is a parameter describing the degree of risk or mortality aversion. Appendix A shows that $\gamma = 0$ implies risk neutrality, $\gamma > 0$ implies risk aversion and

$\gamma < 0$ implies risk loving. The advantage of this formulation is that it separates these two concepts. For example, individuals may be unwilling to accept consumption jumps while alive (σ is large) but willing to accept a consumption jump at the time of death (γ is small). The formulation is also able to replicate standard results from the separable model when $\gamma = \sigma$. Notice also that V_t is increasing in π as long as $V_{t+1} > D$.

The value of D satisfies:

$$D = \left\{ \omega^{1-\sigma} + \beta[\pi D^{1-\gamma} + (1-\pi) D^{1-\gamma}]^{\frac{1-\sigma}{1-\gamma}} \right\}^{\frac{1}{1-\sigma}} = \omega (1-\beta)^{\frac{1}{\sigma-1}}.$$

Parameter γ controls the degree to which c and ω are substitutes. We assume from the start that $\omega = 0$ meaning that any individual with positive consumption would prefer to be alive rather than dead. This assumption would imply $VSL = \infty$ in the separable model but not necessarily in the current model. This is a convenient assumption because it reduces notation significantly without major cost. The reason is because any plausible calibration would require ω to be small anyway. An implication of this assumption, however, is that γ is restricted to be between 0 and 1. Otherwise, if $\gamma > 1$ and $D = 0$, then $V_t \rightarrow 0$ if $\sigma > 1$, or $V_t = c_t$, if $\sigma \in (0, 1)$. As we discuss below, the restriction that γ must be below 1 is not binding because a lower-than-one value of γ is required to match the observed VSL.

The resulting lifetime utility function when $\omega = 0$ is given by:

$$V_t = \left\{ c_t^{1-\sigma} + \beta \pi^{\frac{1-\sigma}{1-\gamma}} V_{t+1}^{1-\sigma} \right\}^{\frac{1}{1-\sigma}}, \quad \sigma \geq 0, \quad 0 \leq \gamma \leq 1, \quad \beta < 1.$$

Repeated substitution allows to rewrite this function as:

$$V_t = \left[\sum_{s=0}^{\infty} \tilde{\beta}(\pi)^s c_{t+s}^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad (13)$$

where $\tilde{\beta}(\pi) = \beta \pi^{\frac{1-\sigma}{1-\gamma}}$. Individuals choose a consumption path to maximize their utility subject to the budget constraint

$$Y \geq \sum_{t=0}^{\infty} \left(\frac{\pi}{1+r} \right)^t c_t, \quad (14)$$

where Y is lifetime income, r is the interest rate and markets are assumed to be complete. The first order condition for consumption reads

$$\tilde{\beta}(\pi)^t c_t^{-\sigma} = \frac{1+r}{\pi} \tilde{\beta}(\pi)^{t+1} c_{t+1}^{-\sigma}$$

or

$$c_{t+s} = c_t \left(\frac{\tilde{\beta}(\pi) (1+r)}{\pi} \right)^{s/\sigma} = c_t \left(\beta \pi^{\frac{\gamma-\sigma}{1-\gamma}} (1+r) \right)^{s/\sigma}. \quad (15)$$

Notice that this condition is identical to the one in the separable model when $\sigma = \gamma$. The equation shows that consumption grows faster the more the individual cares about the future, the larger the return on savings, $1+r$, but also the larger $\pi^{\frac{\gamma-\sigma}{1-\gamma}}$. If $\sigma = \gamma$ then the survival probability does not affect consumption growth because it affects equality the marginal utility of consumption but also the marginal cost as prices reflect the chances of survival. If $\sigma > \gamma$, which is the case we stress below, then higher survival reduces consumption growth for surviving individuals because it further increases the need to smooth consumption.

Substituting (15) into (14) and solving for c_0 results in:

$$c_0 = \left(1 - \mu \pi^{\frac{\gamma}{1-\gamma} \frac{1-\sigma}{\sigma}} \right) Y \quad (16)$$

where $\mu \equiv \beta^{\frac{1}{\sigma}} (1+r)^{\frac{1-\sigma}{\sigma}}$ and $\mu \pi^{\frac{\gamma}{1-\gamma} \frac{1-\sigma}{\sigma}} < 1$ is assumed. This expression is identical to the one in the separable case when $\sigma = \gamma$. Substituting (15) and (16) into (13) and simplifying results in the following indirect utility function:

$$V(Y, \pi) = Y \left(1 - \mu \pi^{\frac{\gamma}{1-\gamma} \frac{1-\sigma}{\sigma}} \right)^{-\frac{\sigma}{1-\sigma}} \quad (17)$$

It is convenient to compare this expression with the one that can be obtained in the separable case:⁴

$$V_{sep} = \frac{Y^{1-\sigma}}{1-\sigma} (1 - \mu \pi)^{-\sigma}$$

which suggests that if $\gamma = \sigma$ then one is a monotonic transformation of the other: $V = (1 - \sigma) V_{sep}^{1/(1-\sigma)}$.

3.2 VLS in the non-separable model

Equation (17) can be used to compute $MRS = |\partial Y / \partial \pi|_V$. It follows that:

$$MRS = \frac{\gamma}{1-\gamma} \frac{\pi^{\frac{\gamma-\sigma}{\sigma(1-\gamma)}}}{1 - \mu \pi^{\frac{\gamma}{1-\gamma} \frac{1-\sigma}{\sigma}}} \mu Y = \frac{\gamma}{1-\gamma} \frac{1}{\pi^{\frac{\sigma-\gamma}{\sigma(1-\gamma)}} / \mu - \pi} \frac{c_0}{1 - \left(\tilde{\beta}(\pi) ((1+r)/\pi)^{1-\sigma} \right)^{1/\sigma}}.$$

The VSL can then be defined as

$$VSL = \left(1 - \mu \pi^{\frac{\gamma}{1-\gamma} \frac{1-\sigma}{\sigma}} \right) MRS = \frac{\gamma}{1-\gamma} \frac{c_0}{\pi^{\frac{\sigma-\gamma}{\sigma(1-\gamma)}} / \mu - \pi}. \quad (18)$$

⁴The corresponding separable case would be one with $\omega = 0$, $\sigma \in (0, 1)$ to avoid $V = \infty$, and an arbitrary interest rate, not just $1+r = 1/\beta$.

Notice the following properties of the VSL described by (18). First, VSL is finite for any value of σ . Remember that in the separable case $V = \infty$ when $\omega = 0$ and $\sigma > 1$. This is a key improvement over the separable case because for sensible parameters, $\omega = 0$ and $\sigma > 1$ the model does necessarily produce an unbounded VSL. Second, γ is the key parameter determining the value of life not σ . Since the separable model forces $\gamma = \sigma$, then σ becomes crucial in that model, but once this assumption is relaxed then σ loses its key importance and the weight of the prediction lies on γ . Notice that (18) with $\sigma = \gamma$ is the same expression obtained for the separable case, (8), when $\omega = 0$ and $\sigma \in (0, 1)$, except that γ has taken the place of σ .

The value of an additional year of life in the non-separable utility representation is given by:

$$P_T = \left| \frac{\partial Y}{\partial T} \right|_V = MRS \cdot \frac{\partial \pi}{\partial T} = \frac{MRS}{T^2} = \frac{VSL}{1 - \mu \pi^{\frac{\gamma}{\sigma} \frac{1-\sigma}{1-\gamma}}} \frac{1}{T^2}.$$

3.3 Calibration and results

We use the same parameters of the benchmark separable model except that ω is set to zero. In particular, we set $r = 3\%$, $\beta = 1/(1+r)$, $\sigma = 1.5$ and country specific values for π and c using the World Development Indicators and the Penn World Tables 7.0. The only new parameter is γ . We calibrate this parameter to obtain the same VSL for the US under the separable model, which is \$4.8 million in 2005. This results in $\gamma = 0.76$. Notice that under this calibration consumption is not constant (see equation 15), but results are similar if the interest rate is set for each country so that consumption is constant.

Figure 5 displays the ratio VSL/c for all countries in the sample in 2005 for both the non-separable and the separable models. Results are quite different for both models, except for richer countries. Different from the separable case, there are no negative VSL for any country under the non-separable model. More importantly, the pattern of ratio VSL/c is quite different: increasing in c under the separable model and decreasing in c under non-separability. An interesting feature of the ratio VSL/c under the non-separable model is its large dispersion for poor countries, specifically those with per capita consumption below \$2,500. This pattern mimics the one in Figure 3, which corresponds to within-US data. Figure 6 is similar to Figure 5 but it displays VSL/c as a function of life expectancy. Overall, Figures 5 and 6 suggest that longer life is valued all around the world, including poorer countries. In fact, as a percentage of per capita annual consumption, the non-separable model implies that life is relatively more valued where it is more scarce: in poorer countries.

Figure 7 complements the message above by portraying the willingness to pay for an extra year of life P_T as a fraction of lifetime income Y for both the separable and non-separable models. Again, results are quite similar for richer countries, but very different for poorer ones. The calibrated models suggest that in richer countries, including those with annual per capita consumption around \$20,000 and up, people are willing to pay around 2% of their lifetime income in order to live one more year. People in countries with per capita consumption around \$5,000 are willing to pay 1% of their lifetime income to live one more year in the separable model, and 3% in the non-separable

one. Finally, while under the separable model people in the poorest countries are willing to pay to live less, they would pay anywhere between 4 and 14% of their lifetime income for an extra year of life in the non-separable model. These different predictions are quite striking.

4 Welfare across countries and time

This section uses the models presented above to compute welfare measures across time and across countries. Specifically, we are interested in using the formula for $V(Y, \pi)$ in order to calculate a "full measure" of income adjusted for changes in life expectancy, $T = 1/\pi$. Let $V_0 \equiv V(Y_0, \pi_0)$ be the welfare in a benchmark situation and $V_i \equiv V(Y_i, \pi_i)$ the welfare in another situation i . For welfare measures across time, or growth calculations, the subscripts 0 and i refer to two different years for a given country, while for cross-country comparisons they refer to two different countries in a given year. The ratio of lifetime incomes is $R_i = Y_i/Y_0$ and it is the typical way to measure proportional welfare differences between both situations.

We now define a more comprehensive ratio of incomes that includes an imputed value for differences in life expectancy. We denote this ratio R_i^F where F stands for "full" income ratio which is defined implicitly by

$$V(R_i^F Y_0, \pi_0) = V(Y_i, \pi_i) \quad (19)$$

so R_i^F is the proportional change in Y_0 required to equate welfare in both situations. Notice that $R_i^F = R_i$ if $\pi_0 = \pi_i$, and $R_i^F \leq R_i$ if $\pi_i \leq \pi_0$.

R_i^F for the separable (s) and non-separable (n) cases can be easily obtained using (3) and (17) for the CRRA utility case. The solutions are given by:

$$R_i^{Fs} = \left[a \left(\frac{Y_i}{Y_0} \frac{1 - \beta\pi_i}{1 - \beta\pi_0} \right)^{1-\sigma} + (1-a) \left(\frac{\omega}{Y_0} \frac{1}{1 - \beta\pi_0} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad (20)$$

where $a = (1 - \beta\pi_0)/(1 - \beta\pi_i)$ and

$$R_i^{Fn} = \frac{Y_i}{Y_0} \left(\frac{1 - \left(\beta(1+r_0)^{1-\sigma} \right)^{\frac{1}{\sigma}} \pi_0^{\frac{\gamma}{1-\gamma} \frac{1-\sigma}{\sigma}}}{1 - \left(\beta(1+r_i)^{1-\sigma} \right)^{\frac{1}{\sigma}} \pi_i^{\frac{\gamma}{1-\gamma} \frac{1-\sigma}{\sigma}}} \right)^{\frac{\sigma}{1-\sigma}} \quad (21)$$

where $\mu_i \equiv \beta^{\frac{1}{\sigma}} (1+r_i)^{\frac{1-\sigma}{\sigma}}$.

The solution for the separable case is a CES function between the situations in the alive and dead states with a weight, a , that measures the relative change in "effective mortality rates". The larger the change in mortality the higher the weight assigned to the alive state. Moreover, the lower the σ the more substitutable is the consumption between the two states and the larger the value imputed to mortality changes.

For welfare calculations across time, we use as situation 0 1970 and situation i 2005 for each country. Figure 8 displays our relative full income measures across time for both the separable and the non-separable models. Specifically, the figure shows R_i^{Fs}/R_i and R_i^{Fn}/R_i as a function of the changes in life expectancy between 2005 and 1970. Notice that countries to the far right of the figure are those which substantially gained life expectancy, generally poorer countries. Those on the far left of the figure are also poorer countries, but they lost life expectancy probably due to war and AIDS. As richer countries had more modest gains in life expectancy, they are concentrated around zero. Figure 8 again indicates that the separable and non-separable models have similar predictions for richer countries. In addition, ratios R_i^{Fs}/R_i and R_i^{Fn}/R_i are closer to one for this set of countries. While the separable model exhibits a decreasing ratio R_i^{Fs}/R_i in Figure 8, ratio R_i^{Fn}/R_i from the non-separable model is increasing. More specifically, for countries that lost life expectancy between 1970 and 2005, $R_i^{Fs}/R_i > 1$ while $R_i^{Fn}/R_i < 1$. This is the case because since the VSL is negative in these countries, then shorter life spans across time increase welfare in a full measure of income under the separable model. Turning now to countries that gained life expectancy between 1970 and 2005, consider first those on the far right of the figure: while R_i^{Fn}/R_i is well above one for these countries, R_i^{Fs}/R_i is slightly below one. The separable model heavily penalizes gains in life expectancy of even as much as 20 to 25 years because per capita consumption remained mostly stagnant, or even decreased in these countries. In contrast, as the VSL is positive in all countries under the non-separable model, large gains in life expectancy do show up as welfare gains even for those countries whose per capita income remained stagnant over the 1970-2005 period. The differences between the separable and non-separable models displayed in Figure 8 are substantial.

For welfare calculations across countries from equations (20) and (21), we label as 0 the US and as i each of the countries in the sample in 2005. Figure 9 reports the results of the cross-country welfare calculations. Specifically, the figure shows R_i^{Fs}/R_i and R_i^{Fn}/R_i as a function of life expectancy in 2005. The remarkable feature of this figure is that for most countries in the sample, the separable model implies $R_i^{Fs}/R_i > 1$ and R_i^{Fs}/R_i is a decreasing function of life expectancy. The reason goes back again to the increasing pattern of the VSL/c ratio as function of life expectancy in the separable model (Figure 1). A high and larger than one R_i^{Fs}/R_i for poor countries means that once life expectancy is taken into account into a full measure of income, poor countries fair better compared to the US than when only per capita consumption is considered. The reason is that life expectancy is much lower in these countries, but in the separable model life is also worth less there (the VSL/c is also low). Things are almost the opposite according to the non-separable model. For almost all countries $R_i^{Fn}/R_i < 1$ and R_i^{Fn}/R_i is a decreasing function of life expectancy. In the non-separable case, poorer countries fair worse relative to the US when a full measure of income is taken into account because life expectancy there is too low. More interestingly, there are a number of countries with a life expectancy higher than the US for which the non-separable model implies $R_i^{Fn}/R_i > 1$. These are plausible predictions and are reminiscent of the effects of ranking countries according to the Human Development Index (HDI) of the United Nations, rather than ranking them only by per capita income. Under the HDI ranking, the US is penalized due to its

relatively lower life expectancy. These predictions do not follow from the separable model.

5 Wars and AIDS

We now explore the differences between the separable and non-separable models in assessing the welfare effects of positive events like the end of wars and devastating events like AIDS. Table 1 compares the predictions of both models for selected countries. We compute welfare across time using equations (20) and (21), selecting year 1990 as situation 0 and 2005 as situation i 2005 for each country. These dates are relevant to the AIDS pandemic. Countries in Table 1 are classified into two groups according to whether they gained or lost life expectancy between 1990 and 2005. An interesting pattern emerges from the table. Countries like Rwanda, Bhutan and Nepal gained 16, 12 and 11 years of life respectively. However, the welfare calculation from the separable model (R^{Fs}) indicates that taking into account these gains in life expectancy makes these countries worse off relative to a measure that only takes income (consumption) into account! Consider the case of Bhutan, where not only life expectancy increased by 12 years, but also per capita consumption multiplied by 2 between 1990 and 2005 (R). For this country $R^{Fs} < R$, while $R^{Fn} > R$. Having gained both life expectancy and income, it is hard to believe the prediction of the separable model.

Table 1. End of wars versus AIDS: 1990-2005

	LE_{2005}	ΔLE	R	R^{Fs}	R^{Fn}
<i>Gains in life expectancy</i>					
Rwanda	48	16	0.72	0.41	8.67
Bhutan	65	12	2.02	1.66	3.75
Nepal	65	11	1.09	0.89	1.87
<i>Losses in life expectancy</i>					
Central Africa	46	-3	0.74	0.80	0.56
South Africa	52	-9	1.31	1.45	0.77
Botswana	51	-13	1.68	1.98	0.49
Zimbabwe	41	-19	0.90	1.54	0.20

Consider now countries that lost years of life, mostly due to AIDS: Central Africa (3 years), South Africa (9), Botswana (13) and Zimbabwe (19). Interestingly, Zimbabwe not only lost years of life but also 10% of their income ($R = 0.90$). However, the separable model implies that taking into account the 19-year life loss in this country, welfare has actually increased ($R^{Fs} = 1.54$). In contrast, the non-separable model intuitively reflects a tremendous welfare loss in Zimbabwe ($R^{Fn} = 0.2$).

6 Concluding comments

[To be completed]

A Mortality aversion

In our model with Epstein-Zin-Weil preferences:

$$V_t = \left\{ c_t^{1-\sigma} + \beta[\pi V_{t+1}^{1-\gamma} + (1-\pi) D^{1-\gamma}]^{\frac{1-\sigma}{1-\gamma}} \right\}^{\frac{1}{1-\sigma}}$$

where normalizing $D = 0$ reduces the expression to

$$V_t = \left\{ c_t^{1-\sigma} + \beta \pi^{\frac{1-\sigma}{1-\gamma}} V_{t+1}^{1-\sigma} \right\}^{\frac{1}{1-\sigma}}.$$

As shown in the text, V_t can be written as:

$$V_t = \left(\sum_{s=0}^{\infty} \tilde{\beta}(\pi)^s c_{t+s}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

where $\tilde{\beta}(\pi) = \beta \pi^{\frac{1-\sigma}{1-\gamma}}$. Suppose that consumption is c if alive and 0 if dead. In this case, the expected consumption at time t is $E_0 c_t = \pi^t c$. In addition,

$$V_0 = c \left(\sum_{s=0}^{\infty} \tilde{\beta}(\pi)^s \right)^{\frac{1}{1-\sigma}} = c \left(1 - \beta \pi^{\frac{1-\sigma}{1-\gamma}} \right)^{\frac{1}{\sigma-1}}. \quad (22)$$

If the individual is risk, or mortality, averse, then she must be better off by receiving the average consumption $E_0 c_t$ for certain, rather than facing the lottery implicit in V_0 . Consider the lifetime utility of receiving $E_0 c_t$ at period t . Denote such utility $V(Ec)$. Then

$$V(Ec) = \left(\sum_{s=0}^{\infty} \beta^s (E_0 c_s)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} = c \left(\sum_{s=0}^{\infty} (\beta \pi^{1-\sigma})^s \right)^{\frac{1}{1-\sigma}} = c (1 - \beta \pi^{1-\sigma})^{\frac{1}{\sigma-1}} \quad (23)$$

so that the individual is mortality averse if $V_0/V(Ec) < 1$. Moreover, the lower this ratio the larger the degree of mortality aversion. Dividing (22) by (23) yields

$$\frac{V_0}{V(Ec)} = \left[\frac{1 - \beta \pi^{\frac{1-\sigma}{1-\gamma}}}{1 - \beta \pi^{1-\sigma}} \right]^{\frac{1}{\sigma-1}} = h(\gamma, \sigma)$$

so $h(\gamma, \sigma)$ becomes one (risk neutrality) in the followings cases: (i) $\pi = 1$; (ii) $\gamma = \sigma = 0$; and (iii) $\gamma = 0$. Since

$$\begin{aligned} \frac{\partial \tilde{\beta}(\pi)}{\partial \gamma} &= \frac{\partial}{\partial \gamma} e^{\ln \beta + \frac{1-\sigma}{1-\gamma} \ln \pi} = e^{\ln \beta + \frac{1-\sigma}{1-\gamma} \ln \pi} \frac{\partial}{\partial \gamma} \left(\ln \beta + \frac{1-\sigma}{1-\gamma} \ln \pi \right) \\ &= \beta \pi^{\frac{1-\sigma}{1-\gamma}} \left(\frac{1-\sigma}{(1-\gamma)^2} \ln \pi \right) \end{aligned}$$

then,

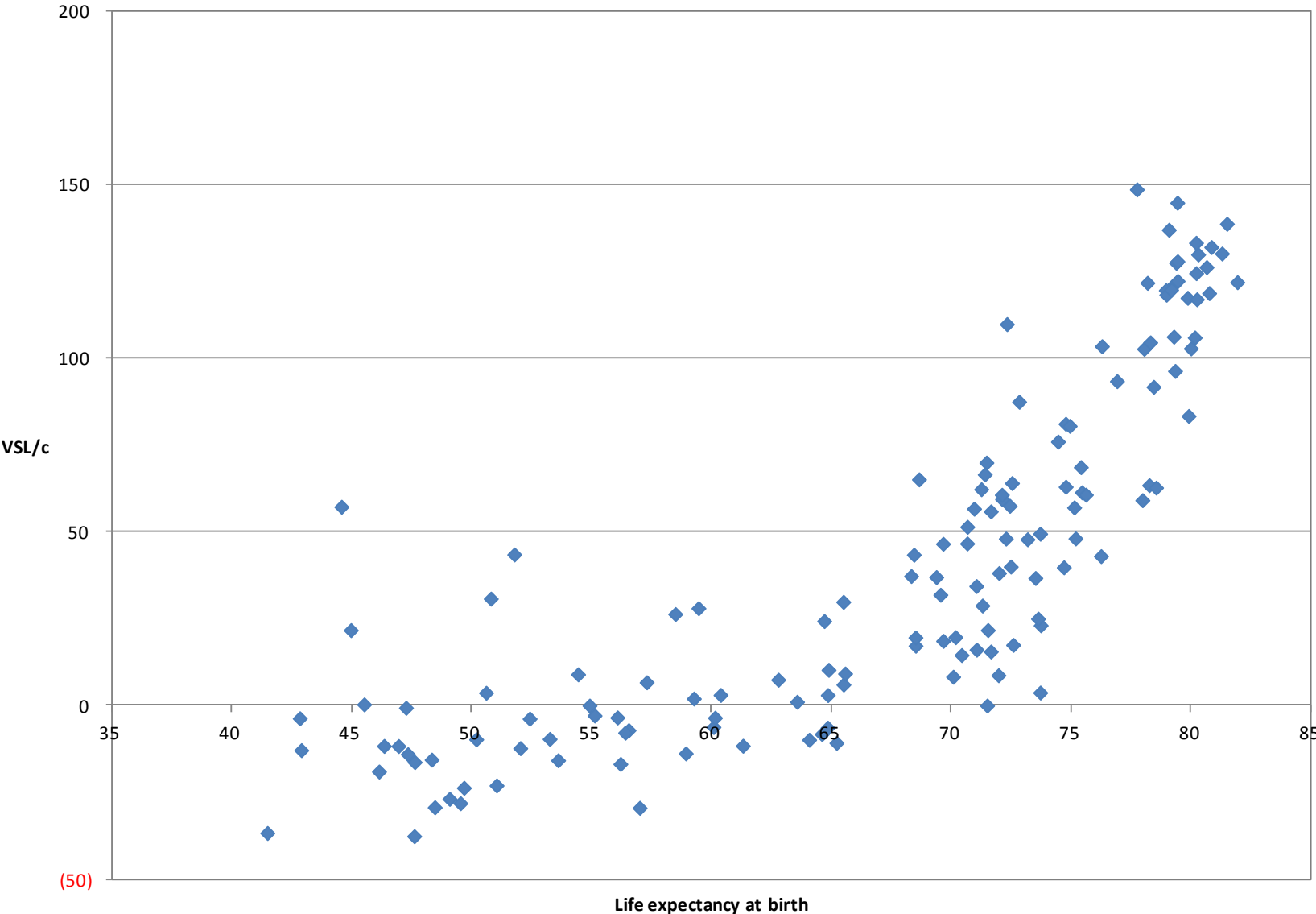
$$\begin{aligned}\frac{\partial h(\gamma, \sigma)}{\partial \gamma} &= -\frac{1}{\sigma - 1} \left[\frac{1 - \beta \pi^{\frac{1-\sigma}{1-\gamma}}}{1 - \beta \pi^{1-\sigma}} \right]^{\frac{1}{\sigma-1}-1} \beta \pi^{\frac{1-\sigma}{1-\gamma}} \left(\frac{1 - \sigma}{(1 - \gamma)^2} \ln \pi \right) \\ &= \left[\frac{1 - \beta \pi^{\frac{1-\sigma}{1-\gamma}}}{1 - \beta \pi^{1-\sigma}} \right]^{\frac{1}{\sigma-1}-1} \beta \pi^{\frac{1-\sigma}{1-\gamma}} \left(\frac{1}{(1 - \gamma)^2} \ln \pi \right) < 0.\end{aligned}$$

We can thus conclude that risk neutrality requires $\gamma = 0$, risk aversion requires $\gamma > 0$ and risk loving requires $\gamma < 0$. Moreover, the higher the γ the higher the mortality aversion.

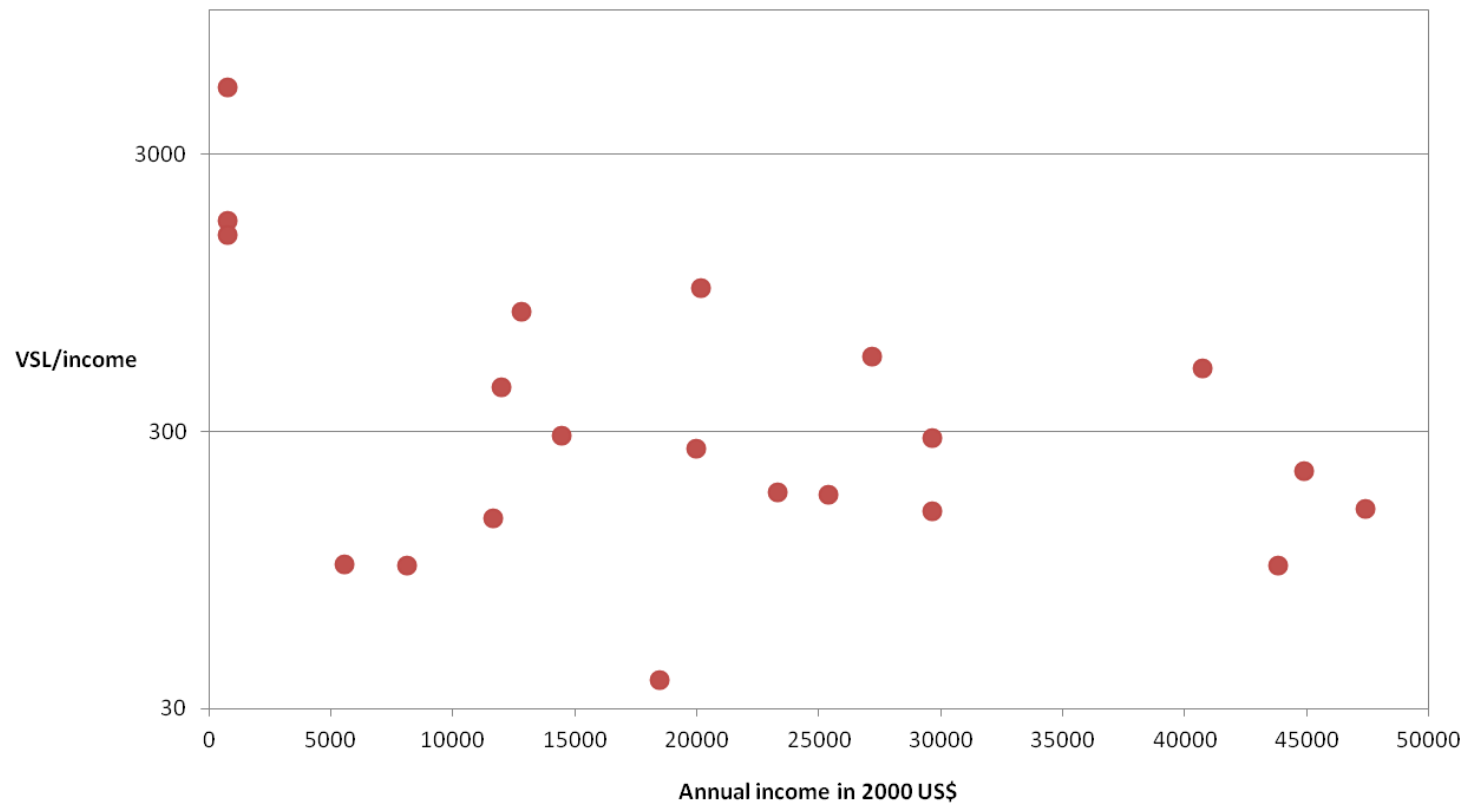
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Figure 1. Value of statistical life and life expectancy under standard model

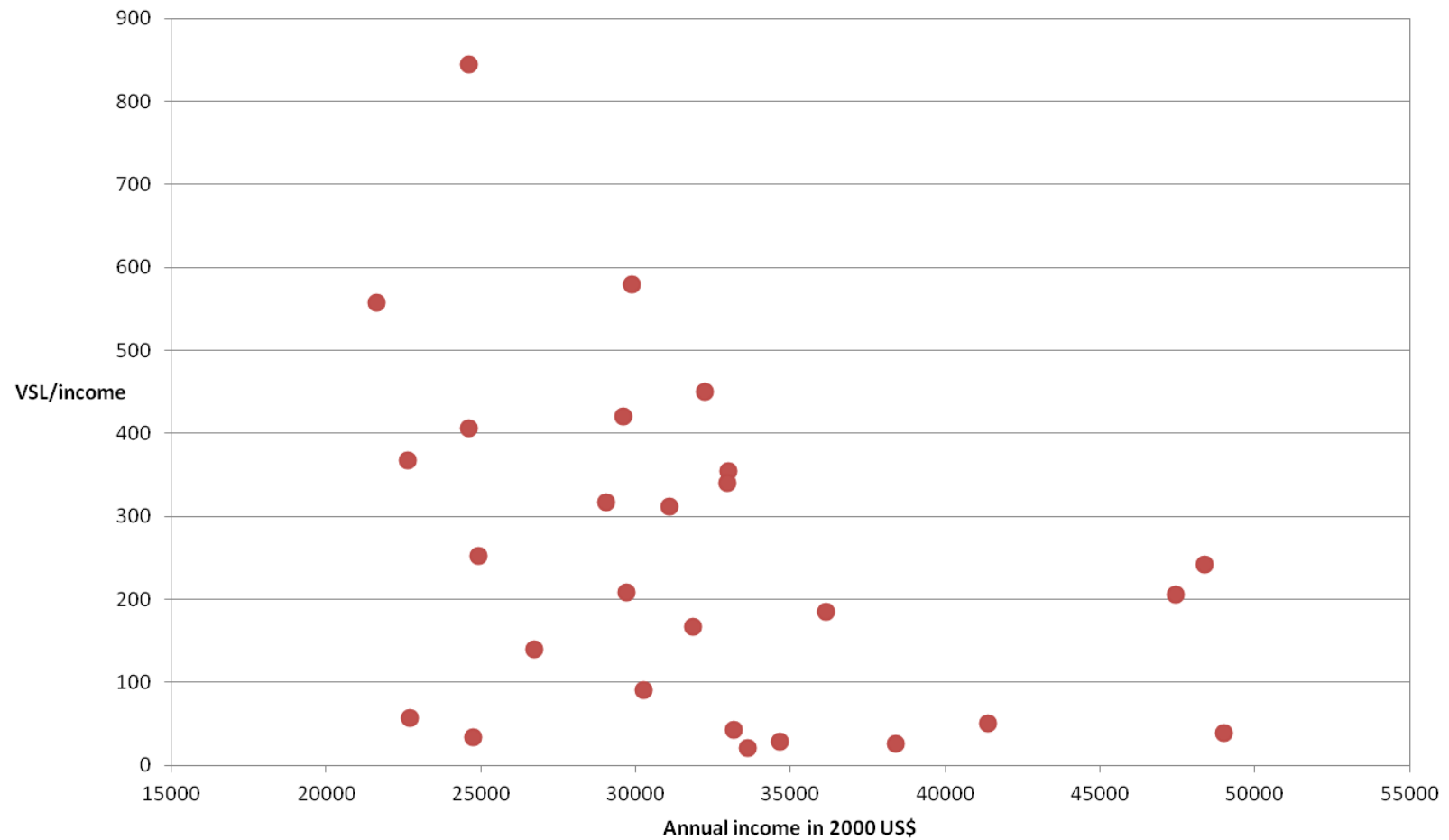


**Figure 2. Value of statistical life and annual income
International data**



Source: Viscusi and Aldy, 2003, Table 4.

**Figure 3. Value of statistical life and annual income
Estimates for the US**



Source: Viscusi and Aldy, 2003, Tables 2 and 3.

Figure 4. Value of statistical life and annual consumption across countries
Separable model

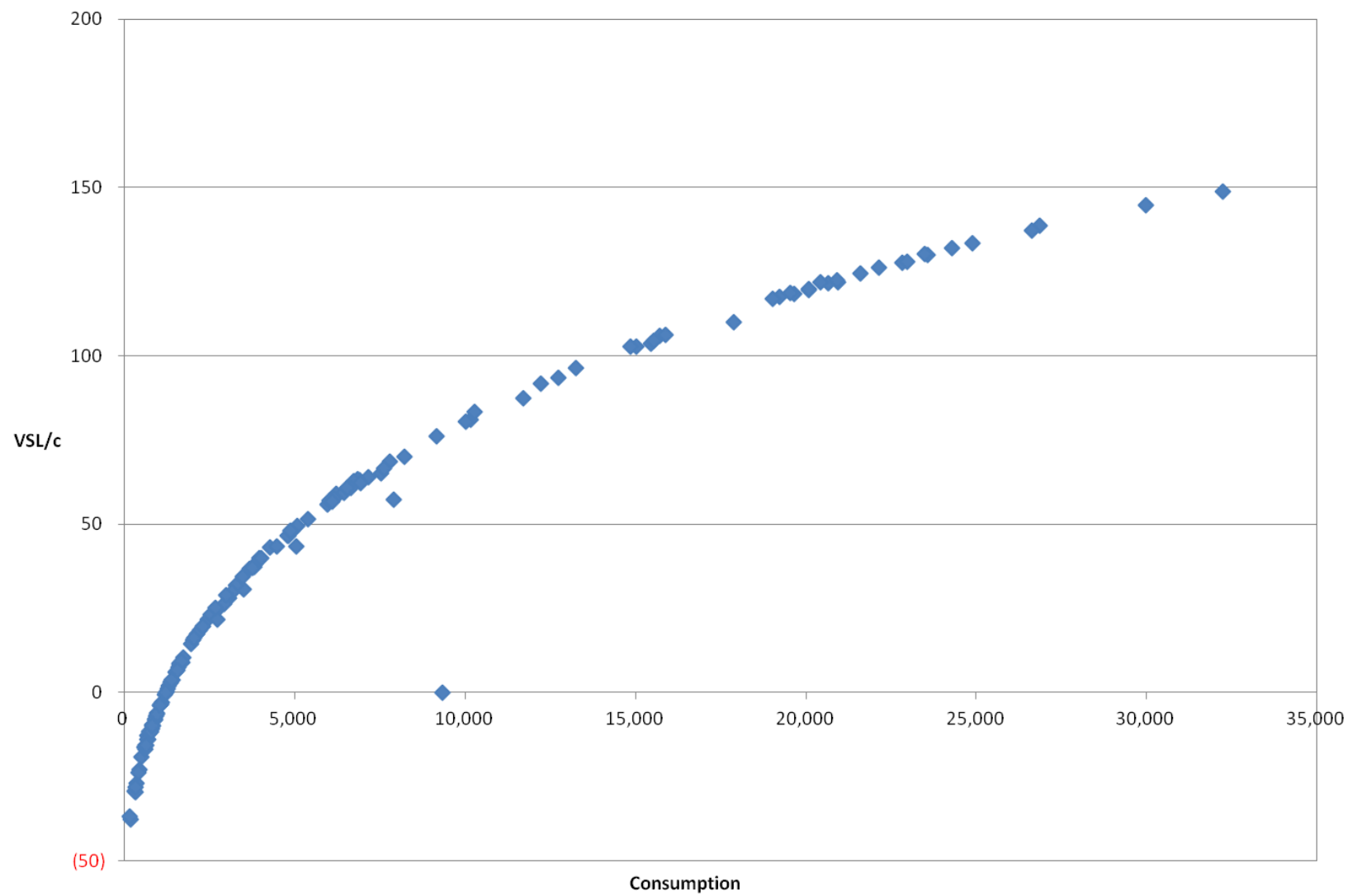


Figure 5. Value of statistical life and annual consumption across countries
Non-separable and separable models

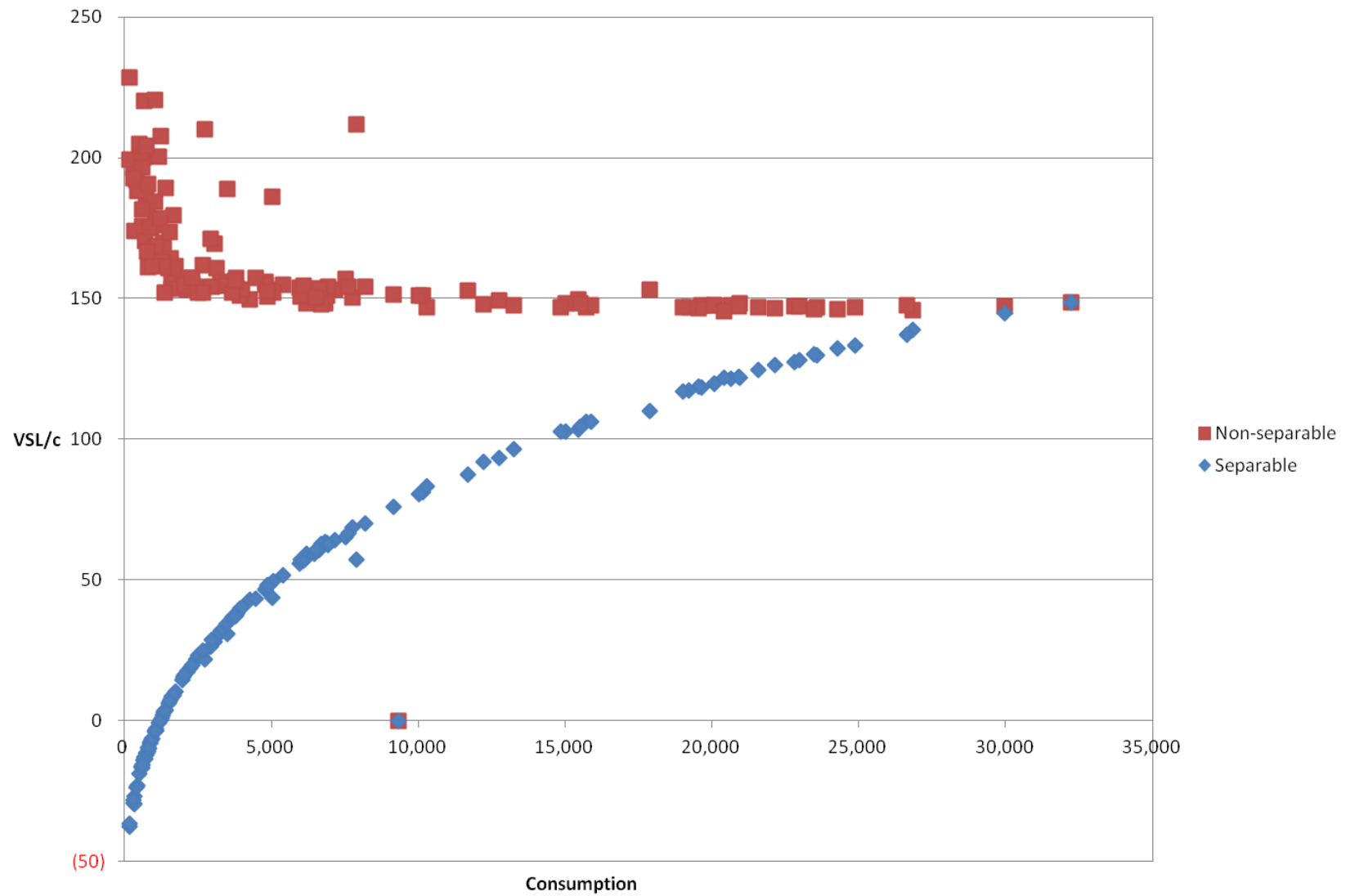


Figure 6. Value of statistical life and life expectancy across countries
Non-separable and separable models

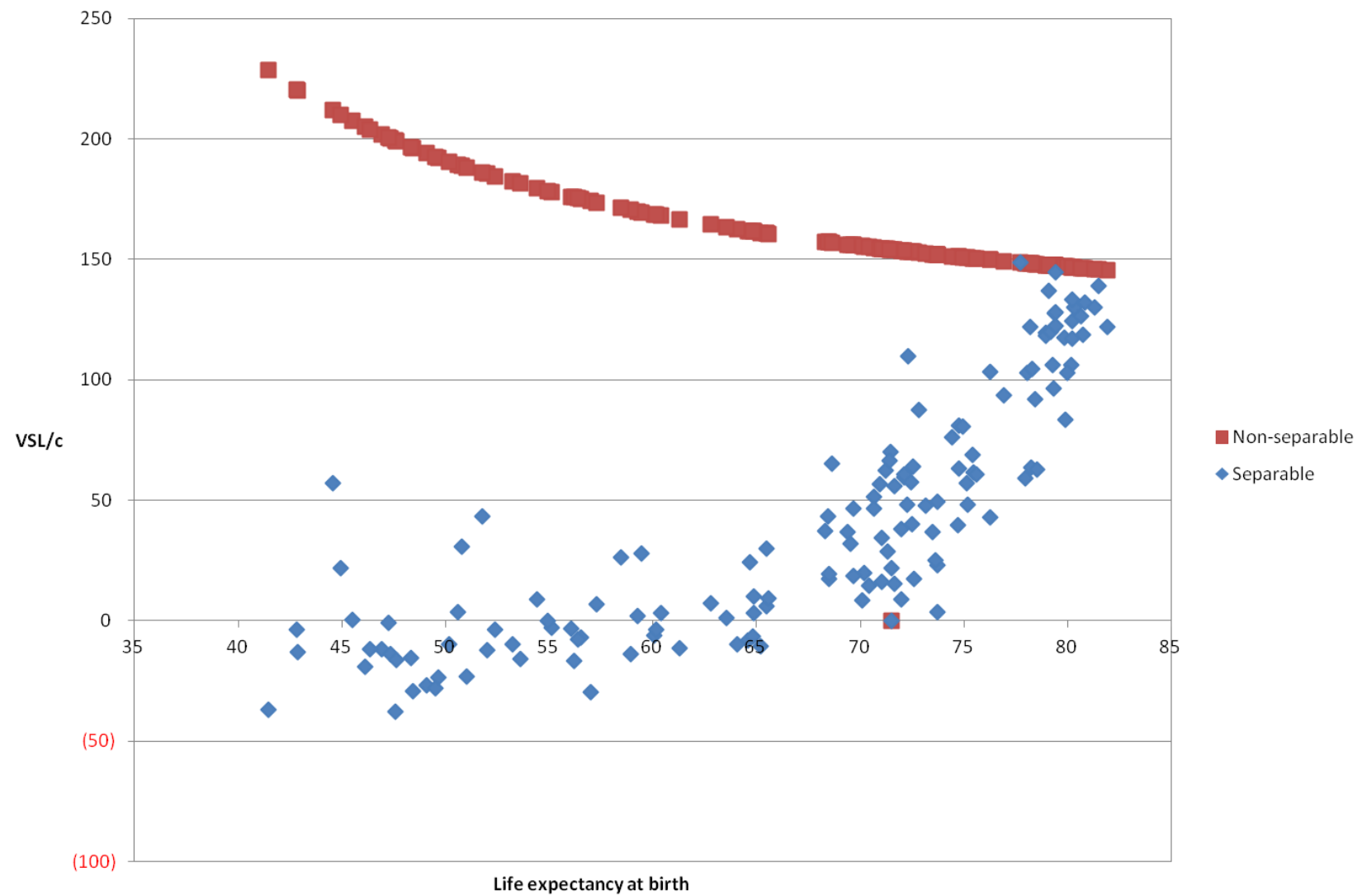


Figure 7. Willingness to pay and lifetime income across countries
Non-separable and separable models

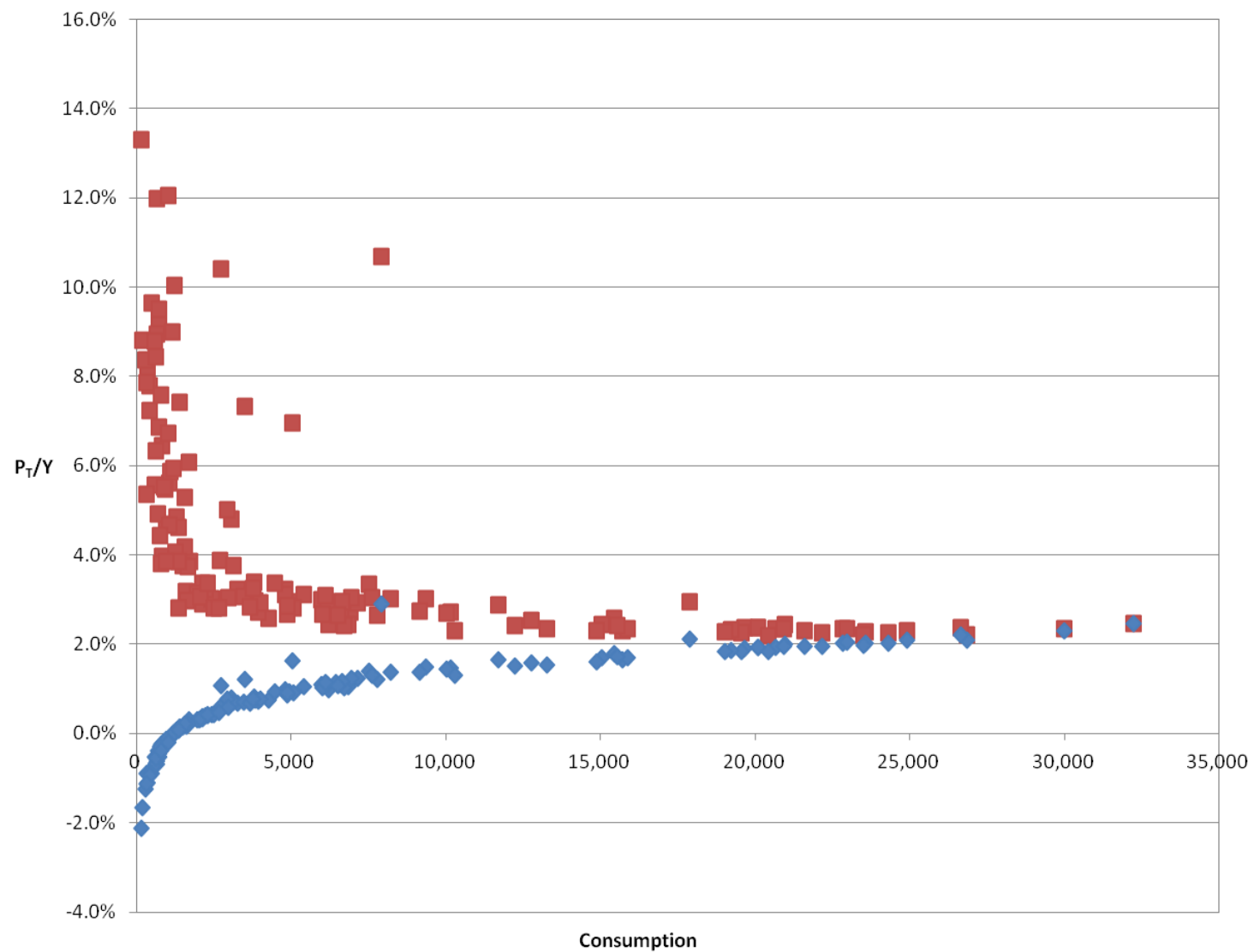


Figure 8. Welfare gains across time - 1970 to 2005
Non-separable and separable models

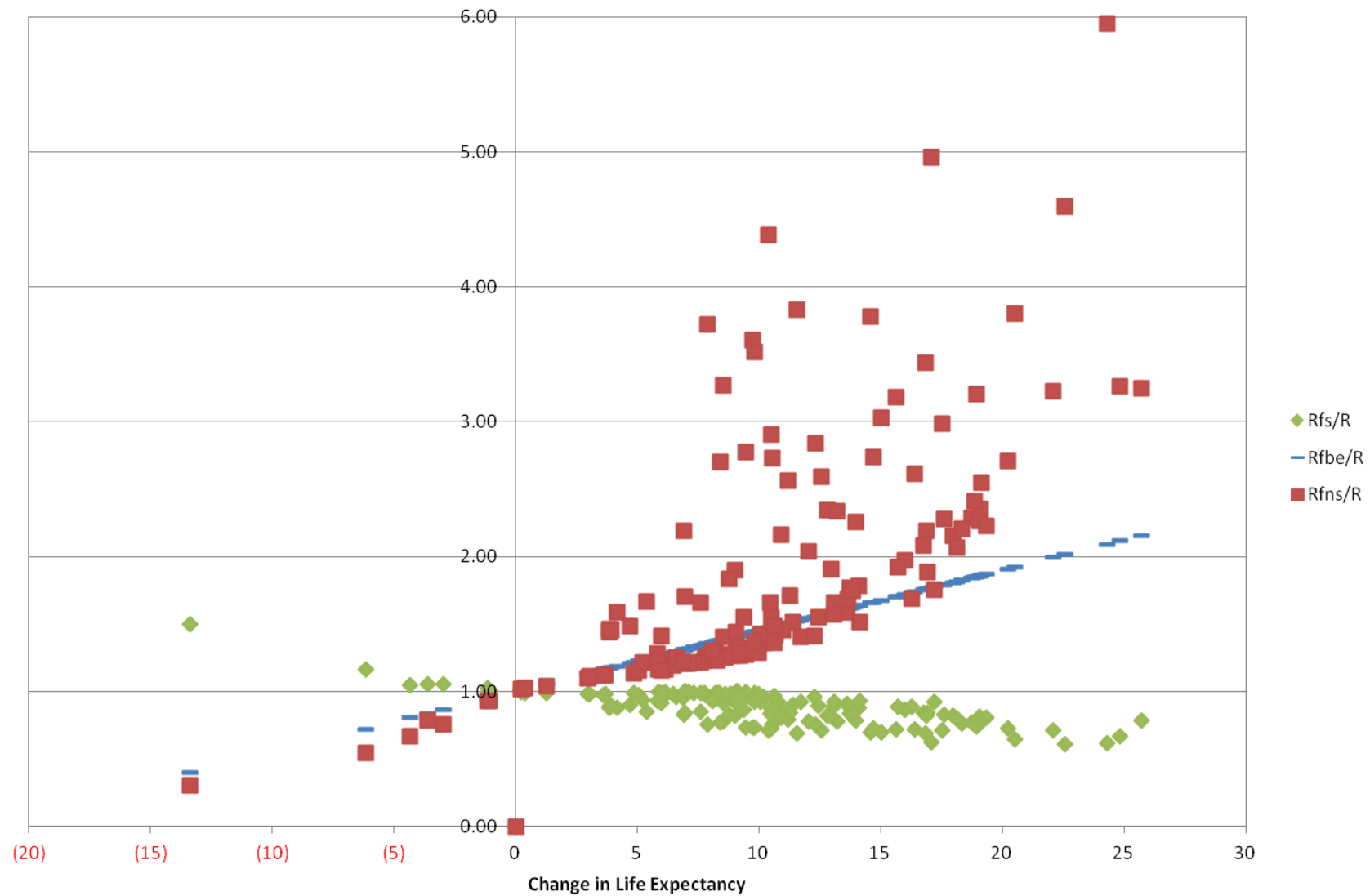


Figure 9. Welfare across countries - 2005
Non-separable and separable models

