

The Role of Housing in Labor Reallocation*

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Abstract: This paper builds a dynamic general equilibrium model of cities and uses it to analyze the role of local housing markets and moving costs in determining the character and extent of labor reallocation in the US economy. Labor reallocation in the model is driven by idiosyncratic city-specific productivity shocks, which we measure using a dataset that we compile using more than 350 U.S. cities for the years 1984 to 2008. Based on this measurement, we find that our model is broadly consistent with the city-level evidence on net and gross population flows, employment, wages and residential investment. We also find that the location-specific nature of housing is more important than moving costs in determining labor reallocation. Absent this quasi-fixity of housing, and under various assumptions governing population flows, population and employment would be much more volatile than observed.

VERY PRELIMINARY AND INCOMPLETE

*The views express here do not necessarily reflect the position of the Federal Reserve Bank of Chicago or the Federal Reserve System.

“The bigger issue is mismatch. Firms have jobs, but can’t find appropriate workers. The workers want to work, but can’t find appropriate jobs. There are many possible sources of mismatch—geography, skills, demography—and they probably interact in non-trivial ways. For example, there may be jobs available in eastern Montana and western North Dakota because of the oil boom. But a household in Nevada that is underwater on its mortgage may find it difficult to move to those locations.”

(Narayana Kochelakota, President of the Federal Reserve Bank of Minneapolis, Sept. 8, 2010)

1 Introduction

The purpose of this paper is to analyze the role of housing in labor reallocation. Understanding this role has become increasingly important in recent times. After a long period of low vacancies and high unemployment in the aftermath of the recent financial crisis, vacancies have increased substantially while unemployment has remained high. This has raised questions about why unemployed workers are not being matched to the additional job openings. A possible answer is the one suggested by the Minneapolis Fed President in the above quote: because of being underwater on their mortgages, moving costs of some homeowners have risen, inducing them to stay in their current homes instead of moving to cities with better job opportunities. Our ultimate goal is to evaluate this possibility. In the current draft we pursue a more modest goal: To construct a framework in which such an analysis can be performed. The basic question that we will address is if the model can reproduce salient features of U.S. data on net and gross population flows, employment, wages, house prices, and residential investment at the city level. We then use our model to assess the relative importance of local housing markets versus moving costs in determining the magnitude of the population flows observed in the U.S. economy.

In order to provide a guide for selecting the model economy and assessing its empirical plausibility we construct an annual panel data set for more than 350 Metropolitan Statistical Areas (MSAs) over the 1984-2008 period. We find that these data have the following properties: 1) wages and employment fluctuate significantly relative to their cross-sectional means, 2) employment fluctuations are double those of population, 3) gross population flows are about ten times larger than net population flows, 4) house prices fluctuate about five times more than population, and 5) residential construction is extremely volatile, fluctuating much more than population. The substantial fluctuations in wages and employment suggest that cities must be subject to significant

idiosyncratic shocks. In turn, the large gross population flows indicate that moving costs must be relatively small. Since net population flows are small this means that cities must have some type of quasi-fixed factor pinning down their sizes. Housing is a natural candidate, since houses cannot be moved across cities and since house prices fluctuate so much. The large volatility in construction seems to contradict the view that housing is in quasi-fixed supply. However, it is consistent with a binding irreversibility constraint on residential investment, which in turn may contribute to the large fluctuations in house prices. Another factor that may contribute to the quasi-fixity of housing and to the large fluctuations in house prices is that housing services are produced not only with construction materials, but also with land, which by nature is in fixed supply. These considerations underly the specification of our model economy.

Our model is a version of the Lucas and Prescott (5) islands economy in which islands are interpreted as cities. Local housing services are produced using structures and local land. Housing structures can be accumulated over time, but are subject to an irreversibility constraint. The local stock of land is in fixed supply. Firms produce city-specific goods using capital and local labor as inputs with a technology that is subject to city-specific idiosyncratic productivity shocks. Capital is freely movable across cities, but labor is not. At the beginning of every period agents must decide whether to stay in their current city or move. Agents must also move when they receive an exogenous reallocation shock. Moving to a different city requires the payment of a fixed cost. Once the moving cost is paid, agents can move to any city of their choice within the period. Once an agent has settled in a city she must decide whether to become employed or not and the amount of local housing services to consume. We assume perfect risk sharing across all agents in the economy.

We calibrate the model to several statistics derived from U.S. data. Some of these are closely related to the neoclassical growth model, such as the interest rate, the labor share, the business capital to output ratio and the business investment to output ratio. Additional observations that we bring to bare on the model are the residential capital to output ratio, the residential investment to output ratio and the share of land in house prices. In our benchmark case we use an elasticity of substitution between goods that is commonly used in the trade literature. However, we consider higher elasticities as well. We estimate the idiosyncratic productivity shock process using the first order conditions determining efficient input use by firms, the technology parameters already calibrated, and our panel data on employment and wages. The utility of leisure and the fixed moving cost are treated as free parameters to match the standard deviations of employment and population.

We find that the model is broadly consistent with the volatility and contemporaneous co-

movement of key model variables. However, the model fails to reproduce the volatility of house prices and the serial correlation in employment, population and house prices observed in U.S. data. This indicates that the model lacks a mechanism at the city level for propagating idiosyncratic productivity shocks. With these caveats in mind we turn to analyze the importance of moving costs versus local housing in determining the size of the net population flows observed in the U.S. economy. We do this by considering two experiments. In the first experiment we set the moving costs to zero. In the second experiment we allow housing to freely reallocate across cities. Abstracting from exogenous population attrition, we find that moving costs are more important than the fixity of housing in determining population volatility. This result is reversed when we introduce exogenous population attrition. A robust finding, though, is that the fixity of housing always play an important role in net population reallocation: In the most conservative case it reduces the standard deviation of population growth rates by 37%.

Our paper is closely related to Van Nieuwerburgh and Weil (6), which also uses a Lucas-Prescott islands economy to study cities and local housing markets. The papers differ in the models used and in the issues analyzed. Van Nieuwerburgh and Weil abstract from labor supply decisions and moving costs, but introduce permanent differences in the ability of agents. Their goal is to study the effect of an increase in wage dispersion across US cities on the corresponding dispersion of house prices. In contrast, we explicitly model labor supply decisions and moving costs (abstracting from any differences in ability levels) in order to understand the role of housing in labor reallocation. The paper is also related to Alvarez and Shimer (1) in that both papers use a Lucas-Prescott islands economy and allow agents to enjoy leisure in the islands where they are located. However, they interpret their islands as industries and use their model to analyze the behavior of wages and unemployment. Finally, the paper is related to Kennan and Walker (4) since both papers analyze migration decisions in the face of wage shocks and moving costs. However, they abstract from local housing markets, assume undirected search, and do not consider any equilibrium interactions. The structural estimation of their model suggest moving costs that are much larger than those obtained in this paper.

The rest of the paper is organized as follows. Section 2 describes the data that we use in our analysis. Section 3 describes the model economy. Section 4 describes a steady state equilibrium. Section 5 characterizes a steady state equilibrium. Section 6 calibrates the model to U.S. data. Finally, Section 7 presents the results.

2 Data

2.1 Sources

To provide an empirical underpinning to our analysis we construct an annual panel data set of population, employment, population inflows and outflows, residential investment, house prices, and average wage per job of a set of Metropolitan Statistical Areas (MSAs) over the 1984-2008 period. The mappings of counties to MSAs we use (except when noted) are consistent with the definitions given by the U.S. Office of Management and Budget (OMB) as of December, 2009.¹ OMB currently defines 366 MSAs in the United States and over our sample period, these MSAs account for about 83 percent of the aggregate population.

Our data on population, employment, and average wage per job are taken from the “Regional Economic Accounts: Local Area Personal Income and Employment” tables as produced by the Bureau of Economic Analysis (BEA).² These annual data are available for all 366 MSAs from 1969 through 2008. The population data are from table CA1-3 and are mid-year estimates from the Census Bureau. For employment, we use “Wage and Salary Employment,” line 20 of Table CA25N). These are counts of full- and part- time jobs of salaried employees.³ We construct nominal average wage per job as the sum of total wage and salary disbursements (line 50, Table CA05) and supplements to wages and salaries (line 60, Table CA05) divided by wage and salary employment.⁴ We convert the nominal estimate to a real measure consistent with our model by deflating using the CPI “All items less shelter.”⁵

For house prices, we use the repeat-sales house price indexes produced by the Federal Home

¹See <http://www.census.gov/population/www/metroareas/lists/2009/List4.txt> for these definitions.

²These data are available at <http://www.bea.gov/regional/reis/>.

³The BEA also publishes an estimate of “Total Employment,” which is the sum of wage and salary employment and proprietors employment. The level and change of the two employment series are highly correlated. We use wage and salary employment to be consistent with our measure of wages, described next; we do not want to include proprietor’s income as part of wages because some of proprietor’s income represents payments to capital.

⁴We construct our own measure because the BEA excludes wage and salary supplements when reporting “average wage per job” in its Table CA34.

⁵The specific CPI series we use is CUSR0000SA0L2. This series is available for download at <http://www.bls.gov/cpi/>.

Finance Agency (FHFA).⁶ These data are quarterly; for each MSA, we construct an annual estimate as the average of all non-missing quarterly observations. All 366 MSAs have price-index data from 2001 through 2009, but prior to 2001, the time span of coverage varies by MSA. Summarizing: Only 14 MSAs have house price indexes starting in 1976; 130 MSAs starting in 1980; 202 MSAs starting in 1985; 326 MSAs starting in 1990, and so forth. As with average wage per job, we use the CPI excluding shelter to convert these nominal indexes to real. For 11 MSAs, the FHFA does not report an MSA-level index but rather a set of indexes corresponding to divisions within the MSA.⁷ For these MSAs, we set the level of the index for the MSA equal to the average of the reported indexes for the underlying divisions.

Data on residential investment at the MSA-level are not available. To proxy for residential investment, we use new housing unit building permits as collected by the Census Bureau. These data have been organized by MSA as part of the “State of the Cities Data System” (SOCDS) that resides on the Department of Housing and Urban Development (HUD) Web Site.⁸ SOCDS reports both monthly and annual data. We use the annual permits data, which begin in 1980 and end in 2009 – the monthly data begin in 2001. Note that the MSA-definitions in the SOCDS data are out of date such that the experiences of only 362 MSAs (363 with one missing) rather than 366 MSAs are available.⁹

We construct data on gross MSA-level population inflows and outflows using county-county migration data based on tax records that is constructed by the Internal Revenue Service. These data are available from 1978 through 1992 at the Inter-University Consortium for Political and Social Research (ICPSR) web site; are available for purchase from the IRS for the 1992 - 2004 period; and are available for free on the IRS web site from 2004 through 2007. The data are annual and cover the “filing year” period, not calendar year. For example, the data for 2007 approximately refer to migration over the period April, 2007 to April, 2008.

For each of the years, the IRS reports the migration data using two files, one for county outflows

⁶These data are available at <http://www.fhfa.gov/Default.aspx?Page=87>.

⁷The MSAs are Boston, Chicago, Dallas, Detroit, Los Angeles, Miami, New York City, Philadelphia, San Francisco, Seattle, and Washington, DC. Taking Boston as an example, the FHFA reports price indexes for each of the the 4 Metropolitan Divisions of the Boston MSA rather than the Boston MSA itself.

⁸The data are available at <http://socds.huduser.org/permits/>.

⁹We are in the process of constructing the appropriate MSA-level data using county-level information, also on the SOCDS web site, but this process is not yet complete.

and one for county inflows, for each county in the United States. Both the inflow and the outflow files report migrants in units of “returns” and in units of “personal exemptions.” According to information from the IRS web site, the returns data approximates the number of households and the personal exemptions data approximates the population.¹⁰ In the data work below, we study the exemptions data.

In the inflow files, for every county the IRS reports the number of people that did not migrate into that county, i.e. lived in that county for two consecutive years. It also reports, for people that migrated into that county, each of the counties sending the migrants and the number of migrants, although not every county sending migrants is reported: All counties sending a relatively small number of migrants are lumped together into an “other county” category. Analogously, in the outflow files, for every county the IRS reports the population that lived in that county for two consecutive years (i.e., the non-migrants), and, for the people that migrated out of that county, each of the counties receiving the migrants and their number. Like the inflows file, counties receiving only a small number of migrants are lumped together into an “other county” category.

We define gross inflows into an MSA as the sum of all migrants into any county in that MSA, as long as the inflows did not originate from a county within the MSA. Analogously, we define gross outflows from an MSA as the sum of all migrants leaving any county in that MSA, as long as the migrants did not ultimately move to another county in the MSA. In the case of gross inflows, the originating counties are not restricted to be part of one of the 366 MSAs; and, in the case of gross outflows, the counties receiving the migrants are not restricted to be included in one of the 366 MSAs.

Table 1 reports means, standard deviations, sample sizes and ranges for the data we use in this study.

2.2 Statistics

2.2.1 Net Expansion and Contraction Rates

We use the BEA population data to measure net expansion e_t and net contraction rates δ_t in the U.S. Define the aggregate population in the middle of year t as P_t and the population in MSA i as

¹⁰See <http://www.irs.gov/taxstats/indtaxstats/article/0,,id=96816,00.html> for details.

of the end of year t as p_{it} .¹¹ Given this, we set:

$$P_t e_t = \sum_{i: p_{it} > p_{i,t-1}} (p_{it} - p_{i,t-1}) \quad (1)$$

$$P_t \delta_t = \sum_{i: p_{it} < p_{i,t-1}} (p_{i,t-1} - p_{it}) \quad (2)$$

Averaging over the 1985-2007 period, our overall sample period, we estimate the average value of e_t to be 1.24 percent and the average value of δ_t to be 0.04 percent. The gap between these two values is average population growth, 1.20 percent over this period.

Our model does not have population growth. Define the realized aggregate population (gross) growth rate in year t as μ_t . To compute net expansion and net contraction rates after abstracting from population growth, $\tilde{e}_t$ and $\tilde{\delta}_t$, we assume MSAs receive new population at the aggregate rate, and in proportion to their existing size. We compute

$$P_t \tilde{e}_t = \sum_{i: \frac{p_{it}}{\mu} > p_{i,t-1}} \left(\frac{p_{it}}{\mu} - p_{i,t-1} \right) \quad (3)$$

$$P_t \tilde{\delta}_t = \sum_{i: \frac{p_{it}}{\mu} < p_{i,t-1}} \left(p_{i,t-1} - \frac{p_{it}}{\mu} \right) . \quad (4)$$

We estimate the average value of $\tilde{e}_t$ and $\tilde{\delta}_t$ to each equal 0.43 percent.

2.2.2 Gross Expansion and Contraction Rates

We use the IRS data to understand gross population flows in general, and more specifically across MSAs. Table 2 provides some detail on these data. Column (1) reports the total number of people migrating into an MSA and column (2) reports the total number of people migrating out of an MSA, both in millions. These data include all within-MSA migrants, including people moving to a different county in the same MSA. Columns (1) and (2) are not exactly equal because not all people move to and from counties included in on of the 366 defined MSAs.

Columns (3) and (4) strip out all within-MSA migration and only report across-MSA migration: New inflows to an MSA (column 3) and outflows from an MSA (column 4), again both in millions of people. Column (5) reports the total sample population corresponding to the IRS data for the

¹¹The BEA reports a middle-of-year estimate for the population. We use this estimate directly in computing P_t . We compute the end-of-year population estimate $p_{i,t}$ as the average value of the middle-of-year population in years t and $t + 1$

366 MSAs.¹² The population reported in column (5) is approximately equal to 65 percent of the entire U.S. population.

Column (6) shows an estimate of the total migration rate (inclusive of both within and across MSA moves), computed as the ratio of the average of columns (1) and (2) to column (5). Summarizing: About 6.2 percent of taxpayers move to a different county in any given year. Columns (7) and (8) report across-MSA migration rates. Columns (7) and (8) strip out all within-MSA moves. Column (7) is our “gross expansions rate” ν , the ratio of new-MSA inflows to population, defined as the ratio of column (3) to column (5). Column (8) reports the gross contractions rate η , the ratio of MSA outflows to population, and defined as the ratio of column (4) to column (5). Columns (7) and (8) show that about 4.3 percent of the population migrate to a different MSA in any given year. Summarizing columns (6)-(8), about 1/3 of all moves are within-MSA moves, and 2/3 of all moves are across-MSA moves. Finally, the last row of the table shows these migration percentages are relatively stable over time: The yearly standard deviations are only about 0.2 percentage points.

Table 3 explores the relationship between net and gross migration a bit further. For each MSA in each year, a net migration (gross inflows less gross outflows) percentage is computed. We then bin these net migration percentages into deciles. The mean net migration rate of each decile is reported in column (1) of Table 3; column (2) reports the mean gross expansion rate for each decile; and column (3) reports the mean gross contraction rate. The table shows that at gross expansion and contraction rates are an order of magnitude higher than net migration rates at all levels of net migration rates. The data also have a distinct “U-shape” for both gross expansions and contractions. That is, the MSAs experiencing either the greatest net loss or greatest net gain of population experience both relatively high gross expansions and contractions rates.

2.2.3 Standard Deviations and Correlations of Detrended Variables

We detrend the logarithm of any variable x_{it} as follows. First, we compute a “hatted” variable, $\hat{x}_{it}$, defined as x_{it} less the yearly sample mean. That is, assuming $i = 1, \dots, N$ MSAs in our sample in each year t , we compute

$$\hat{x}_{it} = x_{it} - \left(\frac{1}{N}\right) \sum_{j=1}^N x_{j,t} .$$

¹²For each MSA, we construct estimates of beginning-of-year population (non-movers plus gross outflows) and end-of-year population (non-movers plus gross inflows). The population of taxpayers is set to the average of these beginning- and end- of year estimates.

The hatted variable is mean zero in every year. Next, we compute the change in the hatted variable, $\Delta\hat{x}_{it}$. In the final step, we subtract the average value of $\Delta\hat{x}_{it}$ for each MSA, that is assuming $t = 1, \dots, T$ observations for each MSA we compute

$$\epsilon_{it} = \Delta\hat{x}_{it} - \left(\frac{1}{T-1}\right) \sum_{t=2}^T \Delta\hat{x}_{it} .$$

We report statistics and correlations of the ϵ_{it} variable in the above equation.

We have in mind that the growth rates of any variable in our sample can be characterized as adhering to the following process:

$$\Delta x_{it} = \xi_i + \epsilon_{it} + a_t , \tag{5}$$

with a_t a shock common to all MSAs, ξ_i an MSA-specific (fixed) component and ϵ_{it} a mean-zero stationary variable that is uncorrelated across MSAs. The “hatting” of variables has the effect of removing the aggregate shock a_t , and the subtraction of the sample average growth rate for each MSA removes an estimate of ξ_i , leaving as a residual a consistent estimate of ϵ_{it} .

Table 4 shows standard deviations and contemporaneous correlations of the detrended variables. The first two columns of the table report the standard deviations of $\Delta\hat{x}_{it}$ and ϵ_{it} . The standard deviations of $\Delta\hat{x}_{it}$ to ϵ_{it} for most variables are similar except in the case of population, where ϵ_{it} has a considerably smaller standard deviation. Focusing on the standard deviations for ϵ_{it} : Population has the smallest standard deviation; the standard deviations of employment and wage per job are roughly double that of population; the standard deviations of house prices, entries and exits are five and seven times that of population; and, the standard deviation of housing permits are 31 times that of population.

While most of the variables we report have a direct mapping to model variables, housing permits, which are available at the MSA-level, are intended to proxy for MSA-level real residential investment, which is not available. The available evidence suggests that permits and residential investment are highly correlated, but that permits are more volatile than residential investment. Over the 1985-2008 period, the correlation of the log differences of aggregate permits and aggregate real residential investment is 0.90. The standard deviation of the log difference of aggregate permits is about 30 percent greater than real residential investment, 13.1% compared to 9.9%. Certainly, a good deal of the volatility of the ϵ_{it} we report for housing permits arises from volatility housing permit data in the smaller MSAs. If we restrict attention to the top-100 largest MSAs in 1984, accounting for 78 percent of the population in our 366 MSA sample, the estimated standard deviation of ϵ_{it} falls from 27.6% to 20.1%.

The rightmost five columns of the table report contemporaneous correlations of all of the the ϵ_{it} variables. Almost all variables are positively correlated, except for lagged exits which is negatively correlated with all variables except house prices. The correlation of average wage per job with most variables is low, potentially signaling to measurement error in wages, a point to which we return later.

Table 5 shows sample autocorrelations of the detrended annual data, the ϵ_{it} variables, at lags 1-4. Summarizing: Lagged entries, housing permits, and average wage per job are not highly serial correlation; population and employment have a more pronounced first-order serial correlation; and house prices are highly serial correlation. Table 6 shows correlations of the ϵ_{it} variable for employment at time t with four lags and leads of the ϵ_{it} for the other variables in our sample. The reported correlations suggest that employment leads population, house prices, and exits (in the sense that the correlation at the first lead of these variables is at least as large as the contemporaneous correlation); and, employment lags housing permits and average wage per job.

3 The model economy

The economy is populated by a representative household that has a continuum of members with names in the unit interval. The preferences of each household member is given by the following utility function:

$$v = \sum_{t=0}^{\infty} \beta^t [\ln c_t + g(l_t) + \alpha \ln (\eta_t)],$$

where c_t is consumption, l_t is leisure, η_t is housing services, $0 < \beta < 1$ is the discount factor, $\varphi \geq 0$ and $\alpha \geq 0$. Since household members are endowed with one unit of time and there is an institutionally determined workweek of length $1 - \bar{l}$, leisure l_t must belong to the set $\{1, \bar{l}\}$. The household's preferences are given by

$$\int_0^1 v(m) dm,$$

where $v(m)$ is the utility of household member m . That is, the household puts equal weights to all of its members.

The consumption good is produced using a continuum of intermediate goods. The production function is given by

$$Y_t = \left[\int_0^1 y_t(j)^\chi dj \right]^{\frac{1}{\chi}}$$

where Y_t is output of the consumption good, $y_t(j)$ is the amount of intermediate good j , and $\chi \leq 1$. Observe that $-1/(1 - \chi)$ is the elasticity of substitution of the intermediate goods.

Each intermediate good is produced by a single city using the following production function:

$$y_t = s_t n_t^\theta k_t^\gamma$$

where y_t is the output of the city-specific intermediate good, $s_t > 0$ is a city specific productivity shock, n_t is labor, k_t is capital, $\theta > 0$, $\gamma > 0$ and $\theta + \gamma \leq 1$. The idiosyncratic productivity shock s_t follows a finite Markov process with transition matrix Q .

Capital is freely movable across cities but labor isn't. At the beginning of every period household members are distributed in some way across all the cities of the economy. The household must decide which household members to leave in the cities where they are initially located and which household members to move. However, a fraction π of all household members are forced to move for exogenous reasons. Whenever a household member moves to a different city the household incurs a moving cost equal to ψ units of the consumption good. The household must also decide which household members to put to work and which ones to leave non-employed. Household members can only obtain employment and housing services from the cities where they are currently located.

Each city produces local housing services according to the following production function:

$$\eta_t = h_t^\varsigma b_t^{1-\varsigma}$$

where η_t is the output of local housing services, h_t is local housing structures, b_t is local land, and $0 \leq \varsigma \leq 1$. While housing structures can be accumulated, cities have a fixed endowment of land $\hat{b}$. The technology for accumulating local housing structures is given by

$$h_{t+1} = (1 - \delta_h) h_t + i_t^h \geq (1 - \delta_h) h_t, \quad (6)$$

where $0 < \delta_h < 1$ is the depreciation rate of housing structures and $i_t^h \geq 0$ is gross residential investment. Observe that local residential investment is subject to an irreversibility constraint.

Aggregate capital is accumulated according to a standard technology given by

$$K_{t+1} = (1 - \delta_k) K_t + I_t^k,$$

where $0 < \delta_k < 1$ is the depreciation rate of capital and I_t^k is gross investment.

4 Steady state equilibrium

In this section we describe a steady state competitive equilibrium. The state of a city z will be given by its idiosyncratic productivity level s , the number of people present at the city at the beginning

of the period x , and the stock of local housing structures h . A time invariant distribution μ will describe the number of cities across the different individual states $z = (s, x, h)$.

The representative household purchases the consumption good, invests in physical capital, invests in housing structures in each city, rents housing services in each city, and pays moving costs. It earns income from the labor services supplied by its household members, from renting the household structures that it owns in each city, from renting the land that it owns in each city, from renting the stock of capital and from the profits produced by the intermediate goods producers. The state of the representative household is given by the stock of physical capital that it owns K , a function λ describing the number of household members that the household has at each type of city z at the beginning of the period, and a function ζ describing the stock of local housing structures that the household owns at each type of city z . The household's problem is described by the following Bellman equation:

$$V(K, \lambda, \zeta) = \max \left\{ \ln C + \varphi \int u(z) d\mu + \alpha \int \ln \left(\frac{\eta(z)}{p(z)} \right) p(z) d\mu + \beta V(K', \lambda', \zeta') \right\}$$

subject to

$$\begin{aligned} & C + I^K + \int i^h(z) d\mu + \int r^n(z) \eta(z) d\mu + \psi \int \max \{ p(z) - (1 - \pi) \lambda(z), 0 \} d\mu \\ & \leq \int w(z) n(z) d\mu + \int r^h(z) h(z) d\mu + \int r^b(z) b d\mu + r^k K + D, \end{aligned}$$

$$\int p(z) d\mu = 1,$$

$$n(z) + u(z) = p(z),$$

$$i^h(z) \geq 0,$$

$$K' = (1 - \delta_k) K + I^K,$$

λ' and ζ' satisfy that for every s' and every Borel set $\mathcal{X}' \times \mathcal{H}'$

$$\int_{\mathcal{X}' \times \mathcal{H}'} \lambda'(s', x', h') \mu(s', dx' \times dh') = \int_{\mathcal{B}} p(s, x, h) Q(s, s') \mu(ds \times dx \times dh),$$

and

$$\int_{\mathcal{X}' \times \mathcal{H}'} \zeta'(s', x', h') \mu(s', dx' \times dh') = \int_{\mathcal{B}} \left[(1 - \delta_h) \zeta(s, x, h) + i^h(s, x, h) \right] Q(s, s') \mu(ds \times dx \times dh),$$

respectively, where

$$\mathcal{B} = \left\{ (s, x, h) : p_\mu(s, x, h) \in \mathcal{X}' \text{ and } (1 - \delta_h) h + i_\mu^h(s, x, h) \in \mathcal{H}' \right\},$$

and where $p_\mu(z)$ is total population in a city of type z , $i_\mu^h(z)$ is total residential investment in a city of type z , $r^\eta(z)$ is the price of housing services in a city of type z , $w(z)$ is the wage rate in city of type z , $r^h(z)$ is the rental price of housing structures in a city of type z , $r^b(z)$ is the rental price of land in a city of type z , r^k is the economy-wide rental price of capital, $p(z)$ is the post-reallocation total number of household members in a city of type z , $n(z)$ is the number of housing members that work in a city of type z , $u(j)$ is the number of housing members that are non-employed in a city of type z , and D are profits. Observe that the household provides full consumption insurance to all of its members. It also provides equal amount of housing services to all the household members residing in a same city.

Intermediate good producers in a city of type z solve the following static maximization problem

$$\max \left\{ q(z) s n^\theta k^\gamma - w(z) n - r^k k \right\},$$

where $q(z)$ is the price of the intermediate good in a city of type z , housing services producers in a city of type z solve the following problem:

$$\max \left\{ r^\eta(z) h^\varsigma b^{1-\varsigma} - r^h(z) h - r^b(z) b \right\},$$

In turn, (economy-wide) producers of the consumption good solve the following problem:

$$\max \left\{ \left[\int y(z)^\chi d\mu \right]^{\frac{1}{\chi}} - \int q(z) y(z) d\mu \right\}.$$

A steady state equilibrium is an allocation $(\bar{\mu}, \bar{p}_\mu, \bar{i}_\mu^h, \bar{K}, \bar{\lambda}, \bar{\zeta}, \bar{C}, \bar{I}^K, \bar{i}^h, \bar{\eta}, \bar{p}, \bar{u}, \bar{n}, \bar{k}, \bar{h}, \bar{y})$, profits $\bar{D}$ and prices $\{\bar{r}^\eta, \bar{w}, \bar{r}^h, \bar{r}^b, \bar{r}^k, \bar{q}\}$ such that:

1) The household chooses $(\bar{C}, \bar{I}^K, \bar{i}^h, \bar{\eta}, \bar{p}, \bar{u}, \bar{n})$ when its individual state is given by $(\bar{K}, \bar{\lambda}, \bar{\zeta})$ and when it takes $(\bar{\mu}, \bar{p}_\mu, \bar{i}_\mu^h)$, $\{\bar{r}^\eta, \bar{w}, \bar{r}^h, \bar{r}^b, \bar{r}^k\}$ and $\bar{D}$ as given,

2) $\bar{\lambda}$ and $\bar{\zeta}$ satisfy that:

$$\bar{\lambda}(s, x, h) = x,$$

$$\bar{\zeta}(s, x, h) = h,$$

for every $z = (s, x, h)$,

3) The intermediate good producers in a city of type z choose $(\bar{n}(z), \bar{k}(z))$ when they take $(\bar{q}(z), \bar{w}(z), \bar{r}^k)$ as given,

4) The housing services producers in a city of type z choose $(\bar{h}(z), \hat{b})$ when they take $(\bar{r}^\eta, \bar{r}^h, \bar{r}^b)$ as given,

5) The consumption good producers choose $\bar{y}$ when they take $\bar{q}$ as given,

6) The markets for intermediate goods clear, i.e.

$$\bar{y}(z) = s\bar{n}(z)^\theta \bar{k}(z)^\gamma,$$

7) The markets for local housing services clear, i.e.

$$\bar{\eta}(z) = \bar{h}(z)^\varsigma \hat{b}^{1-\varsigma}$$

8) The market for the consumption good clears, i.e.

$$\bar{C} + \bar{I}^K + \int \bar{v}^h(z) d\bar{\mu} + \psi \int \max\{\bar{p}(z) - (1 - \pi)x, 0\} d\mu = \left[\int y(z)^\chi d\mu \right]^{\frac{1}{\chi}},$$

9) The market for business capital clears, i.e.

$$\bar{K} = \int \bar{k}(z) d\bar{\mu}$$

10) The stock of capital is stationary, i.e.

$$\bar{I}^K = \delta_k \bar{K}$$

11) The invariant distribution $\bar{\mu}$ satisfies that for every s and every Borel sets $\mathcal{X} \times \mathcal{H}$:

$$\bar{\mu}(s', \mathcal{X} \times \mathcal{H}) = \int_{\mathcal{B}} Q(s, s') \bar{\mu}(ds \times dx \times dh),$$

where

$$\mathcal{B} = \left\{ (s, x, h) : \bar{p}_\mu(s, x, h) \in \mathcal{X} \text{ and } (1 - \delta_h)h + \bar{v}_\mu^h(s, x, h) \in \mathcal{H} \right\},$$

12) The total population and total residential investment rule of cities are generated by the individual decisions of the representative household, i.e.

$$\bar{p}_\mu(s, x, h) = \bar{p}(s, x, h),$$

$$\bar{v}_\mu^h(s, x, h) = \bar{v}^h(s, x, h).$$

5 Characterization

Since the above is a convex economy with no distortions the welfare theorems apply. As a consequence, the equilibrium allocation can be obtained by solving the problem of a social planner that maximizes the utility of the representative household subject to resource feasibility constraints. However, it will be more useful to characterize the equilibrium allocation as the solution to a series of city social planner's problems, one for each city, with some side conditions.

The state of a city social planner is given by the city's state (s, x, h) . His problem is described by the following Bellman equation:

$$V(s, x, h) = \max \left\{ \frac{1}{\chi} Y^{1-\chi} \left[s n^\theta k^\gamma \right]^\chi + C \varphi u + \eta l - (\eta + \psi) a - r k \right. \\ \left. + C \alpha \ln \left[\frac{h^\varsigma \hat{b}^{1-\varsigma}}{p} \right] p - i^h + \beta \sum_{s'} V(s', x', h') Q(s, s') \right\}$$

subject to

$$\begin{aligned} p &= x + a - l, \\ a &\geq 0, \\ l &\geq \pi x, \\ n + u &\leq p, \\ i^h &\geq 0, \\ x' &= p, \\ h' &= (1 - \delta_h) h + i^h, \end{aligned}$$

where l is the number of people that leave the city, a is the number of people that arrive to the city, η is the shadow value of a person that moves to a new city (exclusive of the moving cost ψ), r is the shadow value of capital services, Y is aggregate output of the consumption good, and C is aggregate consumption. The city social planner takes (r, η, Y, C) as given. Observe that the number of people that the city social planner allows to leave the city is bounded below by the fraction π of initial residents x that receive an exogenous moving shock.

The optimal population decision rule $p(s, x, h)$ is characterized by a lower population threshold $\underline{p}(s, h)$ and an upper population rule $\bar{p}(s, h)$ as follows:

$$p(s, x, h) = \max \left\{ \underline{p}(s, h), \min [(1 - \pi) x, \bar{p}(s, h)] \right\}. \quad (7)$$

That is, if beginning population net of exogenous attritions $(1 - \pi) x$ is less than the lower population threshold $\underline{p}(s, h)$, the city social planner brings people into the city until population equals the lower threshold. If $(1 - \pi) x$ is larger than the upper population threshold $\bar{p}(s, h)$, the city social planner takes people out of the city until population equals the upper threshold. Otherwise, the city social planner lets population decline at the exogenous attrition rate π .

Given the optimal population decision rule, entries to the city $a(s, x, h)$ and exits from the city $l(s, x, h)$ are given by

$$a(s, x, h) = \max \{ p(s, x, h) - (1 - \pi) x, 0 \}, \quad (8)$$

$$l(s, x, h) = \max \{x - p(s, x, h), \pi x\}, \quad (9)$$

respectively.

The optimal employment decision rule is characterized by the population decision rule and an employment threshold $\bar{n}(s)$ that solely depends on the idiosyncratic productivity of the city:

$$n(s, x, h) = \min \{\bar{n}(s), p(s, x, h)\}. \quad (10)$$

That is, if the optimal population level is $p(s, x, h)$ is below the employment threshold $\bar{n}(s)$, everybody in the city works. Otherwise, employment is equal to the employment threshold. Non-employment is then given by

$$u(s, x, h) = p(s, x, h) - n(s, x, h). \quad (11)$$

The optimal residential investment decision rule $i^h(s, x, h)$ is also of the (S,s) type. It is characterized by the optimal population rule and a threshold $\hat{h}[s, x']$ as follows:

$$i^h(s, x, h) = \max \left\{ \hat{h}[s, p(s, x, h)] - (1 - \delta_h) h, 0 \right\}. \quad (12)$$

That is, if residential structures net of depreciation $(1 - \delta_h) h$ is below the threshold corresponding to the optimal population size $p(s, x, h)$, residential structures are increased to that threshold level. Otherwise, residential investment is equal to zero.

Conditional on a given employment level n , the shadow value of an employed person is given by:

$$w(s, n) = \left[\hat{Y}^{\frac{1}{\chi} - 1} \right]^{\frac{1}{1 - \gamma\chi}} s^{\frac{\chi}{1 - \gamma\chi}} n^{-\frac{1 - \theta\chi - \gamma\chi}{1 - \gamma\chi}} \theta \left[\frac{\gamma}{r} \right]^{\frac{\gamma\chi}{1 - \gamma\chi}},$$

an expression that takes into account the optimal choice of capital k conditional on the employment level n . The employment threshold $\bar{n}(s)$ then satisfies that

$$\varphi C = w[s, \bar{n}(s)], \quad (13)$$

i.e. $\bar{n}(s)$ is the employment level at which the city social planner would be indifferent between allocating an additional resident to employment or non-employment.

The shadow value of a city resident $\xi(s, x, h)$ solves the following functional equation:

$$\xi(s, x, h) = \max \left\{ \begin{array}{l} \eta, \\ \eta + \psi, \\ -C\alpha + C\alpha \ln \left(\frac{h^\varsigma b^{1-\varsigma}}{(1-\pi)x} \right) + \max \{w[s, (1-\pi)x], \varphi C\} + \pi\beta\eta \\ + (1-\pi)\beta \sum_{s'} \xi(s', (1-\pi)x, (1-\delta_h)h + i^h(s, x, h)) Q(s, s') \end{array} \right\} \quad (14)$$

This equation states that the shadow value of a resident cannot become less than the shadow value of a mover η , otherwise the city planner would take people out of the city until the shadow value of a resident is equal to the shadow value of moving. On the other hand, the shadow value of a resident cannot exceed the shadow value of a mover η plus the cost of moving ψ , otherwise the city planner would bring people into the city until the shadow value of resident is equal to the shadow value of a mover plus the moving cost. If the shadow value of a resident is between η and $\eta + \psi$ the social planner lets the population contract at the exogenous attrition rate π , so population equals $(1 - \pi)x$. In this case, the shadow value of a resident is equal to a current return plus an expected discounted value. The current return has three terms. The first term is the negative impact that an additional resident has in lowering per-capita housing services. The second term is the gain obtained by the additional resident from the existing per-capital housing services. The third term is the marginal value of an employed person, if the additional resident is to become one. Otherwise it is equal to the value of leisure. The expected discounted value has two terms. The first term is the shadow value of a mover η times the probability that the additional resident becomes one at the beginning of the following period. The second term is the expected continuation shadow value of a city resident, given that the idiosyncratic productivity shock of the city will transit next period according to the transition matrix Q and that the optimal residential investment decision is described by equation (12).

The lower population threshold $\underline{p}(s, x, h)$ in equation (7) then satisfies the following equation:

$$\begin{aligned} \eta + \psi = & -C\alpha + C\alpha \ln \left(\frac{h^\varsigma b^{1-\varsigma}}{\underline{p}(s, x, h)} \right) + \max \{ w[s, \underline{p}(s, x, h)], \varphi C \} \\ & + \pi\beta\eta + (1 - \pi)\beta \sum_{s'} \xi(s', \underline{p}(s, h), \max \{ \hat{h}(s, \underline{p}(s, h)), (1 - \delta_h)h \}) Q(s, s'), \end{aligned} \quad (15)$$

and the upper population threshold $\bar{p}(s, x, h)$ satisfies:

$$\begin{aligned} \eta = & -C\alpha + C\alpha \ln \left(\frac{h^\varsigma b^{1-\varsigma}}{\bar{p}(s, x, h)} \right) + \max \{ w[s, \bar{p}(s, x, h)], \varphi C \} \\ & + \pi\beta\eta + (1 - \pi)\beta \sum_{s'} \xi(s', \bar{p}(s, x, h), \max \{ \hat{h}(s, \bar{p}(s, x, h)), (1 - \delta_h)h \}) Q(s, s'). \end{aligned} \quad (16)$$

That is, at the lower population threshold $\underline{p}(s, x, h)$ the city planner is indifferent between bringing an additional resident to the city or not and at the upper population threshold $\bar{p}(s, x, h)$ the city planner is indifferent between taking an additional resident out of the city or not.

The expected discounted shadow value of an additional unit of next period housing structures

$\phi^h(s, x', h)$, conditional on next period population x' , satisfies the following functional equation:

$$\phi^h(s, x', h) = \min \left\{ \begin{array}{c} 1, \\ \left[\begin{array}{c} \beta C \alpha \sum_{s'} \frac{p(s', x', (1-\delta_h)h)}{(1-\delta_h)h} \varsigma Q(s, s') \\ + (1 - \delta_h) \beta \sum_{s'} \phi^h(s', p(s', x', (1 - \delta_h) h), (1 - \delta_h) h) Q(s, s') \end{array} \right] \end{array} \right\}. \quad (17)$$

This equation states that the expected discounted shadow value of an additional unit of next period housing structures cannot exceed 1, otherwise the city planner would invest until that shadow value is brought down to the marginal cost of residential investment (which equals one unit of the consumption good). If the expected discounted shadow value of an additional unit of next period housing structures is less than one, the city planner lets housing structures depreciate at the rate δ_h . In this case, the expected discounted value of an additional unit of next period housing structures equals its expected contribution to increasing next period housing services, plus the continuation expected discounted value of the fraction of the additional unit of housing structures that does not depreciate. Both terms take into account the fact that the idiosyncratic productivity shock of the city transits according to the transition matrix Q and that the optimal population decision is described by equation (7).

The housing structures threshold $\hat{h}(s, x')$ in equation (12) satisfies the following equation:

$$1 = \beta C \alpha \sum_{s'} \frac{p[s', x', \hat{h}(s, x')]}{\hat{h}(s, x')} \varsigma Q(s, s') + (1 - \delta_h) \beta \sum_{s'} \phi^h(s', p[s', x', \hat{h}(s, x')], \hat{h}(s, x')) Q(s, s').$$

That is, at the housing structures threshold $\hat{h}(s, x')$ the city planner is indifferent between investing an addition unit or not.

There are six side conditions that must be satisfied for a solution to the city social planner's problem to represent a steady state competitive equilibrium. First, given the decision rules $p(s, x, h)$ and $i^h(s, x, h)$, the invariant distribution μ must satisfy that for every s and every Borel sets $\mathcal{X} \times \mathcal{H}$:

$$\mu(s', \mathcal{X} \times \mathcal{H}) = \int_{\mathcal{B}} Q(s, s') \mu(ds \times dx \times dh), \quad (18)$$

where

$$\mathcal{B} = \left\{ (s, x, h) : p(s, x, h) \in \mathcal{X} \text{ and } (1 - \delta_h) h + i^h(s, x, h) \in \mathcal{H} \right\}. \quad (19)$$

Second, that the market for the consumption good clears, i.e.

$$C + \delta_k K + \psi \int a(s, x, h) \mu(ds \times dx \times dh) + \int i^h(s, x, h) \mu(ds \times dx \times dh) = Y.$$

Third, that aggregate population is equal to one, i.e.

$$\int [n(s, x, h) + u(s, x, h)] \mu(ds \times dx \times dh) = 1. \quad (20)$$

Fourth, that the rental rate of capital satisfies the standard steady state relationship

$$r = \frac{1}{\beta} - 1 + \delta_k. \quad (21)$$

Fifth, that the rental market for capital clears, i.e.

$$\int k(s, x, h) \mu(ds \times dx \times dh) = K. \quad (22)$$

Sixth, that the aggregate output Y that the city planner takes as given be generated by the optimal decisions:

$$Y = \left\{ \int [sn(s, x, h)^\theta k(s, x, h)^\gamma]^\chi \mu(ds \times dx \times dh) \right\}^{\frac{1}{\chi}}.$$

6 Calibration

In this section we calibrate the steady state competitive equilibrium described above to U.S. data using a model time period equal to one year.

In order to calibrate the model we must first map it to the National Income and Product Accounts (NIPA) appropriately. We identify the model stock of capital K with private nonresidential fixed assets, i.e. with plants, equipment and software. As a consequence, we identify $\delta_k K$ with private nonresidential fixed investment. In turn, we associate $\int i^h.d\mu$ with private residential investment.

Since personal consumption expenditures includes housing expenditures, we associate them with

$$C + \psi \int a.d\mu + r^\eta \eta,$$

i.e. with model consumption, plus moving expenses, plus rental of housing services. However, since model aggregate output Y does not include the production of housing services, we define the empirical counterpart of Y as follows:

$$\begin{aligned} Y &= (C + \psi \int a.d\mu + r^\eta \eta) + \delta_k K + \int i.d\mu - r^\eta \eta \\ &= \text{Total PCE} + \text{non-residential investment} + \text{residential investment} - \text{housing PCE expenditures.} \end{aligned}$$

Observe that we are not incorporating government expenditures and net exports to our definition of Y since our model abstracts from them.

Considering annual nominal data between 1985 and 2008, we calculate an average capital-output ratio K/Y equal to 1.50 and an investment-output ratio $(\delta_k K)/Y$ equal to 0.156. As a consequence, we set the depreciation rate of capital to $\delta_k = 0.104$. For the discount factor β we choose it to generate an annual interest rate $1/\beta = 0.04$, which is a standard value in the literature.

From the first order conditions to the problem of the producers of the consumption good we have that for every intermediate good z :

$$q(z) = Y^{1-\chi} y(z)^{\chi-1}. \quad (23)$$

In turn, the first order conditions to the problem of the intermediate good producers are the following:

$$\frac{w(z) n(z)}{q(z) y(z)} = \theta, \quad (24)$$

$$\frac{r^k k(z)}{q(z) y(z)} = \gamma. \quad (25)$$

From equations (23), (24) and (25) it then follows that

$$\frac{\int w(z) n(z) d\mu}{Y} = \theta$$

and

$$\frac{rK}{Y} = \gamma.$$

That is, the share of labor is equal to θ and the share of capital is equal to γ , independently of the elasticity of substitution parameter χ . We then select $\theta = 0.64$, which is the share of labor commonly used in the literature. In turn we choose $\gamma = 0.217$ which is consistent with the above values for β , δ_k and K/Y and the fact that the rental rate of capital satisfies equation (21).

From the household's first order conditions we have that

$$r^\eta = C\alpha \frac{p}{h^\varsigma b^{1-\varsigma}}.$$

From this equation and the first order conditions for the housing services producers we then have that:

$$\begin{aligned} r^h &= C\alpha \frac{p}{h} \varsigma, \\ r^b &= C\alpha \frac{p}{b} (1 - \varsigma). \end{aligned}$$

The price of one unit of housing structures $\zeta^h(s, x, h)$, which equals the expected discounted value of r^h net of depreciation, is then given by:

$$\zeta^h(s, x, h) = \varsigma C\alpha \frac{p(s, x, h)}{h} + (1 - \delta_h) \beta \sum_{s'} \zeta^h[s', p(s, x, h), i^h(s, x, h)] Q(s, s'), \quad (26)$$

and the price of one unit of land $\zeta^h(s, x, h)$, which equals the expected discounted value of r^b , is given by

$$\zeta^b(s, x, h) = (1 - \varsigma) C \alpha \frac{p(s, x, h)}{\hat{b}} + \beta \sum_{s'} \zeta^b[s', p(s, x, h), i^h(s, x, h)] Q(s, s').$$

The share of land in the total value of housing is then given by

$$\frac{\zeta^b(s, x, h) \hat{b}}{\zeta^h(s, x, h) h + \zeta^b(s, x, h) \hat{b}}.$$

We choose ς to reproduce the average of this share in the U.S. economy, which according to (??) is equal to 0.35.

We associate the aggregate value of housing structures,

$$\int \zeta^h(s, x, h) h d\mu,$$

with total private residential fixed assets in the NIPA accounts, since this measure represents the cost of reproducing the stock of houses. We then choose the value of housing services in the utility function α to generate

$$\frac{\int \zeta^h(s, x, h) h d\mu}{Y} = 1.51,$$

which is the corresponding average ratio in the U.S. economy.

In turn, we select the depreciation rate of housing structures δ_h so that

$$\frac{\int i^h(s, x, h) d\mu}{Y} = 0.062,$$

the average ratio of private residential investment to our measure of output in the U.S. economy.

There is a lot of uncertainty about what value to choose for χ , the parameter determining the elasticity of substitution between city specific goods. In our benchmark case we choose $\chi = 0.667$ to generate an elasticity of substitution equal to 3, which is the median value estimated by Broda and Weinstein (2) using international trade data.¹³ However, since this is an elasticity of substitution between goods and there must be some considerable amount of overlap in the production structure of the different cities, we will consider higher elasticities of substitution as well.

The exogenous population attrition rate π is a crucial determinant of how large gross population flows are in excess of net population flows. To explore the importance of gross flows in the model's ability to match U.S. data, we will consider two different scenarios. The first scenario has $\pi = 0$ so that it generates no population reallocation in excess of population net flows. In the second

¹³This is also the elasticity of substitution between goods in Alvarez and Shimer (1).

scenario, we choose π so that the aggregate gross population expansion rate is about ten times larger than the aggregate net population expansion rate, a feature of U.S. data discussed in Section 2.

In the absence of hard data about moving expenses we will treat the moving cost ψ as a free parameter and choose it to reproduce the volatility of population reported in Section 2. Similarly, we will treat the utility of leisure φ as a free parameter and choose it to reproduce the volatility of employment reported in Section 2. Instead of calibrating φ to some employment/population ratio we choose to treat it as a free parameter because it is not clear what measure of population the model should correspond to. Observe that in the model economy a city's population level increases only after all its residents have become employed, a feature that would never be obtained using some broad population measure (such as total population 16 years or older).

We now turn to a more delicate issue: the selection of the idiosyncratic productivity process.

6.1 Estimation of the idiosyncratic technology process

Contrary to the specification of the model, our data are generated from a growing economy subject to aggregate disturbances and with permanent differences in growth rates among cities. It is therefore necessary to place additional structure on the underlying process driving technology in order to extract a model-consistent measure of the idiosyncratic technology process from the data. A final consideration is that a preliminary analysis of the data suggests the idiosyncratic technology process is very persistent with a root close to unity.¹⁴ Consequently, our empirical specification of technology is

$$\Delta \ln s_{it} = \zeta_i + \varepsilon_{it} + a_t, \quad (27)$$

where $\Delta x_t = x_t - x_{t-1}$. Here ζ_i captures idiosyncratic differences in growth across cities and a_t is the growth rate of the aggregate technology. Since ζ_i is idiosyncratic, $E\zeta_i = 0$. We also assume that ε_{it} is normally distributed with mean zero and variance σ_ε^2 .

We use profit maximization of the representative firm in city i and the fact that productive capital is homogenous and allocated costlessly across cities to identify the idiosyncratic technology process. Combining the first order conditions for labor and capital input in any two cities in any date t yields:

$$\frac{w_{it}}{w_{jt}} = \left(\frac{s_{it}}{s_{jt}} \right)^{\frac{\chi}{1-\gamma\chi}} \left(\frac{n_{it}}{n_{jt}} \right)^{-\frac{1-(\theta+\gamma)\chi}{1-\gamma\chi}}.$$

¹⁴Need to add detail here.

Using the definition

$$\hat{x}_{it} = \ln x_{it} - \int \ln x_{jt} dj$$

the first order conditions can be written as follows:

$$\Delta \hat{s}_{it} = \frac{1 - \gamma\chi}{\chi} \Delta \hat{w}_{it} + \frac{\chi}{1 - (\theta + \gamma)\chi} \Delta \hat{n}_{it}$$

We measure the variable $\Delta \hat{s}_{it}$ using this equation with data on wages and employment by city, using pre-determined values for χ, γ and θ .

Using (27), we have that $\Delta \hat{s}_{it}$ satisfies

$$\Delta \hat{s}_{it} = \zeta_i + \varepsilon_{it}.$$

In practice we find evidence of positive first order serial correlation in $\Delta \hat{s}_{it}$. Therefore we augment our empirical specification with the assumption that idiosyncratic wage growth, $\Delta \hat{w}_{it}$, is subject to first-order moving average classical measurement error. It follows that the variance of the idiosyncratic term is given by

$$\sigma_\varepsilon^2 = E [\Delta \hat{s}_{it} - \zeta_i]^2 - 2 \left[\frac{\chi}{1 - \gamma\chi} \right]^2 E [\Delta \hat{s}_{it} - \zeta_i] [\Delta \hat{s}_{it-1} - \zeta_i].$$

We implement this procedure by identifying ζ_i with the time series average of measured $\Delta \hat{s}_{it}$.

It turns out that under our calibrated values of $\theta = 0.64$ and $\gamma = 0.217$, our estimate of σ_ε is equal to 0.01091 when $\chi = 1$ and equal to 0.01764 when $\chi = 0.667$. Given the measurement errors in both cases, we estimate the standard deviation of wages to be 0.0129 in the first case and 0.0078 in the second. Observe that these standard deviations compare to a standard deviation of 0.0158 of measured wages.

Since our theory requires a stationary process for the idiosyncratic productivity levels, we will approximate our estimated random walk process with the following AR(1) process:

$$\ln s_{i,t+1} = 0.95 \ln s_{it} + \varepsilon_{t+1}, \tag{28}$$

where ε_t is i.i.d., normally distributed, with zero mean and variance σ_ε^2 , where the variance σ_ε^2 is set to our empirical estimate. In addition, we will approximate by quadrature methods the stationary process described by equation (28) using a finite state Markov process in which the idiosyncratic productivity level is allowed to take 9 values.

7 Results

In this section we evaluate how well the model does in reproducing the data described in Section 2. We also use the model to assess the importance of local housing markets vis-a-vis moving costs in shaping population flows in the U.S. economy. As was already mentioned in the previous section we will consider the case of $\chi = 0.667$ (i.e. and elasticity of substitution equal to 3) as our benchmark and we will treat the utility of leisure φ and the moving cost ψ as free parameters. We first consider the case of no exogenous attrition, i.e. the case in which $\pi = 0$. Positive excess population reallocation will be introduced later on. For each of the parameter combinations (φ, ψ, π) that we consider we recalibrate the model to the same observations described in the previous section.

7.1 Case $\pi = 0$

Table 7 reports standard deviations for the log differences of different variables, both for the U.S. economy and a number of different versions of the model. The variables reported are employment n , population p , wages w , housing prices $\frac{\zeta^h h + \zeta^b \hat{b}}{\eta}$, housing rentals r^η , housing structures h , and residential investment i^h . The last two columns, instead of reporting standard deviations, report the average employment/population ratio n/p and the moving cost parameter ψ as a fraction of average wages, respectively. Observe that we define housing prices as the total value of the housing structures and land of a city divided by the total housing services produced by the city, and that we define housing rentals as the price of one unit of housing services produced by the city.

To learn about the role of φ and ψ in our calibration we start by setting the moving cost ψ to zero and choosing φ to generate an aggregate employment-population ratio equal to 0.62, which corresponds to the ratio of employment to population 16 years and over in the U.S. economy. The second row of Table 7 (" $n/p = 0.62$, $\psi = 0$ ") reports the results. We see that in this case employment is significantly more volatile than in the data (2.74% instead of 1.83%). However all other variables have zero volatility. What happens in this case is that, since the employment/population rate is only 62%, the city planner can generate relatively small employment fluctuations in response to the idiosyncratic productivity shock by shifting residents between employment and non-employment, without having to resort to migrations in and out of the city. As a consequence, the population is always equal to one and the stock of housing structures is constant. In fact, since housing structures are constant the irreversibility constraint never binds and the housing structures/population ratio

h/p satisfies the frictionless version of equation (26) given by

$$\frac{1}{\beta} - 1 + \delta_h = \varsigma C \alpha \frac{p}{h}. \quad (29)$$

Observe that the frictionless version of equation (16), which becomes

$$\eta = -C\alpha + C\alpha \ln \left(\frac{h^\varsigma \hat{b}^{1-\varsigma}}{p} \right) + \max \{w[s, p], \varphi C\} + \beta \eta,$$

can be satisfied for $p = 1$ and the solution h/p to equation (29) because the wage rate is always equal to φC (since employment is always less than p).

We now retain the assumption of zero moving costs ψ but set φ equal to zero, such that everybody in a city becomes employed. The third row of Table 7 (“ $n/p = 1$, $\psi = 0$,”) reports the results for this case. We see that employment fluctuates much less than in the previous case (1.92% instead than 2.74%), bringing it closer to the data. The reason for this is that wages never hit the value of leisure φC and thus display enough volatility to dampen the employment fluctuations quite significantly. However, by construction, population growth also has a standard deviation of 1.92%, which is more than twice as large as what is observed in the data.

To learn about the magnitude of the moving cost ψ needed to bring down the standard deviation of population growth to what is seen in the data, we still set φ equal to zero but now choose ψ to reproduce that standard deviation. The fourth row of Table 7 (“ $n/p = 1$, $\psi > 0$ ”) shows that a moving cost ψ equal to 24% of annual wages is required to hit that target. However, now employment, which by construction equals population, now fluctuates too little.

In the fifth column of Table 7 (“free n/p and ψ ”) we choose the disutility of leisure φ and the moving cost ψ to reproduce the standard deviations of employment and population growth rates. Observe that the average employment/population ratio n/p needed to make employment more volatile than population by the factor observed in the data is 0.986. In this case, the moving cost ψ needed to generate the population fluctuations observed in the data is equal to 8.3% of annual wages. Observe that wages fluctuate as much as in the data, indicating that the model generates a reasonable correlation between employment and the idiosyncratic productivity shocks. Where the model fails is in terms of housing prices and housing rentals. There are huge discrepancies in terms of housing prices: Housing prices fluctuate only 0.32% in the model while they fluctuate 4.25% in the data. The discrepancies are not as large in terms of housing rentals, but they are still significant: housing rentals fluctuate 0.96% in the model while they 1.97% in the data. While the growth rate of housing permits is very volatile in the data (its standard deviation is 27.6%) and it must be closely related to the growth rate of residential investment, the standard deviation of the growth

rate in residential investment in the model economy is infinity because of the binding irreversibility constraint. Observe from equation (17) that the price of housing structures (excluding current rental income) is equal to one whenever residential investment is positive, and that it drops below one only when residential investment is zero. Despite of this, the irreversibility constraint does not contribute to generating large fluctuations in the price of housing structures because it binds for only 0.14% of the cities.

In order to generate larger fluctuations in housing prices we introduce quadratic adjustment costs to residential investment. Observe that up to this point the cost of producing the gross increase in housing structures i_t^h described by equation (6) was i_t^h units of the consumption good. Under quadratic adjustment costs, we assume that the same gross increase in housing structures i_t^h costs

$$i_t^h + \frac{\Omega}{2h_t} (i_t^h)^2$$

units of the consumption good.¹⁵ The sixth row of Table 7 (“free n/p and ψ , $\Omega > 0$ ”) shows results for this case. The selected value for Ω is admittedly arbitrary, but represents a compromise between the goal of generating larger fluctuations in house prices and keeping residential investment enough volatile to be broadly consistent with the evidence on housing permits. We see that in this case employment and population fluctuate as much as in the U.S. economy since the value of leisure φ and the moving cost ψ have been selected to this end. Wages also fluctuate as much as in the data, indicating that the model still generates a reasonable correlation between employment and productivity shocks. Observe that housing prices have become significantly more volatile than in the absence of quadratic adjustment costs (0.52% instead of 0.32%) but they are still far less volatile than the data (4.25%). Housing rentals are not much affected, while residential investment display more reasonable fluctuations given that the investment irreversibility constraint no longer binds. Under this parametrization the employment/population ratio is still equal to 98.5% while the moving cost is lowered to only 4.2% of annual wages. In what follows we will treat this last version of the model as our benchmark case when $\pi = 0$.

Table 8 displays contemporaneous correlations between the growth rates of the different variables in this economy. Compared with the empirical correlations displayed in Table 4 we see that the comovements of employment with population (0.64), wages (0.36), residential investment (0.49), and housing prices (0.76) are somewhat stronger, but still broadly in line with the data (0.40, 0.15,

¹⁵We select this specification for the quadratic adjustment costs because it is consistent with balanced growth.

0.25, and 0.38, respectively).¹⁶ The comovements of population with the other variables are also broadly consistent with the data except for the correlation with residential investment, which is 0.43 in the model, while population and housing permits are empirically uncorrelated.¹⁷ The correlation between wages and residential investment (0.34) is slightly higher than the empirical correlation between wages and housing permits (0.10), but the correlation between wages and housing prices is significantly stronger than in the data: 0.82 instead of 0.23. Also, the correlation between housing prices and residential investment is stronger than the corresponding empirical correlation with housing permits: 0.51 instead of 0.22. All considered we conclude that comovements in the model are roughly consistent with those observed in the data: except for the zero correlation between population and housing permits, all other correlations have the correct sign.

The model does not perform as well in terms of autocorrelations, though. Table 5 showed that in the U.S. employment, population and housing prices display sizable positive autocorrelations at the one year horizon. However, Table 9 shows that in the model these autocorrelations are essentially zero (only population exhibits some positive autocorrelation but it is rather small), inheriting the same behavior of the idiosyncratic productivity shocks. On the other hand, wages and residential investment are roughly consistent with the data.

There are also discrepancies in the leads/lags structure. Table 10 reports the correlations of employment growth with the growth rates of different variables at different leads and lags. From Table 6 we know that in the U.S. wages, and to some extent housing permits, lead employment by one year. However, there are no comparable leads in the model economy. The lag structure is also problematic. While Table 6 shows that U.S. employment is positively correlated with lagged population and housing prices, in the model the correlation of employment with lagged population is rather weak and nonexistent with housing prices.

Although the model fails to generate some positive autocorrelations in growth rates and misses the lead/lag structure of employment with some other variables, we have set the model to very high standards. These failures show that the model does not display an internal propagation mechanism at the city level for the idiosyncratic productivity shocks. However the model still captures some salient features of the data, such as volatilities and contemporaneous comovements. Perhaps the most serious deficiency of the model is that it fails to generate the large volatility of housing prices

¹⁶Observe that given the lack of better measures, we associate residential investment with housing permits.

¹⁷The correlation of population growth with housing prices is also somewhat too strong compared with the data: 0.79 instead of 0.33.

observed in U.S. data. However, this problem is not particular to our model: the difficulties of general equilibrium models in generating large asset price fluctuations have become quite familiar to the literature. In the specific case of housing prices, a similar problem has been encountered by Davis and Heathcote (3) in the context of business cycle analysis.

7.1.1 Population flows and local housing markets

With these caveats in mind we now use the model to evaluate the importance of local housing markets vis-a-vis direct moving costs in determining the magnitude of the population flows observed in the U.S. economy. We first evaluate the effects of setting the moving cost parameter ψ to zero. The second row of Table 11 reports the results of this experiment while the first row reproduces statistics for our benchmark economy. We see that removing the moving costs have a relatively small impact on employment volatility: Its standard deviation increases by 16% (from 1.86 to 2.16). However, the impact on population volatility is quite significant: Its standard deviation increases by 60% (from 0.93 to 1.49). The decrease in employment volatility is enough to lower wage volatility by 50% (from 0.72 to 0.36) while the increase in population volatility increases housing prices volatility by 22% (from 0.52 to 0.63) and housing rentals volatility by 63% (from 0.92 to 1.50). Observe that the volatility of housing structures and residential investment also increase slightly to accommodate to the higher population volatility.

We now turn to evaluate the role of local housing markets in determining population flows. In order to do this we consider a version of the model economy in which both housing structures and land are freely mobile. That is, when people move to a different city they can now take their houses with them. Admittedly this is an artificial experiment but it will be extremely useful for assessing the importance of the immobility of houses in determining population flows.

In the economy with freely mobile housing the city planner's problem becomes the following:

$$V(s, x) = \max \left\{ \frac{1}{\chi} Y^{1-\chi} \left[s n^\theta k^\gamma \right]^\chi + C \varphi u + \eta l - (\eta + \psi) a - r k \right. \\ \left. + C \alpha \ln \left[\frac{h^\varsigma b^{1-\varsigma}}{p} \right] p - r^h h - r^b b + \beta \sum_{s'} V(s', x') Q(s, s') \right\}$$

subject to

$$\begin{aligned}
p &= x + a - l, \\
a &\geq 0, \\
l &\geq \pi x, \\
n + u &\leq p, \\
x' &= p,
\end{aligned}$$

where r^h is the economy-wide shadow value of housing structures and r^b is the economy-wide shadow value of land. Observe that at these shadow prices the city planner is free to rent any amount of housing structures h and land b .

The third row of Table 11 reports the effects of converting the benchmark economy into one with perfectly mobile housing (parameter values are exactly the same as those in the benchmark economy, except for the adjustment cost parameter Ω , which is set to zero). We see that the effects on housing structures are large: Its standard deviation increases by a factor of five (from 0.28 to 1.38). However, the effects on employment fluctuations are quite small: its standard deviation increases only by 4% (from 1.86 to 1.93). As a consequence, the effects on wages are also relatively small. Observe that, similarly to the previous experiment, the effects on population fluctuations are significant: the standard deviation of population growth increases by 48% (from 0.93 to 1.38).

The reason why employment volatility increases less than population volatility in both experiments is that in the benchmark case employment has the cushion of unemployment to accommodate to the idiosyncratic shocks, a margin that involves no adjustment costs. On the contrary, population is subject to adjustment costs given by the direct moving cost and the fixity of housing. As a consequence, when one of these adjustment costs is removed the largest effects must fall on population volatility. Employment volatility also increases, however, because the pool of employable people has become more volatile.

Observe from Table 11 that population fluctuates more in the second row than in the third. That is, in the benchmark economy the direct moving costs are a more important factor than the fixity of housing in determining population volatility.

7.2 Case $\pi > 0$

This section introduces explicit excess population reallocation by making the exogenous attrition rate parameter π positive. Before doing so, we would like to point out that the version of the model

with $\pi = 0$ considered in the previous section can actually be reinterpreted as one with implicit excess population reallocation. To see this, consider a version of the model in which population mobility takes place in two stages. In the first stage, a person is forced to move “for family reasons” with probability κ . When this happens the person is randomly assigned to a second person in the economy and must move to the city where the second person is initially located, after payment of a moving cost ψ_f . Observe that because of this random assignment, a city that has x initial residents will lose κx residents for family reasons and will gain κx residents for family reasons, leaving its size unchanged at the end of the first stage. In the second stage, after observing the idiosyncratic productivity shock of the cities where they are subsequently located, people decide whether to stay or reallocate after paying the moving cost ψ , in exactly the same way that has been described in previous sections. Observe that if the moving cost for family reasons ψ_f is equal to zero, the equilibrium of the economy with exogenous family moves will be exactly the same as the one in the previous section, except that gross population flows will be positive. If the moving cost for family reasons ψ_f is positive, aggregate consumption will be affected. However, in this case the equilibrium of the economy with family moves will coincide with the equilibrium of the previous section after the utility of leisure parameter φ and the utility of housing parameter α are appropriately redefined. We conclude that the results of Tables 1-4 also correspond to economies with exogenous family moves and, therefore, that they are consistent with positive gross population flows of (arbitrary) size κ . The general equilibrium effects in Table 11 will generally depend on the size of the moving cost for family reasons ψ_f , because it may affect the size of the wealth effects. However, if ψ_f equals ψ the differences with Table 11 are likely to be small because ψ was only equal to 4% of annual wages.

We now go back to our previous interpretation of the model, which abstracts from family moves, and introduce a positive exogenous attrition rate π . Observe that these exogenous attritions differ from the family moves in that the people making these moves can freely choose where to reallocate depending on local economic conditions (i.e. wages and housing). For the quantitative analysis that follows it will be important to determine what size to introduce for the exogenous attrition rate π .

Recall from Section 2 that the gross population expansions rate is ten times larger than the net population expansions rate in IRS data (4.3% vs. 0.43%). However, net population flows in IRS data are larger than those obtained in BEA data. In particular, the standard deviation of population growth is equal to 1.72% in IRS data while it is equal to 0.89% in BEA data. Since BEA data is based on good quality census data, this suggests introducing lower gross population

flows than those obtained from IRS data. To determine an actual magnitude for the exogenous attrition rate π we thus decide to do the following: We calculate the net expansions rate in the benchmark economy “free n/p and ψ , $\Omega > 0$ ” of Table 7, which has the same standard deviation for population growth rates as BEA data, and pick an exogenous attrition rate that is ten times larger. This delivers a π equal to 0.02.

The first row of Table 12 reports standard deviations for the benchmark economy “free n/p and ψ , $\Omega > 0$ ” when the exogenous attrition rate π is set to $\pi = 0.02$ while all other parameters are left unchanged. Observe that relative to the first row of Table 11 population becomes significantly more volatile: Its standard deviation increases by 42% (from 0.93 to 1.32) This allows employment to fluctuate slightly more (its standard deviation increases by 11%) and this dampens wage volatility. The higher population fluctuations also increase the volatility of housing structures, residential investment and housing prices, but the effects are rather small. The reason why population becomes more volatile when we introduce an exogenous attrition rate is that it creates a large pool of people (more precisely, 2% of the population) that, independently of the size of the moving cost ψ and the state of the cities where they are initially located, are forced to move. Since this pool of people reallocates to cities that receive good idiosyncratic productivity shocks, population becomes more responsive to the shocks and therefore becomes more volatile. For similar reasons as before, the higher population volatility leads to more employment volatility.

The second row of Table 12 reproduces the second row of Table 11. That is, it reports the equilibrium effects of setting the moving cost to zero. Compared to the first row of Table 12 we see that the effects are much smaller when $\pi = 0.02$: The standard deviation of population growth increases by only 13% (from 1.32 to 1.49). The reason is that the pool of people that is forced to move is sufficiently large that the amount of people reallocating to new cities is similar to when the moving cost is equal to zero. Essentially the main factor restricting net population flows in the first row of Table 12 is the fixity of local housing. Indeed, we see in the third row of Table 12 that when housing becomes fully mobile that the effects on population flows are much larger: The standard deviation of population now increases by 37% (from 1.32 to 1.81).

We conclude that when $\pi = 0.02$ the fixity of housing is more important than the moving costs in determining population volatility. This is contrary to the case of $\pi = 0$. What is robust to both cases is that the fixity of housing always plays a significant role in net population reallocation: Even in the most conservative estimate it accounts for a reduction of 37% in the the standard deviation of population growth.

Finally, to make clear that the effects of the moving costs critically depend on π the fourth row

of Table 12 reports the effects of doubling the moving cost parameter ψ so that it equals 8.4% of annual wages. We see that the effects over the benchmark economy with $\pi = 0.02$ (first row of Table 12) are minuscule. The reason for this is that when ψ was 4.2% of annual wages, the economy was already responding to the idiosyncratic productivity shocks by exclusively using people that had to reallocate for exogenous reasons. When the moving cost doubles there are even more reasons for doing this. On the contrary, we see in the fourth row of Table 11 that when the moving costs double in the benchmark economy with $\pi = 0$ (first row of Table 11) that the effects are quite significant. In particular, the standard deviation of population growth now decreases by 17% (from 0.93 to 0.77). Since we have interpreted the economy with $\pi = 0$ as one with exogenous reallocations for “family reasons”, we conclude that determining the exact nature of the gross population flows will be critical for evaluating the effects of moving costs.

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Table 1

Means and Standard Deviations, 1984-2008 (Except Where Noted)

All Variables Except the Real House Price Index in 000s

Variable	Mean	Std. Dev.	Avg. # MSAs
Population _t	607.8	1,423.3	366
Wage Employment _t	294.1	687.2	366
Pop. Inflows _{t-1} *	20.1	30.5	366
Pop. Outflows _{t-1} *	20.0	35.2	366
Housing Permits _t	3.7	7.8	362
Real House Price Index _t	127.3	27.0	334
Real Wage per Job _t **	35.5	5.8	366

* 1986-2008. ** Base Year 2000.

Table 2

MSA Mobility Rates, Aggregated Across MSAs, from IRS Data

	Total Inflow	Total Outflow	Across- MSA Inflow	Across- MSA Outflow	Population	Total Mig. Rate (%)	Across- MSA Inflow Rate (%)	Across- MSA Outflow Rate (%)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1985	9.92	9.74	7.15	6.97	155.05	6.34	4.61	4.49
1986	10.21	10.12	7.25	7.15	157.17	6.47	4.61	4.55
1987	9.99	9.93	7.12	7.05	157.82	6.31	4.51	4.47
1988	10.26	10.22	7.30	7.26	158.15	6.47	4.61	4.59
1989	10.61	10.58	7.60	7.58	162.45	6.52	4.68	4.67
1990	10.13	10.14	7.16	7.17	160.40	6.32	4.46	4.47
1991	10.19	10.11	7.11	7.03	162.41	6.25	4.38	4.33
1992	10.39	10.38	7.17	7.15	162.94	6.37	4.40	4.39
1993	10.31	10.37	7.06	7.13	163.21	6.34	4.33	4.37
1994	10.39	10.45	7.16	7.22	163.16	6.39	4.39	4.43
1995	10.07	10.11	7.00	7.04	165.28	6.10	4.23	4.26
1996	10.37	10.34	7.16	7.13	167.56	6.18	4.28	4.25
1997	10.52	10.44	7.24	7.16	169.53	6.18	4.27	4.22
1998	10.84	10.71	7.39	7.26	172.81	6.24	4.28	4.20
1999	11.00	10.87	7.47	7.34	176.64	6.19	4.23	4.15
2000	11.06	10.87	7.57	7.38	179.20	6.12	4.22	4.12
2001	11.19	11.12	7.48	7.41	183.07	6.09	4.08	4.05
2002	11.04	11.00	7.31	7.27	185.24	5.95	3.95	3.93
2003	11.21	11.16	7.40	7.36	186.59	5.99	3.97	3.94
2004	11.49	11.46	7.68	7.65	188.07	6.10	4.08	4.07
2005	11.98	11.97	8.12	8.11	188.44	6.36	4.31	4.30
2006	11.49	11.42	7.84	7.77	190.50	6.01	4.12	4.08
2007	11.58	11.48	7.87	7.76	197.48	5.84	3.99	3.93
Average						6.22	4.30	4.27
Std. Dev.						0.18	0.21	0.22

Table 3

IRS Data: Binned Net Migration and Gross Expansion and Contraction Rates
(366 MSAs, 1985-2007)

Net Mig Decile (IRS)	Mean Net Mig Pct (IRS)	Mean Gross Exp Pct (IRS)	Mean Gross Contr Pct (IRS)
	(1)	(2)	(3)
0 - 10	-1.62	6.18	7.80
10 - 20	-0.69	4.64	5.33
20 - 30	-0.40	4.49	4.89
30 - 40	-0.17	4.71	4.88
40 - 50	0.03	4.92	4.89
50 - 60	0.23	5.10	4.87
60 - 70	0.47	5.41	4.94
70 - 80	0.77	5.95	5.18
80 - 90	1.26	6.83	5.57
90 - 100	2.96	9.01	6.05

Table 4

Sample Standard Deviations and Correlations
1985-2008

	Std. Dev. of		Contemporaneous Correlations of ϵ_{it}						
	$\Delta \hat{x}_{it}$	ϵ_{it}	p	n	a	l	h	q	w
Population p	1.39	0.89	1.00	0.40	0.43	-0.22	0.08	0.33	0.11
Employment n	2.10	1.83		1.00	0.40	-0.15	0.25	0.38	0.15
Lag Entries a	6.80	6.71			1.00	-0.24	0.20	0.22	0.12
Lag Exits l	6.76	6.63				1.00	-0.06	0.13	0.03
Permits h	27.78	27.61					1.00	0.22	0.10
House Prices q	4.37	4.25						1.00	0.23
Wage per Job w	1.64	1.58							1.00

Table 5

Sample Autocorrelations of ϵ_{it}

1985-2008

Lag	p	n	a	l	h	q	w
0	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1	0.38	0.35	-0.08	-0.24	-0.12	0.67	0.09
2	0.22	0.03	-0.04	-0.01	0.01	0.22	-0.03
3	0.09	-0.06	-0.06	-0.02	-0.03	-0.10	-0.02
4	-0.03	-0.13	-0.14	-0.05	-0.07	-0.22	0.01

Table 6

Correlation of ϵ_{it} for Employment at time t with leads and lags of the other ϵ_{it} variables
1985-2008

Lag or Lead	p_{t-s}	n_{t-s}	a_{t-s}	l_{t-s}	h_{t-s}	q_{t-s}	w_{t-s}
$s = 4$	-0.16	-0.13	-0.08	-0.08	-0.03	-0.26	-0.11
$s = 3$	-0.16	-0.06	-0.01	-0.06	0.04	-0.22	-0.08
$s = 2$	-0.07	0.03	0.10	-0.09	0.14	-0.04	0.04
$s = 1$	0.06	0.35	0.18	-0.06	0.27	0.22	0.21
$s = 0$	0.40	1.00	0.40	-0.15	0.25	0.38	0.15
$s = -1$	0.42	0.35	0.18	0.10	0.10	0.38	0.10
$s = -2$	0.23	0.03	-0.08	0.25	-0.02	0.29	0.10
$s = -3$	0.11	-0.06	-0.11	0.16	-0.07	0.19	0.09
$s = -4$	0.02	-0.13	-0.13	0.09	-0.10	0.07	0.04

Table 7

Standard deviations: Case $\pi = 0$

$$\sigma(\ln x_t - \ln x_{t-1})$$

	n	p	w	$\frac{\zeta^h_h + \zeta^b_b}{\eta}$	r^η	h	i^h	n/p	ψ/w
1) U.S. economy	1.83	0.89	0.78	4.25	1.97	n.a.	27.6*	n.a.	n.a.
2) $n/p = 0.62, \psi = 0$	2.74	0.0	0.0	0.0	0.0	0.0	0.0	0.62	0.0
3) $n/p = 1, \psi = 0$	1.92	1.92	0.56	0.61	2.23	2.33	∞	1.0	0.0
4) $n/p = 1, \psi > 0$	0.88	0.88	1.20	0.32	0.98	0.96	∞	1.0	0.24
5) free n/p and ψ	1.84	0.89	0.82	0.32	0.96	1.07	∞	0.986	0.083
6) free n/p and $\psi, \Omega > 0$	1.86	0.93	0.72	0.52	0.92	0.28	5.46	0.985	0.042

*: standard deviation of housing permits growth rates

Table 8

Contemporaneous correlations: Case $\pi = 0$

$$\rho(\ln x_t - \ln x_{t-1}, \ln y_t - \ln y_{t-1})$$

	n	p	w	$\frac{\zeta^h h + \zeta^b \hat{b}}{\eta}$	r^η	h	i^h
n	1.00	0.64	0.36	0.76	0.65	0.00	0.49
p		1.00	0.47	0.79	0.97	0.16	0.43
w			1.00	0.82	0.51	-0.16	0.34
$\frac{\zeta^h h + \zeta^b \hat{b}}{\eta}$				1.00	0.85	-0.19	0.51
r^η					1.00	-0.08	0.52
h						1.00	-0.37
i^h							1.00

Table 9

Autocorrelations: Case $\pi = 0$

$$\rho(\ln x_t - \ln x_{t-1}, \ln x_{t-s} - \ln x_{t-1-s})$$

Lag	n	p	w	$\frac{\zeta^h h + \zeta^b \hat{b}}{\eta}$	r^η	h	i^h
$s = 0$	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$s = 1$	0.00	0.16	-0.16	-0.01	0.06	0.66	-0.24
$s = 2$	0.00	0.09	-0.10	-0.03	0.02	0.48	-0.10
$s = 3$	-0.01	0.05	-0.06	-0.04	-0.02	0.37	-0.05
$s = 4$	-0.01	0.02	-0.04	-0.04	-0.03	0.29	-0.03

Table 10

Correlations with employment at different leads/lags: Case $\pi = 0$

$$\rho(\ln n_t - \ln n_{t-1}, \ln x_{t-s} - \ln x_{t-1-s})$$

Lead/Lag	p	w	$\frac{\zeta^h h + \zeta^b \hat{b}}{\eta}$	r_t^η	h_t	i_t^h
$s = -4$	-0.01	0.02	0.01	0.00	-0.05	0.0
$s = -3$	0.01	0.03	0.03	0.02	-0.05	0.01
$s = -2$	0.03	0.06	0.05	0.04	-0.04	0.02
$s = -1$	0.06	0.10	0.08	0.07	-0.02	0.03
$s = 0$	0.64	0.36	0.76	0.65	0.0	0.49
$s = 1$	0.16	-0.07	-0.05	0.06	0.39	-0.06
$s = 2$	0.10	-0.06	-0.05	0.02	0.33	-0.06
$s = 3$	0.06	-0.04	-0.05	0.00	0.27	-0.05
$s = 4$	0.04	-0.03	-0.05	-0.02	0.22	-0.04

Table 11

Experiments: Case $\pi = 0$

	n	p	w	$\frac{\zeta^h h + \zeta^b \hat{b}}{\eta}$	r^η	h	i^h	n/p	ψ/w
1) free n/p and ψ , $\Omega > 0$	1.86	0.93	0.72	0.52	0.92	0.28	5.46	0.985	0.042
2) $\psi = 0$	2.16	1.49	0.36	0.63	1.50	0.31	5.73	0.988	0.000
3) mobile housing	1.93	1.38	0.65	n.a.	n.a.	1.38	n.a.	0.991	0.042
4) high ψ	1.81	0.77	0.83	0.45	0.76	0.27	5.26	0.984	0.084

Table 12

Experiments: Case $\pi = 0.02$

	n	p	w	$\frac{\zeta^h h + \zeta^b \hat{b}}{\eta}$	r^η	h	i^h	n/p	ψ/w
1) free n/p and ψ , $\Omega > 0$	2.07	1.32	0.48	0.62	1.33	0.31	5.73	0.988	0.042
2) $\psi = 0$	2.16	1.49	0.36	0.63	1.50	0.31	5.73	0.988	0.000
3) mobile housing	2.30	1.81	0.41	n.a.	n.a.	1.81	n.a.	0.994	0.042
4) high ψ	2.05	1.32	0.49	0.63	1.33	0.30	5.70	0.988	0.084