

A Long-Run Risks Explanation of Predictability Puzzles in Bond and Currency Markets

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Abstract

In the data, we show that bond risk premia increase at times of high uncertainty about expected inflation and decrease with high uncertainty about expected growth; the magnitude of bond return predictability by these two uncertainty measures is similar to that found based on multiple yield factors. Motivated by this evidence, we provide an equilibrium long-run risks model which features time-varying volatilities of expected growth and expected inflation, and non-neutral inflation effect on future growth. We estimate the model and show that it can (i) successfully match the observed bond yield and macroeconomic data, (ii) account for bond return predictability evidence based on real and inflation uncertainties as well as the yield-based projections, and (iii) simultaneously explain for violations of the uncovered interest parity in currency markets. In the model, as in the data, bond risk premia are high in periods of high inflation uncertainty (e.g., 1980s), and are low and even negative in periods of high real uncertainty (e.g., mid-2000). We show that preference for early resolution of uncertainty, time-varying volatilities, and a non-neutral effect of inflation on growth are important to account for these aspects of bond markets.

1 Introduction

Several aspects of bond and currency markets are puzzling from the perspective of equilibrium asset-pricing models. Excess bond returns seem highly predictable using the information in multiple yield variables, as shown in Campbell and Shiller (1991), Dai and Singleton (2002) and Cochrane and Piazzesi (2005) for domestic bond markets, and Fama (1984), Bansal (1997) and Backus, Foresi, and Telmer (2001) for foreign exchange markets. Predictability of excess bond returns implies substantial fluctuation in risk premia over time, and it is a challenge to interpret the economic nature, sources and magnitudes of these risk premium fluctuations in the data. In this paper, we provide direct empirical evidence which tightly connects the fluctuations in bond premia to uncertainties in long-run expectations of real growth and inflation. We find that high expected growth uncertainty lowers nominal bond premia, while high uncertainty about expected inflation raises the premia; as a consequence, the bond risk premia can vary from being positive to negative. The magnitude of bond return predictability based on the uncertainty variables is similar to that found using Cochrane and Piazzesi (2005) yield-based factors to predict excess bond returns. Motivated by this evidence, we develop an inflation-augmented long-run risks model where the risk premia are driven by expected growth and expected inflation uncertainties. We estimate this model and show that it quantitatively captures the documented bond predictability features in the data, and simultaneously accounts for the violations of uncovered interest rate parity in foreign exchange.

Specifically, our model uses a long-run risks framework of Bansal and Yaron (2004). The key ingredients of the model include preference for early resolution of uncertainty, time-variation in expected consumption growth and expected inflation, fluctuations in expected growth and expected inflation volatilities, and inflation non-neutrality. In the model, expected consumption and expected inflation are driven by persistent shocks whose conditional volatilities are time-varying and capture fluctuations in long-run real and inflation uncertainties, respectively. We allow for expected inflation shocks to negatively affect expected consumption dynamics, which reflects inflation non-neutrality: high expected inflation is bad news for future consumption growth. With preference for early resolution of uncertainty, expected consumption and expected inflation shocks are priced in equilibrium, and thus contribute to the bond risk premium. In particular, due to non-neutrality of inflation, the risk compensation for expected inflation shocks plays an important role for an upward slope of the nominal term structure and a significant variation in the bond risk premium over time. The time-variation in expected consumption and inflation risk compensations is driven by the conditional volatilities of expected consumption and expected inflation, respectively, which leads to the predictability of future bond returns by the real and inflation volatilities. We show that an increase in inflation uncertainty raises equilibrium nominal bond risk premia, while an increase in real uncertainty actually lowers

the bond risk premia. This difference in the responses of bond risk premia to real and inflation volatility is a key model implication which also captures an important dimension of the data.

To highlight the model implications for predictability of foreign bond returns, we apply our model to a two-country symmetric set-up similar to the long-run risks framework discussed in Colacito and Croce (2011). In our model, high volatility periods are associated with low short-term interest rates and high risk premium on foreign bonds, so due to the volatility channel, exchange rates are predictable by the interest rate differential between the countries. In particular, in the model high interest-rate countries are expected to appreciate, which allows us to capture the violations of uncovered interest rate parity in the data.

An important goal for the paper is to document a robust link between bond premia and the volatilities of expected growth and expected inflation rate in the data. To this end, we collect quarterly data on consensus forecasts of one-year ahead real GDP growth and inflation rate from the Survey of Professional Forecasts from 1969 to 2010. These average forecasts are very useful as they contain significant information about future macroeconomic variables: for example, real growth forecast predicts year ahead real consumption growth with an R^2 of almost 40%, and inflation forecast predicts a year ahead inflation rate with an R^2 of 60%. Notably, the forecasts are very persistent: at quarterly frequency, the autocorrelation coefficient is 0.866 for expected real growth and 0.986 for expected inflation. We construct measures of conditional volatilities of real growth and inflation forecasts from the survey data, which we use to capture the fluctuations in long-run real and inflation uncertainty in the data. There are interesting differences in movements in real and inflation volatilities across time. Figure 2, which plots the difference between the volatilities of expected inflation and expected growth, shows that inflation uncertainty is much larger than real one in the period from 1980 to 1985, which stands in a sharp contrast to the time period of 2005-2010, where uncertainty about real growth dominates inflation uncertainty. This has important implications for the bond premia in those periods, both in the data and in the model, as we discuss below.

We find that our inflation and growth uncertainty measures contain significant information about bond returns in the data. First, the two uncertainty variables predict about 18% of future excess bond returns, comparable to the amount of predictability by a single bond factor, such as Cochrane and Piazzesi (2005), which is between 15 to 20% in our sample. The slope coefficients in bond return regressions are negative on real uncertainty and positive on inflation uncertainty, consistent with the equilibrium implications of the model. Similar to the bond return predictability evidence, we find that realized term premium decreases with a rise in real uncertainty and increases when inflation uncertainty is high, and the amount of predictability of future realized term premium by the two uncertainty variables reaches 50% at 5-year horizon.

To assess the empirical validity of the various economic channels highlighted in the model, we formally estimate the model using the latent-factor Maximum Likelihood Estimation approach. Specifically, we use observations of 1y to 5y nominal yields and expected inflation from the survey data, and treat expected real growth and real and inflation uncertainties as latent factors. Our quarterly-based estimates imply risk aversion of 20.77 and the inter-temporal elasticity of substitution of 1.86. The expected consumption and expected inflation, which follow a bivariate VAR process, are quite persistent in our estimation, and their estimated autocorrelation coefficients are 0.81 and 0.99, respectively. The estimated amount of persistence in the two factors matches very well the estimates in the survey data. Estimation evidence suggests that inflation is non-neutral, as expected inflation significantly and negatively affects future consumption growth. The real and inflation volatilities are quite persistent in the estimation. Remarkably, the key parameters of the latent real and inflation uncertainties are very close to their estimates based on the survey forecast data.

Using the estimated parameters, we show that the model can reproduce several important features of the nominal term structure data. The unconditional model-implied yields, which are 6.1% and 6.9% at 1-year and 5-year horizons, match their values in the data of 6.1% and 6.8%, respectively. We show that the model generates sizeable variation in the bond premia and can match the bond return predictability evidence in the data. In particular, the model reproduces a negative response of nominal bond premia to real uncertainty, and positive response to nominal uncertainty. The R^2 in bond return predictability regressions is about 20% in the model and in the data. Further, the model captures key aspects of predictability of foreign bond returns. At 1 year horizon, the slope coefficient in foreign bond return regressions is -1.73 in the model, relative to -1.38 (0.71) in the data, and it increases to 0.17 in the model, relative to 0.27 (0.61) in the data at 5 year horizon. Hence, the model matches the evidence on the violation of uncovered interest parity rate condition at a short end and a gradual reduction in violations at a long end, documented in the literature (see Alexius (2001), and Chinn and Meredith (2004)).

Using the parameter estimates of the model, we find that the model and data premia co-move very closely with each other. Both premia fluctuate significantly and can switch sign over over time. The correlation of model-implied and data bond risk premia extracted from Cochrane and Piazzesi (2005) factor is 60%, and it is in excess of 80% for the estimates of the term premia. Both in the model and in the data, the premia are substantially high in 1980-1985 when inflation uncertainty is high. The premia decrease and even become negative by the end of the sample, which is consistent with a relative increase in real uncertainty in mid 2000.

Earlier structural work includes Wachter (2006) and Verdelhan (2010) who explore the implications of the habits model for nominal term structure and currency markets, respectively. In the context of the habits model, Verdelhan (2010) shows

that pro-cyclical interest rates are required to account for foreign exchange violations, however, Wachter (2006) relies on counter-cyclicalities of real interest rates to explain the domestic term structure. Notably, our model is able to explain the predictability of domestic and foreign bonds simultaneously. Piazzesi and Schneider (2005) use recursive-utility framework with consumption and inflation dynamics to evaluate implications for the levels of nominal and real yield curve. Using the rare disasters framework of Rietz (1988) and Barro (2006), Farhi and Gabaix (2008) explore the implications of time-variation in disaster risk for return predictability in currency markets. Bekaert, Hodrick, and Marshall (2001) use a peso-problem argument to evaluate the violations of the expectation hypothesis. Alvarez, Atkeson, and Kehoe (2009) argue that a model of limited and changing participation of agents in financial markets could account for forward premium anomaly at short and long horizons. Earlier equilibrium-model based efforts to explain some of the puzzles in currency markets include Bekaert (1996) and Backus, Gregory, and Telmer (1993), Bansal, Gallant, Hussey, and Tauchen (1995).¹

The rest of the paper is organized as follows. In the next section we document the bond return predictability puzzles and discuss the link of bond premia to long-run real and inflation uncertainties in the data. In Section 3 we setup the model. We present the solution to the model and discuss its theoretical implications for the asset markets in Section 4. In Section 5 we describe model estimation and discuss model implications for bond and currency markets Conclusion and Appendix follow.

2 Bond Return Predictability Evidence

2.1 Evidence Based on Yields as Predictors

Denote $y_{t,n}$ the yield on the real discount bond and $f_{t,n}$ the real forward rate with n months to maturity. We will use variables with a dollar superscript to refer to nominal quantities, e.g., nominal one-month yield $y_{t,1}^{\$}$. To avoid clustering of superscripts, we lay out subsequent discussion using the notation for real variables.

Denote $rx_{t \rightarrow t+m,n}$ the excess log return on buying an n period bond at time t and selling it at time $t+m$ as an $n-m$ period bond:

$$rx_{t \rightarrow t+m,n} = ny_{t,n} - (n-m)y_{t+m,n-m} - my_{t,m}. \quad (1)$$

¹Recent works in no-arbitrage and statistical literature include affine models of Dai and Singleton (2002) and Duffee (2002), regime-switching models of Bansal and Zhou (2002), and macro-finance specifications of the term structure by Ang and Piazzesi (2003), Ang, Dong, and Piazzesi (2005), Rudebusch and Wu (2004) and Bikbov and Chernov (2010).

Using multiple yield variables, Cochrane and Piazzesi (2005) provide strong evidence for the time-variation in bond risk premia. Following their approach, we regress the average of 1-year nominal excess bond returns of 2 to 5 years to maturity on the forward rates of 1 to 5 years to maturity:

$$\frac{1}{4} \sum_{n=2}^5 rx_{t \rightarrow t+4,4n} = \gamma_0 + \gamma_1 f_{t,4} + \gamma_2 f_{t,8} + \gamma_3 f_{t,12} + \gamma_4 f_{t,16} + \gamma_5 f_{t,20} + error. \quad (2)$$

We extract a single bond factor $\widehat{rx}_{t,m}$ from this regression, which is subsequently used to forecast excess bond returns at each maturity n from 2 to 5 years:

$$rx_{t \rightarrow t+m,n} = const + b_{m,n} \widehat{rx}_{t,m} + error. \quad (3)$$

Cochrane and Piazzesi (2005) show that the estimates $b_{m,n}$ are positive and increasing with horizon, and a single factor projection captures a significant portion of the variation in bond returns. We document these results in Table 6 using quarterly observations of bond yields from 1969 to 2010, sampled every second month of the quarter. The slope coefficients increase from 0.44 for 2-year yields to 1.43 for 5-year ones, and the R^2 are 15-20%. The levels of the slope coefficients and the amount of predictability in these regressions provide strong evidence for a substantial time-variation in the risk premium in bond markets². This evidence complements earlier findings on bond return predictability, as in Campbell and Shiller (1991), which focuses on the violations of expectations hypothesis in the data. The expectations hypothesis regressions only use term spread to forecast future bond returns, and therefore leads to somewhat lower R^2 in forecasting bond returns relative to Cochrane and Piazzesi (2005).

In addition to predictability of bond returns in domestic markets, Fama (1984), Hodrick (1987) and others show that the returns on foreign bonds are also predictable by the interest rate variables. Let s_t stand for a real spot exchange rate, in logs, per unit of foreign currency (dollars spot price of one pound). Superscript * will denote the corresponding variable in foreign country, e.g. $y_{t,1}^*$ stands for the foreign risk-free rate. Define a one-period excess dollar return in foreign bonds:

$$rx_{t+1}^{FX} = s_{t+1} - s_t + y_{t,1}^* - y_{t,1}. \quad (4)$$

This corresponds to an excess return on buying foreign currency today, investing the money into the foreign risk-free asset and converting the proceeds back using the spot rate next period.

²Dahlquist and Hasseltoft (2011) find similar evidence for bond return predictability in other countries.

Under the expectations hypothesis in currency markets, the excess returns are constant. Therefore, the slope coefficient in the projection

$$rx_{t+1}^{FX} = \text{const} + \beta^{FX}(y_{t,1} - y_{t,1}^*) + \text{error}. \quad (5)$$

should be equal to zero. In contrast, the regression coefficient in is significantly negative and below one in the data, as shown in Table 6 for UK bonds. This implies that high interest rate bearing countries are expected to appreciate in the future, which violates the predictions of the uncovered interest rate parity condition. As shown in the Table, these violations are less pronounced at long horizons, consistent with the evidence documented in Chinn and Meredith (2004) and Alexius (2001).

All the empirical evidence above suggests that the expected excess returns on domestic and foreign are significantly predictable by the yield variables. The predictability of future bond returns implies significant time-variation in bond and term premia; indeed, their typical estimates in the data are highly volatile and can be of positive and negative sign. The magnitudes and sources of bond return predictability are puzzling from an economic perspective. In the next section, we show that a significant amount of this time-variation can be attributed to the fluctuations in the volatilities of expected real growth and expected inflation in the data.

2.2 Evidence Based on Real and Inflation Uncertainty

An important goal for the paper is to establish a link between bond yields and the uncertainty about the expectations of future real growth and inflation. To obtain direct measures of the expectations of future macroeconomic variables, we collect quarterly data on consensus forecasts of one-year ahead real GDP growth and inflation rate from the Survey of Professional Forecasts from 1969 to 2010; these forecasts are plotted on the top panel on Figure 1.³ The real growth and inflation forecasts contain significant information about future real consumption and inflation in the data. The real forecast predicts next-quarter consumption growth with R^2 of 24%, and the inflation forecast predicts next-quarter inflation with an R^2 of 43%. As shown in Table 1, for 1-year horizon, the amount of predictability increases to 40% and 60%, respectively. Adding additional predictors, such as yields and equity prices, does not significantly improve the R^2 . This empirical evidence strongly motivates our use of the forecasts as empirical proxies for the expectations of future consumption and inflation.

³We use forecasts of real GDP to proxy for forecasts of real consumption, as the data on consumption forecasts starts only in 1985. For the overlapping period, the correlation between real GDP and consumption consensus forecasts is 90%. The forecasts are demeaned and rescaled to predict next-quarter real consumption growth and inflation rate with a loading of one.

Denote $\hat{x}_{ct}$ the real growth forecast, and $\hat{x}_{\pi t}$ the inflation forecast. To highlight persistence and dynamic dependence of the forecasts in the data, we regress each forecast on their lags:

$$\begin{aligned}\hat{x}_{c,t+1} &= \underset{(0.03)}{0.86}\hat{x}_{ct} - \underset{(0.03)}{0.01}\hat{x}_{\pi t} + u_{c,t+1}, \\ \hat{x}_{\pi,t+1} &= \underset{(0.04)}{0.06}\hat{x}_{ct} + \underset{(0.02)}{0.99}\hat{x}_{\pi t} + u_{\pi,t+1}.\end{aligned}\tag{6}$$

As evident from above, both forecasts are very persistent, and much more so than the underlying consumption and inflation variables. Indeed, an autoregressive coefficient is 0.87 for expected real growth and 0.99 for expected inflation, relative to 0.3 and 0.6 for realized consumption growth and inflation rate. Notably, inflation has a non-neutral effect on the real economy: the real growth forecast loads negatively on the lag of the inflation forecast, so that an increase in expected inflation predicts a decline in future consumption; similar evidence is also documented in Piazzesi and Schneider (2005). This evidence on persistence of the forecasts and inflation non-neutrality motivates the specification of our economic model.

To construct measures of uncertainty about expected real growth and inflation, we take the forecast residuals $u_{c,t+1}$ and $u_{\pi,t+1}$ from the regression (6), and regress their annual sum of squares on the current yields of 1, 3 and 5 years to maturity and the market price dividend ratio⁴:

$$\begin{aligned}\frac{1}{h} \sum_{j=1}^h u_{c,t+j}^2 &= const + b_{c1}y_{t,1} + b_{c2}y_{t,12} + b_{c3}y_{t,20} + b_{c4}pd_t + error, \\ \frac{1}{h} \sum_{j=1}^h u_{\pi,t+j}^2 &= const + b_{\pi1}y_{t,1} + b_{\pi2}y_{t,12} + b_{\pi3}y_{t,20} + b_{\pi4}pd_t + error,\end{aligned}\tag{7}$$

where $h = 4$ at an annual horizon. The predictive values from these regressions estimate the conditional variance of expected real growth and expected inflation; this approach of measuring conditional volatilities using financial market data is similar to that in Kandel and Stambaugh (1991)⁵. Indeed, under the null of the model, the four financial variables contain all the information about the economy, so regressing future squared residuals identifies the volatilities of expected real growth and inflation

⁴As the forecasts are released by the middle of the quarter, we align the forecast data with the financial variables as of the second month of the quarter. All our results are robust with respect to the forecast date alignment.

⁵To ensure that variance measures always remain positive, we actually regress the square root of the sum of squared residuals on the financial variables, and then square the fitted value. We rescale the variance measures to match the unconditional variance of the forecast shocks. This has immaterial effect on variance measures.

rate. In the data, the R^2 in these regressions is 40% for the volatility of expected growth and 50% for the volatility of expected inflation, as shown in Table 1.

The two volatility measures are plotted on the bottom panel in Figure 1. The volatilities are quite persistent in the data, with an autocorrelation coefficient of 0.87, and quite volatile. Figure 2 plots the difference between the volatility of expected inflation and volatility of expected growth. As evident from the Figure, in the period from 1980 to 1985 the inflation volatility is sizeably larger than the real one, while from 2005 to 2010 the real uncertainty is measurably larger than the inflation uncertainty; remarkably, there are substantial differences in magnitudes of volatility of expected real growth and expected inflation across time periods.

These moves in real and nominal uncertainty have significant implications for bond risk and term premia, which we document in Tables 2 and 3. Notably, in the regressions of future excess bond returns on the two volatilities, the slope coefficients are negative for the real volatility, and positive for the inflation volatility. That is, in the data, bond premia are high when the uncertainty about expected inflation is high, and are low and even negative when the uncertainty about expected real growth is high. The R^2 s in these regressions are 17-18%, which is similar to the ones using yields as predictive variables. Similarly, as shown in Table 3, in regressions of realized term premia (long-term yield minus the average of the realized future short-term rates over the corresponding horizon), the slope coefficients are negative for the real uncertainty, and positive for inflation uncertainty, and the R^2 reaches 50% at 5-year maturity. The above findings can be used to economically interpret the fluctuations in bond premia and term premia over time (see Figures 4 and 5). In 1970s, both risk and term premia are low and even negative, which is consistent with an observation that there is a high level of real economic uncertainty and relatively low inflation uncertainty. Similarly, both premia increase substantially in 1980s which reflects a significant increase in inflation uncertainty relative to the real one. Finally, the decline in premia towards the end of the sample is consistent with a relatively low level of inflation uncertainty and a sharp increase in the real uncertainty post 2005. Our evidence regarding the role of inflation uncertainty, while using a very different approach, is consistent with the evidence documented in Wright (2011).

For robustness, we also consider alternative measures of expected growth and inflation which do not rely on the forecast data. Specifically, we first construct alternative measures of expected growth and inflation from a projection of future multi-period

realized consumption, Δc_{t+1} , and inflation, π_{t+1} , on the same set of predictors as before:

$$\begin{aligned}\frac{1}{h} \sum_{j=1}^h \Delta c_{t+j} &= const + a_{c1}y_{t,1} + a_{c2}y_{t,12} + a_{c3}y_{t,20} + a_{c4}pd_t + error, \\ \frac{1}{h} \sum_{j=1}^h \pi_{t+j} &= const + a_{\pi1}y_{t,1} + a_{\pi2}y_{t,12} + a_{\pi3}y_{t,20} + a_{\pi4}pd_t + error,\end{aligned}\tag{8}$$

and we take the fitted values from these regressions as our measures of expected consumption, $\hat{x}_{ct}$, and expected inflation, $\hat{x}_{\pi,t}$, respectively. Using these measures, we then proceed in the same way as before to extract the innovations in the expected real growth and expected inflation (Equation (6)), and compute the volatilities of expected real growth and inflation (Equation (7)). Table 1 summarizes the R^2 s in the extractions of expected growth and volatility variables at $h=1$ year horizon. The empirical evidence is very similar using shorter horizons like 1 quarter, or longer horizons such as 8 quarters.

Tables 2 and 3 show the implications for the bond and term premia predictability. Remarkably, our empirical evidence using the alternative volatility measures is very similar to the one based on the survey data. Bond and term premia decrease with positive shocks to real volatility, and increase with positive shocks to inflation volatility. The R^2 s in the bond return regressions are about 20% using these volatility measures, similar to the evidence found in the survey data. The R^2 s for realized term premia regressions increase from 20% for 2-year bond, to about 30% for 5-year bond.

Our third approach of extracting the volatility measures is based on the latent variable estimation where we measure the two volatilities using the MLE approach imposing structural restrictions of the economic model. We later show that with this approach the bond return predicability evidence is also similar to that based on the survey data.

This bond return predictability evidence by the real and inflation uncertainties provides a key motivation for our paper, alongside with the earlier findings on bond return predictability by the yield predictors.

3 Model Specification

3.1 Preferences

We consider a version of a discrete-time real endowment economy developed in Bansal and Yaron (2004). The investors preferences over the uncertain consumption stream C_t are described by the Kreps-Porteus, Epstein-Zin recursive utility function (see Epstein and Zin, 1989; Kreps and Porteus, 1978):

$$U_t = \left[(1 - \delta) C_t^{\frac{1-\gamma}{\theta}} + \delta (E_t U_{t+1}^{1-\gamma})^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}, \quad (9)$$

where δ is the time discount factor, $\gamma \geq 0$ is the risk aversion parameter, and $\psi \geq 0$ is the intertemporal elasticity of substitution (IES). For ease of notations, parameter θ is defined $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$. Note that when $\theta = 1$, that is, $\gamma = 1/\psi$, the above recursive preferences collapse to the standard case of expected utility.

As shown in Epstein and Zin (1989), the logarithm of the real Intertemporal Marginal Rate of Substitution (IMRS) for these preferences is given by

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1}, \quad (10)$$

where $\Delta c_{t+1} = \log(C_{t+1}/C_t)$ is the log growth rate of aggregate consumption and $r_{c,t+1}$ is the log of the return (i.e., continuous return) on an asset which delivers aggregate consumption as its dividends each time period. This return is not observable in the data. It is different from the observed return on the market portfolio as the levels of market dividends and consumption are not equal: aggregate consumption is much larger than aggregate dividends. Therefore, we assume an exogenous process for consumption growth and use a standard asset-pricing restriction

$$E_t[\exp(m_{t+1} + r_{t+1})] = 1 \quad (11)$$

which holds for any continuous return $r_{t+1} = \log(R_{t+1})$ including the one on the wealth portfolio, to solve for the unobserved wealth-to-consumption ratio in the model. This allows us to rewrite the discount factor in Equation (10) in terms of the fundamental state variables and shocks in the economy, and solve for the equilibrium real prices of any assets.

To derive implications for nominal assets, such as nominal bonds, we specify an exogenous inflation process π_{t+1} . Our approach to directly model inflation is similar to that pursued by Wachter (2006) and Piazzesi and Schneider (2005). The nominal

discount factor used to price nominal payoffs is then equal to the real one adjusted by the inflation rate:

$$m_{t+1}^s = m_{t+1} - \pi_{t+1}. \quad (12)$$

An Euler equation in (11) can be used to solve for the equilibrium prices of nominal assets.

3.2 Economy Dynamics

To solve for the equilibrium asset prices we specify exogenously the joint dynamics of real consumption growth and inflation rate. Our goal is to specify the most parsimonious yet flexible dynamics which features persistent fluctuations in expectations and uncertainties about expected real growth and expected inflation.

Specifically, denote x_{ct} and $x_{\pi t}$ the expected consumption growth and expected inflation rate factors in the economy. Then, we write down the consumption and inflation dynamics in the following way:

$$\begin{aligned} \Delta c_{t+1} &= \mu_c + x_{ct} + \sigma_c \eta_{c,t+1}, \\ \Delta \pi_{t+1} &= \mu_\pi + x_{\pi t} + \sigma_\pi \eta_{\pi,t+1}, \end{aligned} \quad (13)$$

where $\eta_{c,t+1}$ and $\eta_{\pi,t+1}$ are standard Normal shocks, and σ_c and σ_π are the conditional volatilities of short-run consumption and inflation shocks. For parsimony, we shut down the time-variation in short-run volatilities as it does not play a significant role for the asset markets relative to the fluctuations in volatilities of the expected growth shocks.

To maintain a parsimonious yet flexible specification, we model the dynamics of the expected growth and expected inflation factors $x_t \equiv [x_{ct} \ x_{\pi t}]'$ as a bivariate VAR(1) process with time-varying conditional volatilities:

$$x_{t+1} = \Pi x_t + \Sigma_t e_{t+1}, \quad (14)$$

where Π is the persistence matrix, and e_{t+1} are Normal innovations. To capture the interaction between the real and nominal economy in the most parsimonious way, in the empirical implementation we zero out the correlation of expected consumption and expected inflation shocks, and allow expected inflation to directly feed in to expected consumption. We define

$$\Pi = \begin{bmatrix} \rho_c & \rho_{c\pi} \\ 0 & \rho_\pi \end{bmatrix}, \quad (15)$$

so that parameters ρ_c and ρ_π capture the persistence of expected real growth and expected inflation, and $\rho_{c\pi}$ reflects the non-neutral effect of expected inflation on future real growth. We expect $\rho_{c\pi} < 0$, so that high expected inflation predicts low future consumption. This is consistent with the data, as real growth forecast in the data loads negatively on lag of expected inflation.

In the expected growth specification in Equation (14), Σ_t captures the time-variation in long-run uncertainty in the economy. In the benchmark specification of the model Σ_t is diagonal:

$$\Sigma_t = \begin{bmatrix} \sigma_{ct} & 0 \\ 0 & \sigma_{\pi t} \end{bmatrix}, \quad (16)$$

so that σ_{ct} and $\sigma_{\pi t}$ reflect the fluctuations in expected growth and expected inflation volatility, respectively. Notably, relative to a standard single volatility specification of the model (see e.g. Bansal and Yaron, 2004), in our model, we disentangle the volatilities of real and nominal factors, because both in the model and in the data the two have very different implications for the term structure. We specify the volatility dynamics in the following way:

$$\begin{bmatrix} \sigma_{c,t+1}^2 \\ \sigma_{\pi,t+1}^2 \end{bmatrix} = (I - \Phi)\Sigma_0 + \Phi \begin{bmatrix} \sigma_{ct}^2 \\ \sigma_{\pi t}^2 \end{bmatrix} + \Sigma_w w_{t+1}, \quad (17)$$

where Σ_0 denotes the level of volatility, and Φ and Σ_w capture persistence and scale of the volatility shocks. For simplicity, the volatility shocks w_{t+1} are Normal in the benchmark application of the model; in Appendix, we present a general solution of the model where these innovations can follow non-Gaussian distribution which guarantees the positivity of volatility processes ⁶.

4 Asset Markets

4.1 Discount Factor

We present a general solution to the model in Appendix A, and we present the key results below.

⁶As volatility shocks are homoscedastic, the assumption on the distribution of these shocks has no impact on the equilibrium bond price loadings, and can only affect price intercepts. We find that for reasonable parameters this impact is quite small at typical bond maturities.

Using the Euler condition (11) and the dynamics of the economy, we show that the equilibrium solution to the price-consumption ratio is linear in expected growth and variance states:

$$pc_t = A_0 + A_{xc}x_{ct} + A_{x\pi}x_{\pi t} + A_{sc}\sigma_{ct}^2 + A_{s\pi}\sigma_{\pi t}^2, \quad (18)$$

where the price-consumption loadings are given by,

$$A_{xc} = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1\rho_c}, \quad A_{x\pi} = \kappa_1\rho_{c\pi} \frac{1 - \frac{1}{\psi}}{(1 - \kappa_1\rho_c)(1 - \kappa_1\rho_\pi)}, \quad (19)$$

$$A_{sc} = \frac{(1 - \gamma)(1 - 1/\psi)}{2(1 - \kappa_1\nu_c)} \left(\frac{\kappa_1}{1 - \kappa_1\rho_c} \right)^2, \quad (20)$$

$$A_{s\pi} = \frac{(1 - \gamma)(1 - 1/\psi)}{2(1 - \kappa_1\nu_\pi)} \left(\frac{\kappa_1^2\rho_{c\pi}}{(1 - \kappa_1\rho_c)(1 - \kappa_1\rho_\pi)} \right)^2, \quad (21)$$

and κ_1 is a linearization parameter.

As in a standard long-run risks model (see Bansal and Yaron, 2004), when the inter-temporal elasticity of substitution parameter ψ is greater than one, the intertemporal substitution effect dominates the wealth effect. So, in response to higher expected growth, agents invest more and consequently, the wealth to consumption ratio rises. Hence, the price-consumption loading on the expected consumption growth is positive, $A_{xc} > 0$, and it is negative on the expected inflation, $A_{x\pi} < 0$, if inflation is bad for consumption ($\rho_{c\pi} < 0$). When the IES and risk aversion are all larger than one, the responses of price-consumption ratio to real and inflation volatility are negative, $A_{sc}, A_{s\pi} < 0$. The persistence of volatility and expected growth shocks magnifies the effects of volatility on valuation ratios as changes in macroeconomic volatility are perceived by investors as being long lasting.

Using the solution for the price-consumption ratio, we provide an analytical expression for the equilibrium marginal rate of substitution in terms of the fundamental state variables and shocks in the economy. The conditional mean of the discount factor is linear in economic states, and the solutions for the discount factor loadings which depend on model and preference parameters are provided in the Appendix. The innovation into the real discount factor, which determine the sources and compensations for risk in the economy, is given by

$$\begin{aligned} m_{t+1} - E_t m_{t+1} = & -\lambda_c \sigma_c \eta_{c,t+1} - \lambda_\pi \sigma_\pi \eta_{\pi,t+1} - \lambda_{xc} \sigma_{c,t} e_{c,t+1} - \lambda_{x\pi} \sigma_{\pi,t} e_{\pi,t+1} \\ & - \lambda_{sc} w_{xc,t+1} - \lambda_{s\pi} w_{x\pi,t+1}, \end{aligned} \quad (22)$$

where the market prices of short-run consumption and inflation risks and expected consumption and expected inflation risks are equal to

$$\begin{aligned}\lambda_c &= \gamma, & \lambda_\pi &= 0 \\ \lambda_{xc} &= \left(\gamma - \frac{1}{\psi}\right) \frac{\kappa_1}{1 - \kappa_1 \rho_c}, & \lambda_{x\pi} &= \left(\gamma - \frac{1}{\psi}\right) \frac{\kappa_1^2 \rho_{c\pi}}{(1 - \kappa_1 \rho_c)(1 - \kappa_1 \rho_\pi)},\end{aligned}\quad (23)$$

and the market prices of volatility risks are provided in the Appendix. The price of short-run consumption risks λ_c is equal to the risk-aversion coefficient γ , and as the immediate inflation innovations are assumed to be independent from the real economy, their price of risk λ_π is zero. When agents have preference for early resolution of uncertainty ($\gamma > 1/\psi$), the market price of expected consumption shock λ_{xc} is positive, and market price of expected inflation shock $\lambda_{x\pi}$ is negative if inflation is bad for consumption ($\rho_{c\pi} < 0$). With preference for early resolution of uncertainty, the market prices of real and inflation volatility risks are both negative.

The solution to the nominal discount factor $m_{t+1}^\$$, specified in Equation (12), takes into account the inflation dynamics. In our specification of the economy, an innovation into the nominal discount factor satisfies

$$\begin{aligned}m_{t+1}^\$ - E_t m_{t+1}^\$ &= -\lambda_c \sigma_c \eta_{c,t+1} - \lambda_\pi^\$ \sigma_\pi \eta_{\pi,t+1} - \lambda_{xc} \sigma_{c,t} e_{c,t+1} - \lambda_{x\pi} \sigma_{\pi,t} e_{\pi,t+1} \\ &\quad - \lambda_{sc} w_{xc,t+1} - \lambda_{s\pi} w_{x\pi,t+1},\end{aligned}\quad (24)$$

where the nominal market price of the inflation risk $\lambda_\pi^\$$ is equal to one. The nominal market prices of next-period expected growth, expected inflation and volatility risks are equal to their real counterparts; however, because of the feedback between expected consumption and expected inflation, the multi-period responses to expected growth shocks are different for real and nominal discount factors.

4.2 Asset Prices

Using the expressions for the real and nominal discount factors in Equations (22) and (24), we can solve for the equilibrium yields in the economy. As shown in the Appendix, the real and nominal yields are linear in economic state variables:

$$y_{t,n} = \frac{1}{n} (B_{0,n} + B_{xc,n} x_{ct} + B_{x\pi,n} x_{\pi t} + B_{sc,n} \sigma_{ct}^2 + B_{s\pi,n} \sigma_{\pi t}^2), \quad (25)$$

$$y_{t,n}^\$ = \frac{1}{n} (B_{0,n}^\$ + B_{xc,n}^\$ x_{ct} + B_{x\pi,n}^\$ x_{\pi t} + B_{sc,n}^\$ \sigma_{ct}^2 + B_{s\pi,n}^\$ \sigma_{\pi t}^2). \quad (26)$$

The bond coefficients, which measure the sensitivity (beta) of bond prices to the aggregate risks, are pinned down by the preference and model parameters – their expressions are presented in the Appendix.

In our model, real yields hedge expected consumption risks. That is, real bond prices rise and real yields fall following negative shock to the expected growth factor ($B_{xc,n} > 0$), or positive shock to the expected inflation factor ($B_{x\pi,n} < 0$) if inflation is bad news for future consumption (i.e., when $\rho_{c\pi} < 0$). Real yields also fall with an increase in real or inflation volatility, $B_{sc,n} < 0$ and $B_{s\pi,n} < 0$, which represents a "flight to quality" effect. An increase in real or inflation volatility (with inflation non-neutrality) increases the uncertainty about future growth, so the demand for real risk-free assets increases, and equilibrium real bond prices go up and their yields fall.

To analyze equilibrium implications for nominal assets, consider Fisher-type equation for nominal bonds:

$$y_{t,n}^{\$} = y_{t,n} + E_t \pi_{t \rightarrow t+n} - \frac{1}{2} Var_t \pi_{t \rightarrow t+n} + Cov_t(m_{t \rightarrow t+n}, \pi_{t \rightarrow t+n}), \quad (27)$$

where $\pi_{t \rightarrow t+n}$ and $m_{t \rightarrow t+n}$ denote the t to $t+n$ multi-period inflation rate and stochastic discount factor, respectively. As shown above, solutions to nominal bond yields take into account expected inflation and inflation premium in the economy. Similar to real bonds, nominal bond yields respond positively to expected consumption, $B_{xc,n}^{\$} > 0$. As high inflation expectations directly impact nominal yields, unlike real yields, nominal yields respond positively to expected inflation shocks, $B_{x\pi,n}^{\$} > 0$. Further, as market price of expected inflation shocks is positive, the inflation premium given by the covariance of future inflation and future discount factor is positive and increases with the volatility of expected inflation. This can offset the "flight to quality" effect for the real bonds, and make longer-term nominal yields respond positively to expected inflation volatility shocks: $B_{s\pi,n}^{\$} > 0$, for high n .

The empirical evidence for the nominal bonds, showed in previous Section, highlighted the importance of uncertainties about expected growth and expected inflation in capturing the variations in bond risk premia. In the model, one-period expected excess return on nominal bonds can be written in the following form:

$$\begin{aligned} E_t(rx_{t \rightarrow t+1,n}^{\$}) + \frac{1}{2} Var_t(rx_{t+1,n}^{\$}) &= -Cov_t(m_{t+1}^{\$}, rx_{t+1,n}^{\$}) \\ &= const^{\$} - B_{xc,n-1}^{\$} \lambda_{xc} \sigma_{ct}^2 - B_{x\pi,n-1}^{\$} \lambda_{x\pi} \sigma_{\pi t}^2. \end{aligned} \quad (28)$$

The bond risk premium consists of risk compensations for volatility risks, which are constant in our setup, and risk compensations for expected consumption and expected inflation, which are time-varying and driven by the real and inflation volatilities, respectively. Recall that the equilibrium market price of expected consumption risk λ_{xc} is positive and that of expected inflation risk $\lambda_{x\pi}$ is negative, and the nominal bond betas to expected consumption and expected inflation risks are positive ($B_{xc,n}^{\$} > 0, B_{x\pi,n}^{\$} > 0$). This implies that nominal bond risk premium increases with inflation uncertainty, while it decreases at times of high real uncertainty. This model

implication is consistent the predictability evidence of future bond returns by the volatilities of expected real growth and expected inflation in the data.

Our model can also account for the bond return predictability by the yield variables as predictors. Indeed, in the model yields are affected by the volatilities of expected real growth and inflation. If the long-short bond yield spread and bond risk premia respond in the same direction to volatility shocks, then the spread forecasts future bond returns with a positive sign. This positive correlation of the term spread and bond risk premia is required to explain the violations of the expectations hypothesis in the data. The actual magnitudes of the slopes coefficients in bond regressions, as well as the amount of predictability of future bond returns, depend on the persistence and variation in the bond risk premium generated by the model.

To show the implications for foreign bonds, note that, as discussed in Backus et al. (2001), with frictionless markets the exchange rate is equal to the difference between the logarithms of the discount factors in the two countries:

$$s_{t+1} - s_t = m_{t+1}^* - m_{t+1}. \quad (29)$$

For tractability, we impose complete symmetry and assume that all the model parameters are identical across the two countries. Consumption and inflation shocks are country-specific, but are allowed to correlate across countries. This is similar to the two-country long-run risks model discussed in Colacito and Croce (2011)⁷. Given the equilibrium solution to the model, we can write down the solution to the nominal short-term interest rate differential across countries in the following way:

$$\begin{aligned} y_{t,1} - y_{t,1}^* &= B_{xc,n}(x_{ct} - x_{ct}^*) + B_{x\pi,n}(x_{\pi t} - x_{\pi t}^*) \\ &\quad + B_{sc,n}(\sigma_{ct}^2 - \sigma_{ct}^{*2}) + B_{s\pi,n}(\sigma_{\pi t}^2 - \sigma_{\pi t}^{*2}) \end{aligned} \quad (30)$$

while the solution to the expected excess return on short-term foreign bonds is just given by the difference between the conditional volatilities of stochastic discount factors at home and abroad:

$$\begin{aligned} E_t r_{t+1}^{FX} &= E_t (s_{t+1} - s_t + y_{t,1}^* - y_{t,1}) = \frac{1}{2} Var_t m_{t+1} - \frac{1}{2} Var_t m_{t+1}^* \\ &= \frac{1}{2} (\lambda_{xc}^2 (\sigma_{ct}^2 - \sigma_{ct}^{*2}) + \lambda_{x\pi}^2 (\sigma_{\pi t}^2 - \sigma_{\pi t}^{*2})). \end{aligned} \quad (31)$$

From Equation (30) it is evident that an increase in domestic volatility, real or nominal, causes the short-term domestic rate to decline; consequently, the interest rate

⁷More extensive international finance puzzles and models which feature trade, multiple consumption goods, financial integration, such as those in Colacito and Croce (2010) and Colacito (2009), are left for future work.

differential $y_{t,1} - y_{t,1}^*$ declines. At the same time, from Equation (31), the risk premium on dollar return on foreign bond rises. Hence, the foreign exchange risk premium and interest rate differential are negatively correlated, which is a necessary condition to explain the violation of uncovered interest rate parity condition. Quantitatively, we show that model-implied magnitudes are large enough to explain these violations in the data. Notably, our model is able to produce the violations of expected hypothesis in foreign exchange, and at the same time it can account for the key features of the nominal term structure in domestic markets. In contrast, habits models require different specifications to explain domestic and foreign bonds; Verdelhan (2010) uses habits specification with pro-cyclical interest rates to account for foreign exchange violations, however, Wachter (2006) needs habits model with counter-cyclical interest rates to explain bond premia in domestic markets.⁸

5 Model Estimation and Output

5.1 Model Estimation

We use quarterly observations of nominal yields of 1 to 5 years to maturity and the SPF survey forecasts from 1969 to 2010. In our benchmark estimation, the consensus inflation forecast from the survey data $x_{\pi t}$ is assumed to measure inflation expectations without an error. Using the inflation expectation directly helps identify real and nominal factors in the estimation. We assume that our measurements of the real growth expectation and real and inflation uncertainties can contain observation noise, so we use a latent-factor approach and extract these states from the nominal yields, similar to the approach in Chen and Scott (1993) and Duffie and Singleton (1997)⁹.

Specifically, the vector of observed data F_t consists of the variables which are assumed to be observed without an error, such as a vector of 3 yields of 1, 3 and 5-years to maturity Y_t^o and the survey expected inflation $x_{\pi t}$, and the variables which contain measurement error, such as the remaining yields Y_t^e and the real and nominal uncertainties Σ_t^e from the forecast data. Given the equilibrium solution to the model, the yields are an affine function of the state variables, so that we can rewrite the

⁸The habits model evidence suggests that it is a challenge to account simultaneously for these puzzles, which we show can be accomplished in a non-neutral inflation augmented model of this paper. In a largely real economy, Bansal and Shaliastovich (2007) and Colacito (2009) show that one can account for the FX puzzle in currency markets.

⁹We also considered an estimation strategy where all four survey variables are treated as measured without an error. Qualitatively, the key implications for the term structure are fairly similar.

expected real growth, expected inflation and the two conditional variances in terms of the variables observed without an error:

$$\begin{bmatrix} x_t \\ \sigma_{ct}^2 \\ \sigma_{\pi t}^2 \end{bmatrix} = K_1^{-1} \left(\begin{bmatrix} Y_t^o \\ x_{\pi t} \end{bmatrix} - K_0 \right), \quad (32)$$

where K_0 and K_1 depend on the model and preference parameters. In practice, we do the inversion under the constraint that the implied volatilities are positive. That is, each period we choose the expected consumption and the volatility factors to minimize the squared difference between the three yields in the data and in the model, imposing positivity on the volatilities. Given the parameters of the model, we then compute the conditionally Normal log-likelihood of the economic states $f(x_t, \sigma_{ct}^2, \sigma_{\pi t}^2 | F_{t-1})$. The remaining yields and the volatility states contain Normal measurement errors which are uncorrelated with the fundamental shocks in the economy, and the errors in yields and volatilities are assumed also to be independent of each other. In this case, the log-likelihood of the observed data satisfies,

$$f(F_t | F_{t-1}) = f(Y_t^o, x_{\pi, t} | F_{t-1}) + f(Y_t^e | x_t, \Sigma_t) + f(\Sigma_t^{2e} | x_t, \Sigma_t). \quad (33)$$

The conditional likelihood of the error-free yields and expected inflation can be calculated from the likelihood of the economic states, $f(Y_t^o, x_{\pi, t} | F_{t-1}) = f(x_t, \sigma_{ct}^2, \sigma_{\pi t}^2 | F_{t-1}) - \log |\det(K_1)|$, while the last two likelihoods are Gaussian and represent measurement errors in remaining yields and long-run uncertainties.

To compute standard errors, we perform a parametric bootstrap where we simulate time-series of model-implied yields and state variables at the estimated parameters of length equal to the data, and use the same MLE method as in the data to re-estimate the parameters in the simulation. The standard errors are then computed as standard deviations of the estimated parameters across the simulations.

5.2 Estimated Parameters and States

For parsimony, in our empirical implementation we assume the real and inflation volatilities are independent from each other, so that the persistence Φ and volatility Σ_w matrices are diagonal:

$$\Phi = \begin{bmatrix} \nu_c & 0 \\ 0 & \nu_\pi \end{bmatrix}, \quad \Sigma_w = \begin{bmatrix} \sigma_{wc} & 0 \\ 0 & \sigma_{w\pi} \end{bmatrix}. \quad (34)$$

The volatility shocks are assumed to follow Gaussian distribution. To further aid identification of the system and decrease the number of estimated parameters, we fix the unconditional consumption growth and inflation rate at $\mu_c = 2\%$ and $\mu_\pi = 4\%$

annualized, respectively, and the volatilities of short-run consumption and inflation shocks at $\sigma_c = \sigma_\pi = 1\%$, annualized. For identification purposes, we fix the subjective discount factor at 0.995.¹⁰

The parameter estimates are reported in Table 4. The risk aversion is estimated at 20.77, and the IES at 1.86. The estimated parameters for the expectations and volatilities of real growth and inflation, reported in Table 4, are consistent with the dynamics of these variables in the survey data. The estimated persistence of expected growth of 0.81 and expected inflation of 0.99 match very well the observed persistence of real GDP and inflation forecasts of 0.87 and 0.99, respectively (see Section 2.2). The expected consumption growth factor loads negatively on the lag of expected inflation, which is also consistent with the data: the model parameter is estimated at -0.05, while it is -0.01 with a standard error of 0.03 based on the survey evidence. The scales of expected consumption and inflation factors of about $1e-03 \times 200 = 0.2\%$, annualized, match nearly exactly the observed means of the real growth and inflation uncertainty in the forecast data. The estimation also matches very well the overall volatility of the real and inflation volatility. The persistence of the volatility measures is estimated at about 0.99, relative to about 0.9 in the survey data.

In Table 5 we document that the model can match the key features of consumption and inflation data. The volatility of annual consumption growth is about 1% in the model and in the data. The autocorrelation of quarterly consumption growth is 0.3 in the data relative to 0.2 in the model. For inflation series, its volatility is 1.6% in the model versus 1.8% in the data, and its persistence is 0.6 in the model and data. The correlation of quarterly consumption and inflation is -0.11 in the data relative to -0.20 in the model. All the model numbers are within one standard deviation of their estimates in the data.

The two extracted volatility factors behave similarly to the ones based on the survey data. The correlation of a real volatility estimated from the data with the one based on the survey data is 45%, while the correlation of the difference between the volatilities of expected consumption and expected inflation in the MLE estimation and the survey data is 65%.

5.3 Asset-Price Implications

Table 6 reports the data and model-implied population values for nominal yields. The model very well matches the levels of yields: the model-implied yields at 1-year and 5-year horizons are 6.1% and 6.9%, relative to 6.1% and 6.8% in the data, and the model fit to 2, 3 and 4 year yields is almost exact. The model implied one-year real

¹⁰This is related to the discussion of identification issues in Kocherlakota (1990) and similar to the approach in Marshall (1992).

yield is 1.9% and its volatility is 0.6%. The real term structure is nearly flat, and at 5 year horizon the level of real yields is about 1.7%.

Figure 3 shows the equilibrium nominal bond yield loadings on expected growth, expected inflation, and the real and inflation uncertainties. Consistent with our discussion in previous section, nominal bond yields respond positively to expected consumption and expected inflation shocks. Bond yields respond negatively to real uncertainty, because of flight to quality effect discussed earlier. The response to inflation uncertainty for nominal bonds is negative at short horizons and becomes positive at longer (above 2 year) maturities. Because of non-neutrality of inflation, inflation uncertainty impacts the uncertainty about real growth rates which depresses real yields (flight to quality). The second impact of inflation uncertainty on nominal bonds comes through inflation risk premium: high inflation uncertainty increases inflation premium. At longer maturities, inflation risk premium channel dominates the flight to quality effect, so longer-term nominal bond yields increase at times of high inflation uncertainty.

In the estimated model, risk aversion is bigger than inverse of the IES, which implies preference for early resolution of uncertainty. These estimates ensure that nominal bond premia loadings on expected growth uncertainty are negative, and on expected inflation uncertainty are positive at all maturities. Thus, the nominal bond premia are high at times of increased inflation uncertainty, and low at times of high real uncertainty, which is consistent with the data (see Table 2). The R^2 in the regressions of future bond returns on the two volatility measures reaches 21%, similar to the data. An important implication of the model, as documented in Table 6, is that it matches very well the bond predictability evidence of Campbell and Shiller (1991) and Cochrane and Piazzesi (2005). Specifically, in the model the slope coefficients in expectation hypothesis projections are negative and decreasing with maturity, as in the data. The slope coefficient for 2-year return is -0.54 in the model versus -0.41 in the data, and they decrease to -0.64 in the model and -1.15 in the data, respectively. The model-implied coefficients are well within one standard deviation of the estimates in the data. To further evaluate the predictability of bond returns using yields, we run regressions using the same approach as in Cochrane and Piazzesi (2005) on the model-simulated data. In the data, the slope coefficients on a single bond factor are 0.44 for 2-year bond returns, 0.85 at 3-year and 1.43 at 5-year horizons. In the model, these slope coefficients are equal to 0.45, 0.84 and 1.52, respectively. The R^2 s in the bond return regressions in the data are about 15-20%, and they are 20% in the model. In sum, the model matches very well the bond return predictability evidence from the expectations hypothesis regressions, as well as that using Cochrane and Piazzesi (2005) bond premium factor.

On Figures 4 and 5 we plot the 3 and 5 year nominal bond premia and term premia in the model, alongside with their estimates in the data. To extract the bond

risk premium we use single-factor projections of Cochrane and Piazzesi (2005). To extract term premium, we estimate a VAR(1) for a short-term rate, 3 bond yields and a Cochrane and Piazzesi (2005) bond risk premium factor, arrive at forecast of accumulated future short rates at different horizons, and subtract this forecast from the long-term yield to estimate the term premium. The model and data premia co-move very closely with each other: the correlation of model-implied and Cochrane and Piazzesi (2005)-based bond risk premia is 60%, and it is in excess of 80% for the term premia. Notably, both in the model and in the data premia fluctuate significantly over time and can switch sign. In the model, as in the data, premia drop substantially and can even be negative prior to 1980, which are the times of significant rise in real uncertainty. Bond and term premia are very high in 1980-1985, as this corresponds to a decrease in real uncertainty and increase in the nominal one. Finally, premia decline substantially and become negative in 2000s. In this period, real uncertainty is very high, and nominal is relatively small though rising at the end of the sample. This suggests that the model captures risk premium and term premium movements quite well. As evident from the graphs, these premia can become positive and negative at different points in time.

To assess the implications of the model for the returns on foreign bonds, we use our estimation results for the US economy, and impose a complete symmetry in model and preference parameters across the two countries. We let the individual shocks to be country-specific, and calibrate the correlation between those shocks. Specifically, as in Colacito and Croce (2011), we set the cross-country correlations between expected consumption factors and its volatilities to one. We assume that the correlation between expected inflation rates is 0.98, and between the volatilities of expected inflation is 0.99. This captures the intuition that the long-run prospects across the two countries are nearly identical, however, there are short-run differences between countries, which are mirrored in the differences in the short-run consumption and inflation shocks. As shown in Table 5, our model can match the evidence on US-UK correlation of real consumption (0.2 in the data and in the model), and inflation rates (0.6 in the data and in the model).

As shown in the Table 6, the model captures the violations of uncovered interest rate parity condition in the data: the slope coefficient in foreign exchange regressions at 1 year horizon is -1.73 in the model versus -1.38 in the data. As the slope coefficient is below negative one, it implies that high interest rate periods at home are associated with an expected appreciation of US dollar. In the model, due to inflation premium, nominal bond beta to inflation volatility becomes positive at longer maturities (see Figure 3). Hence, we expect that these violations become less prominent at longer maturities. Indeed, at 5-year frequency the slope coefficients in foreign exchange regressions reach 0.17 in the model and 0.27 in the data. As in the data, the model R^2 in the foreign exchange regressions are quite small. To sum, our model is consistent with the key predictability dimensions of the foreign exchange returns.

5.4 Importance of Model Ingredients

The key ingredients of the model which are critical for quantitative and qualitative explanation of the predictability of bonds returns in the data include preference for early resolution of uncertainty, time-variation in expectations of consumption growth and inflation, fluctuations in the volatilities of expected growth and inflation, and non-neutrality of money.

Indeed, as can be seen from the expressions for bond premia in Equation (28) and foreign exchange premia in Equation (31), the time-variation in risk premia reflects the time-varying compensation for expected consumption and expected inflation risks, and it is driven by the conditional volatilities of expected consumption and expected inflation. Thus, if two expected growth components are constant, or if two conditional volatilities are constant, the expected excess bond returns in domestic and foreign markets are constant as well. In this case, as we show in Table 7, the expectations hypothesis holds, and bond returns are unpredictable.

Recursive preferences structure which allows for a separation of a risk aversion from an intertemporal elasticity of substitution plays an important role for the signs of market prices of risks, bond loadings on risk factors, and ultimately the level and sensitivity of bond premia to volatility fluctuations. Indeed, as can be seen from the expressions for market prices of risk in Equation (23), with power utility, the expected consumption and expected inflation risks are not priced, λ_{xc} and $\lambda_{x\pi}$ are all zero. Hence, inflation premium is zero, and up to Jensen's adjustment term the bond risk premia in domestic and foreign markets are constant. Indeed, as we show in Table 7, with power utility, the nominal term structure is downward sloping, and there is very little predictability in bond markets in the model. Early preference of resolution ($\psi > 1/\gamma$) is important to ensure that market price of expected consumption risk is positive, and that of the expected inflation is negative. With preference for late resolution of uncertainty, these risk prices switch sign, which implies that nominal bond premia increase at times of high real uncertainty, and bond prices fall at times of high nominal uncertainty, which contradicts the data.

Finally, the non-neutrality of money, which is captured by a negative response $\rho_{c\pi}$ of expected consumption to lags of expected inflation, plays an important role to generate quantitatively large and positive inflation premium to match the key features of the term structure in the data. With money neutrality, the feedback effect from expected inflation to expected consumption is zero, $\rho_{c\pi} = 0$. In this case, price of expected inflation risk $\lambda_{x\pi}$ is zero, so the inflation premium equal to the covariation of real discount factor with inflation is zero at all horizons. Thus, nominal bond yields essentially reflect the real yields plus the expected inflation, as shown in Equation (27). The amount of bond return predictability and variation in bond premia is substantially smaller now: the slope coefficients in Expectations Hypothesis are near

1, and the R^2 are close to zero. This underscores the importance of the inflation channel and monetary non-neutrality to explain the term structure in the data.

6 Conclusion

In the data, we document a link between bond premia and uncertainties about expected real economic growth and expected inflation. Specifically, nominal bond risk premia and term premia are high at times of high inflation volatility, and low at times of high real uncertainty. The amount of predictability of future bond returns by the two uncertainty variables matches that based on the cross-section of yields.

We use an equilibrium long-run risks type model to account for the key features of nominal term structure and the bond return predictability in domestic and foreign markets. The key ingredients of the model critical for a successful explanation of bonds markets include preference for early resolution of uncertainty, time-variation in expected consumption growth and expected inflation, fluctuations in long-run real and inflation uncertainty, and inflation non-neutrality. In the model, bond yields and bond premia are driven by real growth and inflation expectations and long-run real and nominal uncertainties. When investors prefer early resolution of uncertainty and high expected inflation is bad for future consumption, bond premia increase with a positive shock to inflation uncertainty and decrease when real uncertainty goes up. Thus, the model generates predictability of future bond returns by the yield variables, and it accounts for the relationship between the premia on bonds and the expected growth and expected inflation uncertainties in the data.

To empirically evaluate the model, we formally estimate it using the MLE methods. We find that the model matches well the observed consumption, inflation, forecast and yield data. The model delivers sizeable variations in the risk premia and predictability of bond returns, comparable to the data. In the model, as in the data, inflation uncertainty increases bond premia, while real uncertainty decreases it. Further, the model can account for the predictability of foreign bond returns (exchange rates) at short and long horizons.

A Analytical Model Solution

For brevity of exposition, let us consider a general matrix representation of the economy.

Denote $z_t = [\Delta c_t \ \pi_t]'$ a vector of consumption growth and inflation rate. Their joint dynamics satisfies,

$$\begin{aligned} z_{t+1} &= \mu + x_t + S_z \eta_{t+1}, \\ x_{t+1} &= \Pi x_t + S_x \Sigma_t e_{t+1}, \end{aligned} \quad (\text{A.1})$$

where S_z and S_x are volatility scale matrices, and Σ_t is a diagonal matrix with real and nominal volatility factors on the diagonal.

To model volatility process, denote $V_t = \text{diag}(\Sigma_t \Sigma_t')$, and write down:

$$V_{t+1} = V_0 + \Phi(V_t - V_0) + \Sigma_w w_{t+1}, \quad (\text{A.2})$$

where each component in the vector of volatility innovations w_t is independent. We denote by Ψ_i the log moment-generating function of volatility shocks. In case when volatility shocks follow Gamma distribution:

$$w_{t,i} \sim \mathcal{G}(k_i, \theta_i), \quad (\text{A.3})$$

the log moment-generating function of volatility shocks satisfies

$$\Psi_i(u) \equiv \log E e^{u w_{t,i}} = -k_i \log(1 - \theta_i u), \quad \text{for } u < \frac{1}{\theta_i}. \quad (\text{A.4})$$

For normal innovations, $w_{t,i} \sim \mathcal{N}(0, 1)$, so that $\Psi_i(u) = \frac{1}{2}u^2$.

The key ideas of the model rely on solutions which are derived using a standard log-linearization of returns. In particular, the log-linearized return on consumption claim is given by

$$r_{c,t+1} \approx \kappa_0 + \kappa_1 p c_{t+1} - p c_t + \Delta c_{t+1}, \quad (\text{A.5})$$

where $p c_t$ is the log price-to-consumption ratio, and κ_0 and κ_1 are approximating constants based on the endogenous level of price-consumption ratio in the economy. We follow Bansal, Kiku, and Yaron (2007) to solve for these coefficients endogenously inside the model.

In equilibrium, the price-consumption ratio is linear in the expected growth and volatility factors:

$$p c_t = A_0 + A'_x x_t + A'_v V_t, \quad (\text{A.6})$$

where

$$A_x = \left(1 - \frac{1}{\psi}\right) (I - \kappa_1 \Pi')^{-1} i_c, \quad A_v = \frac{1}{2} \theta \kappa_1^2 (I - \kappa_1 \Phi')^{-1} [S'_x A_x]^2, \quad (\text{A.7})$$

where the last square component is taken element-by-element, and the log-linearization coefficient κ_1 satisfies

$$\begin{aligned} \log \kappa_1 = \log \delta + \left(1 - \frac{1}{\psi}\right) i'_c \mu + A'_v((I - \kappa_1 \Phi)V_0 + (1 - \kappa_1)(I - \Phi)^{-1} \Sigma_w Ew) \\ + \frac{1}{2} \theta \left(1 - \frac{1}{\psi}\right)^2 i'_c S_z S'_z i_c + \frac{1}{\theta} \sum_i \Psi_i(\theta \kappa_1 \{A'_v \Sigma_w\}_i), \end{aligned} \quad (\text{A.8})$$

for $i_c = [1 \ 0]'$ and Ew is the unconditional mean of w_t .

The real and nominal discount factor satisfy:

$$\begin{aligned} m_{t+1} &= m_0 + m'_x x_t + m'_v V_t - \lambda'_z S_z \eta_{t+1} - \lambda'_x S_x \Sigma_t e_{t+1} - \lambda'_v \Sigma_w w_{t+1}, \\ m_{t+1}^\$ &= m_0^\$ + m_x^\$ x_t + m'_v V_t - \lambda_z^\$ S_z \Sigma_t \eta_{t+1} - \lambda'_x S_x \Sigma_t e_{t+1} - \lambda'_v \Sigma_w w_{t+1}. \end{aligned} \quad (\text{A.9})$$

The discount factor parameters and market prices of risks equal to:

$$\begin{aligned} m_0 &= \theta \log \delta + (1 - \theta) \log \kappa_1 - \gamma i'_c \mu + (\theta - 1) A'_v((I - \kappa_1 \Phi)V_0 + (1 - \kappa_1)(I - \Phi)^{-1} \Sigma_w Ew), \\ m_x &= -\frac{1}{\psi} i_c, \quad m_v = (\theta - 1)(\kappa_1 \Phi' - I) A_v, \\ \lambda_z &= \gamma i_c, \quad \lambda_x = (1 - \theta) \kappa_1 A_x, \quad \lambda_v = (1 - \theta) \kappa_1 A_v, \end{aligned} \quad (\text{A.10})$$

and the nominal ones are:

$$m_0^\$ = m_0 - i'_\pi \mu, \quad m_x^\$ = m_x - i_\pi, \quad \lambda_z^\$ = \lambda_z + i_\pi, \quad (\text{A.11})$$

for $i_\pi = [0 \ 1]'$.

The log prices of real and nominal bonds are linear in expected growth and variance factors. E.g., for a real bond,

$$p_{t,n} = -B_{0,n} - B'_{x,n} x_t - B'_{v,n} V_t. \quad (\text{A.12})$$

The bond coefficients are given by

$$\begin{aligned} B_{0,n} &= B_{0,n-1} - m_0 + B'_{v,n-1}(I - \Phi)V_0 - \frac{1}{2} \lambda'_z S_z S'_z \lambda_z - \sum_i \Psi_i(-\{(B_{v,n-1} + \lambda_v)' \Sigma_w\}_i), \\ B_{x,n} &= \Pi' B_{x,n-1} - m_x, \\ B_{v,n} &= \Phi' B_{v,n-1} - m_v - \frac{1}{2} [S'_x (\lambda_x + B_{x,n-1})]^2. \end{aligned} \quad (\text{A.13})$$

Similar equations obtain for the nominal bond coefficients using the nominal discount factor parameters.

References

- Alexius, Annika, 2001, Uncovered interest parity revisited, *Journal of International Economics* 9, 505–517.
- Alvarez, Fernando, Andrew Atkeson, and Patrick J. Kehoe, 2009, Time-varying risk, interest rates, and exchange rates in general equilibrium, *Review of Economic Studies* 76, 851–78.
- Ang, A., and M. Piazzesi, 2003, A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables, *Journal of Monetary Economics* 50, 745–787.
- Ang, Andrew, Sen Dong, and Monika Piazzesi, 2005, No-arbitrage taylor rules, working paper, Columbia University and Univesity of Chicago.
- Backus, David K., Silverio Foresi, and Chris I. Telmer, 2001, Affine term structure models and the forward premium anomaly, *The Journal of Finance* 56, 279–304.
- Backus, David K., Allan Gregory, and Chris I. Telmer, 1993, Accounting for forward rates in markets for foreign currency, *The Journal of Finance* 48, 1887–1908.
- Bansal, Ravi, 1997, An exploration of the forward premium puzzle in currency markets, *The Review of Financial Studies* 10, 369–403.
- Bansal, Ravi, A. Ronald Gallant, Robert Hussey, and George Tauchen, 1995, Non-parametric estimation of stuctural models for high-frequency currency market data, *Journal of Econometrics* 66, 251 – 287.
- Bansal, Ravi, Dana Kiku, and Amir Yaron, 2007, Risks for the long run: Estimation and inference, Working paper.
- Bansal, Ravi, and Ivan Shaliastovich, 2007, Risk and return in bond, currency, and equity markets, working paper.
- Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.
- Bansal, Ravi, and Hao Zhou, 2002, Term structure of interest rates with regime shifts, *The Journal of Finance* 57, 1997–2043.
- Barro, Robert, 2006, Rare disasters and asset markets in the Twentieth century, *Quarterly Journal of Economics* 121.
- Bekaert, Geert, 1996, The time variation of risk and return in foreign exchange markets: A general equilibrium perspective, *Review of Financial Studies* 9, 427–470.

- Bekaert, Geert, Robert Hodrick, and David Marshall, 2001, Peso problem explanations for term structure anomalies, *Journal of Monetary Economics* 48, 241–70.
- Bikbov, Ruslan, and Mikhail Chernov, 2010, No-arbitrage macroeconomic determinants of the yield curve, *Journal of Econometrics* 159, 166–182.
- Campbell, John, and Robert Shiller, 1991, Yield spreads and interest rate movements: a bird’s eye view, *Review of Economic Studies* 58, 495–514.
- Chen, Ren-raw, and Louis Scott, 1993, Maximum likelihood estimation for a multi-factor equilibrium model of the term structure of interest rates, *Journal of Fixed Income* 3, 14–31.
- Chinn, Menzie D., and Guy Meredith, 2004, Monetary policy and long horizon uncovered interest parity, *IMF Staff Papers* 51, 409 – 430.
- Cochrane, John, and Monika Piazzesi, 2005, Bond risk premia, *American Economic Review* 95, 138–160.
- Colacito, Riccardo, 2009, Six anomalies looking for a model. A consumption based explanation of international finance puzzle, working paper.
- Colacito, Riccardo, and Mariano Croce, 2010, The short and long run benefits of financial integration, *American Economic Review, Papers and Proceedings* 100, 523–31.
- Colacito, Riccardo, and Mariano M. Croce, 2011, Long run and the real exchange rate, *Journal of Political Economy* 119, 153–181.
- Dahlquist, Magnus, and Henrik Hasseltoft, 2011, International bond risk premia, working paper.
- Dai, Qiang, and Kenneth Singleton, 2002, Expectation puzzles, time-varying risk premia, and affine models of the term structure, *Journal of Financial Economics* 63, 415–442.
- Duffee, Gregory, 2002, Term premia and interest rate forecasts in affine models, *Journal of Finance* 57, 405–43.
- Duffie, Darrell, and Kenneth Singleton, 1997, An econometric model of the term structure of interest-rate swap yields, *Journal of Finance* 52, 1287–1321.
- Epstein, Larry G., and Stanley Zin, 1989, Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937–969.

- Fama, Eugene F., 1984, Forward and spot exchange rates, *Journal of Monetary Economics* 14, 319–338.
- Farhi, Emmanuel, and Xavier Gabaix, 2008, Rare disasters and exchange rates, Working paper.
- Hodrick, Robert, 1987, *The Empirical Evidence on the Efficiency of Forward and Futures Foreign Exchange Markets* (New York: Harwood Academic Publishers).
- Kandel, Shmuel, and Robert F. Stambaugh, 1991, Asset returns and intertemporal preferences, *Journal of Monetary Economics* 27, 39–71.
- Kocherlakota, Narayana, 1990, Disentangling the coefficient of relative risk aversion from the elasticity of intertemporal substitution: An irrelevance result, *The Journal of Finance*, 45, 175–91.
- Kreps, David M., and Evan L. Porteus, 1978, Temporal resolution of uncertainty and dynamic choice theory, *Econometrica* 46, 185–200.
- Marshall, David, 1992, Inflation and asset returns in a monetary economy, *Journal of Finance* 47, 1315–42.
- Piazzesi, Monika, and Martin Schneider, 2005, Equilibrium yield curves, NBER Working paper 12609.
- Rietz, Thomas, 1988, The equity risk premium: A solution, *Journal of Monetary Economics* 22, 117–131.
- Rudebusch, Glenn D., and Tao Wu, 2004, A macro-finance model of the term structure, monetary policy, and the economy, working paper.
- Verdelhan, Adrien, 2010, A habit-based explanation of the exchange rate risk premium, *Journal of Finance* 65, 123–145.
- Wachter, Jessica, 2006, A consumption-based model of the term structure of interest rates, *Journal of Financial Economics* 79, 365–399.
- Wright, Jonathan, 2011, Term premia and inflation uncertainty: Empirical evidence from an international panel dataset, *American Economic Review* 101, 1514–34.

Tables and Figures

Table 1: Consumption Growth, Inflation, and Volatility Predictability

	Expectation	Volatility
Predictive Approach:		
Consumption	0.14	0.08
Inflation	0.45	0.24
Survey Data:		
Consumption	0.38	0.40
Inflation	0.62	0.48

Adjusted R^2 in the expected consumption, expected inflation, and real and inflation volatility regressions. In predictive regression approach, 1-year sum of future consumption growth and inflation rate is regressed on 1y, 3y and 5y yields and price-dividend ratio to measure expected consumption and expected inflation (adjusted R^2 are reported in Expectation column), and then the 1-year sum of squares of expected consumption and expected inflation residuals are regressed on the same variables to measure the volatilities of expected consumption and expected inflation (adjusted R^2 are reported in Volatility column). For the survey data, the expected consumption and expected inflation correspond to the consensus forecasts. Quarterly observations from 1959Q4 to 2010Q3 for predictive regression approach, and from 1968Q4 to 2010Q3 (second month of the quarter) for the survey data.

Table 2: **Excess Bond Return Predictability**

	2y		3y		5y	
Predictive Approach:						
Real vol	-1.73	(0.49)	-3.18	(0.90)	-5.08	(1.57)
Inflation vol	1.40	(0.48)	2.31	(0.89)	3.36	(1.58)
R^2	0.22		0.23		0.22	
Survey Data:						
Real vol	-1.38	(0.52)	-2.39	(0.93)	-3.68	(1.55)
Inflation vol	0.80	(0.56)	1.20	(0.97)	1.48	(1.67)
R^2	0.17		0.18		0.18	

Regressions of excess nominal bond returns on the volatilities of expected consumption and expected inflation constructed from the predictive regressions approach and the survey data. Quarterly observations from 1959Q4 to 2010Q3 for predictive regression approach, and from 1968Q4 to 2010Q3 (second month of the quarter) for the survey data. Volatility measures are standardized to have mean zero and variance one. Standard errors are Newey-West corrected using 12 lags.

Table 3: **Term Premia Predictability**

	2y		3y		5y	
Predictive Approach:						
Real vol	-1.30	(0.38)	-1.83	(0.40)	-2.27	(0.41)
Inflation vol	1.23	(0.36)	1.74	(0.30)	2.26	(0.33)
R^2	0.21		0.25		0.32	
Survey Data:						
Real vol	-1.50	(0.42)	-2.37	(0.54)	-3.43	(0.64)
Inflation vol	1.24	(0.47)	1.96	(0.50)	2.92	(0.64)
R^2	0.20		0.32		0.53	

Regressions of realized term premia (long-term nominal yields minus the average of future realized short-term rates over the corresponding horizon) on the variances of expected consumption and expected inflation constructed from the predictive regressions approach and the survey data. Quarterly observations from 1959Q4 to 2010Q3 for predictive regression approach, and from 1968Q4 to 2010Q3 (second month of the quarter) for the survey data. Volatility measures are standardized to have mean zero and variance one. Standard errors are Newey-West corrected using 12 lags.

Table 4: **Estimated Model Parameters**

Preferences	δ	γ	ψ		
	0.995*	20.77 (0.50)	1.86 (0.04)		
Consumption	μ_c	σ_c	ρ_c	$\rho_{c\pi}$	σ_{xc}
	5.0e-03*	5.0e-03*	0.811 (0.01)	-0.05 (0.01)	1.03e-03 (0.02e-03)
Inflation	μ_π	σ_π	ρ_π	$\sigma_{x\pi}$	
	1.0e-02*	5.0e-03*	0.986 (0.01)	1.04e-03 (0.03e-03)	
Volatility	ν_c	ν_π	σ_{wc}	$\sigma_{w\pi}$	
	0.991 (0.01)	0.988 (0.01)	1.56e-07 (0.03e-07)	1.75e-07 (0.03e-07)	

Estimated parameter values, quarterly frequency. Maximum Likelihood Estimation of the model. Parameters with superscript * are calibrated.

Table 5: **Consumption Growth and Inflation Rate: Data and Model**

	Data		Model
<i>Consumption:</i>			
Mean	1.94	(0.20)	2.00
Std.Dev.	1.03	(0.08)	1.11
AR(1)	0.29	(0.12)	0.17
US-UK corr	0.24	(0.08)	0.19
<i>Inflation:</i>			
Mean	3.61	(0.50)	4.00
Std.Dev.	1.76	(0.21)	1.62
AR(1)	0.56	(0.11)	0.61
Corr($\pi, \Delta c$)	-0.11	(0.10)	-0.20
US-UK corr	0.58	(0.06)	0.61

Properties of consumption growth and inflation rate in the data and the estimated model. Data are quarterly observations of consumption and inflation in US from 1947 to 2010; UK data is from 1957. Population values for the model are based on a long sample of simulated data. Standard errors are Newey-West corrected using 12 lags.

Table 6: **Bond Markets: Data and Model**

	1y	2y	3y	4y	5y
Data:					
Yield Level	6.09	6.33	6.52	6.68	6.79
EH Slope:		-0.41 (0.44)	-0.78 (0.56)	-1.14 (0.63)	-1.15 (0.67)
CP Slope:		0.44 (0.11)	0.85 (0.23)	1.28 (0.33)	1.43 (0.42)
CP R^2		0.15 (0.10)	0.17 (0.10)	0.20 (0.11)	0.17 (0.12)
FX Slope:	-1.38 (0.71)	-1.61 (0.92)	-0.72 (0.71)	-0.18 (0.59)	0.27 (0.61)
Model:					
Yield Level:	6.14	6.31	6.50	6.71	6.92
EH Slope:		-0.54	-0.58	-0.62	-0.64
CP Slope		0.45	0.84	1.19	1.52
CP R^2		0.21	0.21	0.21	0.21
FX Slope:	-1.73	-1.05	-0.54	-0.15	0.17

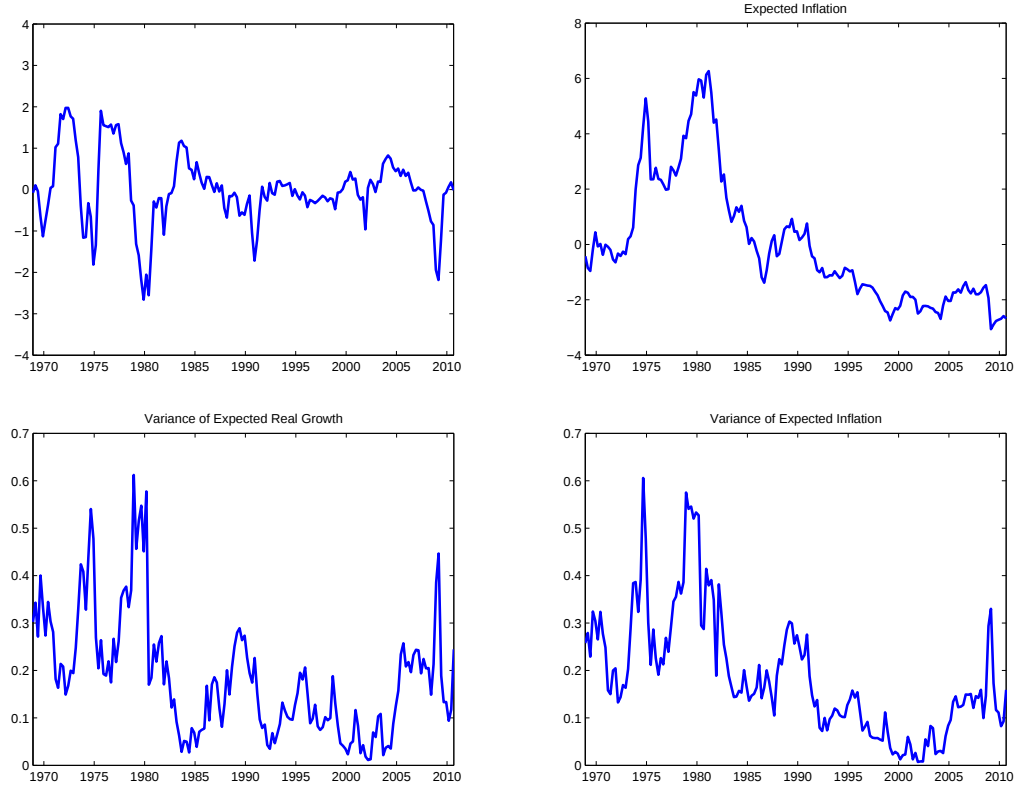
Nominal term structure, slopes in the expectations hypothesis regressions, slopes and R^2 s in Cochrane and Piazzesi (2005) single-factor bond premium regressions, and slopes in regression of excess returns on foreign bonds on the interest rate differential. Data are second-month-of-the-quarter observations of quarterly yields from 1969 to 2010; foreign exchange data are monthly from 1979 to 2010. Population values for the model are based on a long sample of simulated data.

Table 7: **Bond Markets: Importance of Model Ingredients**

	1y	2y	3y	4y	5y
Inflation Neutrality:					
Yield Level:	6.81	6.77	6.75	6.73	6.71
EH Slope:		0.86	0.89	0.92	0.93
CP R^2		0.01	0.00	0.00	0.00
Power Utility:					
Yield Level:	45.15	44.67	44.25	43.91	43.63
EH Slope:		0.95	0.94	0.94	0.94
CP R^2		0.01	0.02	0.02	0.02
Constant Volatility:					
Yield Level:	6.63	6.82	6.99	7.16	7.33
EH Slope:		1.00	1.00	1.00	1.00
CP R^2		0.00	0.00	0.00	0.00

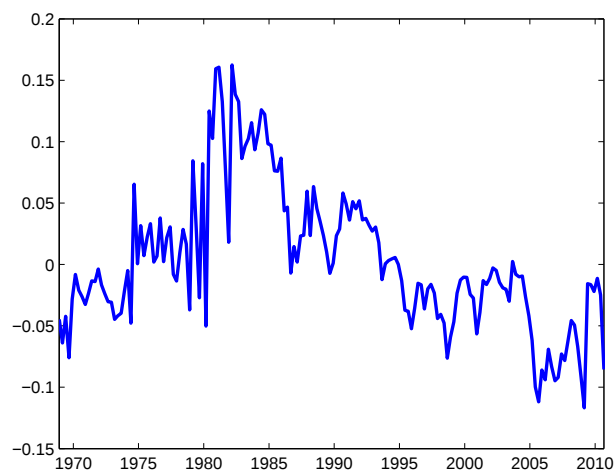
Level of nominal term structure, slope in expectations hypothesis regressions, and R^2 in single-factor bond premium regressions in the model under inflation neutrality, $\rho_{c\pi}$ set to 0, (top panel); under power utility, ψ set to $1/\gamma$ (bottom panel); and under constant variances Σ_w set to zero.

Figure 1: Expected Consumption, Inflation and Volatility Measures



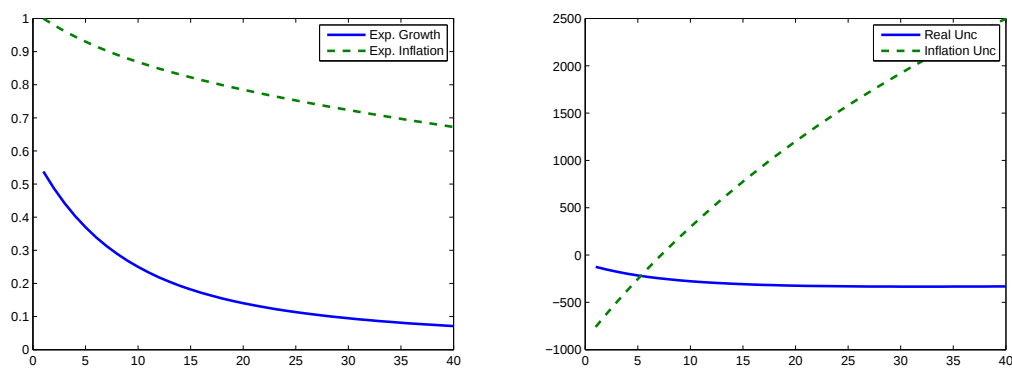
Measures of expected real growth and expected inflation, demeaned, (top panel) and the variances of expected real growth and expected inflation (bottom panel), based on the survey data. Data are quarterly observations from 1968Q4 to 2010Q3, annualized.

Figure 2: **Difference between Inflation and Real Growth Volatility**



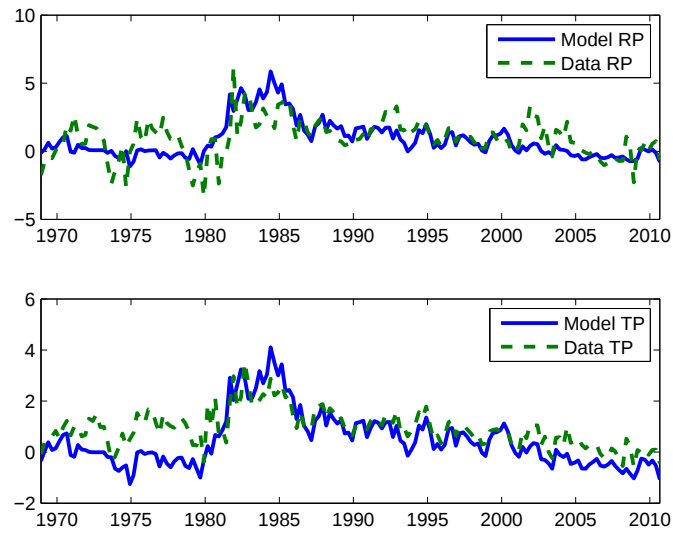
Difference between the variances of expected inflation and expected real growth, based on the survey data. Data are quarterly observations from from 1968Q4 to 2010Q3, annualized.

Figure 3: **Equilibrium Nominal Bond Yield Loadings**



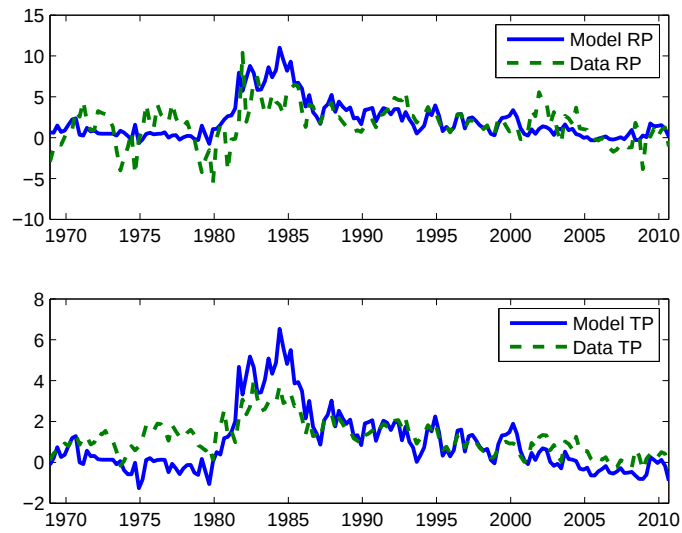
Model-implied nominal bond yield loadings on expected growth and expected inflation (left panel) and real and nominal uncertainty (right panel). Maturity on the horizontal axis is in quarters.

Figure 4: **Bond and Term Premia for 3-year Bond**



Top panel shows the risk premium on a 3-year nominal bond based on the model estimation, alongside with the estimate based on the Cochrane and Piazzesi (2005) factor in the data. Bottom panel shows the term premium on a 3-year bond in the model and in the data using VAR approach.

Figure 5: **Bond and Term Premia for 5-year Bond**



Top panel shows the risk premium on a 5-year nominal bond based on the model estimation, alongside with the estimate based on the Cochrane and Piazzesi (2005) factor in the data. Bottom panel shows the term premium on a 5-year bond in the model and in the data using VAR approach.