

Optimal Retirement Age and Aging Population*

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Abstract

Over recent decades, most developed countries have experienced a fall in fertility and an increase in longevity which have led to a significant increase in the weight of elderly on the population and a decrease in the number of working-age people per elderly population. Economists and politicians are concerned about the aging population process and the need to introduce policy reforms such as fertility enhancing programs and delaying the legal retirement age. This paper introduces a model which determines the optimal retirement age and analyzes the effects of population aging on it. What is revealed is the different role that the drop in the fertility rate and the increase in longevity play in determining the optimal retirement age. While an increase in longevity always implies an increase in the optimal retirement age, a drop in the fertility rate does not. The reason is that a drop in fertility involves three offsetting mechanisms: first, it raises the weight of elders on population increasing the dependency ratio (defined as non working population, children and retirees, over working population), which involves a larger optimal retirement age. Second, it also diminishes the weight of children, and this reduces the dependency ratio, decreasing the optimal retirement age. Finally, a drop in fertility rate increases the weight of older workers in the labor force. If these are more productive than the average, then the drop in the fertility increases the productivity of the labor force and reduces the optimal retirement age. In spite of these counterweighing mechanisms, this paper provides a clear measure to determine the sign of the effect of a drop in the fertility rate over per capita labor and the optimal retirement age. Such measure may be easily obtained from the data and establishes a precise criterion for clarifying the aging population debate

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1. Introduction

The fall in fertility experienced during recent decades and the continuing rise in longevity has led to a significant increase in the proportion of the older population in most developed countries. Thus, social security, health care, long term care system and old age programs, whose expenditures are very much determined by the size of the older population, are coming increasingly under financial stress. As a consequence, measures to alleviate the effects of this situation are priority objectives of current political economic agendas of most governments. Important policy reforms, such as the increase in the legal retirement age or the adoption of policies devoted to rise fertility, have been considered but, is the increase in the retirement age an efficient response to the aging population process? Are really pronatalist policies a right solution? Answers to these questions are more complex than might, *a priori*, looks like. As we will show, results we obtain seems to be running against widespread public opinion and some undertaken policies reforms, which do not take into account some important mechanisms captured in our analysis. This paper develops a framework to answer precisely these questions. The model we present here reveals that the role of a drop in the fertility rate plays in determining the optimal retirement age is different from the role played by an increase in longevity. While an increase in longevity always implies an increase in the optimal retirement age, a drop in the fertility rate does not. This paper provides a clear measure to determine the sign of the effect of a drop in the fertility rate over per capita labor and the optimal retirement age. This is a relevant contribution because this measure, which has the interesting advantage that may be easily obtained from the data, implies a precise criterion for political advice in pronatalist policies and helps greatly to clarify the debate over the retirement age.

On July 7th, 2010, newspapers around the world echoed the latest report from the European Commission that suggests there is a need to raise the average retirement age in the 27-nation bloc from the current age of 60 to 70 by 2060 if workers are to continue supporting retirees at current rates. At the moment there are four working-age people for every person over 65 in the 27-nation EU. That ratio will drop to two for every person over 65 by 2060. EU Employment Commissioner Laszlo Andor said it was urgent for Europe to act now because its working age population will start to shrink from 2012, he stated that “The current situation is simply not sustainable”. This concern about the effects of aging

population is not a new issue ¹ nor something exclusive to Europe. The consequences of the aging of baby-boomers in developed countries has been considered by politicians and economists for years ². However, the recent economic crisis has increased the doubts about the future of the welfare state and the need to introduce policy reforms.

Throughout recent years, developed countries have implemented a range of policies devoted to raising the fertility rate and thus, slowing down the aging of the population and increasing the ratio of workers to retirees. Even the countries that could not be labeled as pronatalist, like the United States, have developed many family or social policies that may lead to higher fertility ³. However, this paper shows that a higher fertility rate would not be the long run solution to reducing the retirement age.

This paper introduces a model in which agents live along three stages of different length: childhood, youth, and old age. Births grow at a constant rate. While during childhood and youth agents survive for sure, during old age stage the surviving probability decreases with age, being zero at a certain threshold. Agents work during the youth and they may retire before arriving the old age. Thus, the retirement age is an endogenous variable. While the amount of efficient units of labor of young agents vary with age due to experience, the subjective cost of working increases with age. This paper determines the optimal retirement age and analyzes the effect of the population aging on it. We also show that a technological improvement produces an increase in the retirement age. The paper shows the different role that play different sources of aging population: while an increase in longevity implies always an increase in the optimal retirement age, a drop in the fertility rate might imply a decrease in the optimal retirement age.

An increase in longevity (an increase in the survival probability), implies an increase in the weight of old agents over total population and an increase in the dependency ratio, defined as the non working population (children and retirees) over working population. In other words, an increase in longevity reduces the participation ratio, defined as working population over total population. Due to the fact that old agents do not work, an increase

¹In recent years Germany, Netherlands and Denmark have deferred the retirement age until 67, while United Kingdom has delayed the retirement age until 68. The government of Spain is considering postponing the retirement age to 67 by around 2040.

²See for example, Velloso (2005), Weller (2005), Boldrin et al (2000), OECD (1998), European Commission Report (2004) and Congress of the United States Report (2004).

³In this respect Grant, Hoorens, Sivadasan, Van Het Loo, Davanzo, Hale and Butz (2006) identifies several forms of policy interventions in family life or population structure. See Kohler, Billari and Ortega (2006) for the case of Europe, Suzuki (2004) for Japan and Fustos (2010) for United States.

in longevity implies a drop in the per capita labor, which works as a negative wealth effect. The consequence of the negative wealth effect is a higher retirement age.

The effect of a decrease in fertility over per capita labor and optimal retirement age is much more complex since involve some offsetting mechanisms. When the fertility rate decreases, the weight that younger people have in population goes down, decreasing the dependency ratio, while the weight that older people have in population goes up, increasing the dependency ratio. Thus, the effect of a drop in fertility over the dependency ratio and the participation ratio is not clear. It turns out that the effect of a drop in fertility over the participation ratio is positive if the average age of working population is larger than the average age of total population. However, the fact that a drop in fertility may produce an increase in the participation ratio does not imply any clear conclusion about the effect of a drop in fertility over per capita labor. The reason is that a drop in fertility increases the weight of older workers in the labor force. If these are more productive than the average, then the drop in fertility increases the productivity of the labor force and, thus, would imply a reduction in the optimal retirement age. In this sense, the model provides another measure that take into account this last mechanism and that determines the effect of a decrease in fertility over per capita labor: the average age of labor force weighted by their contribution to the labor supply (by its productivity). If the average age of workers weighted by their contribution to the total labor supply is higher than the average age of total population, then a drop in fertility would imply an increase in the per capita labor and the optimal response would be an decrease in the retirement age.

We now proceed to calculate empirically the measures we have described before. The first and the second columns in Table 1.1 show the average age of the population and the average age of the working population, respectively for a wide sample of countries in 2005. The result is overwhelming, in almost all countries, the average age of the working population is greater than the average age of the population.⁴ According to the results of the paper, these numbers imply that the effect of a drop in fertility over the participation ratio would be positive in most countries. The third column in Table 1 shows the average age of the labor supply, that is, the average age of workers weighted by their productivity contribution. In all cases, except Italy, the average age of the labor supply is higher

⁴This aging of the work force has been well documented in the literature (see for example, Börsch-Supan, 2002 and Prskawetz, Fent and Guest, 2008).

than the average age of work force. This result implies that the contribution of older workers to the labor supply is greater than others, this is, older workers are the most productive ⁵. More interestingly, is the fact that the average age of the labor supply is higher than the average age of population. According to the results of the paper, a drop in fertility would produce an increase in the per capita labor and the optimal response to it would be a reduction in the retirement age. This conclusion would imply to assert that developed countries, except Italy, that are involved in policy reforms targeted to delaying the retirement age to overcome the drop in fertility, are doing non optimal policies.

Table 1.1: Average age population and average age of work force

Countries ^a	Av. age of population	Av. age of work force	Av. age of labor supply
Australia	37.07	39.60	41.76
Canada	38.65	40.41	42.70
Denmark	39.02	41.30	42.77
Finland	39.97	41.39	43.54
France	38.83	39.75	41.50
Germany	41.74	41.01	42.67
Italy	42.18	39.76	41.30
Japan	42.40	41.80	43.36
Netherlands	38.70	39.89	40.45
Spain	39.96	38.61	41.45
Sweden	40.53	42.03	44.04
United Kingdom	39.00	40.40	41.27
United States	36.40	40.29	42.88

^aSource: Data on population weights were obtained from the US Census Bureau. Working population is measured as the active population aged from 20 to 64 years old. Data on activity rates were obtained from the OECD database. Relative weights of age workers groups used to obtain the average age of the labor supply have been calculated using the relative hourly wages by age group. Data on the wage profiles were obtained from OECD report (1998).

The most closely paper related to ours is the contribution by Crettez and Le Maitre (2002). This paper analyzes the social planner problem in a OLG model where the retirement age is an endogenous variable. The most empirically plausible case is when the elasticity of substitution between old workers' labor and young worker's labor is higher

⁵There exists abundant empirical evidence in favor of a hump-shaped wage-productivity profile in developed countries. The pattern implies that the wage-productivity for older workers is greater than the average of the whole age profile ⁶. Thus, the contribution of one old worker to the total labor supply in the economy is larger than the average age worker.

than one. In this case, if the weight of a population group in the social planner's welfare function depends on its relative size, the effect of fertility on the optimal retirement age is indeterminate. By contrast, our paper offers a precise result: the effect of fertility on the optimal retirement age depends on the value of a well-defined measure, which may be easily constructed from the data in order to give a clear criterion for political advice. Furthermore, the present set-up incorporates some elements which are very relevant for the retirement age debate, which are not possible to analyze in the model of the previous contribution: the effect of a higher life expectancy on the retirement age, the influence of children population to the dependence rate and its consequences for retirement, the effect of wage profile along the life-cycle for optimal retirement, etc.

Many papers in the literature have analyzed the effect of the introduction of a public pension program on the individual retirement decision (see for example Kahn 1988; Fabel 1994). Recent literature has studied the retirement decision from a political economy perspective (see Conde-Ruiz and Galasso 2003, 2004). Other papers have analyzed the retirement decision from a social security reform environment (see Auerbach, Kotlikoff, Hagemann, and G. Nicoletti, 1989 and De Nardi, Imrohoroglu, and Sargent, 1999). Finally, another recent strand of the literature has focused on the impact of changes in fertility and longevity in determining the retirement age. Lacomba and Lagos (2006) analyze the effects of population aging on the retirement age using a life-cycle model. They find that the demographic effects of a decrease in the population growth rate may lead to a delay in the preferred retirement age, when the dependence ratio modifies the contribution rate (see also Bloom, Canning and Graham, 2004 and Fehr, Jokish and Kotlikoff, 2008). While all these previous contributions focus on how different social security schemes affect the individual decisions of retirement, this paper steps back from the social security debate and examines the more general question of how different demographic changes affect the optimal retirement age from social welfare point of view.

Other branch of the literature analyzes different aspects of the economic consequences of aging population, for example Boadway, Marchand and Pestieau (1990 a, b), Marchand, Michel and Pestiau (1990) and Meijdam and Verbon (1997). However, these theoretical papers provide no clear answers which is reflected in the empirical results. Indeed, the empirical literature on the fiscal and economic effects of fertility changes is quite limi-

ted and yields controversial results. For example, Cutler, Poterba Sheiner and Summers (1990), Guest and McDonald (2002), Guest (2006) and Heijdra and Ligthart (2006) find that declining fertility rates have a positive economic impact on future living standards, increasing the per capita consumption. While Berkel, Börsch-Supan, Ludwig and Winter (2004) find that a drop in the fertility rate would worsen the long-run pension finances. All these papers treat retirement age as an exogenous variable and thus, they cannot analyze how aging population affects the optimal retirement age, which is the main goal of our paper.

The contribution of this paper is very significant because it is the first attempt to provide a theoretical analysis of the optimal retirement age determination in a neoclassical growth model with uncertainty in the life horizon. Moreover, this study analyzes the effect of demographic changes on the optimal retirement age and how this effect may result in different conclusions depending on the population composition. In this study, the analysis of the effect of aging population is very meticulous in that it distinguishes the increase in the fertility rate from an increase in the survival probabilities.

The paper is organized as follows. Section 2 develops a representative agent model. Section 3 describes the social planner's problem. Section 4 defines the optimal allocation and section 5 analyzes the dynamical behavior of the economy. Section 6 presents and discusses the effects of an aging population. Section 7 analyzes the effects of an increase in the growth rate, and the last section concludes. The paper also includes a technical appendix.

2. The model

2.1. Demographic Dynamic:

Time is continuous and indexed by $t \in \mathfrak{R}$. Population is composed of agents of different ages, where $a \in [0, \bar{a}]$ is the support of ages; $\bar{a}$ being the upper limit of duration of life. Live agents go through three stages: childhood, when $a \in [0, a_y)$; youth, when $a \in [a_y, a_o]$; and old age, when $a > a_o$, with $a_y < a_o$. Agents are only able to work during youth.

The probability of surviving at age a is denoted by $s(a)$. To simplify, we assume that during childhood and youth, agents survive with probability one: $s(a) = 1, \forall a \leq a_o$; while during old age the probability of being alive at age a is equal to $s(a) = \psi(a; \xi), \forall a > a_o$,

where $\psi : [a_o, \bar{a}] \times \mathfrak{R}_+ \rightarrow [0, 1]$, is a strictly decreasing function in its first argument and strictly increasing in its second argument when $a \in (a_o, \bar{a})$ and such that $\psi(a_o; \xi) = 1$ and $\psi(\bar{a}; \xi) = 0$. Note that when ξ rises, the probability of being alive at age $a \in (a_o, \bar{a})$ also increases. Thus, an increase in ξ implies an increase in life expectancy. In this sense, parameter ξ could be considered as the health status or some biological ingredient that defines individuals' capacity to survive. To simplify, we will interpret an increase in ξ as an increase in life expectancy.

The number of agents of age a at time t is denoted by $N(a, t)$ and is defined as follows:

$$N(a, t) = N(0, t - a)s(a) \quad (2.1)$$

That is, the number of agents of age a at time t is equal to the number of agents born in “ a ” periods before, multiplied by the probability of being alive after “ a ” periods of being born.

We assume that births increase over time at a constant rate denoted by n :

$$\dot{N}(0, t) = nN(0, t) \quad (2.2)$$

Using (2.1) and (2.2) we get:

$$N(a, t) = s(a)N(0, t)e^{-na} \quad (2.3)$$

The total population $N(t)$ can be calculated as:

$$N(t) = \int_0^{\bar{a}} s(a)N(0, t)e^{-na} da$$

and, since in 2.2 it is easy to see that the total population increases over time at the constant rate n :

$$\dot{N}(t) = nN(t) \quad (2.4)$$

Thus, the fraction of agents of age $\hat{a}$ in the whole population, $\mu(\hat{a})$, is defined by:

$$\mu(\hat{a}) = \frac{s(\hat{a})N(0, t)e^{-n\hat{a}}}{\left[\int_0^{\bar{a}} s(a)N(0, t)e^{-na} da \right]} = \frac{s(\hat{a})e^{-n\hat{a}}}{\left[\int_0^{\bar{a}} s(a)e^{-na} da \right]}, \quad \forall \hat{a} \leq \bar{a} \quad (2.5)$$

where:

$$\int_0^{\bar{a}} \mu(a) da = \int_0^{\bar{a}} \frac{s(a)e^{-na}}{\left[\int_0^{\bar{a}} s(a)e^{-na} da \right]} da = 1$$

We can rewrite the number of agents of age a at time t as a fraction of the total population at time t , that is,

$$N(a, t) = \mu(a)N(t) \quad (2.6)$$

2.2. Technology:

There exists a unique good that may be used either as a consumption good or as an investment good. There are two factors: labor L and capital K . The amount of production is given by the Cobb-Douglas production function:

$$\Gamma(t)^{1-\alpha} K(t)^\alpha L(t)^{1-\alpha}$$

There is exogenous technological change:

$$\dot{\Gamma}(t) = \gamma \Gamma(t)$$

The accumulation of capital follows the conventional neoclassical law of motion:

$$\dot{K}(t) = I(t) - \delta K(t)$$

where $I(t)$ denotes gross investment and δ denotes depreciation rate. If we define $k(t) \equiv \frac{K(t)}{N(t)}$ as the per capita capital, we may rewrite the capital accumulation equation as follows:

$$\dot{k}(t) = i(t) - (\delta + n)k(t)$$

where $i(t) \equiv \frac{I(t)}{N(t)}$ denotes the per capita investment. Taking into account the fact that production is devoted to either investment or to consumption, the above equation may be rewritten as follows:

$$\dot{k}(t) = \Gamma(t)^{1-\alpha} k(t)^\alpha l(t)^{1-\alpha} - c(t) - (\delta + n)k(t) \quad (2.7)$$

where $l(t) \equiv \frac{L(t)}{N(t)}$ and $c(t) \equiv \frac{C(t)}{N(t)}$ denote respectively per capita labor and per capita consumption.

2.3. Preferences and endowments:

Households maximize the sum of utility of their members, which depends on consumption:

$$\int_0^\infty \left[\int_{\Lambda(t)} N(a, t) [\ln(c(a, t)) - \phi(a)] da + \int_{[0, \bar{a}] - \Lambda(t)} N(a, t) [\ln(c(a, t))] da \right] e^{-\rho t} dt$$

where $c(a, t)$ denotes the consumption of the agents at age a at time t . $\Lambda(t)$ denotes the correspondence which relates the time index t with a measurable subset of $[a_y, a_o]$ which

defines the ages at which agents work. Thus, if $a \in \Lambda(t)$, then agents at age a work at time t . In other words $\Lambda(t)$ defines the age of the work force in the economy. Agents either work or do not, but they can not choose the amount of time devoted to work which is exogenous. Finally, $\phi(a)$ is the disutility derived from working, which is an increasing function of age a . Furthermore, $\phi(\cdot)$ is continuous and differentiable of second order, convex and $\lim_{a \rightarrow a_o} \phi'(a) = +\infty$. This function could be interpreted as the subjective health cost of working. It is reasonable to assume that older workers have more health problems and thus, they have a bigger cost.⁷

Using the assumptions about population behavior (see equations 2.4 and 2.6), we may rewrite the utility function as follows:

$$N(0) \left[\int_0^\infty \left[\int_0^{\bar{a}} \mu(a) \ln(c(a, t)) da - \int_{\Lambda_t} \mu(a) \phi(a) da \right] e^{-(\rho-n)t} dt \right]$$

Children and old age agents can not work. Young agents can work or not. We denote $l(a)$ the amount of labor that a young worker of age a owns, where $l(a)$ is a function $l : [a_y, a_o] \rightarrow \mathbb{R}_{++}$, which is continuous and differentiable of second degree and quasi-concave. We assume that $\frac{\phi(a)}{l(a)}$ is an increasing function. This assumption implies that the loss of working (the utility cost of working) is growing faster than the gain of working (the endowment of labor). Thus, despite older workers being the most productive, they suffer relatively more health problems. In the end, the cost of working exceeds the gain of doing it.

3. The Social Planner's Problem

We consider a social planner who can choose the support of working ages, the allocation of consumption and capital throughout the time. The social planner chooses among feasible allocation of resources. The social planner's problem is as follows:

$$\begin{aligned} \max_{\{c(a, t)\}_{a=0}^{\bar{a}}, \Lambda(t)} \quad & \int_0^\infty \left[\int_0^{\bar{a}} \mu(a) \ln(c(a, t)) da - \int_{\Lambda(t)} \mu(a) \phi(a) da \right] e^{-(\rho-n)t} dt \\ \text{s.a.} \quad & \dot{k}(t) = \Gamma(t)^{1-\alpha} k(t)^\alpha l(t)^{1-\alpha} - \int_0^{\bar{a}} \mu(a) c(a, t) da - (n + \delta)k(t) \end{aligned} \quad (3.1)$$

$$l(t) = \int_{\Lambda(t)} l(a) \mu(a) da \quad (3.2)$$

⁷Longitudinal data from the federal government's Health and Retirement Survey shows that the onset of major health problems frequently leads directly to withdrawal from the labor force (see Forman and Chen, 2008).

where ρ denotes the subjective discount rate of the utility function, where $\rho > n$. The assumption that the instantaneous utility function derived from consumption $\ln(c)$ is strictly concave implies that consumption across agents will be identical for any optimal solution. The assumption that $\frac{\phi(a)}{l(a)}$ is an increasing function implies that at the optimal solution, younger agents work, while older workers do not. These older agents that do not work are retired. We can define the retirement age, $a_r(t)$, as the age at which agents work if they are at equal or lower than $a_r(t)$ and if not they are retired, that is, older workers do not work while younger worker do. As a consequence, the support of working ages $\Lambda(t)$ can be identified, this is, $\Lambda(t) = [a_y, a_r(t)]$.

All these facts allow us to rewrite the social planner optimization problem as follows:

$$\begin{aligned} \max_{\{c(a,t)\}_{a=0}^{\bar{a}}, a_r(t)} \quad & \int_0^\infty \left[\ln(c(t)) - \int_{a_y}^{a_r(t)} \mu(a)\phi(a)da \right] e^{-(\rho-n)t} dt \quad (3.3) \\ \text{s.a.} \quad & \dot{k}(t) = \Gamma(t)^{1-\alpha} k(t)^\alpha l^s(a_r(t))^{1-\alpha} - c(t) - (n + \delta)k(t) \end{aligned}$$

where $l^s(a_r(t)) = \int_{a_y}^{a_r(t)} l(a)\mu(a)da$ is the amount of per capita labor as a function of the retirement age, this is, the per capita labor supply in the economy⁸. Notice that the per capita labor supply is different from the per capita work force in the economy (active population), $z(a_r(t)) = \int_{a_y}^{a_r(t)} \mu(a)da$.⁹

The first order conditions and transversality conditions of the above problem are:

$$\begin{aligned} \frac{1}{c(t)} e^{-(\rho-n)t} &= \lambda(t) \\ \phi(a_r(t)) e^{-(\rho-n)t} &= \lambda(t) (1 - \alpha) \Gamma(t)^{1-\alpha} \left(\frac{k(t)}{l^s(a_r(t))} \right)^\alpha l(a_r(t)) \\ \dot{\lambda}(t) &= -\lambda(t) \left[\alpha \Gamma(t)^{1-\alpha} \left(\frac{k(t)}{l^s(a_r(t))} \right)^{\alpha-1} - (n + \delta) \right] \\ \lim_{t \rightarrow +\infty} \omega(t) k(t) &= 0 \end{aligned}$$

where $\lambda(t)$ are the Lagrangian multipliers of the associated Hamiltonian with the social

⁸See Appendix A for a detailed description of the properties of labor supply function.

⁹If the endowments of labor over ages are equal to 1, $l(a) = 1, \forall a$, then the per capita labor supply coincides with the per capita work force.

planner optimization problem (3.3). Using the above first order conditions we get:

$$\phi(a_r(t)) = \frac{1}{c(t)}\omega(t)l(a_r(t)) \quad (3.4)$$

$$\frac{\dot{c}(t)}{c(t)} = [r(t) - \rho] \quad (3.5)$$

$$\left[\frac{\phi'(a_r(t))}{\phi(a_r(t))} - \frac{l'(a_r(t))}{l(a_r(t))} \right] \dot{a}_r(t) = -[r(t) - \rho] + \frac{\dot{\omega}(t)}{\omega(t)} \quad (3.6)$$

where

$$\begin{aligned} \omega(t) &= (1 - \alpha)\Gamma(t)^{1-\alpha} \left(\frac{k(t)}{l^s(a_r(t))} \right)^\alpha \\ r(t) &= \alpha\Gamma(t)^{1-\alpha} \left(\frac{k(t)}{l^s(a_r(t))} \right)^{\alpha-1} - \delta \end{aligned}$$

The first equation describes the trade off between the utility of not be working and the loss of labor income when an agent retires: if the social planner decides that the retirement age is $a_r(t)$, the individuals who retire at that age increase their utility when they do not work in $-\phi(a_r(t))$. However, there is a loss in labor income equal to $\omega(t)l(a_r(t))$, which reduces consumption and therefore, the utility derived from consumption in $\frac{1}{c(t)}\omega(t)l(a_r(t))$. The second equation is the typical optimal rule of intertemporal consumption choice: the choice between consumption in the present and consumption in the future depends on the degree at which consumption in the future is valuable, which depends on ρ , and the rate at which consumption in the present may be substituted by consumption in the future. Finally, the third equation describes the trade off between retirement in the present and retirement in the future: if present generations retire later, this allows them to accumulate assets and these assets allow future generation to retire earlier (this is why the return on savings r and the valuation of future generations ρ affects such a choice). The second factor to consider is the growth rate of returns of labor $\frac{\dot{\omega}(t)}{\omega(t)}$, which represents the difference in the loss of labor income that present and future generations are going to suffer when they retire. If returns of labor increase, future generation are going to earn more than present ones, and in this sense it is better for present generations to retire earlier since the opportunity cost of retiring is not as high as in the future. The last factor to take into account is the trend in the net benefit resulting from combining the positive effect on the utility and the negative effect on the labor endowment (opportunity cost) when individuals retire at age $a_r(t)$, $\left[\frac{\phi'(a_r(t))}{\phi(a_r(t))} - \frac{l'(a_r(t))}{l(a_r(t))} \right]$. If this net benefit is increasing at time t , individuals in the

present generation tend to retire later than individuals in future generations, due to the fact that the gains will be greater in the future. Note that one of the conditions (3.5) and (3.6) is redundant, since (3.4) and (3.5) imply (3.6); and (3.4) and (3.6) imply (3.5).

Finally, the transversality condition may be rewritten in the following two alternative ways:

$$\begin{aligned}\lim_{t \rightarrow +\infty} \frac{1}{c(t)} e^{-(\rho-n)t} k(t) &= 0 \\ \lim_{t \rightarrow +\infty} \frac{\phi(a_r(t))}{\omega(t)l(a_r(t))} e^{-(\rho-n)t} k(t) &= 0\end{aligned}$$

and the interpretation is as usual: this means that nothing should be saved in the last period unless it is costless to do so (i.e., $\frac{1}{c(t)} e^{-(\rho-n)t} = \frac{\phi(a_r(t))}{\omega(t)l(a_r(t))} e^{-(\rho-n)t} = 0$).

4. Optimal Solution

Definition: An optimal allocation is an allocation $\{c(t), a_r(t), k(t), l(t), i(t)\}_{t=0}^{\infty}$ such that:

- The social planner maximizes its utility: $\{c(t), a_r(t), k(t)\}_{t=0}^{\infty}$ is the solution of the social planner problem (3.3)
- There exists feasibility: $c(t) + i(t) = \Gamma(t)^{1-\alpha} k(t)^{\alpha} l(t)^{1-\alpha}$ with $l^s(a_r(t)) = l(t)$

5. Dynamic Behavior

It follows from the optimal solution definition that the dynamic behavior of the economy may be described by the following dynamic system:

$$\begin{aligned}\dot{a}_r(t) &= \frac{(1-\alpha)(\delta + \rho + \gamma) + \alpha(\rho - n) - (1-\alpha)\alpha \left(\frac{\Gamma(t)l^s(a_r(t))}{k(t)} \right)^{1-\alpha} \frac{l(a_r(t))}{\phi(a_r(t))l^s(a_r(t))}}{\left[\frac{\phi'(a_r(t))}{\phi(a_r(t))} - \frac{l'(a_r(t))}{l(a_r(t))} + \alpha \frac{\mu(a_r(t))l(a_r(t))}{l^s(a_r(t))} \right]} \\ \dot{k}(t) &= \Gamma(t)^{1-\alpha} k(t)^{\alpha} l^s(a_r(t))^{1-\alpha} \left[1 - (1-\alpha) \frac{l(a_r(t))}{\phi(a_r(t))l^s(a_r(t))} \right] - (\delta + n)k(t) \\ \lim_{t \rightarrow +\infty} \frac{\phi(a_r(t))}{(1-\alpha)} \left(\frac{k(t)}{\Gamma(t)l^s(a_r(t))} \right)^{1-\alpha} e^{-(\rho-n)t} &= 0\end{aligned}$$

If we define $\tilde{k}(t) \equiv \frac{k(t)}{\Gamma(t)}$, we may rewrite the system as follows:

$$\dot{a}_r(t) = \frac{(1-\alpha)(\delta + \rho + \gamma) + \alpha(\rho - n) - (1-\alpha)\alpha \left(\frac{l^s(a_r(t))}{\tilde{k}(t)} \right)^{1-\alpha} \frac{l(a_r(t))}{\phi(a_r(t))l^s(a_r(t))}}{\left[\frac{\phi'(a_r(t))}{\phi(a_r(t))} - \frac{l'(a_r(t))}{l(a_r(t))} + \alpha \frac{\mu(a_r(t))l(a_r(t))}{l^s(a_r(t))} \right]} \quad (5.1)$$

$$\begin{aligned} \dot{\tilde{k}}(t) &= \tilde{k}(t)^\alpha l^s(a_r(t))^{1-\alpha} \left[1 - (1-\alpha) \frac{l(a_r(t))}{\phi(a_r(t))l^s(a_r(t))} \right] - (\delta + n + \gamma)\tilde{k}(t) \quad (5.2) \\ \lim_{t \rightarrow +\infty} \frac{\phi(a_r(t))}{(1-\alpha)} \left(\frac{\tilde{k}(t)}{l^s(a_r(t))} \right)^{1-\alpha} e^{-(\rho-n)t} &= 0 \end{aligned}$$

Figure 5.1 describes the dynamic behavior of the system defined above. We see that the dynamic behavior is the typical saddle point dynamic. So, there is a unique solution path that converges to the steady state, $(\tilde{k}^*, a_r^*)$ ¹⁰. We can also observe that along the solution path the retirement age decreases with capital. This is due to the wealth effect, since early retirement is a normal good, when wealth increases agents retire earlier.

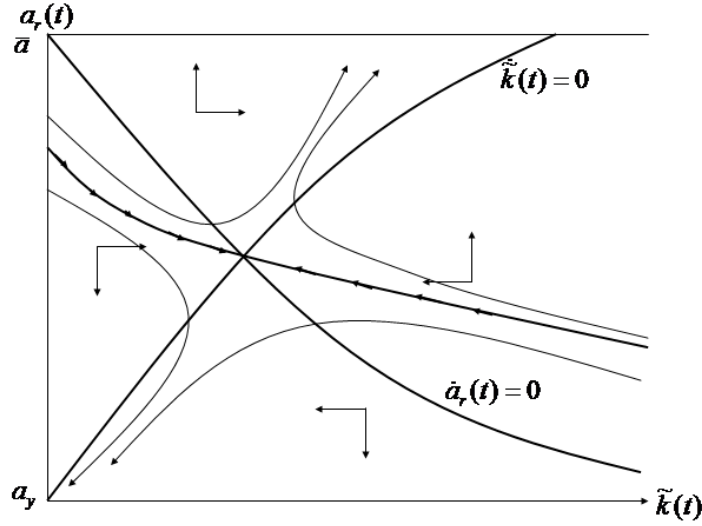


Figure 5.1: Dynamic Behavior

6. The effect of an aging work force

In this section, we study the effect of the aging of the work force, which is due to two main factors: an increase in life expectancy and a drop in the fertility rate. We show that

¹⁰See Appendix B for a detailed construction of the steady state of the economy.

these two factors have, under quite empirically plausible assumptions, exactly the opposite effect on the labor supply and the economy: an increase in the life expectancy increases the optimal retirement age at the steady state; while a drop in the fertility rate reduces the optimal retirement age at the steady state. Considering the retirement aged fixed, a raise in life expectancy increases the portion of old agents and it increases the dependence rate. Thus, it has a negative “wealth effect”: it reduces the per capita consumption level and increases the retirement age at the steady state. The effect of a drop in the fertility rate on the economy is not clear. A drop in the fertility rate decreases the portion of younger people and thus, it increases the portion of retired population, while it decreases the portion of children in the population. However, the effect on the labor supply is ambiguous as it increases the portion of old workers and decreases the portion of young workers. We prove that if the average age of labor supply is greater than the average age of total population then a drop in the fertility rate increases the labor supply in the economy. The reason is that a drop in the fertility rate gives more weight in the active population to older and more experienced and productive workers and thus, it produces an increase in labor productivity and in per capita labor.

6.1. The effect of an increase in life expectancy:

Proposition 6.1. *At the steady state, $\frac{\partial a_r^*}{\partial \xi} > 0$ and $\frac{\partial \tilde{k}^*}{\partial \xi} < 0$.*

Consider an increase in parameter ξ that implies an increase in survival probabilities and therefore, an increase in life expectancy. Notice that the increase in parameter ξ only affects the dynamic system (5.1)-(5.2) through the per capita labor supply. The labor supply is a decreasing function of ξ since an increase in life expectancy increases the old population and therefore, reduces the per capita work force¹¹. Thus, an increase in life expectancy works as a negative wealth effect: it reduces the per capita consumption (and the per capita capital) and it increases the retirement age.

Figure 6.1 shows the effect of an increase of ξ on the steady state and the transitory dynamic. The increase of life expectancy reduces per capita labor supply, and this has a negative effect on per capita resources of households, reducing consumption and increasing the retirement age.

¹¹See Appendix A.

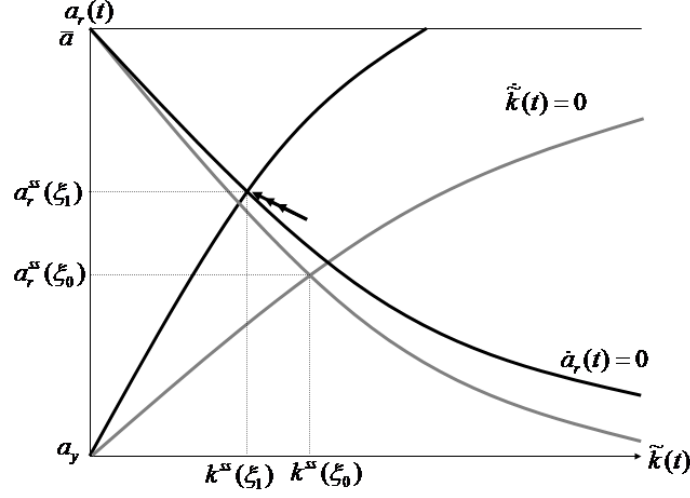


Figure 6.1: The effect of an increase in life expectancy

6.2. The effect of an increase in the fertility rate:

An increase in the fertility rate has two different mechanism: i) When the fertility rate increases, keeping per capita capital constant requires a larger amount of investment. In this sense, the effect is similar to an increase in the depreciation rate. Thus, it is a negative “wealth effect”. ii) The other mechanism is related to the effect of the fertility rate on per capita labor supply. However, such an effect is ambiguous. On the one hand, an increase in the fertility rate reduces the weight of old agents, increasing the portion of active population (work force) over the total population. On the other hand, it increases the weight of children, reducing the portion of active population over total population. Thus, the effect on the dependency ratio would be ambiguous. Furthermore, owing to the fact that workers’ labor productivity increases with experience (age), an increase in the fertility rate reduces the weight of more experienced and productive groups inside the labor market, reducing labor productivity and therefore the per capita labor supply.

In order to analyze the effect of an increase in the fertility rate on per capita labor supply, let’s define the labor supply contribution density function as follows:

$$\nu(a; a_r) = \frac{l(a)\mu(a)}{\int_{a_y}^{a_r} l(a)\mu(a)da}$$

this density function gives the weight that each type- a agent has on the labor supply. Note that the more productive an agent is, the higher her weight in the labor supply.

Proposition 6.2. $\frac{\partial l^s(a_r)}{\partial n} < 0$ if and only if $E_\nu[a/[a_y, a_r]] > E[a]$, where $E_\nu[a/[a_y, a_r]] \equiv \int_{a_y}^{a_r} a \nu(a; a_r) da$ and $E[a] \equiv \int_0^{\bar{a}} a \mu(a) da$.

Proposition 6.2 establishes that if the average age of the work force weighted by its contribution to the labor supply is higher than the average age of total population, then an increase in the fertility rate reduces the capita labor supply. The above necessary condition implies the following sufficient one:

Corollary 6.3. If $E[a/[a_y, a_r]] > E[a]$ and $l(a_r) > E[l(a)/[a_y, a_r]]$ then $\frac{\partial l^s(a_r)}{\partial n} < 0$.

That is, if the average age of the work force is greater than the average age of the total population, and the number of efficiency units of labor of agents at the retirement age is larger than the average number of efficiency units of labor of the work force, then an increase in the fertility rate reduces per capita labor supply. In this proposition, we can see two different effects: first, when $E[a/[a_y, a_r]] > E[a]$, that is, when the average age of the work force is greater than the average age of total population, an increase in the fertility rate reduces the proportion of the work force over the total population. Second, when $l(a_r) > E[l(a)/[a_y, a_r]]$, that is, when the number of efficiency units of labor of agents at retirement age is larger than the average in the age profile then, an increase in the fertility rate reduces the weight of the older and more experienced and productive workers, which implies a drop in the average efficiency units of labor in the economy.

We notice that the necessary condition in proposition 6.2 is very reasonable. It is indeed supported by the existing empirical evidence. As we observed in Table 1.1, for all developed countries in the sample (except one), the average age of the work force weighted by its contribution to the labor supply is higher than the average age of work force. Similarly, assumptions behind corollary 6.3 are also supported by empirical evidence. Table 1.1 showed that in almost all cases, the average age of the working population is higher than the average age of the population. Moreover, the abundant empirical evidence in favor of a hump-shaped wage-productivity profile in developed countries supports the fact that productivity at retirement age is greater than the average productivity throughout the life

cycle.¹² Therefore, we can conclude that the most relevant case from the empirical point of view is the one in which an increase in fertility rate reduces the per capita labor supply. For this reason, we have centered the analysis in this case.

Proposition 6.4. *At the steady state, if $\alpha < \frac{1}{2}$ and $\frac{\partial l^s(a_r)}{\partial n} < 0$ then $\frac{\partial a_r^*}{\partial n} > 0$ and $\frac{\partial \tilde{k}^*}{\partial n} < 0$.*

Considering the propositions above, a drop in the fertility rate has just the opposite effect that an increase in the life expectancy has. If the fertility rate falls, the per capita number of children falls as well and therefore, it increases the per capita work force. Furthermore, the older and more productive workers gain weight in the work force, increasing the number of efficient units of labor per worker, that is, increasing the per capita labor supply. Finally, the amount of investment needed to keep the amount of per capita capital constant is less. All these three effects represent positive wealth effects and therefore, produce a decrease in the optimal retirement age and an increase in the amount of per capita consumption and capital at the steady state. Figure 6.2 shows the new steady state and the dynamic transition due to a reduction in the fertility rate.

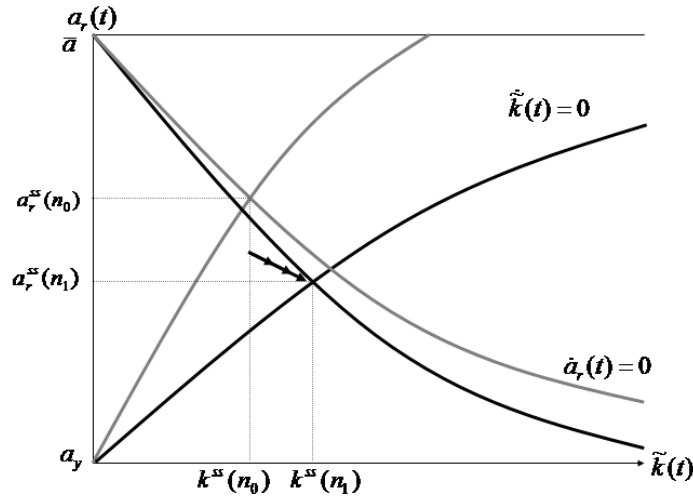


Figure 6.2: The effect of a decrease in the fertility rate

¹²See Section 1 for a more detailed discussion about the suitability of the assumptions.

7. The effect of a technological improvement

Proposition 7.1. *At the steady state, if $\alpha < \frac{1}{2}$ then $\frac{\partial a_r^*}{\partial \gamma} > 0$ and $\frac{\partial \tilde{k}^*}{\partial \gamma} < 0$.*

Imagine that growth rate of the technological change, γ increases. The effect on the capital per efficient unit of labor, $\tilde{k}$ is clear: it produces a negative effect on the level of the capital per efficient unit of labor, equivalent to an increase in the depreciation rate, which implies a decrease in the capital per efficient unit of labor. However, the fact that the capital per efficient unit of labor remains constant at the steady state implies that the per capita capital is growing at the same rate as the technological progress. Thus, the rate of return at the steady state is higher and per capita consumption and wages are also increasing at the new higher rate.

To study the effect on the retirement age we look at equation 3.6. Note that there are two effects coming from different directions: first, the increase in the return on assets implies an increase in the opportunity cost of retirement in the present. Agents would prefer to raise the retirement age in the present and to decrease it in the future. Second, the increase in the wage growth rate makes it more valuable to retire later in the future (with higher gains), so agents would be able to reduce the retirement age now and increase it in future. While the first effect prevails over the second at the beginning of the transition, later they offset each other and the retirement age remains stationary at the steady state. However, since the rate of return at the steady state is higher, the retirement age increases as well.

8. Conclusion

The fall in fertility and the continuation of the rise in longevity experienced during recent decades in developed countries has led to a significant increase in the proportion of the older population and a decrease in the number of working-age people for every elderly person. This increase in the dependency ratio has aroused the debate of economists and politicians about the sustainability of the welfare state and the need to introduce policy reforms, such as fertility enhancing programs and delaying the retirement age.

We have formulated a growth model which determines the optimal retirement age that economies should implement in order to reach the optimal size of the work force relative

to the whole population. Then, we have used the model to analyze the effect of an aging population on the optimal retirement age. We have considered two possible sources of aging: an increase in the life expectancy and a decrease in the fertility rate. We have made clear that these sources of aging population have different effects. In fact, under quite empirically plausible assumptions, these two sources of aging have exactly opposite effects. More precisely, an increase in life expectancy increases the optimal retirement age at the steady state; while a drop in the fertility rate reduces the optimal retirement age at the steady state.

Keeping the retirement age fixed, an increase in life expectancy increases the portion of old agents which implies an increase the dependency rate. This fact produces a negative “wealth effect” in the economy: it reduces per capita consumption and increases the retirement age at the steady state. The effect of a drop in the fertility rate is more complex. A drop in the fertility rate decreases the portion of younger people. Thus, it increases the portion of the retired population and decreases the portion of children in the population. However, the effect on the portion of active population is ambiguous as well as the effect on the dependency ratio. We show that if the life cycle wage profile is hump-shaped and the average age of the work force is larger than the average age of total population, a drop in the fertility rate increases per capita labor in the economy. Furthermore, as a drop in the fertility rate gives more weight to older and more experienced and productive workers in the work force, the drop in the fertility rate may produce an increase in labor productivity and in per capita labor. Thus, the optimal solution for a drop in the fertility rate would be to increase the retirement age.

We have also analyzed an increase in the growth rate of technical progress. It has a positive “wealth effect” implying an increase in the growth rate of per capita consumption and capital. Moreover, the higher rate of return at the steady state makes it more valuable to work and retire later which implies an increase in the retirement age.

The findings of the paper show the relevance of correctly analyzing the population and labor market structure in the design of policies. For example, the popular and widespread pronatalist policies among the developed countries would not be the solution to an increasing dependency ratio. In fact, under plausible assumptions, an increase in the fertility ratio would imply a negative “wealth effect” which would imply an increase in the retirement

age. The empirical evidence we have added supports this result and shows that most developed countries are implementing wrong policies producing the opposite effects to the desired ones.

Appendix

A. The per capita labor supply

In this subsection, we analyze the effect of different exogenous variables on the labor supply. The per capita labor supply is as follows:

$$l^s(a_r(t)) = \int_{a_y}^{a_r(t)} l(a)\mu(a)da = \frac{\int_{a_y}^{a_r(t)} l(a)s(a)e^{-na}da}{\left[\int_0^{\bar{a}} s(a)e^{-na}da\right]} = \frac{\int_{a_y}^{a_r(t)} l(a)e^{-na}da}{\left[\int_0^{a_o} e^{-na}da\right] + \left[\int_{a_o}^{\bar{a}} s(a,\xi)e^{-na}da\right]}$$

We can calculate the growth rate of labor supply as:

$$\frac{\dot{l}^s(a_r(t))}{l^s(a_r(t))} = \frac{l(a_r(t))\mu(a_r(t)) \dot{a}_r(t)}{l^s(a_r(t))}$$

The effect of increasing life expectancy (increase ξ) is to reduce per capita labor supply:

$$\frac{\partial l^s}{\partial \xi} = \frac{-\left[\int_{a_y}^{a_r(t)} l(a)e^{-na}da\right] \left[\int_{a_o}^{\bar{a}} s'_\xi(a,\xi)e^{-na}da\right]}{\left[\int_0^{a_o} e^{-na}da\right] + \left[\int_{a_o}^{\bar{a}} s(a,\xi)e^{-na}da\right]}^2 < 0$$

The effect of an increase in the fertility rate on the labor supply is not clear for two reasons. First, it reduces the portion of young individuals while increasing the portion of old individuals, implying an ambiguous result on the work force, and; second, if labor productivity increases with age ($l(a)$ is an increasing function) then, the increase of the fertility rate reduces the portion of highly productive agents while increasing the portion of the less productive ones, therefore producing, again, an ambiguous result on the labor supply:

$$\begin{aligned} \frac{\partial l^s}{\partial n} &= \frac{\left[\int_{a_y}^{a_r(t)} l(a)s(a)e^{-na}da\right] \left[\int_0^{\bar{a}} a s(a)e^{-na}da\right] - \left[\int_{a_y}^{a_r(t)} a l(a)s(a)e^{-na}da\right] \left[\int_0^{\bar{a}} s(a)e^{-na}da\right]}{\left[\int_0^{\bar{a}} s(a)e^{-na}da\right]^2} = \\ &= \frac{\left[\int_{a_y}^{a_r(t)} l(a)s(a)e^{-na}da\right]}{\left[\int_0^{\bar{a}} s(a)e^{-na}da\right]} \left[\frac{\int_0^{\bar{a}} a s(a)e^{-na}da}{\int_0^{\bar{a}} s(a)e^{-na}da} - \frac{\int_{a_y}^{a_r(t)} a l(a)s(a)e^{-na}da}{\int_{a_y}^{a_r(t)} s(a)l(a)e^{-na}da} \right] = \\ &= l^s(a_r(t)) \left[\left(\int_0^{\bar{a}} a \frac{s(a)e^{-na}}{\left[\int_{a_y}^{a_r(t)} s(a)e^{-na}da\right]} da \right) - \left(\int_{a_y}^{a_r(t)} a \frac{l(a)s(a)e^{-na}}{\left[\int_{a_y}^{a_r(t)} l(a)s(a)e^{-na}da\right]} da \right) \right] \\ &= l^s(a_r(t)) (E[a] - E_\nu[a / [a_y, a_r(t)]]) \end{aligned}$$

Thus the sign of the above derivative depends on the relationship between the average age of the population, $E[a]$, and the average age of the labor supply, $E_\nu[a/[a_y, a_r(t)]]$.

B. The steady state

This appendix describes the construction of the steady state. We first calculate and study the two stable loci ($\dot{a}_r = 0$ and $\dot{\tilde{k}} = 0$) and then, we solve the resulting equations system. We prove that the solution is unique and so, there is only one steady state.

B.1. Locus $\dot{a}_r = 0$

The locus in which the retirement age remains constant is as follows:

$$\tilde{k} = \left[\frac{(1-\alpha)\alpha}{[(1-\alpha)(\delta+\rho+\gamma)+\alpha(\rho-n)]} \frac{l(a_r(t))}{\phi(a_r)[l^s(a_r(t))]^\alpha} \right]^{\frac{1}{1-\alpha}}$$

The slope of the above locus is:

$$\frac{\partial \tilde{k}}{\partial a_r} = \frac{1}{1-\alpha} \tilde{k} \left[\frac{l'(a_r(t))}{l(a_r(t))} - \frac{\phi'(a_r(t))}{\phi(a_r(t))} - \alpha \frac{\mu(a)l(a_r(t))}{l^s(a_r(t))} \right] < 0$$

given that we have assumed that $\frac{\phi(a)}{l(a)}$ is an increasing function in a .

Furthermore, we can obtain the effect of different exogenous variables:

$$\begin{aligned} \frac{\partial \tilde{k}}{\partial \xi} &= -\frac{\alpha}{1-\alpha} \tilde{k} \frac{\frac{\partial l^s(a_r(t))}{\partial \xi}}{l^s(a_r)} > 0 \\ \frac{\partial \tilde{k}}{\partial n} &= \frac{\alpha}{1-\alpha} \tilde{k} \left[-\frac{\frac{\partial l^s(a_r(t))}{\partial n}}{l^s(a_r)} + \frac{1}{[(1-\alpha)(\delta+\rho+\gamma)+\alpha(\rho-n)]} \right] \text{ ambiguous !} \\ \frac{\partial \tilde{k}}{\partial \gamma} &= \frac{\alpha}{1-\alpha} \tilde{k} \left[\frac{1}{[(1-\alpha)(\delta+\rho+\gamma)+\alpha(\rho-n)]} \right] > 0 \end{aligned}$$

B.2. Locus $\dot{\tilde{k}} = 0$:

$$\tilde{k} = l^s(a_r(t)) \left[\frac{\left[1 - (1-\alpha) \frac{l(a_r(t))}{\phi(a_r(t))l^s(a_r(t))} \right]}{(\delta+n+\gamma)} \right]^{\frac{1}{1-\alpha}}$$

The slope of the above locus is:

$$\frac{\partial \tilde{k}}{\partial a_r} = \tilde{k}^\alpha \frac{\phi(a_r(t))l(a_r(t)) [l^s(a_r(t))]^\alpha}{\delta + n + \gamma} \left[-\frac{l'(a_r(t))}{l(a_r(t))} + \frac{\phi'(a_r(t))}{\phi(a_r(t))} + \frac{\mu(a)l(a_r(t))}{l^s(a_r(t))} + \frac{\mu(a_r(t))}{[l^s(a_r(t))]^{2\alpha}} \right] > 0$$

Furthermore, we can obtain the effect of different exogenous variables:

$$\begin{aligned} \frac{\partial \tilde{k}}{\partial \xi} &= \tilde{k}^\alpha \frac{1}{(\delta + n + \gamma) [l^s(a_r(t))]^\alpha} \left[1 + \alpha \frac{l(a_r(t))}{\phi(a_r(t))l^s(a_r(t))} \right] \frac{\partial l^s(a_r(t))}{\partial \xi} < 0 \\ \frac{\partial \tilde{k}}{\partial n} &= \tilde{k}^\alpha \frac{1}{(\delta + n + \gamma) [l^s(a_r(t))]^\alpha} \left[1 + \alpha \frac{l(a_r(t))}{\phi(a_r(t))l^s(a_r(t))} \right] \frac{\partial l^s(a_r(t))}{\partial n} - \frac{\tilde{k}}{(1 - \alpha)(\delta + n + \gamma)} \text{ ambiguous !} \\ \frac{\partial \tilde{k}}{\partial \gamma} &= -\frac{1}{1 - \alpha} \frac{\tilde{k}}{(\delta + n + \gamma)} < 0 \end{aligned}$$

B.3. Steady State

The steady state is the pair $(\tilde{k}^*, a_r^*) : \dot{a}_r = 0$ and $\dot{\tilde{k}} = 0$:

Retirement age:

$$\begin{aligned} \left[\frac{(1 - \alpha)\alpha}{[(1 - \alpha)(\delta + \rho + \gamma) + \alpha(\rho - n)]} \frac{l(a_r(t))}{\phi(a_r(t)) [l^s(a_r(t))]^\alpha} \right]^{\frac{1}{1 - \alpha}} &= l^s(a_r(t)) \left[\frac{1 - (1 - \alpha) \frac{l(a_r(t))}{\phi(a_r(t))l^s(a_r(t))}}{(\delta + n + \gamma)} \right]^{\frac{1}{1 - \alpha}} \\ \frac{(1 - \alpha)\alpha(\delta + n + \gamma)}{[(1 - \alpha)(\delta + \rho + \gamma) + \alpha(\rho - n)]} \frac{l(a_r(t))}{\phi(a_r(t))l^s(a_r(t))} &= \left[1 - (1 - \alpha) \frac{l(a_r(t))}{\phi(a_r(t))l^s(a_r(t))} \right] \\ \frac{\phi(a_r^*)l^s(a_r^*)}{l(a_r^*)} &= \frac{(1 - \alpha)[\alpha(\delta + n + \gamma) + 1]}{[(1 - \alpha)(\delta + \rho + \gamma) + \alpha(\rho - n)]} \end{aligned} \quad (\text{B.1})$$

Capital:

$$\begin{aligned} \tilde{k}^* &= l^s(a_r^*) \left[\frac{[1 - (1 - 2\alpha)(\delta + \rho + \gamma) - \alpha(\rho - n)]}{(\delta + n + \gamma)[\alpha(\delta + n + \gamma) + 1]} \right]^{\frac{1}{1 - \alpha}} = \\ &\frac{(1 - \alpha)[\alpha(\delta + n + \gamma) + 1]}{[(1 - \alpha)(\delta + \rho + \gamma) + \alpha(\rho - n)]} \frac{l(a_r)}{\phi(a_r)} \left[\frac{[1 - (1 - 2\alpha)(\delta + \rho + \gamma) - \alpha(\rho - n)]}{(\delta + n + \gamma)[\alpha(\delta + n + \gamma) + 1]} \right]^{\frac{1}{1 - \alpha}} \end{aligned} \quad (\text{B.2})$$

C. Proof of proposition 6.1

- $\frac{\partial a_r^*}{\partial \xi} > 0$:

From the definition of the loci $\dot{a}_r = 0$ and $\dot{\tilde{k}} = 0$, in the steady state is verified that:

$$\tilde{k} \Big|_{\dot{a}_r=0} - \tilde{k} \Big|_{\dot{\tilde{k}}=0} = 0$$

By the Implicit Function Theorem:

$$\frac{\partial a_r^*}{\partial \xi} = - \frac{\frac{\partial \tilde{k}}{\partial a_r} \Big|_{\dot{a}_r=0} - \frac{\partial \tilde{k}}{\partial a_r} \Big|_{\dot{k}=0}}{\frac{\partial \tilde{k}}{\partial \xi} \Big|_{\dot{a}_r=0} - \frac{\partial \tilde{k}}{\partial \xi} \Big|_{\dot{k}=0}} > 0$$

since $\frac{\partial \tilde{k}}{\partial a_r} \Big|_{\dot{a}_r=0} < 0$, $\frac{\partial \tilde{k}}{\partial a_r} \Big|_{\dot{k}=0} > 0$, $\frac{\partial \tilde{k}}{\partial \xi} \Big|_{\dot{a}_r=0} > 0$, $\frac{\partial \tilde{k}}{\partial \xi} \Big|_{\dot{k}=0} < 0$ (see the definition of the steady state in Appendix B for the calculations of the derivatives).

- $\frac{\partial \tilde{k}^*}{\partial \xi} < 0$:

From eq. B.2 it is easy to see that:

$$\frac{\partial \tilde{k}^*}{\partial \xi} = \left[\frac{[1 - (1 - 2\alpha)(\delta + \rho + \gamma) - \alpha(\rho - n)]}{(\delta + n + \gamma)[\alpha(\delta + n + \gamma) + 1]} \right]^{\frac{1}{1-\alpha}} \frac{\partial l^s(a_r^*)}{\partial \xi}$$

then, using the definition of the retirement age at the steady state (eq. B.1) and the Implicit Function Theorem we get:

$$\frac{\partial l^s(a_r^*)}{\partial \xi} = \frac{(1 - \alpha)[\alpha(\delta + n + \gamma) + 1]}{[(1 - \alpha)(\delta + \rho + \gamma) + \alpha(\rho - n)]} \frac{l(a_r^*)}{\phi(a_r^*)} \left[\frac{l'(a_r)}{l(a_r)} - \frac{\phi'(a_r)}{\phi(a_r)} \right] \frac{\partial a_r^*}{\partial \xi} < 0$$

since $\frac{\partial a_r^*}{\partial \xi} > 0$ as we proved above. Therefore, $\frac{\partial \tilde{k}^*}{\partial \xi} < 0$.

D. Proof of proposition 6.2

From Appendix A we obtain that:

$$\begin{aligned} \frac{\partial l^s}{\partial n} &= l^s(a_r(t)) \left[\left(\int_0^{\bar{a}} a \frac{s(a)e^{-na}}{\left[\int_{a_y}^{a_r(t)} s(a)e^{-na} da \right]} da \right) - \left(\int_{a_y}^{a_r(t)} a \frac{l(a)s(a)e^{-na}}{\left[\int_{a_y}^{a_r(t)} l(a)s(a)e^{-na} da \right]} da \right) \right] = \\ &= l^s(a_r(t)) (E[a] - E_\nu[a/[a_y, a_r(t)]]) \end{aligned}$$

Therefore, if $E_\nu[a/[a_y, a_r(t)]] > E[a]$ then $\frac{\partial l^s}{\partial n} < 0$. ■

E. Proof of corollary 6.3

From Appendix A we obtain that:

$$\begin{aligned} \frac{\partial l^s}{\partial n} &= l^s(a_r(t)) (E[a] - E_\nu[a/[a_y, a_r(t)]]) = \\ &= l^s(a_r(t)) (E[a] - E[a/[a_y, a_r(t)]] + E[a/[a_y, a_r(t)]] - E_\nu[a/[a_y, a_r(t)]]) \end{aligned}$$

By assumption of the corollary $E[a/[a_y, a_r]] - E[a] > 0$, thus to prove the corollary it is only required to prove that $E_\nu[a/[a_y, a_r]] - E[a/[a_y, a_r]] \geq 0$.

$$\begin{aligned}
E_\nu [a / [a_y, a_r]] - E [a / [a_y, a_r]] &= \frac{\int_{a_y}^{a_r(t)} a l(a) \mu(a) da}{\int_{a_y}^{a_r(t)} l(a) \mu(a) da} - \frac{\int_{a_y}^{a_r(t)} a \mu(a) da}{\int_{a_y}^{a_r(t)} \mu(a) da} = \\
&= \frac{\int_{a_y}^{a_r(t)} a l(a) \mu(a) da - \frac{\int_{a_y}^{a_r(t)} l(a) \mu(a) da}{\int_{a_y}^{a_r(t)} \mu(a) da} \int_{a_y}^{a_r(t)} a \mu(a) da}{\int_{a_y}^{a_r(t)} l(a) \mu(a) da} = \tag{E.1}
\end{aligned}$$

$$\begin{aligned}
&= \frac{\int_{a_y}^{a_r(t)} a l(a) \mu(a) da - E [l(a) / [a_y, a_r]] \int_{a_y}^{a_r(t)} a \mu(a) da}{\int_{a_y}^{a_r(t)} l(a) \mu(a) da} = \\
&= \frac{1}{\int_{a_y}^{a_r(t)} l(a) \mu(a) da} \left[\int_{a_y}^{a_r(t)} a (l(a) - E [l(a) / [a_y, a_r]]) \mu(a) da \right] \tag{E.2}
\end{aligned}$$

It follows from Weierstrass Theorem and continuity of $l(a)$ that $l(a)$ reaches a minimum in the interval $[a_y, a_r(t)]$. Since the function $l(a)$ is quasi-concave, the function reaches a minimum either at a_y or at a_r :

$$l(a) = l(\lambda(a)l(a_y) + (1 - \lambda(a))l(a_r)) \geq \min \{l(a_y), l(a_r)\} \quad \forall a \in (a_y, a_r)$$

where $\lambda(a) = \frac{a_r - a}{a_r - a_y}$. Since by assumption $l(a_r) > E [l(a) / [a_y, a_r]]$, the minimum is reached at a_y . Thus, $a_y < E [l(a) / [a_y, a_r]]$. Since $l(\cdot)$ is continuous, there is $\tilde{a} \in (a_y, a_r)$ such that $l(\tilde{a}) = E [l(a) / [a_y, a_r]]$ and $l(a) < E [l(a) / [a_y, a_r]] \quad \forall a \in [a_y, \tilde{a})$. Furthermore it follows from quasi-concavity that:

$$l(a) = l(\tilde{\lambda}(a)l(\tilde{a}) + (1 - \tilde{\lambda}(a))l(a_r)) \geq \min \{l(\tilde{a}), l(a_r)\} = E [l(a) / [a_y, a_r]] \quad \forall a \in (\tilde{a}, a_r] \tag{E.3}$$

where $\tilde{\lambda}(a) = \frac{a_r - a}{a_r - \tilde{a}}$.

Using equation (E.2) yields:

$$\begin{aligned}
&\int_{a_y}^{a_r(t)} a (l(a) - E [l(a) / [a_y, a_r]]) \mu(a) da = \\
&\int_{a_y}^{a_r(t)} a (l(a) - E [l(a) / [a_y, a_r]]) \mu(a) da - \tilde{a} \int_{a_y}^{a_r(t)} \mu(a) da \underbrace{\left[\frac{\int_{a_y}^{a_r(t)} l(a) \mu(a) da}{\int_{a_y}^{a_r(t)} \mu(a) da} - E [l(a) / [a_y, a_r]] \right]}_0 = \\
&\int_{a_y}^{a_r(t)} a (l(a) - E [l(a) / [a_y, a_r]]) \mu(a) da - \tilde{a} \left[\int_{a_y}^{a_r(t)} l(a) \mu(a) da - E [l(a) / [a_y, a_r]] \int_{a_y}^{a_r(t)} \mu(a) da \right] =
\end{aligned}$$

$$\begin{aligned}
& \int_{a_y}^{a_r(t)} a (l(a) - E[l(a)/[a_y, a_r]]) \mu(a) da - \tilde{a} \left[\int_{a_y}^{a_r(t)} (l(a) - E[l(a)/[a_y, a_r]]) \mu(a) da \right] \\
& \int_{a_y}^{a_r(t)} (a - \tilde{a}) (l(a) - E[l(a)/[a_y, a_r]]) \mu(a) da = \\
& \int_{a_y}^{\tilde{a}} \underbrace{(a - \tilde{a})}_{\ominus} \underbrace{(l(a) - E[l(a)/[a_y, a_r]])}_{\ominus} \mu(a) da + \int_{\tilde{a}}^{a_r(t)} \underbrace{(a - \tilde{a})}_{\oplus} \underbrace{(l(a) - E[l(a)/[a_y, a_r]])}_{\oplus} \mu(a) da > 0
\end{aligned}$$

where in the first equality we use the definition of $E[l(a)/[a_y, a_r]]$, and in the last equality we use the definition of $\tilde{a}$ and equation (E.3).

F. Proof of proposition 6.4

- $\frac{\partial a_r^*}{\partial n} > 0$:

From the definition of the loci $\dot{a}_r = 0$ and $\dot{\tilde{k}} = 0$, in the steady state is verified that:

$$\tilde{k} \Big|_{\dot{a}_r=0} - \tilde{k} \Big|_{\dot{\tilde{k}}=0} = 0$$

By the Implicit Function Theorem:

$$\frac{\partial a_r^*}{\partial n} = - \frac{\frac{\partial \tilde{k}}{\partial a_r} \Big|_{\dot{a}_r=0} - \frac{\partial \tilde{k}}{\partial a_r} \Big|_{\dot{\tilde{k}}=0}}{\frac{\partial \tilde{k}}{\partial n} \Big|_{\dot{a}_r=0} - \frac{\partial \tilde{k}}{\partial n} \Big|_{\dot{\tilde{k}}=0}} > 0$$

since $\frac{\partial \tilde{k}}{\partial a_r} \Big|_{\dot{a}_r=0} < 0$, $-\frac{\partial \tilde{k}}{\partial a_r} \Big|_{\dot{\tilde{k}}=0} > 0$, $\frac{\partial \tilde{k}}{\partial n} \Big|_{\dot{a}_r=0} > 0$ thereby $\frac{\partial l^s(a_r^*)}{\partial n} < 0$ and finally, $\frac{\partial \tilde{k}}{\partial n} \Big|_{\dot{\tilde{k}}=0} < 0$ thereby $\frac{\partial l^s(a_r^*)}{\partial n} < 0$ (see the definition of the steady state in Appendix B for the calculations of the derivatives).

- $\frac{\partial \tilde{k}^*}{\partial n} < 0$:

From Figure 1 it is easy to see that

$$\text{if } \left| \frac{\partial \tilde{k}}{\partial n}(\tilde{k}^*) \right|_{\dot{\tilde{k}}=0} > \frac{\partial \tilde{k}}{\partial n}(\tilde{k}^*) \Big|_{\dot{a}_r=0} \text{ then } \frac{\partial \tilde{k}^*}{\partial n} < 0$$

We now proceed to prove that $\left| \frac{\partial \tilde{k}}{\partial n}(\tilde{k}^*) \right|_{\dot{\tilde{k}}=0} > \frac{\partial \tilde{k}}{\partial n}(\tilde{k}^*) \Big|_{\dot{a}_r=0}$. Since $\frac{\partial l^s(a_r)}{\partial n} < 0$ we can rewrite it as

$$\begin{aligned}
& \left| \tilde{k}^\alpha \frac{1}{(\delta + n + \gamma) [l^s(a_r(t))]^\alpha} \left[1 + \alpha \frac{l(a_r(t))}{\phi(a_r(t)) l^s(a_r(t))} \right] \frac{\partial l^s(a_r(t))}{\partial n} \right| + \frac{\tilde{k}}{(1 - \alpha)(\delta + n + \gamma)} > \\
& \frac{\alpha}{1 - \alpha} \tilde{k} \left[- \frac{\frac{\partial l^s(a_r(t))}{\partial n}}{l^s(a_r)} + \frac{1}{[(1 - \alpha)(\delta + \rho + \gamma) + \alpha(\rho - n)]} \right]
\end{aligned}$$

$$\begin{aligned}
& \underbrace{\left| \tilde{k}^\alpha \frac{1}{(\delta + n + \gamma) [l^s(a_r(t))]^\alpha} \left[1 + \alpha \frac{l(a_r(t))}{\phi(a_r(t)) l^s(a_r(t))} \right] \frac{\partial l^s(a_r(t))}{\partial n} \right| - \frac{\alpha}{1 - \alpha} \tilde{k} \frac{\frac{\partial l^s(a_r(t))}{\partial n}}{l^s(a_r(t))}}_A + \\
& \underbrace{\frac{\tilde{k}}{(1 - \alpha)(\delta + n + \gamma)} - \frac{\alpha}{1 - \alpha} \frac{\tilde{k}}{[(1 - \alpha)(\delta + \rho + \gamma) + \alpha(\rho - n)]}}_B > 0
\end{aligned}$$

The strategy we follow is to prove that $A > 0$ and $B > 0$. We start with $B > 0$:

$$\frac{\tilde{k}}{(1 - \alpha)(\delta + n + \gamma)} - \frac{\alpha}{1 - \alpha} \frac{\tilde{k}}{[(1 - \alpha)(\delta + \rho + \gamma) + \alpha(\rho - n)]} > 0$$

Then,

$$\delta + \rho + \gamma > 2\alpha(\delta + n + \gamma)$$

and this is true since $\rho > n$ and $\alpha < \frac{1}{2}$.

Now we prove that $A > 0$:

$$\begin{aligned}
& \left| \tilde{k}^\alpha \frac{1}{(\delta + n + \gamma) [l^s(a_r(t))]^\alpha} \left[1 + \alpha \frac{l(a_r(t))}{\phi(a_r(t)) l^s(a_r(t))} \right] \frac{\partial l^s(a_r(t))}{\partial n} \right| - \frac{\alpha}{1 - \alpha} \tilde{k} \frac{\frac{\partial l^s(a_r(t))}{\partial n}}{l^s(a_r(t))} > 0 \\
& \frac{1 - \alpha}{\alpha(\delta + n + \gamma)} \left[1 + \alpha \frac{l(a_r(t))}{\phi(a_r(t)) l^s(a_r(t))} \right] > \frac{\tilde{k}^{1-\alpha}}{[l^s(a_r(t))]^{1-\alpha}} \\
& l^s(a_r(t)) \left(\frac{1 - \alpha}{\alpha(\delta + n + \gamma)} \left[1 + \alpha \frac{l(a_r(t))}{\phi(a_r(t)) l^s(a_r(t))} \right] \right)^{\frac{1}{1-\alpha}} > \tilde{k}
\end{aligned}$$

Using the definition of capital at the steady state (eq. B.2)

$$\begin{aligned}
& l^s(a_r(t)) \left(\frac{1 - \alpha}{\alpha(\delta + n + \gamma)} \left[1 + \alpha \frac{l(a_r(t))}{\phi(a_r(t)) l^s(a_r(t))} \right] \right)^{\frac{1}{1-\alpha}} > l^s(a_r) \left[\frac{[1 - (1 - 2\alpha)(\delta + \rho + \gamma) - \alpha(\rho - n)]}{(\delta + n + \gamma) [\alpha(\delta + n + \gamma) + 1]} \right]^{\frac{1}{1-\alpha}} \\
& \frac{1 - \alpha}{\alpha(\delta + n + \gamma)} \left[1 + \alpha \frac{l(a_r(t))}{\phi(a_r(t)) l^s(a_r(t))} \right] > \frac{[1 - (1 - 2\alpha)(\delta + \rho + \gamma) - \alpha(\rho - n)]}{(\delta + n + \gamma) [\alpha(\delta + n + \gamma) + 1]}
\end{aligned}$$

Using the expression B.1

$$\begin{aligned}
& \frac{1 - \alpha}{\alpha} \left[1 + \alpha \frac{[(1 - \alpha)(\delta + \rho + \gamma) + \alpha(\rho - n)]}{(1 - \alpha) [\alpha(\delta + n + \gamma) + 1]} \right] > \frac{[1 - (1 - 2\alpha)(\delta + \rho + \gamma) - \alpha(\rho - n)]}{[\alpha(\delta + n + \gamma) + 1]} \\
& \frac{1 - \alpha}{\alpha} + 2[(1 - \alpha)(\delta + \rho + \gamma) + \alpha(\rho - n)] > 1 - (1 - 2\alpha)(\delta + \rho + \gamma) \\
& \frac{1 - 2\alpha}{\alpha} + 2[(1 - \alpha)(\delta + \rho + \gamma) + \alpha(\rho - n)] > -(1 - 2\alpha)(\delta + \rho + \gamma)
\end{aligned}$$

and this is true for $\alpha < \frac{1}{2}$.

- $\frac{\partial l^s(a_r^*)}{\partial n} < 0$:

In subsection 2.2 the per capita income is defined as $y(t) = \Gamma(t)^{1-\alpha} k(t)^\alpha l(t)^{1-\alpha}$. Thus, the per capita income growth rate is: $\frac{\dot{y}(t)}{y(t)} = (1-\alpha) \frac{\dot{\Gamma}(t)}{\Gamma(t)} + \alpha \frac{\dot{k}(t)}{k(t)} + (1-\alpha) \frac{\dot{l}(t)}{l(t)}$. If we evaluate it in the steady state we get: $\frac{\dot{y}(t)}{y(t)} = \gamma$, since $\dot{\tilde{k}} = 0$ which implies $\frac{\dot{k}(t)}{k(t)} = \gamma$ and, $\dot{a}_r = 0$ which implies $\frac{\dot{l}(t)}{l(t)} = 0$ (see Appendix A).

From eq. 2.7 we get $c(t) = y(t) - \dot{k}(t) - (\delta + n)k(t)$. If we evaluate the per capita consumption at the steady state we get $\frac{c(t)}{k(t)} = \frac{y(t)}{k(t)} - (\gamma + \delta + n)$. It is easy to see that $\frac{c(t)}{c(t)} = \frac{k(t)}{k(t)} = \gamma$, since $\frac{\dot{y}(t)}{y(t)} = \frac{\dot{k}(t)}{k(t)} = \gamma$ which implies that the rate $\frac{y(t)}{k(t)}$ is constant and, thus the rate $\frac{c(t)}{k(t)}$ is constant as well.

Therefore, eq. 3.4 can be rewritten as: $r(t) = \gamma + \rho$. If we substitute $r(t)$ defined by eq. ??, then we obtain:

$$\alpha \left(\frac{\Gamma(t)l(t)}{k(t)} \right)^{1-\alpha} = \gamma + \rho + \delta$$

$$\alpha \left(\frac{l(t)}{\tilde{k}(t)} \right)^{1-\alpha} = \gamma + \rho + \delta$$

Thus,

$$l^s(a_r^*) = \left(\frac{\gamma + \rho + \delta}{\alpha} \right)^{\frac{1}{1-\alpha}} \tilde{k}^*$$

Therefore, it is easy to see that:

$$\frac{\partial l^s(a_r^*)}{\partial n} = \left(\frac{\alpha}{\gamma + \rho + \delta} \right)^{\frac{1}{1-\alpha}} \frac{\partial \tilde{k}^*}{\partial n} < 0$$

since $\frac{\partial \tilde{k}^*}{\partial n} < 0$.

G. Proof of proposition 7.1

- $\frac{\partial a_r^*}{\partial \gamma} > 0$:

From the definition of the loci $\dot{a}_r = 0$ and $\dot{\tilde{k}} = 0$, in the steady state is verified that:

$$\tilde{k} \Big|_{\dot{a}_r=0} - \tilde{k} \Big|_{\dot{\tilde{k}}=0} = 0$$

By the Implicit Function Theorem:

$$\frac{\partial a_r^*}{\partial \gamma} = - \frac{\frac{\partial \tilde{k}}{\partial a_r} \Big|_{\dot{a}_r=0} - \frac{\partial \tilde{k}}{\partial a_r} \Big|_{\dot{\tilde{k}}=0}}{\frac{\partial \tilde{k}}{\partial \gamma} \Big|_{\dot{a}_r=0} - \frac{\partial \tilde{k}}{\partial \gamma} \Big|_{\dot{\tilde{k}}=0}} > 0$$

since $\frac{\partial \tilde{k}}{\partial a_r} \Big|_{\dot{a}_r=0} < 0$, $\frac{\partial \tilde{k}}{\partial a_r} \Big|_{\dot{k}=0} > 0$, $\frac{\partial \tilde{k}}{\partial \gamma} \Big|_{\dot{a}_r=0} > 0$, $\frac{\partial \tilde{k}}{\partial \gamma} \Big|_{\dot{k}=0} < 0$ (see the definition of the steady state in Appendix B for the calculations of the derivatives).

- $\frac{\partial \tilde{k}^*}{\partial \gamma} < 0$:

From Figure 1 it is easy to see that

$$\text{if } \left| \frac{\partial \tilde{k}}{\partial \gamma}(\tilde{k}^*) \Big|_{\dot{k}=0} \right| > \left| \frac{\partial \tilde{k}}{\partial \gamma}(\tilde{k}^*) \Big|_{\dot{a}_r=0} \right| \text{ then } \frac{\partial \tilde{k}^*}{\partial \gamma} < 0$$

We now proceed to prove that $\left| \frac{\partial \tilde{k}}{\partial \gamma}(\tilde{k}^*) \Big|_{\dot{k}=0} \right| > \left| \frac{\partial \tilde{k}}{\partial \gamma}(\tilde{k}^*) \Big|_{\dot{a}_r=0} \right|$. Since $\frac{\partial l^s(a_r)}{\partial \gamma} < 0$ we can rewrite it as

$$\frac{1}{1-\alpha} \frac{\tilde{k}}{(\delta+n+\gamma)} > \frac{\alpha}{1-\alpha} \tilde{k} \left[\frac{1}{[(1-\alpha)(\delta+\rho+\gamma) + \alpha(\rho-n)]} \right]$$

simplifying:

$$(1-\alpha)(\delta+\rho+\gamma) + \alpha(\rho-n) > \alpha(\delta+n+\gamma)$$

and this is true since $\rho > n$ and $\alpha < \frac{1}{2}$.

- $\frac{\partial l^s(a_r^*)}{\partial \gamma} > 0$:

From the definition of $l^s(\cdot)$ in Appendix A it is easy to see that:

$$\frac{\partial l^s(a_r^*)}{\partial n} = \frac{\partial l^s(a_r^*)}{\partial a_r^*} \frac{\partial a_r^*}{\partial \gamma} = l(a_r^*) \mu(a_r^*) \frac{\partial a_r^*}{\partial \gamma} > 0$$

since $\frac{\partial a_r^*}{\partial \gamma} > 0$ as we proved in the first part of the proposition.

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