

A Long-Run, Short-Run and Politico-Economic Analysis of the Welfare Costs of Inflation*

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Abstract

This paper assesses the long-run and short-run (i.e. along the transition path) welfare implications of permanent changes in inflation in an environment with essential money and perfectly competitive markets. The model delivers a monetary distribution that matches moments of the distribution seen in the US data. Although there is potential for wealth redistribution to deliver welfare gains from inflation, the (total) costs of 10 percent inflation relative to zero is over 7 percent of consumption. While these results suggest a dominating real-balance effect of inflation, a politico-economic analysis concludes that the prevailing (majority rule) inflation rate is above the Friedman Rule.

Keywords: Inflation; Welfare; Transitions; Voting

JEL: E40; E50

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“Indeed, most central banks around the world aim to set inflation above zero, usually at about two percent.”

- Federal Reserve Chairman Ben Bernanke, April 27, 2011

1. Introduction

While it is well known that many central banks around the world maintain long-run inflation targets above zero, the welfare implications of following such policies are not fully understood. The goal of this paper is to quantitatively assess the welfare costs of inflation in an environment which delivers: (i) a micro-founded rationale for holding money balances; and (ii) a nondegenerate distribution of money holdings across agents that shares properties with the available monetary data in the US.

Since Lucas (2000), quantifying the welfare implications of alternative monetary policies has been a prominent issue in monetary economics.¹ Subsequent papers such as Lagos and Wright (2005) stress the importance of assessing these welfare effects (among other issues) in an environment with sufficient microfoundations making money essential. One common formalization of these microfoundations is through search-theoretic monetary environments with bilateral matching and bargaining. While Lagos and Wright (2005) and others studied the welfare effects in environments with simplifying assumptions that deliver a degenerate monetary distribution across agents, Molico (2006) and Chiu and Molico (2011) have relaxed these assumptions and assessed the distributional welfare effects of inflation.² In a related study, Dressler (2011) avoids the computational burden associated with bilateral bargaining by assuming that agents meet multilaterally in a centralized Walrasian market and take a

¹The literature which theoretically maps out the welfare implications of inflation stems as far back as Bailey (1956) and Friedman (1969).

²Having money be essential in an environment generally results in the set of allocations supported with money being larger (and possibly better) than without. Other examples that allow for a nondegenerate monetary distribution in a search-theoretic environment are Green and Zhou (1998, 2002), Camera and Corbae (1999), Zhou (1999), and Zhu (2003, 2005).

competitively determined price as given.³ While Molico (2006), Chiu and Molico (2011) and Dressler (2011) have shown that interesting policy analyses can be performed in models with essential money delivering a nondegenerate distribution of money holdings across agents, none of these models deliver monetary distributions that match relevant properties of US data.

Assessing the welfare implications of inflation in a model that delivers an empirically plausible monetary distribution is important because there are two off-setting effects of inflation. First, the well known *real-balance effect* implies that inflation is always costly in terms of welfare by serving as a distortionary tax on money holdings. However, there is also a potentially welfare improving, *redistributive effect* that arises from the transfer of liquidity from those with excess liquidity to those that are liquidity constrained. In other words, an increase in inflation serves as a tax (subsidy) to households with above (below) average money holdings. The potential relevance of this second effect can be supported by the empirical distribution of money holdings across US households taken from the 2004 Survey of Consumer Finances (SCF). Figure 1 illustrates the distribution of checking accounts across households from the SCF normalized such that average holdings are equal to one. It is clear from the figure that a large majority of households are holding below average balances, and the resulting median-mean ratio is 0.44.⁴ Therefore, given that the majority of agents can benefit from the redistributive effect of expansionary monetary policies, previous analyses that do not take the empirical distribution into account are potentially ignoring a large portion of the full welfare implications of inflation.⁵

The model studied in this paper extends Dressler (2011) in order to deliver a monetary distribution more in line with the data. Similar to Dressler (2011), agents are allowed to

³This analysis could be viewed as a quantitative extension of Bewley (1980) and Lucas (1980).

⁴The SCF also contains data on all transactions accounts (money market, checking, saving and call accounts). The characteristics of this distribution, the Gini coefficient and median-mean ratio in particular, are surprising similar to the distribution described here.

⁵Welfare benefits of positive inflation due to substantial wealth redistribution for large and moderate inflations have been documented by Doepke and Schneider (2006a and 2006b).

trade multilaterally in a Walrasian market at a competitively determined price.⁶ However, while Dressler (2011) assumes agents face idiosyncratic consumption and production shocks concurrently in order to mimic a search-theoretic environment, the model presented here assumes all agents consume and produce a nonstorable good in every period while some receive a noninsurable preference shock. The shared ability of all agents to produce in every period reduces the liquidity demand for money, but money is still essential and held for self-insurance in the event that an agent receives a shock and would rather purchase consumption than produce it. The two degrees of freedom in the model are the size of this shock and the portion of the population receiving it. These degrees of freedom are used to calibrate the model in order to match a measure of monetary demand used previously in other analyses (i.e. velocity), as well as the median-mean ratio found in the SCF data.

The long-run, average welfare implications of alternative monetary policies are calculated as in previous analyses by comparing the stationary (steady state) monetary distributions from the model under several inflation rates with the zero inflation steady state. Although the distributions display a potential for significant welfare gains from expansionary monetary policy, the welfare costs of inflation are much larger than what has been previously reported. In particular, the costs of 10 percent inflation relative to zero percent is calculated to be 5.10 percent of consumption. This cost is larger than the maximum welfare cost calculated by Lagos and Wright (2005) for various degrees of bargaining power, and much larger than the calculations of Lucas (2000) using a representative-agent model (one percent of income) and of Chiu and Molico (2011) using a variant of the Lagos-Wright model (0.59 percent of consumption).⁷ It is shown through a decomposition of the welfare costs that although the redistributive effect is in operation and does deliver benefits, the costs delivered by the real-balance effect are upwards of 10 times larger. One reason why the real-balance effect is so

⁶Rocheteau and Wright (2005) consider competitive pricing in a version of the Lagos-Wright model, but maintain the assumptions delivering a degenerate monetary distribution.

⁷Lagos and Wright (2005) calculate the welfare gain of going from 10 percent inflation to 0 to be between 3 and 5 percent of consumption, depending on the bargaining power of the buyers and sellers within a pairwise match. The welfare costs of Chiu and Molico (2011) should be viewed as a lower bound since they assume that buyers receive all the surplus from bargaining.

large is that inflation hurts buyers significantly more than sellers, and the model calibration results in a buyer-seller ratio of over 2. Therefore, even though the model captures a key moment of the empirical monetary distribution, the redistributive effect of expansionary monetary policies appear small from an aggregate welfare perspective.

The long-run welfare analysis is extended by assessing the short-run (transitional) welfare implications. In particular, short-run dynamics of the economy are calculated as the economy transitions from a zero-inflation steady state to a nonzero (permanent) inflation steady state. The average welfare costs are calculated by comparing the transition to the new steady state against the alternative of remaining at zero inflation. The results from this exercise essentially mimic the long-run analysis, insofar that transitions to positive (negative) inflation result in average welfare losses (gains). However, these costs can again be large. For example, the welfare costs of *transitioning* to 10 percent inflation relative to staying at zero percent is 2.25 percent of consumption, and this transition takes only five (annual) periods. This implies that the total welfare costs of 10 percent inflation, taking the short and long run together, can be as large as 7.35 percent of consumption.

Despite the relatively small redistributive effect on average welfare in both the long and short runs, there still could be significant welfare implications at the individual level. A final exercise adds a politico-economic component to the model and determines which inflation rate would prevail if the agents were allowed to vote. The value of this exercise lies in the fact that although the welfare maximizing inflation rate in this environment from a planner's perspective is the Friedman Rule (one minus the inverse of the discount rate), this inflation rate may not be in the best interest of a majority of the agents due to the redistributive effect. The steady-state politico-economic recursive competitive equilibrium is calculated using the method of Corbae et al. (2009), and the prevailing inflation rate turns out to be -3.0 percent while the calibrated Friedman Rule is -4.16 percent. The voting outcome being deflation is yet another result of the dominant real-balance effect in the model. However, the fact that the voting outcome is greater than the Friedman Rule suggests that the overall welfare

maximizing inflation rate is not optimal for a majority of the households. This prevailing inflation rate above the Friedman Rule is a significant result of the redistributive effect.

In addition to the monetary literature cited above, this analysis is related to the literature considering politico-economic environments and the inequality effects of inflation. Bhattacharya et al. (2001 and 2005) find a non-monotonic relationship between inflation and inequality in an over-lapping generations framework. Bullard and Waller (2004) find inflationary biases when a central bank applies a majority voting rule. Albanesi (2007) studies the distribution impact of inflation in a model where rich and poor agents Nash-bargain over the inflation rate as opposed to voting. Other studies which consider the inequality effects of inflation without voting are Imrohoroglu (1992) and Erosa and Ventura (2002). These endowment economies use cash-in-advance constraints and assess money's role as a store of value and a medium of exchange, respectively. Finally, Hulagu (2011) extends Imrohoroglu (1992) to include idiosyncratic labor productivity and applies the same voting solution used here. His model predicts a steady-state voting outcome of 1.1 percent, which exceeds the -3.0 percent predicted in this analysis. Several potential reasons for this higher outcome are that money is not essential in his model, he assumes labor preferences that ignore wealth effects, and his model is calibrated to non-monetary features of the data. Nonetheless, these analyses together show that an inflation rate above the Friedman Rule prevails in a variety of politico-economic settings.

The remainder of the paper proceeds as follows. Section 2 describes the model and defines equilibrium. Section 3 describes the parameterization of the environment and presents the quantitative results. Section 4 concludes.

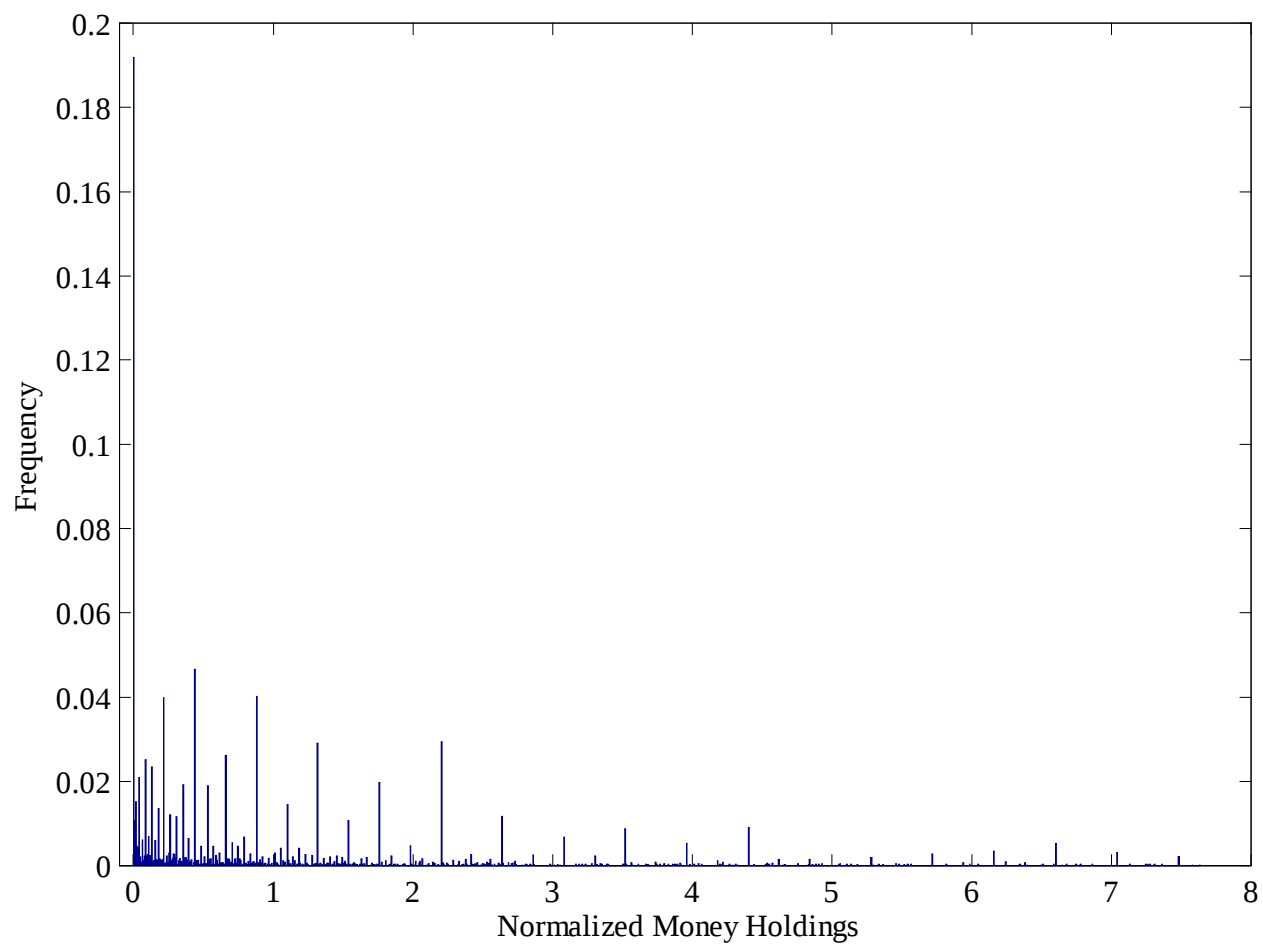


Figure 1: Distribution of (normalized) checking accounts, 2004 SCF data.

2. Model

2.1. Environment

There exists a unit measure of infinitely lived households. Their preferences are given by

$$E \left[\sum_{t=0}^{\infty} \beta^t u(x_t, y_t, e_t) \right] \quad (1)$$

where x_t and y_t denote consumption and production of a perfectly divisible and nonstorable good in period t , $\beta \in (0, 1)$ is the discount factor, and $e_t \in E$ is an uninsurable, idiosyncratic preference shock which evolves according to a finite state Markov process $\Pi(e_{t+1}|e_t)$. Let the set be $E = \{b, s\}$ where $b(s)$ denotes a relatively high (low) preference shock. The period utility function is assumed to be

$$u(x_t, y_t, e_t) = \frac{e_t x_t^{1-\sigma}}{1-\sigma} - \frac{y_t^{1+1/\gamma}}{1+1/\gamma} \quad (2)$$

where σ is the coefficient of relative risk aversion and γ is the intertemporal (Frisch) elasticity of labor supply.⁸

There exists a stock of fiat money that is perfectly divisible, costlessly storable, and unable to be produced or consumed by any private individual. Let $\hat{M}_t$ denote the stock of money available at the beginning of period t . The law of motion for the money stock is given by $\hat{M}_{t+1} = (1 + \mu_t) \hat{M}_t$ where μ_t is the growth rate of the money stock in period t . Agents can hold any nonnegative amount of money ($\hat{m}_t \in \mathbb{R}_+$), and new money is injected into the environment via identical lump-sum transfers τ_t to all agents at the beginning of the period.

After receiving their shocks and transfers, all agents are granted access to a centralized market where they are permitted to buy and/or sell consumption goods at a competitively determined price $\hat{P}_t > 0$. Although there is a single consumption good that can be produced by all agents with equal efficiency, type $b(s)$ agents may wish to consume (produce) more

⁸Production is assumed to be one-for-one with inputted labor, so the disutility of production y_t appears in place of labor supply.

than they wish to produce (consume). In addition, all agents are assumed to be anonymous, which precludes the availability of credit and generates an essential role for money as a medium of exchange.⁹

It should be noted that having buyers produce and sellers consume in the model are motivated by the need to have an amount of consumption be *nonmonetary*. In other words, the consumption of a seller and the production of a buyer can be interpreted as either consumption of their own production or simple one-for-one trades with other agents.¹⁰ Since these exchanges do not involve money, it delivers enough flexibility for the model to be calibrated to match monetary velocity while delivering a distribution with a mass of agents holding near zero money balances.¹¹

2.2. Recursive competitive equilibrium

The environment is rendered stationary by normalizing all nominal variables by the beginning of period stock of money (e.g. $m_t = \hat{m}_t / \hat{M}_t$ and $M_t = \hat{M}_t / \hat{M}_t = 1$). Let the joint distribution of money holdings and preference shocks across agents be denoted $\Gamma_t(m_t, e_t)$ with law of motion $\Gamma_{t+1} = H(\Gamma_t, \mu_t)$. The aggregate money stock is given by

$$M_t = \int m_t d\Gamma_t(m_t, e_t) = 1 \quad (3)$$

and aggregate consumption and production are given by

$$X_t = \int x_t d\Gamma_t(m_t, e_t) \text{ and } Y_t = \int y_t d\Gamma_t(m_t, e_t).$$

⁹These conditions are maintained by Levine (1991), Kotcherlakota (1998), and Rocheteau and Wright (2005).

¹⁰One can argue that the same types of exchanges take place in the centralized markets of search-theoretic monetary models (i.e. Lagos and Wright, 2005).

¹¹If all consumption was purchased with currency, then the inverse relationship between prices and output (given a fixed quantity of money) makes matching velocity very difficult (see Molico, 2006). If buyers could not produce for themselves, then all agents would hold larger money balances for self-insurance and the resulting distribution would not resemble the data.

Letting x and x' denote x_t and x_{t+1} , respectively, the household problem can be stated recursively as

$$V(m, e; \Gamma, \mu) = \max_{x, y, m'} u(x, y, e) + \beta \sum_{e'} \Pi(e'|e) V(m', e'; \Gamma', \mu') \quad (4)$$

s.t.

$$\frac{m + \mu}{1 + \mu} + P(y - x) \geq m'$$

$$x, y, m' \geq 0$$

$$\Gamma' = H(\Gamma, \mu)$$

$$\mu' = \Psi(\Gamma, \mu)$$

where the perceived law of motion for the money growth rate is given by $\mu_{t+1} = \Psi(\Gamma_t, \mu_t)$.

The solution to the household's problem generates decision rules which are denoted by

$$x = \eta(m, e; \Gamma, \mu), \quad y = g(m, e; \Gamma, \mu), \quad \text{and} \quad m' = h(m, e; \Gamma, \mu).$$

Definition: Given $\Psi(\Gamma, \mu)$, a *recursive competitive equilibrium* (RCE) is a set of functions $\{V, \eta, g, h, H, P\}$ such that:

1. Given (Γ, μ, H, Ψ) , the functions $V(\cdot)$, $\eta(\cdot)$, $g(\cdot)$, and $h(\cdot)$ solve the household's problem in (4).
2. The aggregate resource constraint is satisfied

$$X = \int x d\Gamma(m, e) = \int y d\Gamma(m, e) = Y$$

3. Prices are competitively determined such that the markets for goods (condition 2) and money holdings (3) clear.

4. The law of motion for money is satisfied.

5. $H(\Gamma, \mu)$ is given by

$$\Gamma'(m', e') = \int 1_{\{h(m, e; \Gamma, \mu) = m'\}} \Pi(e'|e) d\Gamma(m, e)$$

2.3. Politico-economic recursive competitive equilibrium

Given the definition of a RCE, the money growth rate can be made endogenous via voting. In particular, households are allowed to vote on μ' . Given that households are rational, voters evaluate the equilibrium effects of their choices, calculate the expected discounted utility associated with each μ' , and choose the μ' which gives them highest utility.¹² Since the source of household heterogeneity arises from idiosyncratic shocks to preferences, the median voter is not as readily identified as in models which assume nonstochastic shocks (e.g. Krusell and Ríos-Rull, 1999). Therefore, the analysis follows Corbae et al. (2009) and determines the median of the distribution of ‘most preferred’ inflation rates among households as the winning outcome.¹³

In order to choose the most preferred inflation rate, a household must choose among alternatives. Suppose a household with state vector $(m, e; \Gamma, \mu)$ were to consider a one-period deviation for next period’s money growth rate μ' not necessarily given by $\Psi(\Gamma, \mu)$ while taking as given that all future money growth choices will be given by Ψ . The household’s problem would then be given by

$$\tilde{V}(m, e; \Gamma, \mu, \mu') = \max_{x, y, m'} u(x, y, e) + \beta E_{e'|e} V(m', e'; \Gamma', \mu') \quad (5)$$

¹²It should be noted that the terms money growth rate and inflation rate can be used interchangeably when the economy is in a steady state.

¹³In other words, the selected inflation rate is the Condorcet winner which beats any alternative inflation rate in a pairwise comparison.

s.t.

$$\frac{m + \mu}{1 + \mu} + P(y - x) \geq m'$$

$$x, y, m' \geq 0$$

$$\Gamma' = \tilde{H}(\Gamma, \mu, \mu')$$

where $\tilde{H}$ denotes the law of motion for Γ induced by the deviation, while all future distributions evolve according to H . Note that the future value function is given by the solution to (4). A solution to this problem generates

$$x = \tilde{\eta}(m, e; \Gamma, \mu), \quad y = \tilde{g}(m, e; \Gamma, \mu), \quad \text{and} \quad m' = \tilde{h}(m, e; \Gamma, \mu).$$

Definition: A politico-economic recursive competitive equilibrium (PRCE) is:

1. a set of functions $\{V, \eta, g, h, H, P\}$ that satisfy the definition of a RCE;
2. a set of functions $\{\tilde{V}, \tilde{\eta}, \tilde{g}, \tilde{h}\}$ that solve (5), at a price which clears the market and satisfies the aggregate resource constraint, and $\tilde{H}$ satisfying

$$\Gamma(m', e') = \int 1_{\{\tilde{h}(m, e; \Gamma, \mu) = m'\}} \Pi(e'|e) d\Gamma(m, e)$$

with continuation values satisfying condition 1 above;

3. in individual state $(m, e)_i$, household i 's most preferred growth rate μ^i satisfies

$$\mu^i = \Psi((m, e)_i, \Gamma, \mu) = \arg \max_{\mu'} \tilde{V}((m, e)_i; \Gamma, \mu, \mu') \quad (6)$$

4. the policy outcome function $\mu^m = \Psi(\Gamma, \mu) = \Psi((m, e)_m, \Gamma, \mu)$ satisfies

$$\begin{aligned} \int I_{\{(m,e):\mu^i \geq \mu^m\}} d\Gamma(m, e) &\geq \frac{1}{2} \\ \int I_{\{(m,e):\mu^i \leq \mu^m\}} d\Gamma(m, e) &\geq \frac{1}{2} \end{aligned}$$

Condition 4 above effectively defines the median voter. The median money growth rate is determined by sorting agents by their most preferred money growth rate and then integrating the distribution of most preferred money growth rates over (m, e) using $\Gamma(m, e)$.

3. Quantitative Analysis

This section establishes the calibration of the environment and presents the quantitative results. The results from the long-run analysis of the model are presented first, followed by the results from the short-run and politico-economic analyses.

3.1. Parameterization

Some of the model parameters (β , σ , and γ) are standard. A model period corresponds to one year, implying $\beta = 0.96$. The preference parameters are set to $\sigma = 2$ and $\gamma = 1/2$. Studies by McCurdy (1981) and Altonji (1986) estimate the Frisch elasticity to be between 0 and 0.54. A robustness analysis is performed on these two preference parameters at the end of this section.

It is assumed that shocks are transient across households (i.e. $\Pi(b|e) = \Pi(b)$ and $\Pi(s|e) = \Pi(s) = 1 - \Pi(b)$), and the value of the seller's preference shock is normalized to one. This leaves two parameters to be calibrated: the unconditional probability of receiving a buyer shock ($\Pi(b)$) and the size of the preference shock for a buyer ($e = b$). These two parameters are calibrated so the steady state of the model under $\mu = 0.02$ displays a monetary velocity of 5 and a stationary monetary distribution with a median equal to 0.44.

The choice of the monetary velocity follows Molico (2006) and others, while the median of the monetary distribution is taken from the empirical distribution calculated from the 2004 Survey of Consumer Finances. The resulting parameter values are $\Pi(b) = 0.69$ and $b = 4.76$.

3.2. Findings

This section presents the quantitative results for a long-run, short-run and politico economic analysis of the environment. The long-run analysis compares properties of the invariant monetary distributions for the environment under several constant money growth rates as in the analyses of Molico (2006), Chiu and Molico (2010) and Dressler (2011).¹⁴ The short-run analysis calculates the transition paths from a steady state with $\mu = 0.00$ to the same steady states considered in the long-run analysis and compares the welfare implications along the path and remaining at zero inflation.¹⁵ The politico-economic analysis compares voting outcomes under the assumptions that the median voter chooses a future permanent money growth rate, and that the monetary authority has full commitment. This delivers a simplifying restriction on the sequential PRCE defined above such that all continuation values are evaluated according to an “identity” function. In other words, $\mu_{t+n+1} = \Psi(\Gamma_{t+n}, \mu_{t+n}) = \mu_{t+n}$, for all Γ_{t+n} and μ_{t+n} , $n = 1, 2, \dots$, with $\mu_{t+n} = \Psi^0(\Gamma, \mu) = \arg \max_{\mu'} \tilde{V}((m, e)_m; \Gamma, \mu, \mu')$. It should be noted that this restriction is only on the evolution of money growth rates. The evolution of Γ is given by $H(\Gamma, \mu)$ and the entire transition of the price level must be computed for each possible money growth rate. However, similar to the short-run analysis, the evolution of Γ can be computed by determining the transition from a steady state with an initial money growth rate μ_0 to a new steady state with the proposed money growth rate μ_T where $\mu_t = \mu_T$ for all $t \geq 1$.¹⁶

¹⁴The stationary distributions of the environment can be calculated using standard solution methods. For example, Hugget (1993) and Aiyagari (1994).

¹⁵The transitions between two stationary steady states again follows standard solution methods. For example, Ríos-Rull (1999).

¹⁶While the assumption of commitment is a simplification of the sequential voting problem, it is a natural extension of the (long run) steady-state comparisons performed in much of the monetary literature.

3.2.1. Long-run analysis

The steady state results under zero percent inflation ($\mu = 0.00$) are illustrated in Figure 2. The value functions of buyers and sellers (upper-left panel) illustrate the decreasing marginal value of money in the amount of money holdings, implying that there are declining incentives for self-insurance for wealthier agents. The remaining two panels illustrate the agents' decision rules. The upper-right panel illustrates $m' = h(m, e; \Gamma, \mu)$, while the lower panel illustrates $x = \eta(m, e; \Gamma, \mu)$ and $y = g(m, e; \Gamma, \mu)$. These panels indicate that an agent receiving a buyer shock ($e = b$) uses her money balances to obtain more consumption than what she wishes to produce herself. This results in fewer money balances to bring into the following period. Likewise, an agent receiving a seller shock ($e = s$) produces more consumption than she wishes to consume in order to increase her money holdings for self-insurance. The decreasing incentive to self-insure is also illustrated by the sellers decision rule for new money balances tending towards the 45 degree line. Once a seller becomes *wealthy enough*, she will begin to mimic buyers and begin to deplete her money balances.¹⁷

The kinks in the buyer's decision rules at low money holdings are worth elaborating on. These kinks indicate that the agent chooses to spend all of her money balances and enter the next period with zero. In the economy with $\mu = 0.00$, this decision is shared by all buyers who enter the period with money holdings of around 0.54 or less. These decisions have an intuitive impact on the distribution of money holdings across agents, which is illustrated in Figure 3. A large mass of agents (over 34 percent) enter the period with zero money holdings and must therefore produce for their own consumption. The smaller masses of agents with positive money holdings are those who share similar histories. In other words, these masses of agents have received the same stream of shocks since entering a period with zero money holdings.

Several steady-state statistics of the environment under various inflation rates are reported in Table 1. In general, higher inflation results in a higher market price and monetary

¹⁷This occurs when a seller has roughly 14 times the average amount of money holdings.

velocity, and a lower percentage of consumption goods purchased in the market. These three results are highly connected, since the price level has a direct effect on calculating velocity, while decreasing the amount of consumption able to be purchased with a given money balance due to the real-balance effect. The remaining three columns in the table report characteristics of the monetary distribution which are not monotonic with respect to the inflation rate and depend on how much *deflation* is in the environment. In particular, for inflation rates of -2.0 percent or higher, the median of the monetary distribution is decreasing in the inflation rate while the standard deviation is increasing. This is again a direct effect of the increasing price level. When prices are high, buyers are willing to spend more of their money balances. In addition, high prices also increase the size of the monetary transfer between buyers and sellers, meaning that some sellers are accumulating larger money balances and increasing the standard deviation of the distribution. This increase in dispersion implies that inequality is increasing in the inflation rate within this region. This is also confirmed by the increase in the Gini coefficient as well as the Lorenz curves illustrated in Figure 4. When inflation and the price level are higher, the constraint binds for more buyers and therefore a larger number of agents are willing to hold zero money balances. The sellers are accumulating large amounts of money balances, which adds to the positive correlation between inequality and inflation in this environment.

One final observation to point out in the table is the change in the relationships between the inflation rate and the median and standard deviation of the monetary distribution when the inflation rate gets sufficiently negative. In particular, for inflation rates of -3.0 percent or lower, the median is increasing in the inflation rate while the standard deviation is decreasing. To get some intuition on this result, note that the Friedman Rule in this environment is approximately -4.17 percent. At the Friedman Rule, money would become an unattractive medium of exchange and the price level would be zero. At low enough price levels, fewer buyers choose to deplete their money balances and the resulting monetary distribution becomes smoother than the one for zero inflation. A smoother distribution with

fewer or no mass points can exhibit both a smaller median and a larger standard deviation (with a higher Gini) as reported in the table.

Welfare calculations for this analysis are done in a standard consumption-equivalent manner. First, define the average expected welfare of an agent under inflation rate μ as $W(\mu) = \Pi(b) W(b, \mu) + (1 - \Pi(b)) W(s, \mu)$ where

$$\begin{aligned} W(b, \mu) &= \Phi \int \left(\begin{array}{c} (1 - \beta \Pi(s|s)) u(x_\mu, y_\mu, b) + \\ \beta (1 - \Pi(b|b)) u(x_\mu, y_\mu, s) \end{array} \right) d\Gamma_\mu(m, b) \\ W(s, \mu) &= \Phi \int \left(\begin{array}{c} \beta (1 - \Pi(s|s)) u(x_\mu, y_\mu, b) + \\ (1 - \beta \Pi(b|b)) u(x_\mu, y_\mu, s) \end{array} \right) d\Gamma_\mu(m, s) \end{aligned} \quad (7)$$

and $\Phi = (1 - \beta^2 - \beta(1 - \beta)(\Pi(b|b) + \Pi(s|s)))^{-1}$. The welfare cost in terms of consumption of having inflation rate μ relative to zero inflation is given by $(1 - \Delta_0(\mu)) \times 100\%$ where $\Delta_0(\mu)$ solves $W(\mu) = \Pi(b) W(b, 0) + (1 - \Pi(b)) W(s, 0)$ with

$$\begin{aligned} W(b, 0) &= \Phi \int \left(\begin{array}{c} (1 - \beta \Pi(s|s)) u(\Delta_0(\mu) x_0, y_0, b) + \\ \beta (1 - \Pi(b|b)) u(\Delta_0(\mu) x_0, y_0, s) \end{array} \right) d\Gamma_0(m, b) \\ W(s, 0) &= \Phi \int \left(\begin{array}{c} \beta (1 - \Pi(s|s)) u(\Delta_0(\mu) x_0, y_0, b) + \\ (1 - \beta \Pi(b|b)) u(\Delta_0(\mu) x_0, y_0, s) \end{array} \right) d\Gamma_0(m, s) \end{aligned} \quad (8)$$

and Φ defined above. It should be noted that overall agent welfare is affected by a change in the stationary distribution (Γ_0 versus Γ_μ) as well as a change in the decision rules (e.g. x_0 versus x_μ). This welfare measure can be decomposed by changing either the distribution or decision rules one at a time. In particular, calculating the change in welfare when only changing the distribution (and leaving the decisions rules intact) will emphasize the change in welfare due to the redistributive effect. Conversely, calculating the change in welfare when only changing the decision rules (and leaving the distribution intact) will emphasize the change in welfare which would be captured in a model with a degenerate stationary distribution such as

a representative agent environment. The overall and decomposed welfare calculations are reported in Table 2.

Generally speaking, agents find inflation to be a welfare cost and deflation to be a welfare benefit. While this is a standard result, it is informative to point out several details. For example, the welfare cost of 10 percent inflation relative to zero inflation is 5.10 percent of consumption. This cost is much larger than previous calculations, and appears surprising since a large majority of agents are said to benefit from inflation by holding below average money balances. The primary reasons for this result are: (i) high inflation increases the mass of agents at zero money holdings, making them worse off since they are no longer self-insuring; and (ii) inflation hurts buyers more than sellers, and the buyer-seller ratio is 2.25. To illustrate the latter point, the decision rules for consumption and production are illustrated in Figure 5 for $\mu = 0.00$ (the thick lines) and $\mu = 0.10$ (the thin lines). The figure clearly indicates that higher inflation induces buyers to both consume less and produce more for all money holdings with the exception of a tiny neighborhood near zero. Agents who were holding zero money balances at $\mu = 0.00$ do receive a welfare gain when μ increases to 0.10, but this population is dwarfed by the new agents holding zero money balances at $\mu = 0.10$ who are worse off. Although sellers are receiving welfare gains by consuming more and producing less in response to higher inflation, the high buyer-seller ratio places more weight on the welfare costs of the buyers. The bottom line is that the real-balance effect clearly overpowers the redistributive effect in this environment.

The contrasting decision rules in Figure 5 provide insight into the welfare results where only the decision rules are allowed to change and the average utility is calculated under the initial monetary distribution. This calculation would be closest to a model with a representative agent or a constant (degenerate) monetary distribution, and is larger in absolute value for all inflation rates. When the decision rules are kept constant and only the distribution is allowed to change (third column), one can capture the sole impact of the distributional effect. It is not surprising that the real-balance effect is depicting large welfare costs for high

Table 1: Long-Run Comparison

μ (%)	P	median(m)	Velocity	std(m)	Market %	Gini
-3.95	0.15	0.64	0.20	1.16	16.03	0.51
-3.0	1.28	0.76	1.72	0.92	14.45	0.50
-2.0	1.93	0.80	2.59	1.03	13.53	0.55
0.0	2.94	0.48	3.94	1.17	12.26	0.61
2.0	3.73	0.43	5.00	1.25	11.34	0.64
5.0	4.86	0.27	6.51	1.36	10.23	0.67
10.0	6.68	0.00	8.93	1.51	8.83	0.72

Table 2: Long-Run Welfare Results

μ (%)	Overall (%)	DRs only (%)	Dist only (%)
-3.95	-11.92	-13.43	5.80
-3.0	-4.00	-5.14	1.56
-2.0	-2.23	-2.84	0.75
0	—	—	—
2.0	1.50	1.81	-0.30
5.0	3.18	3.88	-0.55
10	5.10	6.36	-0.61

inflation while the redistributive effect is depicting welfare gains. What is surprising are the magnitudes. A 10 percent inflation only delivers a distributional welfare benefit of 0.61 percent, which is dominated by a real-balance welfare cost of 6.36 percent. For all inflation / deflation rates considered, the redistributive effect is less than half of the real-balance effect in absolute value.

3.2.2. Short-run (transitional) analysis

While the previous section presents a welfare comparison among different steady-state levels of inflation and zero, this section assesses the welfare implications as the economy transitions between these permanent levels of inflation. This requires calculating the transition dynamics for all prices, decision rules and distributions as the economy moves from an initial steady state with $\mu_0 = 0.00$ to a new steady state with $\mu_t \neq 0.00 \forall t \geq 1$. Transition paths for the price level are illustrated in Figure 6. For comparability purposes only, the price paths are normalized so the price level at the new steady state under the nonzero money

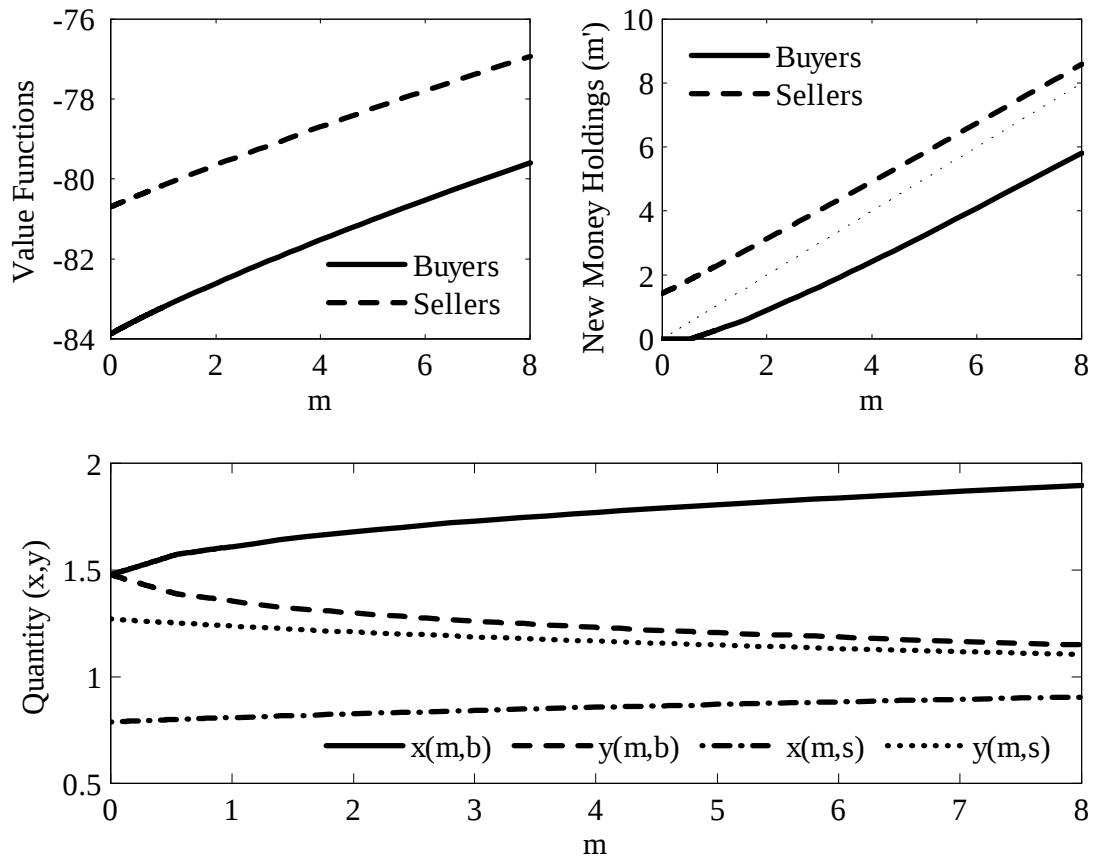


Figure 2: Value Functions and Decision Rules, $\mu = 0.00$

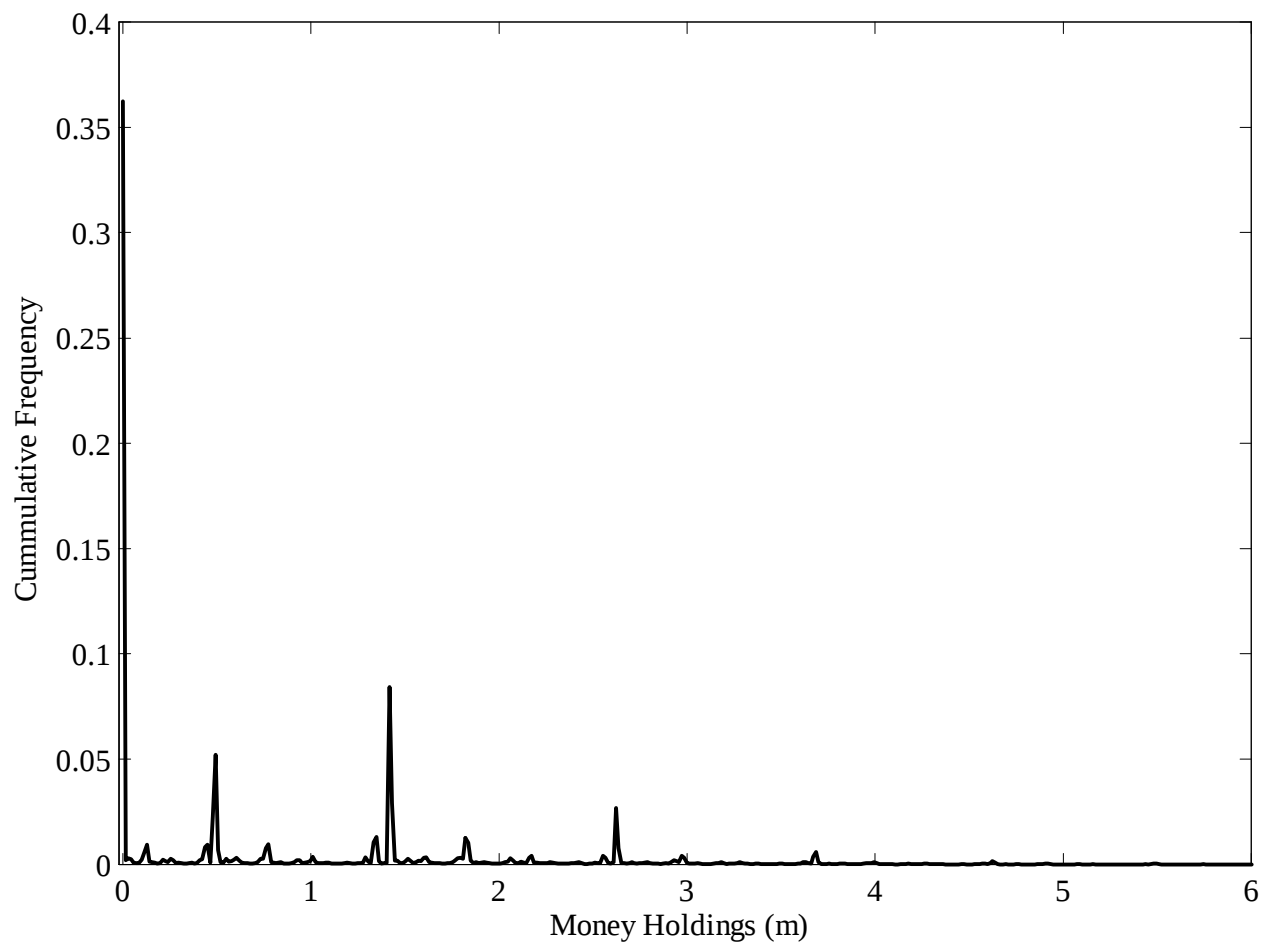


Figure 3: Stationary Distribution of Money Holdings, $\mu = 0.00$

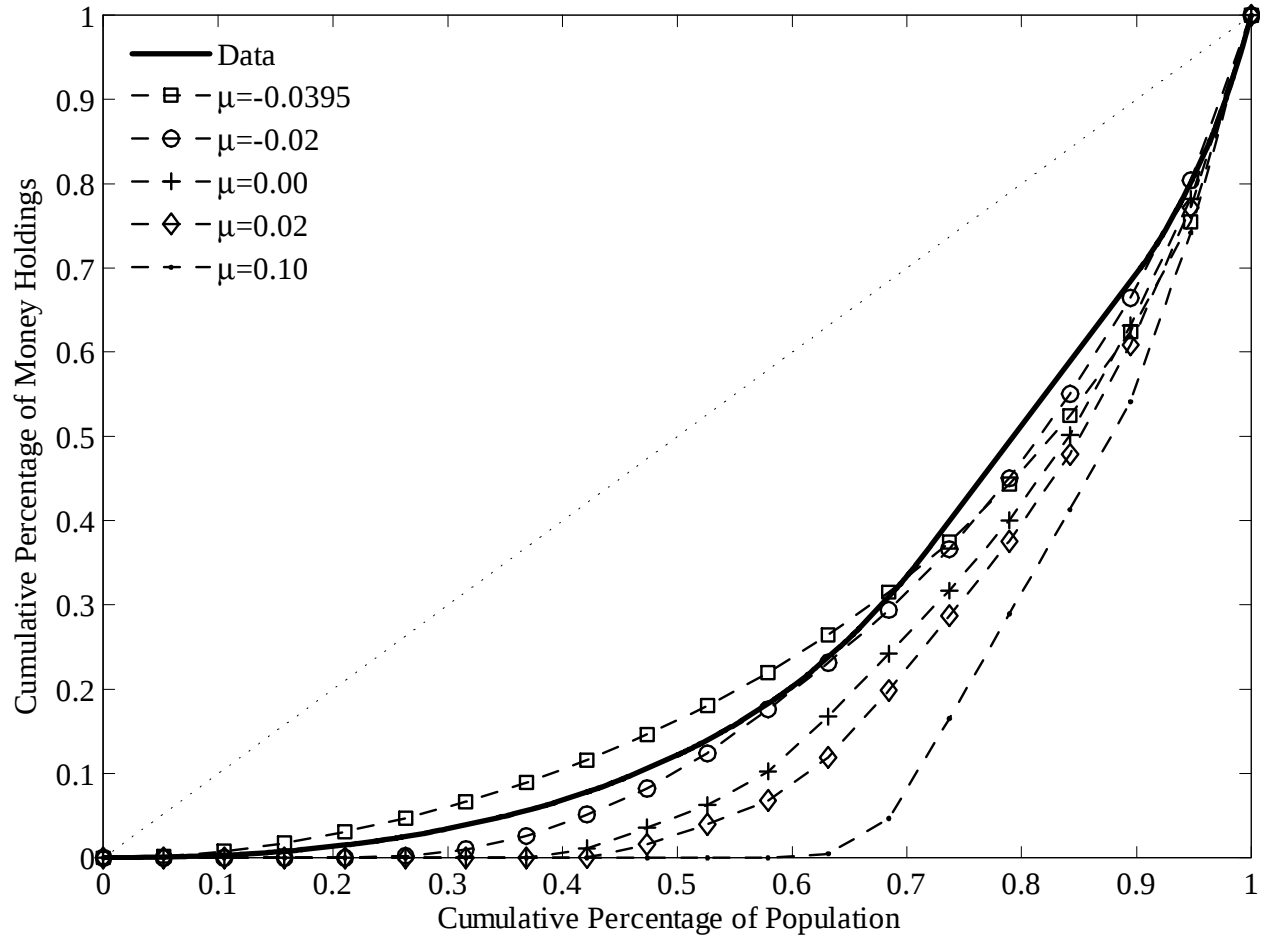


Figure 4: Lorenze Curves of Monetary Distributions with Varying Inflation (μ) .

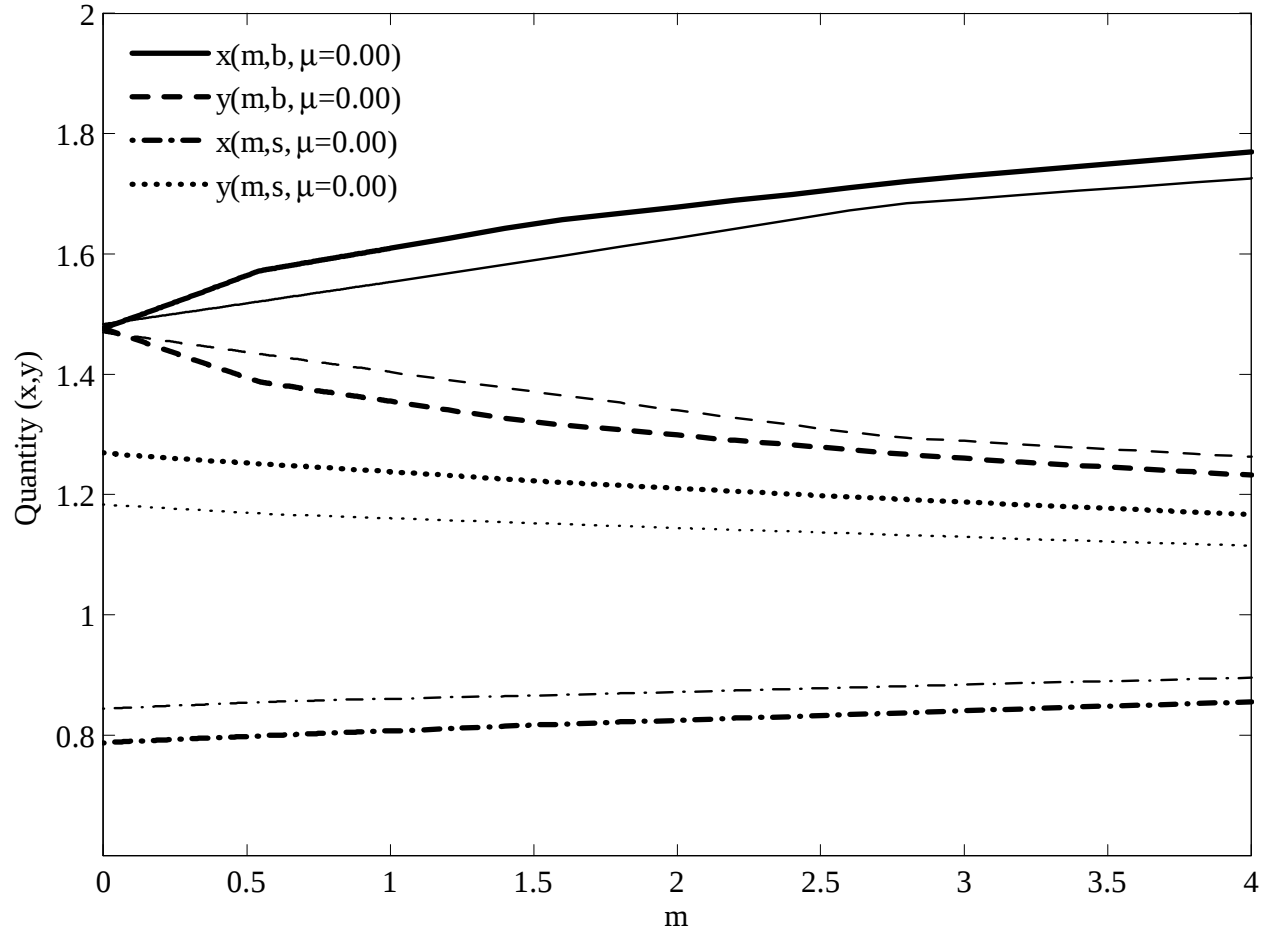


Figure 5: Decision Rules for $\mu = 0.00$ (Thick Lines) and $\mu = 0.10$ (Thin Lines).

growth rate is equal to one. The figure indicates that the economy converges to positive inflation steady states rather quickly because of the increased mass of agents running into the nonnegativity constraint for future money holdings ($m' \geq 0$). In other words, higher inflation is inducing more agents to spend all of their current money balances and leave zero for self-insurance. When the new inflation rate is lower, there are fewer agents running into this constraint which results in a slower transition.

The short-run welfare costs are calculated in a similar way as the long-run costs given by (7) and (8), but with a few exceptions. First, the length of the transition needs to be determined for each new money growth rate and is denoted by T . This time duration needs to be sufficiently long enough such that the economy has reached the new stationary distribution by the end. Second, the decision rules and distributions need to be calculated at every period along the transition. These are denoted by $x_{\mu t}$, $y_{\mu t}$ and $\Gamma_{\mu t}$, respectively. Given T , it must be the case that x_{00} , y_{00} and Γ_{00} are the decision rules and distribution associated with zero inflation at the initial time period ($t = 0$) and $x_{\mu T}$, $y_{\mu T}$ and $\Gamma_{\mu T}$ are the steady state decision rules and stationary distribution associated with the new (permanent) inflation rate μ . Along the transition from $\mu_0 = 0.00$ to $\mu_T = \mu$, define the average expected welfare of an agent as $\hat{W}(\mu) = \Pi(b) \hat{W}(b, \mu) + (1 - \Pi(b)) \hat{W}(s, \mu)$ where

$$\begin{bmatrix} \hat{W}(b, \mu) \\ \hat{W}(s, \mu) \end{bmatrix} = \sum_{t=0}^T \beta^t \Pi^t \begin{bmatrix} \int u(x_{\mu t}, y_{\mu t}, b) d\Gamma_{\mu t}(m, b) \\ \int u(x_{\mu t}, y_{\mu t}, s) d\Gamma_{\mu t}(m, s) \end{bmatrix}. \quad (9)$$

The welfare cost from transitioning to some inflation rate μ (in terms of consumption) relative to remaining at zero inflation is given by $(1 - \hat{\Delta}_0(\mu)) \times 100\%$ where $\hat{\Delta}_0(\mu)$ solves $\hat{W}(\mu) = \Pi(b) \hat{W}(b, 0) + (1 - \Pi(b)) \hat{W}(s, 0)$ with

$$\begin{bmatrix} \hat{W}(b, \mu) \\ \hat{W}(s, \mu) \end{bmatrix} = \sum_{t=0}^T \beta^t \Pi^t \begin{bmatrix} \int u(\hat{\Delta}_0(\mu) x_{0t}, y_{0t}, b) d\Gamma_{0t}(m, b) \\ \int u(\hat{\Delta}_0(\mu) x_{0t}, y_{0t}, s) d\Gamma_{0t}(m, s) \end{bmatrix}. \quad (10)$$

Table 3: Short-Run Welfare Results

μ (%)	Overall (%)	T
-3.95	-0.07	120
-3.0	-1.57	27
-2.0	-0.91	30
0	—	—
2.0	0.64	6
5.0	1.42	5
10	2.25	5

In other words, $\hat{\Delta}_0(\mu)$ delivers the average welfare effects along the transition path relative to how well off the agents would be if the economy remained at zero inflation and never experience a transition.¹⁸ These welfare implications are reported in Table 3 along with the transition length T for each terminal inflation rate μ .

The table suggests that the welfare implications along the transition to a new inflation rate are similar to those reported for the long-run analysis. In particular, a transition to a positive (negative) inflation rate delivers overall welfare costs (gains). For example, transitioning to a permanent inflation rate of 10 percent compared to remaining at zero percent inflation costs 2.25 percent of consumption, and this transition takes only 5 (annual) periods. This transitional cost, combined with the long-run cost reported in Table 1, suggest that the total welfare costs of 10 percent inflation can be as high as 7.35 percent. Although the transition lengths differ for the other inflation rates considered, the short-run welfare implications are of similar sign and generally half the size of the long-run results. One exception is for the transition to -0.0395 percent, which delivers a very small welfare gain of -0.0668 percent. This result could possibly be due to the fact that the steady states under this new inflation rate and zero percent inflation share distributions with very similar levels of dispersion (Table 1 reports standard deviations of 1.16 and 1.17 for $\mu = -0.0395$ and $\mu = 0.00$, respectively). Otherwise, the bigger the increase (decrease) in dispersion observed in the new steady state, the bigger the welfare cost (benefit) along the transition.

¹⁸The expressions above use a ‘^’ simply to differentiate these short-run welfare expressions from the long-run expressions.

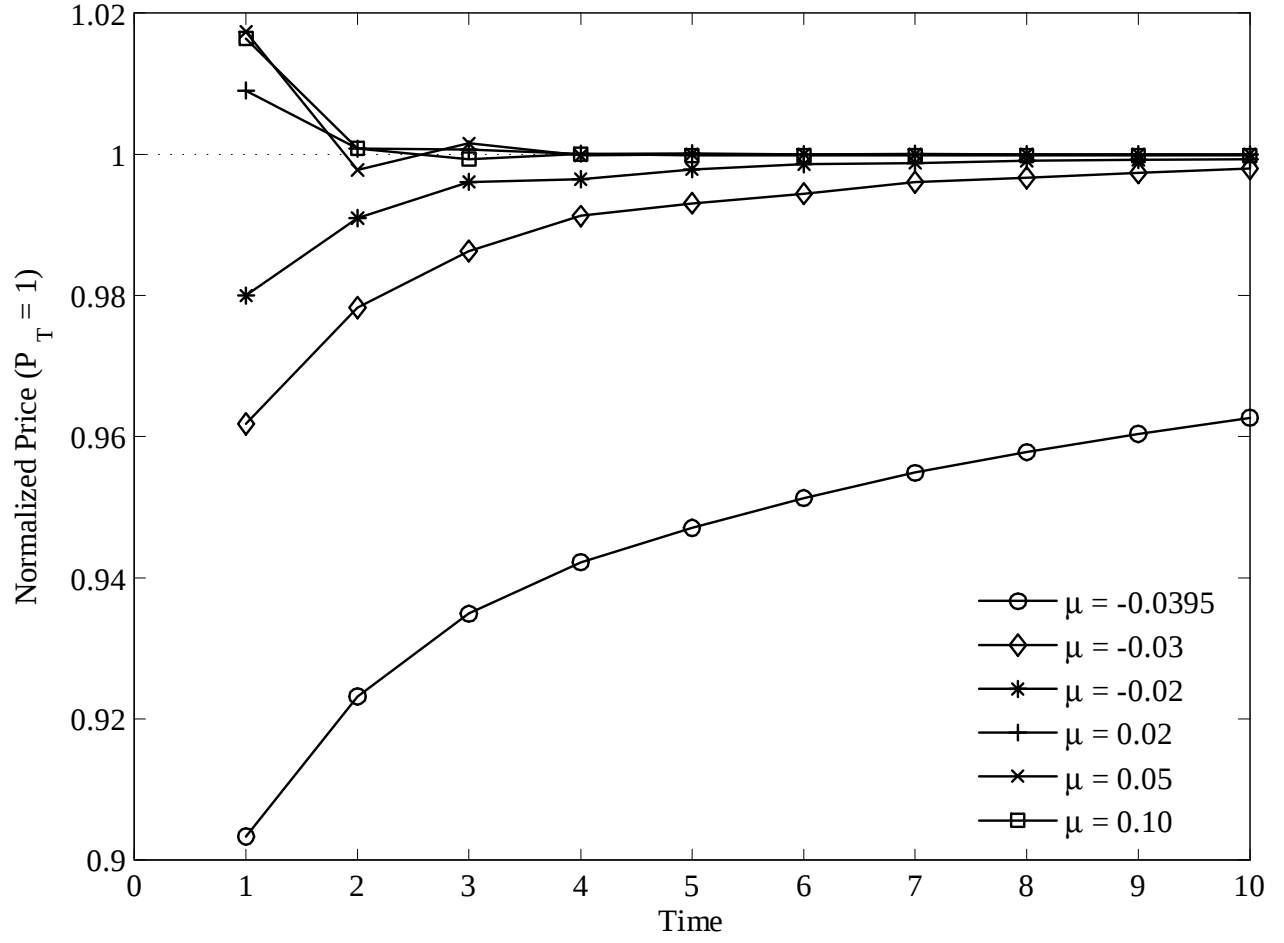


Figure 6: Transition path of normalized price level from $\mu_0 = 0.00$.

3.2.3. Politico-economic analysis

The analysis now turns to an individual welfare analysis and asks which inflation rate would appeal to a majority of the agents. This is accomplished under the assumptions of a one-time vote for the agents and commitment on the part of the monetary authority. This implies that the law of motion for the money growth rate is a one-time jump from the initial value (μ) to the new permanent value (μ') at $t = 1$, and the dynamics amount to solving the transition of the economy in the same way as in the short run analysis.

As in previous analyses of a politico-economic equilibrium, preferences among voting alternatives need to be single-peaked in order to establish that the median voter theorem applies.¹⁹ Since a general proof of single-peakedness of the indirect utility function (5) is unavailable for this environment, it is verified numerically for every (m, e, Γ, μ) including those off-the-equilibrium path. An example of this verification is illustrated in Figure 7 which plots $\tilde{V}(m, e; \Gamma, \mu, \mu')$ over μ' for buyers and sellers holding money balances equal to the quartiles of the stationary monetary distribution Γ associated with $\mu = 0.00$. In addition to confirming the single-peakedness requirement, the figure indicates that poorer agents (Q1) prefer inflation while richer agents (Q3) prefer deflation.²⁰

In order to reach a voting outcome, transitions from every μ to every μ' are used to calculate the indirect utility function (5) for every agent, which are to determine each agent's most-preferred inflation rate following (6). Finally, the prevailing inflation rate is determined by finding the median inflation rate of the resulting distribution of most-preferred inflation rates. A list of prevailing inflation rates given various initial inflation rates are reported in Table 4. For all initial inflation rates of -2.00 percent or higher, the prevailing inflation rates are lower due to the dominating real-balance effect. However, a *steady-state* PRCE exists with $\mu^* = -0.03$. In other words, this is the unique initial inflation rate such that the

¹⁹In particular, one must establish that the voting outcome beats any other feasible alternative in pairwise comparisons.

²⁰It should be noted that although the choice for the agents with Q3 money holdings is not an interior solution, it has been verified for all voting outcomes that the solution for the median lies in the interior.

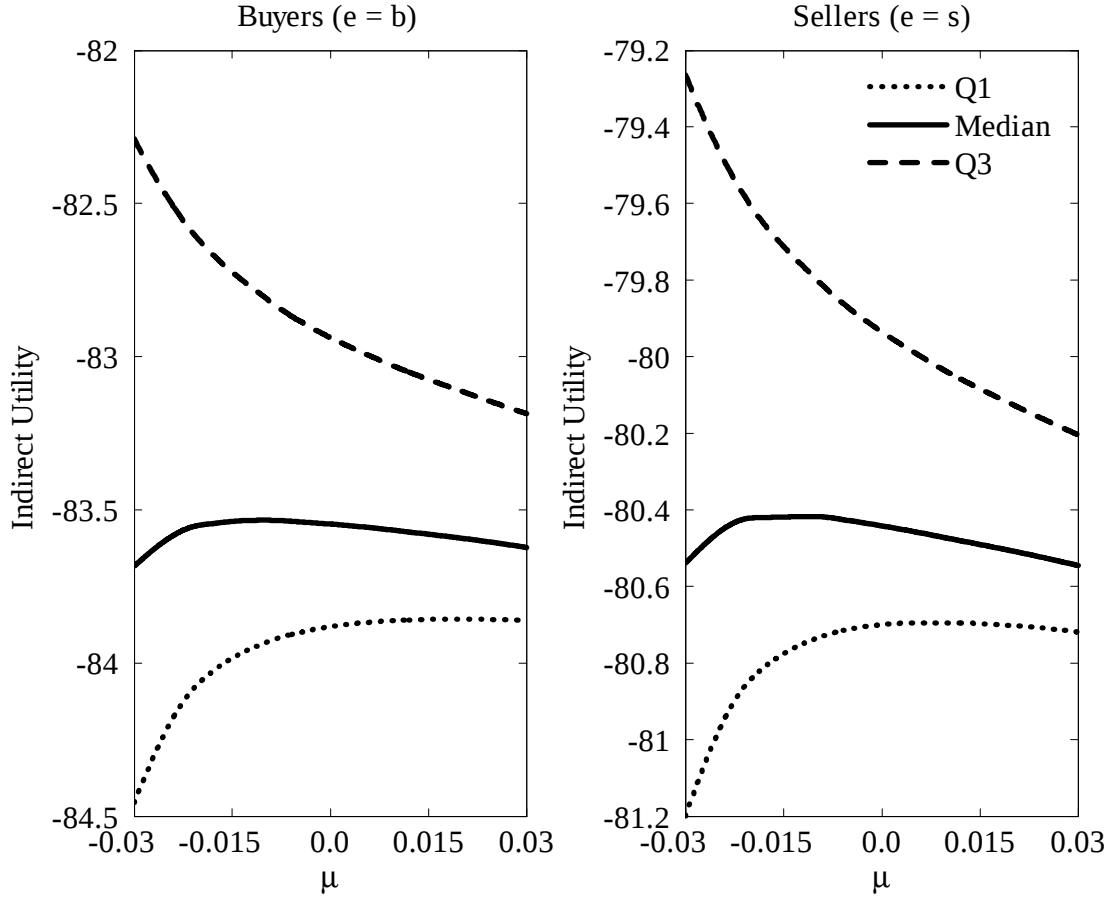


Figure 7: Indirect Utility Functions for $\mu_0 = 0.00$.

prevailing vote is to keep the inflation rate unchanged. While this steady state voting outcome is deflation, it is not the Friedman rule (-4.19) which is the overall welfare-maximizing amount of deflation from a planner's perspective. This prevailing inflation rate above the Friedman Rule is a direct result of the redistributive effect of inflation. Although this effect might be small in the overall welfare analysis, it is large enough on an individual scale to result in a voting outcome with a higher inflation rate than the Friedman Rule.

Table 4: Voting Outcome under Various Initial Inflation Levels

Initial Inflation (μ)	Voting Outcome (μ')
-3.95	-2.00
-3.00	-3.00
-2.00	-3.00
-1.00	-2.00
0.00	-1.01
2.00	-1.00
5.00	0.00

4. Conclusion

This paper set out to assess the long-run, short-run and politico-economic welfare costs of inflation in a model with essential money that delivers a nondegenerate distribution of money holdings across agents in line with the data. The long-run analysis predicts welfare costs that are qualitatively in line with previous findings, but their magnitudes are much larger than previously reported. While these results are surprising given that the stationary distribution indicates that there are a large proportion of agents who stand to benefit from the wealth redistribution brought about by positive inflation, the analysis shows that these benefits are significantly dominated by the costs associated with the real balance effect. The short-run analysis predicts additional costs and benefits which have previously not been considered in models with nondegenerate monetary distributions. These transitional welfare implications can considerably add to the overall costs / benefits of alternative monetary policies. The politico-economic analysis extends the model to determine which inflation rate would prevail if the agents of the model were allowed to vote. This analysis identifies a significant dimension to the redistributive effect of inflation, which is dominated in an aggregate welfare analysis, by predicting a steady-state voting outcome that is higher than the Friedman Rule.

While this analysis predicts large welfare costs of inflationary monetary policies, the model lacks features which may decrease these costs. One feature would be to have a second asset (e.g. capital) that could serve as a useful store of value and shield agents' wealth

from inflation. Another feature would be to allow the preference shocks to be persistent, which would potentially induce buyers to desire more inflation and benefit from the resulting transfers. Unfortunately, the simple model studied here has difficulty matching the key empirical moments (as well as additional calibrating moments) under persistent preference shocks. Therefore, a more sophisticated environment is needed, with possibly more than two types of agents or an additional asset, in order to provide enough flexibility to match a larger set of calibrating moments. An analysis of such an environment is left for future research.

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