

Margin Requirements and Asset Prices*

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Abstract

This paper examines the effect of collateral constraints and margin requirements on asset prices in a general equilibrium setup. We consider a Lucas-style infinite-horizon exchange economy with heterogeneous agents and endogenous collateral constraints. In our calibrated economy with disaster risk, collateral constraints lead to a large increase in the return volatility of long-lived assets, thus regulation of margin requirements on all these assets has a strong stabilizing effect. This finding is in line with the popular sentiment that collateralized borrowing contributes to market volatility, a view supported by previous theoretical analyses. In stark contrast, empirical evidence shows that the regulation of margin requirements for stocks does little to reduce stock market volatility. We provide an explanation for this apparent contradiction between theoretical and empirical results. In a model with different collateralizable assets, stocks constitute only a comparatively small fraction of total margin-eligible assets. The regulation of margin requirements on this fraction of collateralizable assets has no significant impact on its volatility. However, the volatility of other assets decreases monotonically as margins on stocks are increased. These important spillover effects have been neglected in much of the previous literature as well as in the policy debate.

Keywords: General equilibrium, heterogeneous agents, leverage, collateral constraints, endogenous margins, regulated margins, rare disasters.

JEL Classification Codes: D53, E21, G01, G12.

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1 Introduction

In the aftermath of financial crises, it is often argued that excessively low margin requirements play a prominent role in explaining instability in security markets. The 1929 crash led to the Securities Exchange Act of 1934 which granted the Federal Reserve Board (Fed) the power to set margin requirements on all securities traded on a national exchange (under Regulation T). Until the early 1970s, the Fed viewed the regulation of margin requirements as an important policy tool.¹ This changed in the mid 70s when the Fed stopped adjusting margin requirements and instead fixed them at a comparatively low level. The 1987 stock market crash led policy makers and academics to reconsider the by-then-dominant view that regulation of margin requirements is ineffective. In fact, the Presidential Task Force on Market Mechanisms deemed low margin requirements an important factor in the crash.² More recently, in the aftermath of the financial crisis of 2007–2009, policy makers argued that the build-up of collateralized borrowing before the crisis increased financial market procyclicality, thus exacerbating the subsequent downturn; see, for example, CGFS (2010).

Despite the popular view that collateralized borrowing contributes to market volatility, there is surprisingly little empirical evidence suggesting that there is a relationship between margin requirements and the volatility of securities prices. Fortune (2001) claims that “the literature does provide some evidence that margin requirements affect stock price performance, but the evidence is mixed and it is not clear that the statistical significance found translates to an economically significant case for an active margin policy.” Kupiec (1998) is even more negative in his assessment of the empirical results and concludes that “there is no substantial body of scientific evidence that supports the hypothesis that margin requirements can be systematically altered to manage the volatility in stock markets.” Day and Lewis (1997) reach the same conclusion for the case of futures markets. Seguin (1990) even finds a result contrary to the popular view; on average, a security’s volatility decreases after it becomes margin eligible.

In this paper, we reconcile the popular view that borrowing on margin increases asset price volatility with the documented empirical evidence. We develop a quantitative general equilibrium asset pricing model and find that the ability of agents to use long-lived assets as collateral for short-term borrowing increases asset return volatility substantially. In our baseline calibration, volatility increases by almost fifty percent compared to the same model without borrowing or with natural borrowing limits. Thus, our model supports the popular view that collateralized borrowing increases asset price volatility. However, our model also predicts that regulation of margin requirements on stocks (which constitute only a comparatively small fraction of all long-lived assets in the economy) has almost no effect on volatility if there are other unregulated borrowing opportunities. This model prediction is in line with the aforementioned empirical results. In our calibration, the volatility of the stock market remains essentially constant as

¹For example, in a US Senate testimony in 1955, Fed chairman Martin summarized the Fed view on margin policy as follows: “The task of the Board, as I see it, is to formulate regulations with two principal objectives. One is to permit adequate access to credit facilities for securities markets to perform the basic economic functions. The other is to prevent the use of stock market credit from becoming excessive.”

²See Kupiec (1998) or Fortune (2000) for detailed accounts on the history of margin regulation.

margin requirements for borrowing on stocks are changed to lie anywhere between 60 and 100 percent. In our framework we can also replicate the finding of Seguin (1990) that the volatility of a previously margin-ineligible asset decreases as it becomes margin-eligible. For this purpose, we examine a long-lived asset that constitutes only a small fraction of the market and compare its price volatility when it cannot be used as collateral to its price volatility after it becomes margin-eligible. In our calibration, the asset's price volatility decreases substantially while overall market volatility increases slightly. This result is an instance of a more general insight from our analysis. In the presence of several long-lived assets, changes in the regulated margin requirements for one asset have substantial effects on the volatility of other assets. While tightening margins for loans on an asset might very well increase its volatility, it always decreases the volatility of the market as a whole. These important spillover effects have been neglected in much of the previous literature as well as in the policy debate.

We consider a Lucas-style exchange economy with heterogeneous agents and collateral constraints. Agents can only borrow, i.e. take short positions in bonds, if they hold an infinitely-lived asset (a Lucas tree) as collateral. This model was first analyzed by Kubler and Schmedders (2003) and was subsequently used by Cao (2010) and Brumm and Grill (2010). As in Kubler and Schmedders (2003), we assume that agents can default on a short position at any time without any utility penalties or loss of reputation. Financial securities are therefore only traded if the promises associated with these securities are backed by collateral. The collateral (or margin) requirement determines how much agents can borrow using risky assets (trees) as collateral. Following Geanakoplos (1997) and Geanakoplos and Zame (2002), we endogenize the margin requirements by introducing a menu of financial securities. All securities promise the same payoff, but they differ in their respective margin requirement. In equilibrium, only some of them are traded, thereby determining an endogenous margin requirement. This setup implies that for many bonds and many next period's shocks, the face value of the debt falls below the value of the collateral. As a result, there is default in equilibrium. We assume that default is costly by introducing a real cost to the lender. In our calibration, trade in defaultable bonds ceases to exist with moderate default costs. In addition to endogenous margin requirements, we also consider regulated margin requirements. In particular, we want to consider the realistic case where some asset markets are unregulated while in other markets the margin requirements are set by a regulator.

In our calibration of the model there are two types of heterogeneous agents with Epstein-Zin utility. They have identical elasticities of substitution (IES) but differ with respect to their risk-aversion (RA). The agent with the low risk aversion is the natural buyer of risky assets and leverages to finance these investments. The agent with the high risk aversion has a strong desire to insure against bad shocks and thus is a natural buyer of safe bonds. When the economy is hit by a negative shock, the collateral constraint forces the leveraged agent to reduce consumption and to sell risky assets to the risk-averse agent, triggering substantial changes in the wealth distribution, which in turn affect asset prices. To obtain a sizable market price of risk, we follow Barro (2009) and include the possibility of 'disaster shocks'. In particular, we calibrate

the disaster shocks to match the first three moments of the heavy-tailed distribution of disaster shocks as estimated by Barro and Jin (2012).

We start our analysis by considering an economy with a single long-lived asset for which margin requirements are determined endogenously. In this model, collateral constraints lead to a significant increase in the return volatility of the long-lived asset. Thus regulating margin requirements has the potential to reduce this volatility substantially. This observation motivates our subsequent analysis of regulated margin requirements. As margin requirements increase, we observe two opposing effects. On the one hand, the amount of leverage decreases in equilibrium, leading to less de-leveraging after bad shocks which in turn leads to smaller price changes. On the other hand, the collateral constraint is more likely to become binding in equilibrium. This effect increases the probability of de-leveraging episodes which in turn leads to higher asset return volatility. For margin levels between 60 and 90 percent, these two effects approximately offset each other and thus asset return volatility barely changes. For larger margin levels, the first effect dominates which results in a significant drop in asset volatility.

In the next step of our analysis, we examine a model with two long-lived assets, one of them being margin eligible while the other one is not. In this specification, both the average return and the return volatility of the margin-eligible (collateralizable) asset are significantly smaller compared to the corresponding values for the margin-ineligible asset. The margin-eligible asset is more valuable to its owner because it provides value as collateral. When both assets have identical dividends, an agent can only be induced to hold the non-marginable asset if it pays a higher average return. In our calibration this effect is indeed large; the average excess return of the non-marginable asset is more than 80% higher than that of the marginable asset. A key factor, among others, contributing to the different volatility levels of the two assets is that the non-marginable asset is traded much more often and in larger quantities than the marginable one. If the less risk-averse agent, the natural buyer of risky assets, holds both assets and then becomes poorer after a bad shock, the prices of both assets fall. But as the agent sells the non-marginable asset first, its price falls much faster than the price of the marginable asset.

In the final part of our analysis, we assume that margin requirements are exogenously regulated for one long-lived asset (representing stocks) while the margin requirement for a second asset (representing housing and corporate bonds) is endogenous. We find that tighter margins on all stocks have no significant effect on their return volatility. This result is in line with the empirical evidence cited in Fortune (2001) and Kupiec (1998), who document that the relationship between Regulation T margin requirements and the volatility of the stock market is weak. The reason for this result is that an increase in the margin requirement of the regulated tree has two direct effects: First, the regulated asset becomes relatively less attractive as collateral; second, the agents' ability to leverage decreases. While the first effect increases the asset's volatility, the second effect reduces it. In equilibrium, these two effects approximately offset each other. For the second asset with endogenous margins, the two described effects do not counteract but both lead to a reduction of its volatility. So, there are strong spillover effects from the margin regulation of the regulated tree on the return volatility of the unregulated tree. Finally, our model also provides a possible explanation for the empirical result documented in

Seguin (1990), namely that the return volatility of individual stocks fell significantly after becoming margin-eligible. As the margin requirement on a newly regulated small asset decreases, this asset becomes more attractive for the agents, who thus sell it less frequently after a bad shock. Not only does margin eligibility decrease the volatility of a stock that is small relative to the market, but in addition our analysis shows that its volatility is in fact monotonic in its margin requirement.

Most of our paper focuses on the volatility of the long-lived assets that can (or cannot) be used as collateral. We also find that margin-eligibility has a strong effect on the first moment of asset prices. This phenomenon is empirically well documented (see, e.g., Seguin (1990)) and there is a theoretical literature on this issue (see, e.g., Hindy and Huang (1995) and Garleanu and Pedersen (2011)). These papers derive analytically that an asset's excess return is determined by both its cash flow risk as well as its 'collateralizability'. Our results are consistent with this literature.

A string of papers in the economic literature has formalized the idea that borrowing against collateral may increase asset price volatility. In contrast to this study, most of these papers do not consider calibrated models and do not investigate quantitative implications. Prominent early papers include Geanakoplos (1997) and Aiyagari and Gertler (1999). In these models, the market price may deviate substantially from the corresponding price in frictionless markets. Brunnermeier and Pedersen (2009) develop a model where an adverse feedback loop between margins and prices may arise. In their model, risk-neutral speculators trade on margin and margin requirements are determined by a value-at-risk constraint. Fostel and Geanakoplos (2008) apply some of these ideas to emerging market economies.

The paper closest to ours is Coen-Pirani (2005), who also considers a Lucas-style model with agents that differ in risk-aversion but have identical IES. By further assuming that the common IES is equal to one and that all income stems from dividend payments, he can show analytically that collateral constraints have no effect on stock return volatility. We find that this result changes dramatically if one takes into account that labor income finances a large part of aggregate consumption. In this case, collateral constraints substantially increase return volatility.

Following Kiyotaki and Moore (1997) there is also a large literature that examines the effects of collateral constraints on the transmission of TFP shocks in production economies. The focus of our paper is on asset prices but there is some similarity in the mechanism. Cordoba and Ripoll (2004) show that the effects are quantitatively small for a standard calibration of preference parameters and the capital share. Gourio (2012) examines the effect in the presence of disaster shocks similar to those in our calibration. He finds large amplification of shocks through collateral constraints. Brunnermeier and Sannikov (2011) examine a continuous time model with two agents where shifts in the wealth distribution lead to large volatility. While they focus on qualitative results and consider neither a calibrated model nor margin regulation, their economic mechanisms resemble those in this study.

The remainder of this paper is organized as follows. We introduce the model and its cali-

bration in Section 2. In Section 3 we discuss results for economies with a single collateralizable asset. Section 4 focuses on economies with two long-lived assets, only one of which is margin eligible. In Section 5 we consider the effects of margin regulation on volatility and thereby explain findings of the empirical literature. Section 6 concludes. In the Appendix we provide extensive sensitivity analysis.

2 The economy

We examine a model of an infinite horizon exchange economy with infinitely-lived heterogeneous agents, long-lived assets and collateral constraints for short-term borrowing. Section 2.1 describes the model; in Section 2.2 we discuss our calibration.

2.1 The model

Time is indexed by $t = 0, 1, 2, \dots$. A time-homogeneous Markov chain of exogenous shocks (s_t) takes values in the finite set $\mathcal{S} = \{1, \dots, S\}$. The $S \times S$ Markov transition matrix is denoted by π . We represent the evolution of time and shocks in the economy by a countably infinite event tree Σ . The root node of the tree represents the initial shock s_0 . Each node of the tree, $\sigma \in \Sigma$, describes a finite history of shocks $\sigma = s^t = (s_0, s_1, \dots, s_t)$ and is also called date-event. We use the symbols σ and s^t interchangeably. To indicate that $s^{t'}$ is a successor of s^t (or s^t itself) we write $s^{t'} \succeq s^t$. We use the notation s^{-1} to refer to the initial conditions of the economy prior to $t = 0$.

At each date-event $\sigma \in \Sigma$ there is a single perishable consumption good. The economy is populated by H agents, $h \in \mathcal{H} = \{1, 2, \dots, H\}$. Agent h receives an individual endowment in the consumption good, $e^h(\sigma) > 0$, at each node. In addition, at $t = 0$ the agent owns shares in long-lived assets (“Lucas trees”). We interpret these Lucas trees to be physical assets such as firms, machines, land or houses. There are A different such assets, $a \in \mathcal{A} = \{1, 2, \dots, A\}$. At the beginning of period 0, each agent h owns initial holdings $\theta_a^h(s^{-1}) \geq 0$ of tree a . We normalize aggregate holdings in each Lucas tree, that is, $\sum_{h \in \mathcal{H}} \theta_a^h(s^{-1}) = 1$ for all $a \in \mathcal{A}$. At date-event σ , we denote agent h ’s (end-of-period) holding of Lucas tree a by $\theta_a^h(\sigma)$ and the entire portfolio of tree holdings by the A -vector $\theta^h(\sigma)$.

The Lucas trees pay positive dividends $d_a(\sigma)$ in units of the consumption good at all date-events. We denote aggregate endowments in the economy by

$$\bar{e}(\sigma) = \sum_{h \in \mathcal{H}} e^h(\sigma) + \sum_{a \in \mathcal{A}} d_a(\sigma).$$

The agents have preferences over consumption streams representable by the following recursive utility function, see Epstein and Zin (1989),

$$U^h(c, s^t) = \left\{ \left[c^h(s^t) \right]^{\rho^h} + \beta \left[\sum_{s_{t+1}} \pi(s_{t+1} | s_t) \left(U^h(c, s^{t+1}) \right)^{\alpha^h} \right]^{\frac{\rho^h}{\alpha^h}} \right\}^{\frac{1}{\rho^h}},$$

where $\frac{1}{1-\rho^h}$ is the intertemporal elasticity of substitution (IES) and $1 - \alpha^h$ is the relative risk aversion of the agent.

At each date-event, agents can engage in security trading. Agent h can buy $\theta_a^h(\sigma) \geq 0$ shares of tree a at node σ for a price $q_a(\sigma)$. Agents cannot assume short positions of the Lucas trees. Therefore, the agents make no promises of future payments when they trade shares of physical assets and thus there is no possibility of default when it comes to such positions.

In addition to the physical assets, there are J one-period financial securities, $j \in \mathcal{J} = \{1, 2, \dots, J\}$, available for trade. We denote agent h 's (end-of-period) portfolio of financial securities at date-event σ by the vector $\phi^h(\sigma) \in \mathbb{R}^J$ and denote the price of security j at this date-event by $p_j(\sigma)$. These assets are all one-period bonds in zero-net supply; they promise one unit of the consumption good in the subsequent period. Whenever an agent assumes a short position in a financial security j , $\phi_j^h(\sigma) < 0$, she promises a payment in the next period. Such promises must be backed by collateral.

2.1.1 Collateral and Default

At each node σ , we associate with each financial security $j \in \mathcal{J}$ a tree $a(j) \in \mathcal{A}$ and a collateral requirement $k_{a(j)}^j(\sigma) > 0$. If an agent sells short one unit of security j , then she is required to hold $k_{a(j)}^j(\sigma)$ units of tree $a(j)$ as collateral. If an asset a can be used as collateral for different financial securities, the agent is required to buy $k_{a(j)}^j(\sigma)$ shares for each security $j \in \mathcal{J}_a$, where $\mathcal{J}_a \subset \mathcal{J}$ denotes the set of financial securities collateralized by the same tree a .

The reader may be more familiar with the term “margin requirement” used in financial markets and the empirical literature. We relate margin and collateral requirements below. It is notationally simpler to write the model in terms of collateral requirements, $k_{a(j)}^j(\sigma)$.

Following Geanakoplos and Zame (2002), we assume that an agent can default on her earlier promises without declaring personal bankruptcy.³ In this case the agent does not incur any penalties but loses the collateral she had to put up. In turn, the buyer of the financial security receives the collateral associated with the initial promise. Since there are no penalties for default, an agent who sold security j at date-event s^{t-1} defaults on her promise at a successor node s^t whenever the initial promise exceeds the current value of the collateral, that is, whenever

$$1 > k_{a(j)}^j(s^{t-1}) (q_{a(j)}(s^t) + d_{a(j)}(s^t)).$$

The payment by a borrower of security j at node s^t is, therefore, always given by

$$f_j(s^t) = \min \left\{ 1, k_{a(j)}^j(s^{t-1}) (q_{a(j)}(s^t) + d_{a(j)}(s^t)) \right\}.$$

Our model includes the possibility of costly default. This feature of the model is meant to capture default costs such as legal cost or the physical deterioration of the collateral asset. For example, it is well known that housing properties in foreclosure deteriorate because of moral hazard, destruction, or simple neglect. We model such costs by assuming that part of the

³Examples of such arrangements include pawn shops and the housing market in many U.S. states, in which households are allowed to default on their mortgages without defaulting on other debt.

collateral value is lost and thus the payment received by the lender is smaller than the value of the borrower's collateral. Specifically, the loss is proportional to the difference between the face value of the debt and the value of collateral, that is, the loss is

$$l_j(s^t) = \lambda \left(1 - k_{a(j)}^j(s^{t-1}) (q_{a(j)}(s^t) + d_{a(j)}(s^t)) \right)$$

for some parameter $\lambda \geq 0$. The resulting payment to the lender of the loan in security j when $f_j(s^t) < 1$ is thus given by

$$r_j(s^t) = \max \{0, f_j(s^t) - l_j(s^t)\} = \max \left\{ 0, (1 + \lambda) k_{a(j)}^j(s^{t-1}) (q_{a(j)}(s^t) + d_{a(j)}(s^t)) - \lambda \right\}.$$

If $f_j(s^t) = 1$ then $r_j(s^t) = f_j(s^t) = 1$. This repayment function does not capture all costs associated with default. For example, it does not allow for fixed costs which are independent of how much the collateral value falls short of the repayment obligation. However, our functional form offers the advantage that the resulting model remains tractable since the repayment function is continuous in the value of the collateral.

2.1.2 Margin Requirements and Collateral

An agent selling one unit of bond j with price $p_j(s^t)$ must hold collateral worth at least $k_{a(j)}^j(s^t) q_{a(j)}(s^t)$. The difference between the value of the collateral holding and the current value of the loan is the amount of capital an agent must put up to obtain the loan. The collateral requirement $k_{a(j)}^j(s^t)$ thus imposes a lower bound $m_{a(j)}^j(s^t)$ on this capital-to-value ratio,

$$m_{a(j)}^j(s^t) = \frac{k_{a(j)}^j(s^t) q_{a(j)}(s^t) - p_j(s^t)}{k_{a(j)}^j(s^t) q_{a(j)}(s^t)}. \quad (1)$$

Using language from financial markets, we call these bounds ‘margin requirements’ throughout the remainder of the paper. Equation (1) provides the definition of the term ‘margin’ according to Regulation T of the Federal Reserve Board. However, there does not appear to be a unified definition of this term. For example, in CGFS (2010) the term $m_{a(j)}^j(s^t)$ is called a ‘haircut’ and instead the capital-to-loan ratio

$$\frac{k_a^j q_a(s^t) - p_j(s^t)}{p_j(s^t)}$$

is described as a ‘margin requirement.’ Here, we use the definition and terminology according to Regulation T. It should be noted that, contrary to the unbounded capital-to-loan ratio, the capital-to-value ratio is bounded above by one.

To simplify the exposition of our model, we state agents’ trading restrictions as well as the payoff functions of the bonds in terms of the collateral requirements $k_{a(j)}^j(s^t)$. However, only the margin requirements $m_{a(j)}^j(s^t)$ are usually mentioned on financial markets. Therefore, we report these margin requirements in our results section below.

The specification of the margin requirements $m_{a(j)}^j(s^t)$ for bond j across date-events s^t has important implications for equilibrium prices and allocations. In this paper, we examine two

different rules for the determination of margin requirements and the resulting collateral levels. The first rule determines endogenous margin requirements along the lines of Geanakoplos and Zame (2002). The second rule assumes exogenously regulated margin requirements.

2.1.3 Default and Endogenous Margin Requirements

One of the contributions of this paper is to endogenize margin requirements in an infinite-horizon dynamic general equilibrium model. For this purpose, our first collateral rule follows Geanakoplos (1997) and Geanakoplos and Zame (2002) who suggest a simple and tractable way to endogenize margin requirements. They assume that, in principle, financial securities with any margin requirement could be traded in equilibrium. Only the scarcity of available collateral leads to equilibrium trade in only a small number of such securities.

To formalize this approach, recall that the S direct successors of a node s^t are denoted $(s^t, 1), \dots, (s^t, S)$ and that $\mathcal{J}_a$ denotes the set of all bonds collateralized by the same tree a . We define endogenous margin requirements for bonds $j \in \mathcal{J}_a$ collateralized by the same tree $a \in \mathcal{A}$ as follows. For each shock next period, $s' \in \mathcal{S}$, there is a bond which satisfies $k_{a(j)}^j(s^t) (q_{a(j)}(s^t, s') + d_{a(j)}(s^t, s')) = 1$. This bond defaults precisely in those states in which the cum-dividend price of the tree is lower than in the state s' . The bond which defaults in all states but the one with the highest cum-dividend price is redundant because its return (net of default cost) is identical to the tree return. The payoffs of the remaining $S - 1$ bonds and of tree a are independent. Therefore, the defaultable bonds greatly enhance risk-sharing opportunities. In the absence of default cost, agents typically trade in these $S - 1$ bonds in equilibrium.⁴

The inclusion of default cost makes defaultable bonds less attractive. In fact, we show in Appendix B.3 that agents no longer trade default bonds in the presence of moderate default costs. Then only a single bond collateralized by tree a is traded in equilibrium; this bond's collateral requirements are endogenously set to the lowest possible value that still ensures no default in the subsequent period. This specification is similar to the collateral requirements in Kiyotaki and Moore (1997). Formally, the resulting condition for the collateral requirement $k_{a(1)}^1(s^t)$ of this bond is

$$k_{a(1)}^1(s^t) \left(\min_{s^{t+1} \succ s^t} (q_{a(1)}(s^{t+1}) + d_{a(1)}(s^{t+1})) \right) = 1.$$

We refer to this bond as the ‘risk-free’ or ‘no-default’ bond.

2.1.4 Regulated Margin Requirements

The second rule for setting margin requirements relies on regulated capital-to-value ratios. A (not further modeled) regulating agency now requires debtors to hold a certain minimal amount of capital relative to the value of the collateral they hold. Put differently, the regulator imposes

⁴The arguments in Araújo et al. (2010) show that adding additional bonds with other collateral requirements (also only using tree a as collateral) do not change the equilibrium allocation. In the presence of $S - 1$ bonds as specified above, any bond with an intermediate collateral requirement can be replicated by holding a portfolio of the described bonds and tree a using the same amount of collateral.

a margin restriction $m_{a(j)}^j(s^t)$. If the margin requirement is regulated to be $m_{a(j)}^j(s) < 1$ in shock $s \in \mathcal{S}$ and constant over time, then it follows from (1) that the collateral requirement at each node s^t is

$$k_{a(j)}^j(s^t) = \frac{p_j(s^t)}{q_{a(j)}(s^t)(1 - m_{a(j)}^j(s^t))}. \quad (2)$$

Note that, contrary to the exogenously regulated margin requirement, the resulting collateral level $k_{a(j)}^j(s^t)$ is endogenous since it depends on equilibrium prices. If the margin requirement is one, $m_{a(j)}^j(s) = 1$, then the tree cannot be used as collateral.

2.1.5 Financial Markets Equilibrium with Collateral

We are now in the position to formally define the notion of a financial markets equilibrium. To simplify the statement of the definition, we assume that for a set of trees $\hat{\mathcal{A}} \subset \mathcal{A}$ margin requirements are endogenous, that is for each $\hat{a} \in \hat{\mathcal{A}}$, there exist a set $\mathcal{J}_{\hat{a}}$ of $\mathcal{S}$ bonds for which this tree can be used as collateral. For all other trees, margins are exogenously regulated – if they are set to 100 percent, the tree cannot be used as collateral. It is helpful to define the terms $[\phi_j^h]^+ = \max(0, \phi_j^h)$ and $[\phi_j^h]^- = \min(0, \phi_j^h)$. We denote equilibrium values of a variable x by $\bar{x}$.

DEFINITION 1 *A financial markets equilibrium for an economy with initial shock s_0 and initial tree holdings $(\theta^h(s^{-1}))_{h \in \mathcal{H}}$ is a collection of agents' portfolio holdings and consumption allocations as well as security prices, payouts of financial securities to lender and borrower, and collateral requirements for all one-period financial securities $j \in \mathcal{J}$*

$$\left((\bar{\theta}^h(\sigma), \bar{\phi}^h(\sigma), \bar{c}^h(\sigma))_{h \in \mathcal{H}} ; (\bar{q}_a(\sigma))_{a \in \mathcal{A}} , (\bar{p}_j(\sigma))_{j \in \mathcal{J}} ; (\bar{r}_j(\sigma), \bar{f}_j(\sigma))_{j \in \mathcal{J}} ; (\bar{k}_{a(j)}^j(\sigma))_{j \in \mathcal{J}} \right)_{\sigma \in \Sigma}$$

satisfying the following conditions:

(1) *Markets clear:*

$$\sum_{h \in \mathcal{H}} \bar{\theta}^h(\sigma) = 1 \quad \text{and} \quad \sum_{h \in \mathcal{H}} \bar{\phi}^h(\sigma) = 0 \quad \text{for all } \sigma \in \Sigma.$$

(2) *For each agent h , the choices $(\bar{\theta}^h(\sigma), \bar{\phi}^h(\sigma), \bar{c}^h(\sigma))$ solve the agent's utility maximization problem,*

$$\begin{aligned} \max_{\theta \geq 0, \phi, c \geq 0} U_h(c) \quad \text{s.t.} \quad & \text{for all } s^t \in \Sigma \\ c(s^t) &= e^h(s^t) + \sum_{j \in \mathcal{J}} ([\phi_j(s^{t-1})]^+ \bar{r}_j(s^t) + [\phi_j(s^{t-1})]^- \bar{f}_j(s^t)) + \\ & \quad \theta^h(s^{t-1}) \cdot (\bar{q}(s^t) + d(s^t)) - \theta^h(s^t) \cdot \bar{q}(s^t) - \phi^h(s^t) \cdot \bar{p}(s^t) \\ 0 &\leq \theta_a^h(s^t) + \sum_{j \in \mathcal{J}_a} \bar{k}_a^j(s^t) [\phi_j^h(s^t)]^-, \quad \text{for all } a \in \mathcal{A}. \end{aligned}$$

(3) For all s^t :

(i) For each $\hat{a} \in \hat{\mathcal{A}}$, there exists for each state $s' \in \mathcal{S}$ a financial security j such that $\hat{a} = a(j)$ and

$$\bar{k}_{\hat{a}}^j(s^t) (\bar{q}_{\hat{a}}(s^t, s') + d_{\hat{a}}(s^t, s')) = 1.$$

(ii) For each $\tilde{a} \notin \hat{\mathcal{A}}$ the collateral requirement $\bar{k}_{\tilde{a}}^j(s^t)$ of the unique bond j with $\tilde{a} = a(j)$ and the given margin requirement $m_{\tilde{a}}^j(s_t)$ satisfies

$$\bar{k}_{\tilde{a}}^j(s^t) = \frac{\bar{p}_j(s^t)}{\bar{q}_{\tilde{a}}(s^t)(1 - m_{\tilde{a}}^j(s_t))}.$$

(4) The payoffs of the financial securities are given by

$$\bar{f}_j(s^t) = \min \left\{ 1, k_{a(j)}^j(s^{t-1}) (q_{a(j)}(s^t) + d_{a(j)}(s^t)) \right\}$$

and

$$\bar{r}_j(s^t) = \begin{cases} \max \left\{ 0, (1 + \lambda) k_{a(j)}^j(s^{t-1}) (q_{a(j)}(s^t) + d_{a(j)}(s^t)) - \lambda \right\} & \text{if } \bar{f}_j(s^t) < 1 \\ 1 & \text{if } \bar{f}_j(s^t) = 1. \end{cases}$$

The approach in Kubler and Schmedders (2003) can be used to prove existence. The only non-standard part—besides the assumption of recursive utility, which can be handled easily—is the assumption of default costs. Note, however, that our specification of these costs still leaves us with a convex problem and standard arguments for continuity of best responses go through.

To approximate equilibrium numerically, we use the algorithm developed in Brumm and Grill (2010). In Appendix C, we describe the computations and the numerical error analysis in detail. For the interpretation of the results it is useful to understand the recursive formulation of the model. The natural endogenous state-space of this economy consists of all agents' beginning-of-period financial wealth as a fraction of total financial wealth (i.e. value of the trees cum dividends) in the economy. That is, we keep track of the current shock s_t and of agents' wealth shares

$$\omega^h(s^t) = \frac{\sum_{j \in \mathcal{J}} \left([\phi_j^h(s^{t-1})]^+ r_j(s^t) + [\phi_j^h(s^{t-1})]^- f_j(s^t) \right) + \theta^h(s^{t-1}) \cdot (q(s^t) + d(s^t))}{\sum_{a \in \mathcal{A}} (q_a(s^t) + d_a(s^t))}.$$

As in Kubler and Schmedders (2003), we assume that a recursive equilibrium on this state space exists and compute prices, portfolios and individual consumptions as a function of the exogenous shock and the distribution of financial wealth. In our calibration we assume that shocks are i.i.d. and that these shocks only affect the aggregate growth rate. In this case, policy and pricing functions are independent of the exogenous shock, thus depend on the wealth distribution only, and our results can easily be interpreted in terms of these functions.

2.2 The calibration

We calibrate our model to annual US data. The aggregate endowment grows at a stochastic rate, calibrated by six exogenous growth shocks including three disaster shocks. There are two types of agents in the economy. The first type receives fifteen percent of the total labor income and is much less risk-averse than the second type which receives the remaining eighty-five percent of the aggregate labor income. Both types have the same intertemporal elasticity of substitution. Finally, default costs are derived from figures of the U.S. housing market. In the remainder of this section we describe the details of the calibration of our baseline economy which, for simplicity, we call *CC: Collateral Constraints*.

2.2.1 Growth rates

The aggregate endowment at date-event s^t grows at the stochastic rate $g(s_{t+1})$ which (if no default costs are incurred) only depends on the new shock $s_{t+1} \in \mathcal{S}$. So, if either $\lambda = 0$ or $f_j(s_{t+1}) = 1$ for all $j \in \mathcal{J}$, then

$$\frac{\bar{e}(s^{t+1})}{\bar{e}(s^t)} = g(s_{t+1})$$

for all date-events $s^t \in \Sigma$. If there is default in s_{t+1} , then the endowment $\bar{e}(s_{t+1})$ is reduced by the costs of default and the growth rate is reduced accordingly.

There are $S = 6$ exogenous shocks. We declare the first three of them, $s = 1, 2, 3$, to be disasters. We calibrate the disaster shocks to match the first three moments of the continuous distribution of consumption disasters estimated by Barro and Jin (2012) who use data from Barro and Ursúa (2008). Also following Barro and Jin, we choose transition probabilities such that the six exogenous shocks are i.i.d. The non-disaster shocks, $s = 4, 5, 6$, are then calibrated such that their standard deviation matches “normal” U.S. business cycle fluctuations with a standard deviation of 2 percent and an average growth rate of 2.5 percent, which results in an overall average growth rate of about 2 percent. We sometimes find it convenient to call shock $s = 4$ a “recession” since $g(4) = 0.966$ indicates a moderate decrease in aggregate endowments. Table I provides the resulting growth rates and probability distribution for the six exogenous shocks of the economy.

Shock s	1	2	3	4	5	6
$g(s)$	0.566	0.717	0.867	0.966	1.025	1.089
$\pi(s)$	0.005	0.005	0.024	0.065	0.836	0.065

Table I: Growth rates and distribution of exogenous shocks

In our results sections below, we report that collateral requirements have a quantitatively strong impact on equilibrium prices. Clearly, the disaster shocks play an important role in generating these effects. However, the sensitivity analysis in Appendix A shows that also in economies with much less severe disaster shocks, collateral constraints substantially increase

asset return volatility. This fact is present even in simulations of the economy during which no disaster shocks occur.

2.2.2 Dividends

For our quantitative analysis, we need to take a stand on what the Lucas trees in our economy represent. In our model, trees have three distinguishing features: They are a claim to aggregate capital income, they can be traded without transaction costs and some of them can serve as collateral for borrowing on margin. We want to think of trees as including the stock market and aggregate housing as well as being part of the corporate bond market, but want to focus our calibration on the margin-eligibility of trees. For the sake of simplicity, we do not model the trees' dividends to have stochastic characteristics different from aggregate consumption. Formally, for each tree a , we set $d_a(s^t) = \delta_a \bar{e}(s^t)$, where δ_a measures the size of the tree.

To determine the dividend share of aggregate income, $\sum_a \delta_a$, and to gain a sense of the historical margin requirements on different long-lived assets, we follow Chien and Lustig (2010) and use Table 1.2 of the National Income and Product Accounts (NIPA) to determine the share of national income that is derived from collateralizable assets. We use annual NIPA data starting from 1947 (the year when Regulation T was first used to tighten margins for borrowing on stocks) until 2010 and report (unweighted) arithmetic averages below.

Chien and Lustig (2010) define collateralizable income in a narrow sense as the sum of 'rental income of persons with capital consumption adjustment', 'net dividends' and 'net interest'. In NIPA data, rental income includes the imputed rental income of owner-occupants of nonfarm dwellings. This figure is net of mortgage payments which are included in the category interest payments. Net interest also includes net interest paid by private businesses, but does not include interest paid by the government. Between 1947 and 2010, the average share of this narrowly-defined collateralizable income was about 10.5 percent. This definition of collateralizable income does not include proprietary income which constitutes a large share of income (about 10 percent, on average, between 1947 and 2010). However, it is difficult to assess what portion of this income is derived from assets that can be easily traded and fully collateralized. Up until the early 1980s, a significant share of this income was farm income, but nowadays it is almost entirely non-farm income. It obviously includes income from partnerships such as law firms or investment banks, which is certainly neither tradable nor collateralizable, but it also includes income of sole proprietorships and partnerships engaged in the real estate business. In our baseline calibration we partly include this income into the trees' dividends and thus set $\sum_a \delta_a = 0.15$. However, we perform an extensive sensitivity analysis with respect to the aggregate dividend share in Section 4.2.

Margin requirements differ depending on the underlying asset that is used for collateral. Fortune (2000) reviews margin requirements for different financial assets. The Board of Governors of the Federal Reserve System establishes initial margin requirements for stocks under Regulation T. As Fortune points out, amendments to this regulation in 1996 and 1998 also regulate margins on convertible corporate bonds, while the regulation sets no margins on non-convertible corporate bonds and mortgage-related securities. Mortgages are largely unregulated. In NIPA

data, the average share of dividend income for the time period 1947–2010 is about 3.33 percent.⁵ This fraction is smaller than the values typically assumed in the literature (these values range from 4–5 percent, see, e.g., Heaton and Lucas (1996)) since this number does not include retained earnings. However, a large fraction of the stock market is held in retirement accounts which do not allow for margin loans. In our analysis of empirical results in Section 5 below, we assume that one tree represents the aggregate stock market and set its dividend share to four percent. According to NIPA data, rental income constituted, on average, about 2.3 percent and net interest about 4.9 percent of total income for the time period 1947–2010. As mentioned above, rental income is net of mortgage payments which are added to net interest. In our model, this cash flow needs to be counted into the value of housing. In order to simplify the analysis, we aggregate net interest, rental income, and part of the proprietary income to be the dividends of a second tree. This obvious simplification allows for a cleaner analysis of different collateral requirements for different assets.

2.2.3 Endowment shares

There are $H = 2$ types of agents in the economy, the first type, $h = 1$, being less risk-averse than the second. Each agent h receives a fixed share of aggregate endowments as individual endowments, that is, $e^h(s^t) = \eta^h \bar{e}(s^t)$. We abstract from idiosyncratic income shocks because it is difficult to disentangle idiosyncratic and aggregate shocks for a model with two types of agents.

We assume that agent 1 receives 15 percent of all individual endowments, and agent 2 receives the remaining 85 percent of all individual endowments. Since we set $\sum_a \delta_a = 0.15$ this assumption implies that $\eta^1 = 0.1275$ and $\eta^2 = 0.7225$. As we assume below that agent 1 is less risk-averse, she holds the Lucas trees most of the time along the equilibrium path. Therefore the labor income share of agent 1 is chosen to roughly match the fraction of agents in the US population that holds substantial amounts of stocks outside of retirement accounts. It is often claimed, see e.g. Vissing-Jørgensen and Attanasio (2003), that about 20 percent of the US population holds stocks. However, many of these households have only small stock investments, see Poterba et al. (1995). Therefore, we choose 15 percent; however, in Appendix B.2 we report results for an economy in which type 1 agents receive 25 percent of individual endowments. We observe that the qualitative insights of our analysis are robust to this change.

2.2.4 Utility parameters

The choice of an appropriate value for the IES is rather difficult. On the one hand, several studies that rely on micro-data find values of about 0.2 to 0.8; see, for example, Attanasio and Weber (1993). On the other hand, Vissing-Jørgensen and Attanasio (2003) use data on stock owners only and conclude that the IES for such investors is likely to be above one. Barro

⁵During the time period 1947–1974 when there were frequent changes in the margin requirement, the share of the narrowly defined collateralizable income was about 8.5 percent on average. Net dividends constituted on average 33% of this income.

(2009) finds that for a successful calibration of a representative-agent asset-pricing model the IES needs to be larger than one. In our benchmark calibration both agents have identical IES of 1.5, that is, $\rho^1 = \rho^2 = 1/3$.

Agent 1 has a risk aversion of $1/8$ while agent 2's risk aversion is 8. Recall the weights for the two agents in the benchmark calibration, $\eta^1 = 0.1275$ and $\eta^2 = 0.7225$. The majority of the population is therefore very risk-averse, while 15 percent of households have very low risk aversion. Recall that this number is chosen to match observed stock-market participation as we have discussed above. Finally, we set $\beta^h = 0.93$ for both $h = 1, 2$, because it results in a good match for the annual risk-free rate.

In Appendix B we also perform various sensitivity tests. In particular we consider the case of both agents having an IES of 0.5. For this specification, the quantitative results are much weaker compared to the benchmark calibration, but the qualitative insights remain intact. We also consider different specifications for risk aversion and discounting.

2.2.5 Default costs

In our baseline calibration, we assume positive default costs of 25%. In the sensitivity analysis in Appendix B.3, we show that different values for the costs of default do not change our main conclusions. Recall from the description in Section 2.1 that the cost is proportional to the difference of the face value of the bond and the value of the underlying collateral. Therefore, a proportional cost of 25 percent means a much smaller cost as a fraction of the underlying collateral. Campbell et al. (2011) find an average ‘foreclosure discount’ of 27 percent for foreclosures in Massachusetts from 1988 until 2008. This discount is measured as a percentage of the total value of the house. As a percentage of the difference between the house value and face value of the debt this figure would be substantially larger. A value of $\lambda = 0.25$, therefore, certainly seems realistic and is, if anything, too small when we compare it to figures from the U.S. housing market. It is difficult to assess default costs in securities markets. In these markets, agents cannot legally default on individual contracts. However, as Fortune (2000) writes, “Customers typically find reasons to dispute their liability, and while the requirement of binding arbitration of disputes tilts the scales in favor of brokers, it does not always avoid expensive litigation, nor does it always lead to successful recovery. This suggests that margin loans, while legally recourse loans, might be in a limbo, somewhere between recourse and non-recourse.” To simplify the analysis, we set default costs to be identical across the different trees.

3 Identical margin requirements across all assets

We begin our quantitative analysis by first considering economies where all long-lived assets have identical cash flows and margin requirements. Since all trees are identical, we can model them as a single Lucas tree. We show that scarce collateral has a large effect on the return volatility of this tree and examine how the magnitude of this effect depends on the specification of margin requirements. This section sets the stage for our analysis of economies with two trees in Sections 4 and 5 where we assume that assets differ as to their margin-eligibility.

3.1 Collateral and volatility

For an evaluation of the quantitative effects of scarce collateral in our baseline economy *CC: Collateral Constraints*, we benchmark our results against those for two much simpler models. In the model *B1: No bonds* agents cannot borrow. The model *B2: Unconstrained* is an economy in which agents can use their entire endowment as collateral. This model is equivalent to a model with natural borrowing constraints (and without short-sale constraints on the tree). Table II reports four statistics for each of the three economies, see Appendix C for a description of the simulation procedure. Throughout the paper we measure volatility by the average standard deviation of tree returns over a long horizon. Another meaningful measure is the average one-period-ahead conditional price volatility. These two measures are closely correlated for our models. In Table II we report both measures but omit the second one in the remainder of the paper. We also report average risk-free interest rates (RFR) and excess returns (ER). While our paper does not focus on an analysis of these measures, we do check them because we want to ensure that our calibration delivers reasonable values for these measures.

Model	STD returns	1-period price vol.	RFR	ER
B1: No bonds	5.46	4.98	n/a	n/a
B2: Unconstrained	5.51	5.03	8.64	0.17
CC: Collateral Constraints	8.02	7.27	1.62	4.80

Table II: Three economies with a single tree (all figures in percent)

Recall that in our calibration, agents of type 1 are much less risk averse than type 2 agents. In the long run, agent 1 holds the entire Lucas tree in model *B1* with no borrowing and agent 2 effectively consumes his labor-income. As a result, the tree price is determined entirely by the Euler equation of agent 1, and so its volatility is as low as in the model with a representative agent whose preferences exhibit very low risk aversion. The wealth distribution remains constant across all date-events. In the second benchmark model *B2*, the less risk-averse agent 1 holds the entire tree during the vast majority of time periods. A bad shock to the economy leads to shifts in the wealth distribution and a decrease of the tree price. However, these effects are small. Thus the resulting return volatility in model *B2* is only barely larger than the volatility in *B1*. In the model *B2* the risk-free rate is high and the equity premium is very low. Despite the presence of disaster shocks, the market price of risk is low because risk is borne almost entirely by agent 1 who has very low risk aversion.

Table II shows that both first and second moments show substantial differences when we compare models without collateral requirements to our model *CC* with tight collateral constraints. The most important result reported in Table II is that volatility in our baseline economy is 47 percent larger than in the two benchmark models without borrowing (*B1: No bonds*) and with natural borrowing constraints (*B2: Unconstrained*), respectively. The standard deviation of returns is 8.02 percent in the baseline economy *CC* but only 5.46 percent and 5.51

percent for the benchmark models *B1* and *B2*, respectively.

Collateral constraints drastically increase the volatility in the standard incomplete markets model. Figure I displays the time series of four key variables in a simulation for a time window of 200 periods. Recall that we consider a stochastic growth economy. Therefore, we report normalized tree prices, that is, equilibrium tree prices divided by aggregate consumption. Similarly, we report normalized bond positions. The first graph in Figure I shows the normalized tree price. The second graph displays agent 1's holding of the Lucas tree. The last two graphs show the risk-free interest rate and agent 1's (normalized) holding of the risk-free bond, respectively. In the displayed sample, the shock $s = 3$ (drop of aggregate consumption of 13.3 percent) occurs in periods 71 and 155 while shock 2 occurs in period 168 and the worst disaster shock 1 hits the economy in period 50.

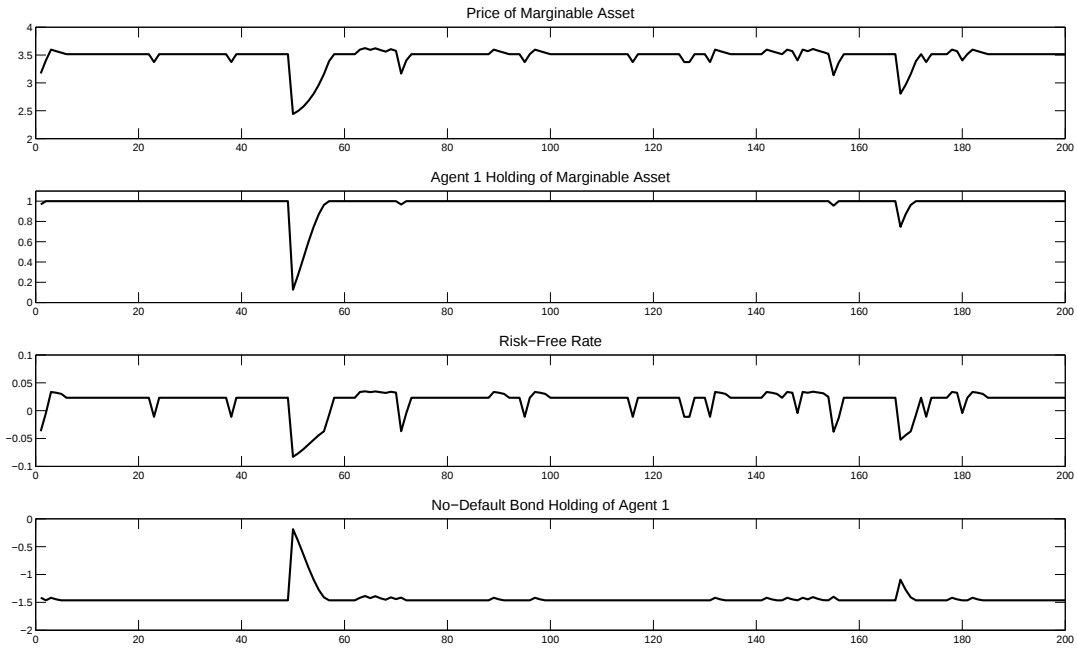


Figure I: Snapshot from a simulation of the baseline model

When a bad shock occurs, both the current dividend and the expected net present value of all future dividends of the tree decrease. As a result, the price of the tree drops, but in the absence of further effects, the normalized price should remain the same as we consider i.i.d. shocks to the growth rate. That is exactly what happens in the benchmark model *B1*. Figure I, however, indicates that additional effects occur in our baseline economy *CC*. First, note that agent 1 is typically leveraged; thus, when a bad shock happens, her beginning-of-period financial wealth falls relative to the financial wealth of agent 2. This effect is the strongest when the worst disaster shock 1 occurs. In this case, the wealth of agent 1 decreases to zero if she was fully leveraged in the previous period. The reason is that the margin requirement is determined such that in the worst shock the collateral is just sufficient to repay the loan. High leverage leads to large changes in the wealth distribution when bad shocks occur. The fact that collateral

is scarce in our economy now implies that these changes in the wealth distribution strongly affect equilibrium portfolios and prices. Since agent 1 cannot borrow against her future labor income, she can only afford to buy a small portion of the tree if her financial wealth is low. In equilibrium, therefore, the price has to be sufficiently low to induce the much more risk-averse agent 2 to buy a substantial portion of the tree. In addition to this within-period effect, there is a dynamic effect at work. As agent 1 is poorer today due to the bad shock, she will also be poorer tomorrow (except for state 1 when she has zero wealth anyway) implying that the price of the tree tomorrow is depressed as well. This further reduces the price that agent 2 is willing to pay for the tree today. Clearly, this dynamic effect is active not only for one but for several periods ahead, which is displayed in Figure I by the slow recovery of the normalized price of the tree after bad shocks. Figure I shows that the total impact of the above described effects is very strong for shock $s = 1$ but also large for shock 2. Note that the prices are normalized prices, so the drop of the actual tree price is much larger than displayed in the figure. In disaster shock 1, agent 1 is forced to sell almost the entire tree and the normalized price drops by almost 30 percent (the actual price drops by approximately 60 percent). In shock 2, she sells much less than half of her tree holdings but the price effect is still substantial. In shock 3, the price effect is still clearly visible, although agent 1 has to sell only very little.

3.2 Volatility with regulated margins

As the next step in the analysis of economies with a single collateralizable tree, we consider the case of regulated margin requirements. We assume that there is a regulatory agency setting minimal margin requirements. Margin requirements are constant across all shocks, so $m_{a(j)}^j(s^t)$ does not depend on the current date-event s^t .

As margin requirements increase, we observe two opposing effects. On the one hand, the amount of leverage decreases in equilibrium, leading to less de-leveraging after bad shocks which in turn leads to smaller price changes. On the other hand, the collateral constraint is more likely to become binding in equilibrium. This effect increases the probability of de-leveraging episodes which in turn leads to a higher tree return volatility. The solid line in Figure II displays the resulting tree return volatility. At a margin level of 60 percent, the implied collateral level uniformly exceeds the corresponding varying levels for the no-default bond under the rule of endogenous margin requirements in our baseline economy analyzed above. Therefore, the regulated bond is default-free for all values of $m_{a(j)}^j$ reported in Figure II. For margin levels between 60 and 90 percent, the return volatility of the long-lived asset exceeds 8 percent, which is the return volatility of the baseline economy *CC* with endogenous margin requirements, see Table II. Interestingly, for values of the margin level between 60 and 90 percent, the asset return volatility varies very little. The asset return volatility only drops significantly once the margin requirement exceeds 90 percent. Of course, as the margin level approaches 100 percent, the economy approaches the benchmark model *B1: No bonds* without borrowing and so volatility becomes small.

In sum, we find that margin regulation has a negligible effect on volatility within a large range of margin requirements. Within that range, volatility is substantially higher than with en-

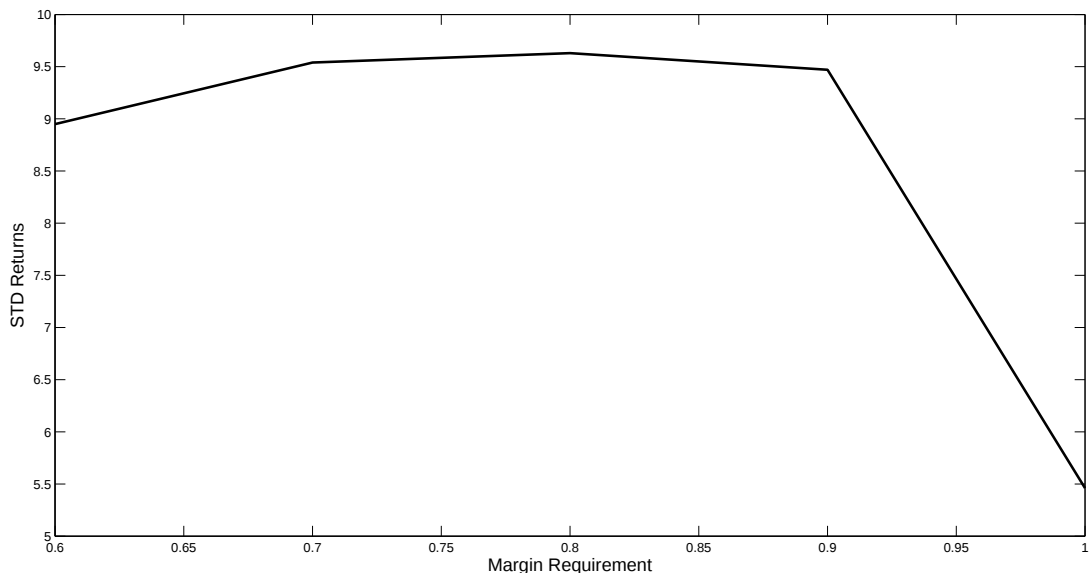


Figure II: Volatility as a function of the margin requirement

ogenous margin requirements and also much higher than in an economy without collateralized borrowing.

4 Marginable and non-marginable assets

Up until now, our analysis has focused on an economy with a single asset representing aggregate collateralizable wealth in the economy. However, households trade in various assets and durable goods. Some of them, e.g. houses, can be used as collateral very easily for loans at comparatively low interest rates. Other assets, e.g. stocks, can only be used as collateral for loans with relatively high margin requirements and comparatively high interest rates (see Fortune (2000)). Moreover, some stocks that were not margin-eligible in the past became margin-eligible in the 1970s and 1980s. Prior to 1968, over-the-counter securities were not eligible for margin trading. Under the Over The Counter Market Act of 1968, the Fed has the authority to determine which OTC securities are margin-eligible (see Seguin (1990)). These empirical facts clearly suggest that it is unrealistic to assume that the entire aggregate tree can be used as collateral. We now examine a model with two trees having identical dividends, one of them being margin eligible and the other one not. The objective of the analysis is to understand the dependence of the assets' excess returns and return volatility on their margin-eligibility in the absence of other factors that may cloud the picture.

4.1 Margin-Eligibility and Volatility

We initially assume that the two trees have identical cash flows ($\delta_1 = \delta_2 = 7.5\%$) and differ only by the extent to which they can be used as collateral. This setup allows for a clean analysis of the effect of margin-eligibility. Margin requirements for one asset are endogenous while the other asset is regulated to be margin-ineligible. As before in the economy with a single asset, default costs of $\lambda = 0.25$ imply that there is no trade in defaultable bonds. We therefore restrict attention to an economy in which only the no-default bond (collateralized by the marginable asset) is traded.

	STD returns	ER	agg STD	agg ER	RFR	STD <i>B1</i>
Marginable asset	6.56	4.81	7.16	6.04	0.20	5.46
Non-marginable asset	8.75	8.81				

Table III: Moments of assets' returns with marginable and non-marginable assets

Table III reports moments of the two assets' returns as well as for the overall market. Compared to the baseline economy with a single tree in Section 3, aggregate return volatility is now considerably lower, so volatility is no longer 47%, but rather 31% higher than in the benchmark *B1: No bonds*. More important, the two trees exhibit substantially different returns despite their identical dividends. The asset that can be used as collateral has a lower return volatility as well as a much lower average excess return than the asset that cannot be used as collateral.

To understand the price dynamics of the two trees, we examine the analogue of Figure I for the same sequence of shocks to the growth rate. Figure III shows the time series of eight key economic variables. The first two graphs show the (normalized) price and the first agent's holding of the marginable tree. The next two graphs display the corresponding values for the non-marginable tree. The fifth graph shows the risk-free interest rate and the sixth graph the first agent's (normalized) bond holding. These graphs illustrate three important features of the equilibrium. First, the volatility of the marginable tree is much lower than that of the single tree in the baseline economy. Secondly, the volatility for the non-marginable tree is larger than for the marginable one and its average price is much smaller. Last, agent 1 holds the marginable tree the entire time (except for a tiny blip after an occurrence of the worst disaster shock in period 50) but frequently sells the asset that is margin-ineligible. The second-to-last graph in the figure shows the endogenous margin requirement and the last graph depicts the collateral premium, which we define as follows: the difference between the price of the marginable tree and the price of a one-period asset which has the same payoff as this tree but cannot be used as collateral. Whenever agent 1 is unconstrained then the collateral premium is zero. However, when agent 1 becomes constrained, the collateral premium is significant.

Our observations lead us to a simple explanation for the prices and excess returns of the two assets. The margin-eligible asset is more valuable to agent 1 because of its collateral value — when agent 1 is fully leveraged the value of the asset exceeds next period's discounted (with

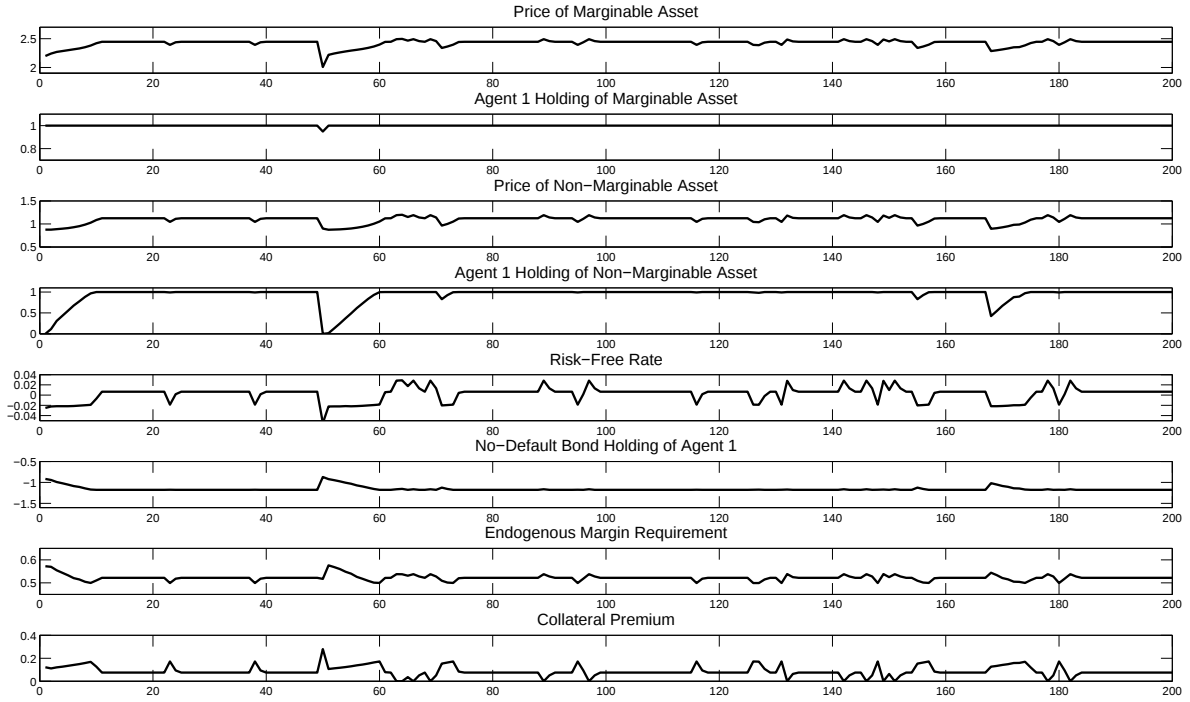


Figure III: A simulation of the model with marginable and non-marginable assets

agent 1's state prices) cum-dividend price, since it provides value for agent 1 as collateral. Since both assets have identical dividends, an agent can only be induced to hold the non-marginable asset if it pays a higher average return. The specific magnitude of the difference between the two asset prices is, of course, a quantitative issue. In our calibration with a reasonable market price of risk, the effect is indeed large: the average excess return of the non-marginable asset is almost 80% higher than that of the marginable asset.

There are several key factors that play a role in asset return volatility in the two-asset economy. For a discussion of these factors it is helpful to consider the policy and price functions in Figure IV. When faced with financial difficulties after a bad shock, agent 1 holds on to the marginable tree as long as possible, because this asset allows her to hold a short position in the bond. Therefore, the collateral value is one of the reasons why the marginable tree is much more valuable to agent 1. So, after suffering a reduction in financial wealth, agent 1 first sells the non-marginable tree. In fact, agent 1 only sells a portion of the marginable tree after she sold the entire non-marginable tree. In our sample path this happens only after the worst disaster shock hits in period 50. Of course, the policy functions in Figure IV show that it would happen in a more pronounced way after two or more consecutive disaster shocks, but such a sequence has extremely low probability. Whenever agent 1 sells a portion of a risky asset to agent 2, the price of that asset must fall, just as in the homogeneous margins baseline economy. So, one key factor contributing to the different volatility levels of the two assets is that the non-marginable tree is traded much more often and in larger quantities than the marginable one. If agent 1

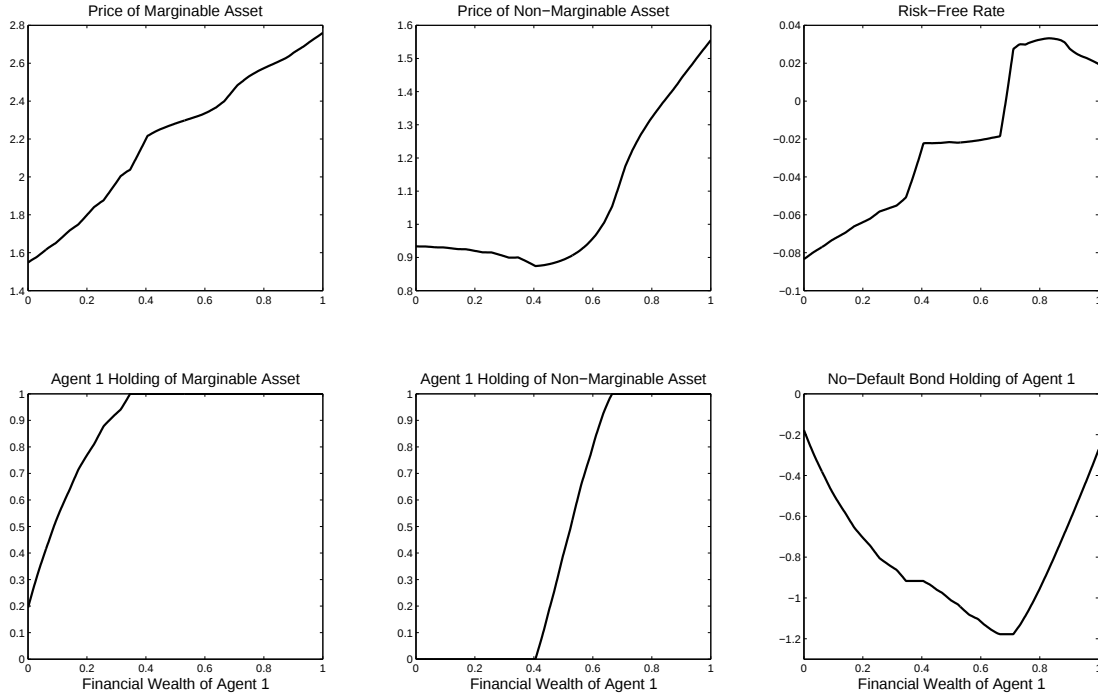


Figure IV: Prices and policy functions of the model with heterogeneous margins

holds both assets and then becomes poorer after a bad shock, the prices of both assets fall. But as she first sells the non-marginable asset, its price falls much faster than the price of the other asset. In fact, the price drop of the marginable asset is dampened by the onset of the collateral premium. This effect also contributes to the difference in the return volatilities of the two trees.

In the two-asset economy, only half of the aggregate tree can be used as collateral. This constraint limits the ability of agent 1 to leverage and consequently makes her less vulnerable to negative aggregate shocks. This factor reduces the individual return volatilities of both assets and thus also the volatility of the aggregated returns.

Finally, there is another key effect that was not present in the baseline economy with homogeneous margins. In ‘normal times’ agent 1 holds both assets but is fully leveraged. In a bad shock, agent 1 must sell part of the non-marginable asset which makes her poorer in the subsequent period. As a consequence, the margin requirement increases despite the fact that the interest rate decreases. This effect is clearly visible in the second-to-last graph of Figure III. Whenever a disaster occurs the endogenous margin requirement increases.

In sum, the fact that only five percent of aggregate output is collateralizable in this economy leads to a decrease in leverage and to smaller movements in the wealth distribution than in the one-tree baseline economy where ten percent of aggregate output is collateralizable. Consequently, the return volatility of the marginable asset is considerably smaller than that of the single asset in the baseline economy. By contrast, for the non-marginable asset this effect is counteracted through two channels. First, its price is not stabilized by a collateral premium

as it cannot be used as collateral. Second, a decrease in the holdings of the non-marginable asset leads to an increase in the margin requirements for loans on the marginable asset, which in turn forces agent 1 to sell more of the non-marginable asset. As a consequence, both the excess return and the return volatility are much larger for the non-marginable asset than for the marginable asset.

4.2 Volatility and the availability of collateral

Up until now, our analysis of the two-tree economy relied on the assumption that the marginable asset and the non-marginable asset have identical dividend shares of 7.5%. While this assumption offers the advantage of allowing for a clean comparison of margin-eligible and margin-ineligible assets, it has the disadvantage that it is not consistent with the stylized facts from Section 2.2.2. In preparation for the explanation of empirical findings in Section 5, we now examine the robustness of the qualitative insights of our analysis for changes in the amount of available collateral. For this purpose, we first show how the return volatility of the assets responds to a change in the aggregate dividend share. Subsequently, we report return volatilities for changes in the relative dividends of the two assets.

For our first robustness check, we change the aggregate dividend share $\delta_1 + \delta_2$ but maintain the assumption that the two individual shares remain identical, that is, $\delta_1 = \delta_2$. Figure V depicts the return volatilities of the two types of assets and of the aggregate market as a function of the aggregate dividend share. For very small dividend shares, there is little collateral in the economy and so the collateral constraint is almost always binding. The combined value of all long-lived assets is so small that agent 1 does not have to sell any assets even if the economy is hit by an extremely bad aggregate shock. The resulting return volatility is relatively small. As the aggregate dividend share increases the probability of the collateral constraint being binding decreases, but the effects of it being binding become stronger. Initially the second effect dominates and so all three volatilities increase until they all peak at dividend shares about 7.5%. As the aggregate dividend share increases further, return volatilities decrease with the return volatility of the marginable asset showing the biggest decrease. For all displayed values of the aggregate dividend share, the volatility of the non-marginable asset is substantially larger than that of the marginable asset.

As a second robustness check, we analyze changes in the fraction of long-lived assets that can be used as collateral while keeping the overall dividend share constant at $\delta_1 + \delta_2 = 15\%$. Our previous finding that the volatility of the non-marginable asset is larger than that of the marginable asset turns out to be robust to changes in the dividend shares of the two assets. However, Figure 6 shows that the difference between the two volatility levels depends on the share of the marginable asset. If this share is very small, there is almost no leverage in the economy and de-leveraging is not a problem, so the return volatilities of the two assets are small and do not differ much. As the share of the marginable asset increases from zero to 20 percent, the volatility of the non-marginable tree increases substantially, because agent 1 has to sell this asset in case of bad shocks. With a further increase from 20 to 50 percent, agent 1 takes on more and more leverage. To still hold on to the marginable tree in bad times, he has to reduce

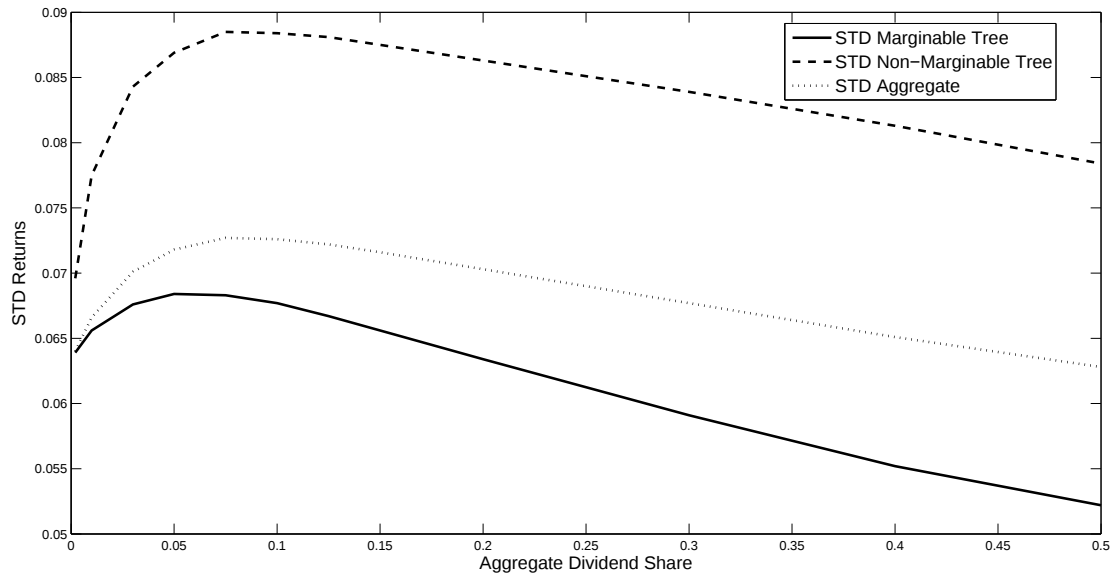


Figure V: Volatility as a function of the aggregate dividend share

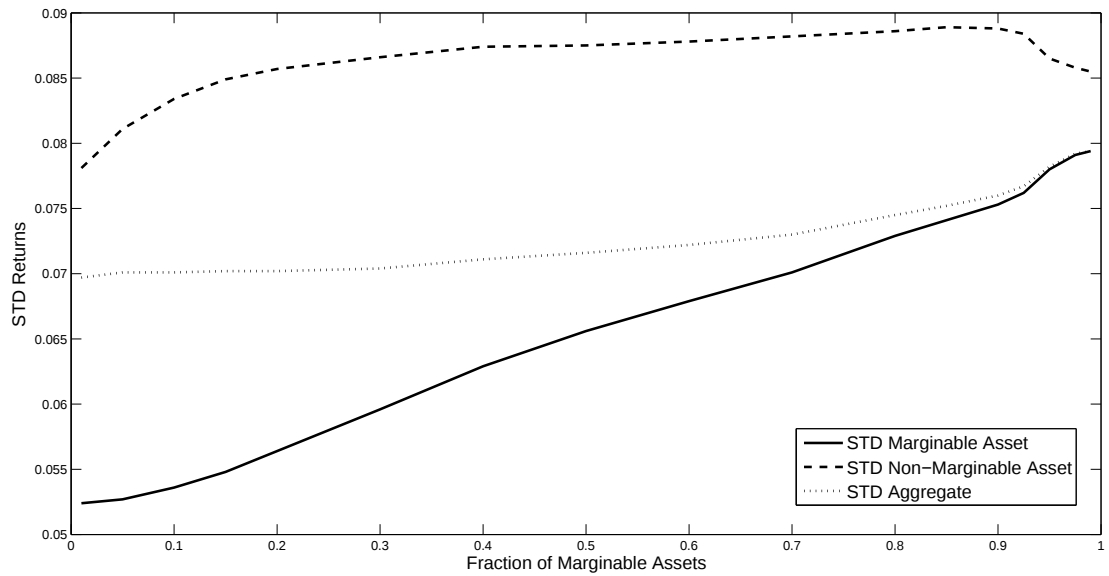


Figure VI: Volatility as a function of the share of marginable assets

consumption substantially, which reduces the price of the marginable tree. As a consequence, the volatility of the marginable tree increases substantially. Starting from a 50 percent share of the marginable tree, agent 1 now has to sell part of this tree in bad times. Therefore, both trees are now priced by agent 2 in bad times and thus their return volatilities differ less. Finally, as the non-marginable tree becomes tiny, it provides almost no buffer against bad shocks, which explains why the volatilities of the two trees approach each other as the share of the marginable tree goes to one.

5 Margin regulation: explaining the empirical findings

Our analysis in Sections 3 and 4 revealed the main mechanisms at work in our economic model of collateral constraints and margin requirements. We now show that our framework can qualitatively replicate the effects of margin regulation on the U.S. stock market and on individual assets as documented in the empirical literature such as, among others, Kupiec (1998), Fortune (2001), and Seguin (1990). For this purpose, we consider a model with two long-lived assets; a regulating agency sets the exogenous margin requirements for one of these assets but does not regulate the other.

5.1 Regulating the stock market

In Section 4 we discovered the significance of disaggregating the aggregate capital income into different long-lived assets. Such a disaggregation is clearly important when the different income streams result from assets with different collateral requirements. When U.S. households borrow against stocks or houses, they indeed face different margin requirements and different interest rates. The Board of Governors of the Federal Reserve System establishes initial margin requirements for broker-dealers under Regulation T. On the contrary, mortgage loans are largely unregulated. Also, interest rates on margin loans against stocks exceed mortgage rates.⁶

We now analyze a two-tree version of our economic model that allows for different margin regimes and takes into account all aspects of the empirical data discussed in Section 2.2. We assume that margins for one tree are set exogenously. We refer to this asset as the ‘regulated tree’ and want to interpret it as the aggregate stock market. Collateral requirements for the other tree are endogenous. We refer to this asset as the ‘unregulated tree’ and interpret its collateral features to resemble those of the housing market and other unregulated asset markets.

We want to explore the effects of a margin regulation in the stock market on stock-market volatility and on the prices of other collateralizable assets. Figure VII is a copy of Figure 1 in Fortune (2000) and shows the Regulation T margin requirements between 1940 and 2000 as set by the Federal Reserve Board.

⁶In 2011 and early 2012 margin rates of the discount broker Charles Schwab & Co. ranged from 8.5% for margin loans below \$25,000 to 6% for loans above \$2,500,000. See <http://www.schwab.com/public/schwab/investing/accounts.products/investment/margin.accounts> (accessed on January 27, 2012) By comparison, standard mortgage rates for 30-year fixed mortgage loans were below 4% in the U.S. in January 2012.

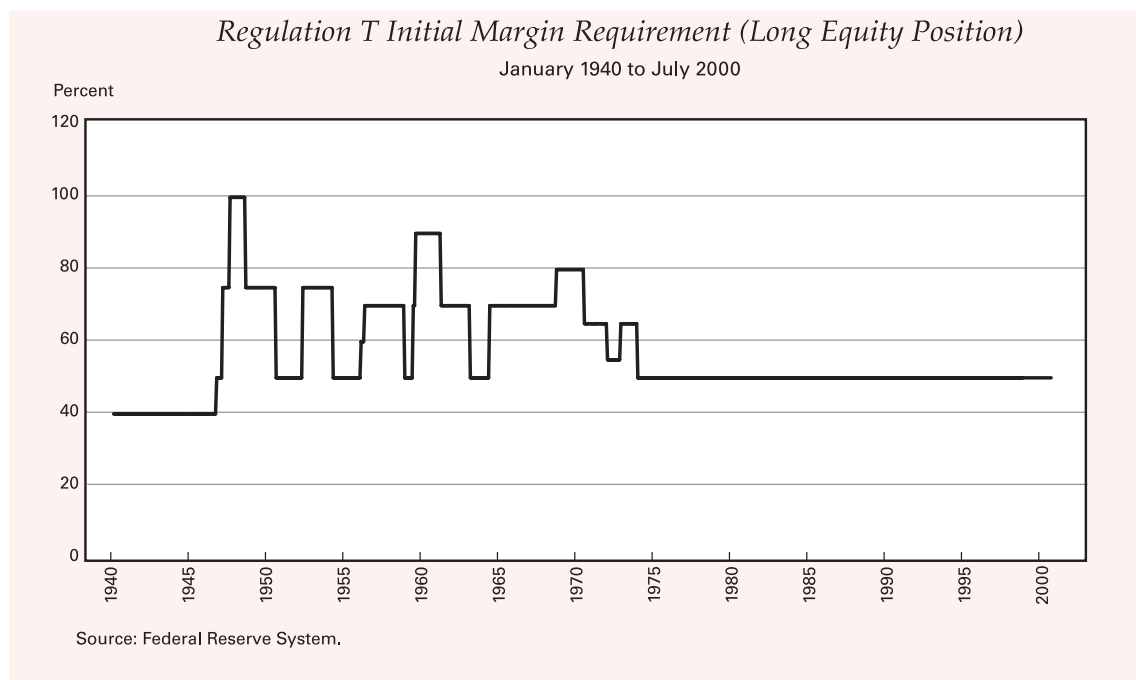


Figure VII: Historical Levels of Margin Requirements (from Fortune (2000))

Until 1974 the Fed changed initial margin requirements frequently in the range of 50 to 100 percent. Naturally, the question arises: What effects do such changes in the margin requirement have in our model? Our analysis in Section 3.2 indicates that for an economy in which the entire aggregate tree only consists of the stock market and no other assets, these changes in margin requirements should have had significant effects on volatility. Recall from Figure II that asset return volatility is flat until a margin requirement of about 90 percent and declines steeply thereafter. However, as we explain in Section 2.2.2, the stock market constitutes only a small part of all collateralizable assets in the US. And our analysis in Section 4 highlights the importance of an explicit consideration of different assets with different margin requirements. Therefore, we now assume that the regulated tree's dividends constitute 4 percent of the aggregate income and the unregulated tree's dividends 11 percent.

Figure VIII displays the volatility of both trees' returns as a function of the margin for the regulated tree. Most interestingly, over the entire range of values for the regulated margin requirement, the volatility of the regulated assets is almost flat. It initially increases slightly from 8.54 percent to 8.98 percent and then decreases very slightly to about 8.84 percent. Changes in the margin requirement of the stock market have a negligible effect on its volatility. As pointed out in the introduction, this result is in line with the empirical evidence cited in Fortune (2001) and Kupiec (1998), who document that the relationship between Regulation T margin requirements and the volatility of the stock market is weak. The result should not be interpreted as to mean that collateral requirements do not lead to excess volatility in asset markets. The point is instead that a uniform bound on margin requirements on some assets has little effect on their

volatility. Combined with the results from Section 3, this shows that margin regulation can substantially decrease volatility only if collateral requirements of many if not all relevant asset classes are regulated tightly.

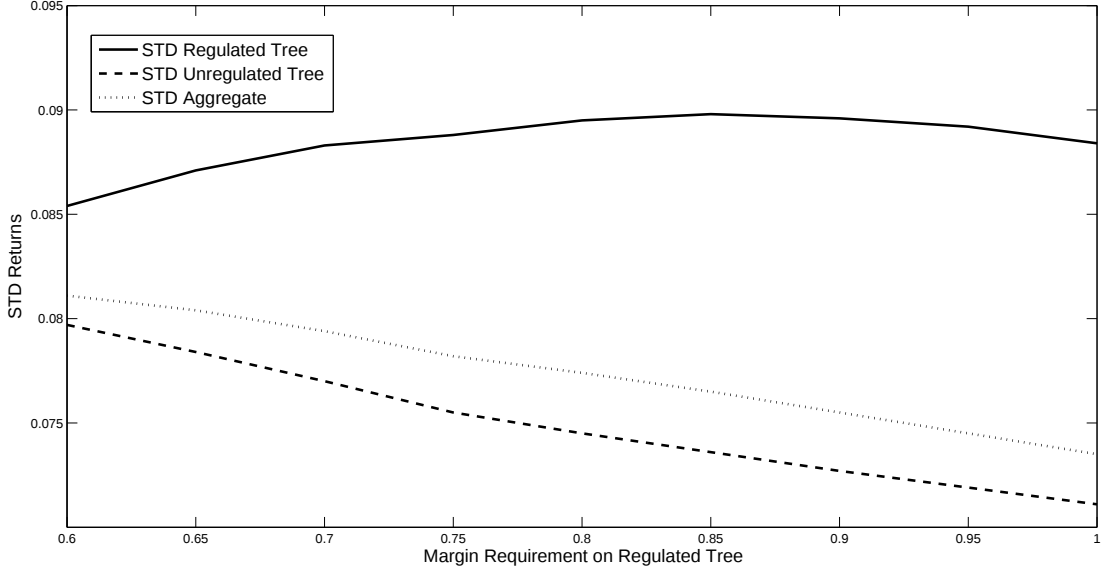


Figure VIII: Volatilities as a function of the margin requirement on the regulated tree

The reason for our findings is that an increase in the margin requirement of the regulated tree has two effects which approximately offset each other. As the margin requirement increases, the regulated tree becomes less attractive as collateral and at the same time the agents' ability to leverage decreases. These two effects influence (the much less risk-averse) agent 1's portfolio decisions after a bad shock occurs. First, when agent 1 must de-leverage her position, she sells the regulated tree first, as it has a lower collateral value than the unregulated tree. In equilibrium, this effect becomes more pronounced as the margin requirement on this regulated tree increases. Initially this effect leads to an increase in the return volatility of the regulated asset. However, the second effect, a reduced ability to leverage, generally decreases the return volatility of all assets and it also becomes more pronounced as the exogenously set margin increases. In our calibration, the two effects roughly offset each other and therefore a change in the margin requirement has almost no observable effect on the volatility of the regulated asset.

Another interesting result emerges when we consider the effects of margin changes on other asset classes. As Figure VIII shows, the volatility of the unregulated tree decreases monotonically as the margin requirement on the other tree is increased. The reason is that for the unregulated asset, both of the above described mechanisms work in the same direction. As agent 1's (aggregate) ability to leverage decreases, the return volatility of the unregulated tree decreases, resembling the effect on the regulated tree. However, as the margin for the regulated

tree increases, the unregulated asset becomes a relatively better collateral for agent 1. Hence, she now has an even stronger motive to hold on to the unregulated asset after a bad shock. So, there are strong spillover effects from the margin regulation of the regulated tree on the return volatility of the unregulated tree.

5.2 Regulating individual assets

As explained in detail in Fortune (2001), until 1968 bank loans against collateral in the form of over-the-counter (OTC) stocks were unrestricted, but brokers could not lend against OTC stocks. In 1969, the Federal Reserve System revised Regulation T to provide margin-eligibility for OTC stocks and to place bank loans under its margin regulation. Since that time, the Fed has written specific criteria for OTC stock margin eligibility, and OTC stocks that satisfy those criteria are placed on the so-called ‘List of Marginable OTC Securities’. Seguin (1990) examined the effect of additions to this list on the volatility of the underlying OTC stocks for the period 1976–1987. He finds that the return volatility of stocks fell significantly after becoming margin eligible.

Our model provides a possible explanation for this empirical finding. In Section 4 we show that the return volatility of margin-ineligible assets is significantly higher than that of margin-eligible assets. If a small stock becomes margin-eligible, the effect on aggregate prices is presumably negligible, but this stock changes from being part of the non-marginable tree with high volatility to being part of the margin-eligible tree with much lower volatility. Therefore, the volatility of the stock decreases substantially as it becomes margin-eligible. Clearly this thought experiment is only an approximation for a more refined examination. Such an analysis would require a model with three trees, one with regulated margins, one with endogenous margins and finally one that is margin-ineligible. We conjecture that the analysis of such a three-tree model would confirm the result of the thought experiment.

Another way to look at Seguin’s findings is to consider a model in which only a very small fraction of the aggregate asset market is regulated. The remaining large portion of the aggregate market is margin-eligible but unregulated, that is, it can be used for collateralized borrowing at endogenous margin levels. To examine such a model, we set the dividend share of the unregulated asset to 14.5% and that of the regulated asset to 0.5%. We vary the regulated margin requirement on the small asset between 60 and 100 percent. Figure IX shows the results of this variation on the return volatility of the regulated asset as well as on the return volatility of the remaining (unregulated) market.

In this setting, an asset’s return becomes less volatile as it becomes margin-eligible. The figure shows that the return volatility of the regulated tree is 8.61% percent when it cannot be used for borrowing on margin, while it decreases to 8.14% percent when it can be used for borrowing at a margin requirement of 60 percent. Based on our previous insights, we can easily explain this effect. As the margin requirement on the regulated asset decreases, this asset becomes more attractive for agent 1, who thus sells it less frequently after a bad shock. Not only does margin-eligibility decrease the volatility of a stock that is small relative to the market, our analysis shows that its volatility is monotonic in the margin requirement set for

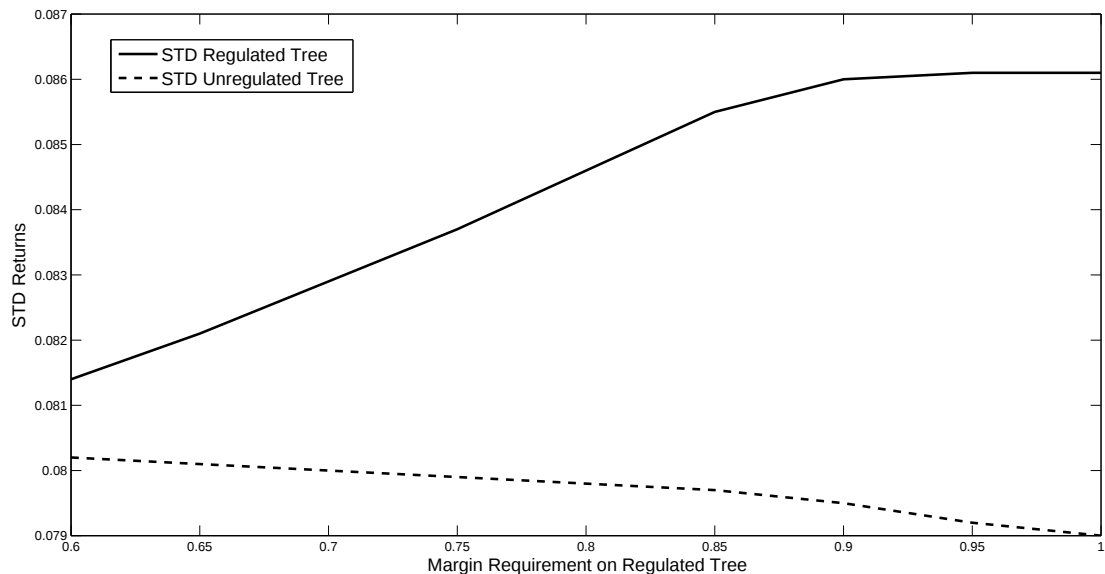


Figure IX: Volatilities as a function of the regulated margin of the small tree

borrowing against this asset.

The main message of this section is as follows. Despite the fact that collateral constraints lead to excess volatility, a tightening of margin requirements might have no effect on the asset's volatility (when the asset is a large part of the market) or might even increase its volatility substantially (when the asset is a small part of the market).

6 Conclusion

The three main contributions of this paper are as follows. First, we show in a calibrated general equilibrium model with disaster risk that collateral and margin requirements play a quantitatively important role for the prices of long-lived assets. Borrowing on margin leads to substantial excess volatility. Second, we reconcile this insight with the empirical evidence that the regulation of stock-market margin requirements under Regulation T had little effect on the volatility of stock prices. We show that tightening margins on all stocks has tiny quantitative effects on the volatility of the stock market. We also show that tightening margins on an individual stock that constitutes only a tiny fraction of all collateralizable assets increases the return volatility of this stock. Third, we discover important spillover effects from a regulation of margins on stocks. While the effect on the volatility of the regulated asset is generally small and its sign ambiguous, the volatility of other unregulated assets decreases monotonically as margins on regulated assets are increased.

The recent financial crisis has led researchers to suggest anew that central banks should

regulate collateral requirements; see, for example, Ashcraft et al. (2010) or Geanakoplos (2009). One of the important lessons of our analysis is that regulating margins, as done in the past via Regulation T, can only have the desired effects if central banks are able to regulate margin requirements for all relevant asset classes and not just of stocks.

The framework presented in this paper provides avenues for much additional analysis of margin policies. For instance, one may explore how effective different kinds of state contingent regulations, e.g. counter-cyclical ones, are in stabilizing asset prices.

Appendix

A The role of disaster shocks

Disaster shocks are a central feature of our calibration. Naturally the question arises how much the reported qualitative and quantitative economic consequences of collateral constraints depend on these extreme shocks. To answer this question, we conduct several analyses. We first document that our key results are even stronger in simulations during which disaster shocks do not occur. We then report results for models with less severe disaster shocks and demonstrate that the results remain qualitatively the same. We perform these robustness checks for the model specification with two trees of equal size; one tree is marginable, while the other tree is not.

A.1 No disaster shocks in simulations

We simulate our economy but assume that disaster shocks do not occur along the simulation paths. Table IV summarizes the results.

	STD returns	ER	agg STD	agg ER	RFR	STD B1
Marginable Tree	3.16	5.05	3.82	6.24	0.65	2.41
Non-marginable Tree	5.35	8.85				

Table IV: No disaster shocks in simulations (one tree is marginable, the second tree is not)

Not surprisingly, all return standard deviations are smaller than in the previously reported simulations (where disaster shocks occur), see Tables II and III. More strikingly, however, the aggregate return volatility increases by 59% as compared to the benchmark economy *B1*. This is almost twice the increase of about 31% that we get when disaster shocks occur along equilibrium paths, see Table III. In the absence of disaster shocks, endogenous price movements due to a binding collateral constraint are relatively more important than exogenous movements due to shocks. In the benchmark economy *B1*, endogenous price movements are absent and only exogenous shocks matter for volatility. In the two-tree economy with collateral constraints, however, the occurrence of standard recession shocks in the presence of binding collateral constraints force the first agent to reduce her consumption and to sell some of her assets. These trades in turn contribute significantly to the aggregate return volatility in the economy.

A.2 Halved probability of disaster shocks

As a second robustness check with respect to the assumption of disaster risk, we hold the magnitude of the disaster shocks constant but reduce the overall probability of a disaster by 50 percent: Instead of setting the probabilities of the disaster shocks 1, 2, and 3 to 0.005, 0.005, and 0.024, respectively, we set them equal to 0.0025, 0.0025, and 0.012, respectively, and increase the probability of shock 5 accordingly. Table V shows the analogous results to Table IV for this second robustness check.

	STD returns	ER	agg STD	agg ER	RFR	STD B1
Marginable Tree	4.88	3.57	5.37	4.75	2.03	4.23
Non-marginable Tree	6.47	7.17				

Table V: Halved probability of disaster shocks (one tree is marginable, the second tree is not)

Recall that the disaster states play two roles in our model. First, they exogenously lead to high return volatility of the trees. Second, they imply high endogenous margin requirements which endogenously increases the trees' return volatility further. As we decrease the probability of disaster, the second effect remains unchanged as long as the disaster probability remains nonzero. Only the first effect is diminished. As a consequence, the standard deviations and excess returns are lower, but the presence of collateral constraints still increases the standard deviation of returns by about thirty percent relative to the benchmark *B1*.

A.3 Half-sized disaster shocks

In a third and final robustness check, we reduce the severity of the three disaster shocks. In particular, we replace the growth rates in shocks 1, 2, and 3 of 0.566, 0.717, and 0.867, respectively, by 0.783, 0.8585, and 0.9335, respectively. In each of the three states the decline in the aggregate endowment is half as severe as before. We recalibrate the discount factor β to be 0.99. Table VI reports results from simulations for the altered economy.

	STD returns	ER	agg STD	agg ER	RFR	STD B1
Marginable Tree	4.76	0.44	5.18	0.52	1.87	3.36
Non-marginable Tree	6.11	0.68				

Table VI: Half-sized disaster shocks (one tree is marginable, the second tree is not)

The trees' return volatilities as well as their excess returns are substantially smaller than before. Contrary to a change in the probabilities, the change in the support of the disaster shocks mitigates both aforementioned effects of these bad shocks. However, collateral constraints still increase the standard deviation of returns substantially.

We briefly summarize the insights of our robustness checks on the impact of the disaster shocks in our economic model. The presence of disaster shocks has two key effects. First, the large exogenous volatility of the growth rate results in high excess returns of the trees. This effect is sensitive to both the likelihood as well as the magnitude of the bad shocks. When these

shocks become considerably less likely or less severe, then this effect is substantially reduced. Secondly, the mere possibility of a disaster leads to endogenously high margin requirements. This second effect is not sensitive to the likelihood of a disaster; however, it becomes less salient as the magnitude of the worst shock decreases.

B Sensitivity analysis

In this appendix, we first discuss how our results depend on the preferences and on the endowment shares of the two types of agents. In addition, we compute equilibria for varying degrees of default costs. These robustness checks deepen our understanding of the model mechanisms further. They are all carried out for homogeneous unregulated margin requirements.

B.1 Preferences

As a robustness check for the results in our baseline model (with one tree and one bond) from Section 3, we consider different specifications for the IES, the coefficients of risk aversion, and the discount factor, β . Obviously, changes in the IES and the risk aversion coefficients affect the risk-free rate. For these cases, we also examine specifications with an adjusted β so that the risk-free rates remain comparable. Table VII reports asset-price moments for several different combinations of these parameters. For convenience, we also state the results for our baseline model, $(IES, RA, \beta) = ((1.5, 1.5), (1/8, 8), (0.93, 0.93))$, and report them as the case (P1). For each model specification, we also report the standard deviation of returns for the benchmark case *B1: No bonds*.

$(IES^1, IES^2), (RA^1, RA^2), (\beta^1, \beta^2)$	STD returns	RFR	ER	STD in B1
(P1): (1.5,1.5),(1/8,8),(0.93,0.93)	8.02	1.62	4.80	5.46
(P2): (0.5,0.5),(1/8,8),(0.93,0.93)	5.96	1.94	4.18	5.34
(P3): (1.5,1.5),(1/8,8),(0.91,0.91)	7.81	3.40	4.88	5.57
(P4): (1.5,1.5),(1/8,8),(0.95,0.95)	8.31	0.09	4.71	5.34
(P5): (1.5,1.5),(1/10,10),(0.93,0.93)	8.93	-2.26	7.96	5.45
(P6): (1.5,1.5),(1/10,10),(0.88,0.88)	8.32	1.15	8.54	5.76
(P7): (1.5,1.5),(1/6,6),(0.93,0.93)	7.06	4.97	2.44	5.45
(P8): (1.5,1.5),(1/6,6),(0.97,0.97)	7.40	1.13	2.48	5.29
(P9): (1.5,1.5),(2,8),(0.93,0.93)	7.12	1.05	5.59	5.46

Table VII: Sensitivity analysis for preferences (all reported figures in percent)

In case (P2), a model in which both agents have an IES of 0.5, the tree return volatility is considerably lower than in the baseline case (P1). However, it is still higher than in an economy with the same preferences but without borrowing, see column *B1* of (P2). We also computed results for other values of the IES below 1.5 and always observed the same phenomenon: Volatility effects are qualitatively similar but quantitatively less pronounced.⁷

⁷For low values of the IES, there is an additional unwanted effect. As one agent holds most of the wealth

Next we consider a change in the discount factor β . For the benchmark case *B1*, a higher β decreases return volatility simply because it decreases levels of returns and we report absolute volatility as opposed to the coefficient of variation. The effects in our model with one tree and one bond are quite different. As β increases from 0.93 in our baseline case (P1) to 0.95 in (P4), the return volatility increases from 8.02 to 8.31. The reason for this increase is simple. As β increases and the stock becomes more expensive, it is more difficult for agent 1 to buy a significant portion of the stock when she is in financial difficulties. This fact depresses the price of the stock when agent 1 is poor. Changes in the wealth distribution are large when agent 1 is fully leveraged and now lead to larger swings in the tree price.

In the next sensitivity exercise, we change the risk-aversion of the two agents. In light of the intuition that we developed for the baseline case in Section 3, we expect an increase in the risk aversion of agent 2 to lead to both a higher return volatility and a higher equity premium. This intuition is strongly confirmed by the comparison of (P1) and (P5). However, the increase in the second agent's risk aversion also leads to a large reduction of the interest rate to unrealistically low levels. In (P6) we recalibrate the model to obtain a positive interest rate and we find that the previously described effect of a smaller β dampens the impact of a higher risk aversion. Still, overall volatility increases substantially once the risk aversion and β are changed simultaneously: For risk aversions of 4, 6, and 10, (cases (P8), (P1) and (P6)) the return volatility is 7.40, 8.02, and 8.32 respectively. Finally, in case (P9), we increase the risk aversion of the first agent to 2. This agent now demands a higher equity premium to hold the tree, and as she invests less aggressively volatility is eleven percent below its value in our baseline calibration. However, volatility is still thirty percent above the benchmark *B1: No bonds*. Thus, the strong impact of collateralized borrowing on volatility does not hinge on the risk aversion of agent 1 being below one.

B.2 Endowment shares

To understand the role played by the endowment share of the risk-tolerant agent, we increase this share from 15% to 25%, i.e. we compute the case $\eta^1 = 0.2125$, $\eta^2 = 0.6375$. Table VIII reports first and second moments of returns. Compared to the baseline economy (first line in Table VIII), the excess returns and the volatility decrease. As the debt position of the risk-tolerant agent becomes smaller relative to her endowments, the economy becomes less fragile.

(η_1, η_2)	STD returns	RFR	ER	STD in B1
(0.1275, 0.7225)	8.02	1.62	4.80	5.46
(0.2125, 0.6375)	6.80	2.93	3.51	5.46

Table VIII: Sensitivity analysis for endowment shares (all figures are in percent)

(that is, as the other agent becomes poor), asset prices increase because of the desire of the rich agent to save. This effect on the boundary of the state space is absent when the IES is set to 1.5, as in our baseline.

B.3 Default costs

Up until now, we assumed default cost of $\lambda = 25\%$. Table IX shows how the volatility of the tree and the trading volume of the default bonds depends on this cost parameter. The reported trading volume is the average absolute bond holding of agent 1 (which is the same as that of agent 2) over the simulation paths.

	$\lambda = 0$	$\lambda = 0.01$	$\lambda = 0.03$	$\lambda = 0.10$	$\lambda = 0.15$	$\lambda = 0.25$
STD returns	7.84	7.89	7.97	8.08	8.03	8.02
Total trading	1.534	1.497	1.457	1.421	1.438	1.433
No-default bond	1.437	1.418	1.389	1.380	1.422	1.433
1-default bond	0.051	0.048	0.046	0.041	0.016	0
2-default bond	0.024	0.029	0.022	0	0	0
3-default bond	0.019	0.002	0	0	0	0
4-default bond	0.003	0	0	0	0	0

Table IX: The effect of default costs on tree-return volatility and bond trading volume

In the absence of default costs ($\lambda = 0$), the average trading volume of all bonds is nonzero. Proportional default costs of as low as 10 percent ($\lambda = 0.1$) result in zero trade for the bonds defaulting in two or more states. For default costs of 25 percent, trade in any type of default bond ceases to exist. Therefore, we do not need to consider higher default costs, as only the risk-free bond is traded and the resulting equilibrium prices and allocations are identical to the case of $\lambda = 0.25$. Table IX also shows that the volatility of the tree returns remains almost stable as the costs of default change. Thus, the effect of default costs on volatility is comparably small in our setup.

C Details on computations

The algorithm used to solve all versions of the model is based on Brumm and Grill (2010). Equilibrium policy functions are computed by iterating on the per-period equilibrium conditions, which are transformed into a system of equations which we solve at each grid point. Policy functions are approximated by piecewise linear functions. By using fractions of financial wealth as the endogenous state variables, the dimension of the state space is equal to the number of agents minus one. Hence with two agents, the model has an endogenous state space of one dimension only. This makes computations much easier than in Brumm and Grill (2010), where two- and three-dimensional problems are solved. In particular, in one dimension reasonable accuracy may be achieved without adapting the grid to the kinks. For the reported results we used 640 grid points, which results in average (relative) Euler errors with an order of magnitude of 10^{-4} , while maximal errors are about ten times higher. If the number of gridpoints is increased to a few thousands, then Euler errors fall about one order of magnitude. However, the moments under consideration only change by about 0.1 percent. Hence, using 640 points provides a solution which is precise enough for our purposes. Compared to other models the

ratio of Euler errors to the number of grid points used might seem large. However, note that due to the number of assets and inequality constraints our model is numerically much harder to handle than standard models. For example, in the version with one tree and five bonds, eleven assets are needed (because long and short positions in bonds have to be treated as separate assets) and we have to impose eleven inequality constraints per agent.

The moments reported in the paper are averages of 100 different simulations with a length of 10,000 periods each (of which the first 100 are dropped). This is enough to let the law of large numbers do its job, even for the rare disasters.

D Equilibrium conditions

We state the equilibrium equations for economies with a single tree and a single bond. For our computation of financial markets equilibria we normalized all variables by the aggregate endowment $\bar{e}$. To simplify the notation, we drop the dependence on the date-event s^t and, in an abuse of notation, denote the normalized parameters and variables by e_t, d_t and c_t, q_t, p_t, r_t, f_t , respectively. Similarly, we normalize both the objective function and the budget constraint of agents' utility maximization problem. The resulting maximization problem is then as follows (index h is dropped).

$$\begin{aligned} \max \quad & u_t(c_t) = \left\{ (c_t)^\rho + \beta [E(u_{t+1}g_{t+1})^\alpha]^\frac{\rho}{\alpha} \right\}^\frac{1}{\rho} \\ \text{s.t.} \quad & 0 = c_t + \phi_t p_t + \theta_t q_t - e_t - [\phi_{t-1}]^+ \frac{r_t}{g_t} + [\phi_{t-1}]^- \frac{f_t}{g_t} - \theta_{t-1} (q_t + d_t) \\ & 0 \leq \theta_t + k_t [\phi_t]^-, \quad 0 \leq [\phi_t]^+, \quad [\phi_t]^- \leq 0, \end{aligned}$$

The latter two inequalities are imposed because, for the computations, we treat the long and short position in the bond, $[\phi_t]^+$ and $[\phi_t]^-$, as separate assets. Note that $\phi_t = [\phi_t]^+ + [\phi_t]^-$. Let λ_t denote the Lagrange multiplier on the budget constraint. The first-order condition with respect to c_t is as follows,

$$0 = (u_t)^{1-\rho} (c_t)^{\rho-1} - \lambda_t.$$

Next we state the first-order condition with respect to c_{t+1} .

$$0 = \beta u_t^{1-\rho} [E(u_{t+1}g_{t+1})^\alpha]^\frac{\rho-\alpha}{\alpha} (u_{t+1}g_{t+1})^{\alpha-1} g_{t+1} (u_{t+1})^{1-\rho} (c_{t+1})^{\rho-1} - \lambda_{t+1}.$$

Below we need the ratio of the Lagrange multipliers,

$$\frac{\lambda_{t+1}}{\lambda_t} = \beta [E(u_{t+1}g_{t+1})^\alpha]^\frac{\rho-\alpha}{\alpha} (u_{t+1})^{\alpha-\rho} (g_{t+1})^\alpha \left(\frac{c_{t+1}}{c_t} \right)^{\rho-1}$$

Let μ_t denote the multiplier for the collateral constraint and let $\hat{\mu}_t = \frac{\mu_t}{\lambda_t}$. We divide the first-order condition with respect to θ_t ,

$$0 = -\lambda_t q_t + \mu_t + E(\lambda_{t+1} (q_{t+1} + d_{t+1}))$$

by λ_t and obtain the equation

$$0 = -q_t + \hat{\mu}_t + \beta [E(u_{t+1}g_{t+1})^\alpha]^\frac{\rho-\alpha}{\alpha} E \left((u_{t+1})^{\alpha-\rho} (g_{t+1})^\alpha \left(\frac{c_{t+1}}{c_t} \right)^{\rho-1} (q_{t+1} + d_{t+1}) \right)$$

Similarly, the first-order conditions for $[\phi_t]^+$ and $[\phi_t]^-$ are as follows,

$$\begin{aligned} 0 &= -p_t + \nu_t^+ + \beta [E(u_{t+1}g_{t+1})^\alpha]^{\frac{\rho-\alpha}{\alpha}} E \left((u_{t+1})^{\alpha-\rho} (g_{t+1})^\alpha \left(\frac{c_{t+1}}{c_t} \right)^{\rho-1} \left(\frac{r_{t+1}}{g_{t+1}} \right) \right) \\ 0 &= -p_t + \hat{\mu}_t k_t - \nu_t^- + \beta [E(u_{t+1}g_{t+1})^\alpha]^{\frac{\rho-\alpha}{\alpha}} E \left((u_{t+1})^{\alpha-\rho} (g_{t+1})^\alpha \left(\frac{c_{t+1}}{c_t} \right)^{\rho-1} \left(\frac{f_{t+1}}{g_{t+1}} \right) \right), \end{aligned}$$

where ν_t^+ and ν_t^- denote the multipliers on $0 \leq [\phi_t]^+$ and $[\phi_t]^- \leq 0$, respectively.

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