

The effect of options on coordination^{*}

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December 2011

Abstract

In some coordination problems, an agent's payoff depends on what other agents will do in the future. This paper studies how constraints on the timing of actions affect equilibrium in those problems. While the possibility of waiting longer for others' actions helps agents to coordinate in the good equilibrium, the option of delaying one's actions harms coordination. In a symmetric case, the risk-dominant equilibrium of the corresponding one-shot game is selected.

JEL Codes: C72, C73, D84

Keywords: Coordination; option; risk dominance; dynamic games

1 Introduction

In some coordination problems, payoffs depend on what others will do in later periods. For example, investment in a new technology might be profitable only if others also choose to invest, hence coordination failures might have important effects on industrialization in different countries (Murphy, Shleifer and Vishny (1989), Rodrik (1999)).¹ But what will influence coordination?

This paper studies how coordination in dynamic models is affected when agents have the option to delay their actions or to wait longer for others' actions. For instance, it is often argued that

^{*}PRELIMINARY VERSION

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¹For example, the internet would probably be less present in our lives weren't for the possibility of transmitting large amounts of information in little time (broadband technology), and the technology that allows wireless transmission of data (routers). Those would probably not have been present in every coffee house if tablets and smart phones had not been invented, whose popularity depends to a large extent on the availability of the so called app's – which will be really profitable once people start to buy things through their app's, which requires further developments.

limited competition in developing countries might hinder industrialization, but why would that be the case? Monopolies won't choose an inferior technology just because there is no competition, but perhaps the lack of competition, by allowing a firm to delay its investments, might affect coordination. Property rights believed to be enforced in the long term might also affect coordination since agents might be able to wait for longer until others invest. In general, having the option to move later might affect whether agents coordinate.

The contribution of this paper is a model to understand the effect of options on coordination. For that, we build on the literature on equilibrium selection in coordination games, particularly the literature on dynamic games with complete information where a state variable is subject to shocks (Frankel and Pauzner (2000), Burdzy, Frankel, and Pauzner (2001)).² Our main departure from this literature is the notion that coordination problems in settings with strategic complementarities are inherently dynamic in the sense that agents always contemplate the "option" to anticipate or to delay their actions. By pursuing this view, and in contrast to the previous literature, we offer a clear map between the availability of "options" and the equilibrium behavior.

Our model works as follows. Consider a discrete time infinite horizon economy with two types of agents, type 0 (say, young entrepreneurs) and type 1 (say, old entrepreneurs). Every period, type 0 agents are randomly and pairwise matched with type 1 agents, and choose whether to exert effort at cost c . Type 1 agents make no choice. If the type 0 agent chooses effort, he becomes a type 1 agent. In turn, the type 1 agent receives a benefit b and is replaced by a new type 0 agent. If the type 0 agent does not exert effort, he is replaced by a new type 0 agent with probability $1 - p_0$, while the corresponding type 1 agent is replaced by a new type 1 agent with probability $1 - p_1$. Thus, p_0 captures the option to delay the effort decision while young, while p_1 captures the probability that the old agent will remain active in the economy before he gets the chance to receive the benefit for his past effort. We assume that $\beta b > c$, where β is the agent's discount factor. Irrespective of the values of p_0 and p_1 , this model exhibits multiple equilibria: there is an equilibrium where all agents choose effort and an equilibrium where all agents choose no effort. We then follow Frankel and Pauzner (2000), and Burdzy, Frankel, and Pauzner (2001) and introduce a random variable z that describes the state of the economy at any point in time. We make two assumptions: (i) z evolves according to a random walk where the change in states follows a symmetric and continuous probability distribution that is independent of time; (ii) z has no influence on payoffs if $z \in [-Z, Z]$,

²Our analysis also relates to the literature on static games with incomplete information (global games). The key references are Carlsson and Van Damme (1993), Morris and Shin (2003), and Frankel, Morris and Pauzner (2003).

z is such that effort is a strictly dominant action if $z > Z$, and z is such that no effort is a strictly dominant action if $z < -Z$.

If we remove the options from our model, by setting $p_0 = p_1 = 0$, our model can be seen as a sequence of 2×2 coordination games, where payoffs of an agent deciding at present depend on the action of an agent deciding next period. In this case, consistent with the literature of global games and dynamic games, we show that there is a unique equilibrium, and in this equilibrium the risk dominant action of the 2×2 coordination game is selected. We then generalize this result for the case where $p_0 = p_1 > 0$. Precisely, we show that there is a unique equilibrium, and in this equilibrium the risk-dominant action of the 2×2 coordination game continues to be selected, irrespective of the values of β and of the process for z . We conclude by showing that the region where effort is the unique equilibrium decreases with p_0 and increases with p_1 . In particular, we show that, even if the cost c is negative and there is a large benefit associated with becoming a type 1 agent, the type 0 agent may never choose effort if $p_0 = 1$ and $p_1 < 1$. This result illustrates the strength of the incentives for delaying when p_0 is larger than p_1 , and shows how inefficient the outcome might be.

In many models of coordination failures, strategic complementarities leads to multiple equilibria due to self-sustaining beliefs. A number of papers suggests that multiplicity is natural, and reproduces the fact that, in many cases, the same set of fundamentals is related with quite distinct outcomes. For instance, in the context of development, Matsuyama (1991) argues that “the diversity of per capita income levels across countries suggests the presence of some sort of multiplicity” (page 619). In turn, Murphy, Shleifer and Vishny (1989) relate the big push into industrialization, and the fact that not all countries experience the big push, with the existence of multiple equilibria (see also Rodrik (1999)). In general, papers within this tradition argue that the selection of which equilibria will eventually play out depends on features that are outside of the model such as history and policy. We do not debate the view that history and policy may very well matter for equilibrium behavior. However, we believe that there is a lot to be gain if one produces a clear map between the fundamentals of the economy and the equilibrium behavior.

As is clear from the discussion above, our model is abstract and not built to match the specifics of any particular example. It is used to analyze the effect of options on coordination more generally. However, we highlight the implications of our model by considering a simplified version of Matsuyama (1991) and Murphy, Shleifer and Vishny (1989) that collapses into our model. In particular, we consider a setting where agents have the option between a mature and a new

technology and the economy exhibits multiple equilibria. We then eliminate the multiplicity of equilibria by introducing the random variable z and the assumptions (i) and (ii) above. We show that the emergence of an equilibrium where all agents adopt the superior technology depends not only on fundamentals such as the cost and benefit of acquiring the new technology (this point is already present in Frankel and Pauzner (2000)), but also on "options" and, in particular, on the option of an agent with the mature technology to postpone acquisition of the new technology. The sheer ability of postponing investment, which might be interpreted as lack of competition, hinders coordination. Hence we offer a novel, policy-based explanation to why coordination might happen in some countries but not in others.

Our paper proceeds as follows. In the next section, we present the model and the results. In section 3 we present an example, and in section 4 we conclude. Proofs omitted in the main text are relegated to the appendix.

2 Model

2.1 Environment

Time is discrete and the discount factor is $\beta \in (0, 1)$. The economy is initially populated by an equal number $N \geq 1$ of two types of agents, type 0 and type 1. In the first period, each type 0 agent is randomly and pairwise matched with a type 1 agent. In the match, the type 0 agent decides between effort (e) and no effort (n), while the type 1 agent does not make any decision. If the type 0 agent chooses effort, he incurs a cost c and, in the next period, he becomes a type 1 agent. In turn, the type 1 agent receives a benefit b , leaves the economy and, in the next period, is replaced by a new type 0 agent with probability one. If, instead, the type 0 agent chooses no effort, in the next period he continues as a type 0 agent with probability $p_0 \in [0, 1]$, and is replaced by a new type 0 agent with probability $1 - p_0$. As for the type 1 agent, if he does not receive the benefit, in the next period he continues as a type 1 agent with probability $p_1 \in [0, 1]$ and is replaced by a new type 1 agent with probability $1 - p_1$. We assume that $\beta b > c$.

In every period, the economy is in some state $z \in \mathbb{R}$. This state has no influence on payoffs in $z \in [-Z, Z]$. However, for $z < -Z$, the type 0 agent incurs a cost $C > \beta b$ if he exerts effort, and it is thus strictly dominant to choose no effort. In turn, for $z > Z$, the type 0 agent suffers a disutility $D > c$ if he does not exert effort, and it is thus strictly dominant to choose effort. States evolve according to a random process $z_t = z_{t-1} + \Delta z_t$, where $z_0 \in [-Z, Z]$ and Δz_t follows a

continuous probability distribution that is independent of z and t with probability density $f(\Delta z)$. The process for Δz is symmetric around 0 and non-degenerate, hence $f(\Delta z) = f(-\Delta z)$, $E(\Delta z) = 0$ and $\text{var}(\Delta z) > 0$. We are interested in the agents' behavior in the region $[-Z, Z]$ with Z arbitrarily large.

2.2 Equilibrium

First, consider the limit case where $\Delta z = 0$ and the dominant regions where $z \notin [-Z, Z]$ are never reached. In this case, $\beta b > c$ leads to the existence of multiple equilibria. In particular, there is a no-coordination equilibrium, where type 0 agents never exert effort and a coordination equilibrium where type 0 agents always exert effort. Importantly, note that the existence of these equilibria does not depend on the "option" values p_0 and p_1 .³

In what follows we are interested in the agent's behavior in the general case where $\text{var}(\Delta z) > 0$, and the dominant regions where $z \notin [-Z, Z]$ are reached with positive probability. In this case, let $V_e(z, z^*)$ be the value function of an agent that exerts effort at state z and holds the belief that $z^* \in [-Z, Z]$ denotes a (hypothetical) state such that everyone exerts effort if $z > z^*$ and no one exerts effort if $z < z^*$. The value function of an agent that does not exert effort, denoted $V_n(z, z^*)$, is defined in a similar way. Note that, since Δz follows a continuous probability distribution, $V_e(z, z^*)$ varies continuously with z and z^* . Moreover, since Δz is independent of z , $V_e(z', z') = V_e(z'', z'')$ for all $z', z'' \in [-Z, Z]$. The same properties apply to $V_n(z, z^*)$.

Proposition 1 *Fix any $Z > 0$. For all states $z \in [-Z, Z]$, exerting effort is the only rationalizable action if*

$$V_e(z, z) > V_n(z, z) \tag{1}$$

and not exerting effort is the only rationalizable action if the inequality is reversed.

Proof. First, we prove that effort is exerted if $V_e(z, z) > V_n(z, z)$. Consider the problem of a type 0 agent in state $z = Z$. The agent knows that for all $z > Z$, $D > c$ implies that it is strictly dominant to exert effort. Since the choice of effort exhibits strategic complementarities, $V_e(z, z) > V_n(z, z)$ then implies that exerting effort at $z = Z$ is a strictly dominant action. Intuitively, the case considered in the computation of $V_e(z, z)$ and $V_n(z, z)$ where all agents choose no effort when $z < Z$, is the worst possible scenario for an agent that chooses effort in state Z . Now, since

³There also exists a mixed strategy equilibrium where type 0 agents are indifferent and always exert effort with probability $\frac{c}{\beta b} \frac{1-\beta p_1}{1-\frac{c}{\beta b} p_1}$. In this equilibrium, the probability of effort does not depend on p_0 but it is decreasing in p_1 .

$V_e(z, z^*)$ varies continuously with z , if the type 0 agent strictly prefers to exert effort at $z = Z$, then there exists ε small enough, say $\varepsilon = \widehat{\varepsilon}$, so that the agent also strictly prefers to exert effort at $z > Z - \widehat{\varepsilon}$, that is, $V_e(Z - \widehat{\varepsilon}, Z) > V_n(Z - \widehat{\varepsilon}, Z)$. In words, effort is a strictly dominant action for all $z > Z - \widehat{\varepsilon}$. Consider now the problem of a type 0 agent in state $z = Z - \widehat{\varepsilon}$. Since it is strictly dominant to exert effort for all $z > Z - \widehat{\varepsilon}$, this agent faces *exactly* the same problem as that of a type 0 agent in state $z = Z$. Thus, $V_e(z, z) > V_n(z, z)$ implies that exerting effort at $z \equiv Z' = Z - \widehat{\varepsilon}$ is a strictly dominant action. We can then apply the same reasoning as above and conclude that, if the type 0 agent strictly prefers to exert effort at $z = Z'$, the fact that $V_e(z, z^*)$ varies continuously with z implies that he must also strictly prefer to exert effort at $z > Z' - \widehat{\varepsilon}$, that is, $V_e(Z' - \widehat{\varepsilon}, Z) > V_n(Z' - \widehat{\varepsilon}, Z)$. Successive iterations of this argument lead to the conclusion that effort is the only action that survives iterated elimination of strictly dominated strategies for all $z \in [-Z, Z]$. Finally, the same induction argument can be used to show that if $V_e(z, z) < V_n(z, z)$, then no effort is the only strategy that survives iterated elimination of strictly dominated strategies for all $z \in [-Z, Z]$. ■

According to Proposition 1, the equilibrium condition depends solely on the choices of an agent in a hypothetical state $z^* \in [-Z, Z]$ with the belief that the state space is divided in two regions: everyone exert effort for all $z > z^*$ and no one exerts effort for all $z < z^*$. If an agent in this hypothetical state strictly prefers to exert effort, then effort is the only rationalizable action in all states $z \in [-Z, Z]$. Conversely, if an agent in this hypothetical state prefers not to exert effort, then no effort is the only rationalizable action in all states $z \in [-Z, Z]$.

The proof of Proposition 1 uses an induction argument, where at each step strictly dominated strategies are eliminated. The argument of the proof is essentially the same as the one in Araujo and Guimaraes (2011).⁴ The intuition runs as follows. Consider an agent in the hypothetical state z^* and assume that this agent strictly prefers to exert effort. Then, by continuity, there exists some $\epsilon > 0$ such that the agent strictly prefers to exert effort in state $z^* - \epsilon$. Now, incentives for an agent to exert effort at a given period are increasing in the likelihood that others will choose effort in subsequent periods. Thus incentives to exert effort in the hypothetical state z^* could only become stronger if the other agents were to exert effort for some $z < z^*$. In consequence, once not-exerting effort has been ruled out for all $z > z^*$, effort is a dominant strategy for an agent in any state $z > z^* - \epsilon$ that strictly prefers to exert effort in the hypothetical state z^* .

⁴In Araujo and Guimaraes (2011) we study equilibrium selection in a random search model of money along the lines of Kiyotaki and Wright (1993). We do not explicitly model the role of options, and our focus is on the coordination role played by the intrinsic and the extrinsic properties of money.

Note that the above argument applies to any state $z^* \in [-Z, Z]$ satisfying the condition that effort is chosen for all $z > z^*$. Now, the assumption that an agent suffers a disutility $D > c$ if he does not exert effort in states $z > Z$ implies that $z = Z$ satisfies this condition. Thus if an agent at Z obtains a strictly positive payoff if he exerts effort, then not exerting effort for all $z > Z - \epsilon$ is a strictly dominated strategy, and can be eliminated. The argument can then be repeated assuming that all agents exert effort for all $z > Z - \epsilon$ and so on. Successive iterations of this argument lead to the conclusion that exerting effort is the only strategy that survives iterated elimination of strictly dominated strategies for all $z \in [-Z, Z]$. The same induction argument can be used to show that if the agent in the hypothetical state z^* prefers not to exert effort, then no effort is the only strategy that survives iterated elimination of strictly dominated strategies for all $z \in [-Z, Z]$.

Proposition 1 implies that a complete characterization of behavior in equilibrium requires a comparison between $V_{0e}(z^*)$ and $V_{0n}(z^*)$. In order to do so, we first introduce some notation. Suppose the economy is at state z at time s . For $x > 0$ and $t > 0$, define $\phi(x, t)$ as the probability density that the economy will be at a state larger than z at time $s + t$ but not before and that at time $s + t$ the economy will be at state $z + x$. Note that

$$\int_0^\infty \left(\sum_{t=1}^\infty \phi(x, t) \right) dx = 1$$

In words, since the process for z is symmetric, eventually the economy will be at a state larger than z . We define the functions Γ_{0x} and Γ_{1x} as the sum of probability densities that the economy will be at the state $z + x$ the first time it reaches a state larger than z discounted, respectively, by $(\beta p_0)^t$ and $(\beta p_1)^t$.

$$\Gamma_{0x} = \sum_{t=1}^\infty (\beta p_0)^t \phi(x, t), \quad (2)$$

$$\Gamma_{1x} = \sum_{t=1}^\infty (\beta p_1)^t \phi(x, t). \quad (3)$$

We also define Ω_{1x} as the sum of probabilities that the economy will be at a state larger than $z + x$ for the first time discounted by $(\beta p_1)^t$. The function Ω_{1x} can be recursively written as:

$$\Omega_{1x} = \int_x^\infty \Gamma_{1w} dw + \int_0^x \Gamma_{1w} \Omega_{1x-w} dw. \quad (4)$$

The first integral considers all processes such that the economy is at a state larger than $z + x$ the first time it is at a state larger than z . The second integral considers occurrences where the

economy reaches a state between z and $z + x$ before eventually reaching a state larger than $z + x$ for the first time.

An agent exerting effort at a state z^* such that everyone exerts effort if $z > z^*$ and no one exerts effort if $z < z^*$ obtains payoff

$$V_{0e}(z^*) = -c + \frac{1}{2}\beta b + \left[\int_0^\infty f(x)\Omega_{1x}dx \right] \beta b. \quad (5)$$

In words, an agent exerting effort at the state z^* pays cost c at time s . At time $s + 1$, the economy will be at a state larger than z^* with probability $\frac{1}{2}$, in which case the agent gets payoff βb and is replaced by a new type 0 agent with probability one, or it may be at a state smaller than z^* . The last term in equation (5) corresponds to the expected gain in case the economy is at a state $z < z^*$ at time $s + 1$. For each non-negative value of x , the economy will be at the state $z^* - x$ with probability density $f(x)$, and Ω_{1x} is the sum of probabilities that the economy will be at a state larger than z^* discounted by $(\beta p_1)^t$. The discount factor takes into account the time discount factor β and the probability the agent remains in the economy if the corresponding type 0 agent makes no effort.

An agent not exerting effort at a state z^* such that everyone exerts effort if $z > z^*$ and no one exerts effort if $z < z^*$ obtains payoff

$$V_{0n}(z^*) = \int_0^\infty \Gamma_{0x} \left\{ -c + \beta b \left[F(x) + \int_0^\infty f(w+x)\Omega_{1w}dw \right] \right\} dx. \quad (6)$$

In words, the agent will exert effort when the economy reaches a state larger than z^* , and the term Γ_{0x} is the discounted sum of probability densities that the economy will be at the state $z + x$ the first time it reaches a state $z > z^*$. The discount factor takes into account both the time discount factor β and the probability p_0 that the agent remains in the economy conditional on having made no effort. At the state $z + x$, the agent exerts effort incurring a cost c . In the following period, the economy will still be at a state larger than z^* as long as $\Delta z > -x$, which occurs with probability $F(x)$ given that the process for Δz is symmetric. The last integral accounts for the cases when $\Delta z < -x$, so that the economy jumps to a state $z^* - w$, and the term Ω_{1w} is the discounted sum of probabilities that the economy will be back to a state larger than z^* conditional on starting at the state $z^* - w$.

2.2.1 The case $p_0 = p_1 = 0$

If $p_0 = p_1 = 0$, equations (2) and (3) imply that $\Gamma_{0x} = \Gamma_{1x} = 0$. From equation (4), $\Omega_{1x} = 0$ as well. As a result,

$$V_{0e}(z^*) = -c + \frac{1}{2}\beta b,$$

and

$$V_{0n}(z^*) = 0.$$

An agent exerting effort obtains the benefit b with probability $\frac{1}{2}$ and leaves the economy without reaping any benefit with probability $\frac{1}{2}$. An agent that exerts no effort leaves the economy with zero payoff. Hence to exert effort is the optimal decision if

$$\frac{1}{2} > \frac{c}{\beta b}.$$

It turns out that this result has a natural interpretation. In particular, if $p_0 = p_1 = 0$, our economy can be thought of as a sequence of one-shot coordination games between pairs of type 0 agents, where the type 0 agent in the current period chooses between effort and no effort given his belief about the behavior of the type 0 agent he may be paired with in the following period (see Figure 1). In this game, effort is the risk dominant action if and only if $\frac{1}{2} > \frac{c}{\beta b}$, which is the same condition for effort to be the optimal decision in our economy. In other words, we obtain that, if $p_0 = p_1 = 0$, the risk dominant action of the one-shot coordination game in Figure 1 is the only rationalizable outcome in our dynamic game.

		type 0 agent	
		e	n
type 0 agent	e	$\beta b - c, \beta b - c$	$-c, 0$
	n	$0, -c$	$0, 0$

Figure 1

2.2.2 General case

In general the option of delaying effort (as captured by p_0) and the possibility of having the benefit of effort deferred into the future (as captured by p_1) will have important effects on agents' payoffs. To observe that, note that equation (6), which gives the payoff of not exerting effort, can be written

as

$$V_{0n}(z^*) = \int_0^\infty \Gamma_{0x} dx \left(-c + \frac{1}{2}\beta b \right) + \left\{ \int_0^\infty \Gamma_{0x} \left[F(x) - \frac{1}{2} + \int_0^\infty f(w+x)\Omega_{1w}dw \right] dx \right\} \beta b. \quad (7)$$

This equation differs from (5), which gives the payoff of exerting effort, in a number of ways. For instance, the term $-c + \frac{1}{2}\beta b$ is discounted by Γ_{0x} , as costs and benefits are postponed if the agent does not exert effort. Moreover, if effort is delayed and exerted at a state $z^* + x$, the probability that the agent will be rewarded in the next period is given by $F(x)$, which is higher than $\frac{1}{2}$. Intuitively, while it is true that postponing the decision to exert effort implies that benefits will only be accrued later in the future, delaying this decision reduces the time between the effort and the benefit.

The balance between the costs and the benefits of postponing effort depends on how the second integral in (7) compares to the integral in (5). The following lemma provides a key result that greatly simplifies the analysis.

Lemma 1 *For all $\beta \in [0, 1]$ and for all $p_1 \in [0, 1]$,*

$$\int_0^\infty \Gamma_{1x} \left[F(x) - \frac{1}{2} + \int_0^\infty f(w+x)\Omega_{1w}dw \right] dx = \int_0^\infty f(x)\Omega_{1x}dx. \quad (8)$$

Proof. First, using the definition of Ω_{1x} , we can write the right-hand side of (8) as

$$\int_0^\infty f(x)\Omega_{1x}dx = \int_0^\infty \int_x^\infty f(x)\Gamma_{1w}dw dx + \int_0^\infty \int_0^x f(x)\Gamma_{1w}\Omega_{1x-w}dw dx. \quad (9)$$

Manipulating the first term on the right-hand side of (9), we get:

$$\int_0^\infty \int_x^\infty f(x)\Gamma_{1w}dw dx = \int_0^\infty \int_0^w \Gamma_{1w}f(x)dx dw = \int_0^\infty \Gamma_{1w} \left[F(w) - \frac{1}{2} \right] dw, \quad (10)$$

where the first equality comes from changing the order of variables in the double integral. In turn, manipulating the second term on the right-hand side of (9), we get

$$\int_0^\infty \int_x^\infty f(x)\Gamma_{1w}\Omega_{1x-w}dw dx = \int_0^\infty \int_w^\infty \Gamma_{1w}f(x)\Omega_{1x-w}dx dw = \int_0^\infty \int_0^\infty \Gamma_{1w}f(y+w)\Omega_{1y}dy dw, \quad (11)$$

where the first equality comes from changing the order of variables in the double integral and the second line comes from making $y = x - w$. We can thus rewrite the left-hand side of (9) as

$$\int_0^\infty f(x)\Omega_{1x}dx = \int_0^\infty \Gamma_{1x} \left[F(x) - \frac{1}{2} \right] dx + \int_0^\infty \int_0^\infty \Gamma_{1x}f(w+x)\Omega_{1w}dw dx,$$

which yields the claim. ■

The right-hand side of (8) (multiplied by βb) is the payoff (excluding the cost c) of an agent who exerts effort when the economy is at the state z^* at time s and is in the region $z < z^*$ at time $s + 1$. In (9), we separate this payoff into two parts, a direct payoff that considers only cases where the economy goes directly to the region $z > z^*$ at time $s + 2$; and an indirect payoff that considers only cases where the economy goes at least to one additional state $z^- < z^*$ before reaching the region $z > z^*$. Equation (10) shows that the direct payoff is equivalent to the payoff of a type 1 agent who is at the state z^* at time s , perturbed by the factor $F(x) - \frac{1}{2}$, which is increasing in the distance between the state of the economy at the time the benefit is delivered ($z^* + x$) and the state z^* . In turn, equation (11) shows that the indirect payoff is equivalent to the payoff of a type 1 agent who is at the state z^* at time s and only receives the benefit the second time the economy reaches some state $z^+ > z^*$, and only if the economy does not stay for two consecutive periods in the region $z > z^*$. A key implication of Lemma 1 is that, when $\Gamma_{0x} = \Gamma_{1x}$, the right-hand side of (8) is equal to the second integral in Equation (7). In this case the benefits of delaying effort and having less expected time between effort and rewards in the future are equivalent to the benefits of anticipating effort, thus anticipating the reaping of its benefits.

We can now provide a precise characterization of equilibrium in terms of the parameters of the economy. However, before doing so, it is convenient to define a variable λ such that exerting effort is an equilibrium if and only if

$$\lambda > \frac{c}{\beta b}.$$

The variable λ summarizes the equilibrium selection. If $\lambda = 0$, effort is never exerted for all $z \leq Z$, and if $\lambda = 1$, effort always is exerted for all $z \geq -Z$. Lemma 1 allows for a simpler characterization of λ .

Proposition 2 *In the unique equilibrium of the game, for any $z \in [-Z, Z]$, effort is made if and only if $\lambda > \frac{c}{\beta b}$, where*

$$\lambda = \frac{1}{2} + \frac{\int_0^\infty (\Gamma_{1x} - \Gamma_{0x}) \left(F(x) - \frac{1}{2} + \int_0^\infty f(w+x) \Omega_{1w} dw \right) dx}{1 - \int_0^\infty \Gamma_{0x} dx}. \quad (12)$$

If $p_0 = p_1$, we have

$$\lambda = \frac{1}{2}.$$

Proof. Substituting equation (8) into the expression for $V_{0e}(z^*)$ in (5) and making $V_{0e}(z^*) = V_{0n}(z^*)$, we obtain

$$-c + \frac{1}{2}\beta b + \left\{ \int_0^\infty (\Gamma_{1x} - \Gamma_{0x}) \left[F(x) - \frac{1}{2} + \int_0^\infty f(w+x) \Omega_{1w} dw \right] dx \right\} \beta b = \int_0^\infty \Gamma_{0x} dx \left(-c + \frac{1}{2}\beta b \right),$$

that can be rewritten as

$$\lambda \equiv \frac{1}{2} + \frac{\int_0^\infty (\Gamma_{1x} - \Gamma_{0x}) \left[F(x) - \frac{1}{2} + \int_0^\infty f(w+x) \Omega_{1w} dw \right] dx}{1 - \int_0^\infty \Gamma_{0x} dx} = \frac{c}{\beta b}.$$

Thus, for any $z \in [-Z, Z]$, effort is made if $\lambda > \frac{c}{\beta b}$, the agent is indifferent if $\lambda = \frac{c}{\beta b}$, and effort is never exerted if $\lambda < \frac{c}{\beta b}$. If $p_0 = p_1$, $\Gamma_{1x} = \Gamma_{0x}$ for all x and the expression simplifies to $\lambda = \frac{1}{2}$. ■

Proposition 2 generalizes the risk-dominance result obtained when $p_0 = p_1 = 0$ to any situation with $p_0 = p_1$, irrespective of the value of β and of the process Δz . When $p_0 = p_1$, the gains from a smaller time between effort and rewards exactly cancel the gains from getting rewards sooner. What is left is the term $-c + \frac{1}{2}\beta b$ in $V_{0e}(z^*)$ and the same term weighted by $\int_0^\infty \Gamma_{0x} dx$ (which is smaller than one) in $V_{0n}(z^*)$. Hence, effort is the optimal decision if $\frac{1}{2} > \frac{c}{\beta b}$.

A corollary from proposition 2 is that λ will be larger than $\frac{1}{2}$ if $p_1 > p_0$ and smaller than $\frac{1}{2}$ if the inequality is reversed, because $p_1 > p_0$ implies $\Gamma_{1x} - \Gamma_{0x} > 0$ for all x . Actually, we can prove a stronger result:

Proposition 3 *Comparative statics and limiting cases:*

1. λ is decreasing in p_0 .
2. For distributions of Δz such that $\Delta z > \Delta z' > 0 \implies f(\Delta z') \geq f(\Delta z)$, λ is increasing in p_1 .
3. If $p_0 = 1$ and $p_1 < 1$, $\lim_{\beta \rightarrow 1} \lambda \rightarrow -\infty$, and effort is never undertaken for any $z \in [-Z, Z]$.
4. If $p_0 < 1$ and $p_1 = 1$, $\lim_{\beta \rightarrow 1} \lambda = 1$, and effort is always undertaken for any $z \in [-Z, Z]$.

Proof. 1. The expression for $V_{0e}(z^*)$ in (5) if an agent is indifferent between effort and no effort is given by

$$\frac{V_{0e}(z^*)}{\beta b} = -\lambda + \frac{1}{2} + \int_0^\infty f(x) \Omega_{1x} dx, \quad (13)$$

since $\lambda = \frac{c}{\beta b}$ if the agent is indifferent. This expression is a function of λ , p_1 , β , b , and the parameters of the stochastic process for Δz . Lemma 3 in the appendix shows that $\frac{V_{0e}(z^*)}{\beta b}$ in (13) is increasing in p_0 when expressed as a function of p_0 , p_1 , β and the parameters of the stochastic process for Δz . Hence, λ has to be decreasing in p_0 .

2. The expressions for $V_{0e}(z^*)$ in (5) and the expression for $V_{0n}(z^*)$ in (7) if an agent is indifferent between effort and no effort are given by

$$\frac{V_{0e}(z^*)}{\beta b} = -\lambda + \frac{1}{2} + \int_0^\infty f(x) \Omega_{1x} dx,$$

and

$$\frac{V_{0n}(z^*)}{\beta b} = \int_0^\infty \Gamma_{0x} dx \left(-\lambda + \frac{1}{2} \right) + \int_0^\infty \Gamma_{0x} \left[F(x) - \frac{1}{2} + \int_0^\infty f(w+x) \Omega_{1w} dw \right] dx.$$

Since $V_{0e}(z^*) = V_{0n}(z^*)$, we have

$$\left(-\lambda + \frac{1}{2} \right) \left(1 - \int_0^\infty \Gamma_{0x} dx \right) = \int_0^\infty \Gamma_{0x} \left[F(x) - \frac{1}{2} + \int_0^\infty f(w+x) \Omega_{1w} dw \right] dx - \int_0^\infty f(w) \Omega_{1w} dw. \quad (14)$$

Lemma 4 in the appendix shows that the right-hand side of (14) is decreasing in p_1 under the conditions on $f(\cdot)$. Since the left-hand side of (14) is decreasing in λ , by the implicit function theorem, λ is increasing in p_1 .

3. If $p_0 = 1$, $\lim_{\beta \rightarrow 1} \int_0^\infty \Gamma_{0x} dx \rightarrow 1$. Hence as β approaches 1, the denominator of equation (12) approaches $+\infty$. Since $p_1 < 1$, $\Gamma_{1x} - \Gamma_{0x} < 0$, hence the numerator is negative, which yields the result.

4. If $p_1 = 1$, $\lim_{\beta \rightarrow 1} \Omega_{1x} \rightarrow 1$. Thus $\int_0^\infty f(w+x) \Omega_{1w} dw$ becomes $1 - F(x)$. Then λ can be written as

$$\lambda = \frac{1}{2} + \frac{1}{2} \frac{\int_0^\infty \Gamma_{1x} dx - \int_0^\infty \Gamma_{0x} dx}{1 - \int_0^\infty \Gamma_{0x} dx}$$

If $p_1 = 1$, $\lim_{\beta \rightarrow 1} \int_0^\infty \Gamma_{1x} dx \rightarrow 1$, which yields the claim. ■

The comparative statics with respect to p_0 means that if agents have an option to act later, they might never act. The possibility of acting later might prevent agents from coordinating in the good equilibrium as it increases the incentives for delays. Anticipating that other agents will only act when it is “safe enough”, the no-action equilibrium will be played. The condition for λ increasing in p_1 holds for the usual symmetric distributions, including normal and uniform. Simply derivating the expression for λ in (12) with respect to p_1 shows that λ is increasing in p_1 for any distribution as long as $p_1 \geq p_0$.

The limiting results show that the effect is very strong when agents are patient and p_0 or p_1 are very high. For instance, if $p_1 < 1$, $p_0 = 1$ and agents are extremely patient, effort will only be exerted when the economy reaches the region where effort is the dominant strategy. Those agents are willing to wait a large amount of time to become type 1 agents so that they are sure they won't lose the opportunity of getting b . Such an agent at a state z^* that divides the set of states in two (everyone exerts effort if $z > z^*$ and no one exerts effort if $z < z^*$) will never choose to exert effort (even if the cost c is actually negative and there is a large benefit associated with becoming a type-1 agent!). Everyone knows others will act like this, hence effort is never made. While this is a only

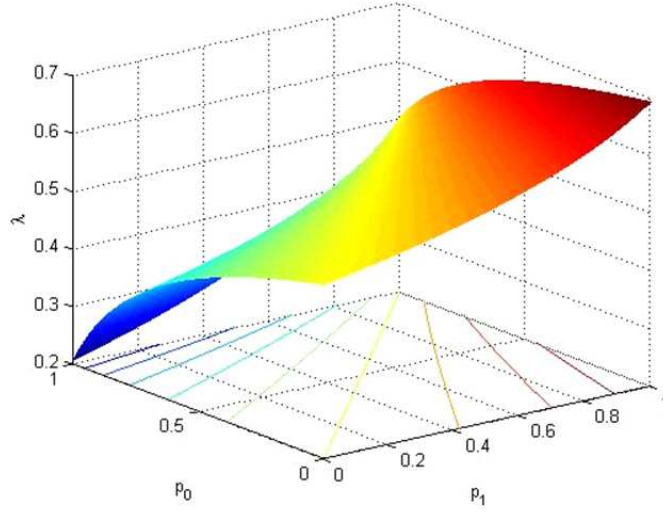


Figure 1: Values of λ for $\beta = 0.8$

limiting result, it illustrates the strength of the incentives for delaying when p_0 is larger than p_1 , and shows how inefficient the outcome might be.

2.2.3 Numerical examples

In general, the value of λ depends on the process for z . Here we assume Δz is normally distributed. Using numerical techniques, the values of $\phi(x, t)$ can be found and equation (12) yields the value of λ .

Figures 1 and 2 show the value of λ as a function of p_0 and p_1 for $\beta = 0.8$ and $\beta = 1$, respectively (negative values of λ are shown as $\lambda = 0$). The lines below the graph are iso-lambda curves. As one would expect from the analytical results, p_0 and p_1 have a larger influence on the equilibrium when β is higher.

2.3 Interpreting p_0 as lack of competition

The parameter p_0 can be interpreted as reflecting the (lack of) competition that threatens the position of a type-0 agent. This section formalizes this claim and clarifies which aspects of competition can be precisely represented by a lower p_0 .

As in the basic model, the economy is initially populated by two types of agents, type 0 and

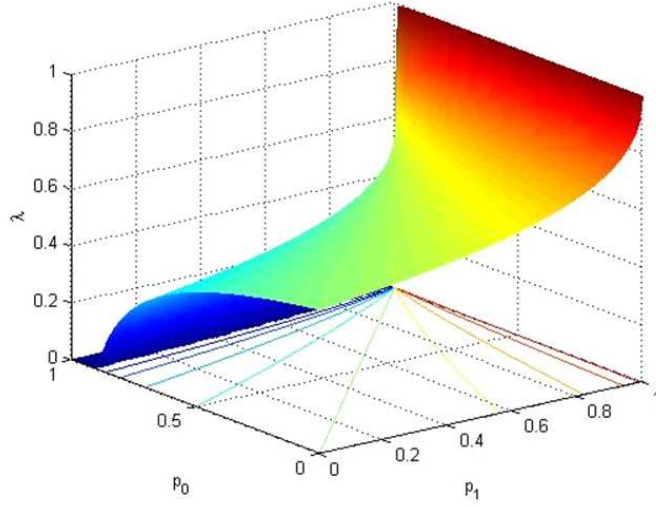


Figure 2: Values of λ for $\beta = 1$

type 1, and the type 0 agent decides between effort (e) and no effort (n). The difference here is that if the type 0 agent chooses no effort, in the next period he continues as a type 0 agent with probability 1, but he is joined by N other type 0 agents. There is a pecking order among the type 0 agents. The one that tops the list is called the “best” one. Each of the new type 0 agents is better than the previous “best one” with probability γ and is somewhere else in the pecking order with probability $1 - \gamma$.

At the start of every period, the type-0 agents have opportunities to plan investment. That has a cost ε and allows them to invest later in that period. ε is a small number. All type 0 agents observe whether other agents have planned to invest, and there are further rounds of opportunities to plan investment, until no other type 0 plans to invest. Then all agents that planned investment have the opportunity to pay a cost $c - \varepsilon$ which transforms their plans to invest on real investment. Among those who paid the cost, only the highest in the pecking order will become a type 1 agent. All the other type 0 agents leave the game. If noone pays the cost $c - \varepsilon$, they all leave the game.

All other aspects of the game are unchanged.

Lemma 2 *In equilibrium:*

1. Only the “best” type 0 agent invests; whenever he plans investment, he also invests.

2. *His problem is the same as that of an agent in the basic model with $p_0 = (1 - \gamma)^N$, which is decreasing in N .*

Proof. *See appendix* ■

The first statement in the lemma shows that competition from “worse” type-0 agents does not play a role in this model. Owing to the structure of the model, there is no way a low-ranked type-0 agent can threaten the position of the top ranked agent. That is an important aspect of competition that doesn’t fit precisely in our framework. This simple example of competition was designed to rule that out because it is hard to fit together competition from existing agents and a constant p_0 at every period – which makes the problem recursive and tractable.

Competition plays an important role though because a type-0 agent is afraid someone better than him will arrive in the market and take his place. That happens with probability $1 - (1 - \gamma)^N$, hence $(1 - \gamma)^N$ can be interpreted as p_0 in our model. The larger N (flow of potential competitors), the lower p_0 .

3 Examples

3.1 Protection and investment

Consider the following environment, broadly based on Murphy, Shleifer and Vishny (1989). There are two technologies, mature and new. Type 0 agents are initially endowed with the mature technology, but they have the ability/opportunity to acquire the new technology if they incur a cost C and become a type 1 agent. The mature technology yields y_0 every period, while the new technology yields y_1 every period, and we assume that $y_1 > y_0$ and $-C + \frac{y_1}{1-\beta} < \frac{y_0}{1-\beta}$. Thus, even though the new technology is superior to the mature technology, by himself a type 0 agent has no incentive to choose the new technology. However, if a type 1 agent meets a type 0 agent that chooses to incur the cost C , from that period on the type 1 agent obtains a yield of $y > y_1$. Thus, if y is high enough and the type 0 agent expects to meet another type 0 agent in the near future that incurs the cost C , he is also willing to incur the same cost and to become a type 1 agent. In other words, the progression to the new technology requires an intertemporal coordination between type 0 agents. Finally, we assume that the new technology might depreciate. Precisely, every period, there is a probability δ that the new technology becomes obsolete, in which case the type 1 agent will only receive y_1 from that period on, irrespective of the behavior of the type 0 he may be

paired with. Future periods are discounted at rate β . Lastly, we assume that, at the beginning of every period, a type 0 agent may lose the ability/opportunity to acquire the new technology with probability ρ .

In this example, the cost of investing is given by the fixed cost plus the opportunity cost,

$$C + \frac{y_0}{1 - \beta},$$

while the payoff of a type 1 agent is at least

$$\frac{y_1}{1 - \beta}.$$

Moreover, if a type 1 agent meets a type 0 agent that invests, his payoff from that period on is increased by

$$\sum_{t=0}^{\infty} \beta^t (1 - \delta)^t (y - y_1) = \frac{y - y_1}{1 - \beta(1 - \delta)}.$$

Now, define

$$c = C + \frac{y_0 - y_1}{1 - \beta},$$

and

$$b = \frac{y - y_1}{1 - \beta(1 - \delta)}.$$

If we let $p_1 = 1 - \delta$ and $p_0 = 1 - \rho$, this example collapses into our model. Precisely, if $\beta b > c$, in the case where $\Delta z = 0$, the economy exhibits multiple equilibria. In particular, there is an inferior equilibrium where only the mature technology is active and the agent's payoff is $\frac{y_0}{1 - \beta}$, and a superior equilibrium where only the new technology is active and the agent's payoff is $-C + \frac{y_1}{1 - \beta} + \beta \frac{y - y_1}{1 - \beta(1 - \delta)}$. In turn, in the general case where $\text{var}(\Delta z) > 0$ there is a unique equilibrium: only the new technology is active if

$$\lambda > \frac{1 - \beta(1 - \delta)}{1 - \beta} \frac{(1 - \beta) C + y_0 - y_1}{\beta (y - y_1)},$$

and only the mature technology is active if the inequality is reversed, where λ is given by (12).

An interesting implication of our model is that a reduction in the probability δ that the technology becomes obsolete has two effects. It has the standard effect of increasing the payoff of investing in the new technology (b is decreasing in δ) but it also helps agents to coordinate on the superior equilibrium (λ increases when δ decreases). We also obtain that an increase in ρ , which can be thought of as an increase in the competition for the new technology, helps agents in their effort to

coordinate on the superior equilibrium (λ increases when ρ increases), even though changes in ρ have no impact on the values of c and βb .

One can also think of our example as a model of patents, where a good enforcement of patents could be interpreted as a low value for δ and/or a low value for ρ : patents allow agents to profit longer from their inventions, and prevent others from investing and taking their places. Patents do increase the fundamental value of an invention, but once we consider that the value of inventions might also depend on coordination, patents have other important effects. On the one hand, they increase p_1 , but on the other hand, as they also increase p_0 , they might harm coordination for providing incentives for agents to delay their investments. In other words, even though specific policy implications would require a specific model and a serious estimation of the parameters, the general idea is that when coordination is an important issue, protection conditional on the sunk investment being taken helps coordination, but once it provides an option to delay, it harms coordination.

Lastly, it is interesting to compare our analysis with the one carried out in Jones (2011).⁵ Jones considers a model with technological complementarities and proposes that linkages in the production chain explain differences in per capita income across countries. The argument is that, if an intermediate good has a high productivity, this high productivity increases the productivity of the next good in the chain. As a result, linkages introduce a multiplier effect where increases in productivity leads to increases in the production of both final goods and intermediate goods, and increases in the latter further increase the production of final goods. The focus of Jones (2011) is on the impact of misallocations along the input chain on the aggregate income. In particular, no attention is given to the role of strategic complementarities along the input chain, i.e., the idea that a firm's decision on whether to choose a new technology may depend on her expectation that the next firm in the chain will also do the same. If coordination is important, a small multiplier effect might arise not due to misallocations along the input chain but because poor competition (high p_0) or badly regulated property rights (low p_1) may cause firms to delay the adoption of new technologies. Thus, our model can offer an alternative interpretation to the determinants of multiplier effect that emerges when one takes into account linkages in the production process.

⁵See also Ciccone (2002).

3.2 Securitization

Securitization is the process of creating loans (origination), arranging these loans in portfolios (tranching), and selling the claims on the cash flows associated to these portfolios as securities in the market (funding). Securitization allows for funds to be transferred from savers to borrowers and, in this sense, it is not much different from traditional banks. However, unlike traditional banks, where credit intermediation is performed under one roof – the bank – securitization works through a chain of non-bank financial intermediaries. In particular, there are usually several steps from origination to funding, with different actors involved at each step.⁶ Thus, it is natural to consider that coordination should play an important role in this process.

Now, even though securitization has played a key and relatively well understood role in the recent financial crisis, the origins of this phenomenon are much less clear (Gorton (2010), page 40). Ongoing explanations usually involve three elements: (i) the increase in capital requirements and changes in regulation which reduced the profitability of banks and led to an exit of capital from the traditional banking industry; (ii) the increase in the demand for collateral, which made securitization profitable; (iii) entry of specialist non-banks into the businesses of credit intermediation.

In terms of our framework, one could think of c as the opportunity cost of engaging in securitization. In this case, increases in capital requirements and increases in restrictions imposed by regulators on traditional banking corresponds to a reduction in c . Proposition 2 then suggests that, given λ , the decrease in $\frac{c}{\beta b}$ implied by the reduction in c may facilitate securitization by mitigating the underlying coordination problems that it involves. Note that this reasoning is different from saying that a reduction in the profitability of traditional banking explains the exit of capital from the traditional banking industry and into securitization. For the latter reasoning to make sense it must be the case that regulators are imposing capital requirements on traditional banks that are binding. Indeed, Gorton (2010, page 55) argues that: “we do not know for certain if (binding capital requirements), in fact, caused capital to exit regulated banking for the shadow banking system. But, in any case, the shadow banking system developed coincidentally with the disappearance of charter value.” What we suggest is that coordination may have something to do with this coincidence. Note also that, in terms of our framework, one could think of b as the benefit of securitization. In this case, an increase in the demand for collateral can then be thought of as an increase in b . This effect reinforces the decrease in $\frac{c}{\beta b}$ implied by the reduction in c .

More interestingly, we can offer a novel interpretation of the effect on securitization of the entry

⁶A detailed description of the different steps involved in securitization is given by Pozsar et alli (2010).

of specialist non-banks into the businesses of credit intermediation. This increased competition has been extensively documented (see Adrian et al (2010) and Gorton (2010)) and it came from money market mutual funds on the liability side of banks and from junk bonds on the asset side of banks. In terms of our framework, this increased competition could be interpreted as a reduction in p_0 . Given c and b , the reduction in p_0 increases λ and thus facilitates the coordination required for securitization to take place.

4 Conclusion

To be written

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A Proofs

A.1 Proof of Proposition 1

Proof. First, we prove that effort is exerted if $V_{0e}(z^*) > V_{0n}(z^*)$. Consider the problem of a type 0 agent in state $z = Z$. The agent knows that for all $z > Z$, $D > c$ implies that it is strictly dominant to exert effort. Thus, irrespective of his belief about the behavior of type 0 agents in states $z \leq Z$, $V_{0e}(z^*) > V_{0n}(z^*)$ implies that exerting effort at $z = Z$ is a strictly dominant action. Now, since Δz follows a continuous probability distribution, if the type 0 agent strictly prefers to exert effort at $z = Z$, then there exists an ε small enough, say $\varepsilon = \widehat{\varepsilon}$, so that this agent also strictly prefers to exert effort at $z > Z - \widehat{\varepsilon}$. This implies that no effort is a strictly dominated action for all $z > Z - \widehat{\varepsilon}$. Consider now the problem of a type 0 agent in state $z = Z - \widehat{\varepsilon}$. The above reasoning implies that it is strictly dominant to exert effort for all $z > Z - \widehat{\varepsilon}$. Thus, this agent faces *exactly* the same problem as that of a type 0 agent in state $z = Z$. That is, irrespective of his belief about the behavior of type 0 agents in states $z \leq Z - \widehat{\varepsilon}$, $V_{0e}(z^*) > V_{0n}(z^*)$ implies that exerting effort at $z \equiv Z' = Z - \widehat{\varepsilon}$ is a strictly dominant action. We can then apply the same reasoning as above and conclude that, if the type 0 agent strictly prefers to exert effort at $z = Z'$, he must also strictly prefer to exert effort at $z > Z' - \widehat{\varepsilon}$. Successive iterations of this argument lead to the conclusion that effort is the only action that survives iterated elimination of strictly dominated strategies for all $z \in [-Z, Z]$. Finally, the same induction argument can be used to show that if the agent in the hypothetical state z^* prefers not to exert effort, that is if $V_{0e}(z^*) > V_{0n}(z^*)$, then no effort is the

only strategy that survives iterated elimination of strictly dominated strategies for all $z \in [-Z, Z]$. ■

A.2 Proofs of proposition 3

A.2.1 First statement

Lemma 3 *The value of $V_{0e}(z^*)$ that makes an agent indifferent between effort and no effort is increasing in p_0 .*

Proof. Combining equation (13) with the expression for λ given by 12, we get that the agent is indifferent when

$$\frac{V_{0e}(z^*)}{\beta b} = -\frac{\int_0^\infty (\Gamma_{1x} - \Gamma_{0x}) \left[F(x) - \frac{1}{2} + \int_0^\infty f(w+x)\Omega_{1w}dw \right] dx}{1 - \int_0^\infty \Gamma_{0x}dx} + \int_0^\infty f(x)\Omega_{1x}dx.$$

Using equation (8) and rearranging,

$$\frac{V_{0e}(z^*)}{\beta b} = \frac{\int_0^\infty \Gamma_{0x} \left[F(x) - \frac{1}{2} + \int_0^\infty f(w+x)\Omega_{1w}dw \right] dx - \int_0^\infty \Gamma_{0x}dx \int_0^\infty f(w)\Omega_{1w}dw}{1 - \int_0^\infty \Gamma_{0x}dx},$$

which can be further rearranged to

$$\frac{V_{0e}(z^*)}{\beta b} = \frac{\int_0^\infty \Gamma_{0x} \left[F(x) - \frac{1}{2} + \int_0^\infty f(w+x)\Omega_{1w}dw - \int_0^\infty f(w)\Omega_{1w}dw \right] dx}{1 - \int_0^\infty \Gamma_{0x}dx}.$$

The denominator is decreasing in p_0 . The term inside brackets in the numerator can be written as

$$\int_0^x f(w)dw + \int_x^\infty f(y)\Omega_{1y-x}dy - \int_0^\infty f(w)\Omega_{1w}dw = \int_0^x f(w)(1-\Omega_{1w})dw + \int_x^\infty f(y)(\Omega_{1y-x}-\Omega_{1y})dy,$$

which is positive for all x since $\Omega_w < 1$ and Ω_y is decreasing in y . Hence the numerator is increasing in p_0 , which completes the proof. ■

A.2.2 Second statement

Lemma 4 *The derivative of the right-hand of (14) with respect to p_1 is negative.*

Proof. The derivative of the right-hand of (14) with respect to p_1 is given by

$$\int_0^\infty \Gamma_{0x} \left(\int_0^\infty f(w+x) \frac{\partial \Omega_w}{\partial p_1} dw \right) dx - \int_0^\infty f(w) \frac{\partial \Omega_w}{\partial p_1} dw.$$

Note that the condition $f(w) > f(w')$ for all $w' > w > 0$, implies that this expression is smaller than or equal to

$$\int_0^\infty \Gamma_{0x} \left(\int_0^\infty f(w) \frac{\partial \Omega_{1w}}{\partial p_1} dw \right) dx - \int_0^\infty f(w) \frac{\partial \Omega_{1w}}{\partial p_1} dw,$$

which is equal to

$$\left(\int_0^\infty f(w) \frac{\partial \Omega_{1w}}{\partial p_1} dw \right) \left(\int_0^\infty \Gamma_{0x} dx \right) - \int_0^\infty f(w) \frac{\partial \Omega_{1w}}{\partial p_1} dw.$$

The last expression is negative since $\int_0^\infty \Gamma_{0x} dx < 1$ as long as $\beta p_0 < 1$. This completes the proof. ■

A.3 Proof of Lemma 2

Proof. Consider the (hypothetical) problem of an agent with only 2 possibilities: invest at the current period and become a type1 agent next period with probability 1; or leave the game. Say agents are expected to invest if and only if $z > z^*$ and the economy is at state x .

Suppose investing at the current period is not the best option for this agent. Then noone will have any incentive to plan investment, since paying the ε cost is worse than leaving the game with nothing.

Suppose investing at the current period is the best option for this agent. Now note that this is exactly the problem faced by the “best” type-0 agent once someone else has planned investment: the type-0 agent can plan investment, invest and become a type 1 or leave the game. Hence if any other type-0 agent plans investment, the “best one” will follow suit an invest. But that implies that no other agent has an incentive to plan investment (it costs ε and yields 0).

Therefore type-0 agents below the top of the pecking order play no role in this game, which is the first statement.

Now the best type 0 agent knows he wont be the best forever. He will remain as the best type 0 agent, and hence will remain in the game, with probability $(1 - \gamma)^N$. With probability $1 - (1 - \gamma)^N$, he leaves the game. That proves the second statement. ■