Monetary Shocks with Observation and Menu Costs (Preliminary) *

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Abstract

We compute the impulse response to an aggregate monetary shock in a general equilibrium model where firms set prices subject to observation and menu costs. The firm optimally decides when to "review" costly information on the adequacy of its price. Upon each review, the firm chooses whether to adjust its price, one or more times, before the next price review. Each price adjustment entails paying a menu cost. In Alvarez, Lippi, and Paciello (2011) we show that the firm's choices map into several statistics: the frequency of price reviews, the frequency of price adjustments, the size-distribution of price changes. In this paper we use those results to obtain a measure of the relative size of observation and adjustment costs from available micro data in the United States. We study how the predictions about the aggregate effects of monetary shocks depend on the relative size of the information and adjustment frictions. We find that unexpected monetary shocks have sizable and persistent impact on aggregate output. We find that menu costs account for about one quarter of the cumulated output response to monetary shocks. We show that menu costs affect the transmission of monetary shocks to aggregate output through two channels: on one side menu costs affect the decision of adjusting prices conditional on an observation; on the other side, menu costs affect the decision of how frequently to observe the state. We find that these two channels are equally important in determining the impact of menu costs on the transmission of monetary shocks to aggregate output in the United States.

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1 Introduction

The implications of micro level price rigidity for aggregate dynamics have given rise to a long standing debate in macroeconomics. In particular, a large literature has investigated the implications of sticky prices for the response of real aggregate variables to nominal aggregate shocks. This is an important topic in the macroeconomic literature as, for instance, it has consequences for the ability of the monetary authority to reduce business cycle fluctuations through the control of the level of nominal money supply. Despite the importance of this question, substantial disagreement still exists on the role of price rigidity for the transmission of aggregate shocks. An important reason behind this disagreement is in the mechanism generating the observed price rigidity: while, on one side, models of price rigidity arising from the presence of fixed non-convex cost of price adjustment have difficulties generating substantial money non-neutrality when calibrated to match the micro data, models of price rigidity arising from the presence of costly information acquisition predict larger and more persistent responses of aggregate output to nominal shocks.[1] In particular, following the seminal work of Golosov and Lucas (2007), two parallel but highly related literatures have developed. On one side, many authors such as Midrigan (2011), Kehoe and Midrigan (2010), Nakamura and Steinsson (2010), Gertler and Leahy (2008), have revisited the ability of menu cost models of price rigidity to account for substantial money non-neutrality, mostly by focusing on the role of strategic complementarity in price setting, multi-product firms and different assumptions about the distribution of innovations. On the other side, models of aggregate price rigidity arising from information frictions as the ones suggested by Mankiw and Reis (2006) and Mackowiak and Wiederholt (2009) have been much more successful at generating substantial output effects after monetary shocks. However, the absence of adjustment costs makes models with only information frictions generally unable to explain infrequent price adjustments at the micro level. This is potentially a problem because statistics on infrequent price adjustments provide useful information on firm's pricing behavior and, as Golosov and Lucas (2007) have shown, these statistics provide discipline in the choice of parameters that might be fundamental for the predictions about the aggregate effects of nominal shocks.

This paper bridges the gap between the two literatures, and derives the equilibrium response of aggregate variables to aggregate nominal shocks in a model where firms set prices under two types of frictions: a standard fixed cost of adjusting the state and a fixed cost of observing the state. Our model embeds the two "polar" cases of menu cost (e.g. Barro (1972); Dixit (1991)) and observation cost only (e.g. Caballero (1989); Reis (2006)). These models produce different policy rules: the menu cost yields "state-dependent" rules, as in e.g. Golosov and Lucas (2007), while the observation cost yields "time-dependent" rules, as in e.g. Mankiw and Reis (2002, 2006, 2007). This paper delivers several novel results. First, this paper allows to compare the predictions about the aggregate effects of nominal shocks of models of price setting with adjustments cost and models of price setting with observation cost in a unified framework.

Second, most importantly, our framework allows to study how the size and shape of output and inflation response to nominal shock depend on the relative size of adjustment and observation costs. We show that economies that are observationally equivalent in terms

[1] See Caplin and Spulber (1987) and Golosov and Lucas (2007) for the case of menu cost models, and Mankiw and Reis (2010) for a review of models of price setting under imperfect information.

of frequency and size of price adjustments might still experience very different consequences from nominal shocks depending on the relative size of adjustment and observation cost. In particular, the larger the observation friction relatively to the adjustment friction, the larger the effects of nominal shocks on aggregate output.

Third, when calibrated to match salient moments of the distribution and timing of price changes in the U.S. we find that unexpected nominal aggregate shocks have sizable and persistent impact on aggregate real variables.

Moreover, our framework allows us to quantify the contribution of adjustment and observation costs to the cumulated output response after a nominal shock. We find that adjustment costs contributes for about one quarter to the cumulated output response to a monetary shock. Interestingly, we emphasize that in our model menu costs affect the transmission of nominal shocks to aggregate output both *directly*, through the impact on the decision to adjust conditional on an observation, and *indirectly*, through the impact on the decision of how frequently to observe the state. The larger the menu cost, the less frequent the observation of the state. We find that the latter accounts for a half of the overall effect of menu costs on the output response to the monetary shock.

Finally, we compare predictions about aggregate effects of nominal shocks from our model calibrated to match U.S. firms' pricing behavior against predictions from our model calibrated to match Euro area firms' pricing behavior. Despite, the average frequency of price changes in the Euro area is substantially lower than in the U.S., our model with both observation and adjustment costs predict similar effects of nominal shocks on output in the two economies. We find that this result is due to the fact that firms in the Euro area are characterized by a higher frequency of observations than firms in the U.S.. This result highlights the importance of modeling both the adjustment and information acquisition decisions of firms in order to correctly assess the aggregate consequences of nominal shocks. In fact, we show that models with adjustment cost only calibrated to match the frequency and size of price changes would predict that monetary shocks have substantially larger effect on output in the Euro area than in the U.S.

The economy is modeled along the lines of Golosov and Lucas (2007), but in addition of the menu cost that firms pay to change prices, we incorporate costly observation of their cost, as a way to model rational inattention, as we did in Alvarez, Lippi, and Paciello (2011). Each firm is a monopolistically producer of a single product, and maximizes the expected discounted value of future profits. Firms in the economy are subject to permanent idiosyncratic productivity shock z, for each product. Additionally we assume that the life of each product is exponentially distributed, with average duration $1/\lambda$. All new products have productivity $z_t = 1$. This assumption helps obtaining a stationary distribution of relative prices, as well as capturing product substitution observed in the micro data. After time zero the log of the money supply is assumed to grow with drift parameter μ. When the product is exogenously replaced, the firm knows the productivity of the new product and has to set its price. In each period, conditional on survival, the firm faces two different choices: the first choice is whether to pay the observation cost and observe the level of productivity; the second choice is whether and to what level to adjust its price upon payment of the menu cost. Both observation and menu costs are assumed to be proportional to the level of wages in the economy, as well as to firm's specific productivity. As in standard models of price setting with menu costs, the decision rule is characterized by an inaction range with a minimum size of the price change, giving rise to the"state-dependent" component of the price adjustment decision. As in models of price setting with costly information acquisition, the decision rule is characterized by infrequent information gathering, giving rise to the"time-dependent" component of the price adjustment decision.

This paper builds on results we obtained in Alvarez, Lippi, and Paciello (2011) to identify the relevant parameters of the model from available data. In particular, we use that the ratio of menu to observation cost can be identified from the ratio of the frequency of observations to price changes. For given ratio of menu to observation costs, then the frequency and size of price changes can be used to identify the level of the two fixed costs as well as the volatility of innovations to productivity. Applying those results to survey evidence on U.S. firms from Blinder et al. (1998) we obtain that observation costs are twice as large as adjustment costs. We use these results to address the following question: how do the aggregate effects of nominal shocks depend on the combination of adjustment and observation costs?

This paper relates to the the literature on price setting with imperfect information, a good summary of which can be found is in Section 7.1 of Mankiw and Reis (2010). A strand of this literature studies price-setting decisions in models where both the size of the price change and its timing are endogenous in the presence of costly observation. In Bonomo and Carvalho (2004) and Woodford (2009) the firm optimally chooses the times of observation and adjustment, under the assumption that the information and the menu cost are lumped together. As in our setup, these models predict infrequent information review and price adjustment. But since the observation and menu cost are lumped, every price review triggers a price adjustment: this differs from our model and, as a consequence, yields different predictions concerning pricing behavior as observed in the micro data. In Bonomo and Carvalho observations/adjustments are equally spaced in time, the distribution of price changes is unimodal, and the hazard-rate of price changes is constant at zero with a spike at the time of the observation. In Woodford's model it is assumed that the firm cannot keep track of the time elapsed since the last observation/adjustment, but that it receives noisy signals on the price gap. This implies that price adjustments are triggered by the signals (by construction they are independent of the time elapsed since the last price change). The distribution of price changes has either one or two modes, depending on the size of the observation/adjustment cost. For small levels of the cost, which are needed to reproduce a large mass of small price adjustments, the distribution of price changes has a single mode. By contrast, our model with separate observation and menu costs produces a distribution of price changes that is bi-modal and features a large mass of small adjustments and large standard deviation of price changes. Gorodnichenko (2008) presents numerical solutions of a model where the firm faces a fixed cost to acquire information and a fixed cost to change the price. Differently from our paper, he focuses on aggregate uncertainty, and also assumes that after one period the firm learns the true value of the state for free.

Our paper relates to the recent work by Hellwig and Venkateswaran (2011) who study the interactions of menu cost and imperfect information in price setting, and derive the aggregate response to nominal shocks. Differently from this paper, they assume that firms can perfectly observe transaction outcomes, such as wages and demand, in each period at no cost, but cannot distinguish between different sources of shocks. In this environment, at their favorite calibration, they find that adding their information structure to adjustment frictions does not improve substantially the ability of adjustment cost models of price setting to generate substantial real effects to nominal shocks. TO BE CONTINUED.

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2 The model

The economy is modeled along the lines of Golosov and Lucas (2007) but where in addition of the menu cost that firms pay to change prices we incorporate costly observation of their cost, as a way to model rational inattention, as we did in Alvarez, Lippi, and Paciello (2011). Our objective is to study the aggregate impact of a monetary shock, in the form of a time zero, unexpected permanent increase in money supply. As in Golosov and Lucas (2007), firms in the economy are subject to persistent idiosyncratic productivity shock z, for each product. Additionally we assume that the life of each product is exponentially distributed, with average duration $1/\lambda$. After time zero the log of the money supply is assumed to grow with drift parameter μ.

$$d\ log(m_t) = \mu\ dt. \tag{1}$$

The monetary shock takes the form of an unexpected (unforeseen) one time increase in m_t, i.e. we will start the economy with the price level and cross sectional distribution corresponding to an steady state, and set m_0 to value that is higher than the one corresponding to steady state.

The firm-specific shocks are such that productivity z evolves according to,

$$d\ log(z_t) = \gamma\ dt + \sigma\ dB_t \tag{2}$$

where B_t is a standard brownian motion with zero drift and unit variance. With probability λ per unit of time a product disappears and is replaced by a new one, i.e. product life are exponentially distributed. We will refer to this event as a product replacement or substitution.[2] All new products have productivity $z_t = 1$.

2.1 Household Problem

We assume that (real) aggregate consumption c_t is given by the Spence-Dixit-Stiglitz consumption aggregate

$$c_t = \left[\int_0^1 C_t(i)^{(\eta-1)/\eta} di\right]^{\eta/(\eta-1)} \quad \text{with} \quad \eta > 1\ ,$$

where $C_t(i)$ denotes the consumption of variety i at time t, and all the varieties enter symmetrically. The household chooses a buying strategy $C_t(\cdot)$ where, abusing notation, $C_t(p)$ denotes the number of units of the consumption good that it buys from a seller who charges nominal price p at date t. It also chooses a labor supply strategy l_t and a money-holding strategy $\hat{m}_t$, where l_t is the units of labor supplied and $\hat{m}$ is dollar balances held. For any buying strategy $C_t(\cdot)$,

[2] We will set $\gamma = -\sigma^2/2$ so that for a product not replaced during s periods, the level of productivity is a martingale: $\mathbb{E}[z_{t+s}] = z_t$.

We index the goods by a triplet $(p, \hat{\text{z}}, \hat{\sigma}^2)$ which consists of the nominal price at which consumer can buy these goods, as well as the mean and precision of the firm's belief about the log of their productivity, $\log z$. We let $\phi_t(p, \hat{\text{z}}, \hat{\sigma}^2)$ be the measure of these triplets at time t, so that we can write:

$$c_t = \left[\int_{p,\hat{\text{z}},\hat{\sigma}^2} C_t(p)^{(\eta-1)/\eta} \phi_t(dp, d\hat{\text{z}}, d\hat{\sigma}^2) \right]^{\eta/(\eta-1)} \tag{3}$$

where $\phi_t(\cdot)$ is the distribution of products according to their nominal price and associated productivity at time t. Current-period utility depends on c_t and also on labor supply l_t and cash holdings $\hat{m}$, deflated by the ideal price index P_t:

$$P_t = \left[\int_{p,\hat{\text{z}},\hat{\sigma}^2} p^{(1-\eta)} \phi_t(dp, d\hat{\text{z}}, d\hat{\sigma}^2) \right]^{1/(1-\eta)} \tag{4}$$

Preferences over time are given by

$$\int_0^\infty e^{-\rho t} \left[\frac{c_t^{1-\epsilon}}{1-\epsilon} - \xi \; l_t + log\left(\frac{\hat{m}_t}{P_t}\right) \right] dt \tag{5}$$

Financial markets are complete, in the sense that all profits of firms are held in a diversified mutual fund. Since all aggregate quantities are deterministic, the budget constraint of the representative agent is a simple present value constraint:

$$\hat{m}_0 \;\; \geq \;\; \int_0^\infty Q_t \left[\int_{p,\hat{\text{z}},\hat{\sigma}^2} p \; C_t(p) \; \phi_t(dp, d\hat{\text{z}}, d\hat{\sigma}^2) + R_t \hat{m}_t - \mu m_t - W_t l_t - \bar{\Pi}_t \right] dt, \tag{6}$$

$$\text{where } Q_t = \exp\left(- \int_0^t R(s) ds \right)$$

where R_t is the instantaneous risk-free net nominal interest rate −and hence the opportunity cost of holding money−, and Q_t is the implied time zero price of a dollar delivered at time t, W_t is time t the nominal wage, and $\bar{\Pi}_t$ are the aggregate nominal time t net profits rebated from all firms to households. The household chooses goods demand, labor supply, and money-holding strategies $C_t(\cdot), l_t, \hat{m}_t$ so as to maximize equation (5), subject to equation (6), taking $Q_t, P_t, R_t, W_t, \hat{m}_0$ as given.

The first order condition, after setting $\hat{m}_t = m_t$ for money holdings:

$$e^{-\rho t}/m_t = \zeta \; Q_t R_t, \tag{7}$$

where we have used $\hat{m} = m_t$, and ζ is the Lagrange multiplier of equation (6). The first order condition for consumption and labor are:

$$e^{-\rho t} \, c_t^{1/\eta - \epsilon} \, C_t(p)^{-1/\eta} = \zeta \; Q_t \; p, \tag{8}$$

$$e^{-\rho t} \xi = \zeta \; Q_t \; W_t, \tag{9}$$

Taking logs and differentiating w.r.t. to time equation (7) one obtains the following o.d.e.:

$$\dot{R}(t) = R(t) \left(R(t) - \mu - \rho \right) \tag{10}$$

which has two steady states, zero and $\rho + \mu > 0$. The steady state $\rho + \mu$ is unstable: if $0 < R(0) < \rho + \mu$ then it converges to zero, and if $R(0) > \rho + \mu$ it diverges to $+\infty$. Thus, there exists an equilibrium where, regardless of the initial condition for $\phi_0(\cdot)$:

$$R_t = R = \rho + \mu \quad , \quad W_t = \xi \; R \; m_t \; , \quad \text{and} \quad \zeta = \frac{1}{m_0 R} \tag{11}$$

We will use this characterization into the firm's problem.

2.2 The Individual Firm Problem

In this section we set up and analyze the price setting problem of a firm until its product is replaced. First we set up the notation for the period profits, second we describe the different adjustment cost, third we set up the dynamic programming problem, and finally we restate this problem simplifying its state and describing the optimal policies.

The firm's nominal per period profits, scaled by the economy wide money supply, are given by

$$\Pi_t(p,z) = C_t(p) \left(\frac{p}{m_t} - \frac{W_t/z}{m_t} \right)$$

where $\Pi_t^*(z)$ is the level of profits at the period profit maximizing price, i.e.

$$p_t^* = \frac{\eta}{\eta - 1} \frac{m_t R \xi}{z} = \arg\max_p \Pi_t(p,z) \; . \tag{12}$$

After using the first order conditions above, we obtain:

$$\Pi_t(p,z) \;\; = \;\; c_t^{1-\epsilon\eta} R \left(\frac{p}{R m_t} \right)^{-\eta} \left(\frac{p}{R m_t} - \frac{\xi}{z} \right) = c_t^{1-\epsilon\eta} z^{\eta - 1} \; F(p/p_t^*) \tag{13}$$

$$\Pi_t^*(z) \;\; = \;\; c_t^{1-\epsilon\eta} R \left(\frac{\eta}{\eta - 1} \frac{\xi}{z} \right)^{-\eta} \left(\frac{\eta}{\eta - 1} \frac{\xi}{z} - \frac{\xi}{z} \right) = c_t^{1-\epsilon\eta} z^{\eta - 1} \; F(1) \tag{14}$$

where $F(\cdot)$ is given by

$$F(g) \equiv R \; \xi^{1-\eta} \left(g \; \frac{\eta}{\eta - 1} \right)^{-\eta} \left(g \; \frac{\eta}{\eta - 1} - 1 \right) \quad \text{and where} \quad g \equiv \frac{p}{p^*} \tag{15}$$

We assume that firms do not observe their profits and the value of the productivity z_t, unless they decide to undertake a costly action, which we refer to as an observation. Immediately after the observation firms pay the following cost, expressed in dollars and divided by the economy wide money supply:

$$\theta_t(z) = \bar{\theta} \; c_t^{1-\epsilon\eta} \; z^{\eta - 1} \; \frac{W_t}{m_t} \; , \tag{16}$$

where $\bar{\theta} > 0$ is a parameter. After paying the observation cost the firm learns perfectly the current value of z. The first three terms in equation (16) give the number of units of labor required to make an observation, which depend on c_t and z, where z is the value of productivity that has been just observed. This specification of the cost scales in the same way as the flexible price profits Π_t^*. Furthermore we assume that after paying the observation cost the firms can change its price, subject to a menu cost, which is independent of the size of the price change, also expressed in nominal terms relative to the economy wide money supply is:

$$\psi_t(z) = \bar{\psi} \ c_t^{1-\epsilon\eta} \ z^{\eta-1} \ \frac{W_t}{m_t} \ , \tag{17}$$

where $\bar{\psi} > 0$ is a parameter. As in the case of the observation cost, this specification scales up as the flexible price profits Π^*. Technically, the purpose of having the menu and observation costs indexed by $z^{\eta-1}$ is to induce stationarity in the problem, this is a common feature in many models.

First, we describe the problem for a firm right after an observation at time $t \geq 0$. At this time the firm learns the value of z_t as well as its current profits. The firm chooses three objects: times at which to observe $\{\tau_i\}$, whether or not to adjust prices $\{\chi_i\}$, and if so by how much p_{τ_i}. We describe each of them in more detail.

The stopping times $\{\tau_i\}$ denote the date where the subsequent observations will take place, conditional on that the product has not been replaced. These stopping times satisfy $t = \tau_0 \leq \tau_1 \leq \tau_2 \leq \cdots$. At an observation time τ_i the firm learns z_{τ_i}, so that the information available at time τ_i is given by $\{z_{\tau_0}, z_{\tau_1}, \cdots, z_{\tau_i}\}$. Given the structure of our productivity shock, this is summarized by z_{τ_i}. Until the next observation date, no new information arrives. Thus, the next time to gather information, τ_{i+1}, is decided with the information obtained at the previous observation date τ_i, namely z_{τ_i}.

Immediately after an observation at time τ_i the firm decides whether to pay the menu cost and change its nominal price, using all the information available at that time. We use the indicator $\chi_i = 1$ to denote the decision of paying the menu cost and changing price, and $\chi_i = 0$ if no adjustment takes place. We let p_{τ_i} be the price chosen in the case of an adjustment. The optimal value for a firm that maximizes expected discounted profits net of observation and menu costs after observing at date t and up to the time the product is replaced is

$$\begin{aligned} V_t(p,z,m) = \max_{\{\tau_i, p_{\tau_i}, \chi_{\tau_i}\}_{i=0}^{\infty}} \mathbb{E}\Bigg\{ \sum_{i=0}^{\infty} e^{-(\mu+\rho+\lambda)(\tau_i - t)} \Big[m_{\tau_i} \ \theta_{\tau_i}(z_{\tau_i}) \quad + \\ (1-\chi_{\tau_i}) \int_{\tau_i}^{\tau_{i+1}} e^{-(\mu+\rho+\lambda)(s-\tau_i)} \ \mathbb{E}\left[m_s \ \Pi_s\left(p_{\tau_{i-1}}, z_s\right) \big| z_{\tau_i} \right] ds \quad + \\ \chi_{\tau_i} \left(\ m_{\tau_i} \ \psi_{\tau_i}(z_{\tau_i}) + \int_{\tau_i}^{\tau_{i+1}} e^{-(\mu+\rho+\lambda)(s-\tau_i)} \ \mathbb{E}\left[m_s \ \Pi_s\big(p_{\tau_i}, z_s\big) \big| z_{\tau_i} \right] \ ds \right) \Big] \ \Big| z_t = z \Bigg\} \end{aligned} \tag{18}$$

where $m_t = m$ and $\tau_0 = t$. The firm price at all dates $s \geq t$ is thus given by

$$\begin{aligned} p_s &= p_{\tau_{i-1}} \text{ for } s \in [\tau_i, \tau_{i+1}) \text{ if } \chi_{\tau_i} = 0 \text{ no adjustment takes place at } \tau_i \ , \\ p_s &= p_{\tau_i} \quad \text{ for } s \in [\tau_i, \tau_{i+1}) \text{ if } \chi_{\tau_i} = 1 \text{ an adjustment takes place at } \tau_i \ . \end{aligned}$$

Profits, observation cost and menu cost are expressed in nominal terms, and hence so is the value function V_t. The discounting uses the result that the nominal interest rate equals

$\rho + \mu$ as well as the fact that products die at rate λ. Since no information is gathered between observation dates, expectations of profits and adjustment cost are conditional on the information gathered at the last observation date. No decision takes place between observation dates. Thus, at a typical time period, most firms will be making no decision, i.e. neither to observe nor to adjust, but they will be in between two observation dates.

Using the notation of time to review we can write a specific expression for the beliefs of firms between observations conditional on the product not being replaced. In a sample path where $\tau_i \leq t < \tau_{i+1}$

$$
\begin{aligned}
\hat{\mathrm{z}}_t &\equiv \mathbb{E}\left[\log z_t \mid z_{\tau_i}\right] = \gamma(t-\tau_i) + \log z_{\tau_i} \ , \\
\hat{\sigma}_t^2 &\equiv \mathbb{V}ar\left[\hat{\mathrm{z}}_t \mid z_{\tau_i}\right] = (t-\tau_i)\sigma^2 \ .
\end{aligned}
$$

Time $t = 0$ is a somewhat special date because we need to examine the problem for all the firms, and describe their beliefs about their initial z_0. Most of the firms will choose not to observe at $t = 0$, i.e. will choose $\tau_0 > 0$ but of course this is a result to be established, which requires some notation. At time $t = 0$ each firm is endowed with the triplet: $(p, \hat{\mathrm{z}}, \hat{\sigma}^2)$ describing the current nominal price p, the expected value of $\log z_0$, and the variance of their best forecast of $\log z_0$. Given the normality of the distribution of $\log z$, the values of $\hat{\mathrm{z}}, \hat{\sigma}^2$ completely characterize the beliefs of the firm. Notice that, as the shocks for a given product are permanent, $\hat{\sigma}^2 = (t-\tau_i)\,\sigma^2$ where $t - \tau_i$ is the time since the last observation. Each of these firms solves the following problem which determines the first date at which they observe:

$$
\begin{aligned}
\tau_0(p, \hat{\mathrm{z}}, \hat{\sigma}^2) \;&=\; \arg\max_T \int_0^T e^{-(\mu+\rho+\lambda)s}\ \mathbb{E}\left[m_s\ \Pi_s\left(p, z_s\right) \middle| \hat{\mathrm{z}}, \hat{\sigma}^2\right] ds \\
&\quad + \; e^{-(\mu+\rho+\lambda)T}\ \mathbb{E}\left[\ V_T(p, z_T, m_T) \middle| \hat{\mathrm{z}}, \hat{\sigma}^2\right] \ ,
\end{aligned}
\tag{19}
$$

where the value function $V_t(\cdot)$ is given in equation (18).

Proposition 1. Consider the normalized value function: $v_t(g) \equiv \frac{V_t(p,z,m)}{m\,\Pi_t^*(z)}$ where g is the price gap defined in equation (15). This normalized, single-state, value function solves:

$$
\begin{aligned}
v_t(g) \quad &= \quad \max\left\{\hat{v}_t \ , \ \bar{v}_t(g)\right\} \ \text{ where} \\
\hat{v}_t \quad &= \quad -(\theta+\psi) + \max_{T,\hat{g}} \int_0^T e^{-(\rho+\lambda)s} \left(\frac{c_{t+s}}{c_t}\right)^{1-\epsilon\eta} f(s,\hat{g})\, ds \ + \\
&+ \quad e^{-(\rho+\lambda)T}\left(\frac{c_{t+T}}{c_t}\right)^{1-\epsilon\eta} \int_{-\infty}^{\infty} e^{(\eta-1)\left(\gamma T + \sigma\sqrt{T}\,x\right)}\ v_{t+T}\left(\hat{g}\ e^{(\gamma-\mu)T+\sigma\sqrt{T}\,x}\right)\ dN(x) \ , \\
\bar{v}_t(g) \quad &= \quad -\theta + \max_T \int_0^T e^{-(\rho+\lambda)s}\left(\frac{c_{t+s}}{c_t}\right)^{1-\epsilon\eta} f(s,g)\, ds \ + \\
&+ \quad e^{-(\rho+\lambda)T}\left(\frac{c_{t+T}}{c_t}\right)^{1-\epsilon\eta} \int_{-\infty}^{\infty} e^{(\eta-1)\left(\gamma T + \sigma\sqrt{T}\,x\right)}\ v_{t+T}\left(g\ e^{(\gamma-\mu)T+\sigma\sqrt{T}\,x}\right)\ dN(x) \ , \\
\text{where} \quad & f(s,g') \equiv \int_{-\infty}^{\infty} e^{(\eta-1)\left(\gamma s + \sigma\sqrt{s}\,x\right)}\ \frac{F\left(g'\ e^{(\gamma-\mu)s+\sigma\sqrt{s}\,x}\right)}{F(1)} dN(x) \ ,
\end{aligned}
\tag{20}
$$

and where $N(\cdot)$ is the CDF of a standard normal and where $\theta = \bar{\theta}\ \xi^\eta\ \eta^\eta\ (\eta-1)^{1-\eta}$ and $\psi = \bar{\psi}\ \xi^\eta\ \eta^\eta\ (\eta-1)^{1-\eta}$.

See Appendix A for the proof. The function $f(s,g)$ is the ratio of the expected value of the nominal profits s periods from now (conditional on the current price gap being g) to the current flexible price nominal profits. Since $F(\cdot)$ is the sum of two power functions, f is known in closed form. The cost are also divided by the current flexible price nominal profits, and thus the value function is expressed in "real terms". The discount rate in equation (20) is $\rho+\lambda$, reflecting the steady state real rate as well as the probability that the good disappears. The term involving $\{c_{t+s}/c_t\}$ reflects the effect of the real interest rate, as well as the shift on the demand of the product coming from aggregate demand. In a steady state, i.e. when c_t is constant, the solution of the stationary value function characterizes completely the dynamics of price changes across firms. The value function v_t that solves equation (20) depends on time *only* because of the effect of the path of $\{c_{t+s}/c_t\}$. We study a simplified, steady-state version of equation (20) in Alvarez, Lippi, and Paciello (2011).

The optimal decision rules for each time t are described by three values for the price gap $\underline{g}_t < \hat{g}_t < \bar{g}_t$, and a function $\mathrm{T}_t(\cdot)$. A firm that immediately after observing its price gap at t to be g leaves its price unchanged, if $g \in (\underline{g}_t\ ,\ \bar{g}_t)$, and otherwise changes its price gap to $\hat{g}_t$. The function $\mathrm{T}_t(g)$ gives the time elapsed until the next observation. This form of the decision rules assumes that $v_t(\cdot)$ is single peaked. In the closely related model in Alvarez, Lippi, and Paciello (2011) we show that this is the case, and we also provide an analytical approximation for these objects.

Using the notation of Proposition 1 the next proposition rewrites the problem for the time of the first observation described in equation (19) as

Proposition 2. *The first observation time for a firm that at time $t=0$ has nominal price p, beliefs on its idiosyncratic productivity given by $(\hat{\mathrm{z}},\hat{\sigma}^2)$, and faces economy wide money m_0 and a path of aggregate consumption $\{c_t\}_{t\geq 0}$, solves:*

$$\tau_0(p,\hat{\mathrm{z}},\hat{\sigma}^2) = \arg\max_T \int_0^T e^{-(\rho+\lambda)s}\left(\frac{c_s}{c_0}\right)^{1-\epsilon\eta} f_0\left(s,\frac{p}{\hat{p}_0^*},\hat{\mathrm{z}},\hat{\sigma}^2\right)\,ds\ + \tag{21}$$

$$+e^{-(\rho+\lambda)T}\left(\frac{c_T}{c_0}\right)^{1-\epsilon\eta}\int_{-\infty}^{\infty} e^{(\eta-1)\left(\hat{\mathrm{z}}+\gamma T+\sigma\sqrt{T+\frac{\hat{\sigma}^2}{\sigma^2}}\ x\right)}\ v_{t+T}\left(\frac{p}{\hat{p}_0^*}\ e^{(\gamma-\mu)T+\sigma\sqrt{T+\frac{\hat{\sigma}^2}{\sigma^2}}\ x}\right)\ dN(x)$$

$$\text{where } f_0(s,g,\hat{\mathrm{z}},\hat{\sigma}^2) \equiv \int_{-\infty}^{\infty} e^{(\eta-1)\left(\hat{\mathrm{z}}+\gamma s+\sigma\sqrt{s+\frac{\hat{\sigma}^2}{\sigma^2}}\,x\right)}\ \frac{F\left(g\ e^{(\gamma-\mu)s+\sigma\sqrt{s+\frac{\hat{\sigma}^2}{\sigma^2}}\,x}\right)}{F(1)}dN(x)\ ,$$

$$\text{and where } \hat{p}_0^* = \frac{R\xi\eta}{\eta-1}\,m_0\,e^{-\hat{\mathrm{z}}}\ .$$

In equation (21) we use that the belief about $\log z_0$ is normal with parameters $(\hat{\mathrm{z}},\hat{\sigma}^2)$. In addition, after T periods have elapsed, log productivity will accumulate normal innovations with variance σ^2 per unit of time. Hence, the expected values of profits and the value function take this into account. For instance, the term $\sigma\sqrt{T+\hat{\sigma}^2/\sigma^2}$ is the standard deviation of the distribution of the log of the price gap after T periods, which includes both the cumulative effect of the innovations, as well as the uncertain initial level of the price gap.

We will consider two extreme versions of the beliefs of firms at time $t = 0$ with respect to the aggregate "shocks", i.e. the initial value of the economy wide money supply m_0 and the path of aggregate consumption $\{c_t\}_{t\geq 0}$. In the first case we assume that all firms learn at time $t = 0$ the new level of the money supply and the correct path of aggregate output $\{c_t\}_{t\geq 0}$ (but no new information on their own price gap is obtained at this time). This case represents an environment where firms continually pay attention to information about aggregates and where this information is readily available. In the second case we assume that until firms make their first observation they do not learn that there is a new value for the money supply or aggregate output. In this case we assume that until the first observation occurs firms use the decision rules that correspond to the steady state, which they believe, incorrectly, will describe the behavior of aggregates for the indefinite future. After the first observation, we assume they find out the value of the aggregate level of money m_{τ_0} as well as the path of aggregate consumption $\{c_t\}_{t\geq\tau_0}$. The second case represents an environment where firms are mostly concern with idiosyncratic shocks, so they only learn about aggregates as a consequence of observing their own cost and profits. This case corresponds closely to the assumptions in the rational inattention modeling by Mankiw and Reis (2006).

2.3 Equilibrium

In this section we define an equilibrium, and specify a map from the set of paths of aggregate consumptions into itself whose fixed point characterizes an equilibrium.

Proposition 1 describes a maximization problem, given a path for aggregate consumption $\{c_t\}_{t\geq 0}$, whose solution is described by $\{\underline{g}_t, \hat{g}_t, \bar{g}_t, \mathrm{T}_t(\cdot)\}_{t\geq 0}$. Now we describe a consistency condition implied by equilibrium, i.e. a mapping from policies $\{\underline{g}_t, \hat{g}_t, \bar{g}_t, \mathrm{T}_t(\cdot)\}_{t\geq 0}$ to a path of aggregate consumption $\{c_t\}_{t\geq 0}$. Given an initial distribution ϕ_0 and policies $\{\underline{g}_t, \hat{g}_t, \bar{g}_t, \mathrm{T}_t(\cdot)\}_{t\geq 0}$ we will get a path for the distributions $\{\phi_t\}_{t>0}$. These distributions must satisfy

$$c_t = \left(\int \left(\frac{p}{m_t\ (\mu+\rho)} \right)^{1-\eta} \phi_t(dp, d\hat{\mathrm{z}}, d\hat{\sigma}^2) \right)^{\frac{1}{\epsilon\,(\eta-1)}} \tag{22}$$

which is a consequence of the definition of $g_t = p_t/p^*$ where p^* is given by equation (12), as well as the first order conditions equation (8) and equation (7) and the definition of c_t. An equilibrium consists of a fixed point on sequences $\{c_t\}_{t\geq 0}$.

The law of motion for the triplet $(p_t, \hat{\mathrm{z}}_t, \hat{\sigma}^2_t)$, given a policy $\{\underline{g}_t, \hat{g}_t, \bar{g}_t, \mathrm{T}_t(\cdot)\}_{t\geq 0}$, is as follows. There are three cases. First, if t is an observation date, i.e. $t = \tau_i$, then $\tau_{i+1} = \mathrm{T}_t(p_t/p^*_t)$

$$(p_t\ ,\ \hat{\mathrm{z}}_t\ ,\ \hat{\sigma}^2_t) = \begin{cases} (\hat{g}_t\ p^*_t\ ,\ \log z_t\ ,\ 0) & \text{if } p_t/p^*_t \notin (\underline{g}_t\ ,\ \bar{g}_t) \\ (p_t\ , \log z_t\ ,\ 0) & \text{if } p_t/p^*_t \in (\underline{g}_t\ ,\ \bar{g}_t) \end{cases} \tag{23}$$

Second, if t is not an observation date and the product has not been replaced, so that $\tau_i < t < \tau_{i+1}$, then $\tau_{i+1} = \mathrm{T}_{\tau_i}(p_{\tau_i}/p^*_{\tau_i})$ and

$$(p_t\ ,\ \hat{\mathrm{z}}_t\ ,\ \hat{\sigma}^2_t) = \big(p_{\tau_i}\ ,\ \log z_{\tau_i} + \gamma(t-\tau_i)\ ,\ \sigma^2(t-\tau_i)\big) \tag{24}$$

Third, if t is not an observation date, but the product has been replaced at date $\tau_0 < t < \tau_1$, then $\tau_1 = \mathrm{T}_{\tau_0}(p_{\tau_0}/p^*_{\tau_0})$ and

$$(p_t\ ,\ \hat{\mathrm{z}}_t\ ,\ \hat{\sigma}^2_t) = \big(p_{\tau_0}\ ,\ \log z_{\tau_0} + \gamma(t-\tau_0)\ ,\ \sigma^2(t-\tau_0)\big) \tag{25}$$

In the case where $\eta\epsilon = 1$ the equilibrium fixed point simplifies vastly. In particular, the problem for the firm described in Proposition 1 and Proposition 2 is independent of the path for aggregate consumption c_{t+s}, and thus equal to the one for the steady state, which is simpler to characterize. Hence the aggregate shocks affect the path of aggregate variables only through the aggregation of fixed firms' decision rules. The assumption that $\eta\epsilon = 1$ means that the substitution elasticity of a firm product is the same across varieties than it is across time, so that the consumption of each variety enters in the utility in an additively separable way. As can be seen from the consumers' first order conditions (equation (8)) the additive separability implies that the demand faced by each firm is independent of the aggregate consumption at that date. Instead, when varieties are more substitutable across them than aggregate output across time, i.e. when $\eta > 1/\epsilon$, an increase of the aggregate consumption c_t reduces the household demand for good $C_t(p)$, so that the demand for the firms producing this good shifts down.

3 Computation of Impulse Response

We discretize time into intervals of length Δ, so now calendar dates, and hence choices of times to review $t \in \mathbb{T} \equiv \{0, \Delta, 2\Delta, ...\}$. Let ϕ_0 be the initial distribution, which is set to the stationary distribution for constant money growth rate, and whose computation is described below.

The numerical procedure to compute the path for equilibrium quantities consists of iterating on the following two steps until convergence.

1. Solve for the path for aggregate consumption $\{c_t\}_{t\in\mathbb{T}}$ assuming the economy starts with distribution ϕ_0, receives a monetary shock ϵ_m at $t = 0$, and the firms use decision rules $\{\underline{g}_t, \hat{g}_t, \bar{g}_t, \mathrm{T}_t\}_{t\in\mathbb{T}}$.

2. Solve the problem of the firm for all times $t \in \mathbb{T}$ to obtain the decision rules $\{\underline{g}_t, \hat{g}_t, \bar{g}_t, \mathrm{T}_t\}_{t\in\mathbb{T}}$ given a path for the aggregate consumption $\{c_t\}_{t\in\mathbb{T}}$.

The initial condition for this process is given by $c_t = \bar{c}$ for all $t \in \mathbb{T}$. Also we impose that the equilibrium path of consumption $c_t = \bar{c}$ for $t \geq \hat{T}$ where $\hat{T}$ is a large number. Below we give more details about steps 1 and 2.

3.1 Computation of the Stationary Equilibrium

First we discuss the computation of the decision rules. We use standard value function iterations in the problem described in equation (20), for $c_t = \bar{c}$, with the restriction that time elapsed between reviews $T \in \mathbb{T}$. In this case, the decision rules and value function do not depend on calendar time. In each of the value function iterations we solve the value function in a grid of price gaps $g \in G$ containing n points, and we interpolate to all $g \in \mathbb{R}$. This gives the time independent decision rules $\underline{g}, \hat{g}, \bar{g}, \mathrm{T}(\cdot)$ and the corresponding, time independent, value function $v(\cdot)$.

Using the time independent optimal decision rules we compute the stationary distribution ϕ_0 by simulating the discrete time analog of the triplets $(p, \hat{z}, \hat{\sigma}^2)$ for N firms for $\bar{T}/\Delta$ model periods. For each firm i we simulate:

$$\log z_{t+\Delta,i} = \begin{cases} \log z_{t,i} + \gamma\Delta + \sigma\sqrt{\Delta}\ x & \text{with prob. } e^{-\Delta\lambda} \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

where x is a standard normal and where the starting date is $t = -\bar{T}/\Delta$ up to $t = 0$. Using this realization for z_{ti} and the times at which products have been replaced, we can find the sequence for the triplet $(p_{it}, \hat{\text{z}}_{it}, \hat{\sigma}^2_{it})$ using the decision rules as described in Section 2.3 in equation (23) - equation (25). The outcome of this process is to have N time series, each of length $\bar{T}/\Delta$ of the triplets $(p_{it}, \hat{\text{z}}_{it}, \hat{\sigma}^2_{it})$. The number of firms N and the number of periods $\bar{T}$ are chosen to be large enough so that the cross section of this triplets at time $t = 0$ stabilizes, and hence represents ϕ_0 accurately.

3.2 Computing the consumption path implied by the decision rules

We use the time dependent decision rules to simulate the price gaps, productivity $\{g_{it}, z_{it}\}$ and the triplets $\{p_{it}, \hat{\text{z}}_{it}, \hat{\sigma}^2_{it}\}$ for each of the N firms for $\hat{T}$ periods, as explained in Section 2.3, and equation (23)-equation (25). Each of the N firms is initialized from the sample of initial distribution ϕ_0 of the triplets $(p, \hat{\text{z}}, \hat{\sigma}^2)$, $(p_{i0}, \hat{\text{z}}_{i0}, \hat{\sigma}^2_{i0})$ described above. Then for each cross section at times $t \geq 0$ we compute the implied aggregate consumption as:

$$c_t = \left(\sum_{i=1}^{N} \left(\frac{p_{it}}{m_t\ (\mu + \rho)} \right)^{1-\eta} \right)^{\frac{1}{\epsilon\,(\eta-1)}} \tag{27}$$

3.3 Computing the decision rules implied by the consumption path

The decision rules are the solution of the dynamic programming problem described in equation (20) where time elapsed between reviews are restricted to $T \in \mathbb{T}$. The value function and decision rules are computed by backward induction, taking as given the stationary value function v at $t = \hat{T}$. In each of the value function iterations we solve for the value function on a grid of price gaps $g \in G$ containing n points, and we interpolate to all $g \in \mathbb{R}$. This gives a path of decisions rules $\{\underline{g}_t, \hat{g}_t, \bar{g}_t, \mathrm{T}_t(\cdot)\}_{t \in \mathbb{T}, t \leq \hat{T}}$.

4 Choosing the parameters of the model

We calibrate the parameters of our model to match salient statistics from the U.S. economy. Later in the paper we will consider a different parametrization of the model that is tailored to the Euro area. We set $\eta = 4$ so that the average markup is roughly one third, between the values used Midrigan (2007) and Golosov and Lucas (2007). Following Golosov and Lucas (2007), we set $\epsilon = 2$ so to have an intertemporal elasticity of substitution of 1/2, and $\xi = 6$ so that households allocate approximately 1/3 of the unit time endowment to work in steady state. We set the yearly discount rate $\rho = 0.02$, and yearly growth rate of money supply $\mu = 0.02$, so to imply an annual average inflation of 2%. We set the drift in the cost process so that Z_t is a martingale, i.e. $\gamma = -\sigma^2/2$. We set the parameter λ to match the average frequency of price adjustments per year due to product substitutions estimated by Nakamura and Steinsson (2008), implying $\lambda = 0.25$. The remaining parameters, θ, ψ and σ are key to determine firms's pricing decisions and, therefore, to determine the aggregate effects of nominal shocks. In Alvarez, Lippi, and Paciello (2011) we show that these parameters can be uniquely identified through the model implied statistics about (i) the ratio of the average number of price reviews to the average number of adjustments, (ii) the average number of price adjustments, and (iii) the average size of price changes. We use estimates obtained by Nakamura and Steinsson (2008) on U.S. CPI data from 1998 to 2005 for the frequency and size of price adjustments. In particular, we target the average number of price adjustments per year in our model to be 1.3 adjustments per year excluding sales, and the average size of price changes conditional on adjustment to be 0.085. As we show in Alvarez, Lippi, and Paciello (2011), these two statistics allow to identify the *level* of the two fictions, θ and ψ, as well as the volatility of the state, σ. The ratio of the average number of price reviews to the average number of adjustments determines instead the relative size of the two frictions, $\alpha \equiv \psi/\theta$. Our benchmark parametrization of $\alpha \equiv \psi/\theta$ comes from matching the ratio of the average number of price reviews to the average number of price adjustments estimated by Blinder et al. (1998) for the U.S., which is equal to 1.4. This procedure gives estimates of $\theta = 0.88\%$, $\psi = 0.44\%$ and $\sigma = 0.12$. The value ψ so estimated correspond to 0.11% of steady state revenues, and is in between the menu costs estimated directly by Zbaracki et al. (2004) and Levy et al. (1997), who find that menu costs amount to 0.04% and 0.70% of revenues respectively.[3] The value of θ so estimated correspond to 0.22% of steady state revenues, and is very close to estimates by Zbaracki et al. (2004) who find that managerial cost of price changes amount to 0.28% of revenues.

Our framework nests two popular models which have been extensively analyzed in the literature to study the implications of aggregate nominal shocks: the model with only menu cost and the model with only observation cost. A separate contribution of this paper to the existing literature is to provide a comparison between the predictions of these two different models in a unified framework. When firms only face observations costs, i.e. $\psi/\theta = 0$, the price adjustment rule is a time-dependent one, so that price adjustments coincide with observations.[4] When firms only face an adjustment type of friction and observe the state continuously, i.e. $\psi/\theta = \infty$, the posted price is adjusted whenever it is far enough from the optimal price, so that the price adjustment rule is a state-dependent one.[5] Therefore, we derive the aggregate responses of output and prices to monetary shocks at $\alpha = 0$ and at $\psi/\theta = \infty$, corresponding to a ratio of average frequencies of price reviews to adjustments, n_r/n_a, of $n_r/n_a = 1$ and $n_r/n_a = \infty$ respectively. To study more generally the sensitivity of our results to the choice of α, and to highlight the importance of this parameter for the predictions about the aggregate effects of monetary shocks, we also consider other alternative parametrizations. In particular, we compute impulse responses in economies where the ratio of the average frequency of price reviews to price adjustments is equal $n_r/n_a = 2$ and $n_r/n_a =$

[3]Revenues are measured in steady state and at the profit-maximizing price in absence of frictions in price setting, i.e. $\Pi^*(z)$, evaluated at $z = 1$. We have used the fact that revenues are equal to $\eta\, \Pi^*(z)$.

[4]The predictions for the aggregate effects of monetary shocks of this class of models have been extensively studied in the literature, and are discussed by Reis (2006).

[5]The predictions for the aggregate effects of monetary shocks of this class of models have been extensively studied in the literature, and are discussed by Dixit (1991).

3, respectively, corresponding to $\psi/\theta = 2$ and $\psi/\theta = 9$ respectively. While these economies have different *ratios* of the frequency of price reviews to price adjustments, i.e. different $\alpha = \psi/\theta$, they are parametrized to predict the same average frequency and size of price adjustments. As a consequence, the economies we consider are characterized by different triplets of ψ, θ and σ. We think this is an interesting exercise because it allows us to compare predictions of models that are observationally equivalent in terms of two popular statistics summarizing pricing behavior, frequency and size of price changes, but that can yet deliver very different predictions about the effects of monetary shocks. Table 1 reports the values of ψ, θ and σ corresponding to these different economies.

Table 1: Parametrization of the economy as a function of the ratio of the average frequencies of reviews to adjustments

Ratio of the average frequencies of reviews to adjustments, n_r/n_a	ψ	θ	σ
$n_r/n_a = 1.0$ (Observation Cost Only)	0.00%	2.98%	0.120
$n_r/n_a = 1.4$ (Benchmark)	0.44%	0.88%	0.110
$n_r/n_a = 2.0$	0.62%	0.31%	0.107
$n_r/n_a = 3.0$	0.65%	0.07%	0.105
$n_r/n_a = \infty$ (Menu Cost Only)	0.85%	0.00%	0.102

The statistics about the average frequencies of reviews and adjustments are obtained from simulating the model in steady state. In all parametrizations the average frequency of price adjustment is equal to 1.3 adjustments per year, while the average size of price changes is equal to 0.085. All other parameters are constant.

5 Impulse responses to nominal shocks

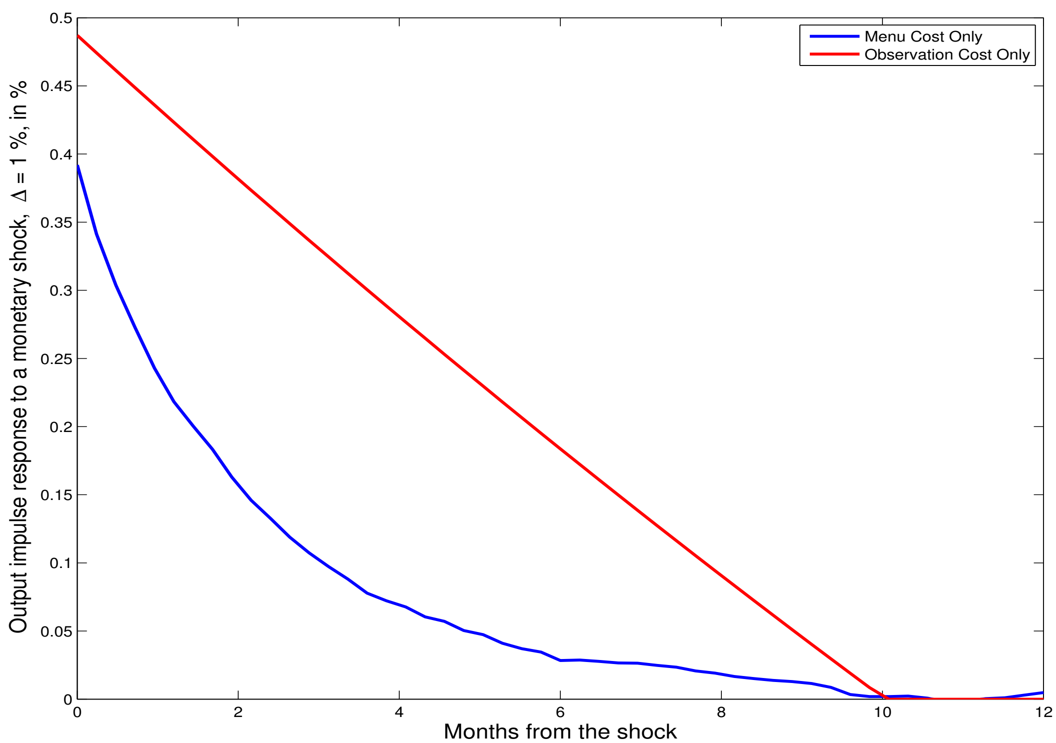
In this section we solve the model numerically, and compute aggregate output and price responses to one time, unforeseen, permanent increase in money supply, m_t, of size δ. We discretize time with intervals of approximately two weeks. We assume that firms are unaware of the monetary shock until they make an observation. We think this is a reasonable assumption for relatively small monetary shocks. Later in the paper we will show results for the case in which firms are instead perfectly informed about the realization and size of the aggregate shock.

Before presenting results for our benchmark model with both adjustment and observation costs, we derive impulse responses in the two polar cases where firms face either adjustment cost only, or observation cost only. The comparison of impulse responses predicted by these two extreme cases is particularly interesting to illustrate the different implications of the two types of frictions for aggregate responses.

5.1 Impulse responses in models with observation and adjustment costs only

We consider two polar cases in the parametrization of our model. The first alternative is a model where observation costs are zero, i.e. $\theta = 0$, and adjustment costs are positive, $\psi > 0$. The implications of the latter for output responses to nominal shocks have been widely studied in the literature. The second alternative is a model where menu costs are zero, i.e. $\psi = 0$, but observation costs are positive, $\theta > 0$. We choose the parameters of each of these models so that they are observationally equivalent in steady state, and in particular imply the same frequency and size of price adjustment of our benchmark model where both costs are allowed to be strictly positive.[6] This procedure implies $\psi = 0.0085$, $\theta = 0$ and $\sigma = 0.10$ for the adjustment cost only model, and $\psi = 0$, $\theta = 0.0298$ and $\sigma = 0.12$ for the observation cost only model. All other parameter values are identical to our benchmark calibration with both costs strictly positive.

Figure 1: Impulse response of $\log C_t$ to a monetary shock $\delta = 1\%$, observation and menu cost only models

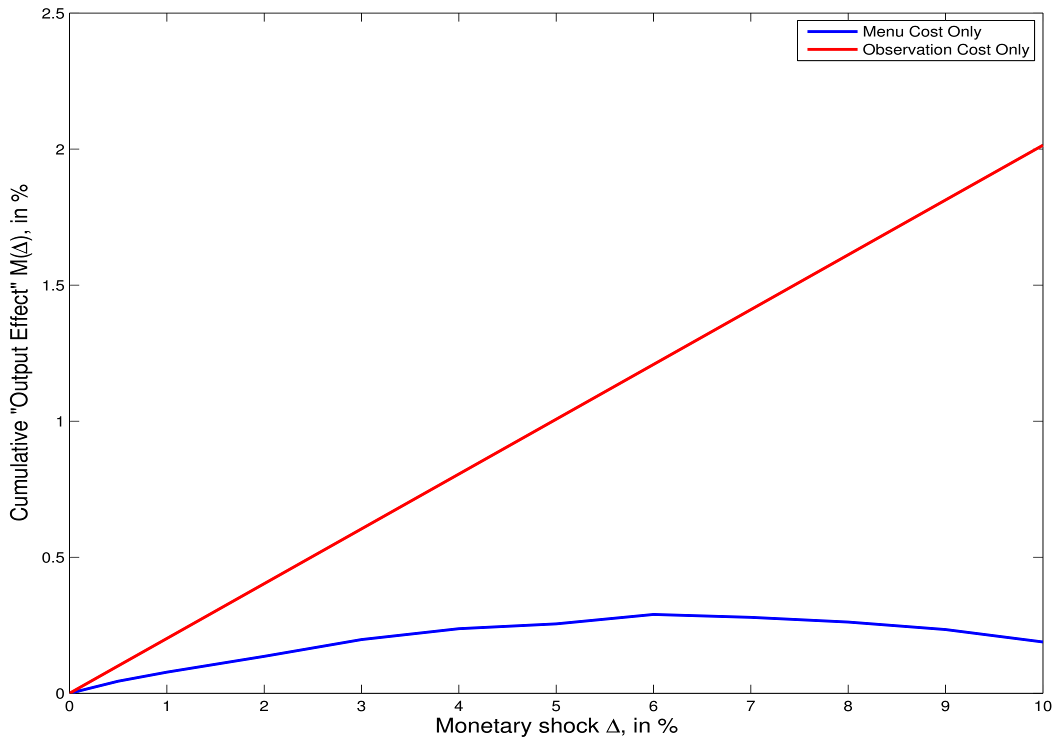


Note: impulse responses of $\log C_t$ to a 1% one-time permanent increase in m_t, in % deviation from steady state.

[6]In the case of observation cost only, we consider a technology for changing prices where prices can be adjusted only upon observation.

Figure 1 shows output impulse responses to a one time unexpected permanent increase in money supply equal to $\delta = 1\%$ predicted by the models with observation and adjustment cost only. Despite being characterized by the same frequency and size of price adjustments, these two different models have substantially different predictions about the aggregate output response to the same monetary shock. The output response to the monetary shock is larger on impact in the model with observation cost only than in the model with adjustment cost only. The output response is linear in the time elapsed since the shock in the model with observation cost only, while it is exponentially decreasing in the time elapsed since the shock in the model with adjustment cost only. As a consequence of the different impact and different slope of the output response as a function of the time elapsed since the shock, the output response to the monetary shock in the model with observation cost only is above the response predicted by the model with adjustment cost only for about ten months.

Figure 2: Cumulated output response as a function of the size of the shock δ



Note: impulse responses of $\log C_t$ to a 1% one-time permanent increase in m_t, in % deviation from steady state.

Figure 2 plots the cumulated output response as a function of the size of the monetary shock, δ, in the models with adjustment and observation cost only. The cumulated output response, $\mathcal{M}(\delta)$ is defined as

$$\mathcal{M}(\delta) = \int_{t_0}^{\infty} (\log(c_t(\delta)) - \log(c_{t_0-\epsilon}))\ dt \tag{28}$$

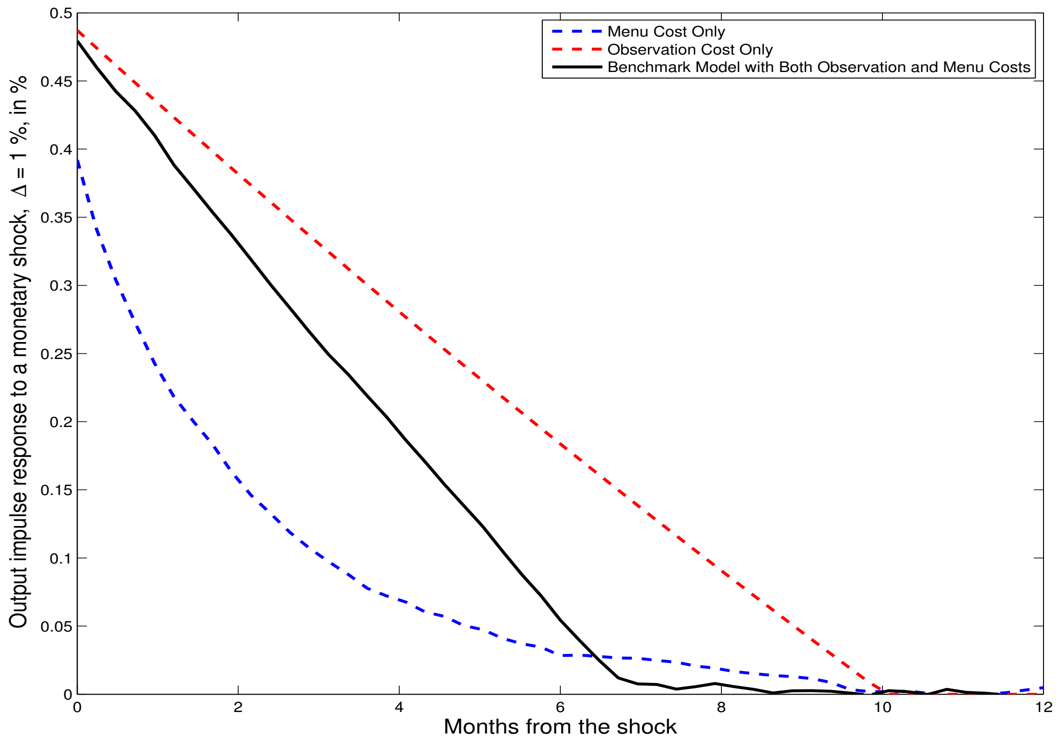
where $c_t(\delta)$ is the equilibrium level of aggregate output in period $t \geq 0$, after a monetary shock of size δ realized in period t_0, and $c_{t_0-\epsilon}$ is the equilibrium level of aggregate output in the steady state. A striking difference arises between the two models. The cumulated output response increases linearly in the size of the monetary shock in the model with observation cost only, while it has an inverse-U shape in the model with menu cost only, increasing in the size of the shock for small values of monetary shocks, but eventually decreasing towards zero as monetary shocks become large enough. As a consequence, the difference in the predictions of the two models about the impact of monetary shocks on cumulated output increase with the size of the the monetary shock. In other words, the larger the monetary shock is, the more important distinguishing between the two models.

The different predictions of the two models about the impact of nominal shocks arise from the different firms' decision rules for adjusting prices. As we show in Alvarez, Lippi, and Paciello (2011), the model with observation cost only implies that firms plan to observe the state at equal and constant intervals of time. Given that firms only adjust prices once they observe the state, this implies that the price setting will follow a time-dependent rule. Therefore, in each period of time, there will be the same fraction of firms adjusting prices, independently of the size of the shock. The latter implies that the output response is linear in the time elapsed since the shock, and that the cumulated output response is linear in the size of the shock. In contrast, in the model with adjustment cost only, the firms' price setting is state-dependent. Firms adjust prices whenever the price gap g_t crosses the thresholds $\{\underline{g}_t, \bar{g}_t\}$. A monetary shock causes a shift of price gaps of all firms in the same directions causing some firms, that would otherwise have not adjusted, to adjust their price. The larger the mass of firms that is moved outside of the inaction region upon impact of the shock, the larger the response of prices, the smaller the output response of the monetary shock. As time elapses, more and more firms will be hit by idiosyncratic shocks that pushes them outside of the inaction region, but the mass of firms that have not yet adjusted their price after the monetary shocks, and have a price gap outside of the inaction region is decreasing in the time elapsed from the shock.

5.2 Impulse responses in the model with observation and adjustment costs

Figure 3 plots impulse responses of output in percentage deviations from the steady state to a permanent and one time 1% increase in m_t, in the benchmark model with two costs, as well as in the models with one cost only. The benchmark model with both observation and menu costs predicts a sizable and persistent response of output to the monetary shock. In fact, at our benchmark parametrization, we obtain an impulse response of output that is, not surprisingly, in between the impulse responses obtained in the model with observation cost only, i.e. $\psi = 0$, and in the model with menu cost only, i.e. $\theta = 0$. The shape of the output response is roughly linear for about seven months, which is the time it takes before all firms have observed the state of the economy, i.e. the time to next observation upon an adjustment, $\text{T}_{\hat{T}}(\hat{g}_t)$. After, $\text{T}_{\hat{T}}(\hat{g}_t)$ periods, the shape of the impulse response displays more non-linearity. In fact, this non-linear component of the response comes from the state-dependent component of the firm adjustment policy due to the presence of the menu cost. Therefore, the time-dependent component of the adjustment rule dominates over the state-dependent component

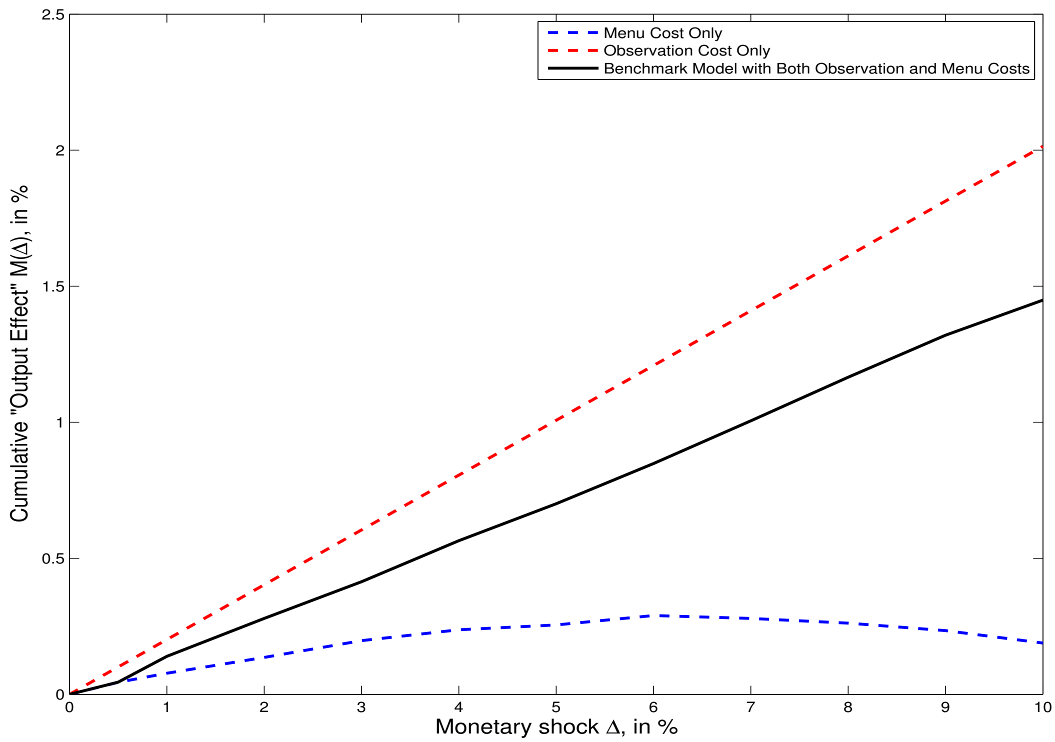
Figure 3: Impulse response of $\log C_t$ to an unexpected 1% increase in m_t, $\delta = 1\%$



Note: impulse responses of $\log C_t$ to a 1% one-time permanent increase in m_t, in % deviation from steady state.

of the adjustment rule in determining the shape of the output response to a $\delta = 1\%$ monetary shock. This is because, the invariant distribution of price gaps conditional on observing the state displays a relatively large mass of firms close to the adjustment thresholds so that a monetary shock moves a relatively large fraction of these firms outside of the inaction region. The smaller the menu cost relatively to the observation cost, the larger the fraction of firms adjusting to the monetary shock upon observation.

Figure 4: Cumulated output response as a function of the size of the shock δ



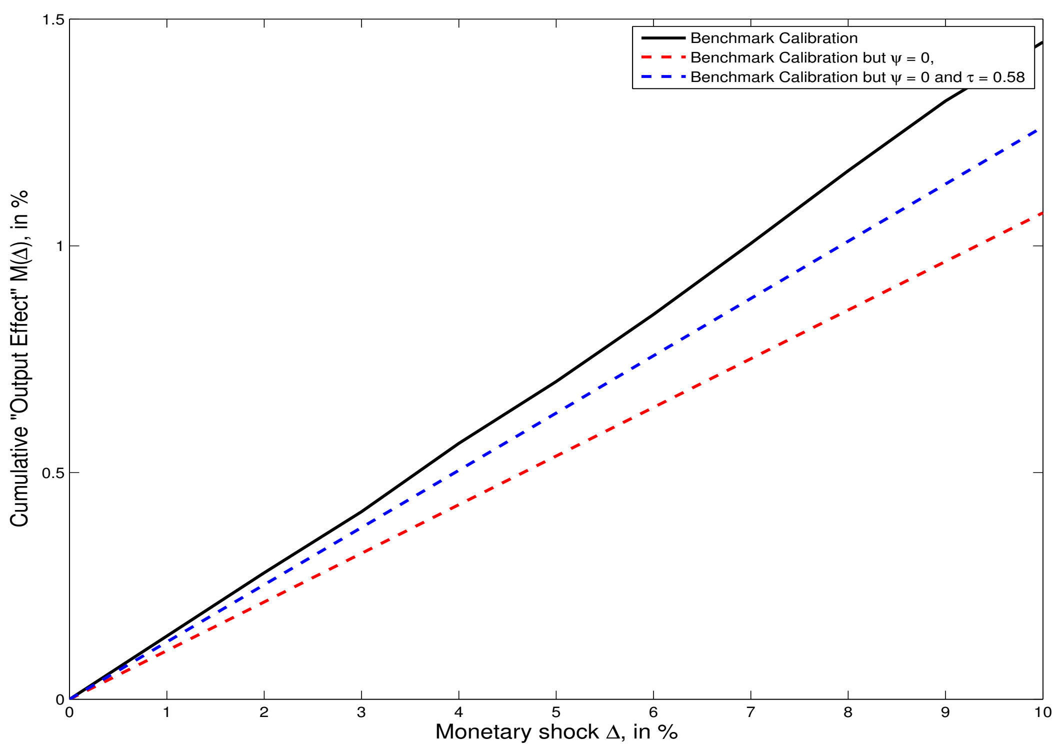
Note: impulse responses of $\log C_t$ to a 1% one-time permanent increase in m_t, in % deviation from steady state.

Figure 4 plots the cumulated output response defined in equation equation (28), as a function of the size of the monetary shock, under our benchmark parametrization with both adjustment and observation costs. The cumulated output response is increasing in the size of the shock, and this is a property inherited from the the time-dependent component of firms' price setting behavior due to the presence of the observation cost. The figure also makes clear that a model with only adjustment costs, and observationally equivalent in steady state to the model with both adjustment and observation costs, would understate the impact of a nominal shock, for all sizes. In relative terms, the extent of the understatement is increasing in the size of the nominal shock: the menu cost only model would predict a cumulated output response to a 1% and to a 10 % increase in money supply that is respectively about 45% and 85% smaller than the one predicted by the benchmark model with both costs. In contrast, a model with only observation costs, but observationally equivalent in steady state to the model with both adjustment and observation costs, would overerstate the impact of a nominal shock, for all sizes. In relative terms, the extent of the overstatement is decreasing in the size of the nominal shock: the menu cost only model would predict a cumulated output response to a 1% and to a 10 % increase in money supply that is respectively about 45% and 39% larger than the one predicted by the benchmark model with both costs.

5.2.1 The contribution of menu costs to money non-neutrality

How much does the menu cost account for the overall output response to a nominal shock? What is the channel through which the menu costs affect the aggregate response to nominal shocks? These are an interesting question because of the interaction between observation and menu costs in determining the optimal price setting of firms. As shown by Alvarez, Lippi, and Paciello (2011), menu costs can affect the price setting decision through two channels: directly, by inducing an inaction region conditional on observations; indirectly, by affecting the decision of the time to the next observation. In order to investigate this issue, we run

Figure 5: Cumulated output response as a function of the size of the shock δ, Benchmark calibration with and without menu cost



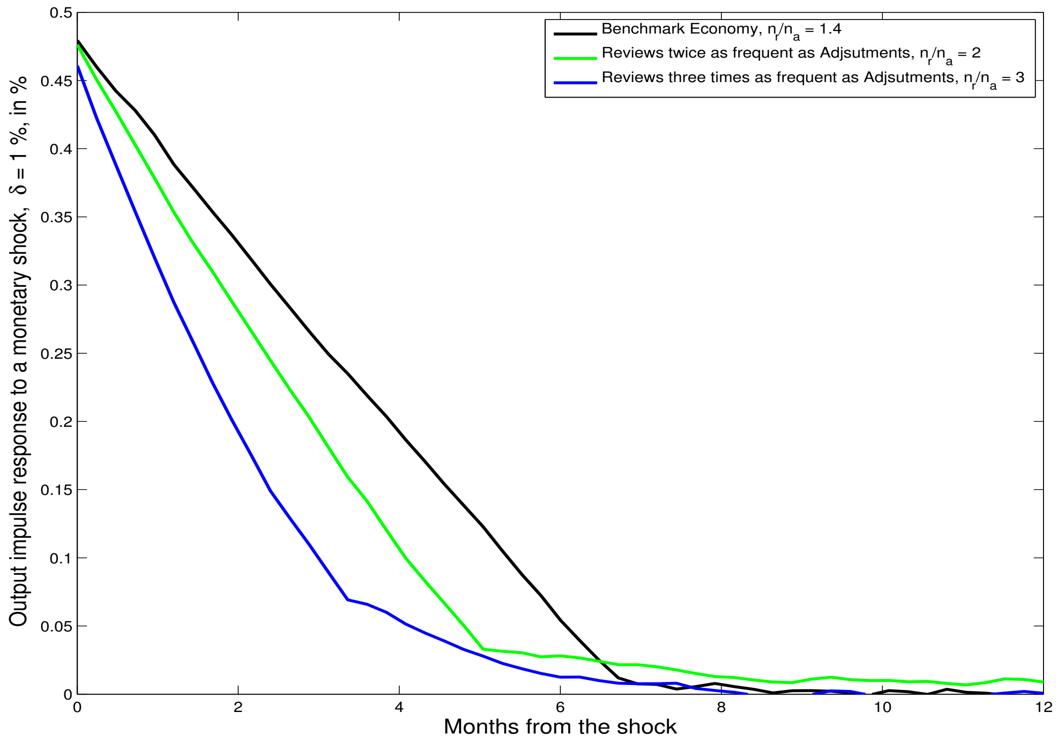
Note: impulse responses of $\log C_t$ to a 1% one-time permanent increase in m_t, in % deviation from steady state.

the following experiments. We solve for the cumulated output responses for different sizes of the monetary shock in a model where: (i) we completely shut down the menu cost friction, i.e. we keep all parameters equal to the ones used for our benchmark parametrization but we set $\psi = 0$, solve for the optimal firms' decision rule and derive the aggregate implications; (ii) we set $\psi = 0$, but we keep the decision of the time to the next observation unchanged from the optimal decision rule derived under the benchmark parametrization. The first experiment captures the overall impact of menu costs on the impulse response of output to the monetary shock through both the *direct* and *indirect* channels. The second experiment captures the role of the *indirect* channel. Figure 5 plots the cumulated output responses from these experiments. According to the first experiment, the cumulated output response is smaller in absence of the menu cost. In fact, it ranges from about 77% of the cumulated response in the benchmark model to a 1% shock, to about 74% in the case of a 10% shock. According to the second experiment, the cumulated output response ranges from about 90% of the cumulated response in the benchmark model to a 1% shock, to about 87% in the case of a 10% shock. Therefore, the impact of menu costs on the decision about the frequency of observations matters for about a half of the overall cumulated output response due to menu cost.

5.3 Sensitivity of impulse responses to the choice of $\alpha = \psi/\theta$

We have seen how sensitive predictions about the output responses to a monetary shock can be when we have compared the two polar cases with observation cost only and adjustment cost only. In this section we compute impulse responses for intermediate parametrizations of the relative size of the two frictions as described in Table 1, and compare them to predictions obtained under our benchmark parametrization for the U.S.

Figure 6: Impulse response of $\log C_t$ to an unexpected 1% increase in m_t, for different values ofψ/θ



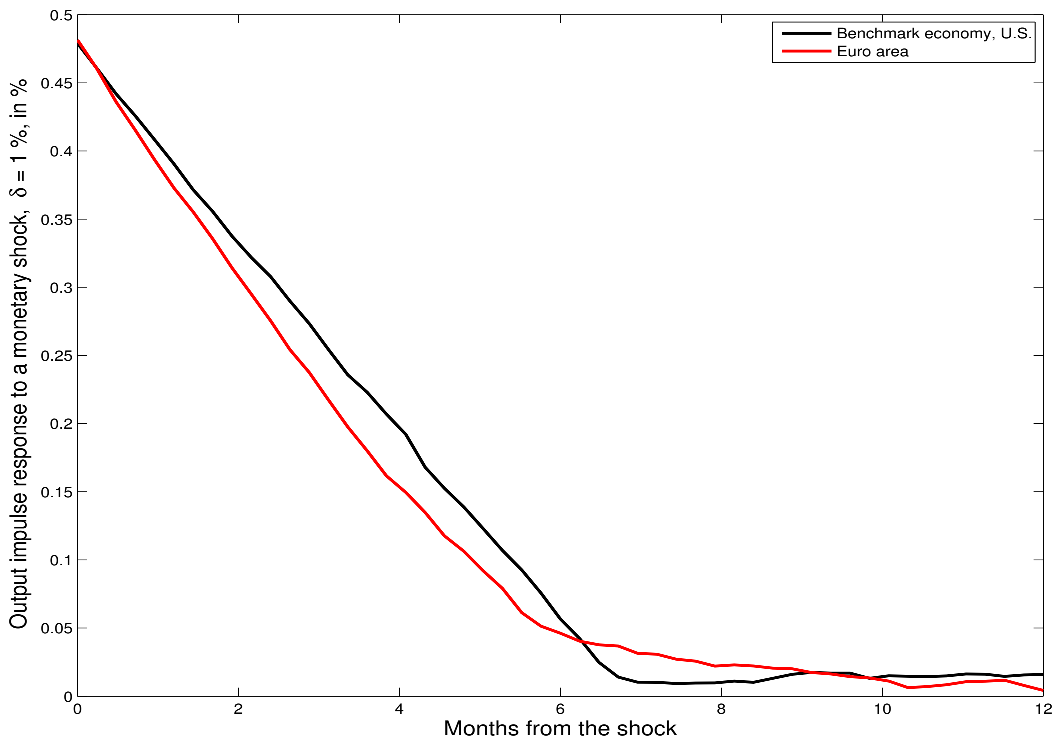
Note: impulse responses of $\log C_t$ to a 1% one-time permanent increase in m_t, in % deviation from steady state. All models have the same average frequency and size of price adjustments. The parameters σ, ψ and θ are set according to Table 1.

Figure 6 plots impulse responses of output to a 1% increase in m_t in our benchmark model where the average frequency of price reviews is 40% larger than the average frequency of price adjustments, and in two alternative economies where the average frequency of price reviews is respectively 100% and 200% larger than the average frequency of price adjustments. The larger the ratio of frequencies of reviews to adjustments, the larger the implied ratio of menu cost to observation cost. The larger the size of adjustment cost relatively to observation costs, the smaller the impact of the monetary shock on impact, the the more non-linear the shape of the impulse response, and the smaller the overall cumulated output response.

5.4 Output responses to monetary shocks in the Euro area

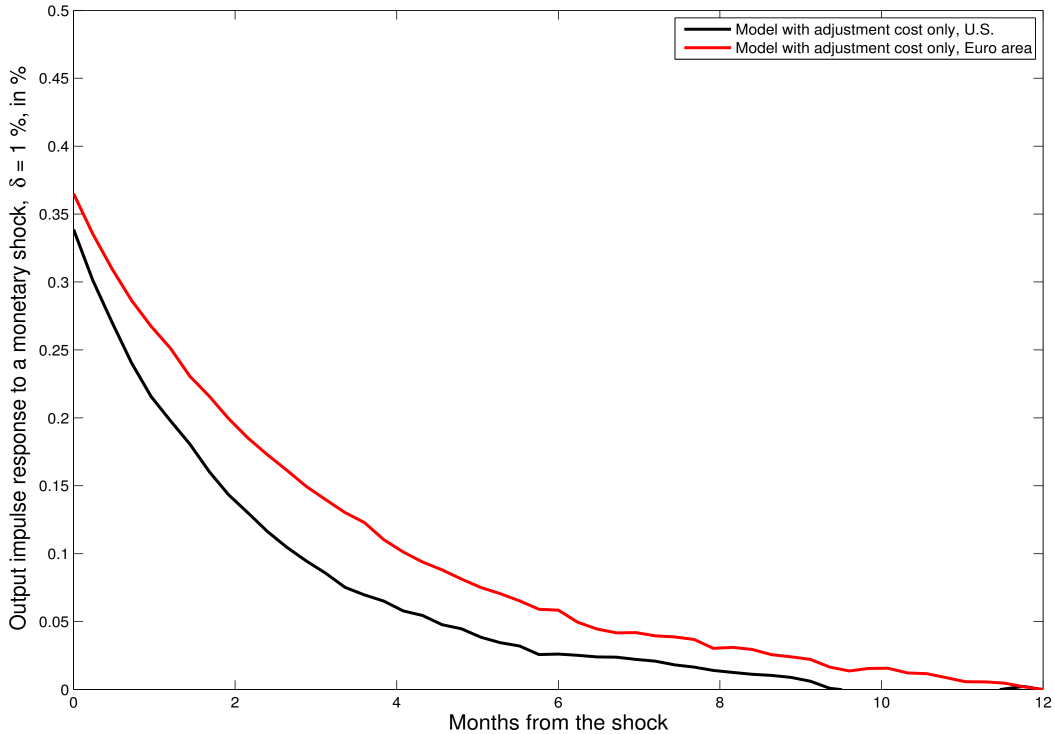
Do monetary shocks have larger or smaller output effects in the Euro area than in the U.S.? In this section we choose parameters ψ, θ and σ to match the ratio of the average frequencies of reviews to adjustment, the average frequency and size of price adjustment estimated in the Euro area by Fabiani et al. (2007). In particular, the ratio of price reviews to price adjustments is roughly equal to 2.5, while the frequency of price adjustments is equal to 1 adjustment per year and the average size of price adjustments is equal to 0.088. These statistics imply parameter values equal to $\theta = 0.27\%$, $\psi = 1.08\%$ and $\sigma = 0.096$. Figure 7 plots output responses to a 1% monetary shock in the Euro area against the predicted output response from our benchmark economy calibrated to match statistics from the U.S. economy. The predicted impact of monetary shocks in the two economies are very similar. The output response in the Euro are decays faster than in the U.S. in the first months after the shock, but it displays relatively more persistence at longer horizons. These differences partially offset, and in fact the cumulated output response in the two economies is very similar. The reason why the output response display different patterns has to do with the different frequencies of adjustment and observations in the two economies. The fact that observations are more frequent in the Euro area makes the output response to decay faster in the first months after the shock, as a larger fraction of firms becomes aware of it, but the fact that the frequency of price adjustments is smaller in the Euro area implies that conditional on observing the state a smaller fraction of firm adjusts, which makes the output response to be relatively more persistent than in the U.S. economy at longer horizons. Notice also, that the average size of price changes is very much similar in the two economies, so that any difference in the shape of impulse responses come from the different combinations of the frequencies of price reviews and adjustments. Therefore, despite the frequency of price adjustments is larger in the U.S. economy that in the Euro area, monetary shocks have relatively similar impact on the economy. This prediction highlights the importance of computing impulse responses in a model where both observation and adjustment types of frictions are present. This is evident from Figure 8 where we compute output responses to the same 1% monetary shock predicted in the U.S. and in the Euro area from a model where only adjustment costs are present. The relevant statistics used to calibrate the two different economies are the average frequencies of price adjustments, equal to 1.3 and 1 adjustments per year in the U.S. and Euro are respectively, and the average size of price changes, equal to 0.085 and 0.088 in the U.S. and in the Euro are respectively. As we have previously shown, the model with menu cost only would predict smaller output responses to monetary shocks in both economies. However, in relative terms, the model with adjustment cost only would predict that the cumulated output response to a 1% permanent and unexpected increase in money supply in the Euro is 45% larger than in the U.S. Therefore, the predictions obtained from standard menu cost models where only adjustment costs are present might lead to the misleading conclusion that monetary shocks have substantially larger real effects in the Euro area than in the U.S. economy.

Figure 7: Impulse response of $\log C_t$ to an unexpected 1% increase in m_t: firms perfectly observing the shock



Note: impulse responses of $\log C_t$ to a 1% one-time permanent increase in m_t, in % deviation from steady state in our benchmark model, and in the case where monetary shocks are perfectly observed upon impact.

Figure 8: Impulse response of $\log C_t$ to an unexpected 1% increase in m_t: economies with adjustment cost only



Note: impulse responses of $\log C_t$ to a 1% one-time permanent increase in m_t, in % deviation from steady state in the model with adjustment cost only.

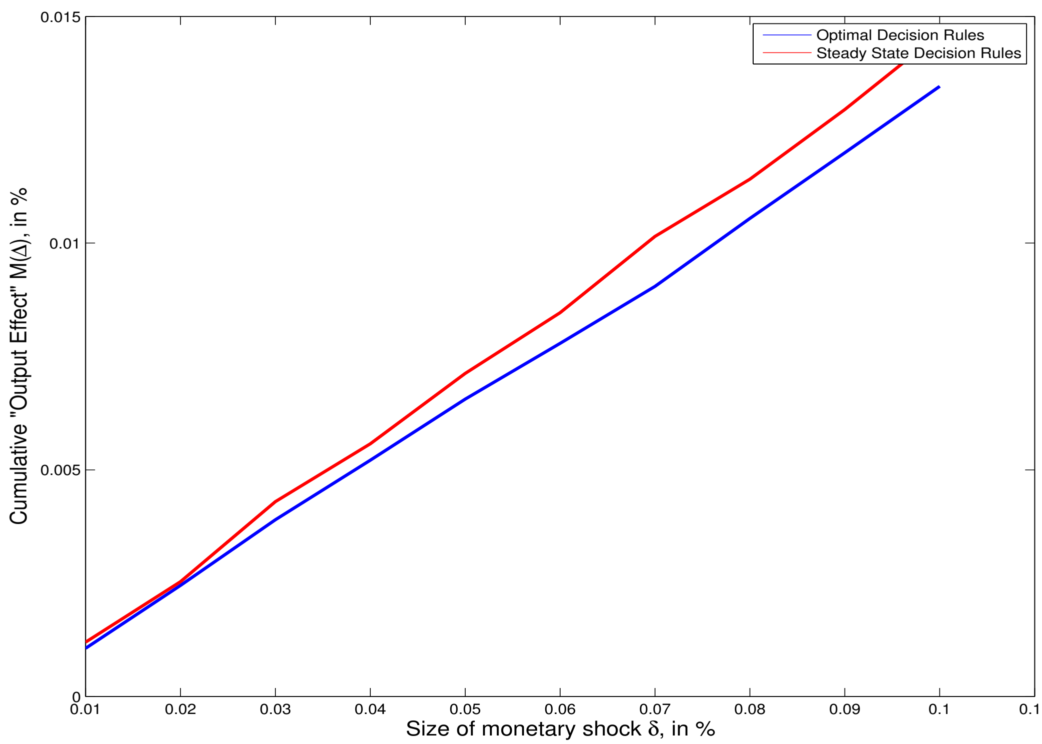
6 General equilibrium feedbacks and firms' decision rule

Solving for impulse responses in our model with both observation and menu costs can be numerically demanding, as the decision rule of the firm at each point in time depends on the equilibrium level of aggregate consumption, which in turn depends on the entire distribution of firms. However, in this section we show that the decision rules computed under the invariant distribution of prices in absence of aggregate shocks is a very good approximation of the decision rule computed instead by taking into account the general equilibrium feedback effects. In fact, the aggregate state matters for firms' decision only through its impact on how firms discount future profits, as evident from equation (20). In our environment, aggregate shocks have effects on firms' decision rules that are similar to shocks to the discount factor. We find that these types of effects are second order. We illustrate this finding in Appendix B where we report firm's decision rules computed numerically after a 1% increase in money supply. This is an interesting result because, combined with the analytical approximations to steady state firms' decision rules that we have derived in Alvarez, Lippi, and Paciello (2011), it provides a relatively easily implementable and fast algorithm to obtain a good approximation of impulse responses to aggregate shocks in our model.

The extent to which the steady state decision rules are a good approximation of the optimal decision rules after an aggregate shock depend on size of the shock. The latter is illustrated in Figure 9 where we plot the cumulated output response, $\mathcal{M}(\delta)$ as a function of the size of the shock, computed numerically in the model where firms optimally change their behavior after the aggregate shock, and in the model where instead we keep firms' decision rule invariant from the steady state.

A limitation of our result is that in our model there is no complementarity in the pricing decisions. The latter is evident from the fact that optimal price target, p^*, does not depend on other firms' actions. In an environment where this type of complementarity in price setting arises, as for instance when the production function has decreasing returns to scale, the steady state decision rules might not be a good approximation of the optimal decision rules after an aggregate shock anymore

Figure 9: Cumulated output response as a function of the size of the shock δ, computed under optimal and steady state decision rules



Note: Impulse responses to a monetary shock of size δ are obtained numerically. The blue solid line refers to the cumulated output response obtained in the model where the firms' decision rules are kept invariant to the aggregate shock; the red solid line refers to the cumulated output response obtained in the model where the firms' decision rules are computed optimally after the aggregate shock.

7 Perfectly observing the monetary shock

So far we have assumed that firms observe the realizations of monetary shocks upon observation of the state. In this section we instead compute impulse responses under the assumption that monetary shocks are immediately observed as in the case with menu cost only, but the firm needs to pay the observation cost to observe the idiosyncratic state. Notice that in steady state these two models are equivalent.

TO BE CONTINUED

8 Concluding remarks

TBA

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A Proofs

Proof. (of Proposition 1) The optimal value of the sequence problem in equation (18) solves the following functional equation:

$$
\begin{aligned}
\frac{V_t(p,z,m)}{m} &= \max\left\{\frac{\hat{V}_t(z,m)}{m}\ ,\ \frac{\bar{V}_t(p,z,m)}{m}\right\}\\
\frac{\hat{V}_t(z,m)}{m} &= -\theta_t(z)-\psi_t(z)+\max_{T,\hat{g}}\int_0^T e^{-(\rho+\lambda)s}\ c_{t+s}^{1-\epsilon\eta}\ \mathbb{E}\left[z_s^{\eta-1}\ F\left(\frac{\hat{g}}{p^*_{t+s}}\right)\big|z_0=z\right]ds+\\
&+\ e^{-(\rho+\lambda)T}\ c_{t+T}^{1-\epsilon\eta}\ \mathbb{E}\left[\frac{V_{t+T}\left(\hat{g},z_T\right)}{m\,e^{\mu T}}\ \big|z_0=z\right]\ ,\\
\frac{\bar{V}_t(p,z,m)}{m} &= -\theta_t(z)+\max_{T}\int_0^T e^{-(\rho+\lambda)s}\ c_{t+s}^{1-\epsilon\eta}\ \mathbb{E}\left[z_s^{\eta-1}\ F\left(\frac{p}{p^*_{t+s}}\right)\ \big|z_0=z\right]ds+\\
&+\ e^{-(\rho+\lambda)T}\ c_{t+T}^{1-\epsilon\eta}\ \mathbb{E}\left[\frac{V_{t+T}\left(p,z_T\right)}{m\,e^{\mu T}}\ \big|z_0=z\right]\ ,
\end{aligned}
$$

From this recursion we can show the following homogeneity for the nominal expected discounted net profits:

$$
V_t(p,z,m)\ =\ m\ z^{\eta-1}\ V_t\left(\frac{p\ z}{m},1,1\right)\quad \text{for all}\quad p,z,m,t\geq 0 \tag{29}
$$

and hence we define the expected discounted profits relative to the economy money supply, for a normalized shock and a price relative to the flexible optimal price. This value function, scaled by an index of real aggregate demand, turns out to be equal to the ratio of the nominal value function to the nominal flexible price profits, which gives an economic interpretation to the normalization:

$$
v_t(g)\ \equiv\ \frac{V_t\left(R\alpha\frac{\eta}{\eta-1}\ g,1,1\right)}{c_t^{1-\epsilon\eta}\ F(1)}=\frac{V_t(p,z,m)}{m\ \Pi^*_t(z)} \tag{30}
$$

We can write a one state Bellman equation associated to the real net profits immediately after the time of an observation as:

$$
\begin{aligned}
v_t(g) &= \max\left\{\hat{v}_t\ ,\ \bar{v}_t(g)\right\}\quad\text{where}\\
\hat{v}_t &= -(\theta+\psi)+\max_{T,\hat{g}}\int_0^T e^{-(\rho+\lambda)s}\left(\frac{c_{t+s}}{c_t}\right)^{1-\epsilon\eta}\mathbb{E}\left[\left(\frac{z_s}{z_0}\right)^{\eta-1}\frac{F\left(g_s\right)}{F(1)}\ \big|\ g_0=\hat{g}\right]ds+\\
&+\ e^{-(\rho+\lambda)T}\left(\frac{c_{t+T}}{c_t}\right)^{1-\epsilon\eta}\mathbb{E}\left[\left(\frac{z_T}{z_0}\right)^{\eta-1}v_{t+T}\left(g_T\right)\ \big|\ g_0=\hat{g}\right]\ ,\\
\bar{v}_t(g) &= -\theta+\max_{T}\int_0^T e^{-(\rho+\lambda)s}\left(\frac{c_{t+s}}{c_t}\right)^{1-\epsilon\eta}\mathbb{E}\left[\left(\frac{z_s}{z_0}\right)^{\eta-1}\frac{F\left(g_s\right)}{F(1)}\ \big|\ g_0=g\right]ds+\\
&+\ e^{-(\rho+\lambda)T}\left(\frac{c_{t+T}}{c_t}\right)^{1-\epsilon\eta}\mathbb{E}\left[\left(\frac{z_T}{z_0}\right)^{\eta-1}v_{t+T}\left(g_T\right)\ \big|\ g_0=g\right]\ ,
\end{aligned}
$$

The dynamics of the state g in the inaction region are given by $d\log g = (\gamma - \mu)dt + \sigma dB$. Thus for each s the distributions of g_s and z_s/z_0 are given by:

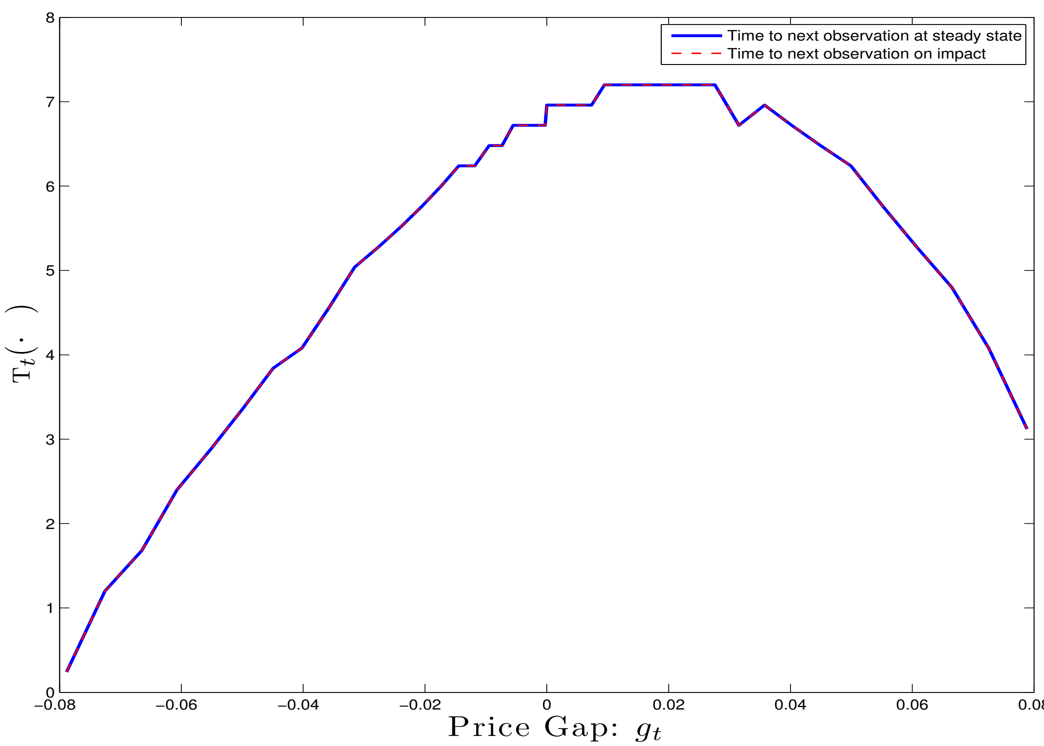
$$\frac{g_s}{g_0} = e^{(\gamma-\mu)s+\sigma\sqrt{s}\,x} \quad \text{and} \quad \frac{z_s}{z_0} = e^{\gamma s+\sigma\sqrt{s}\,x}$$

where x is a standard normal random variable. $\square$

B Firms' decision rules after the monetary shock

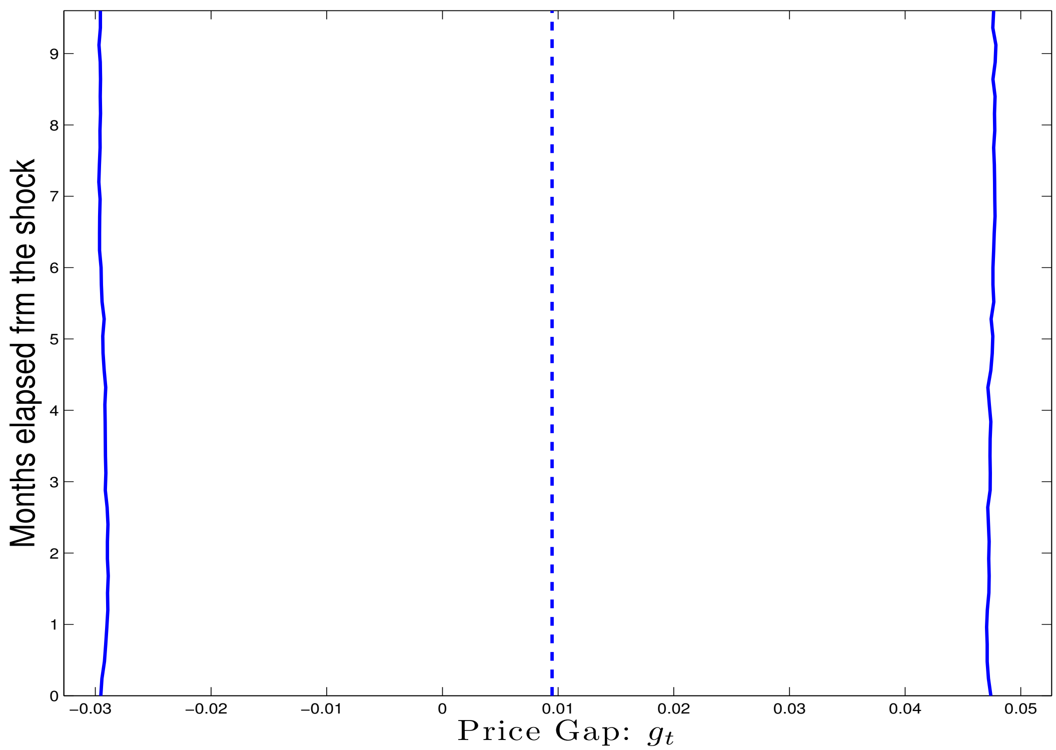
Figure 10 plots the optimal time to the next observation $\tau_t(\cdot)$ at the time of impact of the monetary shock of size $\delta = 1\%$, t_0, as well as at steady state obtained under our benchmark parametrization. Firms' decision rules have been solved numerically. The two functions overlap. Similarly, Figure 11 plots the inaction region and the optimal return point, $\{\underline{g}_t, \hat{g}_t, \bar{g}_t\}$, as a function of the time elapsed since the impact of the aggregate shock. We find that the inaction region changes very little in response to the aggregate shock, and in particular both the left and right thresholds move towards the optimal return point on impact, while we find that the optimal return point does not change.

Figure 10: Optimal time to next observation as a function of the price gap



Note: red dashed line plots $\tau_t(\cdot)$ at the time of impact of the aggregate shock, $t = 0$; blu solid line plots $\tau_t(\cdot)$ at steady state, $t = \hat{T}$.

Figure 11: Optimal inaction region and return point as a function of the time elapsed from the shock



Note: solid lines plot the two thresholds determining the inaction region; dashed line plots the optimal return point.