

Corporate Debt Structure and the Financial Crisis*

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Preliminary

Abstract

We present a DSGE model where firms optimally choose among alternative instruments of external finance. The model is used to explain the evolving composition of corporate debt during the financial crisis of 2007-09, namely the observed shift from bank finance to bond finance despite the increasing cost of debt securities relative to bank loans. We show that substitutability among instruments of external finance is important to shield the economy from the adverse effects of a financial crisis on investment and output

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1 Introduction

During the financial crisis of 2007-09, European banks experienced major difficulties to finance themselves in money markets. Starting in August 2007, concerns about their exposure to the US sub-prime market enhanced the perception of counterparty risk in the interbank market and triggered a drying-up of liquidity. Banks refrained from lending to each other and began to hoard liquidity. Their funding difficulties were soon passed on to the corporate sector. Euro area non-financial corporations - traditionally heavily dependent on bank-finance - faced progressively tightening lending standards.

Early in 2008, non-financial corporations started shifting the composition of their debt from bank loans towards debt securities (figure 1). At the same time, the cost of market debt raised above the cost of bank loans, where it remained throughout the crisis (figure 2). Despite the increase in the cost of external finance, aggregate debt to equity kept rising and only stabilized in 2009, while the default rate of non-financial corporations increased sharply.

Overall, the implication of the turmoil for economic activity was a drop in investment and output that was unprecedented since the introduction of the euro.

In this paper, we propose a model that can account for the stylized facts observed during the crisis both on the composition of corporate debt and on aggregate economic activity. We use the model to evaluate the role played by the composition of corporate debt in determining the response of investment and output during the crisis. More generally, we assess whether abstracting from an endogenously evolving debt structure - as done in most models currently used for policy analysis - can bias the predictions on the effects of shocks on aggregate economic activity.

The framework we consider is a stochastic dynamic general equilibrium model where lenders and borrowers face agency costs, and where heterogeneous firms can choose among alternative instruments of external finance. The model is a monetary version of the setup analyzed in De Fiore and Uhlig (2011), the main difference being that here credit is denominated in nominal terms. As a consequence, monetary policy affects economic activity by influencing the terms at which financial intermediaries are able to lend.

The model generates an endogenous corporate debt structure as a result of two key features.

The first is the existence of two types of financial intermediaries, where banks (which intermediate loan finance) are willing to spend resources to acquire information about an unobserved productivity factor, while "capital mutual funds" (which intermediate bond finance) are not. Because information acquisition is costly, bond issuance is a cheaper - although riskier - instrument of external finance.

We view banks as financial intermediaries that build a much closer relationship with borrowers than dispersed investors. They assess and monitor information about firms' uncertain productive prospects and are ready to adapt the terms of the loans accordingly. Our modelling of banks builds on theories of financial intermediation that stress the higher flexibility provided by banks relative to the market (Chemmanur and Fulghieri (1994) and Boot, Greenbaum and Thakor (1993)). It is also consistent with the recent role taken by banks as originators of asset-backed securities, which requires screening of applicants' projects.

The second key feature of the model is a sequence of three idiosyncratic productivity shocks hitting each firm during the period. The first shock, ε_1 , is publicly observed and realizes before firms take any decision. It introduces ex-ante heterogeneity in firms' risk of default. The second shock, ε_2 , is not observed by anyone. Information can be acquired by banks at a cost, in exchange of an up-front fee paid by the firm. In combination with the information role of banks, it provides a rationale for firms facing high risk of default to choose bank finance. The third shock, ε_3 , realizes during production and is observable to the entrepreneur only. It can be monitored at a cost by financial intermediaries at the end of the period. This shock introduces a costly-state-verification environment (as in Townsend (1979)), where risky debt arises as the optimal financial contract between borrowers and lenders.

In equilibrium, firms experiencing high risk of default choose to abstain from production and not to raise external finance. This choice enables them to retain their net worth, which would otherwise get sized by financial intermediaries in case of bankruptcy. Firms with relatively low risk of default choose instead to issue debt securities because this is the cheapest form of external finance. Only firms with intermediate risk of default decide to approach banks, as they highly value the option of getting further information before deciding whether or not to produce. The model delivers a distribution of firms among financing choices (whether or not to raise external finance) and among debt instruments (bank loans or debt securities) that reacts to aggregate conditions and evolves endogenously over the cycle.

We obtain three sets of results.

First, we show that the model can qualitatively replicate the observed changes both in the composition of corporate debt and in aggregate variables, in response to a shock that increases information acquisition costs and reduces the efficiency of banks as financial intermediaries.

This shock induces a fall in the ratio of bank loans to debt securities, as a larger share of firms with high ex-ante risk of default now finds the cost of external finance too high, and choose to abstain from production. Similarly, a larger share of firms experiencing intermediate realizations of the first productivity shock find the flexibility provided by banks too costly, and decides to issue bonds instead.

The shift in the composition of debt in turn affects the cost of external finance. Bond finance becomes more costly as the average risk of default for the new pool of market-financed firms is higher. The cost of bank finance also rises. On the one hand, the share of firms with high risk of default that move out of banking and decides not to produce reduces the average default risk of bank-financed firms and generates downward pressure on the cost of bank finance. However, this effect is more than compensated by the share of firms with low risk of default that decides to shift from bank-finance to bond-finance. Overall, the average increase in bond yields is higher than the average increase in lending rates. Also, the higher cost of external finance increases the average default rate.

Finally, the shock exerts contractionary effects on real activity as a consequence of the reduction in the fraction of producing firms. More firms decide not to approach a financial intermediary and a larger share of bank-financed firms decide not to produce, conditional on obtaining information on the uncertain productivity factor. The aggregate level of credit and investment fall, together with output.

Our second result relates to the ability of the model to match quantitatively the responses observed during the financial crisis. We show that the peak effects (relative to post-EMU averages) can be broadly replicated when the shock to bank efficiency is combined with two other shocks that have been proposed in the literature as possible drivers of the financial crisis: a shock that reduces the ability of firms to transform profits into capital (similar to the capital quality shock in Gertler and Karadi (2011)) and a shock to the overall risk faced by firms (similar to the risk shock in Christiano, Motto and Rostagno (2010)).

Our third finding is that firms' ability to shift among alternative instruments of external finance has important implications for the effects of shocks on aggregate activity. We compare the real effects of a shock to bank costs when corporate debt structure is endogenous to the

effects obtained when the debt structure is kept unchanged. Consistent with recent empirical evidence documented in Becker and Ivashina (2011), we find that the effects on the cost of external finance, investment and output are greatly amplified when debt structure is exogenous relative to the case when it reacts to aggregate conditions.

Our paper relates to recent work by Adrian, Colla and Shin (2011). As we do, they document and explain the fall in bank finance during the 2007-09 crisis, the compensating increase in bond finance, and the rising price of both instruments. Different from us, in order to account for this evidence, they present a model that builds around a procyclical behaviour of leverage for commercial banks. In a recession, banks sharply contract lending through deleveraging. Risk-averse bond investors need to increase their credit supply to fill the gap in demand. In order to induce this outcome, risk premiums need to rise. In their model, a contraction in economic activity arises because of the rising premiums, rather than because of a contraction in total credit.

Our paper is also related to an older literature that models the endogenous choice between bank finance and market finance. For instance, Holstrom and Tirole (1997) and Repullo and Suarez (1999) analyse this choice for firms that are heterogeneous in the amount of available net worth. In those models, moral hazard arises because firms can divert resources from the project to their private use. In Holstrom and Tirole (1997), moral hazard applies to both firms and banks, while it applies only to firms in Repullo and Suarez (1999). In both cases, it is assumed that monitoring is more intense under bank finance. The papers find that, in equilibrium, firms with large net worth choose to raise market finance, firms with intermediate levels of net worth prefer to raise bank finance, and firms with little net worth do not obtain credit. One implication of their model is that a contraction in net worth, as observed during the crisis, leads to a reduction of bond finance, at odds with the evidence observed during the recent financial crisis. In our model, firms financing choices depend on their observed productivity or, equivalently, on their risk of default. Hence, a fall in net worth needs not to produce a reduction in the share of bond-financed firms. A second main difference relative to this literature is that we cast the analysis of corporate finance into a fully general equilibrium model. This enables us to relate the equilibrium choice of the instrument of external finance to the behaviour of real aggregate variables in the economy.

The paper proceeds as follows. In section 2, we describe the environment. In section 3, we present the analysis and describe the equilibrium of the model. We refer to the appendix

for a description of the methodology we use to log-linearize the equilibrium conditions. The additional complication arises because of the need to aggregate across heterogeneous firms and because of the presence of endogenously changing regions of integration. In section 4, we show results from the numerical analysis. We first document the response of financial and real variables under a temporary shock to bank information acquisition costs. Then, we document the ability of the model to match the peak effects observed during the crisis. Finally, we evaluate the importance of considering firms' endogenous debt structure for assessing the investment and output effects of shocks. In section 5, we conclude. In the appendix, we provide details of the aggregation across firms; we define the financial variables used in the numerical analysis; we collect the conditions that characterize a competitive equilibrium in the model; we characterize the stochastic steady state and describe the numerical procedure used to compute it; and we illustrate how to obtain the coefficients of the log-linearized equilibrium conditions.

2 The model

We extend the model presented in De Fiore and Uhlig (2011, henceforth DFU). The main difference is that we consider here a monetary economy and analyze the case where firm debt is denominated in nominal terms. As a consequence, the nominal interest rate affects marginal costs and expected profits of firms, thus influencing their financing decisions.

The timeline of events is as follows.

At the beginning of period t , after the realization of aggregate shocks, the financial market opens. Holding cash balances carried from the previous period, households receive interest on their holdings of nominal assets and a monetary transfer from the central bank. They choose how to allocate their nominal wealth among cash, state-contingent bonds, and one-period deposits. They also plan, how much labor to supply, and how much to consume and invest in capital. Before the opening of the goods market, households receive nominal factor income from firms.

Entrepreneurs enter the period holding their previously accumulated stock of capital. After the realization of aggregate shocks, they calculate the end-of-period value of their capital holdings and take financial decisions, which unfold over three stages. Each stage corresponds to the realization of one of the three idiosyncratic productivity shocks that hit firms sequentially during the period.

In the first stage, $\varepsilon_{1,it}$ is realized and publicly observed. Conditional on its realization, entrepreneurs decide whether to: a) abstain from production; b) approach a bank and possibly receive bank loans to produce; or c) borrow from capital mutual funds (henceforth CMFs) and produce.

In the second stage, $\varepsilon_{2,it}$ is realized but not observed by anyone. Information can be acquired by banks at a cost and communicated to entrepreneurs, which are indexed by $i \in [0, 1]$. Conditional on this information, entrepreneurs who have approached banks choose whether to: d) abstain from production and rent out capital, retaining their remaining net worth until the end of the period; or e) borrow from banks and produce. Producing firms hire labor and rent capital from the households against payment of these services.

In the third stage, $\varepsilon_{3,it}$ is realized and observed by the entrepreneur only. Entrepreneurs produce, keep part of output for own consumption and investment, and sell the rest. They repay loans or default. Conditional on this decision, the intermediaries decide whether to monitor.

At the end of the period, households and entrepreneurs consume and invest.

2.1 Households

At the beginning of period t , after the realization of aggregate shocks, the financial market opens. Households receive the interest on nominal financial assets acquired at time $t - 1$ and the monetary transfer distributed by the central bank, $P_t \theta_t$. The households choose to allocate their nominal wealth among existing nominal assets, namely cash M_t , nominal state-contingent bonds B_{t+1} paying a unit of currency in a particular state in period $t + 1$, and one-period deposits denominated in units of currency D_t paying a gross interest R_t^d in period $t + 1$. Furthermore, the household carries forward cash balances $\tilde{M}_{t-1}$ from the previous period. Households's cash balances to be used for transactions purposes are then given by $M_t = B_t + R_{t-1}^d D_{t-1} + \tilde{M}_{t-1} + P_t \theta_t - D_t - E_t [Q_{t,t+1} B_{t+1}]$. Before the opening of the goods market, households receive factor income from firms. The amount of resources available in the goods market is therefore given by $M_t + P_t (1 - \delta) k_t + P_t (w_t h_t + r_t k_t)$, where P_t is the consumer price, w_t is the real wage, r_t is the rental rate on capital and k_t is the stock of capital held by the household. In the goods market, firms sell their goods and households spend their resources in consumption and investment expenditures. The amount of cash balances brought

into period $t + 1$, is then given by

$$\widetilde{M}_t \equiv M_t - P_t [c_t + k_{t+1} - (1 - \delta) k_t] + P_t (w_t h_t + r_t k_t) \geq 0. \quad (1)$$

The household's problem is to maximize preferences, defined as

$$E_o \left\{ \sum_0^{\infty} \beta^t [u(c_t) - \eta v(h_t)] \right\}, \quad (2)$$

where β is the households' discount rate, c_t is consumption, h_t denotes working hours and η is a preference parameter. The problem is solved subject to (1) and to the budget constraint

$$M_t + D_t + E_t [Q_{t,t+1} B_{t+1}] \leq W_t, \quad (3)$$

where nominal wealth at the beginning of period $t + 1$ is given by

$$W_{t+1} = B_{t+1} + R_t^d D_t + P_{t+1} \theta_{t+1} + \widetilde{M}_t. \quad (4)$$

2.2 Entrepreneurs

Each entrepreneur i operates a CRS technology described by

$$y_{it} = A_t \varepsilon_{1,it} \varepsilon_{2,it} \varepsilon_{3,it} H_{it}^\alpha K_{it}^{1-\alpha}, \quad (5)$$

where K_{it} and H_{it} denote the firm-level capital and labor and A_t is an aggregate technology shock.

The shocks $\varepsilon_{1,it}$, $\varepsilon_{2,it}$ and $\varepsilon_{3,it}$ are random and mutually independent disturbances. They have mean unity and aggregate distribution functions denoted by Φ_1 , Φ_2 and Φ_3 , respectively. The shocks occur sequentially during the period. $\varepsilon_{1,it}$ is publicly observed and realizes at the time when the aggregate shocks occur, before firms take financial and production decisions. We assume it to be autocorrelated, i.e. $\log(\varepsilon_{1,it}) = \rho_{\varepsilon_1} \log(\varepsilon_{1,it-1}) + \log(v_{it})$.¹ $\varepsilon_{2,it}$ is not observed by anyone but information on its realization can be acquired by the financial intermediaries upon payment of an up-front fee τ_t that is proportional to the value of the firm. $\varepsilon_{3,it}$ realizes after borrowing occurs and is observable to the entrepreneur only. It can be monitored by

¹Under the assumption that $\varepsilon_{1,it}$ is iid, firms would experience high volatility in ex-ante productivity and would frequently move from one instrument of external finance to the other. Assuming an AR1 process for $\varepsilon_{1,it}$ generates persistence both in firms' productivity and in the choice of the instrument of external finance, as reflected by the parameter ρ_{ε_1} . This latter, however, has no implications for the equilibrium allocations in the aggregate.

financial intermediaries at the end of the period, at a cost that is a fraction of the value of firm's production. We assume that τ_t is stochastic.

The entrepreneur faces the constraint that the available funds, x_{it} , need to equal the initial downpayment to the factors of production,

$$x_{it} = w_t H_{it} + r_t K_{it}. \quad (6)$$

Entrepreneurs have linear preferences over consumption with rate of time preference β^e , and they die with probability γ . We assume β^e sufficiently high so that the return on internal funds is always higher than the preference discount, $\frac{1}{\beta^e} - 1$. It is thus optimal for entrepreneurs to postpone consumption until the time of death. When they die or default on the debt, entrepreneurs receive an arbitrarily small transfer from the government to restart productive activity.

2.3 Agency costs and financial intermediation

Entrepreneur i rents labor and capital inputs from the households against an up-front payment $P_t x_{i,t}$. Since the firm's nominal net worth, $P_t n_{it}$, is not sufficient to cover for this initial payment, the entrepreneur needs to raise external finance from either a bank or a CMF. Loans are stipulated in nominal terms, after all aggregate shocks have occurred.²

To ensure that all firms raise finite amounts of external finance, despite the presence of the ex-ante heterogeneity introduced by the shock $\varepsilon_{1,it}$, we assume that financial intermediaries finance projects whose size is a fixed proportion of the internal funds invested,³ $\widehat{n}_{it}$,

$$x_{it} = \xi \widehat{n}_{it}, \quad \xi \geq 1. \quad (7)$$

After the realization of the uncertain productivity factor, the entrepreneur observes the actual production in units of goods, y_{it} , and announces to the financial intermediary repayment of the debt or default. Production is only known to the firm unless there is costly monitoring,

²We rule out the possibility that entrepreneurs enter actuarially fair gambles with financial intermediaries.

³Restriction (7) is necessary to ensure that all firms raise finite amounts of external finance despite the presence of ex-ante heterogeneity. Without that restriction, only the top firms would receive financing, creating a homogenous pool of firms with a potentially high leverage ratio. This would be inconsistent with the evidence in Kurshev and Strebulaev (2006) that rather different firms feature reasonably similar and modest leverage ratios.

which requires destroying a fraction μ of the value of output. After the announcement of the entrepreneur, the intermediary decides whether or not to monitor.⁴

2.4 Monetary policy

Monetary policy occurs through central banks' liquidity injections, carried out with nominal transfers $P_t \theta_t$ to households. The total amount of liquidity injections in the economy is

$$P_t \theta_t = M_t^s - M_{t-1}^s, \quad (8)$$

where M_t^s denotes money supply. We assume that the latter grows at the exogenous rate ν , $M_t^s = \nu M_{t-1}^s$.

3 Analysis

3.1 Households

Define real balances as $m_t \equiv M_t/P_t$ and the inflation rate as $\pi_t \equiv P_t/P_{t-1}$. The safe nominal rate satisfies $R_t = (E_t[Q_{t,t+1}])^{-1}$. A comparison with the equation for the interest rate on deposits shows that $R_t = R_t^d$. Since we concentrate on equilibria with $R_t > 1$, the first-order conditions of the household problem simplify to

$$\begin{aligned} \frac{\eta}{u_c(c_t)} &= w_t \\ u_c(c_t) &= \beta R_t E_t \left[\frac{u_c(c_{t+1})}{\pi_{t+1}} \right] \\ u_c(c_t) &= \beta E_t [(1 - \delta + r_{t+1}) u_c(c_{t+1})]. \end{aligned}$$

3.2 The firm-specific markup

Each entrepreneur's net worth, n_{it} , is given by the market value of his capital stock, z_{it} ,

$$n_{it} = (1 - \delta + r_t) z_{it}. \quad (9)$$

The firm's demand for labor and capital is derived by solving the problem

$$\max \mathcal{E}^j [A_{t \in 1, it} \varepsilon_{2, it} \varepsilon_{3, it} H_{it}^\alpha K_{it}^{1-\alpha} - w_t H_{it} - r_t K_{it}]$$

⁴The informational structure at contracting time corresponds to the costly state verification framework of Townsend (1979).

subject to the financing constraint (6). Here the expectation $\mathcal{E}^j[\cdot]$ is taken with respect to the productivity variables yet unknown at the time of the factor hiring decision. Define

$$\varepsilon_{it}^j \equiv \mathcal{E}^j[A_t \varepsilon_{1,it} \varepsilon_{2,it} \varepsilon_{3,it}] = \begin{cases} A_t \varepsilon_{1,it} & \text{for } j = c \\ A_t \varepsilon_{1,it} \varepsilon_{2,it} & \text{for } j = b \end{cases} \quad (10)$$

Denote the Lagrange multiplier on the financing constraint (6) as $q_{it}^j - 1$. The Lagrangean can be written as

$$L = \mathcal{E}^j [A_t \varepsilon_{1,it} \varepsilon_{2,it} \varepsilon_{3,it} H_{it}^\alpha K_{it}^{1-\alpha} - w_t H_{it} - r_t K_{it}] - (q_{it}^j - 1) [w_t H_{it} + r_t K_{it} - x_{it}].$$

FOCs:

$$\begin{aligned} (1 - \alpha) \frac{\mathcal{E}^j [y_{it}]}{K_{it}} &= r_t q_{it}^j \\ \alpha \frac{\mathcal{E}^j [y_{it}]}{H_{it}} &= w_t q_{it}^j \end{aligned}$$

These can be used to obtain

$$\mathcal{E}^j [y_{it}] = q_{it}^j x_{it}$$

and

$$\begin{aligned} \mathcal{E}^j [y_{it}] &= \varepsilon_{it}^j A_t H_{it}^\alpha K_{it}^{1-\alpha} \\ &= \varepsilon_{it}^j A_t \left[\frac{\alpha \mathcal{E}^j [y_{it}]}{w_t q_{it}^j} \right]^\alpha \left[\frac{(1 - \alpha) \mathcal{E}^j [y_{it}]}{r_t q_{it}^j} \right]^{1-\alpha} \\ &= \varepsilon_{it}^j A_t \left[\frac{\alpha}{w_t} \right]^\alpha \left[\frac{(1 - \alpha)}{r_t} \right]^{1-\alpha} \frac{\mathcal{E}^j [y_{it}]}{q_{it}^j}. \end{aligned}$$

Define

$$q_t \equiv A_t \left(\frac{\alpha}{w_t} \right)^\alpha \left(\frac{1 - \alpha}{r_t} \right)^{1-\alpha} \quad (11)$$

From the last condition, it follows that

$$q_{it}^j = \varepsilon_{it}^j q_t.$$

Thus, optimality implies that

$$\begin{aligned} K_{it} &= (1 - \alpha) \frac{x_{it}}{r_t} \\ H_{it} &= \alpha \frac{x_{it}}{w_t} \\ \mathcal{E}^j [y_{it}] &= \varepsilon_{it}^j q_t x_{it}. \end{aligned}$$

where

$$q_t \equiv A_t \left(\frac{\alpha}{w_t} \right)^\alpha \left(\frac{1 - \alpha}{r_t} \right)^{1-\alpha}. \quad (12)$$

3.3 Financial intermediaries and the optimal contract

The financial contract is intra-period but the game between firms and financial intermediaries unfolds over three stages. The endogenous financial structure can be solved using backward induction.

3.3.1 Stage III: costly state verification

Provided the entrepreneur goes ahead with some sort of financing, the optimal financial decisions are given by the solution to a costly state verification problem. Define the available net worth as $\tilde{n}_{it}^j$ and the unexpected productivity factor as ω_{it}^j . Then,

$$\tilde{n}_{it}^j = n_{it} \text{ and } \omega_{it}^j = \varepsilon_{2,it}\varepsilon_{3,it}, \text{ for } j = c,$$

and

$$\tilde{n}_{it}^j = (1 - \tau_t)n_{it} \text{ and } \omega_{it}^j = \varepsilon_{3,it}, \text{ for } j = b.$$

The optimal contract sets a threshold $\bar{\omega}_{it}^j$ corresponding to a fixed repayment of $P_t \varepsilon_{it}^j \bar{\omega}_{it}^j q_t x_{it}$ units of currency. If the entrepreneur announces a realization of the uncertain productivity factor $\omega_{it}^j \geq \bar{\omega}_{it}^j$, no monitoring occurs. If $\omega_{it}^j < \bar{\omega}_{it}^j$, the intermediary monitors the entrepreneur, at the cost of destroying a proportion $0 \leq \mu \leq 1$ of the firm output. Let Φ_j and φ_j be respectively the distribution and density function of ω_{it}^j . Define

$$f_j(\bar{\omega}_{it}^j) = \int_{\bar{\omega}}^{\infty} (\omega - \bar{\omega}_{it}^j) \varphi_j(\omega) d\omega \quad (13)$$

$$g_j(\bar{\omega}_{it}^j) = \int_0^{\bar{\omega}_{it}^j} (1 - \mu) \omega \varphi_j(\omega) d\omega + \bar{\omega}_{it}^j \left[1 - \Phi_j(\bar{\omega}_{it}^j) \right] \quad (14)$$

as the expected shares of final output accruing respectively to an entrepreneur and to a lender, after stipulating the contract.

Since the contract is intra-period, we can drop the price level. The optimal contract is the pair $(x_{i,t}, \bar{\omega}_{it}^j)$ that solves the following costly state verification problem:

$$\max \varepsilon_{it}^j q_t f_j(\bar{\omega}_{it}^j) x_{it} \quad (15)$$

subject to

$$A_t \varepsilon_{it}^j q_t g_j(\bar{\omega}_{it}^j) x_{it} \geq R_t (x_{it} - \hat{n}_{it}^j) \quad (16)$$

$$0 \leq x_{it} \leq \xi \hat{n}_{it}^j \quad (17)$$

$$\varepsilon_{it}^j q_t f_j(\bar{\omega}_{it}^j) x_{it} \geq \hat{n}_{it}^j \quad (18)$$

$$f_j(\bar{\omega}_{it}^j) + g_j(\bar{\omega}_{it}^j) = 1 - \mu \int_0^{\bar{\omega}_{it}^j} \omega \varphi_j(\omega) d\omega \quad (19)$$

Restrictions to the problem are on the financial intermediary being repaid an amount not lower than the repayment due to the household, (16), the project size, (17), the entrepreneur being willing to sign the contract, (18), and the feasibility condition (19) which result from the shares (13) and (14) summing up to one.

Since the problem is linear in $\hat{n}_{it}^j$, the solution is such that the entrepreneur either invest nothing and does not produce, $\hat{n}_{it}^j = 0$, or invest everything and produce, $\hat{n}_{it}^j = \tilde{n}_{it}^j$. Entrepreneurs that produce only raise costly external finance to cover what is needed in excess of the internal funds, $x_{it} - \hat{n}_{it}^j = (\xi - 1) \tilde{n}_{it}^j$.

Notice that $f_j' < 0$, and $\lim_{\omega} f_j(\omega) = 0$. If an interior solution to the problem exists, $g'(\bar{\omega}_{it}^j) > 0$. Suppose this was not the case. It would then be optimal to set $\bar{\omega}_{it}^j = 0$. Banks would make zero profits and would not be able to repay depositors. A solution to the contract would not exist.

The optimal contract satisfies $x_{it} = \xi \tilde{n}_{it}^j$ and sets the threshold as the solution to

$$g_j(\bar{\omega}_{it}^j) = \frac{R_t}{\varepsilon_{it}^j q_t} \left(1 - \frac{1}{\xi}\right). \quad (20)$$

From now on, assume that $\bar{\omega}_{it}^j$ be the minimal among all solutions. It follows that $\bar{\omega}_{it}^j = \bar{\omega}(\varepsilon_{it}^j, q_t, R_t)$ is increasing in R_t and decreasing in ε_{it}^j and q_t . The terms of the contract can be written as

$$\bar{\omega}_{it}^j = \bar{\omega}^j(\varepsilon_{it}^j; q_t, R_t), \quad j = b, c, \quad (21)$$

where ε_{it}^j satisfies (10). The loan rate R_{it}^j , defined as the nominal interest rate that is charged for the use of external finance, is implicitly given by the condition

$$R_{it}^j (x_{it} - \tilde{n}_{it}^j) = \mathcal{E}^j[\varepsilon_{1,it} \varepsilon_{2,it} \varepsilon_{3,it}] q_t \bar{\omega}_{it}^j x_{it}. \quad (22)$$

It follows that the risk premium on the external finance of a firm i , which has chosen to use instrument j , is given by

$$rp_{it}^j = \frac{\left(\frac{\xi}{\xi-1}\right) \mathcal{E}^j[\varepsilon_{1,it} \varepsilon_{2,it} \varepsilon_{3,it}] q_t \bar{\omega}_{it}^j}{R_t} - 1. \quad (23)$$

3.3.2 Stage II: bank loan continuation

At the beginning of stage II, the second productivity shock, $\varepsilon_{2,it}$, is realized. Information on this shock is acquired by banks and communicated to the entrepreneur. Conditional on the available information, the entrepreneur chooses whether to abstain from production or to obtain credit and produce. The expected utility of an entrepreneur, who proceeds with production conditional on the realization of $\varepsilon_{1,it}$ and $\varepsilon_{2,it}$, is $F^d(\varepsilon_{1,it}\varepsilon_{2,it}; q_t, R_t)(1 - \tau_t)n_{it}$, where

$$F^d(\varepsilon_{1,it}\varepsilon_{2,it}; q_t, R_t) \equiv \varepsilon_{1,it}\varepsilon_{2,it}q_t f(\bar{\omega}^b(\varepsilon_{1,it}\varepsilon_{2,it}; q_t, R_t))\xi. \quad (24)$$

A value for the second firm-specific shock $\varepsilon_{2,it}$, below which the entrepreneur does not proceed with the bank loan, exists and is unique. It is given by the threshold $\bar{\varepsilon}_{it}^d$, which satisfies

$$F^d(\varepsilon_{1,it}, \bar{\varepsilon}_{it}^d; q_t, R_t) = 1. \quad (25)$$

From (25), $\bar{\varepsilon}_{it}^d = \bar{\varepsilon}_d(\varepsilon_{1,it}; q_t, R_t)$.

3.3.3 Stage I: the choice of the financing instrument

In stage I, after $\varepsilon_{1,it}$ realizes, the entrepreneur chooses whether or not to produce and, if he does, how to finance production. The expected utility of an entrepreneur, who proceeds with bank finance conditional on the realization of ε_1 , is $F^b(\varepsilon_{1,it}; q_t, R_t, \tau_t)n_{it}$, where

$$F^b(\varepsilon_{1,it}; q_t, R_t, \tau_t) \equiv (1 - \tau_t) \left(\int_{\bar{\varepsilon}_d(\varepsilon_{1,it}; q_t, R_t)}^{\varepsilon_{1,it}\varepsilon_2} q_t f(\bar{\omega}^b(\varepsilon_{1,it}\varepsilon_2; q_t, R_t))\xi \Phi_2(d\varepsilon_2) + \Phi_2(\bar{\varepsilon}_d(\varepsilon_{1,it}; q_t, R_t)) \right) \quad (26)$$

The expected utility of an entrepreneur, who proceeds with CMF finance conditional on the realization of $\varepsilon_{1,it}$, is $F^c(\varepsilon_{1,it}; q_t, R_t)n_{it}$, where

$$F^c(\varepsilon_{1,it}; q_t, R_t) \equiv \varepsilon_{1,it}q_t f(\bar{\omega}^c(\varepsilon_{1,it}; q_t, R_t))\xi. \quad (27)$$

Finally, the expected utility of an entrepreneur, who abstains from production, is n_{it} .

Knowing $\varepsilon_{1,it}$, each entrepreneur chooses his or her best option, leading to the overall payoff $F(\varepsilon_{1,it}; q_t, R_t)n_{it}$, where

$$F(\varepsilon_{1,it}; q_t, R_t, \tau_t) \equiv \max\{1; F^b(\varepsilon_{1,it}; q_t, R_t, \tau_t); F^c(\varepsilon_{1,it}; q_t, R_t)\}. \quad (28)$$

In the analysis below, we assume that A1) $\frac{\partial F^b(\cdot)}{\partial \varepsilon_{1,it}} \geq 0$ and A2) $\frac{\partial F^b(\cdot)}{\partial \varepsilon_{1,it}} < \frac{\partial F^c(\cdot)}{\partial \varepsilon_{1,it}}$, for all $\varepsilon_{1,it}$.

Under (A1), a threshold for $\varepsilon_{1,it}$, below which the entrepreneur decides not to raise external finance, exists and is unique. We denote it as $\bar{\varepsilon}_{it}^b$. It is implicitly defined by the condition

$$F^b(\bar{\varepsilon}_{it}^b; q_t, R_t, \tau_t) = 1. \quad (29)$$

The unique cutoff point is a function of aggregate variables only, $\bar{\varepsilon}_{bt} = \bar{\varepsilon}^b(q_t, R_t, \tau_t)$, and hence is identical for all firms.

Under A1) and A2), a threshold for $\varepsilon_{1,it}$ above which entrepreneurs sign a contract with the CMF, also exists and is unique. We denote it as $\bar{\varepsilon}_{it}^c$. It is implicitly defined by the condition

$$F^b(\bar{\varepsilon}_{it}^c; q_t, R_t, \tau_t) = F^c(\bar{\varepsilon}_{it}^c; q_t, R_t) \quad (30)$$

and it is thus identical across firms, $\bar{\varepsilon}_{ct} = \bar{\varepsilon}^c(q_t, R_t, \tau_t)$.

Conditional on q_t , R_t and τ_t , entrepreneurs split into three sets that are intervals in terms of the first idiosyncratic productivity shock $\varepsilon_{1,it}$. Denote the firm's decision on whether to produce with a dummy variable Θ_{it} . Then,

$$\Theta_{it} = \begin{cases} 1 & \text{if } \varepsilon_{1,it} > \bar{\varepsilon}_{ct} \text{ or if } \bar{\varepsilon}_{bt} \leq \varepsilon_{1,it} \leq \bar{\varepsilon}_{ct} \text{ and } \varepsilon_{2,it} > \frac{\bar{\varepsilon}_{it}^d}{\varepsilon_{1,it}} \\ 0 & \text{else} \end{cases}.$$

The functions $s^a(\cdot)$, $s^b(\cdot)$, $s^c(\cdot)$ and $s^{bp}(\cdot)$ measure respectively the shares of firms that abstain from producing, approach a bank, raise CMF finance, and produce conditional on having approached a bank,

$$s^a(q_t, R_t, \tau_t) = \Phi_1(\bar{\varepsilon}^b(q_t, R_t, \tau_t)) \quad (31)$$

$$s^b(q_t, R_t, \tau_t) = \Phi_1(\bar{\varepsilon}^c(q_t, R_t, \tau_t)) - \Phi_1(\bar{\varepsilon}^b(q_t, R_t, \tau_t)) \quad (32)$$

$$s^c(q_t, R_t, \tau_t) = 1 - \Phi_1(\bar{\varepsilon}^c(q_t, R_t, \tau_t)) \quad (33)$$

$$s^{bp}(q_t, R_t, \tau_t) = \int_{\bar{\varepsilon}^b(q_t, R_t, \tau_t)}^{\bar{\varepsilon}^c(q_t, R_t, \tau_t)} \int_{\bar{\varepsilon}^d(\varepsilon_1; q_t, R_t)} \Phi_2(d\varepsilon_2) \Phi_1(d\varepsilon_1). \quad (34)$$

3.4 Entrepreneurs

Because the return on internal funds is always higher than the rate of time preference, entrepreneurs accumulate wealth and only consume before dying. It follows that in the aggregate, entrepreneurs consume each period a fraction γ of their accumulated wealth. Entrepreneurial consumption and accumulation of capital are then given by

$$e_t = (1 - \gamma) \psi^f(q_t, R_t, \tau_t) n_t, \quad (35)$$

$$z_{t+1} = \gamma \varkappa_t \psi^f(q_t, R_t, \tau_t) n_t, \quad (36)$$

where $\psi^f(q_t, R_t, \tau_t) n_t$ are aggregate profits of the entrepreneurial sector, and $\psi^f(q_t, R_t, \tau_t)$ is defined in appendix A. $\varkappa_t$ is a shock to net worth accumulation, which follows an AR1 process. It affects the ability of firms to transform period t profits into period $t + 1$ capital, and can be thought of as a shock to the quality of the existing capital (as in Gertler and Karadi (2011)).

3.5 Aggregation

Aggregate demand for funds, x_t , output y_t , and output lost to agency costs y_t^a are given by:

$$x_t = \left[(1 - \tau_t) s^{bp}(q_t, R_t, \tau_t) + s^c(q_t, R_t, \tau_t) \right] \xi n_t \quad (37)$$

$$y_t = \psi^y(q_t, R_t, \tau_t) \xi q_t n_t \quad (38)$$

$$y_t^a = \left[\tau_t s^b(q_t, R_t, \tau_t) + \psi^m(q_t, R_t, \tau_t) \mu \xi q_t \right] n_t \quad (39)$$

where the functions $s^b(\cdot)$, $s^c(\cdot)$ and $s^{bp}(\cdot)$ are given by (32)-(34). The function $\psi^y(\cdot)$ aggregates the realized productivity factors across all producing firms. The terms $\tau_t s^b(q_t, R_t, \tau_t)$ and $\psi^m(\cdot) \mu \xi q_t$ measure the loss of resources due respectively to bank information acquisition and to monitoring costs, per unit of net worth. All these functions are defined in Appendix A.

Aggregate factor demands are given by

$$w_t H_t = \alpha x_t \quad (40)$$

$$r_t K_t = (1 - \alpha) x_t. \quad (41)$$

In appendix B, we provide analytical expressions for the aggregate financial variables that we use in our numerical analysis, namely the ratio of bank finance to bond finance, ϑ_t , the average risk premium for bank-financed firms, rp_t^b , and for CMF-financed firms, rp_t^c , the aggregate debt to equity ratio, χ_t , the default rate on corporate bonds, ϱ_t^c , the average default across firms, ϱ_t , and the net expected return to entrepreneurial capital, r_t^z .

3.6 Market clearing

Market clearing for money, assets, labor and capital requires that $M_t^s = M_t + D_t$, $B_t = 0$, $K_t = k_t + z_t$ and $H_t = l_t$, respectively. Market clearing conditions for loans and output are, respectively,

$$D_t = P_t \left[(1 - \tau_t) s^{bp}(q_t, R_t, \tau_t) + s^c(q_t, R_t, \tau_t) \right] (\xi - 1) n_t, \quad (42)$$

$$y_t^a = y_t - c_t - e_t - K_{t+1} + (1 - \delta) K_t. \quad (43)$$

3.7 Competitive equilibrium and log-linearization

We collect the equations that characterize a competitive equilibrium in appendix C.

In appendix D, we characterize the steady state and describe the procedure we use to compute it.

In appendix E, we show how to log-linearize the equilibrium conditions around a stochastic steady state. This latter is a steady state where firms are hit by idiosyncratic shocks but aggregate shocks are set to their long-run values. The log-linearization is complicated by the need to aggregate across firms and by the presence of endogenously evolving regions of integration.

4 Numerical analysis

In the numerical analysis, we illustrate the ability of the model to replicate the evidence observed during the crisis on corporate debt and macroeconomic activity, both from a qualitative and quantitative perspective.

We then use the model to evaluate the importance for aggregate activity of firms' ability to shift among alternative instruments of external finance.

The model is calibrated in line with the long-run evidence for the euro area documented in DFU. The dynamics of the system is then solved using Uhlig (1999)'s toolkit.

4.1 Calibration

We assume the functional form $u(c_t) - \eta v(h_t) = \log(c_t) - \eta h_t$. We calibrate the model quarterly in order to match in steady state the financial facts documented for the euro area in DFU. We set $\beta = .99$ and the inflation rate to 0.5 percent per quarter, corresponding to the annual average over the period 1999-2007 in the euro area. The corresponding nominal risk-free rate is $R = 1.015$. The depreciation rate is set at $\delta = .02$ and the discount factor at $\beta = .99$, implying a rental rate for capital of 3 percent. We choose $\alpha = .64$ in the production function and a coefficient in preferences η so that labor equal .3 in steady state. We set $\mu = .15$, a value commonly assumed in related literature.

The iid productivity shocks $v = \varepsilon_2, \varepsilon_3$ are lognormally distributed. $\log(v)$ is normally distributed with mean $-\sigma_v^2/2$ and variance σ_v^2 , so that $E(v) = e^{\mu + \sigma^2/2} = e^{-\sigma_v^2/2 + \sigma_v^2/2} = 1$.

The shock ε_1 is autocorrelated and such that $\log(\varepsilon_{1,it}) = \rho_{\varepsilon_1} \log(\varepsilon_{1,it-1}) + (1 - \rho_{\varepsilon_1}) \log(\kappa_{it})$, where $\log(\kappa_{it})$ is normally distributed with mean $-\sigma_{\kappa}^2/2$ and variance σ_{κ}^2 . It follows that $E(\kappa_{it}) = E(\varepsilon_{1,it}) = 1$.

We set the remaining six parameters, $\xi, \tau, \gamma, \sigma_{\varepsilon_1}, \sigma_{\varepsilon_2}$ and σ_{ε_3} as to replicate the [...] to values that jointly minimize the squared log-deviation of the model-based predictions from their empirical counterparts for the following six financial facts : i) the ratio of aggregate bank loans to debt securities for non-financial corporations, ϑ ; ii) the ratio of aggregate debt to equity, χ ; iii) the average risk premium on debt securities, rp^c ; iv) the average risk premium on bank loans, rp^b ; v) the average default rate on debt securities, ϱ^c ; vi) and the expected return to entrepreneurial capital, r_t^z .⁵ The parameter values selected from our calibration procedure are $\tau = .017$, $\gamma = .977$, $\xi = 2.28$, $\sigma_{\varepsilon_1} = .007$, $\sigma_{\varepsilon_2} = .03$, $\sigma_{\varepsilon_3} = .237$.

The stochastic processes for τ_t and $\varkappa_t$ are assumed to have a persistence parameter of 0.9. The standard deviations are calibrated as to replicate, respectively, the maximum deviation observed during the 2007-2009 crisis of the ratio of bank loans to debt securities and of investment from their average over the post-EMU period.

4.2 Steady state

In order to understand the response of the composition of corporate debt to a shock to bank fees, it is useful to consider how a permanent reduction in τ affects firms' financing choices and risk premia in the steady state of our economy.

In the model, an increase in bank fees τ induces a change in the expected profit function $F^b(\varepsilon_{1,it}; q_t, R_t, \tau_t)$. The higher the τ , the lower the advantage of approaching a bank and obtaining additional information on $\varepsilon_{2,it}$, before deciding whether or not to produce and raise external finance. From equations (29) and (30), it follows that an increase in τ shifts the thresholds $\bar{\varepsilon}_{bt}$ and $\bar{\varepsilon}_{ct}$, thus modifying the share of firms approaching banks and the share of firms raising external finance from CMFs. On the contrary, equation (25) shows that the level of τ does not affect firms' choice of proceeding with production, conditional on having approached a bank. The share of bank-financed firms that decide to drop out after observing the shock $\varepsilon_{2,it}$ remains unaffected.

⁵The (annual) averages observed over the period 1999-2007 are respectively: 5.48, 0.64, 143 bps, 119 bps, 4.96 percent, and 9.3 percent. See DFU for a description of the data.

Figure 1 plots the effect of a 40 percent permanent increase in τ on the share of firms choosing to abstain, to approach a bank and wait, and to raise CMF finance and produce.

The black solid line shows the density function $\varphi(\varepsilon_1)$. The red and purple dashed lines show respectively the threshold for bank-finance, $\bar{\varepsilon}_{bt}$, and the threshold for CMF finance, $\bar{\varepsilon}_{ct}$, when τ equals its benchmark value of .016. The green and pink dashed-dotted lines show the same thresholds when τ is increased to .023.

At $\tau = .016$, firms experiencing a value of ε_1 at the left of the red dashed line find it optimal to abstain from production and to retain their net worth n_{it} . Their risk of default at the end of the period in case of production is too high. Firms experiencing a value of ε_1 between $\bar{\varepsilon}_{bt}$ and $\bar{\varepsilon}_{ct}$ rather find it optimal to raise external finance from banks. Their risk of default is sufficiently high that the "wait and see" option provided by banks compensate the extra-fee being charged. Only firms at the right of $\bar{\varepsilon}_{ct}$ are sufficiently safe to choose CMF finance.

Under the larger fee, $\tau = .023$, the thresholds $\bar{\varepsilon}_{bt}$ and $\bar{\varepsilon}_{ct}$ shift inwards. Firms facing a realization of ε_1 between the red dashed and the green dash-dotted lines now find the flexibility of banks too costly relative to the benefit. At the prevailing price of bank finance, their risk of default is sufficiently high to make it optimal for them to abstain from production. Similarly, the share of firms that experience a shock between the purple dashed line and the pink dashed-dotted line now find it optimal to shift from bank finance to bond finance. The higher τ induces them to face the higher risk of default associated with CMF finance.

Because the average creditworthiness (as measured by the realization of the first shock, $\varepsilon_{1,it}$) of CMF-financed firms falls, the average risk premium on bonds rises. The average risk premium on bank finance increases but not as much. The reduction in average creditworthiness due to some firms with high $\varepsilon_{1,it}$ moving to CMF-finance just more than compensate the improved risk prospects due to firms with low $\varepsilon_{1,it}$ moving out of banking. Overall, the increase in the average risk premium is larger for bonds than for loans.

4.3 Impulse responses

In order to capture the evidence observed during the financial crisis, we need to account for the observed fall in bank loans relative to debt securities and the simultaneous rise in the cost of market finance relative to bank finance. There are two possible explanations.

The first is that the shift in the composition of corporate debt was induced by an expansion of the demand for bonds during the crisis, which exerted upward pressure on their yields. We

rule this out as inconsistent with the output contraction which characterized the crisis. During a recession, the average default risk of firms increases. According to our model, and to most theories of financial intermediation, firms should then value more the flexibility provided by banks and shift instead to bank loans.

A second possibility is that the shift was induced by a negative shock to bank profitability, which was then passed on to lending rates, increasing the cost of bank-finance relative to its benefit.

We explore this explanation through the lenses of our model. We consider a shock that increases bank information acquisition costs, τ_t , thus reducing the efficiency of banks as financial intermediaries. The shock can be seen as capturing the difficulties in raising liquidity faced by euro area banks in 2007-2009.⁶ It is calibrated as to generate a fall on impact of the ratio of loans to bonds of 16 percent, in line with the peak effect observed during the crisis.⁷

Figure 4 shows that the response of the economy is qualitatively consistent with the evidence. As the cost of information acquisition increases, firms move away from bank finance. A larger share of firms facing low realizations of ε_1 find the cost of external finance too high, and choose to abstain from production. A larger share of firms experiencing high realizations of ε_1 find the flexibility provided by banks too costly, and decides to issue bonds instead. The ratio of bank loans to corporate bonds falls.

As in the data, the cost of both bank finance and bond finance rise, and the latter increases to a greater extent than the former. The risk premium on bond finance unambiguously increases because the pool of CMF-financed firms now presents a higher average risk of default. The risk premium on loans also increases on impact (although to a lower extent than bond finance) because the share of firms with low risk of default that move from bank-finance to CMF-finance more than compensates the share of firms with high risk of default that move out of banking and decides not to produce.

⁶The shock is consistent with the sharp increase observed in the item "commissions and fees" of pre-provisioning profits of euro area monetary and financial institutions, which raised by 40 percent in the period 2007-2008. See Financial Stability Review (2011).

⁷The evidence we refer in this section is based on data from the database used for the *Financial Stability Review* of the European Central Bank. The peak effects produced by the model are compared to the peak effects in the data, calculated as the maximum deviation (over the period 2007-2009) of each series relative to the post-EMU average.

The shock increases the aggregate default rate and the debt to equity ratio, as observed during the crisis. More frequent bankruptcies result from the larger cost of external finance, which increases due to higher banking fees and risk premia. The aggregate debt to equity ratio rises because the reduction in aggregate net worth, due to lower available net worth of bank-financed firms, is larger than the reduction in aggregate debt due to the shrinking share of producing firms.

The real effects of the shock to bank costs arise as a consequence of the reduction in the fraction of producing firms. As more firms decide not to approach a financial intermediary (the share of *abstain* increases) and a larger share of bank-financed firms decide to drop out after obtaining information on the second productivity shock, the aggregate level of credit and investment fall, together with output.

How well does the model capture the observed magnitude of the responses?

Under the shock to information costs τ , the model generates too large volatility in the ratio of bank loans to corporate bonds, relative to other variables. Aggregate default increases in the order of 0.4 percentage point, while bankruptcies have almost doubled during the crisis, relative to their long run average value. Also, the debt to equity ratio rises by around 0.4 percent, well below the observed 15 percent. The investment to output ratio and output fall respectively by .05 and .02 percent in the model (vs 1.7 and 6.8 percent in the data).

We consider two alternative shocks that have been proposed in the literature as possible drivers of the financial crisis: a shock that reduces the ability of firms to transform profits into capital (as in Gertler and Karadi (2011)) and a shock to the risk faced by firms (as in Christiano, Motto and Rostagno (2010)).

Figure 5 shows the impulse responses to a reduction in capital quality, $\varkappa_t$, that is normalized to produce the observed peak fall in the ratio of aggregate investment to GDP of 1.7 percent.

Also this shock generates responses that are qualitatively in line with the evidence. The shock reduces firms' capital and net worth in period $t + 1$. It also reduces output, but not as much because a large fraction of the capital stock is owned by households and it is unaffected by the shock. Because leverage is constant for each producing firm, an equilibrium requires inducing a larger share of firms to borrow and produce. The share of producing firms indeed raises because the diminished net worth increases the average financial distortion, as measured by the markup q , contributing to raise expected profits from production. The higher profitability also explains why some of the firms which would otherwise be borrowing from banks

now shift to bond finance. For those firms, improved production prospects reduce default risk and the value of the "wait-and-see " option offered by banks. The average risk premium rises both on bonds and on loans, reflecting the inclusion of new firms with high default risk in the share of both bank-financed firms and CMF-financed firms. As a consequence, the economy faces a higher average risk of default.

Relative to a shock to banking fees, a reduction in capital quality generates more sizeable effects on real and aggregate financial variables. A shock normalized to replicate the peak effect observed on investment generates an increase in the ratio of aggregate debt to equity close to the 15 percent observed in the data. The fall in GDP and the increase in the spreads and aggregate default rate are larger, but still far from the levels observed during the crisis. Also, the shift from bank finance to bond finance is too mild (0.4 percent).

A combination of τ_t and $\varkappa_t$ better captures the magnitude of the responses observed during the crisis. The experiment is illustrated in figure 6, where the shock to bank efficiency is calibrated as to replicate the 16 percent drop in the ratio of bank loans to debt securities, while the shock to capital quality is set to generate an impact reduction in investment of 1.7 percent. The combined shock produces the observed increase in the debt to equity ratio and a more severe output contraction (although milder than in the data). Nonetheless, it generates too little movements in the average risk premia and in the aggregate default rate.

4.4 Permanent shocks

Next, we consider the ability of the model to replicate the magnitude of the responses observed during the crisis by adding a shock that increases the standard deviation of ε_{3t} . By affecting the default risk faced by all producing firms, this shock can produce large effects on risk premia and default rates.

Recent evidence has shown that uncertainty dramatically increase after major economic and political shocks. For instance, the standard deviation of a monthly U.S. index of stock market volatility jumped from 10 percent to around 50 percent during the 2007-2009 crisis (Bloom (2009)). Clearly, risk can be captured by a range of lternative measures, but it has been shown that they generally tend to move together (Bloom, Bond and Van Reenen (2007)). Firm-level share returns volatility is significantly correlated with proxies such as real sales growth volatility and the cross-sectional distribution of financial analysts' forecasts. Shocks to uncertainty have been found to be relevant drivers of business cycle fluctuations in DSGE

models estimated on both US and euro area data (see e.g. Christiano, Motto and Rostagno (2010)).

In order to avoid the complication that this would add to the log-linearization of the model and to our solution method, we analyze the effects of a combined *permanent* shock to τ , $\varkappa$ and σ_{ε_3} . The experiment is conducted by assuming that the economy starts from the calibrated steady state and converges to a new steady state where the three parameters τ , $\varkappa$ and σ_{ε_3} take up their "post-crisis" level.

Figure 7 shows the responses of the economy. The shock to σ_{ε_3} is normalized to replicate the observed increase in the cost of bond issuance (80 percent). The response is computed in percentage deviations from the old steady state.

A combination of these three shocks better replicates the responses observed during the crisis from a quantitative point of view. The increase in σ_{ε_3} produces large effects on both risk premia and the aggregate default rate, which almost double, and a deeper contraction of output, although still milder than observed. The main shortcoming is that the debt to equity ratio falls rather than to increase. The reason is that a higher σ_{ε_3} reduces expected profits and the share of firms that decide to produce. As a consequence, total debt as a share of net worth is reduced. Also, the shock to σ_{ε_3} exerts equally large effects on the risk premium on loans and on the risk premium on bonds. They both increase by around 80 percent. In the data, they increase by 30 and 80 percent, respectively.

4.5 Exogenous thresholds

We evaluate the importance for the aggregate economy of firms' ability to shift among alternative instruments of external finance. We do so by comparing the impulse responses to a τ shock when thresholds $\bar{\varepsilon}_{bt}$, $\bar{\varepsilon}_{ct}$ and $\bar{\varepsilon}_{it}^d$ are endogenous to the case when they are fixed at their steady state level.

Figure 8 shows the results for the case of exogenous thresholds. The shares of firms that abstain, approach a bank, raise bank-finance and produce, and raise bond-finance and produce, remain constant. Nonetheless, the ratio of total bank loans to corporate bonds fall, because the available net worth for bank-financed firms is reduced, together with the amount of finance these firms can raise from banks. For the same reason, the overall debt to equity ratio falls. The reduction in available net worth and total credit, together with the fall in the markup q_t , is also responsible for the fall in investment and output. Risk premia on loans and on bonds

rise because the overall share of producing firms is larger than what would be optimal at this higher level of bank fees. The average risk of producing firms increases together with the risk premia.

Interestingly, the effects of the shock on risk premia, investment and output are amplified relative to the case when the thresholds are endogenous (reported in figure 4). The risk premium on loans and the risk premium on bonds increase by 13 and 11 percent, relative to .06 and .5 percent, respectively, in the case of endogenous thresholds. Output and investment to GDP fall by 0.5 and 2.3 percent, relative to .02 and .05 percent when thresholds are endogenous. The contractionary effect of the shock is much larger when firms are unable to substitute instruments of external finance.

In the case of a combined shock to τ and $\varkappa$ (not reported), the discrepancies between the effects obtained when thresholds are endogenous or exogenous are even larger.

Our results are consistent with recent empirical evidence documented in Becker and Ivashina (2011). Using firm-level data on US firms over the period 1990Q2:2010Q4, the authors show that the effect of a reduction in loan supply on investment is positive and significant for firms that raise debt finance and have access to both bond and loan markets. For firms that are excluded from bond markets, the contractionary effect is even larger.

Our results suggest that sluggish adjustment in financing choices could provide an endogenous propagation mechanism of shocks and large movements in credit spreads and real activity, without the need to assume exogenously changing risk. Our conjecture provides an interesting avenue for future research.

5 Conclusions

We propose a general equilibrium model that enables to assess the macroeconomic consequences of firms' financial choices and of the evolving composition of corporate debt.

In response to a shock that increases banking costs and reduces bank efficiency in financial intermediation, the model replicates qualitatively the main facts observed during the crisis, namely the shift in corporate debt from bank finance to bond finance together with an increasing cost of debt securities relative to bank loans, and a contraction in investment and output.

The model points to an important role played by the composition of corporate debt in determining the response of real activity during the crisis. When firms have no access to the bond market, the negative effects on investment and output of a shock that reduces bank profitability are amplified. These findings suggest that abstracting from an endogenous corporate debt structure - as generally done in models that assess the impact of financial market imperfections - may overstate the negative consequences of adverse shocks on real activity.

These results also suggest that the post-crisis policy debate in Europe might have been too narrowly focused on banks and financial intermediaries. Notwithstanding their central role for ensuring financial stability, policy measures aimed at achieving easier substitutability of bank loans for other instruments of external finance may be equally important, as they reduce the adverse consequences on economic activity of periods of financial distress.

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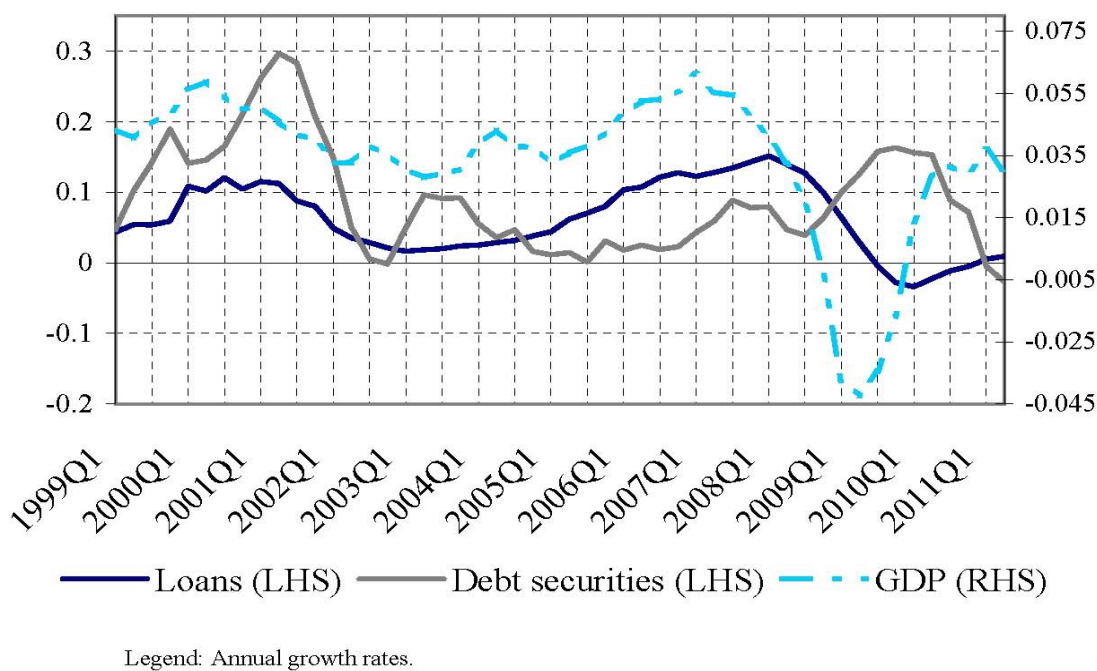


Figure 1: Bank loans and debt securities of non-financial corporations in the euro area.

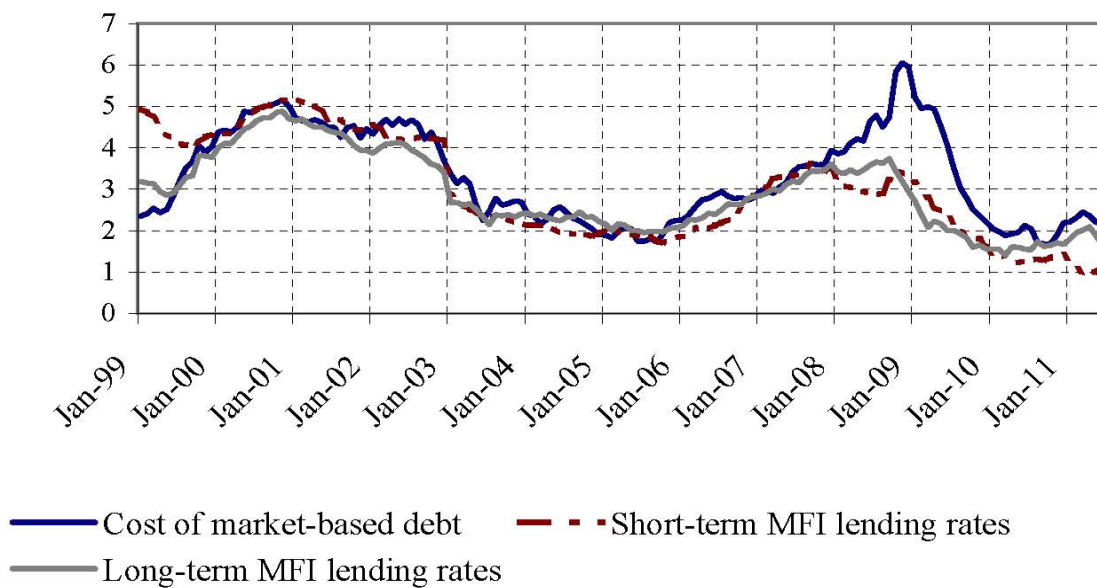


Figure 2: Cost of bank financing and bond financing in the euro area.

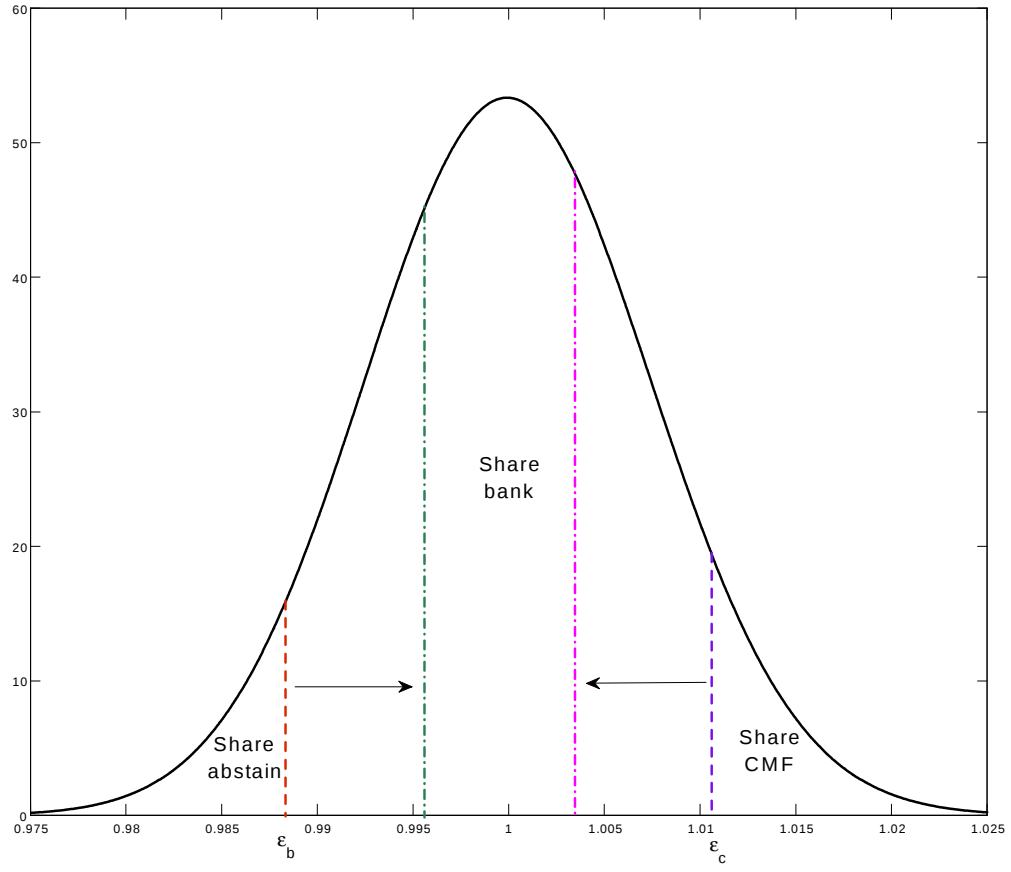


Figure 3: Impact on the steady state distribution of firms of an increase in τ

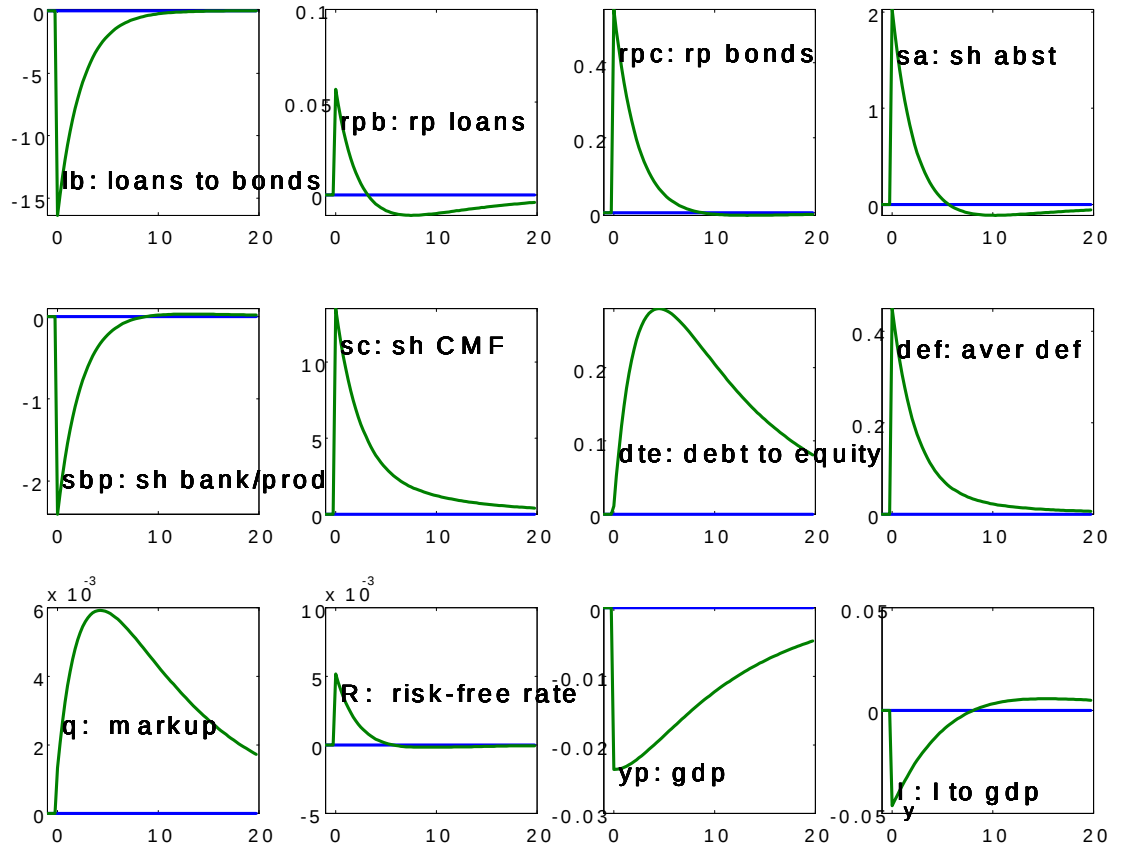


Figure 4: Impulse responses to an increase in bank costs, τ .

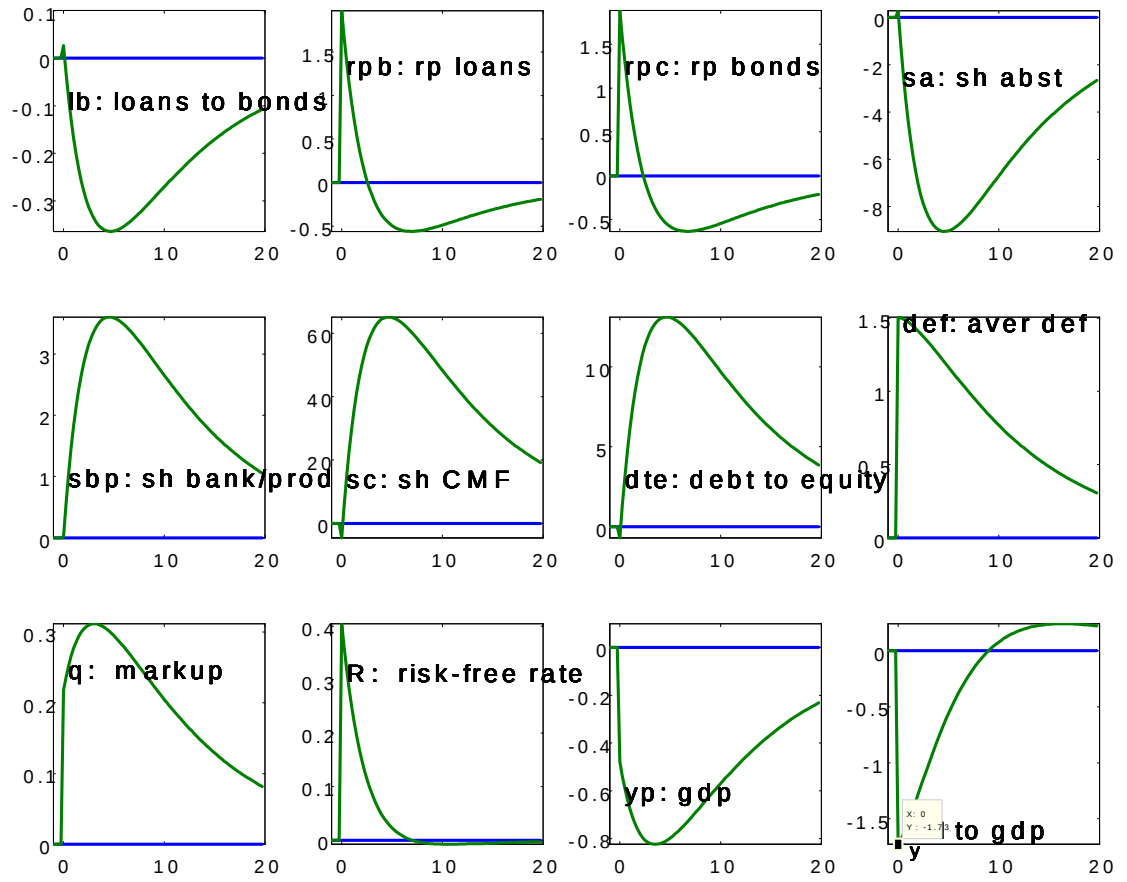


Figure 5: Impulse responses to a negative shock to capital quality, z .

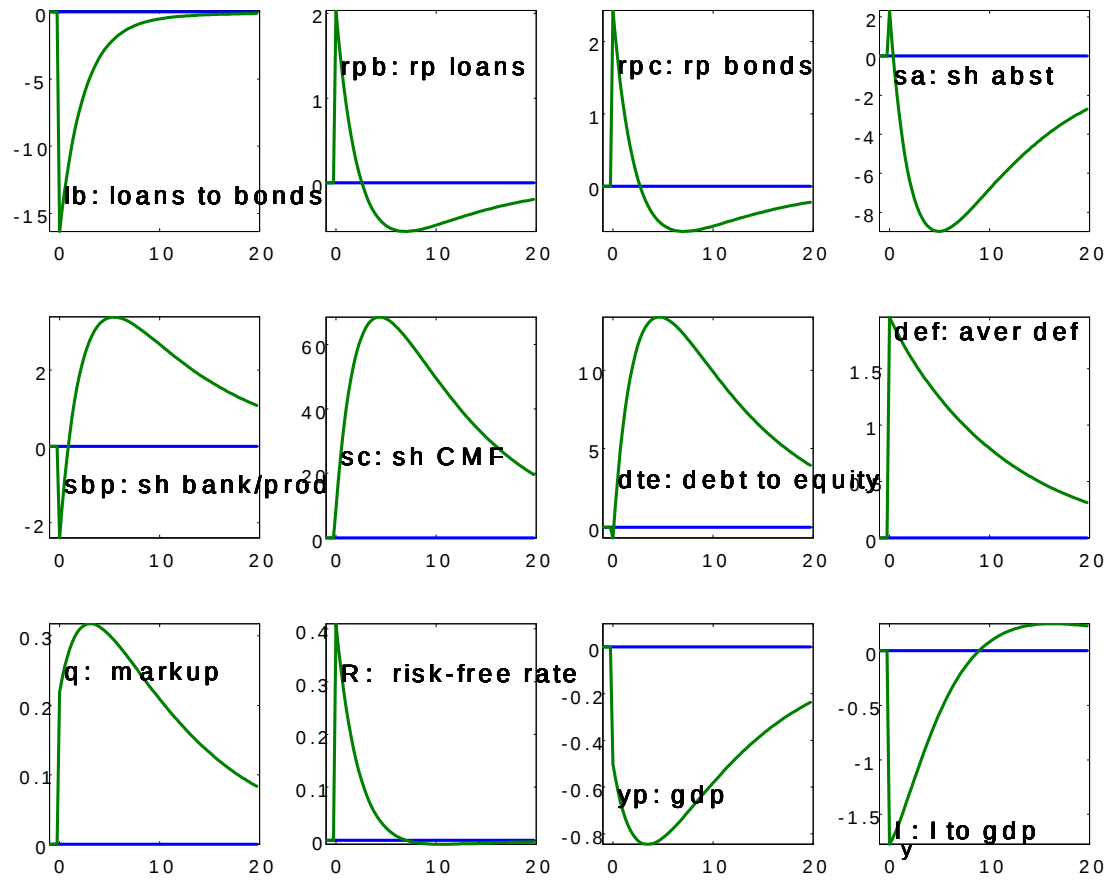


Figure 6: Impulse responses to a combined shock to τ and ξ .

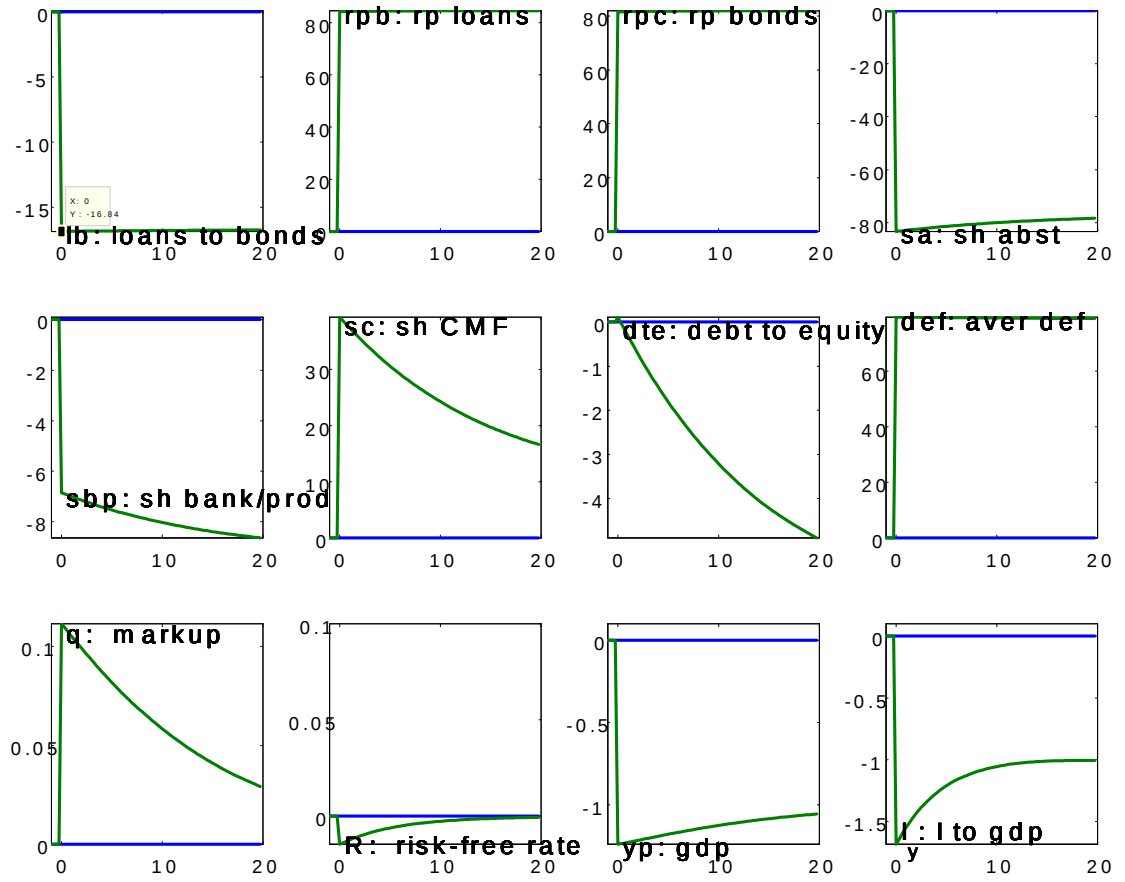


Figure 7: Impulse responses to a permanent combined shock to τ , κ and σ_{ε_3} .

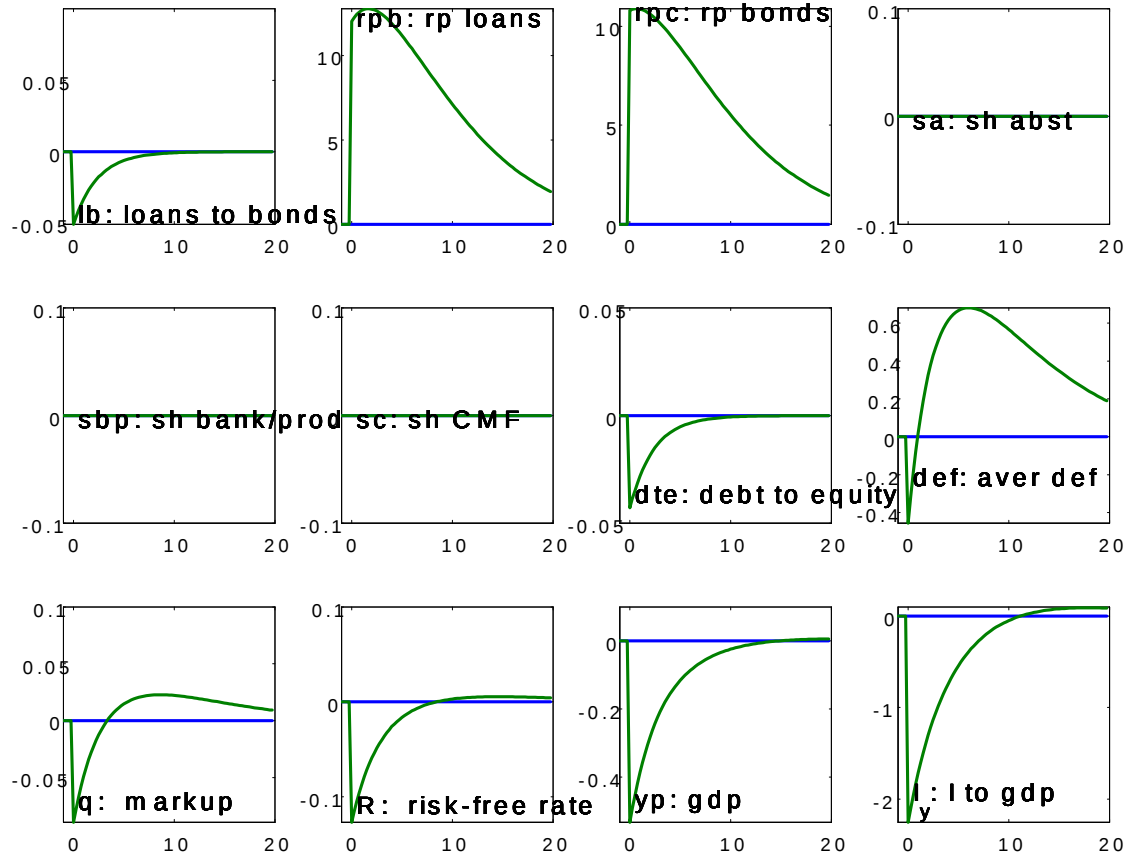


Figure 8: Impulse responses to an increase in bank fees, τ : exogenous thresholds.

APPENDIX

A Aggregating across firms

Aggregate profits of the entrepreneurial sector are given by $\psi^f(q_t, R_t, \tau_t)n_t$, where

$$\psi^f(q_t, R_t, \tau_t) \equiv \int F(\varepsilon_1; q_t, R_t, \tau_t) \Phi_1(d\varepsilon_1),$$

or, equivalently, by

$$\begin{aligned} \psi^f(q_t, R_t, \tau_t) &= s^a(q_t, R_t, \tau_t) + \int_{\bar{\varepsilon}_b(q_t, R_t, \tau_t)}^{\bar{\varepsilon}_c(q_t, R_t, \tau_t)} F^b(\varepsilon_1; q_t, R_t, \tau_t) \Phi_1(d\varepsilon_1) \\ &\quad + \int_{\bar{\varepsilon}_c(q_t, R_t, \tau_t)} F^c(\varepsilon_1; q_t, R_t) \Phi_1(d\varepsilon_1). \end{aligned}$$

Entrepreneurial consumption and accumulation of capital can then be written as equations (35) and (36) in the text.

Define

$$\psi^y(q_t, R_t, \tau_t) = (1 - \tau_t) \int_{\bar{\varepsilon}_b(q_t, R_t, \tau_t)}^{\bar{\varepsilon}_c(q_t, R_t, \tau_t)} \varepsilon_1 \int_{\bar{\varepsilon}_d(\varepsilon_1; q_t, R_t)} \varepsilon_2 \Phi_2(d\varepsilon_2) \Phi_1(d\varepsilon_1) + \int_{\bar{\varepsilon}_c(q_t, R_t, \tau_t)} \varepsilon_1 \Phi_1(d\varepsilon_1) \quad (44)$$

and

$$\psi^m(q_t, R_t, \tau_t) = (1 - \tau_t) \psi^{mb}(q_t, R_t, \tau_t) + \psi^{mc}(q_t, R_t, \tau_t),$$

where

$$\begin{aligned} \psi^{mb}(q_t, R_t, \tau_t) &= \int_{\bar{\varepsilon}_b(q_t, R_t, \tau_t)}^{\bar{\varepsilon}_c(q_t, R_t, \tau_t)} \int_{\bar{\varepsilon}_d(\varepsilon_1; q_t, R_t)} \Phi_3(\bar{\omega}^b(\varepsilon_1 \varepsilon_2; q_t, R_t)) \Phi_2(d\varepsilon_2) \Phi_1(d\varepsilon_1), \\ \psi^{mc}(q_t, R_t, \tau_t) &= \int_{\bar{\varepsilon}_c(q_t, R_t, \tau_t)} \Phi_{2*3}(\bar{\omega}^c(\varepsilon_1; q_t, R_t)) \Phi_1(d\varepsilon_1), \end{aligned}$$

and Φ_{2*3} is the distribution function for the product $\omega^c = \varepsilon_2 \varepsilon_3$. Then, total output, y_t , and total output lost to monitoring costs, y_t^a , are given by equations (38) to (39) in the text.

B Financial variables

We provide analytical expressions for financial variables used in the numerical analysis.

The ratio of bank finance to bond finance, ϑ_t , is defined as the ratio of the funds raised by bank-financed firms to the funds raised by CMF-financed firms, and is given by

$$\vartheta_t = \frac{(1 - \tau_t) s^{bp}(q_t, R_t, \tau_t)}{s^c(q_t, R_t, \tau_t)}. \quad (45)$$

Recall that the risk premium for a firm i , which has chosen to use instrument j , is given by (23). Let $\psi^{rb}(q_t, R_t, \tau_t)$ and $\psi^{rc}(q_t, R_t, \tau_t)$ be

$$\begin{aligned}\psi^{rb}(q_t, R_t, \tau_t) &= \int_{\bar{\varepsilon}_b(q_t, R_t, \tau_t)}^{\bar{\varepsilon}_c(q_t, R_t, \tau_t)} \int_{\bar{\varepsilon}_d(\varepsilon_1; q_t, R_t)} \left[\frac{\left(\frac{\xi}{\xi-1}\right) q_t \varepsilon_1 \varepsilon_2 \bar{\omega}^b(\varepsilon_1 \varepsilon_2; q_t, R_t)}{R_t} - 1 \right] \Phi_2(d\varepsilon_2) \Phi_1(d\varepsilon_1), \\ \psi^{rc}(q_t, R_t, \tau_t) &\equiv \int_{\bar{\varepsilon}_c(q_t, R_t, \tau_t)} \left[\frac{\left(\frac{\xi}{\xi-1}\right) q_t \varepsilon_1 \bar{\omega}^c(\varepsilon_1; q_t, R_t)}{R_t} - 1 \right] \Phi_1(d\varepsilon_1).\end{aligned}$$

The average risk premia for bank-financed firms, rp_t^b , and for CMF-financed firms, rp_t^c , are then given by

$$rp_t^b \equiv \frac{\psi^{rb}(q_t, R_t, \tau_t)}{s^{bp}(q_t, R_t, \tau_t)}, \quad (46)$$

$$rp_t^c \equiv \frac{\psi^{rc}(q_t, R_t, \tau_t)}{s^c(q_t, R_t, \tau_t)}. \quad (47)$$

Although the debt to equity ratio (leverage) is fixed at the firm level and given by $\frac{\xi-1}{\xi}$, the aggregate debt to equity ratio for the corporate sector, χ_t , is endogenous and depends on the share of firms that decide to produce. It is defined as the ratio of all debt instruments used by producing firms to the aggregate net worth of all firms,

$$\chi_t = (\xi - 1) \left[(1 - \tau_t) s^{bp}(q_t, R_t, \tau_t) + s^c(q_t, R_t, \tau_t) \right]. \quad (48)$$

The default rate on bonds, ϱ_t^c , is given by the share of firms which borrow from CMFs but cannot repay the debt,

$$\varrho_t^c = \frac{\psi^{mc}(q_t, R_t, \tau_t)}{s^c(q_t, R_t, \tau_t)}. \quad (49)$$

The average default amounts to the share of firms which sign a contract with either a bank or a CMF but cannot repay the debt,

$$\varrho_t = \frac{\psi^{mb}(q_t, R_t, \tau_t) + \psi^{mc}(q_t, R_t, \tau_t)}{s^{bp}(q_t, R_t, \tau_t) + s^c(q_t, R_t, \tau_t)}. \quad (50)$$

Finally, we define the net expected return to entrepreneurial capital as

$$r_t^z = \psi^f(q_t, R_t, \tau_t) (1 - \delta + r_t) - 1 \quad (51)$$

C Competitive equilibrium

For the convenience of further analysis, we collect the relevant equations here.

1. (a) Households:

$$m_{t+1} + d_{t+1} = \frac{R_{t-1}}{\pi_t} d_t + \theta_t \quad (52)$$

$$0 = m_{t+1} + w_t h_t + r_t k_t - c_t - k_{t+1} + (1 - \delta) k_t \quad (53)$$

(b) Entrepreneurs:

$$n_t = (1 - \delta + r_t) z_t \quad (54)$$

(c) Monetary authority:

$$\theta_t = (\nu - 1) \frac{m_{t-1}^s}{\pi_t} \quad (55)$$

$$m_t^s = \nu \frac{m_{t-1}^s}{\pi_t} \quad (56)$$

(d) Market clearing:

$$y_t^a = y_t - c_t - e_t - (k_{t+1} + z_{t+1}) + (1 - \delta) (k_t + z_t) \quad (57)$$

$$m_t^s = m_t + d_t \quad (58)$$

$$d_t = \left[(1 - \tau_t) s^{bp} (q_t, R_t, \tau_t) + s^c (q_t, R_t, \tau_t) \right] (\xi - 1) n_t \quad (59)$$

(e) Production and aggregation:

$$x_t = \left[(1 - \tau_t) s^{bp} (q_t, R_t, \tau_t) + s^c (q_t, R_t, \tau_t) \right] \xi n_t \quad (60)$$

$$y_t = \psi^y (q_t, R_t, \tau_t) q_t \xi n_t \quad (61)$$

$$y_t^a = \left[\tau_t s^b (q_t, R_t, \tau_t) + \psi^m (q_t, R_t, \tau_t) \mu \xi q_t \right] n_t \quad (62)$$

2. First-order conditions.

(a) Household:

$$\frac{\eta}{u_c(c_t)} = w_t \quad (63)$$

$$u_c(c_t) = \beta R_t E_t \left[\frac{u_c(c_{t+1})}{\pi_{t+1}} \right] \quad (64)$$

$$u_c(c_t) = \beta E_t [(1 - \delta + r_{t+1}) u_c(c_{t+1})]. \quad (65)$$

(b) Entrepreneurs:

$$q_t = A_t \left(\frac{\alpha}{w_t} \right)^\alpha \left(\frac{1-\alpha}{r_t} \right)^{1-\alpha} \quad (66)$$

$$r_t (k_t + z_t) = (1-\alpha)x_t \quad (67)$$

$$w_t h_t = \alpha x_t \quad (68)$$

$$e_t = \gamma \psi^f(q_t, R_t, \tau_t) n_t \quad (69)$$

$$z_{t+1} = \varkappa_t (1-\gamma) \psi^f(q_t, R_t, \tau_t) n_t \quad (70)$$

$$1 = F^d(\varepsilon_{1t}, \bar{\varepsilon}_t^d; q_t, R_t) \quad (71)$$

$$1 = F^b(\bar{\varepsilon}_t^b; q_t, R_t, \tau_t) \quad (72)$$

$$F^b(\bar{\varepsilon}_{ct}; q_t, R_t, \tau_t) = F^c(\bar{\varepsilon}_t^c; q_t, R_t) \quad (73)$$

where the functions F^b, F^c and F^d are defined in equations (26), (27) and (24). Note that these definitions require knowledge of the function $\bar{\omega}^b(\cdot)$ and $\bar{\omega}^c(\cdot)$, which are defined in equation (21) as solution to (20).

3. Financial structure:

$$\vartheta_t = \frac{(1-\tau_t) s^{bp}(q_t, R_t, \tau_t)}{s^c(q_t, R_t, \tau_t)}, \quad (74)$$

$$rp_t^b \equiv \frac{\psi^{rb}(q_t, R_t, \tau_t)}{s^{bp}(q_t, R_t, \tau_t)} \quad (75)$$

$$rp_t^c \equiv \frac{\psi^{rc}(q_t, R_t, \tau_t)}{s^c(q_t, R_t, \tau_t)}, \quad (76)$$

$$\chi_t = (\xi - 1) \left[(1-\tau_t) s^{bp}(q_t, R_t, \tau_t) + s^c(q_t, R_t, \tau_t) \right], \quad (77)$$

$$\varrho_t^c = \frac{\psi^{mc}(q_t, R_t, \tau_t)}{s^c(q_t, R_t, \tau_t)}, \quad (78)$$

$$\varrho_t = \frac{\psi^{mb}(q_t, R_t, \tau_t) + \psi^{mc}(q_t, R_t, \tau_t)}{s^{bp}(q_t, R_t, \tau_t) + s^c(q_t, R_t, \tau_t)}. \quad (79)$$

4. Exogenous variables:

(a) Information acquisition costs

$$\log \tau_t - \log \tau = \rho_\tau (\log \tau_{t-1} - \log \tau) + \varepsilon_{\tau,t}, \quad \varepsilon_{\tau,t} \sim \mathcal{N}(0, \sigma_\tau^2),$$

(b) Net worth

$$\log \varkappa_t = \rho_\varkappa \log \varkappa_{t-1} + \varepsilon_{\varkappa,t}, \quad \varepsilon_{\varkappa,t} \sim \mathcal{N}(0, \sigma_\varkappa^2),$$

where we assume the shocks $(\tau_t, \varkappa_t)$ to be drawn at t and i.i.d. across time.

Given the exogenous variables τ_t and $\varkappa_t$, equations (52) to (79) need to be solved for the variables characterizing the households choices, $(m_t, d_t, c_t, k_t, h_t)$, the entrepreneurs choices $(e_t, z_t, n_t, \bar{\varepsilon}_t^b, \bar{\varepsilon}_t^c, \bar{\varepsilon}_t^d)$, the choices of the monetary authority (θ_t, m_t^s) , aggregate quantities (y_t, y_t^a, x_t) , financial variables $(\vartheta_t, rp_t^b, rp_t^c, \chi_t, \varrho_t^c, \varrho_t)$, and prices and returns $(\pi_t, R_t, r_t, q_t, w_t)$.

This is a system of 28 equations in 27 unknowns. Indeed, one equation is superfluous. By Walras' law, fulfillment of the budget constraints of the entrepreneurs and market clearing on all markets implies fulfillment of the budget constraints of the households as well.

D The stochastic steady state

We compute a steady state where we shut down the aggregate shocks, i.e. $\tau_t = \tau$ and $\varkappa_t = \varkappa$, for all t . We denote steady state variables by dropping the time subscript.

We find it convenient to specify one of the endogenous variables, q , as exogenous and to treat γ as endogenous. Under the assumed specification of the utility function, the unique steady state can be obtained as follows. For each value of q , we can compute π, r, w , and c by solving the equations

$$\begin{aligned}\pi &= \beta R \\ r &= \frac{1}{\beta} - 1 + \delta \\ w &= \left(\frac{A}{q}\right)^{\frac{1}{\alpha}} \alpha \left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}} \\ c &= \left(\frac{w}{\eta}\right)^{\frac{1}{\zeta}}.\end{aligned}$$

To compute the overall expected profits $F(\varepsilon_1; q, R, \tau)$, given by the steady state version of (28), we use the following procedure. First, under our distributional assumptions about the productivity shocks $\varepsilon_1, \varepsilon_2$ and ε_3 , we know that

$$\begin{aligned}\varphi(\bar{\omega}^j) &= \varphi(x_j) \frac{1}{\bar{\omega}^j \sigma_j} \\ f_j(\bar{\omega}^j) &= 1 - \Phi(x_j - \sigma_j) - \bar{\omega}^j [1 - \Phi(x_j)], \\ g_j(\bar{\omega}^j) &= (1 - \mu) \Phi(x_j - \sigma_j) + \bar{\omega}^j [1 - \Phi(x_j)].\end{aligned}$$

where φ and Φ denote the standard normal, $x_j = \frac{\log \bar{\omega}^j + \frac{\sigma_j^2}{2}}{\sigma_j}$ and $j = b, c$. Second, we solve numerically the condition $\varepsilon^j q g_j(\bar{\omega}^j) \xi = R(\xi - 1)$ to obtain the function $\bar{\omega}^j(\varepsilon^j; q, R)$. The function $\bar{\omega}^b(\varepsilon_1 \varepsilon_2; q, R)$ for bank-financed firms is derived by using the variance $\sigma_{\varepsilon_3}^2$ of the log-normal distribution. The function $\bar{\omega}^c(\varepsilon_1; q, R)$ for CMF-financed firms is derived by using the variance $\sigma_{\varepsilon_2}^2 + \sigma_{\varepsilon_3}^2$. The cutoff value $\bar{\varepsilon}^d$ for proceeding with the bank loan is found by solving numerically the condition $F^d(\varepsilon_1, \bar{\varepsilon}^d; q, R, \tau) = 1$. Using $\bar{\varepsilon}^d$, it is then possible to compute the expected utility per unit of net worth for the bank-financed entrepreneur, $F^b(\varepsilon_1; q, R, \tau)$. The expected utility per unit of net worth for the CMF-financed entrepreneur can be computed as $F^c(\varepsilon_1; q, R) = \varepsilon_1 q f(\bar{\omega}^c(\varepsilon_1; q, R)) \xi$. With this, it is possible to calculate the overall return $F(\varepsilon_1; q, R, \tau)$ to entrepreneurial investment, the thresholds $\bar{\varepsilon}^b$ and $\bar{\varepsilon}^c$, and the ratios $\frac{x}{z}$, $\frac{K}{x}$ and $\frac{l}{x}$, as given by

$$\begin{aligned} \frac{x}{z} &= \left[(1 - \tau) s^{bp} + s^c \right] \xi (1 - \delta + r) \\ \frac{K}{x} &= \frac{1 - \alpha}{r} \\ \frac{l}{x} &= \frac{\alpha}{w}. \end{aligned}$$

Notice that in steady state,

$$\begin{aligned} m &= \left(\frac{R}{\pi} - 1 \right) d + \theta = c + \delta k - (wh + rk) \\ d &= \left[(1 - \tau) s^{bp} + s^c \right] (\xi - 1) (1 - \delta + r) \varkappa z \\ \theta &= (\nu - 1) \frac{m^s}{\pi} = \left(\frac{\pi - 1}{\pi} \right) m^s, \end{aligned}$$

and

$$m^s = m + d = c - wh - (r - \delta) k + \left[(1 - \tau) s^{bp} + s^c \right] (\xi - 1) (1 - \delta + r) \varkappa z.$$

Now write the budget constraint of the household as

$$c = \left(\frac{R}{\pi} - 1 \right) d + \theta + wh + (r - \delta) k$$

or as

$$\frac{c}{z} = (R - 1) \left[(1 - \tau) s^{bp} + s^c \right] (\xi - 1) (1 - \delta + r) \varkappa + w \frac{l}{z} + (r - \delta) \frac{k}{z}.$$

Using the solution obtained, calculate z as $z = c / \frac{c}{z}$ and then compute the aggregate variables n, x, K, l and k . Then, use

$$z = \gamma \psi^f(q, R, \tau) n$$

to compute γ , the steady state version of equations (38) and (35) to compute y and e , and of the resource constraint (43) to compute y^a .

Finally, we use these results to compute the financial variables, given by (45)-(50), and the net expected return to entrepreneurial capital, given by (51), in steady state.

E Log-linearization

The equilibrium can be obtained by solving the system of equilibrium conditions, log-linearized around a stochastic steady state where $\pi = 1$ and the aggregate shocks are set to their steady state values. The log-linearized equations are standard and are therefore omitted here.

The difficulty arises in the computation of the coefficients multiplying the variables in the log-linearized equations. We illustrate here how they can be obtained. A detailed appendix with all the log-linearized equations and relative coefficients is available from the authors upon request.

Consider the log-linearized condition corresponding to equation (38),

$$\hat{y}_t = \left[\frac{\psi_q^y(\cdot)q}{\psi^y(\cdot)} + 1 \right] \hat{q}_t + \frac{\psi_R^y(\cdot)R}{\psi^y(\cdot)} \hat{R}_t + \frac{\psi_\tau^y(\cdot)\tau}{\psi^y(\cdot)} \hat{\tau}_t + \hat{n}_t.$$

From equation (44), evaluated at the stochastic steady state, we obtain

$$\begin{aligned} \psi_v^y(q, R, \tau) = (1 - \tau) & \left[\begin{aligned} & \frac{\partial \bar{\varepsilon}_c(\cdot)}{\partial v} \bar{\varepsilon}_c \varphi_1(\bar{\varepsilon}_c) \int_{\bar{\varepsilon}_d(\bar{\varepsilon}_c; q, R)} \varepsilon_2 \Phi_2(d\varepsilon_2) \\ & - \frac{\partial \bar{\varepsilon}_b(\cdot)}{\partial v} \bar{\varepsilon}_b \varphi_1(\bar{\varepsilon}_b) \int_{\bar{\varepsilon}_d(\bar{\varepsilon}_b; q, R)} \varepsilon_2 \Phi_2(d\varepsilon_2) \\ & - \int_{\bar{\varepsilon}_b}^{\bar{\varepsilon}_c} \frac{\partial \bar{\varepsilon}_d(\cdot)}{\partial v} \Big|_{(\varepsilon_1; q, R)} \varepsilon_1 \bar{\varepsilon}_d(\varepsilon_1; q, R) \varphi_2(\bar{\varepsilon}_d(\varepsilon_1; q, R)) \Phi_1(d\varepsilon_1) \end{aligned} \right] \\ & - \frac{\partial \bar{\varepsilon}_c(q, R, \tau)}{\partial v} \bar{\varepsilon}_c \varphi_1(\bar{\varepsilon}_c), \end{aligned}$$

for $v = q, R$, and

$$\begin{aligned} \psi_\tau^y(q, R, \tau) = & - \int_{\bar{\varepsilon}_b(q, R, \tau)}^{\bar{\varepsilon}_c(q, R, \tau)} \int_{\bar{\varepsilon}_d(\varepsilon_1; q, R)} \varepsilon_1 \varepsilon_2 \Phi_2(d\varepsilon_2) \Phi_1(d\varepsilon_1) \\ & + (1 - \tau) \left[\begin{aligned} & \frac{\partial \bar{\varepsilon}_c(\cdot)}{\partial \tau} \bar{\varepsilon}_c \varphi_1(\bar{\varepsilon}_c) \int_{\bar{\varepsilon}_d(\bar{\varepsilon}_c; q, R)} \varepsilon_2 \Phi_2(d\varepsilon_2) \\ & - \frac{\partial \bar{\varepsilon}_b(\cdot)}{\partial \tau} \bar{\varepsilon}_b \varphi_1(\bar{\varepsilon}_b) \int_{\bar{\varepsilon}_d(\bar{\varepsilon}_b; q, R)} \varepsilon_2 \Phi_2(d\varepsilon_2) \end{aligned} \right] \\ & - \frac{\partial \bar{\varepsilon}_c(q, R, \tau)}{\partial \tau} \bar{\varepsilon}_c \varphi_1(\bar{\varepsilon}_c) \end{aligned}$$

To compute the value of $\psi_v^y(q, R, \tau)$ and $\psi_\tau^y(q, R, \tau)$, we now need to compute the derivatives of the thresholds $\bar{\varepsilon}_b, \bar{\varepsilon}_c, \bar{\varepsilon}_d$.

Consider first the threshold at stage II, $\bar{\varepsilon}_d(\varepsilon_1; q, R)$, which is implicitly defined by

$$F^d(\varepsilon_1, \bar{\varepsilon}_d; q, R) = 1.$$

Using the implicit function theorem, we have that

$$\left. \frac{\partial \bar{\varepsilon}_d(\cdot)}{\partial \varepsilon_1} \right|_{(\varepsilon_1; q, R)} = - \frac{F_1^d(\varepsilon_1, \bar{\varepsilon}_d; q, R)}{F_2^d(\varepsilon_1, \bar{\varepsilon}_d; q, R)} \quad (80)$$

$$\left. \frac{\partial \bar{\varepsilon}_d(\cdot)}{\partial v} \right|_{(\varepsilon_1; q, R)} = - \frac{F_v^d(\varepsilon_1, \bar{\varepsilon}_d; q, R)}{F_2^d(\varepsilon_1, \bar{\varepsilon}_d; q, R)}. \quad (81)$$

Using equation (24), we obtain

$$\begin{aligned} F_1^d(\varepsilon_1, \varepsilon_2; q, R) &= \varepsilon_2 q \xi \left[f(\bar{\omega}^b(\varepsilon_1, \varepsilon_2; q, R)) + \varepsilon_1 f'(\bar{\omega}^b(\varepsilon_1, \varepsilon_2; q, R)) \left. \frac{\partial \bar{\omega}^b(\cdot)}{\partial \varepsilon_1} \right|_{(\varepsilon_1, \varepsilon_2; q, R)} \right] \\ F_2^d(\varepsilon_1, \varepsilon_2; q, R) &= \varepsilon_1 q \xi \left[f(\bar{\omega}^b(\varepsilon_1, \varepsilon_2; q, R)) + \varepsilon_2 f'(\bar{\omega}^b(\varepsilon_1, \varepsilon_2; q, R)) \left. \frac{\partial \bar{\omega}^b(\cdot)}{\partial \varepsilon_2} \right|_{(\varepsilon_1, \varepsilon_2; q, R)} \right] \\ F_q^d(\varepsilon_1, \varepsilon_2; q, R) &= \varepsilon_1 \varepsilon_2 \xi \left[f(\bar{\omega}^b(\varepsilon_1, \varepsilon_2; q, R)) + q f'(\bar{\omega}^b(\varepsilon_1, \varepsilon_2; q, R)) \left. \frac{\partial \bar{\omega}^b(\cdot)}{\partial q} \right|_{(\varepsilon_1, \varepsilon_2; q, R)} \right] \\ F_R^d(\varepsilon_1, \varepsilon_2; q, R) &= \varepsilon_1 \varepsilon_2 q \xi f'(\bar{\omega}^b(\varepsilon_1, \varepsilon_2; q, R)) \left. \frac{\partial \bar{\omega}^b(\cdot)}{\partial R} \right|_{(\varepsilon_1, \varepsilon_2; q, R)}. \end{aligned}$$

Computation of the derivatives of $F^d(\cdot)$ requires computing also the derivatives $\frac{\partial \bar{\omega}^b}{\partial \varepsilon_1}$, $\frac{\partial \bar{\omega}^b}{\partial \varepsilon_2}$, and $\frac{\partial \bar{\omega}^b}{\partial v}$, for $v = q, R$. Define

$$\tilde{\omega}^b(\varepsilon_1, \varepsilon_2; q, R) \equiv \frac{g(\bar{\omega}^b(\varepsilon_1, \varepsilon_2; q, R))}{g'(\bar{\omega}^b(\varepsilon_1, \varepsilon_2; q, R))}.$$

Then, from condition (16), we get

$$\left. \frac{\partial \bar{\omega}^b(\cdot)}{\partial \varepsilon_1} \right|_{(\varepsilon_1, \varepsilon_2; q, R)} = - \frac{\tilde{\omega}^b(\varepsilon_1, \varepsilon_2; q, R)}{\varepsilon_1} \quad (82)$$

$$\left. \frac{\partial \bar{\omega}^b(\cdot)}{\partial \varepsilon_2} \right|_{(\varepsilon_1, \varepsilon_2; q, R)} = - \frac{\tilde{\omega}^b(\varepsilon_1, \varepsilon_2; q, R)}{\varepsilon_2} \quad (83)$$

$$\left. \frac{\partial \bar{\omega}^b(\cdot)}{\partial q} \right|_{(\varepsilon_1, \varepsilon_2; q, R)} = - \frac{\tilde{\omega}^b(\varepsilon_1, \varepsilon_2; q, R)}{q} \quad (84)$$

$$\left. \frac{\partial \bar{\omega}^b(\cdot)}{\partial R} \right|_{(\varepsilon_1, \varepsilon_2; q, R)} = \frac{\tilde{\omega}^b(\varepsilon_1, \varepsilon_2; q, R)}{R}. \quad (85)$$

We can then write

$$\begin{aligned}
F_1^d(\varepsilon_1, \bar{\varepsilon}_d; q, R) &= \frac{F^d(\varepsilon_1, \bar{\varepsilon}_d; q, R)}{\varepsilon_1} \left[1 - \frac{f'(\bar{\omega}^b(\varepsilon_1, \bar{\varepsilon}_d; q, R))}{f(\bar{\omega}^b(\varepsilon_1, \bar{\varepsilon}_d; q, R))} \bar{\omega}^b(\varepsilon_1, \bar{\varepsilon}_d; q, R) \right] \\
F_2^d(\varepsilon_1, \bar{\varepsilon}_d; q, R) &= \frac{F^d(\varepsilon_1, \bar{\varepsilon}_d; q, R)}{\bar{\varepsilon}_d} \left[1 - \frac{f'(\bar{\omega}^b(\varepsilon_1, \bar{\varepsilon}_d; q, R))}{f(\bar{\omega}^b(\varepsilon_1, \bar{\varepsilon}_d; q, R))} \bar{\omega}^b(\varepsilon_1, \bar{\varepsilon}_d; q, R) \right] \\
F_q^d(\varepsilon_1, \bar{\varepsilon}_d; q, R) &= \frac{F^d(\varepsilon_1, \bar{\varepsilon}_d; q, R)}{q} \left[1 - \frac{f'(\bar{\omega}^b(\varepsilon_1, \bar{\varepsilon}_d; q, R))}{f(\bar{\omega}^b(\varepsilon_1, \bar{\varepsilon}_d; q, R))} \bar{\omega}^b(\varepsilon_1, \bar{\varepsilon}_d; q, R) \right] \\
F_R^d(\varepsilon_1, \bar{\varepsilon}_d; q, R) &= \frac{F^d(\varepsilon_1, \bar{\varepsilon}_d; q, R)}{R} \frac{f'(\bar{\omega}^b(\varepsilon_1, \bar{\varepsilon}_d; q, R))}{f(\bar{\omega}^b(\varepsilon_1, \bar{\varepsilon}_d; q, R))} \bar{\omega}^b(\varepsilon_1, \bar{\varepsilon}_d; q, R).
\end{aligned}$$

and

$$\left. \frac{\partial \bar{\varepsilon}_d(\cdot)}{\partial \varepsilon_1} \right|_{(\varepsilon_1; q, R)} = -\frac{\bar{\varepsilon}_d}{\varepsilon_1} \quad (86)$$

$$\left. \frac{\partial \bar{\varepsilon}_d(\cdot)}{\partial q} \right|_{(\varepsilon_1; q, R)} = -\frac{\bar{\varepsilon}_d}{q} \quad (87)$$

$$\left. \frac{\partial \bar{\varepsilon}_d(\cdot)}{\partial R} \right|_{(\varepsilon_1; q, R)} = \frac{\bar{\varepsilon}_d}{R} \frac{1}{\left[1 - \frac{f(\bar{\omega}^b(\varepsilon_1, \bar{\varepsilon}_d; q, R))}{f'(\bar{\omega}^b(\varepsilon_1, \bar{\varepsilon}_d; q, R)) \bar{\omega}^b(\varepsilon_1, \bar{\varepsilon}_d; q, R)} \right]} \quad (88)$$

We now need to obtain derivatives of the threshold $\bar{\varepsilon}_b(q, R, \tau)$. This latter is implicitly defined by condition (29) evaluated at the steady state. Using the implicit function theorem, we have that

$$\begin{aligned}
\frac{\partial \bar{\varepsilon}_b(\cdot)}{\partial v} &= -\frac{F_v^b(\bar{\varepsilon}_b; q, R, \tau)}{F_1^b(\bar{\varepsilon}_b; q, R, \tau)} \\
\frac{\partial \bar{\varepsilon}_b(\cdot)}{\partial \tau} &= -\frac{F_\tau^b(\bar{\varepsilon}_b; q, R, \tau)}{F_1^b(\bar{\varepsilon}_b; q, R, \tau)}
\end{aligned}$$

for $v = q, R$. Now, using condition (26), we get

$$\begin{aligned}
F_1^b(\varepsilon_1; q, R, \tau) &= (1-\tau) \left(\begin{aligned} & -\left. \frac{\partial \bar{\varepsilon}_d(\cdot)}{\partial \varepsilon_1} \right|_{(\varepsilon_1; q, R)} \varepsilon_1 \bar{\varepsilon}_d(\cdot) q f(\bar{\omega}^b(\varepsilon_1 \bar{\varepsilon}_d(\cdot); q, R)) \xi \varphi_2(\bar{\varepsilon}_d(\cdot)) \\ & + \int_{\bar{\varepsilon}_d(\varepsilon_1; q, R)} \left[\begin{aligned} & \varepsilon_2 f(\bar{\omega}^b(\varepsilon_1 \varepsilon_2; q, R)) \\ & + \varepsilon_1 \varepsilon_2 f'(\bar{\omega}^b(\varepsilon_1 \varepsilon_2; q, R)) \left. \frac{\partial \bar{\omega}^b(\cdot)}{\partial \varepsilon_1} \right|_{(\varepsilon_1 \varepsilon_2; q, R)} \end{aligned} \right] q \xi \Phi_2(d\varepsilon_2) \\ & + \varphi_2(\bar{\varepsilon}_d) \left. \frac{\partial \bar{\varepsilon}_d(\cdot)}{\partial \varepsilon_1} \right|_{(\varepsilon_1; q, R)} \end{aligned} \right) \\
F_q^b(\varepsilon_1; q, R, \tau) &= (1-\tau) \left(\begin{aligned} & -\left. \frac{\partial \bar{\varepsilon}_d(\cdot)}{\partial q} \right|_{(\varepsilon_1; q, R)} \varepsilon_1 \bar{\varepsilon}_d(\cdot) q f(\bar{\omega}^b(\varepsilon_1 \bar{\varepsilon}_d(\cdot); q, R)) \xi \varphi_2(\bar{\varepsilon}_d(\cdot)) \\ & + \int_{\bar{\varepsilon}_d(\varepsilon_1; q, R)} \left[\begin{aligned} & \varepsilon_1 \varepsilon_2 f(\bar{\omega}^b(\varepsilon_1 \varepsilon_2; q, R)) \\ & + \varepsilon_1 \varepsilon_2 q f'(\bar{\omega}^b(\varepsilon_1 \varepsilon_2; q, R)) \left. \frac{\partial \bar{\omega}^b(\cdot)}{\partial q} \right|_{(\varepsilon_1 \varepsilon_2; q, R)} \end{aligned} \right] \xi \Phi_2(d\varepsilon_2) \\ & + \varphi_2(\bar{\varepsilon}_d(\cdot)) \left. \frac{\partial \bar{\varepsilon}_d(\cdot)}{\partial q} \right|_{(\varepsilon_1; q, R)} \end{aligned} \right)
\end{aligned}$$

$$F_R^b(\varepsilon_1; q, R, \tau) = (1 - \tau) \left(\begin{aligned} & - \frac{\partial \bar{\varepsilon}_d(\cdot)}{\partial R} \Big|_{(\varepsilon_1; q, R)} \varepsilon_1 \bar{\varepsilon}_d(\cdot) q f(\bar{\omega}^b(\varepsilon_1 \bar{\varepsilon}_d(\cdot); q, R)) \xi \varphi_2(\bar{\varepsilon}_d(\cdot)) \\ & + \int_{\bar{\varepsilon}_d(\varepsilon_1; q, R)} \varepsilon_1 \varepsilon_2 q f'(\bar{\omega}^b(\varepsilon_1 \varepsilon_2; q, R)) \frac{\partial \bar{\omega}^b(\cdot)}{\partial R} \Big|_{(\varepsilon_1 \varepsilon_2; q, R)} \xi \Phi_2(d\varepsilon_2) \\ & + \varphi_2(\bar{\varepsilon}_d(\cdot)) \frac{\partial \bar{\varepsilon}_d(\cdot)}{\partial R} \Big|_{(\varepsilon_1; q, R)} \end{aligned} \right).$$

$$F_\tau^b(\varepsilon_1; q, R, \tau) = - \frac{F^b(\varepsilon_1; q, R, \tau)}{(1 - \tau)}.$$

Notice that $\frac{\partial \bar{\varepsilon}_d(\cdot)}{\partial \varepsilon_1}$ and $\frac{\partial \bar{\varepsilon}_d(\cdot)}{\partial v}$ are given by (80)-(81). Moreover, $\frac{\partial \bar{\omega}^b}{\partial \varepsilon_1}$, $\frac{\partial \bar{\omega}^b}{\partial q}$ and $\frac{\partial \bar{\omega}^b}{\partial R}$ are given by (82), (84) and (85). It follows that

$$F_1^b(\bar{\varepsilon}_b; q, R, \tau) = (1 - \tau) \left(\begin{aligned} & - \frac{\partial \bar{\varepsilon}_d(\cdot)}{\partial \varepsilon_1} \Big|_{(\bar{\varepsilon}_b; q, R)} \bar{\varepsilon}_b \bar{\varepsilon}_d(\bar{\varepsilon}_b; \cdot) q f(\bar{\omega}^b(\bar{\varepsilon}_b \bar{\varepsilon}_d(\bar{\varepsilon}_b; \cdot); q, R)) \xi \varphi_2(\bar{\varepsilon}_d(\bar{\varepsilon}_b; \cdot)) \\ & + \int_{\bar{\varepsilon}_d(\bar{\varepsilon}_b; q, R)} \left[\begin{aligned} & \varepsilon_2 f(\bar{\omega}^b(\bar{\varepsilon}_b \varepsilon_2; q, R)) \\ & + \bar{\varepsilon}_b \varepsilon_2 f'(\bar{\omega}^b(\bar{\varepsilon}_b \varepsilon_2; q, R)) \frac{\partial \bar{\omega}^b(\cdot)}{\partial \varepsilon_1} \Big|_{(\bar{\varepsilon}_b \varepsilon_2; q, R)} \end{aligned} \right] q \xi \Phi_2(d\varepsilon_2) \\ & + \varphi_2(\bar{\varepsilon}_d(\bar{\varepsilon}_b; \cdot)) \frac{\partial \bar{\varepsilon}_d(\cdot)}{\partial \varepsilon_1} \Big|_{(\bar{\varepsilon}_b; q, R)} \end{aligned} \right)$$

$$F_q^b(\bar{\varepsilon}_b; q, R, \tau) = (1 - \tau) \left(\begin{aligned} & - \frac{\partial \bar{\varepsilon}_d(\cdot)}{\partial q} \Big|_{(\bar{\varepsilon}_b; q, R)} \bar{\varepsilon}_b \bar{\varepsilon}_d(\bar{\varepsilon}_b; \cdot) q f(\bar{\omega}^b(\bar{\varepsilon}_b \bar{\varepsilon}_d(\bar{\varepsilon}_b; \cdot); q, R)) \xi \varphi_2(\bar{\varepsilon}_d(\bar{\varepsilon}_b; \cdot)) \\ & + \int_{\bar{\varepsilon}_d(\bar{\varepsilon}_b; q, R)} \left[\begin{aligned} & \bar{\varepsilon}_b \varepsilon_2 f(\bar{\omega}^b(\bar{\varepsilon}_b \varepsilon_2; q, R)) \\ & + \bar{\varepsilon}_b \varepsilon_2 q f'(\bar{\omega}^b(\bar{\varepsilon}_b \varepsilon_2; q, R)) \frac{\partial \bar{\omega}^b(\cdot)}{\partial q} \Big|_{(\bar{\varepsilon}_b \varepsilon_2; q, R)} \end{aligned} \right] \xi \Phi_2(d\varepsilon_2) \\ & + \varphi_2(\bar{\varepsilon}_d(\bar{\varepsilon}_b; \cdot)) \frac{\partial \bar{\varepsilon}_d(\cdot)}{\partial q} \Big|_{(\bar{\varepsilon}_b; q, R)} \end{aligned} \right)$$

$$F_R^b(\bar{\varepsilon}_b; q, R, \tau) = (1 - \tau) \left(\begin{aligned} & - \frac{\partial \bar{\varepsilon}_d(\cdot)}{\partial R} \Big|_{(\bar{\varepsilon}_b; q, R)} \bar{\varepsilon}_b \bar{\varepsilon}_d(\bar{\varepsilon}_b; \cdot) q f(\bar{\omega}^b(\bar{\varepsilon}_b \bar{\varepsilon}_d(\bar{\varepsilon}_b; \cdot); q, R)) \xi \varphi_2(\bar{\varepsilon}_d(\bar{\varepsilon}_b; \cdot)) \\ & + \int_{\bar{\varepsilon}_d(\bar{\varepsilon}_b; q, R)} \bar{\varepsilon}_b \varepsilon_2 q f'(\bar{\omega}^b(\bar{\varepsilon}_b \varepsilon_2; q, R)) \frac{\partial \bar{\omega}^b(\cdot)}{\partial R} \Big|_{(\bar{\varepsilon}_b \varepsilon_2; q, R)} \xi \Phi_2(d\varepsilon_2) \\ & + \varphi_2(\bar{\varepsilon}_d(\bar{\varepsilon}_b; \cdot)) \frac{\partial \bar{\varepsilon}_d(\cdot)}{\partial R} \Big|_{(\bar{\varepsilon}_b; q, R)} \end{aligned} \right)$$

$$F_\tau^b(\bar{\varepsilon}_b; q, R, \tau) = - \frac{F^b(\bar{\varepsilon}_b; q, R, \tau)}{(1 - \tau)}.$$

Similar expressions hold below for $F_1^b(\bar{\varepsilon}_c; q, R, \tau)$, $F_q^b(\bar{\varepsilon}_c; q, R, \tau)$, $F_R^b(\bar{\varepsilon}_c; q, R, \tau)$ and $F_\tau^b(\bar{\varepsilon}_c; q, R, \tau)$.

Consider now the threshold for the first stage, $\bar{\varepsilon}_c(q, R, \tau)$. It is implicitly defined by condition (30), evaluated at the steady state. Using the implicit function theorem, we have that

$$\frac{\partial \bar{\varepsilon}_c(\cdot)}{\partial v} = - \left(\frac{F_v^b(\bar{\varepsilon}_c; q, R, \tau) - F_v^c(\bar{\varepsilon}_c; q, R)}{F_1^b(\bar{\varepsilon}_c; q, R, \tau) - F_1^c(\bar{\varepsilon}_c; q, R)} \right),$$

$$\frac{\partial \bar{\varepsilon}_c(\cdot)}{\partial \tau} = - \left(\frac{F_\tau^b(\bar{\varepsilon}_c; q, R, \tau)}{F_1^b(\bar{\varepsilon}_c; q, R, \tau) - F_1^c(\bar{\varepsilon}_c; q, R)} \right).$$

for $v = q, R$. Using condition (27), we get

$$\begin{aligned} F_1^c(\varepsilon_1; q, R) &= \frac{F^c(\varepsilon_1; q, R)}{\varepsilon_1} \left[1 + \varepsilon_1 \frac{f'(\bar{\omega}^c(\varepsilon_1; q, R))}{f(\bar{\omega}^c(\varepsilon_1; q, R))} \frac{\partial \bar{\omega}^c(\cdot)}{\partial \varepsilon_1} \Big|_{(\varepsilon_1; q, R)} \right] \\ F_q^c(\varepsilon_1; q, R) &= \frac{F^c(\varepsilon_1; q, R)}{q} \left[1 + q \frac{f'(\bar{\omega}^c(\varepsilon_1; q, R))}{f(\bar{\omega}^c(\varepsilon_1; q, R))} \frac{\partial \bar{\omega}^c(\cdot)}{\partial q} \Big|_{(\varepsilon_1; q, R)} \right] \\ F_R^c(\varepsilon_1; q, R) &= F^c(\varepsilon_1; q, R) \frac{f'(\bar{\omega}^c(\varepsilon_1; q, R))}{f(\bar{\omega}^c(\varepsilon_1; q, R))} \frac{\partial \bar{\omega}^c(\cdot)}{\partial R} \Big|_{(\varepsilon_1; q, R)}. \end{aligned}$$

Define $\tilde{\omega}^c(\varepsilon_1; q, R) \equiv \frac{g(\bar{\omega}^c(\varepsilon_1; q, R))}{g'(\bar{\omega}^c(\varepsilon_1; q, R))}$. From condition (16), we get

$$\begin{aligned} \frac{\partial \bar{\omega}^c}{\partial \varepsilon_1} &= -\frac{\tilde{\omega}^c(\varepsilon_1; q, R)}{\varepsilon_1} \\ \frac{\partial \bar{\omega}^c}{\partial q} &= -\frac{\tilde{\omega}^c(\varepsilon_1; q, R)}{q} \\ \frac{\partial \bar{\omega}^c}{\partial R} &= \frac{\tilde{\omega}^c(\varepsilon_1; q, R)}{R}. \end{aligned}$$

It follows that

$$\begin{aligned} F_1^c(\bar{\varepsilon}_c; q, R) &= \frac{F^c(\bar{\varepsilon}_c; q, R)}{\bar{\varepsilon}_c} \left[1 - \frac{f'(\bar{\omega}^c(\bar{\varepsilon}_c; q, R))}{f(\bar{\omega}^c(\bar{\varepsilon}_c; q, R))} \tilde{\omega}^c(\bar{\varepsilon}_c; q, R) \right] \\ F_q^c(\bar{\varepsilon}_c; q, R) &= \frac{F^c(\bar{\varepsilon}_c; q, R)}{q} \left[1 - \frac{f'(\bar{\omega}^c(\bar{\varepsilon}_c; q, R))}{f(\bar{\omega}^c(\bar{\varepsilon}_c; q, R))} \tilde{\omega}^c(\bar{\varepsilon}_c; q, R) \right] \\ F_R^c(\bar{\varepsilon}_c; q, R) &= \frac{F^c(\bar{\varepsilon}_c; q, R)}{R} \frac{f'(\bar{\omega}^c(\bar{\varepsilon}_c; q, R))}{f(\bar{\omega}^c(\bar{\varepsilon}_c; q, R))} \tilde{\omega}^c(\bar{\varepsilon}_c; q, R) \end{aligned}$$

from which we can compute $\frac{\partial \bar{\varepsilon}_c(\cdot)}{\partial q}$, $\frac{\partial \bar{\varepsilon}_c(\cdot)}{\partial R}$ and $\frac{\partial \bar{\varepsilon}_c(\cdot)}{\partial \tau}$.