

Linkages across Sovereign Debt Markets*

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Abstract

This paper studies linkages across sovereign debt markets when debt is unenforceable and countries choose to default and renegotiate. In the model countries are linked to one another by borrowing from a common lender. Borrowing from a common lender connects borrowing rates across countries as well as the renegotiation arrangements. Default of one country lowers the lender's wealth which in turn increases the borrowing rate for the other countries. Higher interest rates could then lead to a second default and an even lower wealth for the lender. Foreseeing these events, the lender accepts a lenient haircut from the first defaulter country. The model can rationalize some of the recent events in Europe.

*The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. E-mails: arellano.cristina@gmail.com; yanbai06@gmail.com

1 Introduction

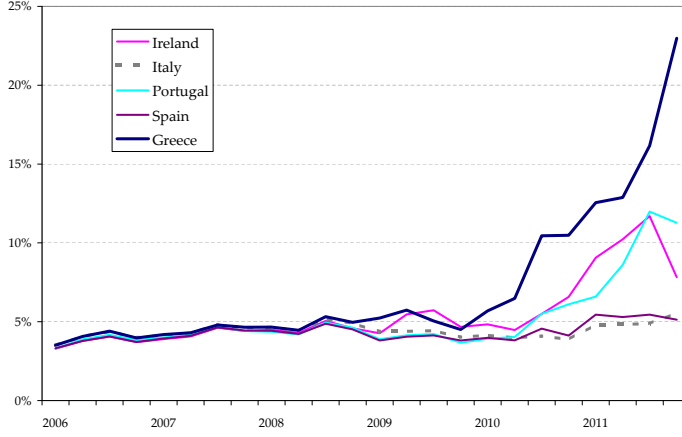
Sovereign debt crises tend to happen in bunches. During the 1980s almost all Latin American countries defaulted and subsequently renegotiated their sovereign debt. The increases in U.S. interest rates in the early 1980s are often cited as contributing to the crisis. During the recent European debt crises, questions about the debt sustainability arose for multiple countries including Greece, Ireland, Italy, Portugal, and Spain. During the crises, interest rates for these countries increased simultaneously. Figure 1 illustrates the co-movement in country's interest rates since 2009. Moreover, discussions about renegotiations and bailouts to Greece are often justified as a way to prevent further contagion in the region. Despite sovereign debt crises happening in tandem, theoretical work on sovereign default has often been restricted to study countries in isolation.

This paper studies linkages across sovereign debt markets when debt is unenforceable and countries choose to default and renegotiate. In the model countries are linked to one another by borrowing from a common lender. Borrowing from a common lender connects borrowing rates across countries as well as the renegotiation arrangements. Default of one country lowers the lender's wealth which in turn increases the borrowing rate for the other countries. Higher interest rates could then lead to a second default and an even lower wealth for the lender.

The model economy consists of three countries, where two symmetric countries borrow from the third country. The borrowing countries can default on their debt. Default entails costs in terms of access to financial market and direct output costs. After default, borrowing countries choose to renegotiate the debt and bargain with the lender over the haircut. After paying the haircut, sanctions are lifted for defaulters and they regain access to financial markets. The price of debt reflects the risk-adjusted compensation for the loss in case of default. The price of debt incorporates three main elements: the risk-free rate, the risk-adjusted default probability, and the risk-adjusted recovery rate, all of which are endogenous.

The representative agent in the lender country is risk averse and has limited wealth. The debt, default, and renegotiation decisions of one of the borrowing coun-

Figure 1: Country Interest Rates in European Countries



tries change the lender country's wealth and hence affect the price of debt for the other borrowing country.

We solve the model numerically and study its implications for the linkages in sovereign debt markets across different countries. Our model predicts that country interest rates co-move because default risk in one country lowers the lender's wealth, which in turn raises the risk free rate and can trigger a default in the second country. The model matches the empirical facts that country interest rates are correlated across countries even when shocks across countries are uncorrelated.

The model also predicts that haircuts are more lenient for one country when the other country has large debt and faces the risk of default. Reaching an agreement with one defaulter country has positive spillover effects for the second country. Specifically, the lender's surplus from reaching an agreement with the defaulter country is larger when the second country has large debt because renegotiating can prevent a second default. If the lender would not reach an agreement with one country, it would have a lower wealth and would charge a higher risk free rate. Such rate could then lead to a default from the second country which is costly to the lender. Anticipating this event, the lender accepts a higher haircut.

The model provides a theory of the linkages in renegotiation procedures when multiple countries borrow from a single lender and can default. The theory implies

that renegotiations with one country can have positive spillovers to other countries by reducing the risk of default.

The model in this paper builds on the work of Aguiar and Gopinath (2006) and Arellano (2008), who model equilibrium default with incomplete markets, as in the seminal paper on sovereign debt by Eaton and Gersovitz (1981). These papers analyze the case of risk neutral lenders, abstract from recovery, and focus on default experiences of single countries. Borri and Verdelhan (2009), Presno and Puzo (2011) and Lizarazo (2010a) study the case of risk averse lenders. They show that risk aversion allows the model to generate spreads larger than default probabilities, which is a feature of the data. Borri and Verdelhan also show empirically that a common factor drives a substantial portion of the variation observed. Lizarazo (2010b) studies contagion in a model similar to ours where multiple borrowers trade with a risk averse lenders. Her model can generate co-movement in spreads across borrowing countries however she abstracts from any debt renegotiation. Yue (2010), D’Erasmus (2011), and Benjamin and Wright (2009) study debt renegotiation in a model with risk neutral lenders. They find that debt renegotiation allows the model to match better the default frequencies and the debt to output ratios.

2 Model

Consider an economy with three countries where two symmetric countries borrow from the third country. Debt contracts are unenforceable and countries can choose to default on their debt whenever they want. Countries that default are get a bad credit standing, are excluded from borrowing, and suffer a direct output cost. Countries in default can renegotiate their debt. During renegotiation the defaulting country and the lender bargain over the haircut. After renegotiation is complete, countries regain a good credit standing.

We consider an economy where each borrowing country receives a stochastic endowment y_i each period which follows a Markov process with transition probabilities $\pi_y(y'_i|y_i, \sigma)$. The stochastic process for the endowment of each country contain a com-

mon stochastic volatility term that is also Markov with transition matrix $\pi_\sigma(\sigma'|\sigma)$.

The timing of events in this economy is as follows. Each borrowing country i for $i = \{1, 2\}$ start each period with a level of debt b_i and a credit standing h_i . Countries with good credit standing $h_i = 0$, decide whether to default or repay their debts. If they repay, they maintain their good standing for next period and choose new debt choices b'_i . If they default, they don't pay their debt and start next period with bad credit standing. Countries with bad credit standing $h_i = 1$ decide whether to renegotiate or not. $z_i = 1$ if country i renegotiates and $z_i = 0$ if it doesn't. If they renegotiate, then they bargain with the lender over the fraction of debt to be paid, ϕ_i . Countries that renegotiate start the next period with good credit standing and zero debt. If they don't renegotiate, then they maintain their bad credit standing. Let $y = \{y_1, y_2\}$ be the countries' endowment shocks, σ the volatility shock, $b = \{b_1, b_2\}$ be the vector of debt holdings, and $h = \{h_1, h_2\}$ be the state of credit standing. The economy wide state is labeled as $s = \{b, h, y, \sigma\}$.

2.1 Borrowing Countries

The representative household in each borrowing country i receives utility from consumption c_{it} and has preferences given by

$$E \sum_{t=0}^{\infty} \beta^t u(c_{it}), \quad (1)$$

where $0 < \beta < 1$ is the time discount factor and $u(\cdot)$ is increasing and concave.

The government of the borrowing country is benevolent and its objective is to maximize the utility of households. The government trades bonds with the lender country. Bonds are one period discount bonds with a face value b'_i with discount price $q(b'_i, s)$. The government also decides whether to repay or default on its debt. The indicator function $d_i = 0$ if it repays and $d_i = 1$ if it defaults. While in default, the government is in bad credit standing and it decides whether to renegotiate or not and bargain with the lender over the fraction of debt to be repaid. The haircut is the

fraction of debt that lenders forgo during the renegotiation. The indicator function $h_i = 0$ if the country is in good credit standing, and $h_i = 1$ if it is in bad credit standing.

The bond price function $q(b'_i, s)$ is endogenous to the government's incentives to default and renegotiate as well as the haircut and compensates the lender for the risk adjusted loss in case of default. The price on debt depends on the size of the bond b'_i and the aggregate state $s = \{b, y, h\}$ because default, renegotiation, and the haircut depend on all these. The government rebates back to households all the proceedings from its credit operations in a lump sum fashion.

When the government is in good credit standing $h_i = 0$ and chooses to repay its debts $d_i = 0$, the resource constraint for borrowing country i is the following

$$c_i = y_i - b_i + q(b'_i, s)b'_i \quad (2)$$

If the government with $h_i = 0$ defaults, the government doesn't pay its outstanding debt b_i , it is excluded from trading international bonds, and it incurs output costs y_{it}^{def} . Consumption equal output during these periods.

$$c_{it} = y_{it}^{def}. \quad (3)$$

Following Arellano (2008) we assume that borrowers lose a fraction λ of output if output is above a threshold: $y_t^{def} = \begin{cases} y_t & \text{if } y_t \leq (1 - \lambda)\bar{y} \\ (1 - \lambda)\bar{y} & \text{if } y_t > (1 - \lambda)\bar{y} \end{cases}$, where $\bar{y}$ is the mean level of output.

Default changes the credit standing of the country to $h_i = 1$. Every period a government with $h_i = 1$ chooses to renegotiate its debts or not. The indicator function $z_i = 0$ if it doesn't renegotiate and $z_i = 1$ if it renegotiates. In periods when the government doesn't renegotiate, consumption equals output $c_{it} = y_{it}^{def}$.

If the government renegotiates, then it bargains with the lender over the recovery $\phi(b_i, s)$. The recovery is the percent of the face value of the defaulted debt b_i that the government pays back the lender to regain its good credit standing. We label the haircut as $1 - \phi$, or the percent reduction in the face value after default. The haircut

depends on the state s because the bargaining power of the borrower country and the lender are functions of the state. When $z_i = 1$ the resource constraint for the economy is

$$c_{it} = y_{it} - \phi(b_i, s)b_i \quad (4)$$

After renegotiating, the borrower country starts next period with zero debt due $b'_i = 0$ and good credit standing $h_i = 0$.

We represent the borrowing country's problem as a recursive dynamic programming problem. Let $v_i(b_i, s)$ be the value function of the borrowing country that has good credit standing $h_i = 0$. We make explicit the country's debt b_i although it's part of s for expositional purposes. Borrowing country i that starts with debt b_i decides whether to default or not after endowment shocks are realized

$$v_i(b_i, s) = \max_{d_i \in \{0,1\}} \{d_i v_i^{nd}(b_i, s) + (1 - d_i) v_i^d(b_i, s)\} \quad (5)$$

where $v_i^{nd}(b_i, s)$ is the value to the country conditional on not defaulting and $v_i^d(b_i, s)$ is the value of default. $d(b_i, S) = 1$ if the country chooses default and zero otherwise.

If the country repays the debt, then it chooses optimal consumption and savings

$$v_i^{nd}(b_i, s) = \max_{c_i, b'_i} \{u(c) + \beta \sum_{s'} \pi(s', s) v_i(b'_i, s')\} \quad (6)$$

subject to (2), and the law of motion of the other country's debt and credit standing

$$\begin{aligned} b_{-i} &= B(s) \\ h_{-i} &= H(s) \end{aligned} \quad (7)$$

Choosing to repay implies that tomorrow the country starts with good credit standing and has to option to default again.

If the country defaults then it is does not pay the debt, cannot borrow and

consumes output y^d

$$v_i^d(b_i, s) = \{u(y_i^d) + \beta \sum_{s'} \pi(s', s) w_i(b_i, s')\} \quad (8)$$

subject to (7). After default the country starts with bad credit standing $h_i = 1$ and maintains the level of the defaulted debt b_i . Let w_i be the value function associated with being in bad credit standing.

Once $h_i = 1$ the country decides whether to renegotiate or not

$$w_i(b_i, s) = \max_{z_i \in \{0,1\}} \{z_i w_i^r(b_i, s) + (1 - z_i) w_i^{nr}(b_i, s)\} \quad (9)$$

$z_i(b_i, S) = 1$ if the country chooses renegotiate and zero otherwise. Let $w_i^r(b_i, s)$ be the value associated with renegotiation and $w_i^{nr}(b_i, s)$ the value of not renegotiating the debt.

If the country renegotiates, then it has to repay the recovery rate $\phi(b_i, s)$ which will be derived below. Renegotiation allows the country to avoid the output cost. Following renegotiation the country starts with zero debt and with good credit standing

$$w_i^r(b_i, S) = \{u(y_i - \phi(b_i, S) b_i) + \beta \sum_{s'} \pi(s', s) v_i(0, s')\} \quad (10)$$

subject to (7). If the country does not renegotiate, then it remains excluded from financial markets and consuming y_i^d

$$w_i^{nr}(b_i, s) = \{u(y_i^d) + \beta \sum_{s'} \pi(s', s) w_i(b_i, s')\}$$

subject to (7). Note that $w_i^{nr}(b_i, s) = v_i^d(b_i, s)$.

This problem delivers value functions v_i and w_i and decision rules for debt $\tilde{b}_i'(b_i, s)$, default $\tilde{d}_i(b_i, s)$, and repayment $\tilde{z}_i(b_i, s)$. These characterize default sets D_i and renegotiation sets Z_i such that

$$D_i(b_i, b_{-i}) = \{\{y_i, y_{-i}, \sigma\} : d_i(b_i, s) = 1\}$$

$$Z_i(b_i, b_{-i}) = \{\{y_i, y_{-i}, \sigma\} : z_i(b_i, s) = 1\}$$

These sets depend on the debt holdings of the two borrowing countries.

2.2 Lender Country

The representative household in the lender country receives utility from consumption c_{Lt} and has preferences given by

$$E \sum_{t=0}^{\infty} \delta^t u(c_{Lt}), \quad (11)$$

where $0 < \delta < 1$ is the time discount factor and $u(\cdot)$ is increasing and concave. We assume that $\beta < \delta$.

Households receive a constant endowment y_L every period. They have access to state contingent assets that are traded domestically as well as foreign savings with the borrowing countries. We assume that the lender country honors all financial contracts.

Each period households choose optimal consumption c_L and state contingent assets $a'(s', s)$. They also choose loans to the borrowing countries in periods where the borrowing countries are in good credit standing $b_1^{g'}$, and $b_2^{g'}$. The value function for the representative household is given by

$$v^L(s) = \max_{c_L, a', b_1^{g'}, b_2^{g'}} \{u(c_L) + \delta \sum_{s'} \pi(s', s) v^L(s')\} \quad (12)$$

subject to its budget constraint and the laws of motion of credit standings for the two borrowing countries

$$\begin{aligned} h'_1 &= H_1(s) \\ h'_2 &= H_2(s) \end{aligned}$$

The budget constraint of the lender country depends on the credit standing of each

borrowing country and whether they default or renegotiate. The budget constraint can be written in a compact way as

$$c_L = y_L + \sum_{i=1,2} (1-h_i) [1 - d_i(s)] (b_i - Q_i(s)b_i^{g'}) + \sum_{i=1,2} h_i z_i(s) \phi(b_i, s) b_i - \sum_{s'} m(s, s') a'(s', s) + a(s). \quad (13)$$

The evolution of the borrowing countries' debt depend on their renegotiation and default decisions

$$b'_i = \left\{ \begin{array}{l} b_i \text{ if } (h_i = 0 \text{ and } d_i(s) = 1) \text{ or } (h_i = 1 \text{ and } z_i(s) = 0) \\ 0 \text{ if } (h_i = 1 \text{ and } z_i(s) = 1) \\ b_i^{g'} \text{ otherwise} \end{array} \right\} \quad (14)$$

It is useful to define the lenders' pricing kernel $m(s', s)$ as the marginal rate of substitution for the lender across periods as follows

$$m(s', s) = \frac{\delta \pi(s', s) u_c(s')}{u_c(s)}. \quad (15)$$

2.3 Bond Price Function

Households in the lender country are competitive and provide loans to borrowing countries as long as they are compensated for the risk adjusted loss in the case of default. Given that lenders are risk averse, they discount future states with their pricing kernel $m(s, s')$ in (15).

For each loan of size b_i lenders receive b_i the following period if the borrower repays. If the borrower defaults, lenders receive the recovery ϕ_i in the period when the borrower renegotiates. Let's define $\zeta(b_i, s)$ to be the risk adjusted present discounted value of recovery. $\zeta_i(b_i, s)$ can be defined recursively with the functional equation

$$\zeta_i(b_i, s) = z_i(b_i, s) \phi(b_i, s) b_i + (1 - z_i(b_i, s)) \sum_{s'} \pi(s', s) \zeta_i(b_i, s') m(s', s) \quad (16)$$

with (7) specifying the evolution of the other country's debt. If the country renego-

tiates this period $z_i(b_i, s) = 1$, and the value for recovery is $\phi(b_i, s)b_i$. If the country doesn't renegotiate, the present value of recovery is given by the discounted value of future recovery given by $\zeta_i(b_i, s')$. These future recovery values are weighted by the pricing kernel $m(s', s)$ which implies that recovery values are weighted more heavily for states s' that feature a higher pricing kernel.

The bond price function for each borrowing country equals the risk adjusted discounted present value of future payments

$$q(b'_i, s)b'_i = b'_i \sum_{s'} \pi(s', s)(1 - d_i(b'_i, s'))m(s', s) + \sum_{s'} \pi(s', s)d_i(b'_i, s')m(s', s) \sum_{s''} \pi(s'', s')m(s'', s')\zeta_i(b'_i, s'') \quad (17)$$

The bond price contains two elements: the payoff in non-default states $d_i(b'_i, s') = 0$ and in default states $d_i(b'_i, s') = 1$. The lender discounts cash flows by the pricing kernel $m(s', s)$ and hence states are weighted by $\pi(s', s)m(s', s)$. In non-default states, the lender gets the face value of the debt b'_i . In default states, the lender gets nothing, but the following period he can get the present value of recovery $\zeta_i(b'_i, s'')$ defined in (16). If default happens in states when $m(s', s)$ is high, the price contains a positive risk premia for the default event. The bond price also compensates for any covariation between recovery value and the pricing kernel.

Borrowing countries do not interact directly with one another. However, they borrow from a common risk averse lender whose wealth fluctuates. Linkages across the borrowing countries are encoded in the bond price schedules $q(b'_i, s)$ which depend on the lender's conditions and are summarized by the aggregate state $s = \{b_1, b_2, h_1, h_2, y_1, y_2, \sigma\}$.

2.4 Renegotiation procedure

After default, the defaulting country decides when to renegotiate by its choice of $z_i(b_i, S)$. The length of renegotiation is endogenous and depends on the amount of time that the borrower takes to choose to renegotiate. However, when the defaulter

country chooses $z_i(b_i, S) = 1$, both countries decide immediately on the fraction $\phi(b_i, S)$ that will be repaid through Nash bargaining. We assume that if the two countries do not reach an agreement, the defaulting country is in permanent financial autarky and $y_i = y_i^d$. The threat value for the defaulter country is

$$v_i^{aut}(y_i, \sigma) = \{u(y_i^d) + \beta \sum_{y'_i, \sigma'} \pi_y(y'_i | y_i, \sigma) \pi_\sigma(\sigma' | \sigma) v_i^{aut}(s')\} \quad (18)$$

In case of no agreement, the lender country receives zero of the debt and will be permanently in financial autarky with the defaulter country. The lender will still have access to financial trading with the other non-defaulting country. Let $v^{L,1}(s)$ be the value to the lender of trading with only one borrowing country which is specified below.

The recovery rate $\phi(b_i, s)$ maximizes the weighted surplus for the defaulter and the lender. The bargaining power for the borrower is θ and that for the lender is $(1 - \theta)$. The haircut $\phi(b_i, s)$ solves

$$\phi(b_i, s) = \max_{\phi \in [0,1]} \{ [w^r(b_i, s; \phi) - v^{aut}(y_i, \sigma)]^\theta [v^L(s; \phi) - v^{L,1}(s)]^{1-\theta} \} \quad (19)$$

subject to both parties receiving a non-negative surplus from the renegotiation

$$\begin{aligned} w^r(b_i, s; \phi) - v^{aut}(y_i, \sigma) &\geq 0 \\ v^L(s; \phi) - v^{L,1}(s) &\geq 0 \end{aligned}$$

In considering the threat point for renegotiating the debt, we assume that the lender country will trade only with the non-defaulting country from then on. The value to the lender country of trading only with one country is similar to the problem above except that it only trades with country $-i$.

$$v^{L,1}(s) = \max_{c_L, a', b_{-i}^{g'}} \{u(c_L) + \delta \sum_{s'} \pi(s', s) v^{L,1}(s')\}$$

subject to its budget constraint

$$c = y_L + (1 - h_{-i}) [1 - d_{-i}(s)] (b_{-i} - Q_{-i}(s)b_{-i}^{g'}) + h_{-i}z_{-i}(s)\phi_{-i}(b_{-i}, S)b_{-i} - \sum m(s', s)a' + a(s)$$

and (14).

2.5 Equilibrium

We focus on recursive Markov equilibria in which all decision rules are functions only of the state variable $s = \{b_1, b_2, h_1, h_2, y_1, y_2, \sigma\}$. A recursive equilibrium for this economy consists on (i) the policy functions for every borrowing country i for consumption $\tilde{c}_i(s)$, debt choices $\tilde{b}'_i(s)$, default and renegotiation decisions $\tilde{d}_i(s)$ and $\tilde{z}_i(s)$, the borrowing countries' value functions $v_i(s)$, $v_i^{nd}(s)$, $v_i^d(s)$, $w_i(s)$, $w_i^{nr}(s)$, and $w_i^r(s)$, (ii) the lender country's policy functions for consumption $c_L(s)$, debt $\tilde{b}'_{L,1}(s)$, and $\tilde{b}'_{L,2}(s)$, and state contingent assets $a(s', s)$, and value functions $v^L(s)$ and $v^{L,1}(s)$, (iii) the recovery function $\phi(b_i, s)$, (iv) the bond price function $q(b'_i, s)$, and (v) the equilibrium price of debt $Q_i(s)$ and state contingent assets $m(s', s)$, such that:

1. Taking as given the bond price function $q(b'_i, s)$ and the recovery function $\phi(b_i, s)$, the policy and value functions functions $\tilde{c}_i(s)$, $\tilde{b}'_i(s)$, $\tilde{d}_i(s)$, $\tilde{z}_i(s)$, $v_i(s)$, $v_i^{nd}(s)$, $v_i^d(s)$, $w_i(s)$, $w_i^{nr}(s)$, and $w_i^r(s)$ satisfy the borrowers' optimization problems.
2. Taking as given the bond price function $q(b'_i, s)$, the recovery function $\phi(s)$, and the state contingent prices $m(s', s)$, the policy functions and value functions $c_L(s)$, $\tilde{b}'_{L,1}(s)$, $\tilde{b}'_{L,2}(s)$, $a(s', s)$, $v^L(s)$ and $v^{L,1}(s)$ satisfy the lender's optimization problem.
3. The recovery function $\phi(s)$ solves the Nash Bargaining problem (19)
4. The bond price functions $q(b'_1, s)$ and $q(b'_2, s)$ satisfy equation (17)
5. The price of debt $Q_i(s)$ clears the bond market for every i

$$q(b'_i, s)\tilde{b}'_i = Q_i(s)\tilde{b}'_{L,i}(s) \tag{20}$$

6. State contingent prices $m(s', s)$ clear the lender country domestic asset market such that $a(s', s) = 0$.

7. The goods market clears

$$c_1 + c_2 + c_L = y_1 + y_2 + y_L \quad (21)$$

3 Quantitative Analysis

We solve the model numerically and analyze the linkages across the two borrowing countries in country interest rates and renegotiation procedures. The model predicts that borrowing rates and default probabilities are higher for one of the borrowing countries when the other country has a high risk of default. The model also predicts that haircuts are more lenient and renegotiations faster for one of the borrowing countries when the other country has a high risk of default.

3.1 Parameterization

The utility function for all the countries is $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. We set the risk aversion coefficients σ is set to 2, which is a common value used in real business cycle studies. The length of a period is one year. The stochastic process for output for the borrowing countries is independent from one another and follow a log-normal AR(1) process, $\log(y_{t+1}) = \rho \log(y_t) + \varepsilon_{t+1}$ with $E[\varepsilon^2] = \eta^2$. [For now we are assuming constant volatility]. We discretize the shocks into a four-state Markov chain using a quadrature-based procedure (Tauchen and Hussey, 1991). We use annual series of linearly detrended GDP for Greece for 1960–2011 taken from the World Development Indicators to calibrate the volatility and persistence of output. We set the output of the lender country to be 6.5 times the output of the borrowing country which we normalize to 1. This corresponds to roughly the average output across Greece and Italy relative to Germany's output. For the borrower's bargaining parameter θ we follow D'Erasmus (2011) and set it equal to 0.58.

We calibrate the lender and borrowers' discount rates δ and β as well as the default cost λ to match an average risk free rate of 5% and an average default probability of 6%. Table 1 summarizes the parameter values.

Table 1: Parameters Values		
	Value	Target
Countries' risk aversion	$\sigma = 2$	Standard value
Stochastic structure for shocks	$\rho_y = 0.93, \eta_y = 0.02$	Greece output
Bargaining power	$\theta = 0.58$	D'Erasmus (2011)
Lender's output	$y_L = 6$	Italian and Greek average output relative to Germany
Calibrated parameters		
Output cost after default	$\lambda = 0.01$	Default probability 7% Risk free rate 5%
Borrowers' discount factor	$\beta = 0.90$	
Lender's discount factor	$\delta = 0.95$	

3.2 Results

We simulate the model and report statistics summarizing debt markets for one of the borrowing countries. Due to symmetry, statistics for the second country are equal.

Borrowing countries do not interact directly with one another or have common shocks. Linkages across borrowing countries are encoded in the bond price and recovery schedules $q(b'_i, s)$ and $\phi(b_i, s)$ because these depend on the lender's conditions which are summarized in $s = \{b_1, b_2, h_1, h_2, y_1, y_2\}$.

Table (2) reports interest rates, default probabilities, length of renegotiation and haircuts. The model predicts an average country interest rate defined as $(1/q - 1)$ of 7.1% . The country interest rate is composed of the risk free rate, the risk-adjusted default probability, and the risk-adjusted present value of recovery, which in turn depends on the length of renegotiation and the haircut. The risk free rate equals on average 5.3%, the average default probability equals 6.6%, the length of renegotiation equals 1.9 years, and the average haircut is 14%. Risk premia on both default and recovery is positive in the model but account for a modest fraction of the country

interest rate. The actuarially fair interest rate defined with a risk neutral pricing kernel equals 7.0%, showing that the majority of the interest rate is accounted for by default probabilities and recovery.

Table 2: Debt Linkages

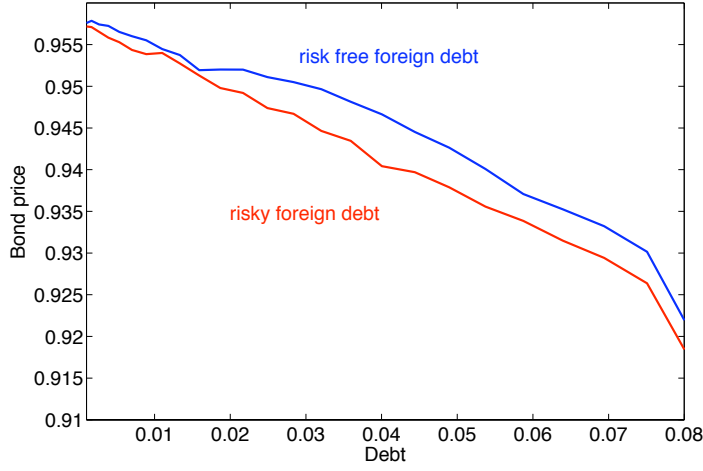
	Overall	Conditional: Country 2	
	Average	$r_2 > \text{Median}$	$r_2 \leq \text{Median}$
Default probability	6.6	7.4	5.9
Country 1 interest rate	7.1	7.4	6.9
Risk free rate	5.3	5.5	5.2
Length renegotiation	1.90	1.83	2.06
Haircut	0.14	0.15	0.14

To illustrate debt market linkages across borrowing countries, table (2) also reports debt market statistics for country 1 conditional on whether the interest rate of country 2 is high or low. We find that country interest rates are correlated: the average interest rate for country 1 equals 7.4% when the interest rate for country 2 is above its median while it is 6.9% when country 2's interest rate is below the median. The main reason for this correlation is that when interest rates in country 2 are high, country 1 defaults more often. The default rate of country 1 equals 7.4% when $r_2 > \text{median}$ while it equals 5.9% when $r_2 \leq \text{median}$.

Renegotiation terms for country 1 also depend on country 2's interest rates. When interest rates in country 2 are high, haircuts for country 1 are bigger and renegotiations lengths are shorter. Conditional on renegotiating, country 1 pays less of the defaulted debt when $r_2 > \text{median}$. Hence, country 1 has more incentive to renegotiate during these times shortening the duration of renegotiations. Larger haircuts when $r_2 > \text{median}$ lead to positive co-movement in interest rates cross countries, but shorter renegotiations when $r_2 > \text{median}$ lead to a negative co-movement.

To illustrate the mechanisms behind these results, we plot some of the decision rules from the model. Figure 2 plots the bond price function $q(b'_1, s)$ for country 1 as a function of the loan size b'_1 (relative to the borrowing country's mean income) when $h_2 = 0$, for a high and a low value of country 2's debt level b_2 . With a high

Figure 2: Bond Price Function



b_2 country 2 faces a high default probability and a high interest rate. With a low b_2 country 1 faces the risk free interest rate.

As in standard models of borrowing and default, bond price fall with the size of the loan because default probabilities rise with larger loans. In our model with endogenous risk free rates, the bond price falls with loans even absent default risk. The bond price schedule depends on country 2's debt; it is tighter and falls faster with b'_1 when country 2 has a high level of debt and high default risk.

An important force behind these schedules is the differences in the risk free rate across b_2 states. When country 2 has a high level of debt, it tends to borrow more than when it has a low level of debt because of consumption smoothing. This implies that the lender experiences larger capital outflows to country 2 and consumes less. A lower consumption for the lender today, pushes the risk free rate up because the lender is compensated for a higher expected consumption growth. In the figure, these forces shift the bond price down. As loans b'_1 increase, the bond price falls to compensate the lender for larger capital outflows to country 1 and a lower consumption. When b'_1 is larger than 0.075, country 1 starts to default the following period and hence the bond price compensates for this loss to the lender.

Higher default probabilities in country 2 also lead to more default for country 1.

Figure 3: Default Decision

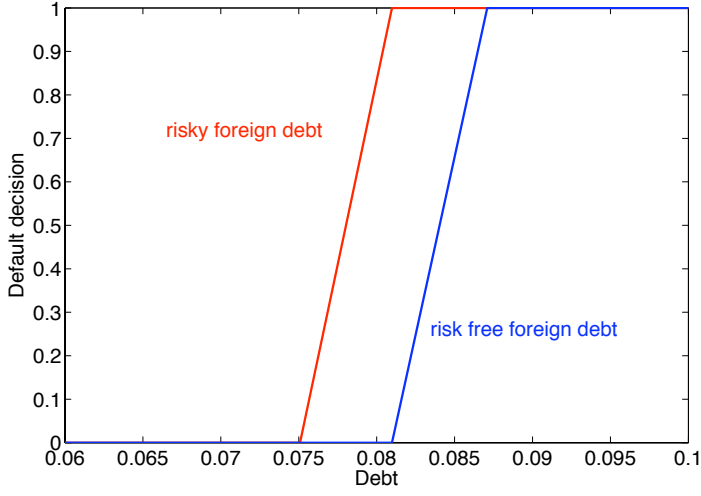


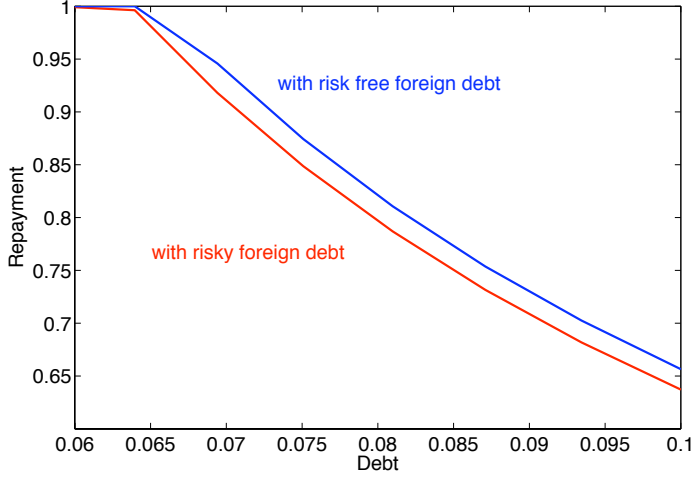
Figure 3 plots the default decision for country 1 $d_1(b_1, s)$ as function of its level of debt b_1 for a high and a low value of country 2's debt level b_2 . Default is chosen for a smaller level of debt when country 2 has high debt. The reason is that the benefits from repaying the debt and accessing new loans are smaller when b_2 is high. As shown above, bond price schedules are tighter when country 2 has a high debt, and hence defaulting becomes more attractive in these states.

Country 2 debt conditions also affect recovery values. Figure 4 plots the recovery function $\phi(b_1, s)$ as a function of country 1's debt b_1 for a high and a low value of country 2's debt b_2 . Lenders recover less of the debt from country 1 when country 2 has a high debt. The reason is that the surplus from the renegotiation is higher for lenders when b_2 is high relative to when it is low because reaching an agreement with country 1 prevents country 2 from defaulting. When the surplus for the lender from the agreement is larger, Nash bargaining implies that the recovery rate is lower because lenders effectively share more of the surplus with the borrower.

The recovery rate ϕ solves the problem (19), which implies that in equilibrium ϕ satisfies:

$$\theta u_c(y - \phi b) [v^L(s; \phi) - v^{L,1}(b_2, y_2)] = (1 - \theta) u_c(c_L; \phi) [w^r(s; \phi) - v^{aut}(y)]. \quad (22)$$

Figure 4: Recovery Function



The left hand side of this expression depends on the borrower's marginal utility times the surplus from the renegotiation for the lender. If an agreement is not reached with country 1, the lender trades only with country 2. The left hand side is increasing in ϕ because both the borrower's marginal utility and the lender's utility from the renegotiation are increasing in ϕ . The right hand side depends on the lender's marginal utility times the surplus from the renegotiation for the borrower which are both decreasing in ϕ .

Country 2's debt level has first order effects on the lender's surplus from the agreement and the lender's marginal utility. In general, high b_2 increases the lender's surplus because reaching an agreement could prevent a default in country 2. The idea is that the lender's wealth is larger when an agreement is reached relative when it is not reached and thus risk free rates are lower. A lower risk free rate in turn can induce country 2 to repay the debt relative to default. Given that default is costly for the lender, it is willing to accept a higher haircut when reaching an agreement has this extra benefit. A high b_2 also tends to make c_L lower which increases the lender's marginal and pushes up ϕ . However, the first effect dominates and recovery rates are smaller with higher b_2 .

4 Conclusion

We developed a multi-country model of sovereign default and renegotiation. Debt market conditions for borrowing countries are linked to one another because they borrow from a common risk averse lender. In our model country interest rates are correlated and renegotiations with one country have spillover effects to other countries. The model provides a rationale for shorter renegotiations and larger haircuts with one country to prevent default in other countries. Our model provides a framework to study some of the recent events in Europe.

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